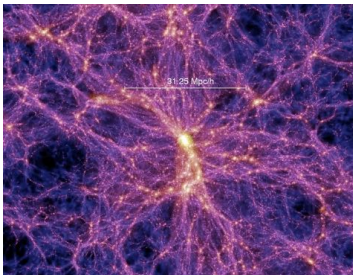
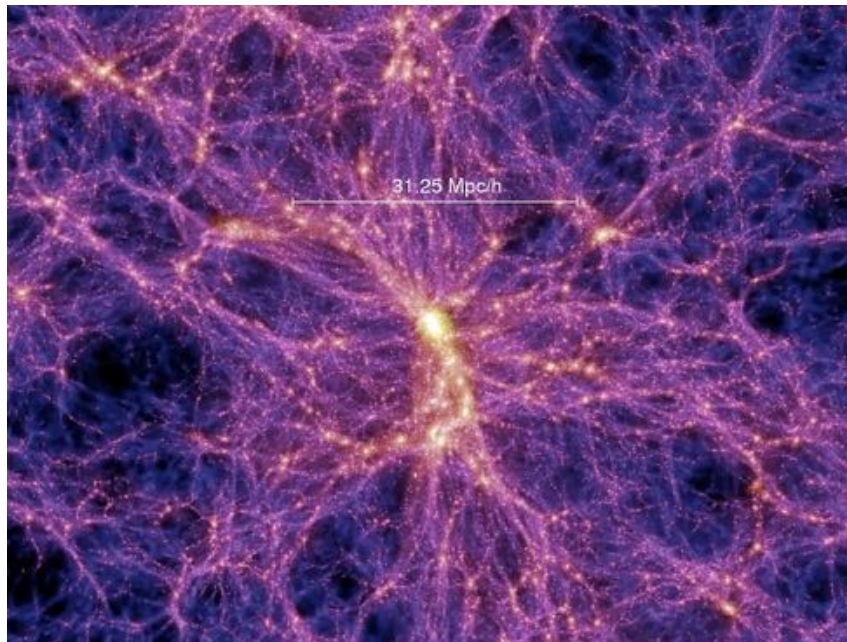




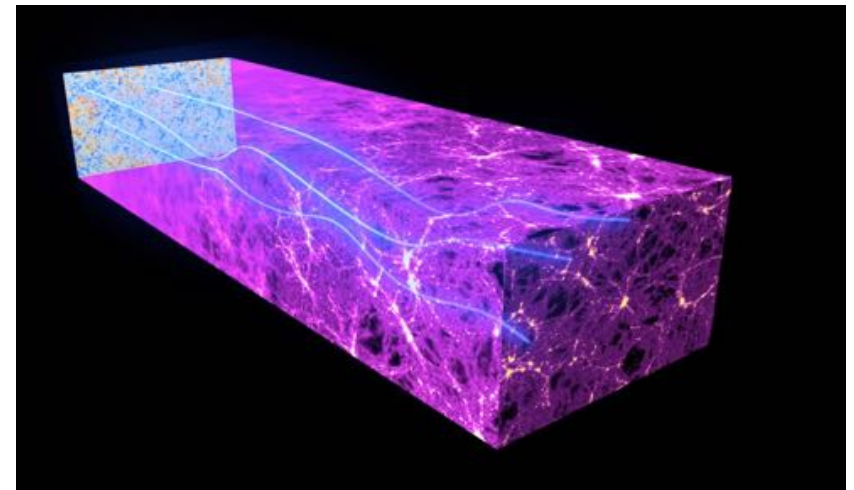
O Universo em grande escala: das flutuações primordiais à formação das estruturas

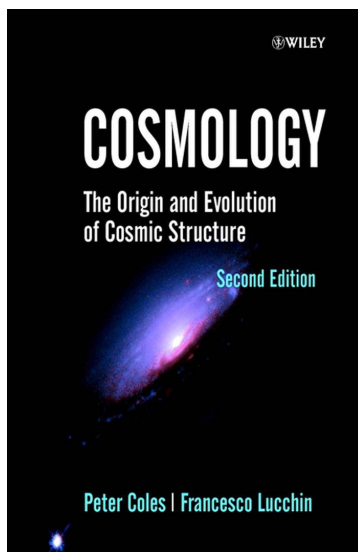


What
Why
How



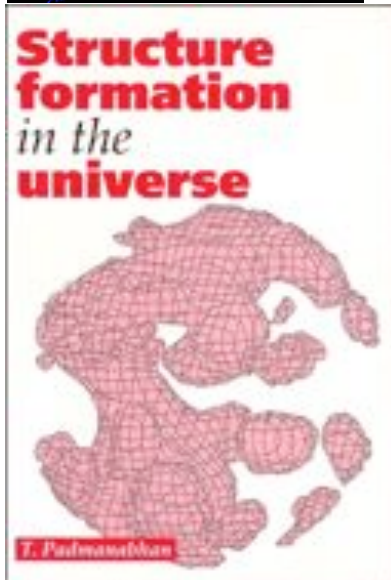
Armando Bernui



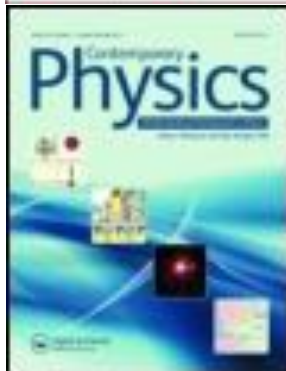


https://fma.if.usp.br/~mlima/teaching/PGF5292_2021/Coles_COECS.pdf

Peter Coles &
Francesco Lucchin



Thanu Padmanabhan



Contemporary Physics

Publication details, including instructions for authors and subscription information:

<http://www.tandfonline.com/loi/tcph20>

The large-scale structure of the universe

Peter Coles^a

^a Astronomy Unit, School of Mathematical Sciences , Queen Mary and Westfield College , Mile End Road, London, E1 4NS, UK

Published online: 20 Aug 2006.

References for cosmological perturbations

TASI Lectures on Primordial Cosmology; Daniel Baumann, arXiv:1807.03098

LSS mini-course; W. Percival, arXiv:1312.5490

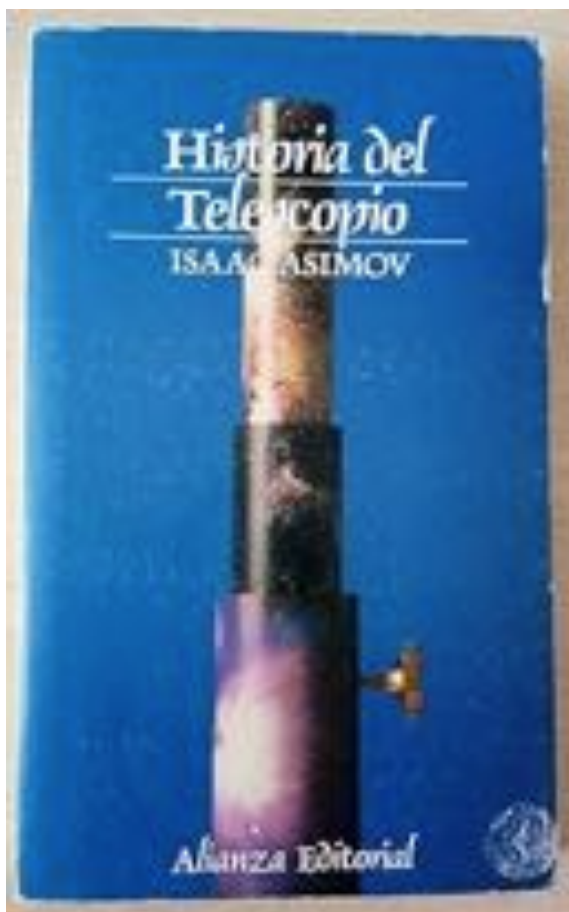
<https://cosmologist.info/teaching/EU/>





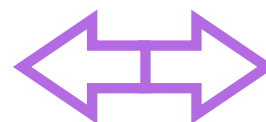
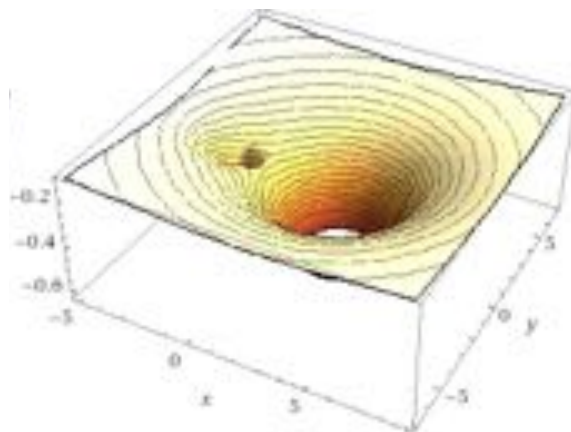
Antes de começar deixem-me contar para vocês:
as perguntas que começam com “Por que?”
são as mais difíceis de responder.

1

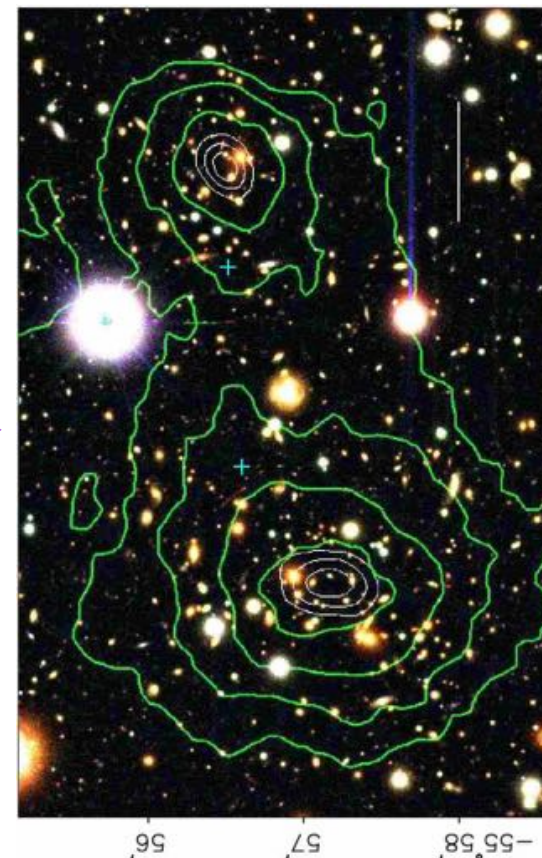


2

“Por que faliu o
Imperio Romano?”



3

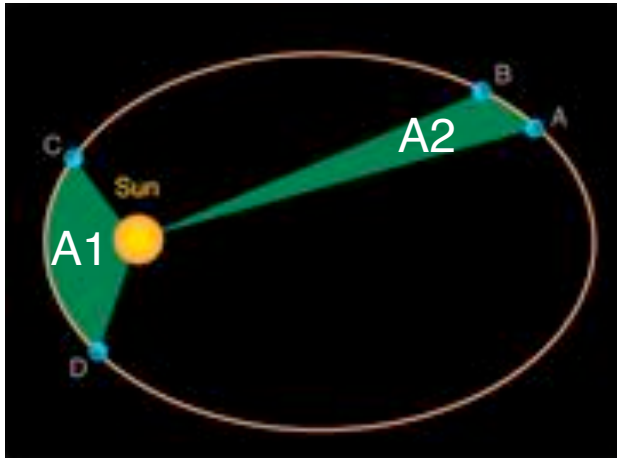
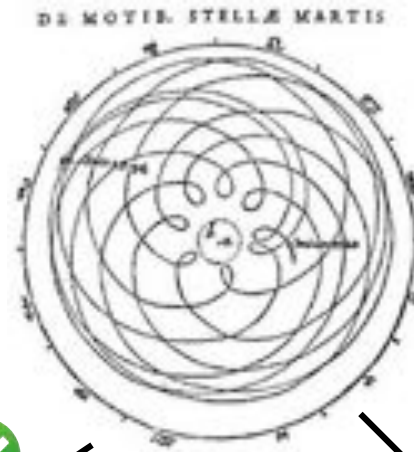


LESSON 2

The background and the clumpy spacetimes

0

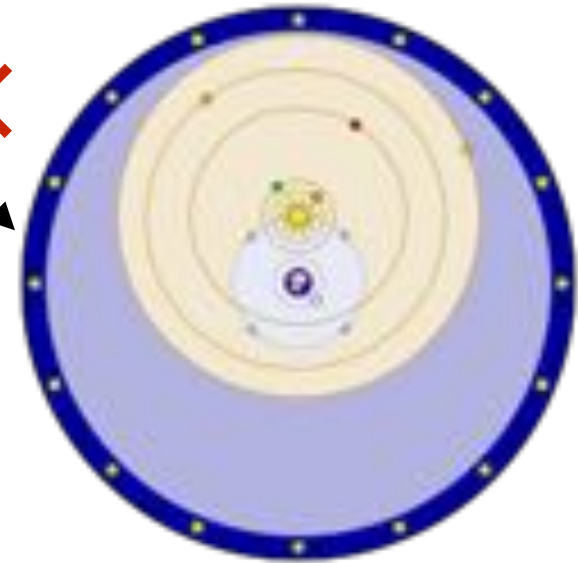
Observations



Model



1. Orbits are ellipses
2. $A1=A2$
3. $T^2 \propto \bar{R}^3$



Johannes Kepler

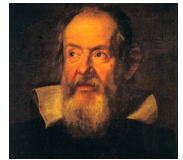
Tycho Brahe

Observations determine the correct Model!

1

Is gravity a special force? ➡ Yes!

Gravity is **distinguished** from all other forces: all bodies given the same initial velocity follow the same trajectory in a gravitational field, regardless of their internal composition



Conclusion 0:

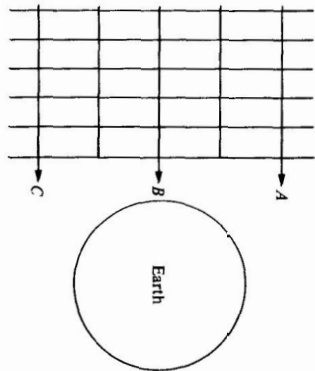
There is no single freely falling **global** frame which is everywhere freely falling in Earth's gravitational field and which is still **rigid**

Conclusion I:

Neither Minkowski spacetime nor the theory of special relativity works to describe (accelerated particles in) gravitational fields!

Conclusion II:

We need to construct a curved spacetime to describe gravitational fields!



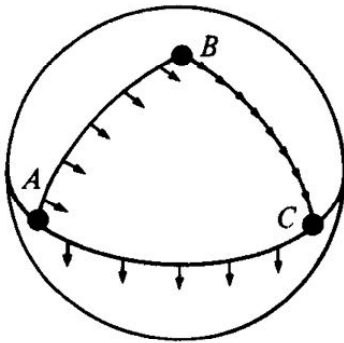
2

G.F.B. Riemann (1826–1866)



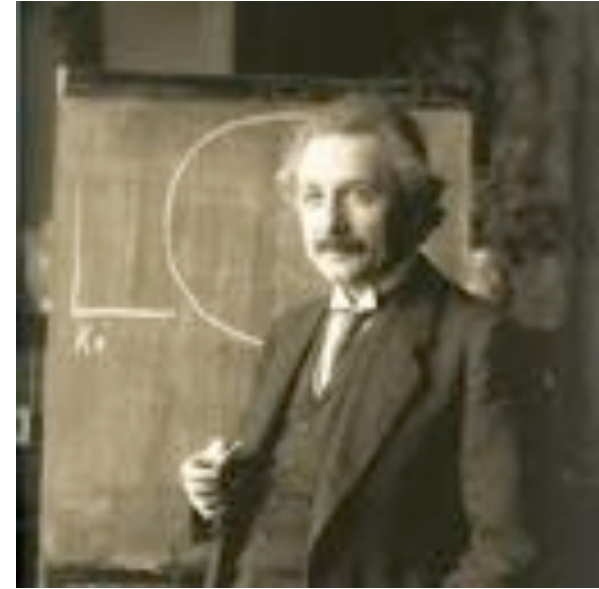
Differential geometry

$$R^{\alpha}_{\beta\mu\nu}$$



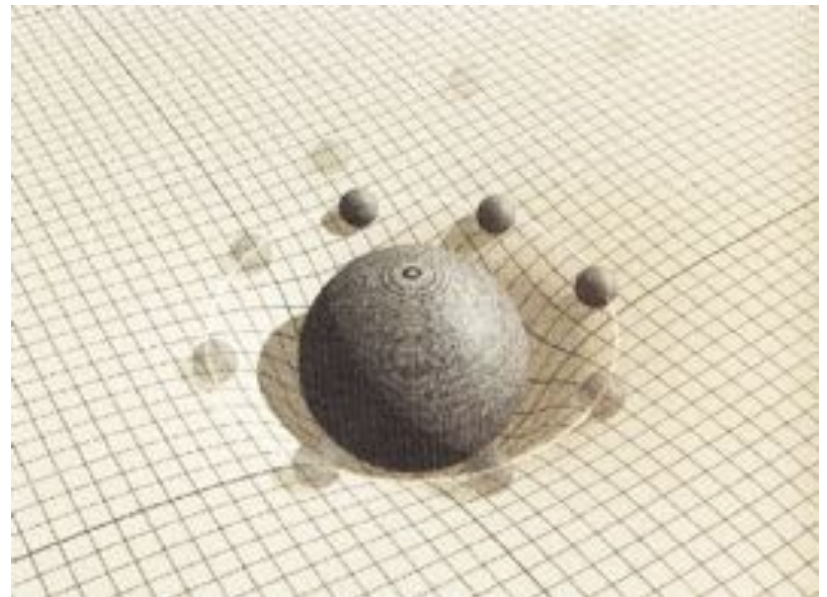
$$R^{\alpha}_{\beta\mu\nu} \equiv \Gamma^{\alpha}_{\beta\nu,\mu} - \Gamma^{\alpha}_{\beta\mu,\nu} + \Gamma^{\alpha}_{\sigma\mu}\Gamma^{\sigma}_{\beta\nu} - \Gamma^{\alpha}_{\sigma\nu}\Gamma^{\sigma}_{\beta\mu}$$

A. Einstein (1879–1955)



General Relativity

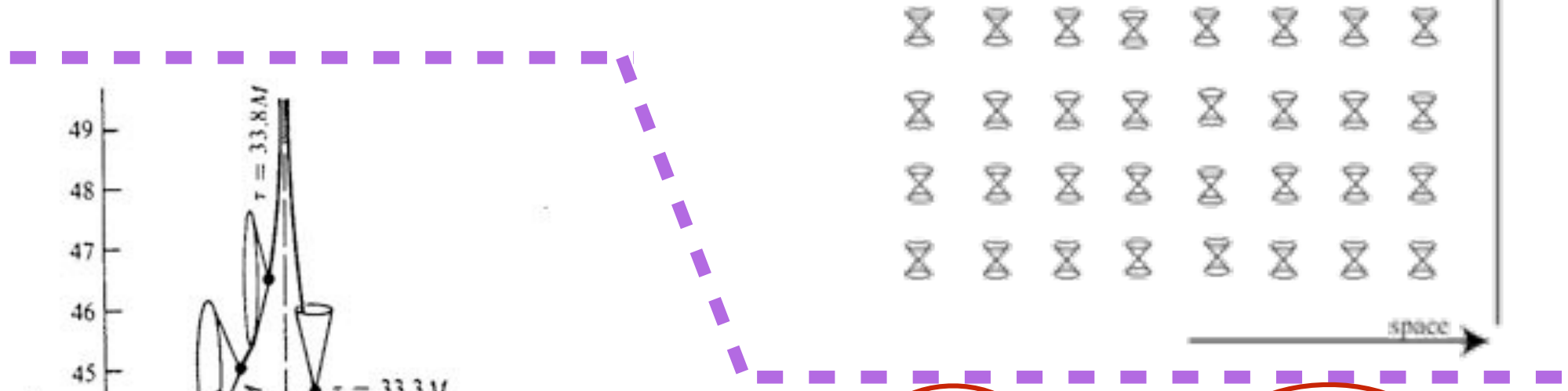
$$G^{\alpha\beta} \equiv R^{\alpha\beta} - \frac{1}{2} R g^{\alpha\beta} = \frac{8\pi}{c^4} T^{\alpha\beta}$$



3

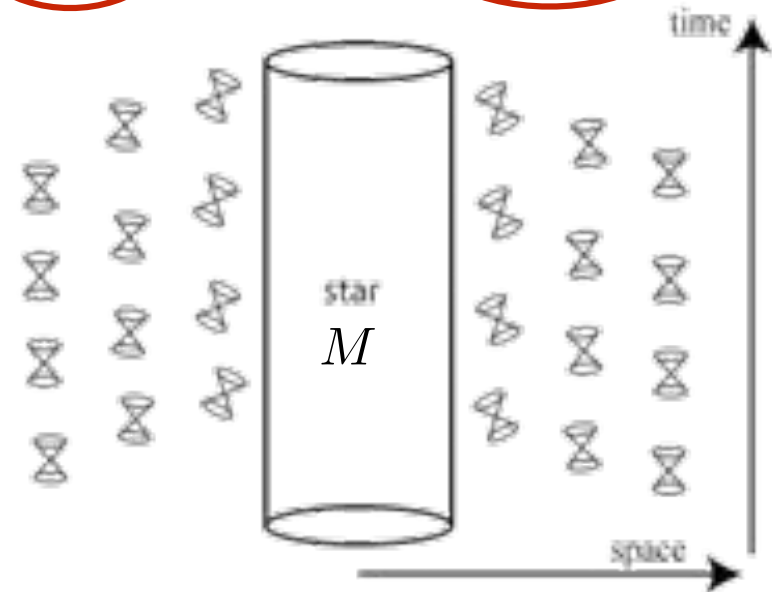
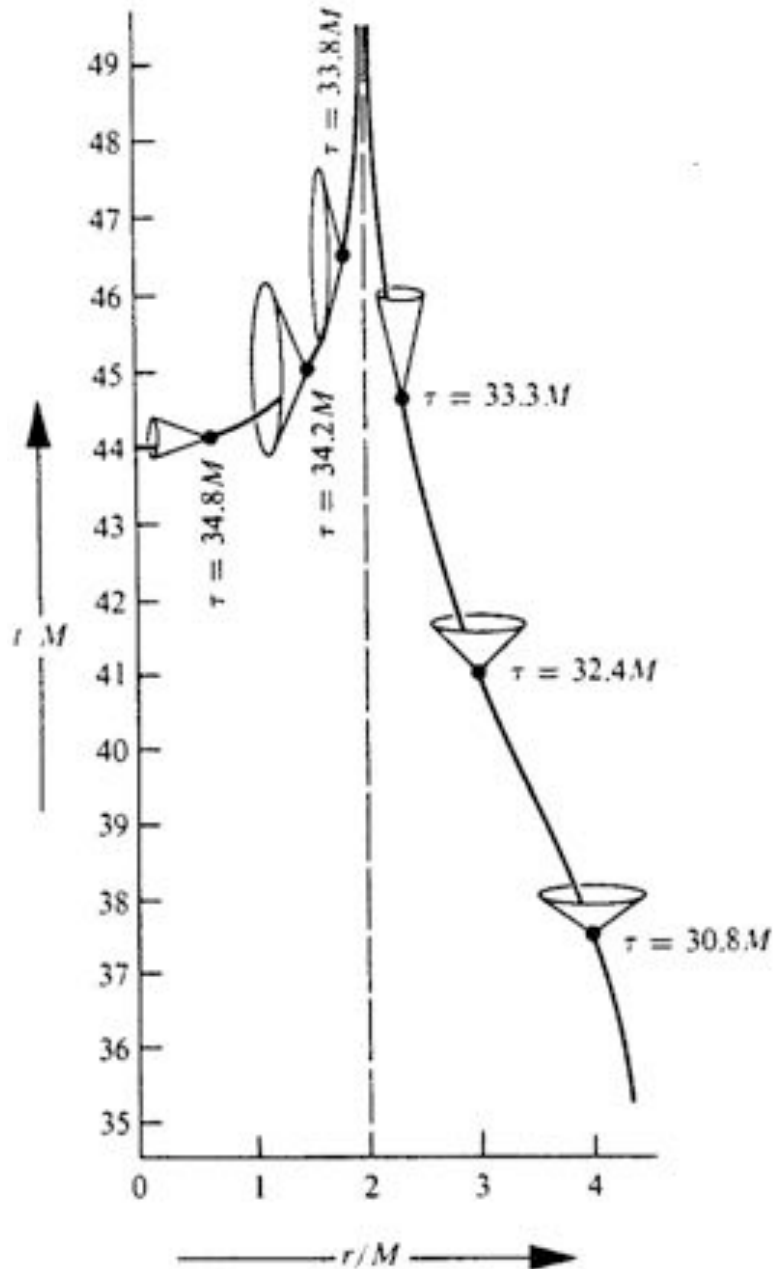
Minkowski spacetime

$$ds^2 = -c^2 dt^2 + d\mathbf{x}^2$$



$$ds^2 = -c^2 \left(1 - \frac{2GM}{c^2 r}\right) dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} d\mathbf{x}^2 *$$

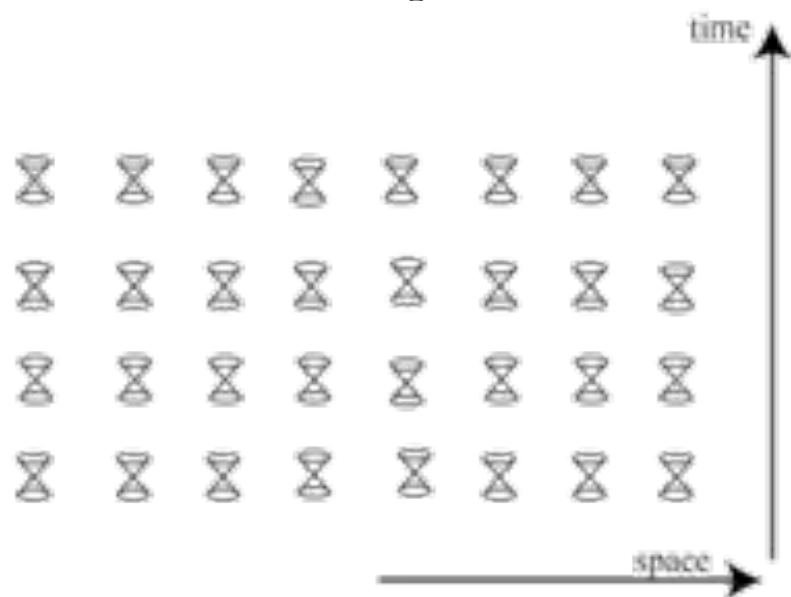
Curved
spacetime
of a mass M



* Schwarzschild metric

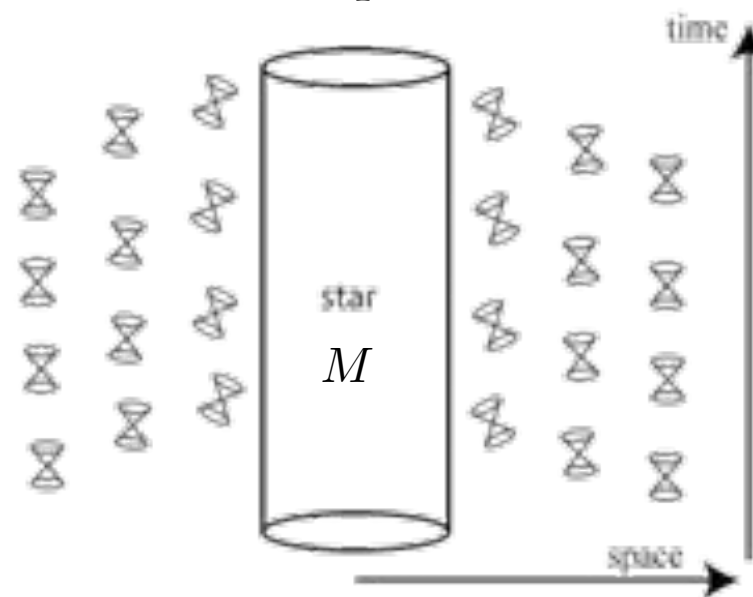
4

Minkowski spacetime



?

Curved spacetime



More generally, the perturbed **metric** describes the clumpy spacetime:

$$ds^2 = -c^2 dt^2 + d\mathbf{x}^2 \longrightarrow ds^2 = -c^2 \left(1 + \frac{2\phi}{c^2}\right) dt^2 + \left(1 - \frac{2\phi}{c^2}\right) d\mathbf{x}^2$$

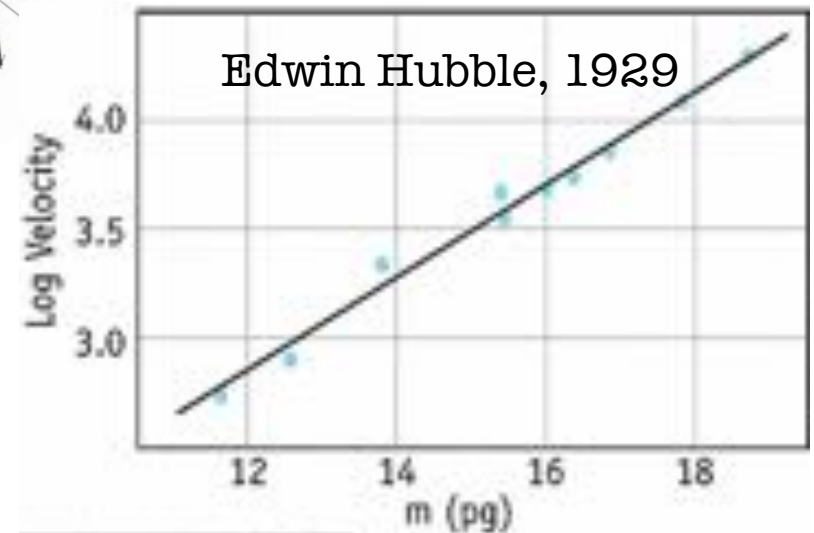
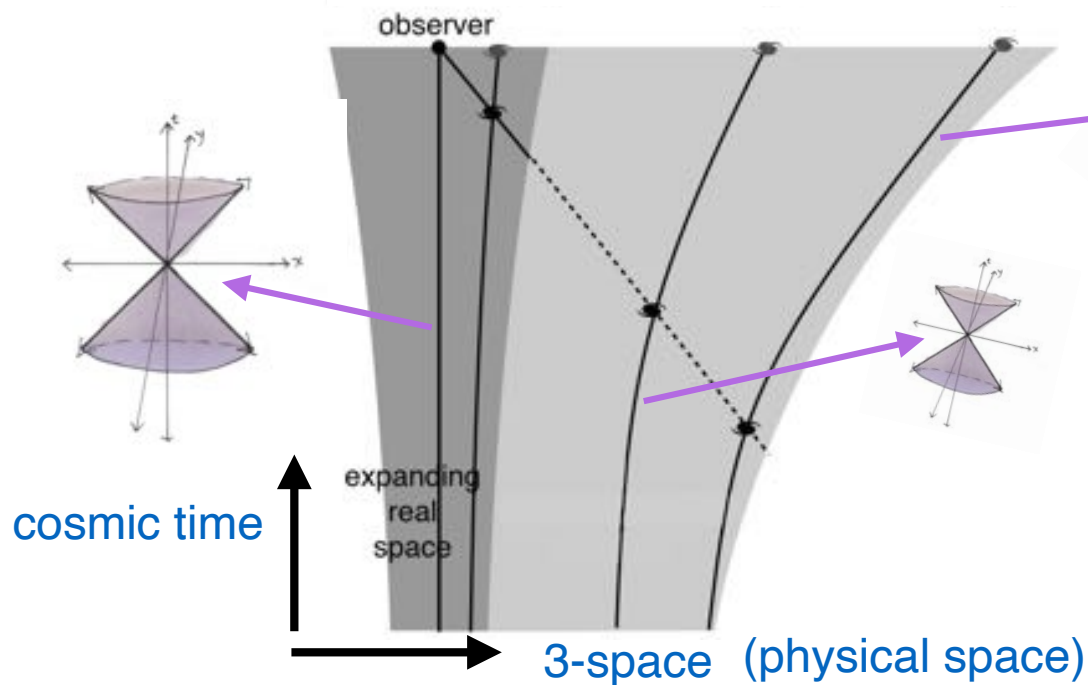
where $\phi = \phi(\mathbf{x})$, is the gravitational potential of a mass-density fluctuation $\delta\rho(\mathbf{x})$

As a particular case $\phi = -GM/r$, is the gravitational potential of a mass M.

5

Expanding spacetime: the Hubble flow

$$ds^2 = -c^2 dt^2 + a^2(t)[dr^2 + r^2 d\Omega^2]$$



- ◆ (iv) Freely falling particles move on timelike geodesics of the spacetime.

Newtonian theory of gravitation

Newton eq. for a particle, mass m , in a central field

$$m \ddot{\vec{r}} = F(r) \vec{e}_r = -\frac{GMm}{r^2} \vec{e}_r$$

$$\vec{r} = r \vec{e}_r \qquad \dot{\vec{e}}_r = \dot{\theta} \vec{e}_\theta \qquad \dot{\vec{e}}_\theta = -\dot{\theta} \vec{e}_r$$

$$\dot{\vec{r}} = r \dot{\vec{e}}_r + \dot{r} \vec{e}_r = r \dot{\theta} \vec{e}_\theta + \dot{r} \vec{e}_r$$

$$\ddot{\vec{r}} = (\ddot{r} - r \dot{\theta}^2) \vec{e}_r + (2 \dot{r} \dot{\theta} + r \ddot{\theta}) \vec{e}_\theta = -\frac{GM}{r^2} \vec{e}_r$$

$$\Rightarrow 2 \dot{r} \dot{\theta} + r \ddot{\theta} = 0$$

$$2 \dot{r} \dot{\theta} + r \ddot{\theta} = \frac{\dot{\ell}}{mr} = 0 \quad \Rightarrow \quad \ell = m r^2 \dot{\theta} = \text{const.}$$

↑ angular momentum conservation

particle in a central grav. potential

$$\Rightarrow \ddot{r} = r \dot{\theta}^2 - \frac{GM}{r^2} = \frac{\ell_0}{m^2 r^3} - \frac{GM}{r^2}$$

$$\ddot{r} = \frac{dr}{dt} = \frac{dr}{dr} \frac{dr}{dt} = \dot{r} \frac{dr}{dr}$$

performing the anti-derivative, one obtains:

$$\dot{r} dr = \left(\frac{\ell_0}{m^2 r^3} - \frac{GM}{r^2} \right) dr \quad \Rightarrow \quad \frac{1}{2} \dot{r}^2 + \left(\frac{\ell_0}{2 m^2 r^2} - \frac{GM}{r} \right) = \text{const.}$$

$$\underbrace{\frac{1}{2} m \dot{r}^2}_{E_k} + \underbrace{\left(\frac{\ell_0}{2 m r^2} - \frac{GmM}{r} \right)}_{U_{\text{eff}}} = E_k + U_{\text{eff}} = E_0$$

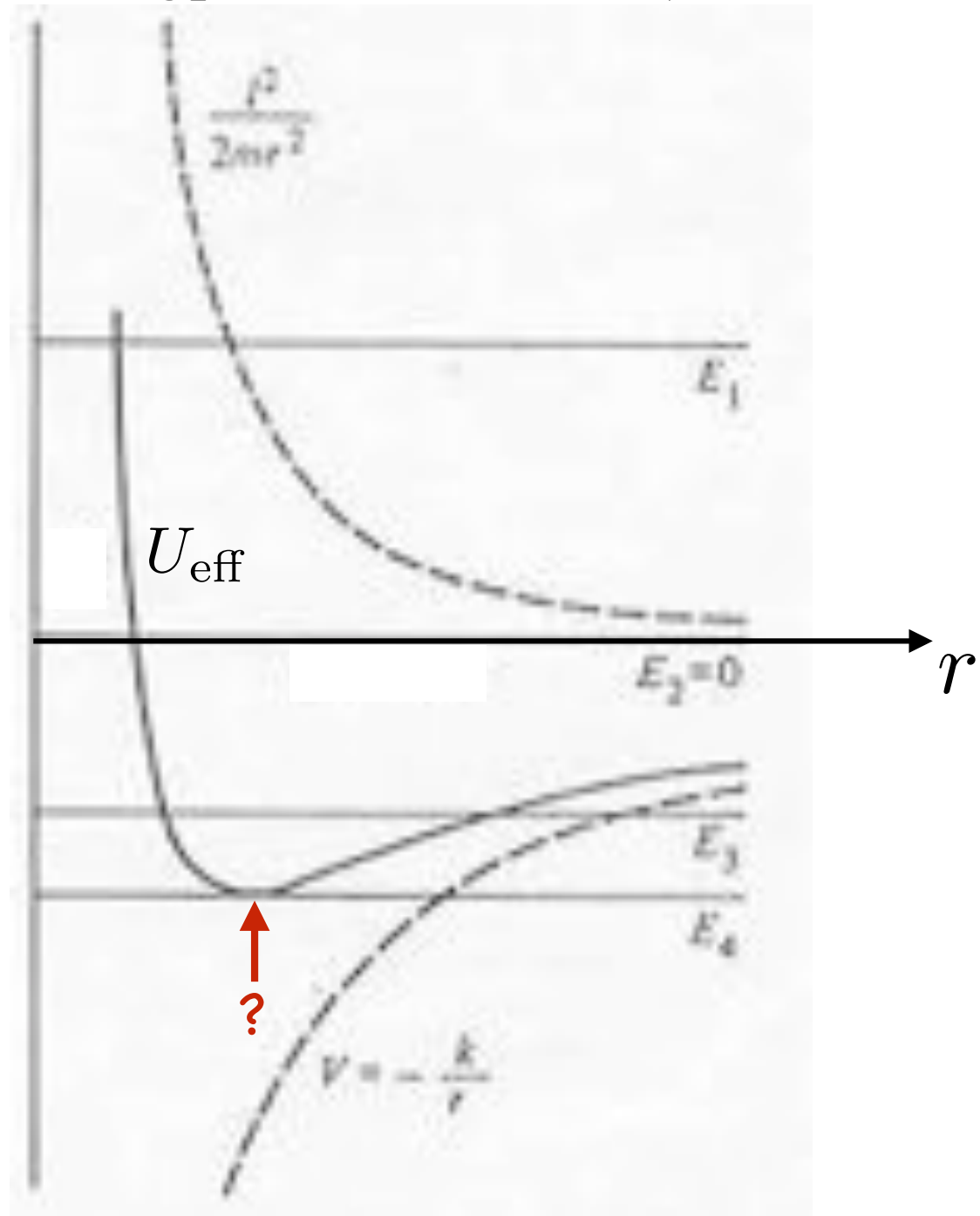
energy conservation

Motion information without formulas (from energy conservation)

Perguntas:

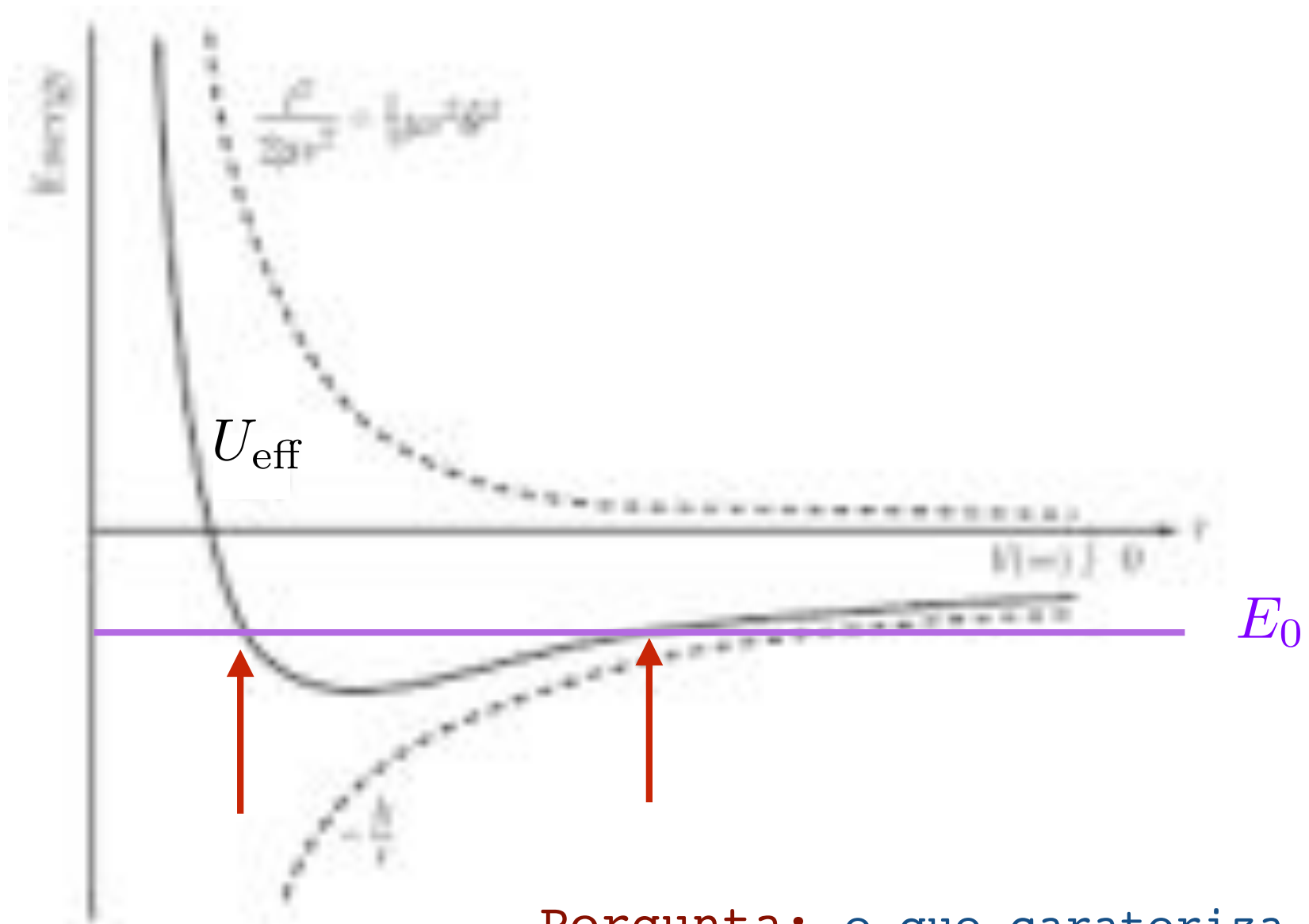
o que representam,
para a partícula m
num campo central,
ter energia total:
 E_1, E_2, E_3, E_4 ?

- mov. limitado?
- mov. ilimitado?



Motion information without formulas (from energy conservation)

miscelânea:



$$E_k + U_{\text{eff}} = E_0$$

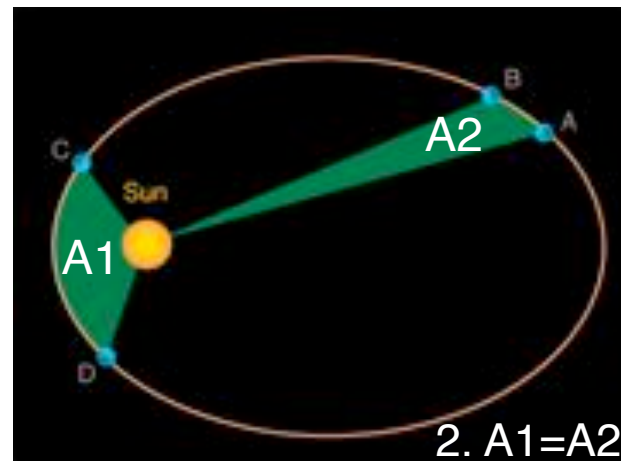
Pergunta: o que caracteriza os pontos de interseção da energia total com o potencial efetivo?

Perguntas:

1) Quem descobriu a lei da conservação do momentum angular?

Dica: Se lembram que $\ell = m r^2 \dot{\theta} = \text{const.}$

e de:



2) será que estão relacionados?

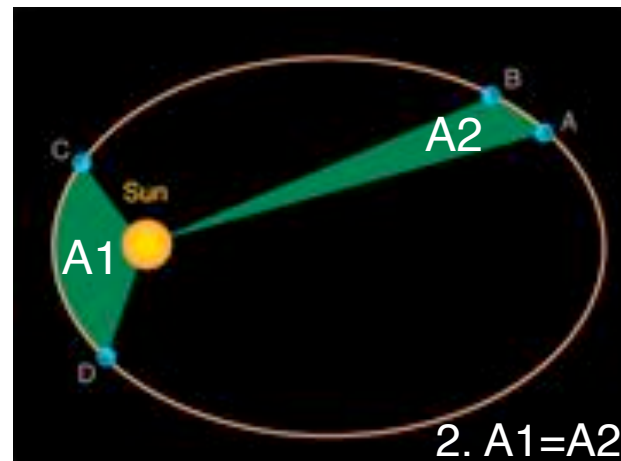
Perguntas:

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Dica: Se lembram que $\ell = m r^2 \dot{\theta} = \text{const.}$

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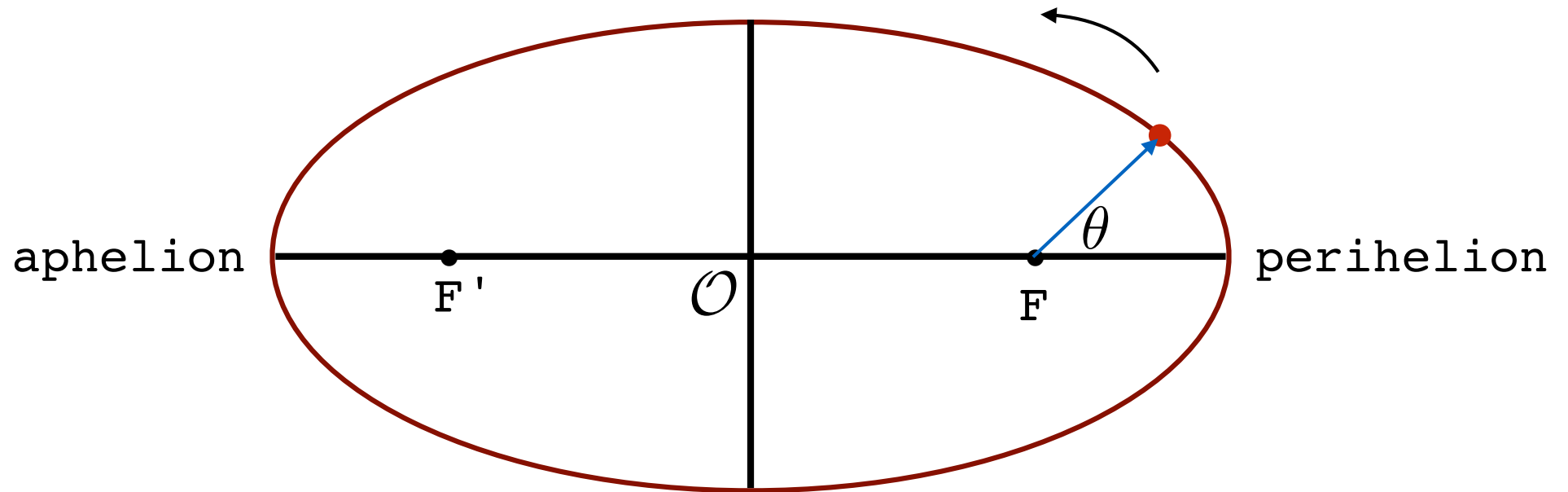
2) será que estão relacionados?

Exercício: M'q' $\frac{\Delta A}{\Delta t} = \frac{\ell}{2m} = \text{const.}$

With a change of variable, $u=1/r$, one can solve the diff. eq. of particle's motion:

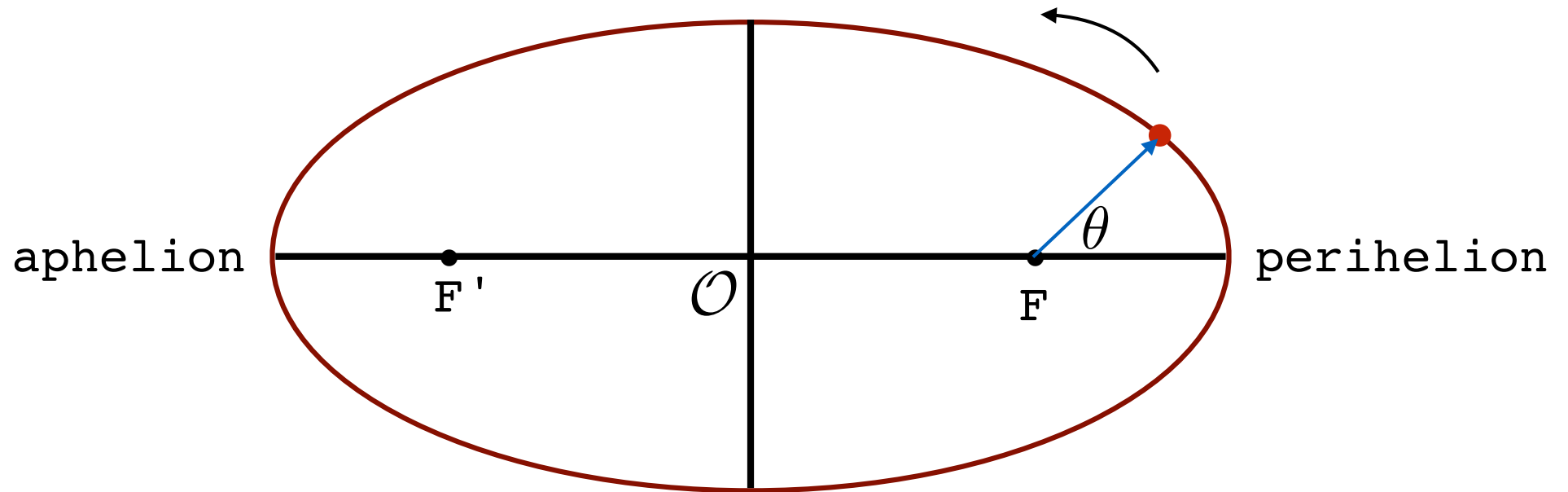
$$\frac{1}{2} \dot{r}^2 + \left(\frac{\ell_0}{2 m^2 r^2} - \frac{GM}{r} \right) = \text{const.} \quad \Rightarrow \quad r(\theta) = \frac{A}{B + \epsilon \cos \theta(t)}$$

$$r(\theta) = \frac{A}{B + \epsilon \cos \theta(t)}$$



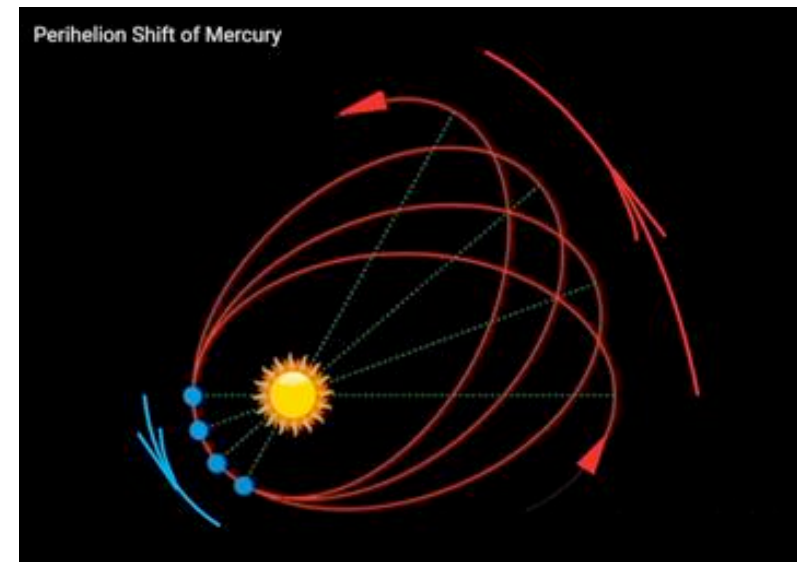
But, astronomical observations
show us a problem:

$$r(\theta) = \frac{A}{B + \epsilon \cos \theta(t)}$$

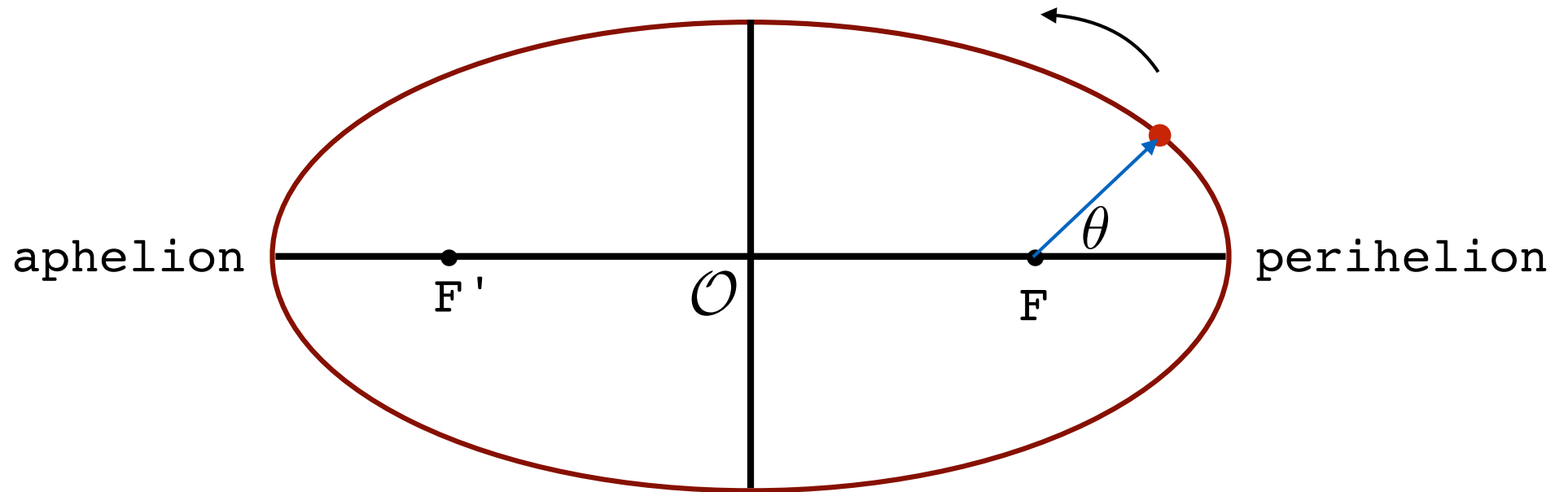


But, astronomical observations
show us a problem:

Orbital precession of Mercury

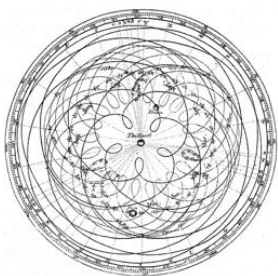


$$r(\theta) = \frac{A}{B + \epsilon \cos \theta(t)}$$

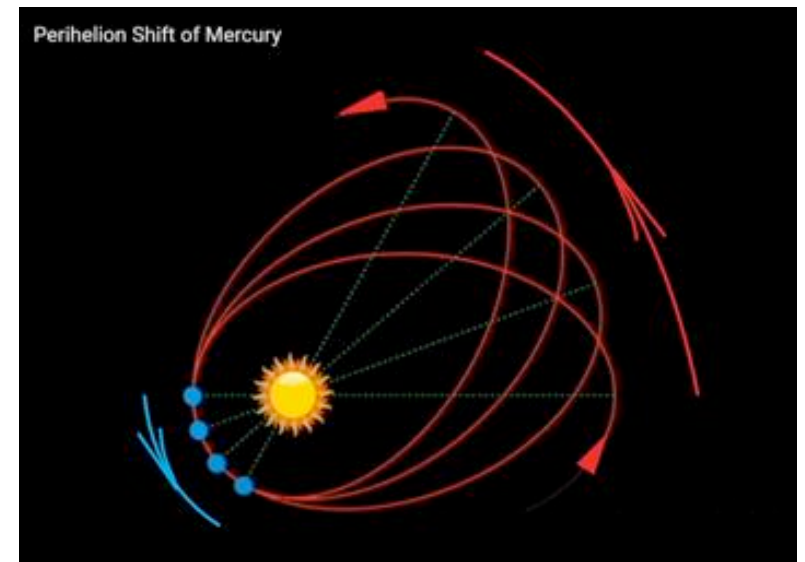


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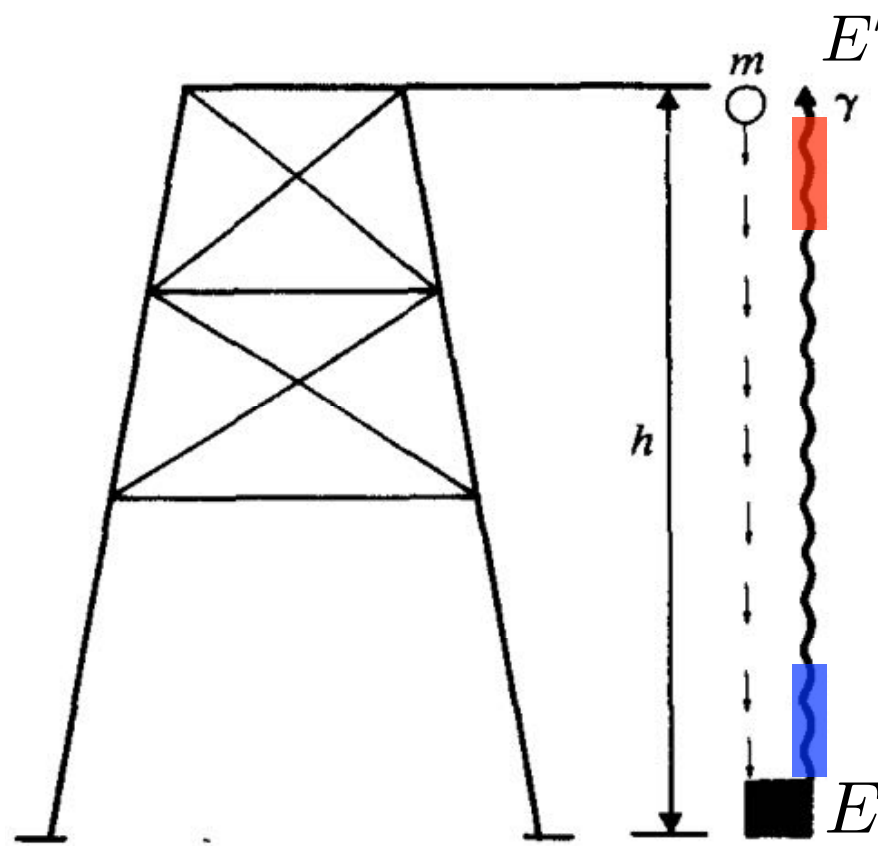
Orbital precession of Mercury



Why?



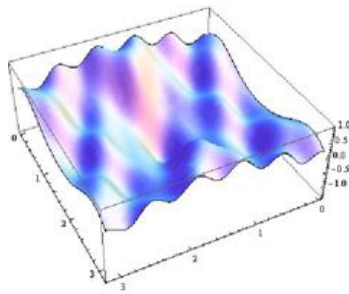
Because space and time (vulgo: spacetime) are not Newtonian!



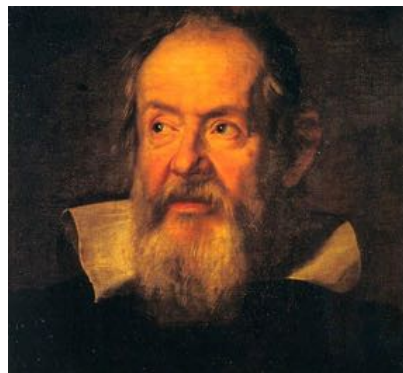
Corollary: A gravitational field induces a time dilation in the spacetime metric

$$\frac{\delta t}{t} = \frac{gh}{c^2}$$

From Newtonian theory of gravitation we learned:
(i) the universality of conservation laws;
(ii) to develop perturbative calculus of Newton eqs.
(iii) the (concept of) gravitational potential is more useful than force, ...



the concept of field $\phi(\vec{r})$ is born!



miscelânea:

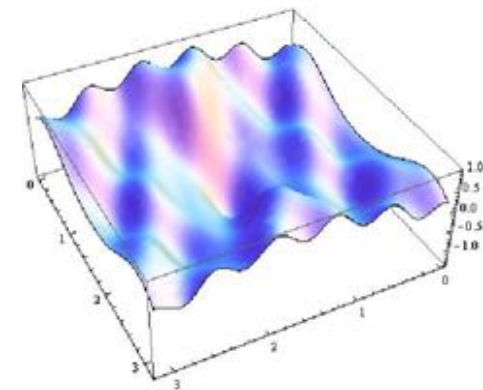
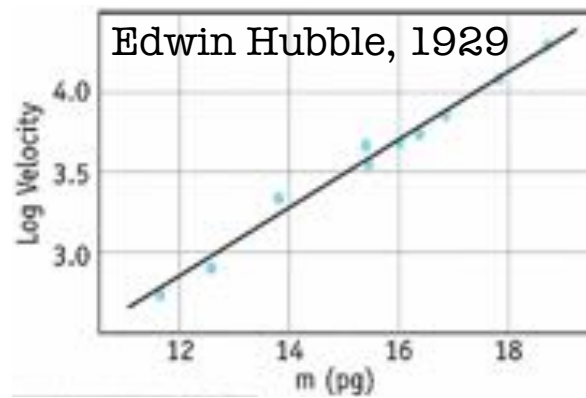


Siméon-Denis Poisson (1781 - 1840)
matemático e físico francês

Laplace e Lagrange foram professores de Poisson

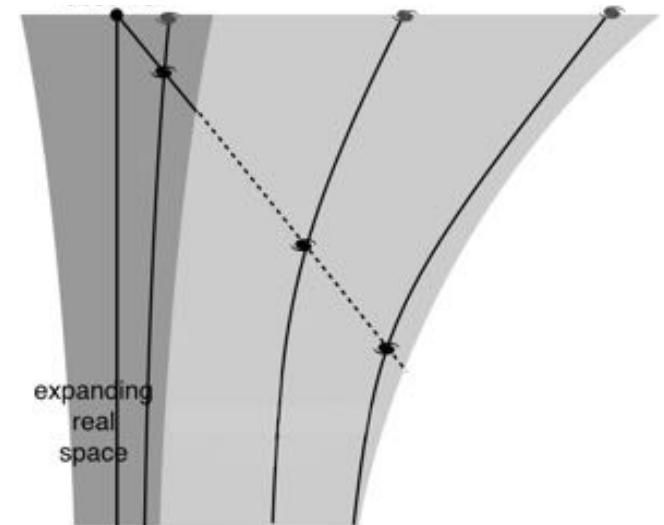
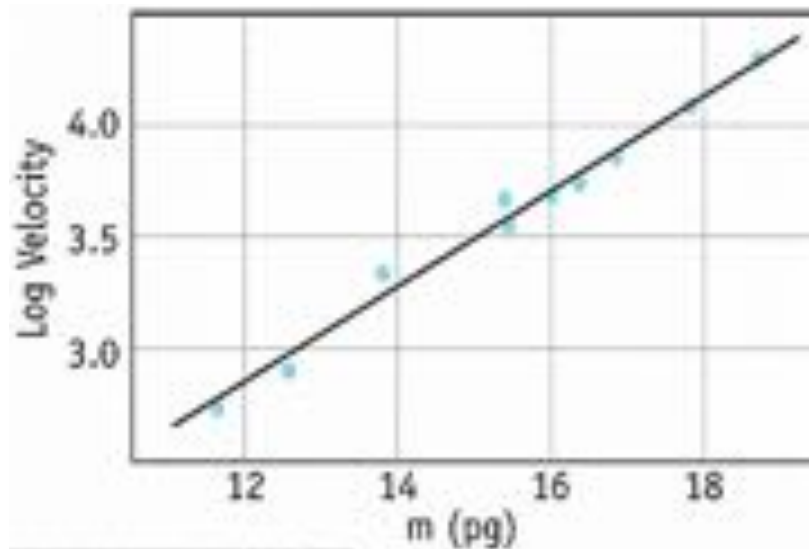
$$\nabla^2 \phi = 4\pi G \rho$$

The background spacetime and the clumpy Universe

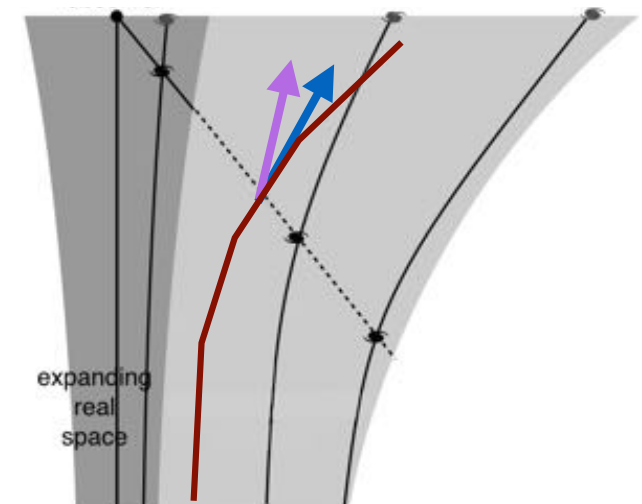
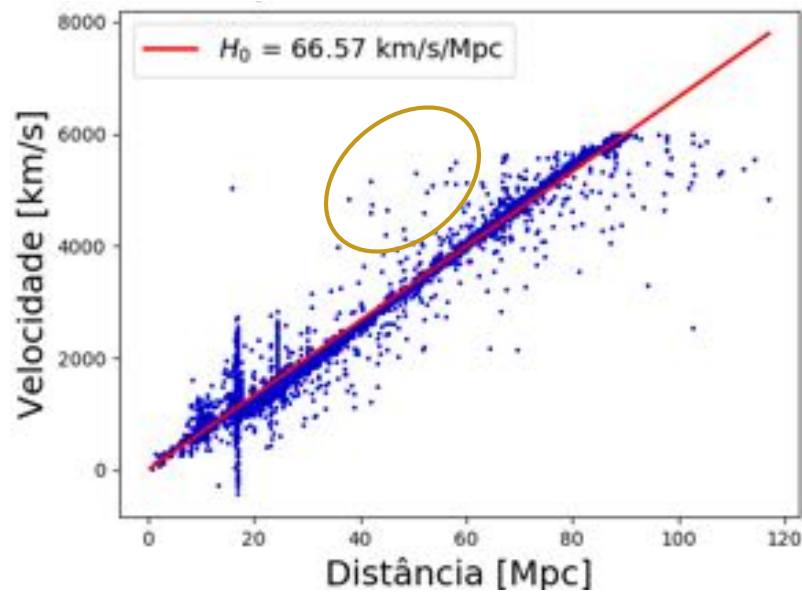


The problem:

expected



observed



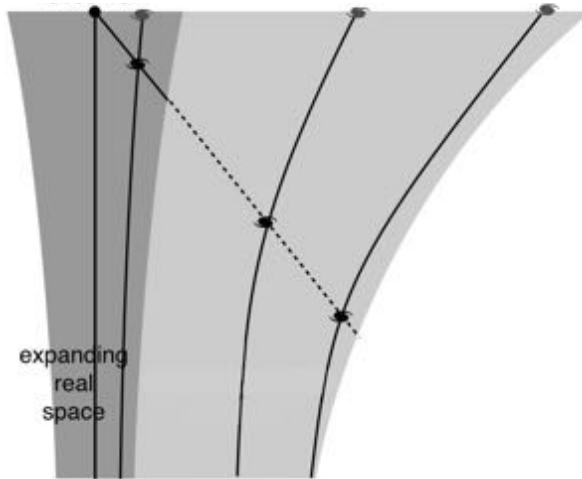
velocidade esperada  velocidade observada 

The proposal:

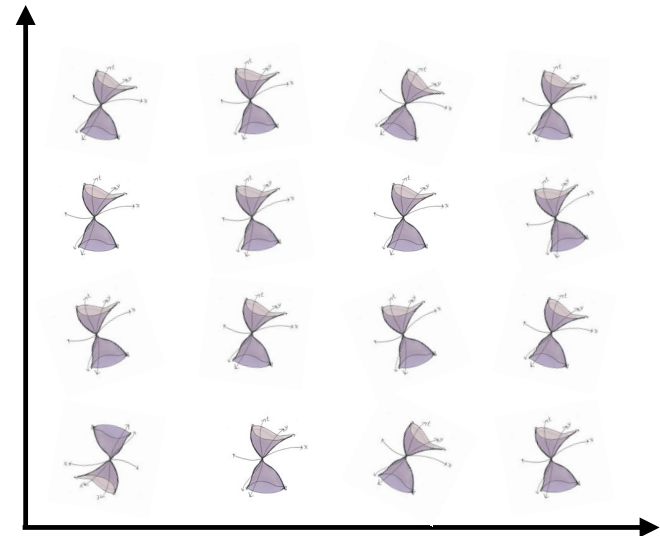
spacetime
metric

$$ds^2 = c^2 dt^2 + a(t)^2 d\mathbf{x}^2 \quad + \quad ds^2 = -c^2 \left(1 + \frac{2\phi}{c^2}\right) dt^2 + \left(1 - \frac{2\phi}{c^2}\right) d\mathbf{x}^2$$

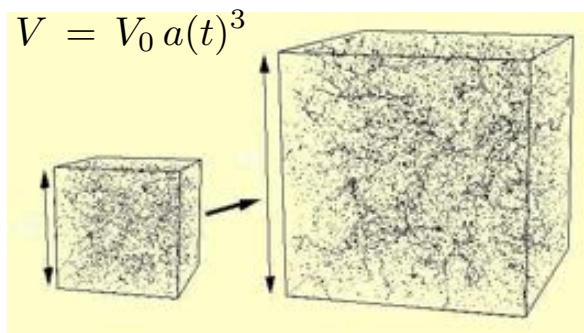
spacetime



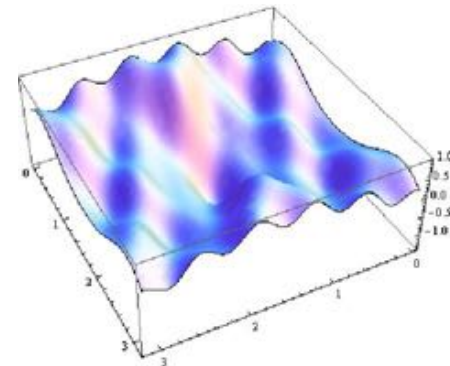
+



3D-space

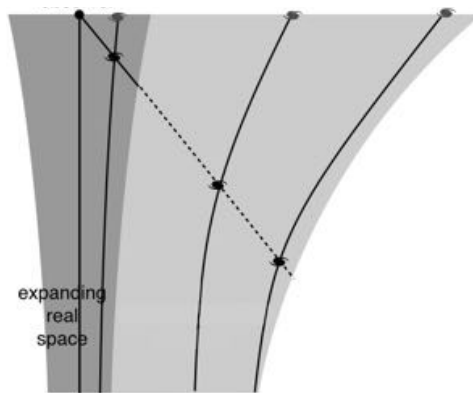


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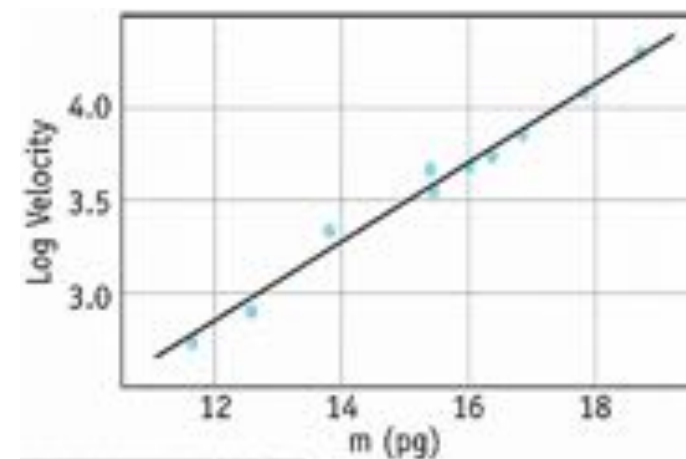


Remember: $a = a(t)$ is the scale factor

The expanding Universe: Hubble flow



$$v = H_0 r$$

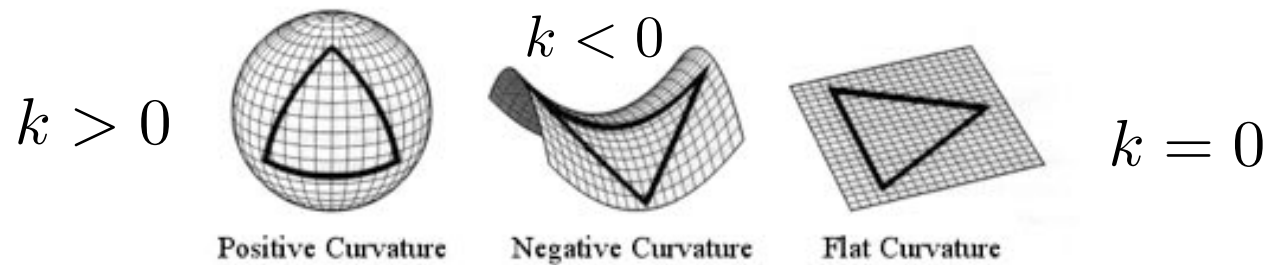


Hubble flow: the Friedmann-Lemaître-Robertson-Walker spacetime

The Robertson-Walker (RW) metric of a **homogeneous** and **isotropic** spacetime is

$$ds^2 = -c^2 dt^2 + a^2(t) \gamma_{ij} dx^i dx^j ,$$

where γ_{ij} denotes the **metric** of a maximally symmetric 3-space, and $a = a(t)$ is the scale factor. For flat 3-space, $\gamma_{ij} = \delta_{ij}$ and define $d\mathbf{x}^2 \equiv \delta_{ij} dx^i dx^j$

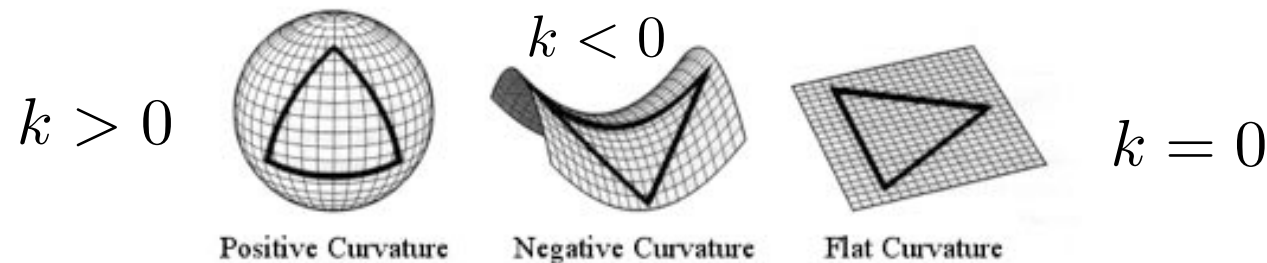


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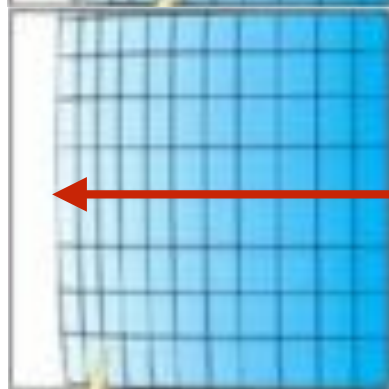
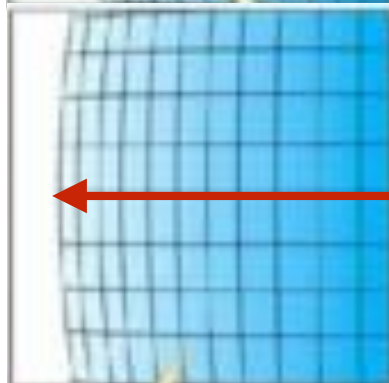
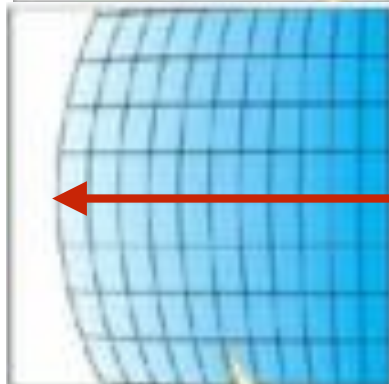
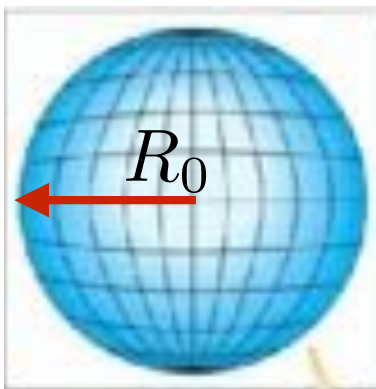


One can also write the RW **metric** in the form

$$ds^2 = c^2 dt^2 - a^2(t) \left(\frac{dr^2}{1 - k (r^2/R_0^2)} + r^2 [d\theta^2 + \sin^2 \theta d\varphi^2] \right)$$

(r, θ, φ) are the spherical coordinates

Gaussian curvature: $\frac{k}{R_0^2}$



Johann Carl Friedrich Gauss

R_0

$R_0 \sim \infty$

= zero curvature

miscelânea:



A child prodigy: Carl Friedrich Gauss



<https://www.youtube.com/watch?v=cD9rI4wSc7o> (4 minutes)

Einstein equations

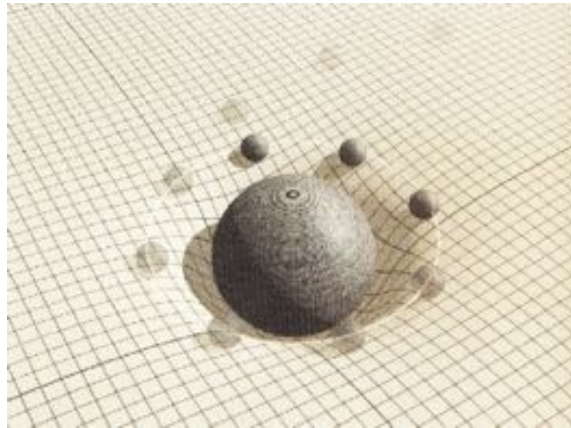
$$G^{\alpha\beta} = \frac{8\pi}{c^4} T^{\alpha\beta}$$

4 spacetime geometry:
metric, metric derivatives,
curvature (intrinsic!)

Geometry

matter-energy content:
physical prop.,
conservation laws

Dynamics



If $T^{\alpha\beta}$ represents a perfect fluid* $\Rightarrow$ Friedmann equations

* see the [Appendix Perfect Fluids](#)

Perfect Fluids

Fluid equation of state dependence: thermodynamic variables

$$p = p(\rho, S, \kappa, \eta, \nu, \sigma, \dots)$$

thermal conductivity κ

viscosity η

kinematic viscosity $\nu = \eta/\rho$

shear σ

....

For a perfect fluid*: $p = p(\rho), p = p(t), \rho = \rho(t)$

$$\begin{aligned} \text{Eq. of State: } p = \omega \rho &\quad \Rightarrow \quad T^{\alpha\beta} = \text{diag}(-c^2 \rho, p, p, p) \\ &\quad T^{\alpha\beta}_{;\beta} = 0 \end{aligned}$$

*perfect fluid: homogeneous, isotropic, adiabatic, $\kappa = 0$, non-viscous, non-shear, etc.

Friedmann equation(s)



Alexander Friedmann

$$H(t)^2 = \frac{8\pi G}{3} \rho(t) - \frac{k c^2}{R_0^2 a(t)^2}$$

Assuming that the Universe is filled with a perfect fluid, one solves the Einstein equations and find the Friedmann eq.

$$\frac{\dot{a}(t)^2}{a(t)^2} = \frac{8\pi G}{3} \rho(t) - \frac{k c^2}{R_0^2 a(t)^2}$$

defining: $\dot{a} \equiv \frac{da}{dt}$ and $H(t) \equiv \frac{\dot{a}(t)}{a(t)}$

$$H(t)^2 = \frac{8\pi G}{3} \rho(t) - \frac{k c^2}{R_0^2 a(t)^2}$$

Friedmann equation

desfiando eq-Friedmann

$$\frac{\dot{a}(t)^2}{a(t)^2} = \frac{8\pi G}{3} \rho(t) - \frac{k c^2}{R_0^2 a(t)^2}$$

Understanding, physically, the Friedmann eq.

$$\frac{\dot{a}(t)^2}{a(t)^2} - \frac{8\pi G}{3} \rho(t) = -\frac{k c^2}{R_0^2 a(t)^2}$$

$$\times R_0^2 a(t)^2 / 2 \quad \longrightarrow \quad \underbrace{\frac{1}{2} R_0^2 \dot{a}(t)^2}_{\text{Kinetic energy?}} - \underbrace{\frac{8\pi G}{6} R_0^2 a(t)^2 \rho(t)}_{\text{Potential energy?}} = -\underbrace{\frac{1}{2} k c^2}_{\text{Total energy?}}$$

$\Rightarrow$ “Analogous”^{*} to the Newtonian energy conservation law

^{*} Lembrando que toda analogia é limitada.

$$E_k + U_{\text{eff}} = E_0$$

desafiando eq-Friedmann

$$\frac{\dot{a}(t)^2}{a(t)^2} = \frac{8\pi G}{3} \rho(t) - \frac{k c^2}{R_0^2 a(t)^2}$$

Understanding, physically, the Friedmann eq.

$$\frac{\dot{a}(t)^2}{a(t)^2} - \frac{8\pi G}{3} \rho(t) = - \frac{\overset{?}{k} c^2}{\underset{?}{R_0^2} \overset{\text{dimensão?}}{a(t)^2}}$$

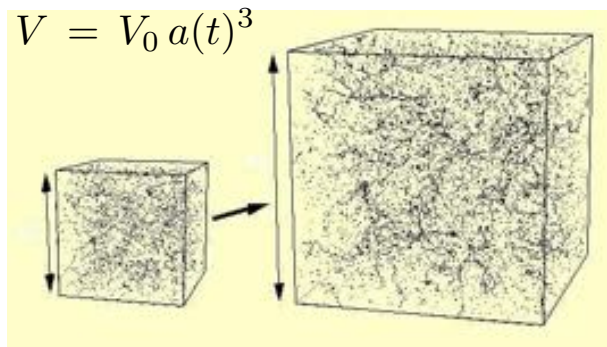
$$\times R_0^2 a(t)^2 / 2 \quad \longrightarrow \quad \underbrace{\frac{1}{2} R_0^2 \dot{a}(t)^2}_{\text{Kinetic energy?}} - \underbrace{\frac{8\pi G}{6} R_0^2 a(t)^2 \rho(t)}_{\text{Potential energy?}} = - \underbrace{\frac{1}{2} k c^2}_{\text{Total energy?}}$$

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Matter-energy conservation equation



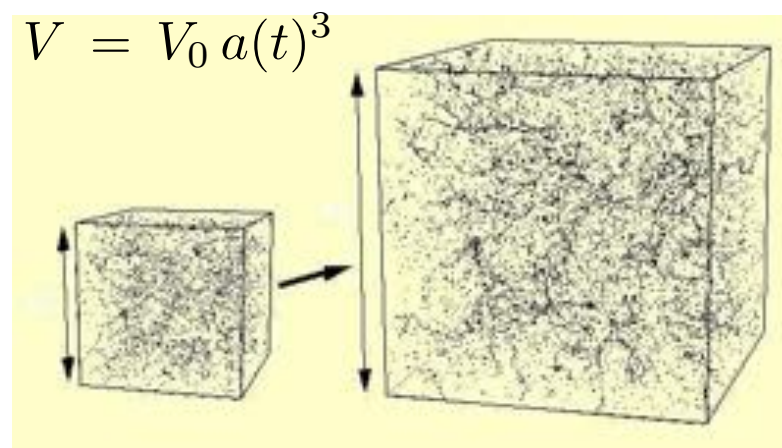
$$\dot{\rho} = -3 \frac{\dot{a}}{a} \left(\rho + \frac{p}{c^2} \right)$$

From the 1st law of thermodynamics:

$$dQ = dE + p dV$$

$$dQ = 0 \quad \text{Why?}$$

$$dE = -p dV$$



$$E = \varepsilon V = c^2 \rho V \longrightarrow dE = c^2 d(\rho V) = c^2 \rho dV + c^2 V d\rho$$

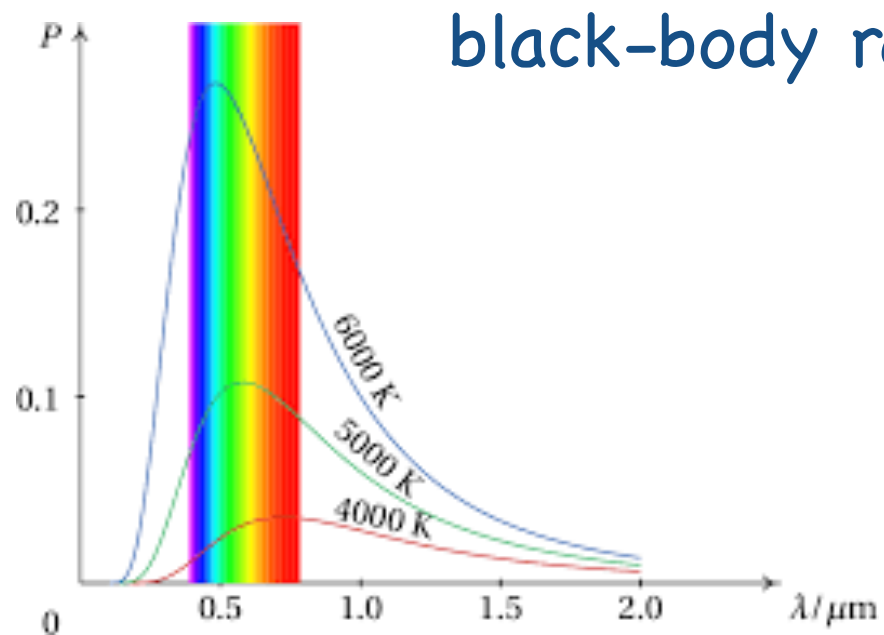
$$\Rightarrow d\rho = -\frac{dV}{V} \left(\rho + \frac{p}{c^2} \right) \xrightarrow{V = V_0 a(t)^3} d\rho = -3 \frac{da}{a} \left(\rho + \frac{p}{c^2} \right)$$

$$\dot{\rho} = -3 \frac{\dot{a}}{a} \left(\rho + \frac{p}{c^2} \right)$$

matter-energy
conservation equation

$$T^{\alpha\beta}_{;\beta} = 0$$

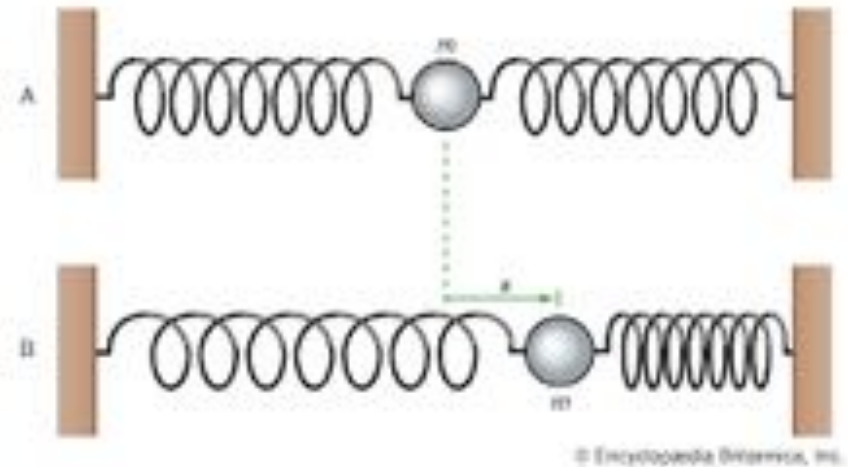
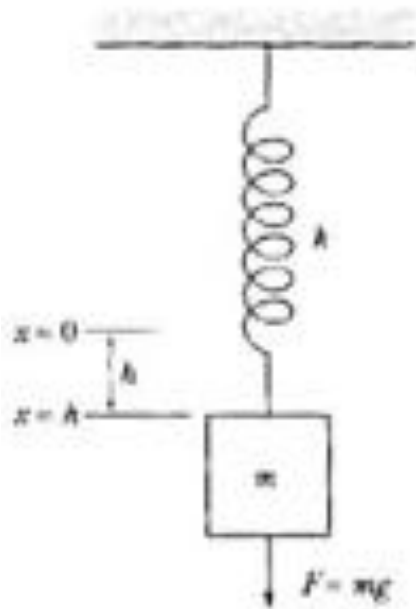
CMB black-body radiation



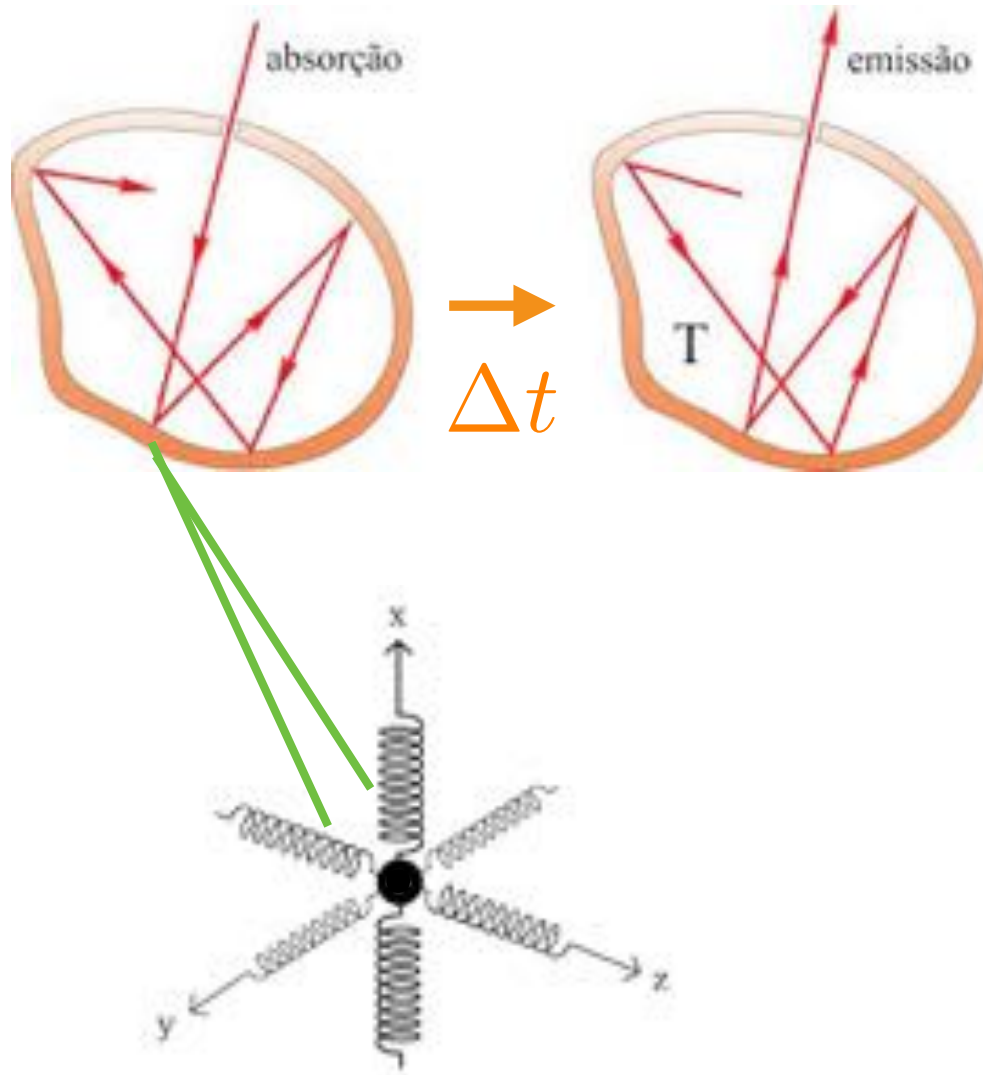
$$\varepsilon(f) = \frac{8\pi h}{c^3} \frac{f^3}{\exp(hf/kT) - 1}$$

Black-body Radiation

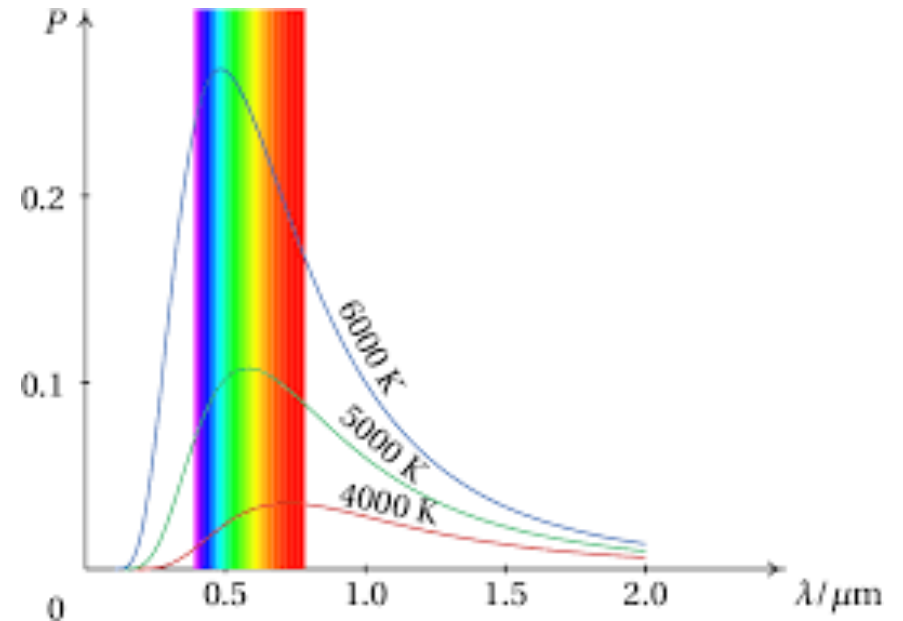
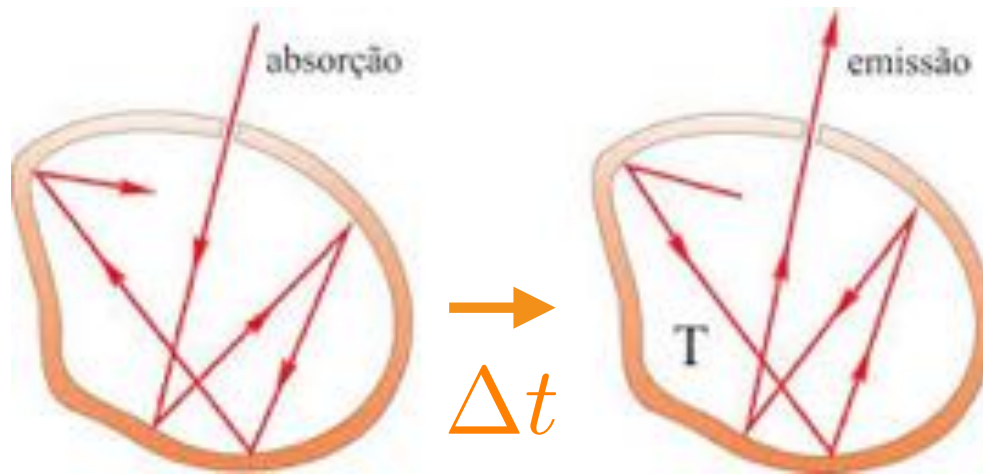
What a harmonic oscillator does?



Vocês sabem o que é um corpo negro termodinâmico?
(i.e., um objeto que radia com o espectro típico de corpo negro)



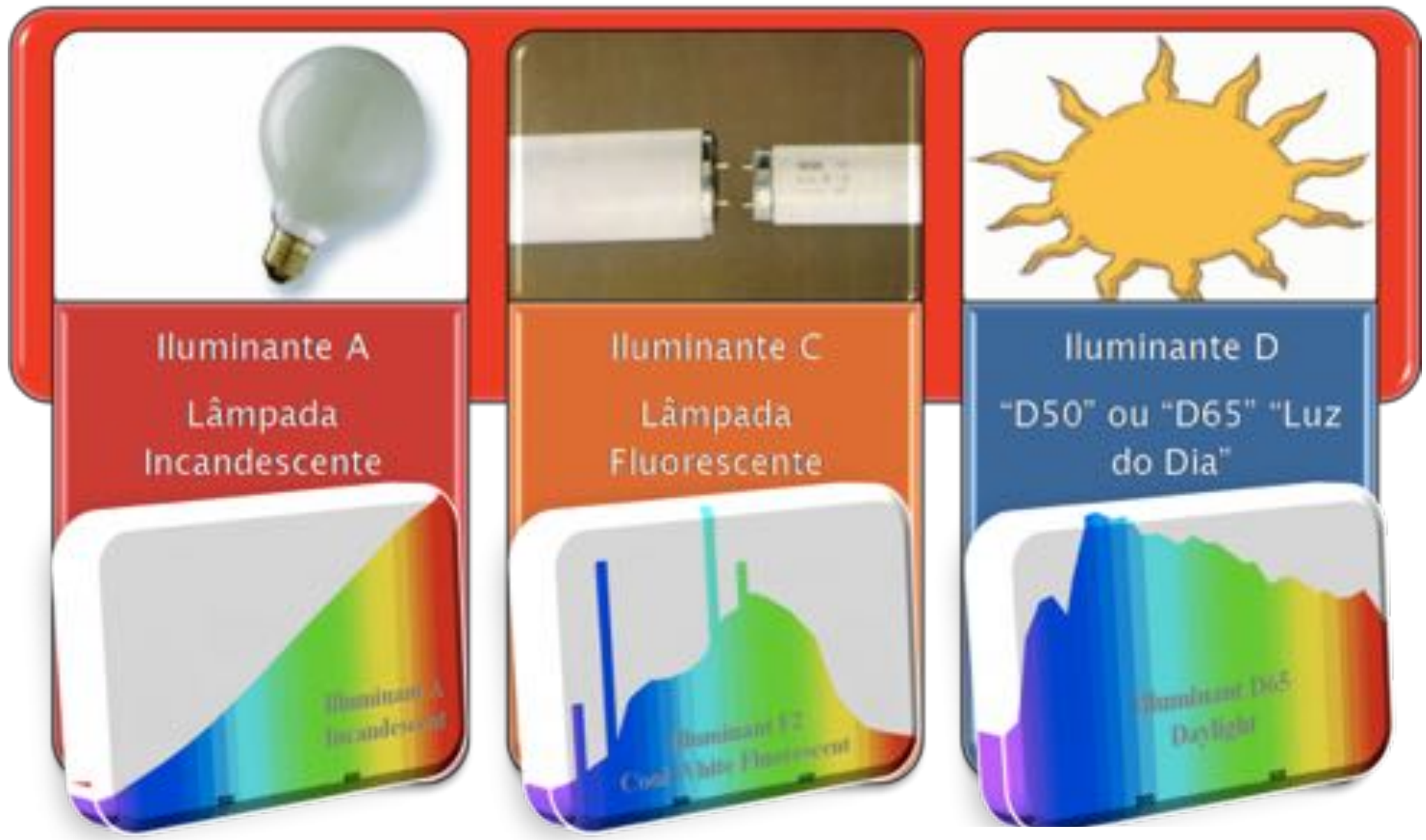
Vocês sabem o que é um corpo negro termodinâmico?
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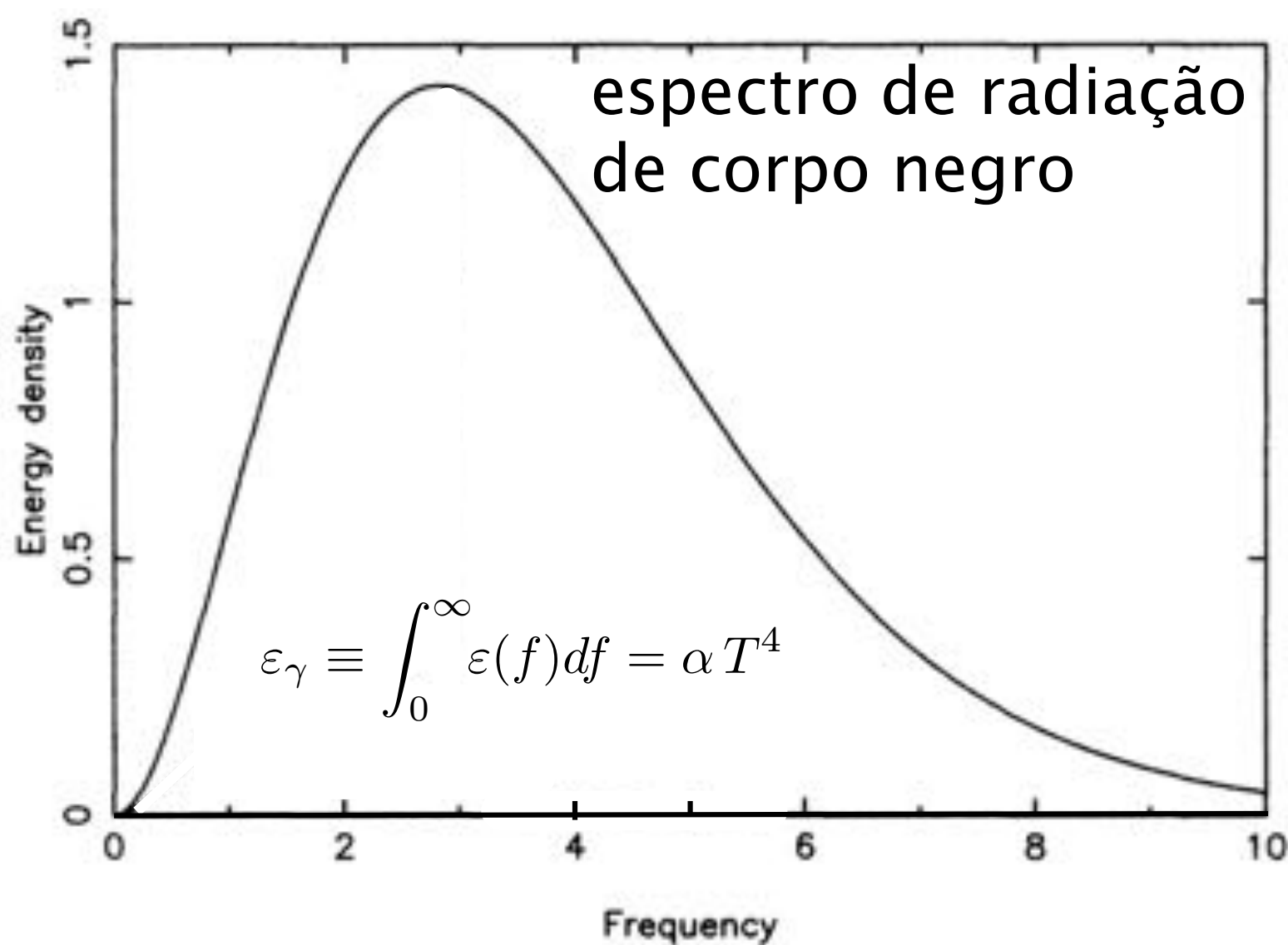
F. Reif, Física Estatística (vol.5, coleção Berkeley)

F. Reif, Fundamentos de física estatística e térmica

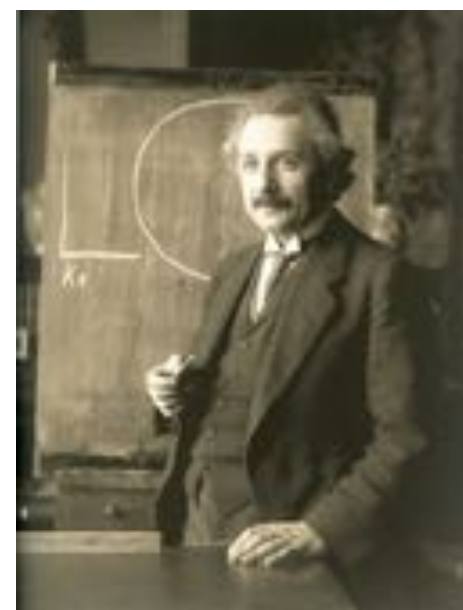
Exemplos de espectros



espectro de radiação de corpo negro



$$\epsilon(f) df = \frac{8\pi h}{c^3} \frac{f^3 df}{\exp(hf/k_B T) - 1}$$



The metric of the RW space-time is

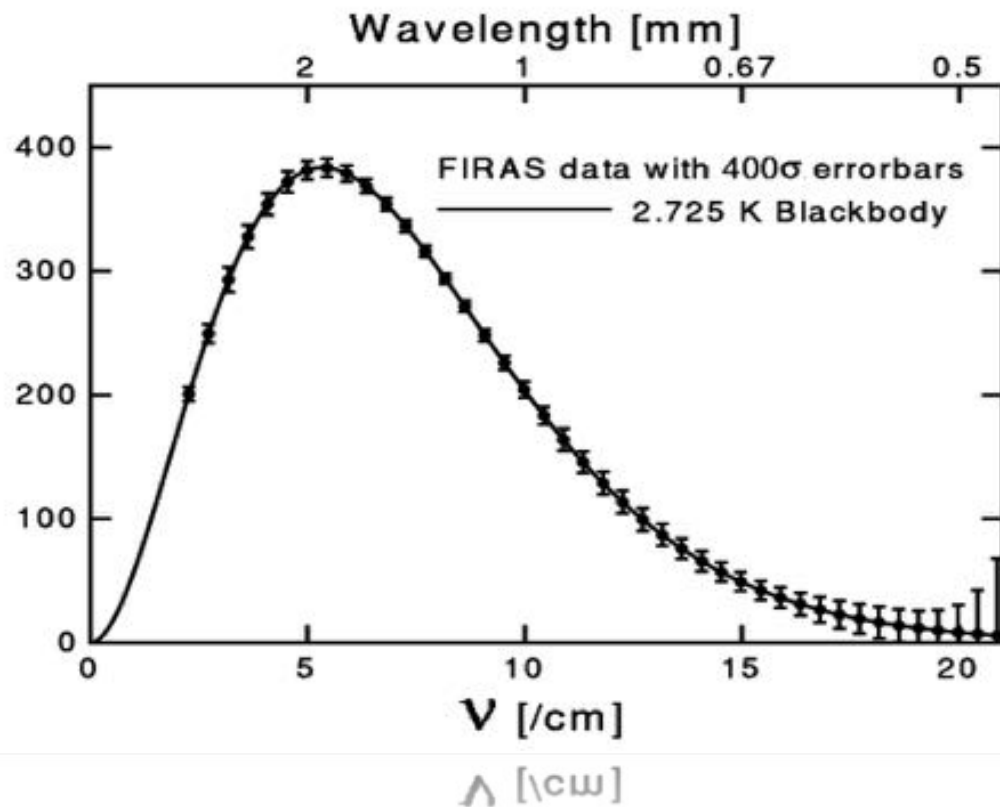
$$ds^2 = -c^2 dt^2 + a(t)^2 [dr^2 + r^2 d\Omega^2]$$

where the function, $a = a(t)$, termed the scale factor, is a well-defined function of the cosmic time t , why?

The metric of the RW space-time is

$$ds^2 = -c^2 dt^2 + a(t)^2 [dr^2 + r^2 d\Omega^2]$$

where the function, $a = a(t)$, termed the scale factor, is a well-defined function of the cosmic time t , why?



Due to the
black-body
nature of
the CMB

$$\Rightarrow T(t) = \frac{a_0 T_0}{a(t)}$$

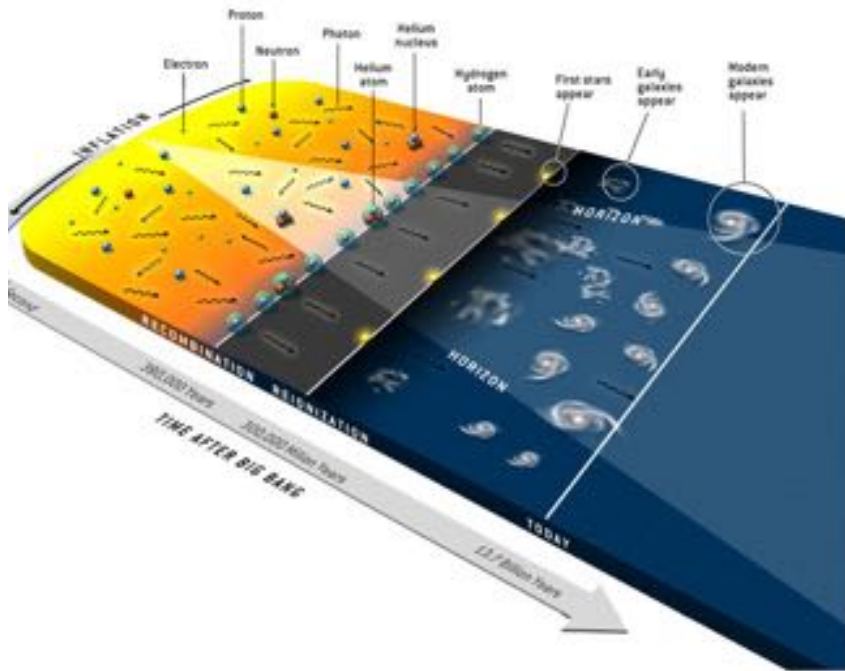
In other words for the RW metric hypothesis, the fact that the CMB radiation is of a black-body type implies:

$$T(t) = \frac{a_0 T_0}{a(t)} \Rightarrow a(t) = \frac{a_0 T_0}{T(t)} \Rightarrow t = a^{-1}[1/T]$$

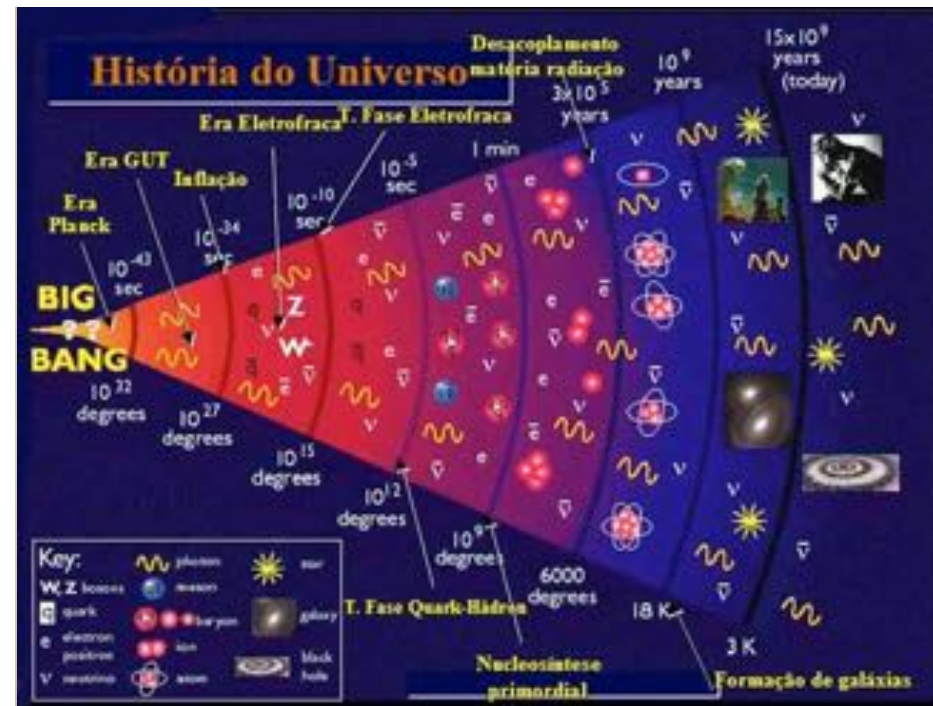
➡ t = the cosmic time is well-defined!



for this reason, it makes sense to talk about the

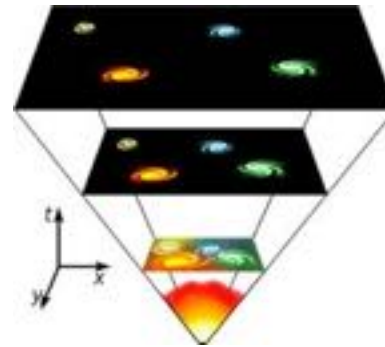
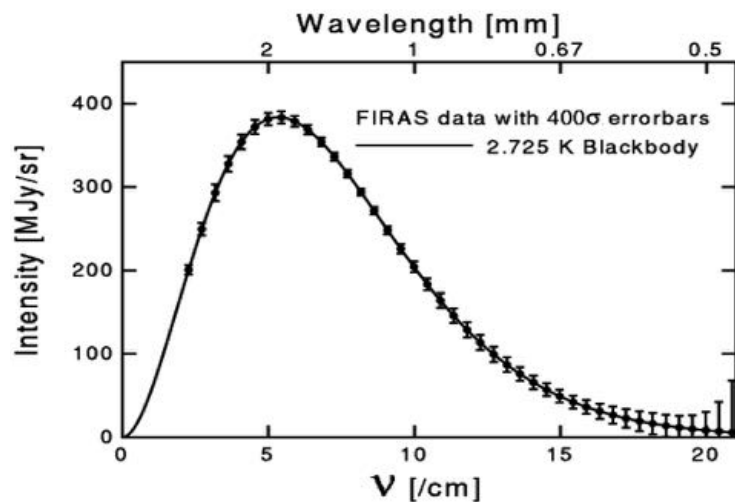


Thermal history of the Universe



The expanding Universe and the fluid thermodynamics

$$\varepsilon_\gamma = c^2 \rho_\gamma = \sigma T^4$$



$$T(t) = \frac{a_0 T_0}{a(t)}$$

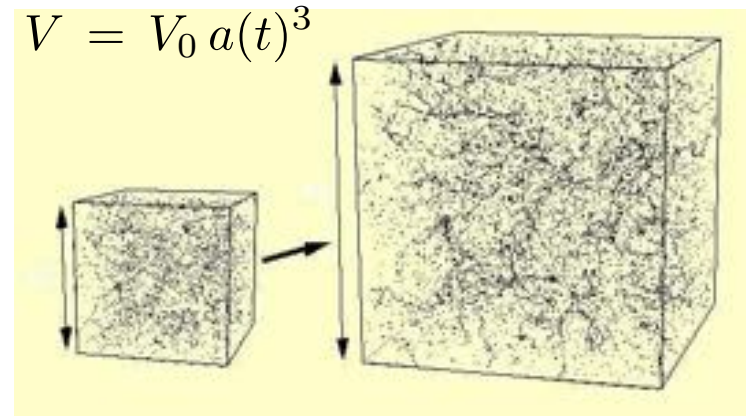
Corollary:

$$\rho_\gamma \propto a(t)^{-4}$$

Universe expansion and Thermodynamics

Consider a gas of photons in thermodynamical equilibrium at T :

$$\varepsilon_\gamma = \sigma T^4$$



in a volume V that expands with the Universe:

$$V = V_0 a(t)^3$$

energy density of
the photons gas

$$\Rightarrow E = E_\gamma = \varepsilon_\gamma V = \sigma T^4 V$$

$$\Rightarrow p = p_\gamma = \omega \varepsilon_\gamma = \frac{1}{3} \sigma T^4$$

energy of the
photons gas

equation of state
(of the fluid)

Excercise: What does this relation $\varepsilon_\gamma = \sigma T^4$ means?

From the 1st law of thermodynamics in an adiabatic process ($dQ = 0$)

$$dQ = dE + p dV \quad \Rightarrow \quad -p_\gamma \frac{dV}{dt} = \frac{dE_\gamma}{dt}$$

$$V = V_0 a(t)^3; \quad p_\gamma = \frac{\sigma}{3} T(t)^4; \quad E_\gamma = \sigma T(t)^4 V_0 a(t)^3; \quad \dot{X} \equiv \frac{dX}{dt}$$

$$-\frac{\sigma}{3} T^4 V_0 3 a^2 \dot{a} = \sigma T^4 V_0 3 a^2 \dot{a} + \sigma V_0 a^3 4 T^3 \dot{T}$$

dividing all terms by: $\sigma V_0 a(t)^2 T^3$

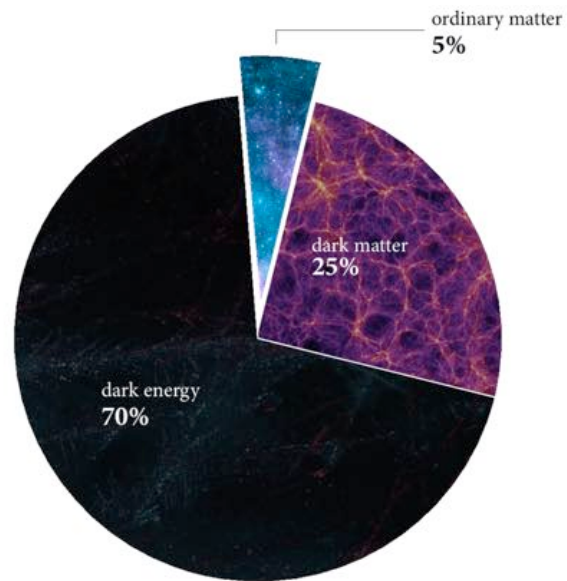
$$-T \dot{a} = 3 T \dot{a} + 4 a \dot{T} \quad \Rightarrow \quad [a(t) T(t)]^\cdot = 0$$

$$\Rightarrow a(t) T(t) = \text{const.}$$

$$\Rightarrow$$

$$T(t) = \frac{a_0 T_0}{a(t)}$$

Simplifying the Friedmann eq.: Universe contents



$$\Omega_0 + \Omega_k = 1$$

$$\Omega_0 \equiv \Omega_r + \Omega_m + \Omega_\Lambda$$

$$H(t)^2 = \frac{8\pi G}{3} \rho(t) - \frac{k c^2}{R_0^2 a(t)^2}$$

Critical density

We define the critical density as that required to the 3-space be flat,
i.e., the density such that the curvature of the 3-space be $k=0$

$$\rho_c(t) \equiv \frac{3}{8\pi G} H(t)^2$$

dividing the Friedmann eq.
by $H(t)^2$

$$1 = \frac{8\pi G}{3} \frac{\rho(t)}{H(t)^2} - \frac{k c^2}{R_0^2 a(t)^2 H(t)^2}$$

$$\Rightarrow 1 = \frac{\rho(t)}{\rho_c(t)} - \frac{k c^2}{R_0^2 a(t)^2 H(t)^2}$$

Defining: $\Omega(t) \equiv \frac{\rho(t)}{\rho_c(t)} \Rightarrow 1 - \Omega(t) = - \frac{k c^2}{R_0^2 a(t)^2 H(t)^2}$

which shows the case for one single component ρ ; what happens for more than one?

$$\Omega \rightarrow \Omega \equiv \Omega_r + \Omega_m + \Omega_\Lambda$$

$$1 - \Omega(t) = -\frac{k c^2}{R_0^2 a(t)^2 H(t)^2}$$

$$\Omega_k(t) \equiv -\frac{k c^2}{R_0^2 a(t)^2 H(t)^2}$$

$$1 - \Omega(t) = \Omega_k(t) \quad \text{ou} \quad \Omega(t) + \Omega_k(t) = 1$$

For $\mathfrak{t} = \text{today}$:

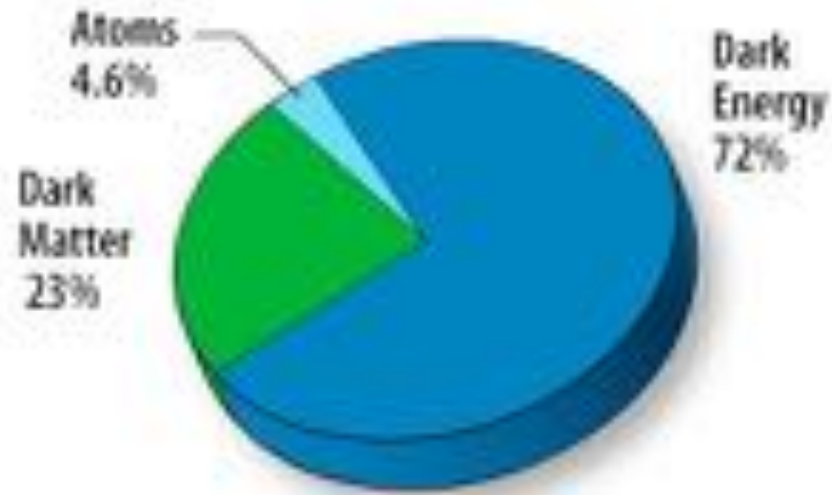
$$\Omega_0 + \Omega_k = 1$$

$\Leftrightarrow$

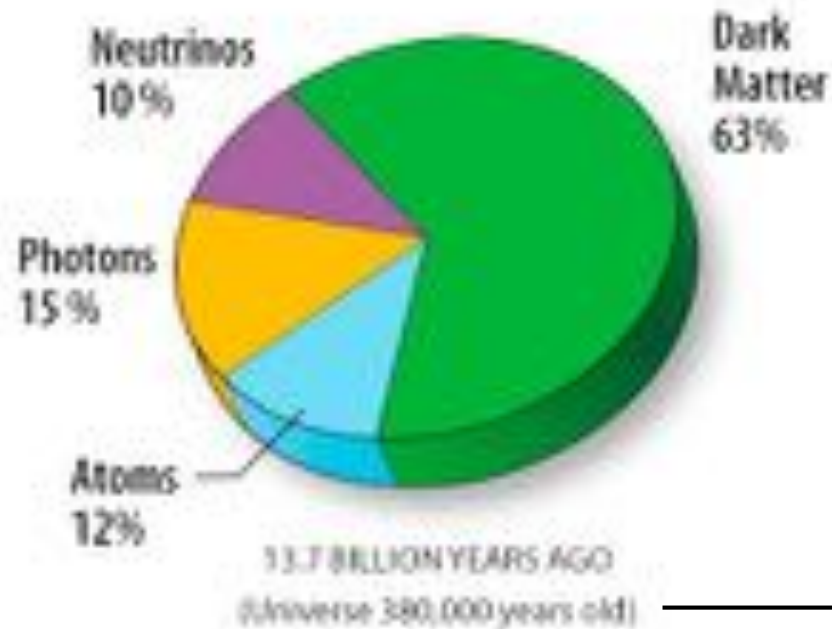
$$1 - \Omega_0 = \Omega_k$$

where $\Omega_0 \equiv \Omega_r + \Omega_m + \Omega_\Lambda$

Note that the symbols $\Omega_m, \Omega_r, \Omega_\Lambda, \Omega_k$ are defined for $\mathfrak{t} = \text{today}$, but we do not write them with sub-index zero



TODAY → **LD**



13.7 BILLION YEARS AGO
(Universe 380,000 years old) → **MD**

$$\Omega_0 + \Omega_k = 1$$

$$\Omega_k \simeq 10^{-2} \Rightarrow \Omega_0 \simeq 1$$

$$\Omega_0 \equiv \Omega_r + \Omega_m + \Omega_\Lambda$$

$\sim 10^{-5}$ ~ 0.3 ~ 0.7

The Friedmann eq. and the Universe epochs

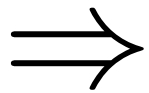
$$H(t)^2 = \frac{8\pi G}{3}\rho(t) - \frac{k c^2}{R_0^2 a(t)^2}$$

Equation of state for a
perfect fluid ($\omega = \text{const.}$): $p = \omega \rho$

Matter-energy conservation eq. $\dot{\rho} = -3 \frac{\dot{a}}{a} (\rho + p)$

For Radiation: $\rho_{\gamma} = \rho_0 a^{-4}$

Exercise



$$\omega = \frac{1}{3}$$

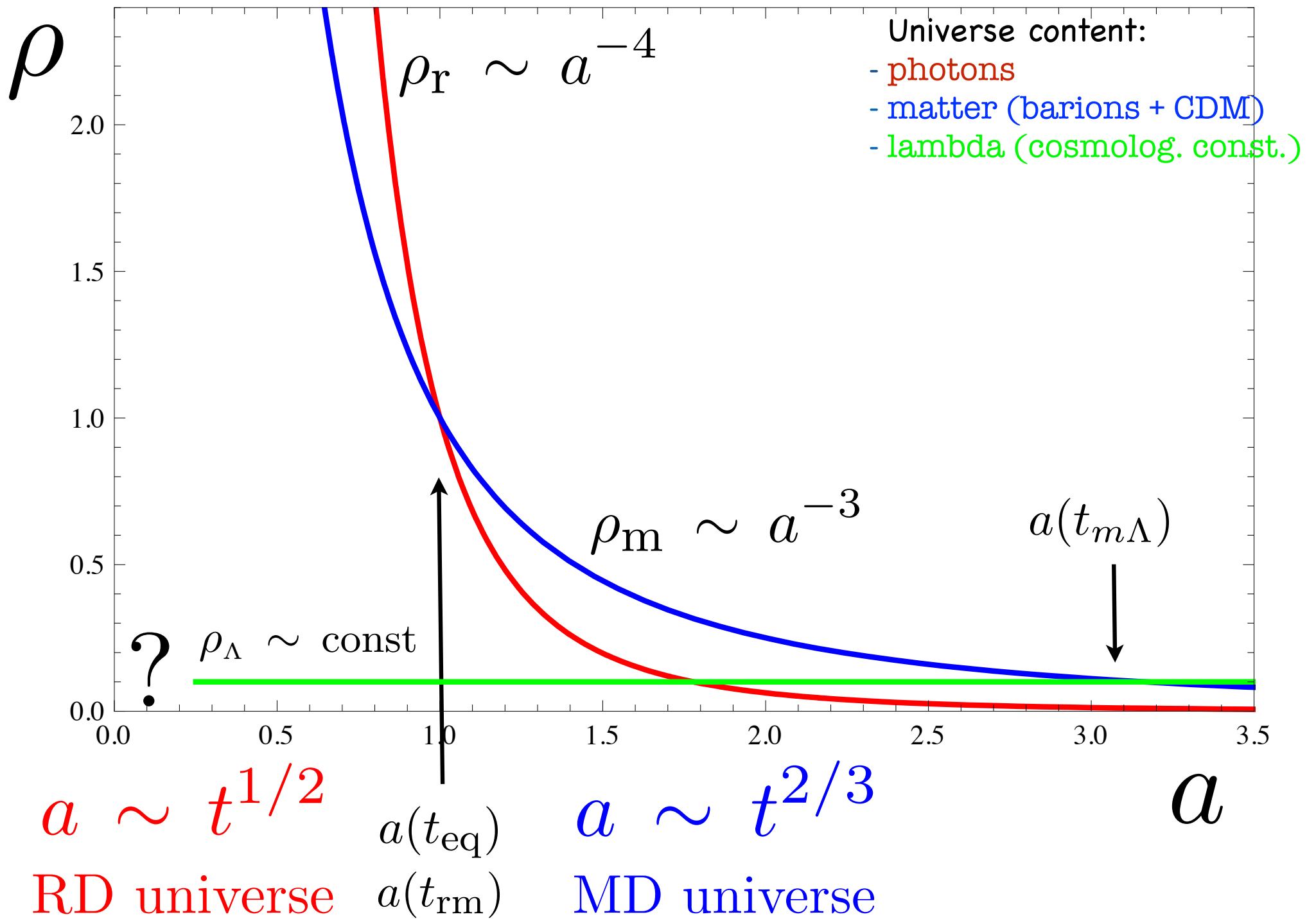
Equation of state for a
perfect fluid ($\omega = \text{const.}$): $p = \omega \rho$

Matter-energy conservation eq. $\dot{\rho} = -3 \frac{\dot{a}}{a} (\rho + p)$

For Matter: $\rho_m = \rho_0 a^{-3}$

Exercise

$$\Rightarrow \omega = 0$$



Epochs of the primordial Universe:

Exercices: since the scale factor can be written as indicated, find $t_0 = t_0(H_0)$

Radiation dominated (RD): $\rho_r \gg \rho_m \longrightarrow a = a_0 \left(\frac{t}{t_0} \right)^{1/2}$

Matter dominated (MD): $\rho_m \gg \rho_r \longrightarrow a = a_0 \left(\frac{t}{t_0} \right)^{2/3}$

Exercice: verify that

$$a(t) \sim t^{\frac{2}{3(1+\omega)}}$$

$$\omega = 1/3, 0 \quad (\text{RD, MD})$$

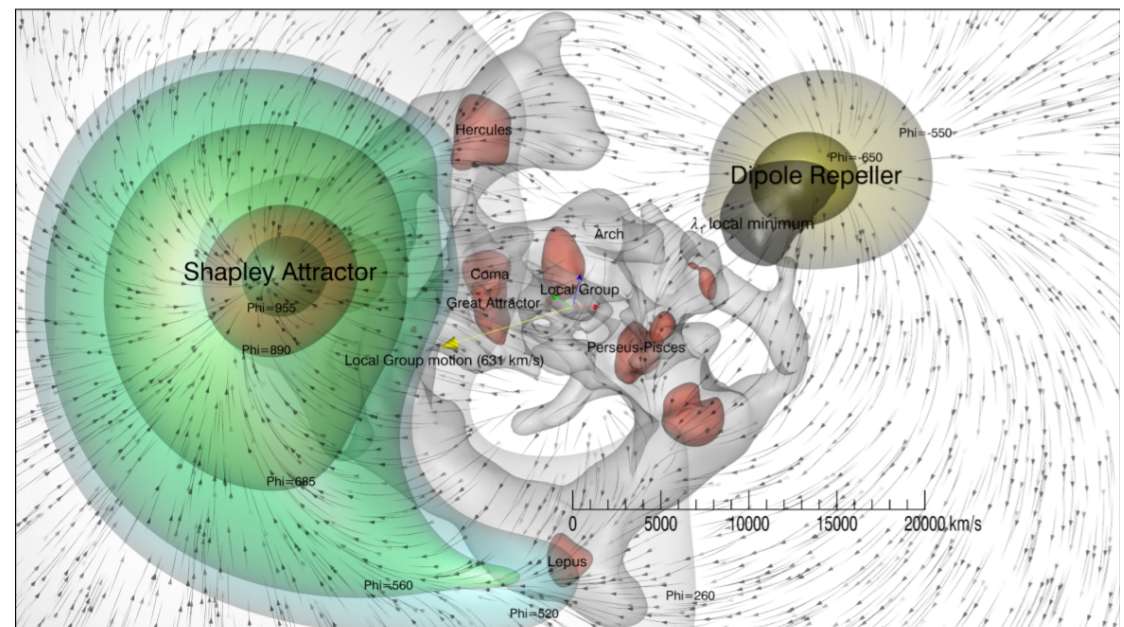
Exercice: verify that

$$\rho \sim a^{-3(1+w)}$$

$$\omega = 1/3, 0 \quad (\text{RD, MD})$$

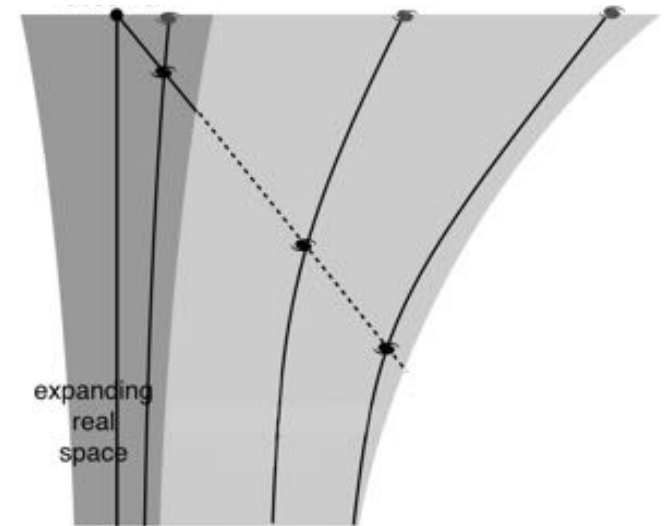
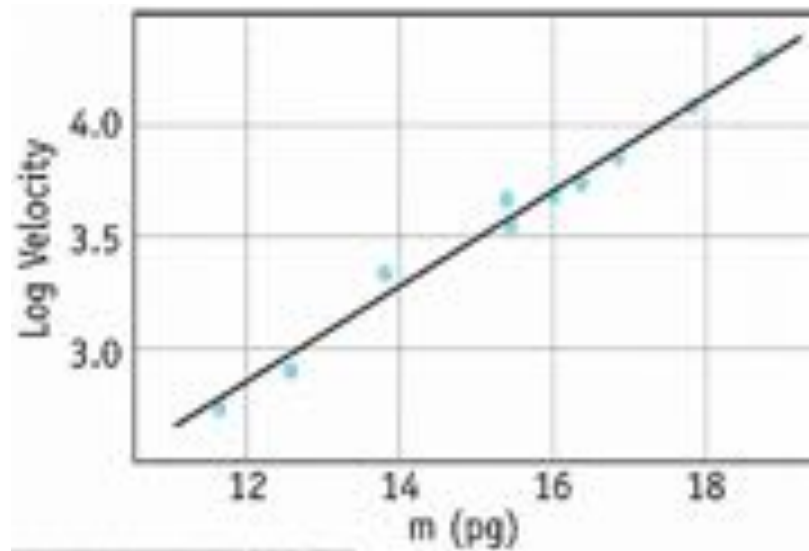
Until now we are talking about the background.

The real Universe:
how can one discover that the Universe is clumpy?

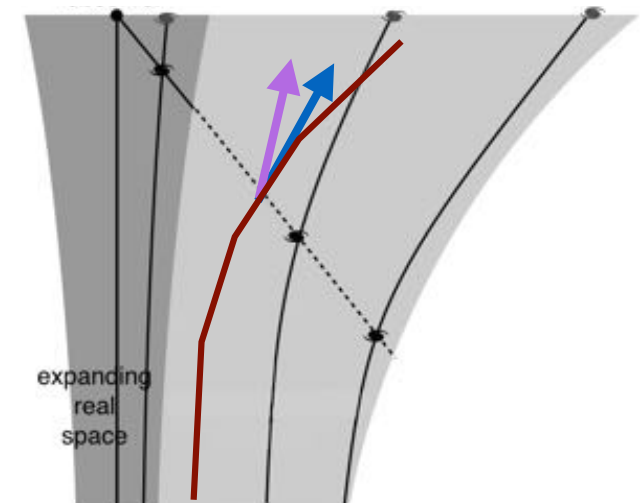
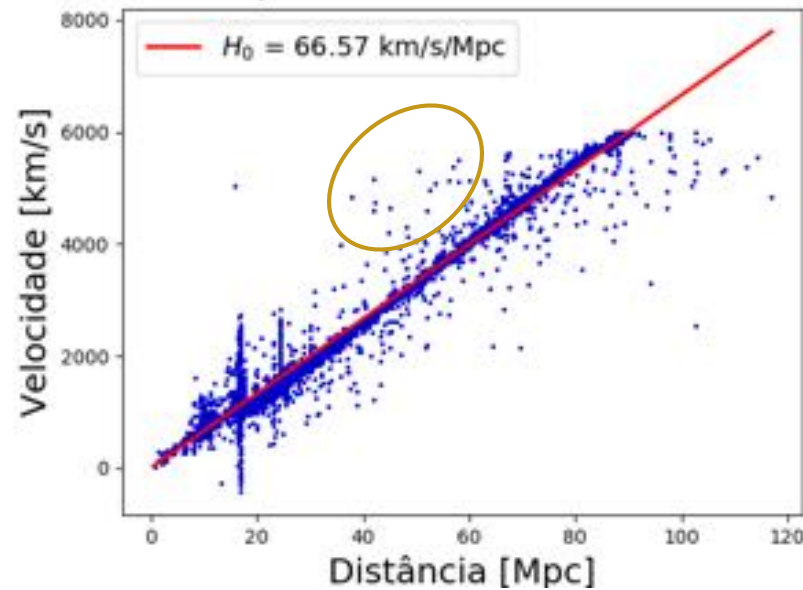


The problem and The solution:

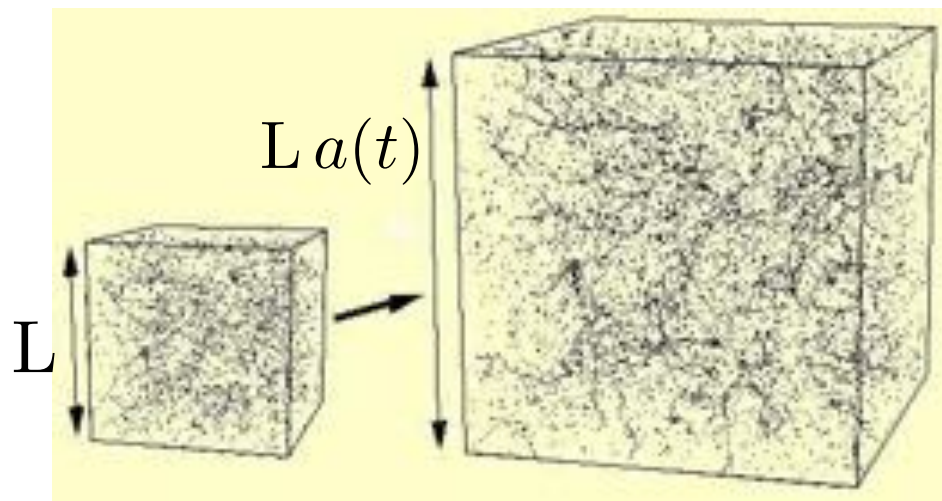
expected



observed



velocidade esperada  velocidade observada 

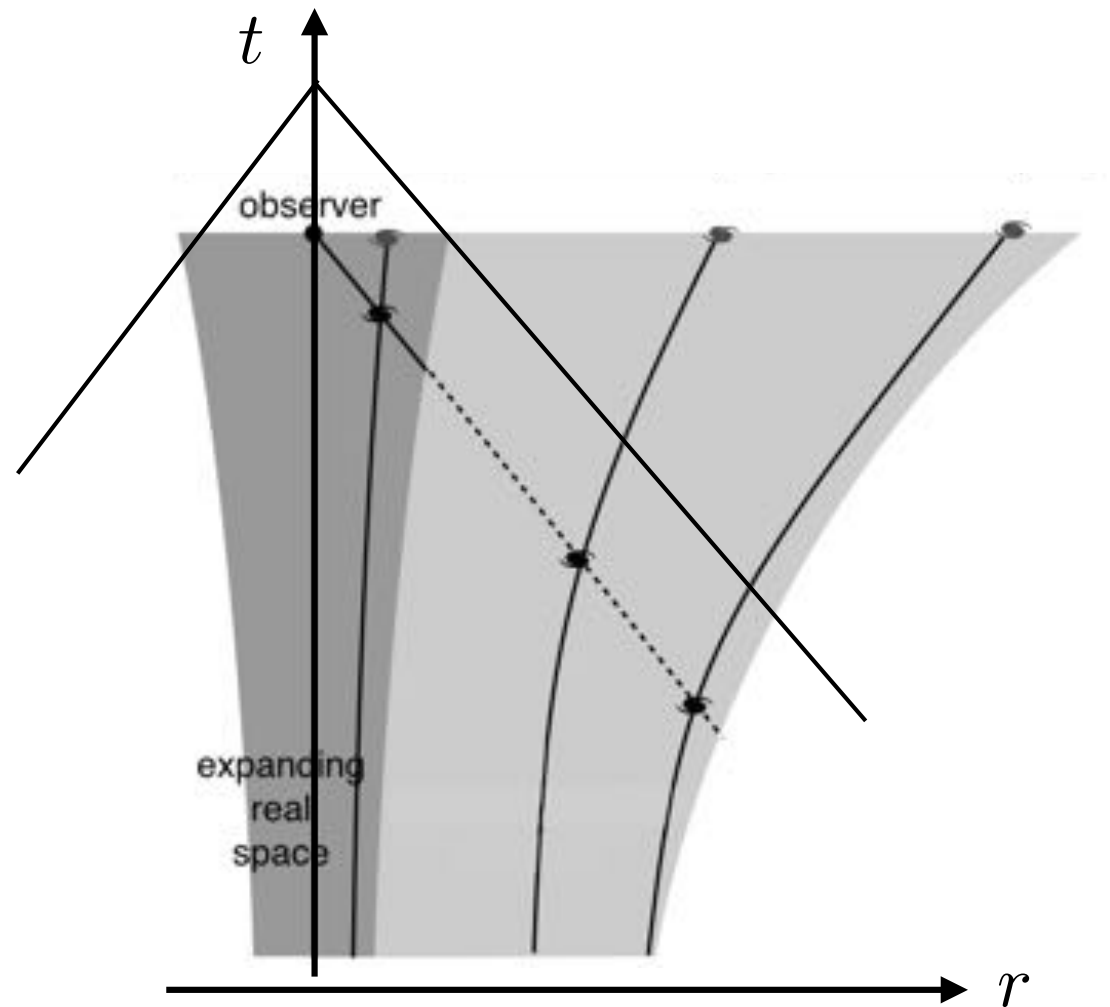


+ Cosmological Principle =

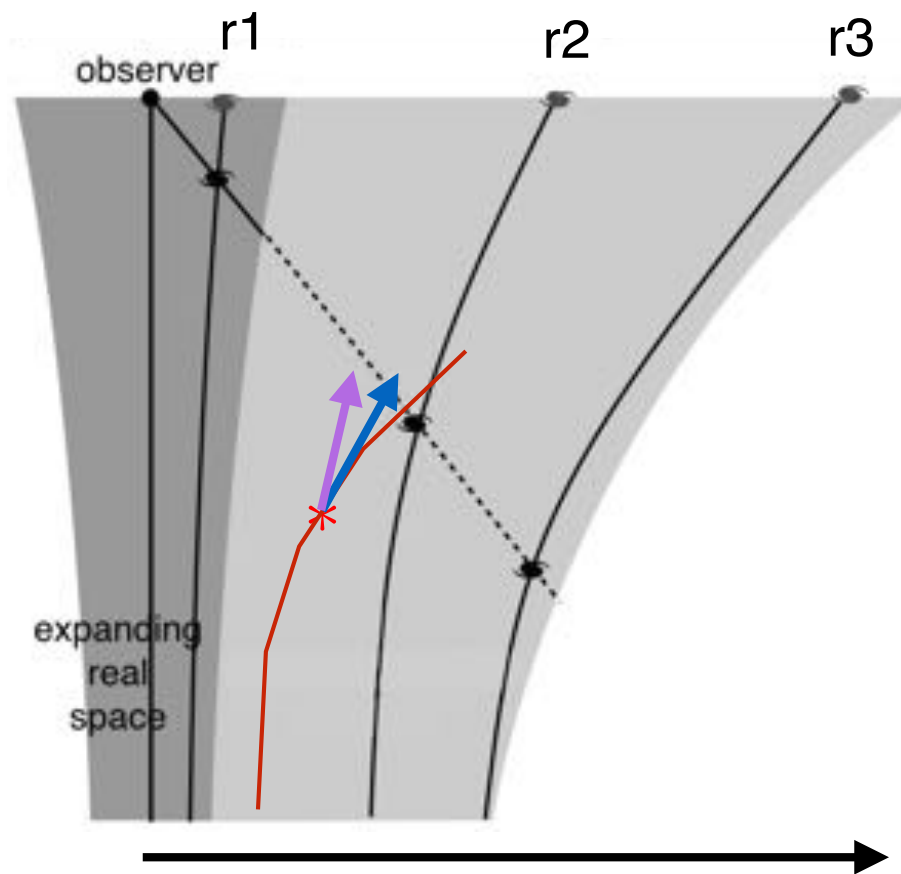
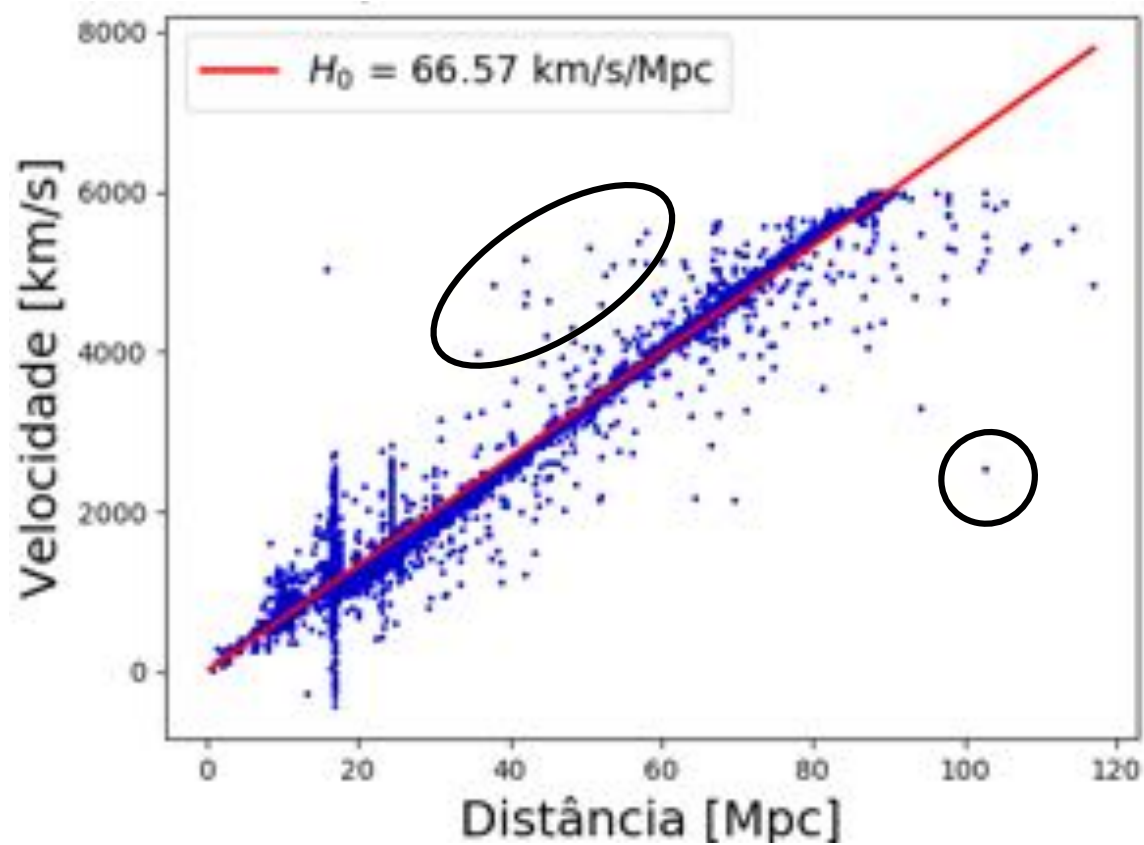
Hubble flow



isotropic expansion
of the Universe

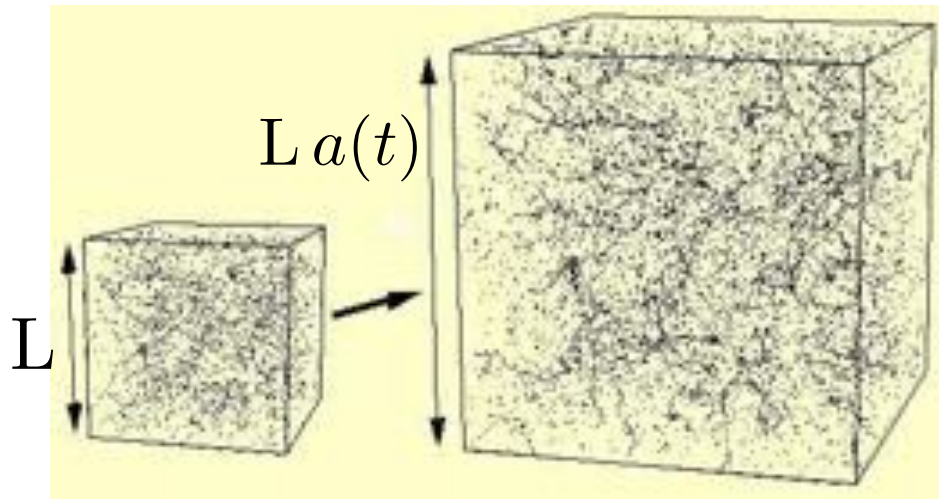


Outside the Hubble flux & Inside the Hubble flux



velocidade
esperada

velocidade
observada



+ Cosmological Principle =

Como assim: pontos fora da curva se a expansão é isotrópica?

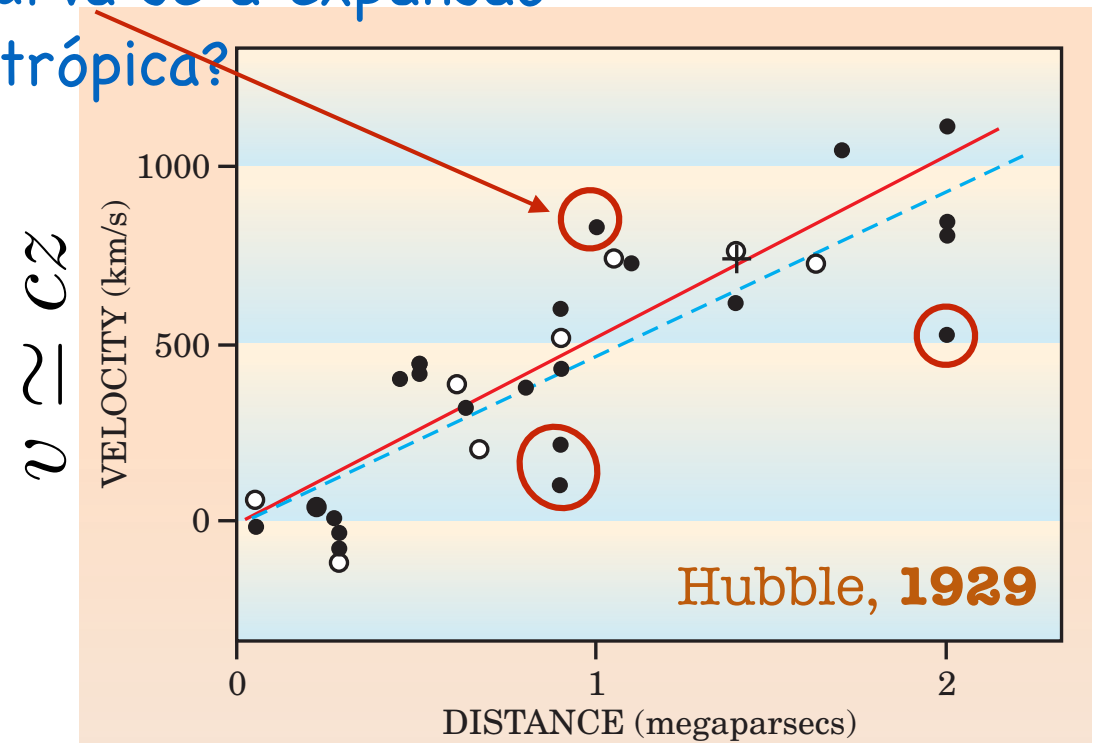
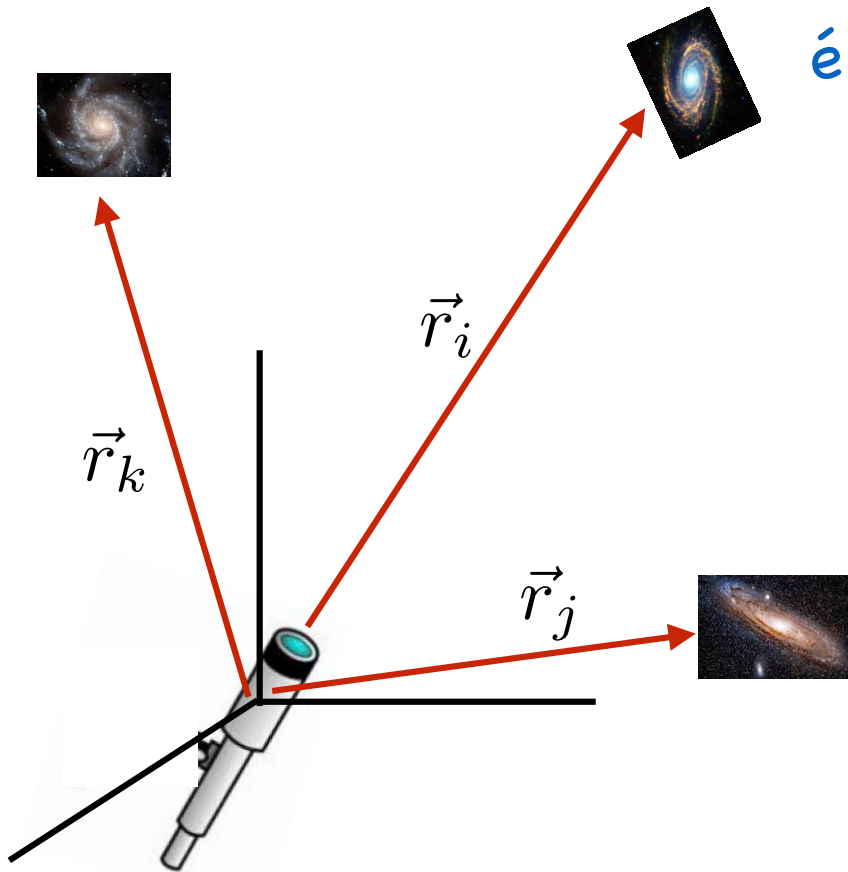
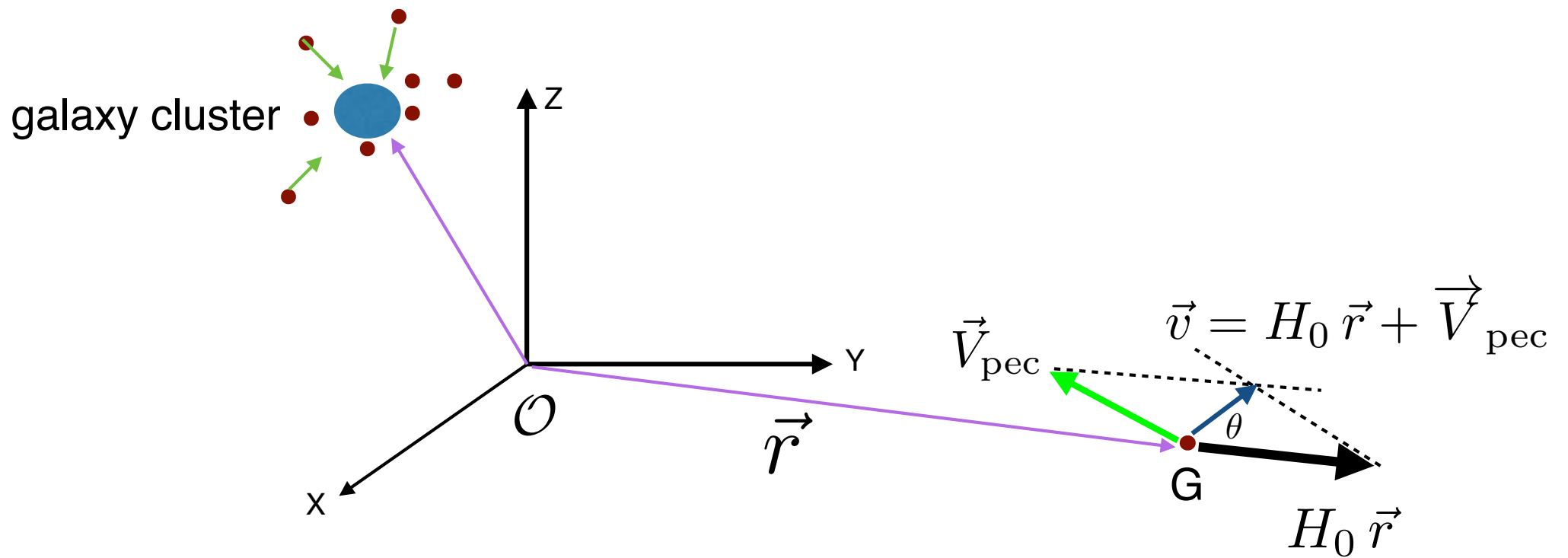


Figure 2. Recessional velocities of nearby galaxies vary linearly with distance as Edwin Hubble demonstrated in these data from 1929. The graph's filled circles and solid



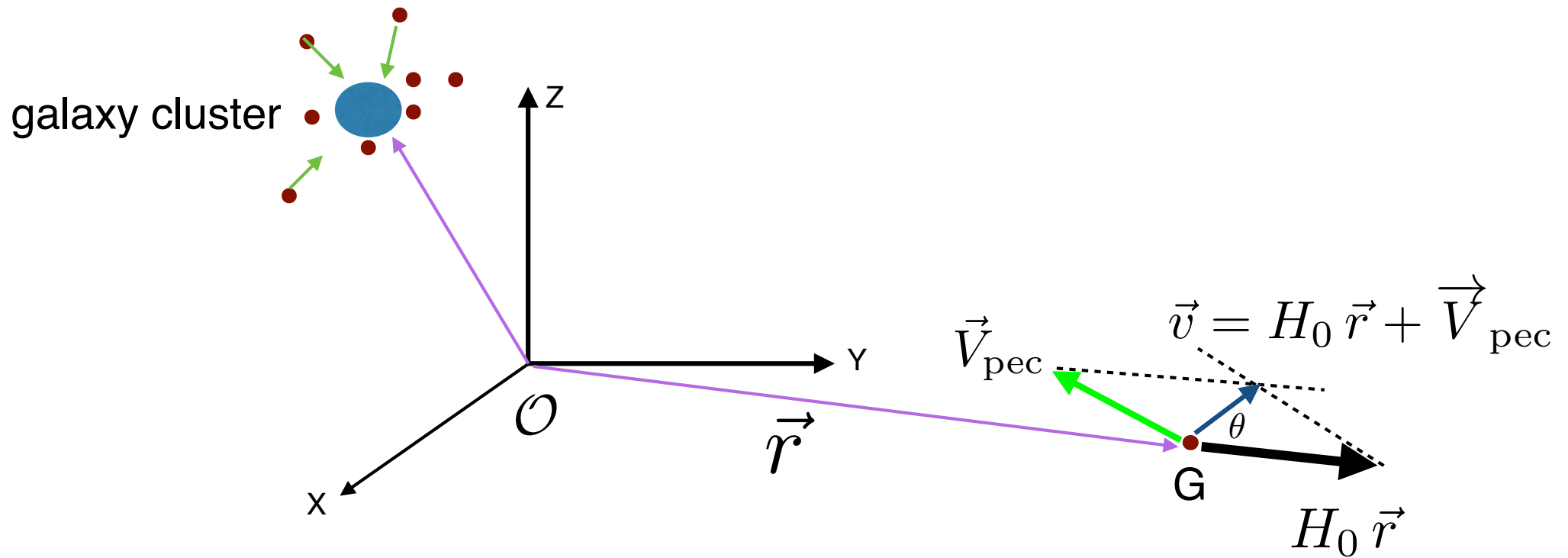
$$\vec{v} = \vec{v}_{\perp} + \vec{v}_r$$

A blue arrow pointing up and to the right is followed by an equals sign, then a blue arrow pointing straight up, followed by a plus sign, and then a blue arrow pointing straight to the right.

Question: what recession velocity ($= c z$) we measure for galaxy G?

Answer A: $H_0 r$

Answer B: v_r



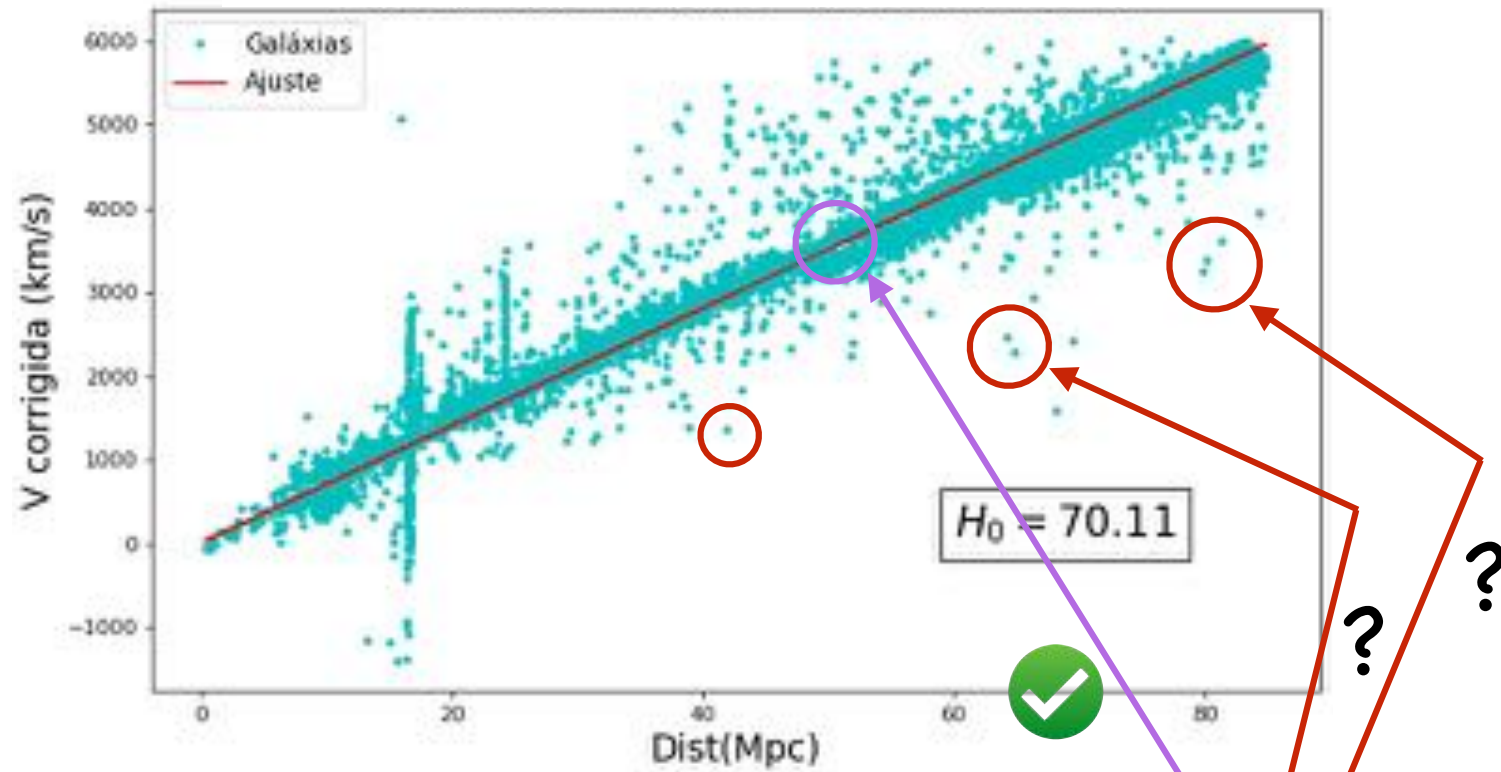
$$\vec{v} = \vec{v}_{\perp} + \vec{v}_r$$

Question: what recession velocity ($= c z$) we measure for galaxy G?

Answer A: $H_0 r$ ~~X~~

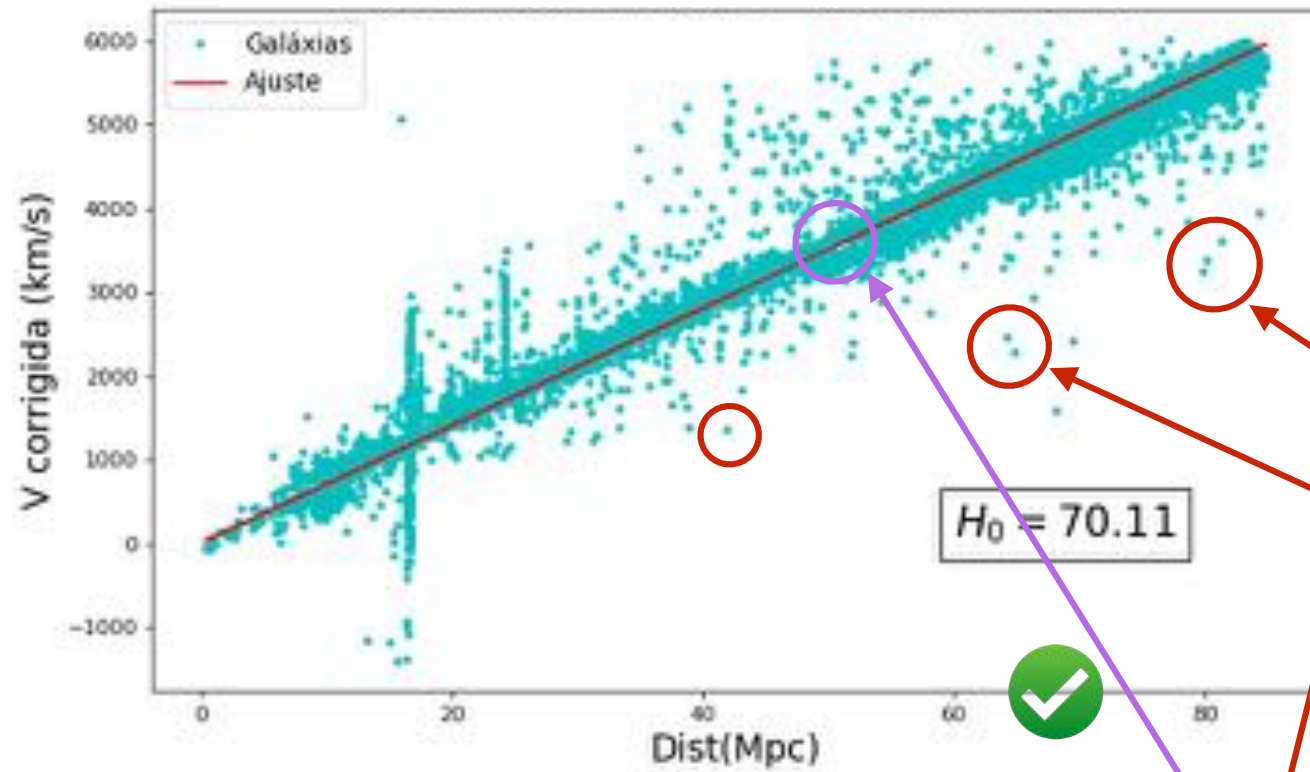
Answer B: $v_r \longrightarrow v_r = |H_0 \vec{r} + \vec{V}_{\text{pec}}| \cos \theta$





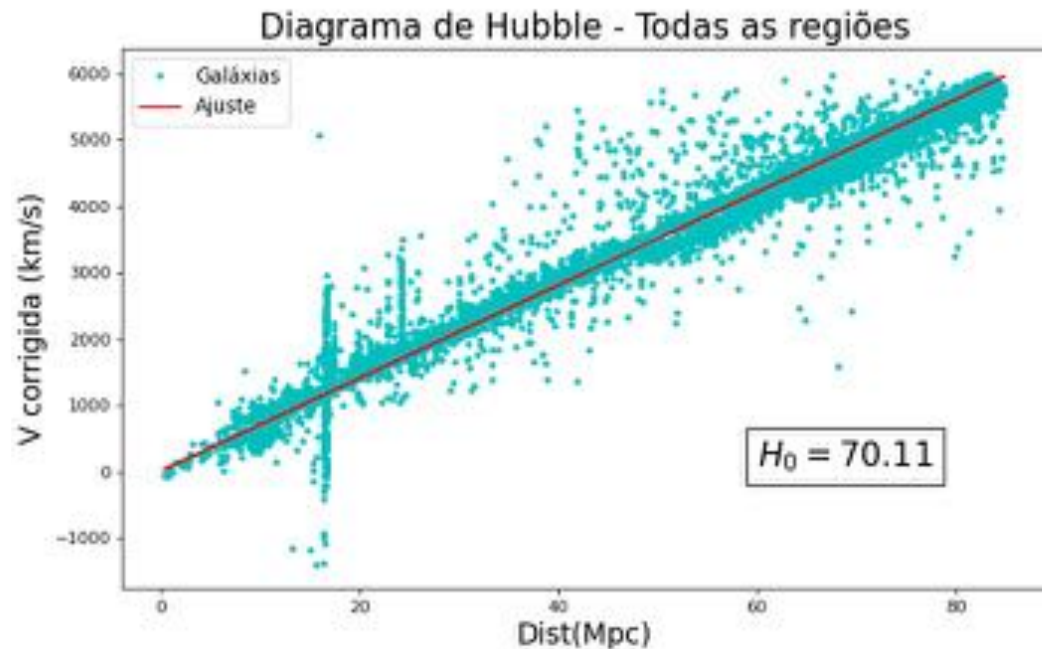
$$v = H_0 r$$

How do we solve this?



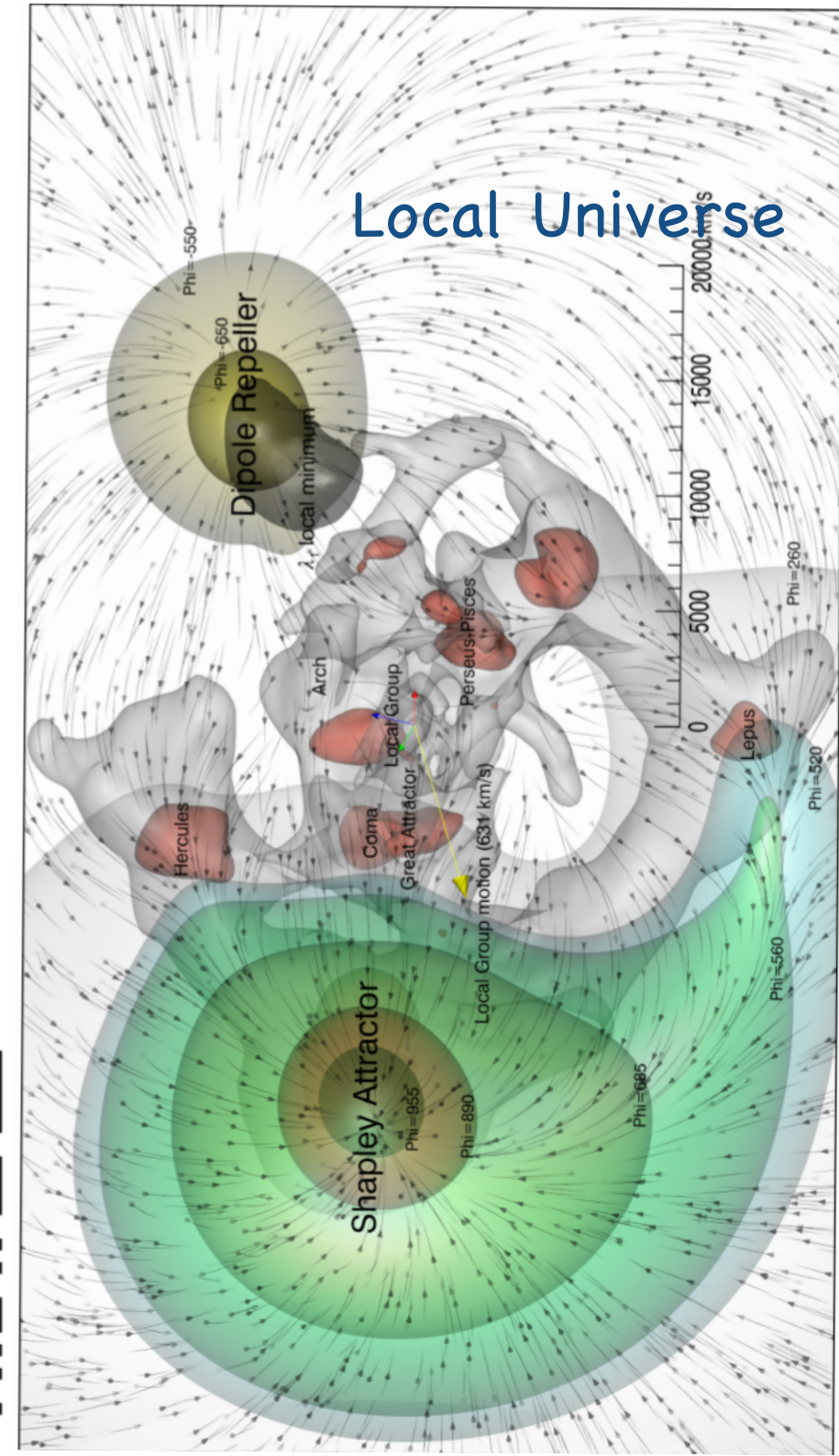
$$\vec{v} = H_0 \vec{r} + \vec{V}_{\text{pec}} \longrightarrow v = H_0 r + \langle V_{\text{pec}} \rangle$$

Conclusion: If deviations from the Hubble flow are due to clustered matter, **then** peculiar velocities traces the clumpy Universe!*

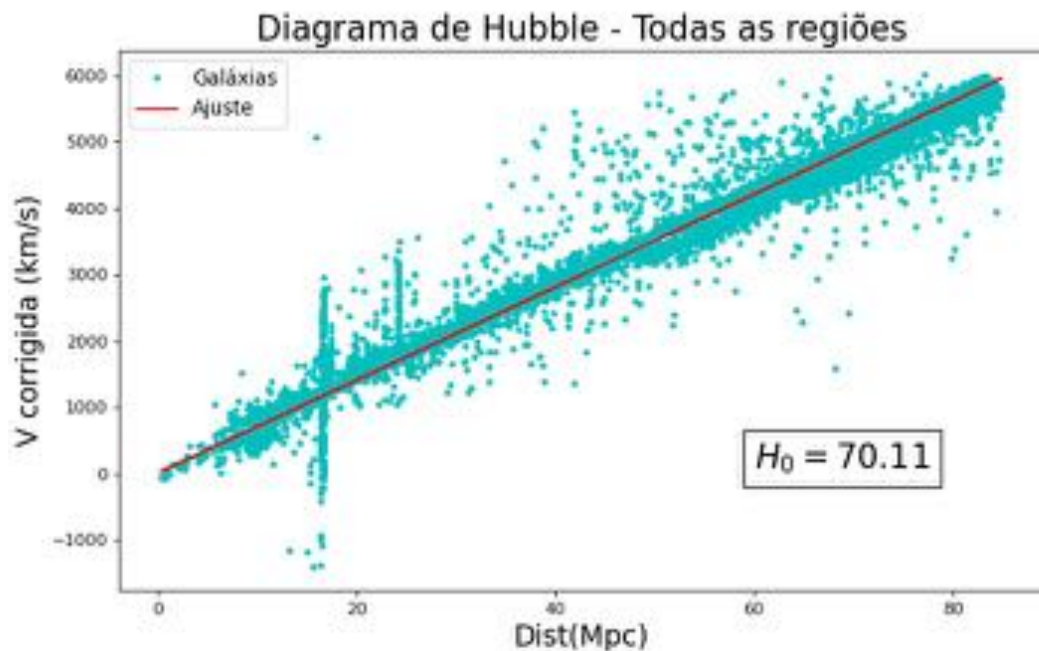


* Nick Kaiser, MNRAS, 1987

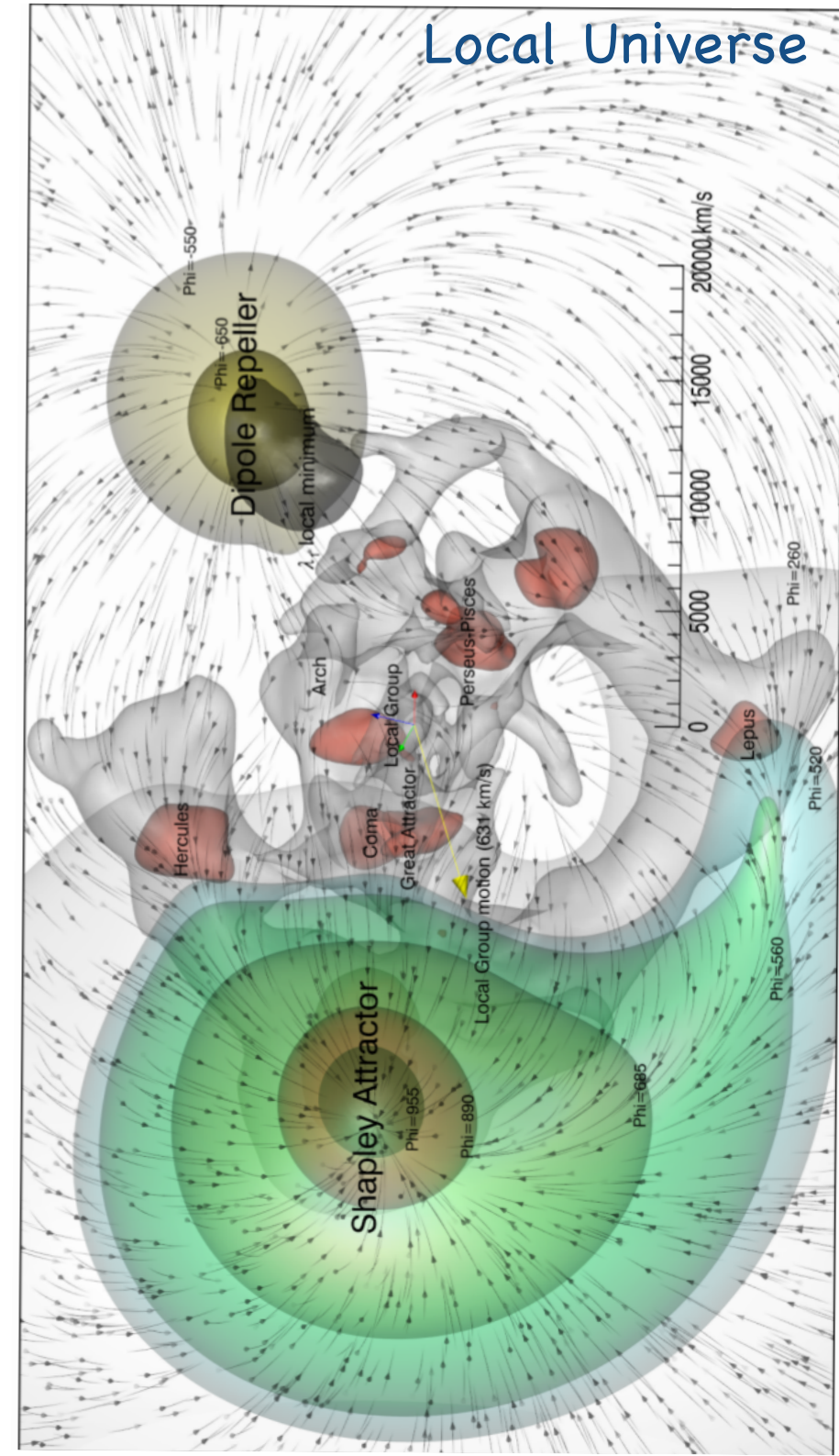
<https://ui.adsabs.harvard.edu/abs/1987MNRAS.227....1K/abstract>



Conclusion: If deviations from the Hubble flow are due to clustered matter, then peculiar velocities traces the clumpy Universe!



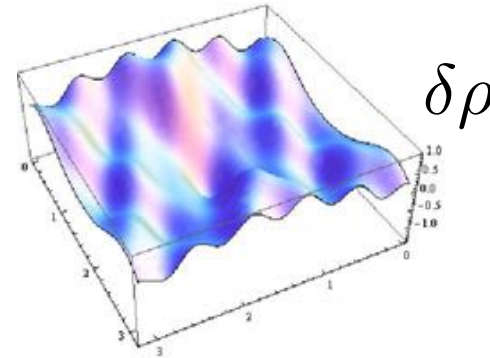
Still, we need a model to interpret these data



The solution:

Cosmological Perturbations of Einstein eqs.

Any mass fluctuation modifies
the space-time curvature



grav. field (due to $\rho(t, \mathbf{r})$) $\Longleftrightarrow$ space-time curvature

$$G_{\alpha\beta} = G_{\alpha\beta}(g_{\mu\nu}); \quad ds^2 = g_{\mu\nu}dx^\mu dx^\nu$$

$$g_{\mu\nu}(t, \mathbf{r}) = g_{\mu\nu}^0(t) + h_{\mu\nu}(t, \mathbf{r}); \quad |h_{\mu\nu}| \ll |g_{\mu\nu}^0|$$

$$h_{\mu\nu} \propto \text{Scalar} \oplus \text{Vector} \oplus \text{Tensor}$$

$\Rightarrow$ $ds^2 = g_{\mu\nu}^0 dx^\mu dx^\nu + h_{\mu\nu} dx^\mu dx^\nu$

$$ds^2 = \overset{\text{background}}{g_{\mu\nu}^0} dx^\mu dx^\nu + \overset{\text{perturbation}}{h_{\mu\nu}} dx^\mu dx^\nu$$

$$g_{\mu\nu}^0 dx^\mu dx^\nu = -c^2 dt^2 + a(t)^2 \gamma_{ij} dx^i dx^j$$

$$\gamma_{ij} dx^i dx^j = \frac{dr^2}{1 - k (r^2/R_0^2)} + r^2 [d\theta^2 + \sin^2 \theta d\varphi^2]$$

$$k = -1, 0, +1$$

Perturbed metric

$$\mathcal{H}^3, \mathcal{R}^3, \mathcal{S}^3$$

After many calculations, the perturbed metric in Newtonian gauge is

$$ds^2 = \overset{\text{background}}{g_{\mu\nu}^0} dx^\mu dx^\nu + \overset{\text{perturbation}}{h_{\mu\nu}} dx^\mu dx^\nu$$

$$g_{\mu\nu}^0 dx^\mu dx^\nu = -c^2 dt^2 + a(t)^2 \gamma_{ij} dx^i dx^j$$

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$$\mathcal{H}^3, \mathcal{R}^3, \mathcal{S}^3$$

After many calculations, the perturbed metric in Newtonian gauge is

$$ds^2 = -\left(1 + \frac{2\Phi}{c^2}\right) c^2 dt^2 + a(t)^2 \left(1 - \frac{2\Phi}{c^2}\right) \gamma_{ij} dx^i dx^j$$

where the perturbation field Φ plays the role of the gravitational potential

$$\nabla^2 \Phi = 4\pi G \delta\rho$$

Poisson eq.

$$ds^2 = \overset{\text{background}}{g_{\mu\nu}^0} dx^\mu dx^\nu + \overset{\text{perturbation}}{h_{\mu\nu}} dx^\mu dx^\nu$$

$$g_{\mu\nu}^0 dx^\mu dx^\nu = -c^2 dt^2 + a(t)^2 \gamma_{ij} dx^i dx^j$$

$$\gamma_{ij} dx^i dx^j = \frac{dr^2}{1 - k(r^2/R_0^2)} + r^2 [d\theta^2 + \sin^2 \theta d\varphi^2]$$

$$k = -1, 0, +1$$

Perturbed metric

After many calculations, the perturbed metric in Newtonian gauge is

$$ds^2 = -\left(1 + \frac{2\Phi}{c^2}\right) c^2 dt^2 + a(t)^2 \left(1 - \frac{2\Phi}{c^2}\right) \gamma_{ij} dx^i dx^j$$

where the perturbation field Φ plays the role of the gravitational potential

background

perturbation

$$[\Phi] = \text{L}^2 \text{T}^{-2} = [c^2]$$

$$\nabla^2 \Phi = 4\pi G \delta\rho$$

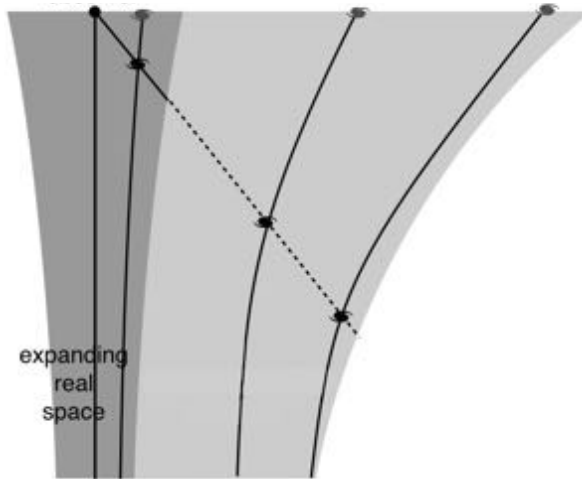
Poisson eq.

The proposal:

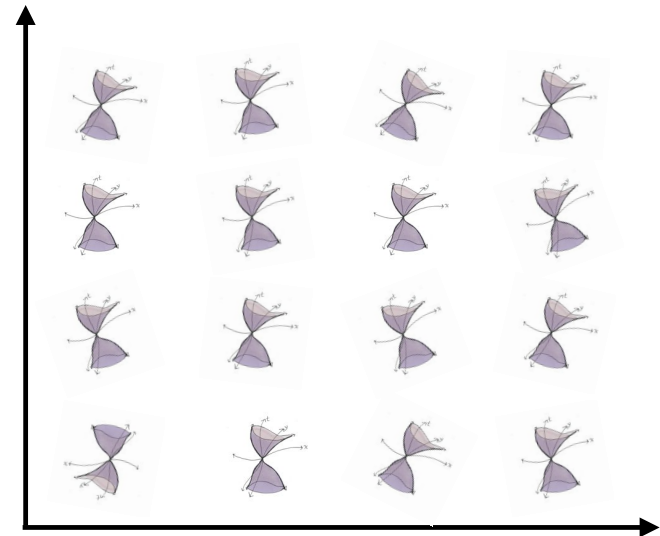
spacetime
metric

$$ds^2 = c^2 dt^2 + a(t)^2 d\mathbf{x}^2 \quad + \quad ds^2 = -c^2 \left(1 + \frac{2\phi}{c^2}\right) dt^2 + \left(1 - \frac{2\phi}{c^2}\right) d\mathbf{x}^2$$

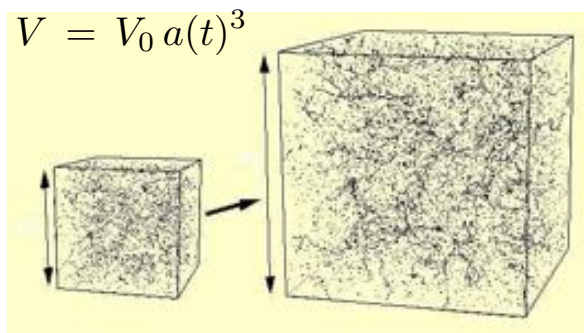
spacetime



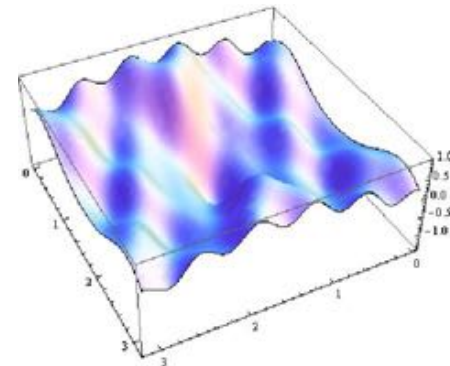
+



3D-space



+



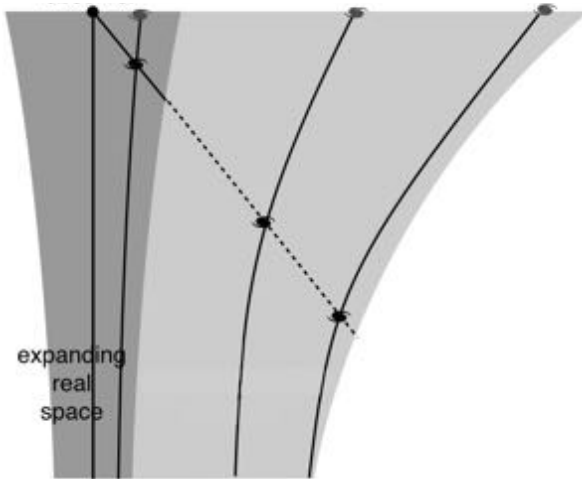
$a = a(t)$ is the scale factor

The solution:

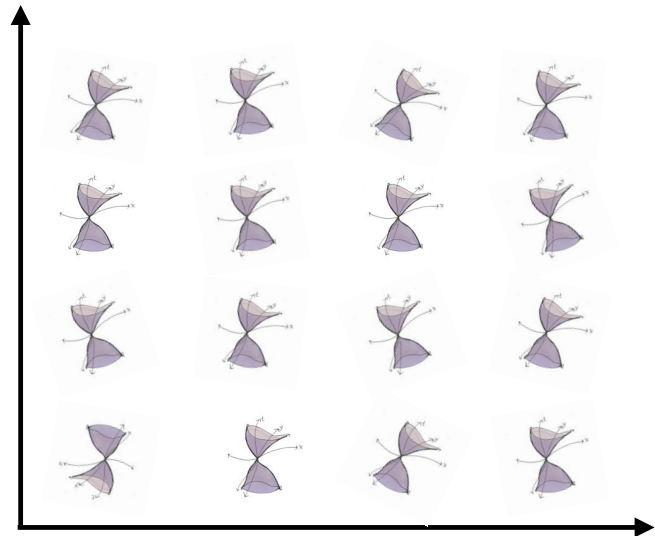
spacetime
metric

$$ds^2 = -\underbrace{\left(1 + \frac{2\Phi}{c^2}\right)}_{\text{blue}} c^2 dt^2 + \underbrace{a(t)^2}_{\text{red}} \underbrace{\left(1 - \frac{2\Phi}{c^2}\right)}_{\text{blue}} \gamma_{ij} dx^i dx^j$$

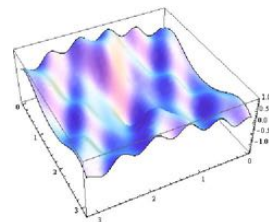
spacetime



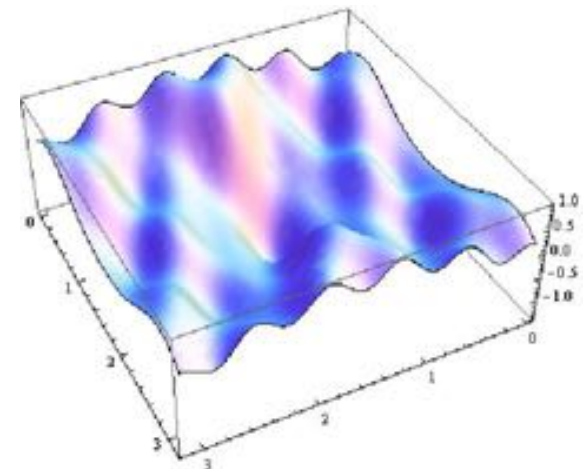
+



3D-space



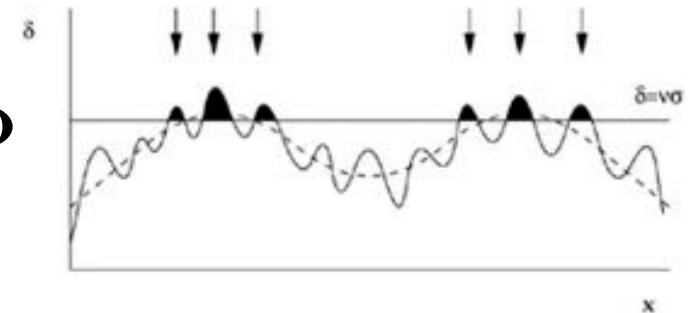
$$V = V_0 a(t)^3$$



$a = a(t)$ is the scale factor

Spacetime density fluctuations:

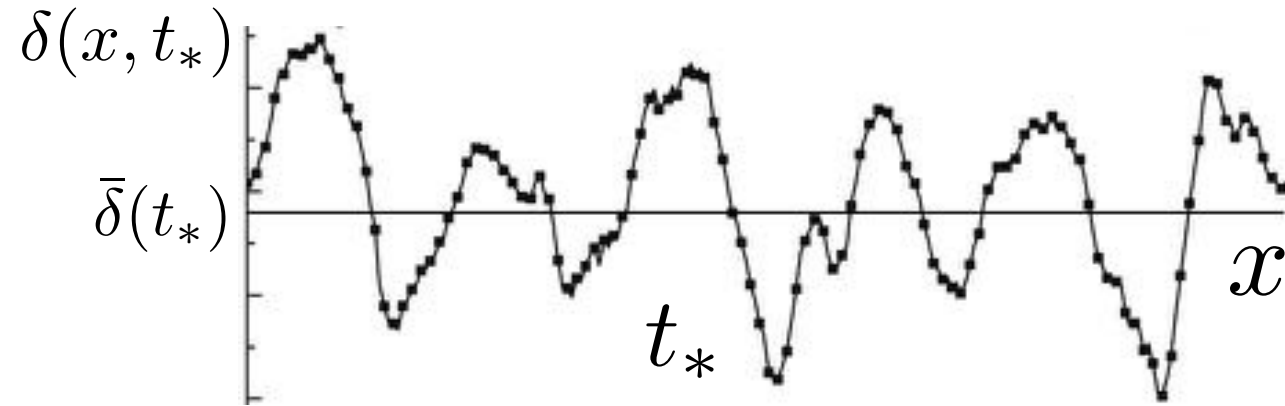
$$\delta\Phi \Longrightarrow \delta T_{\mu\nu} \Longrightarrow \delta g_{\mu\nu} \Longrightarrow \delta\Phi$$



➡ Perturbed Einstein eqs.: Continuity, Euler, Laplace

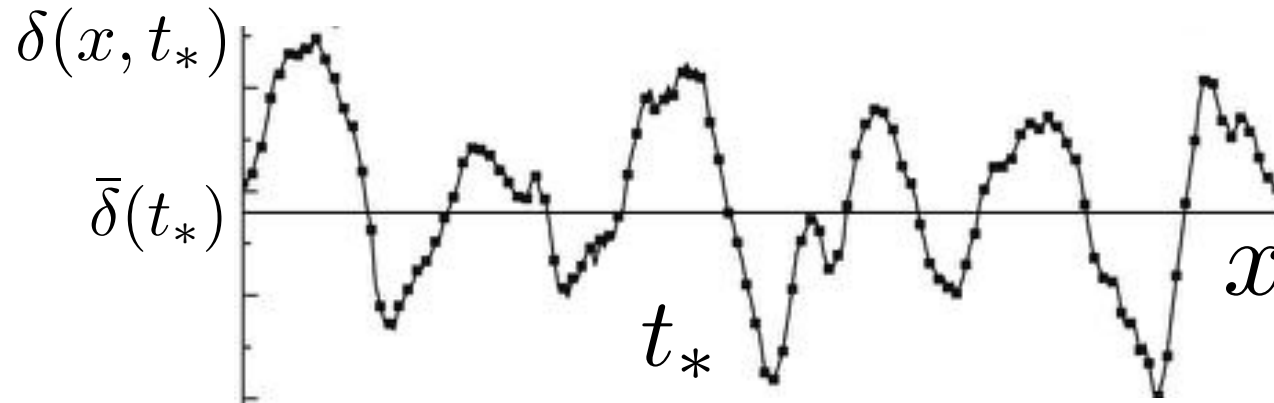
density contrast

$$\delta(x, t_*) \equiv \frac{\rho(x, t_*) - \bar{\rho}(t_*)}{\bar{\rho}(t_*)}$$



$$\delta(x, t_0) = \sum_i c_i \lambda_i(x, t_0)$$

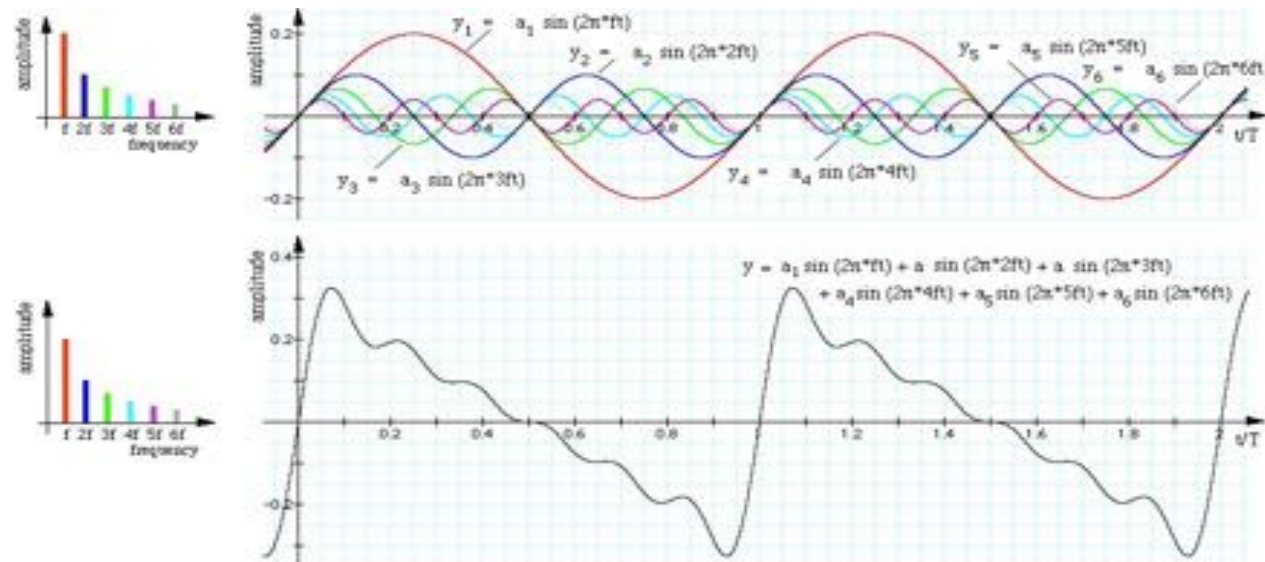
density contrast $\delta(x, t_*) \equiv \frac{\rho(x, t_*) - \bar{\rho}(t_*)}{\bar{\rho}(t_*)}$



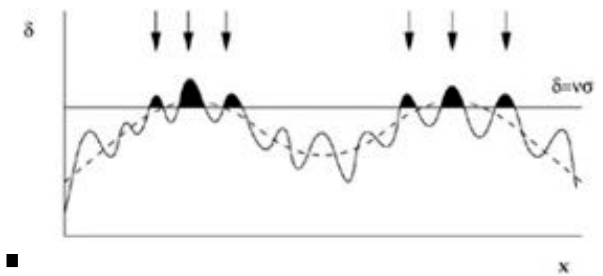
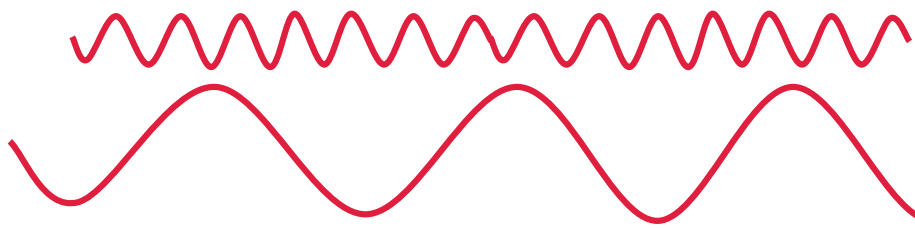
$$\delta(x, t_0) = \sum_i c_i \lambda_i(x, t_0)$$

Harmonic analyses in 1-d

A vibrating string with fixed boundaries: it can vibrate in **fundamental harmonics (FH)**: 1st harm., 2nd harm., etc. Then, an arbitrary **perturbation** of the string can be represented as a superposition of the **FH**:

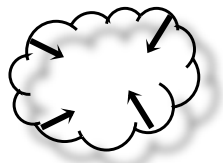
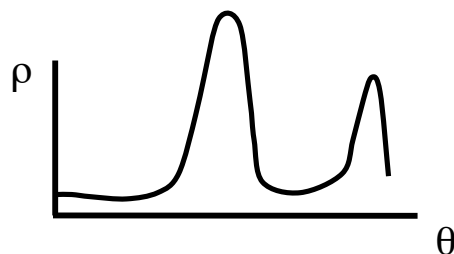
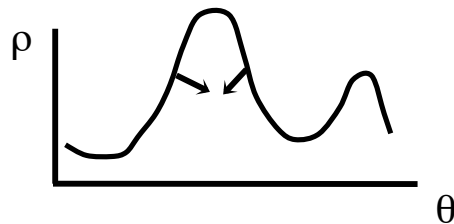
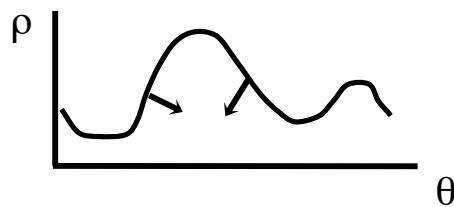


the amplitude for each frequency = **spectrum of the perturbation**

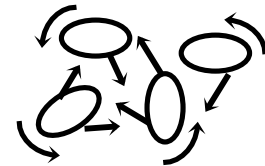
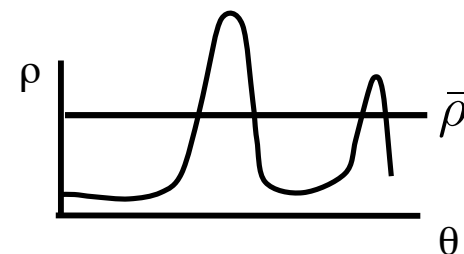
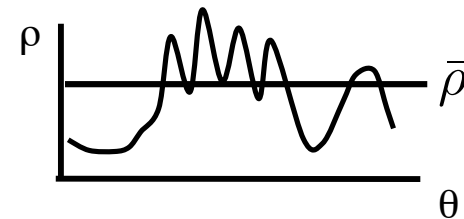
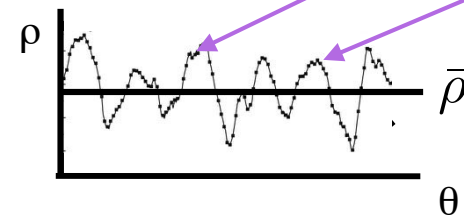


Two collapse scenarios:

Initial collapse
(top down)



Hierarchical merging
(bottom up)

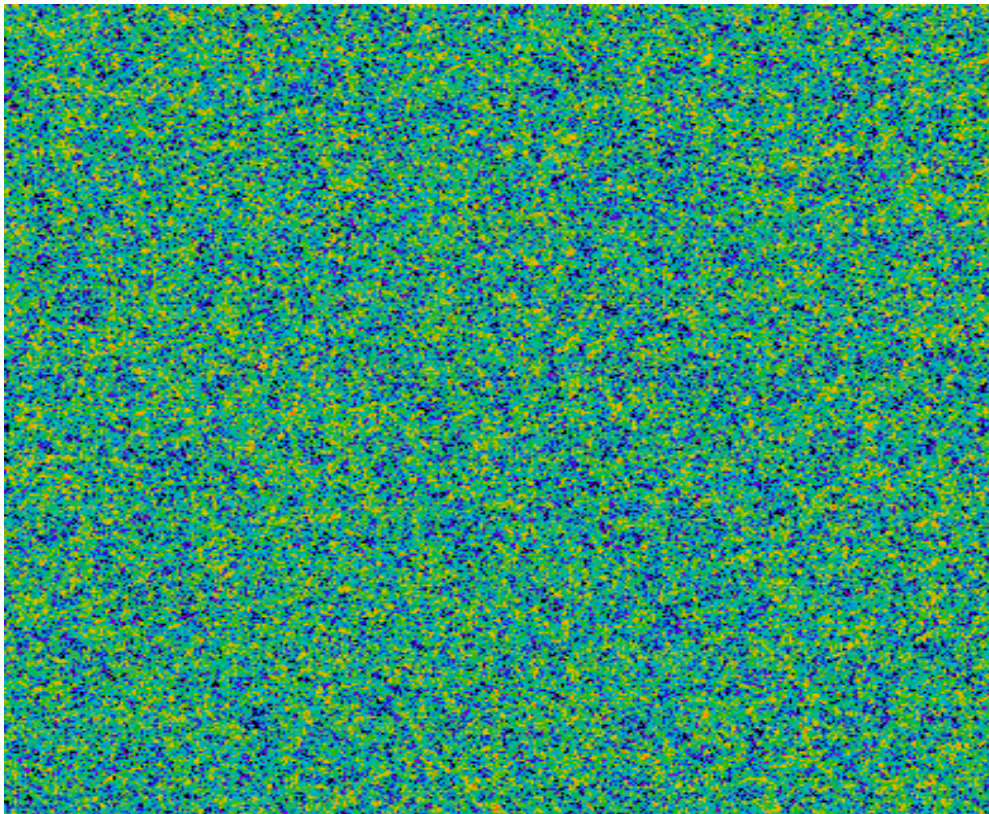


Merging

Nature of density fluctuations (intuitive, phenomenological)

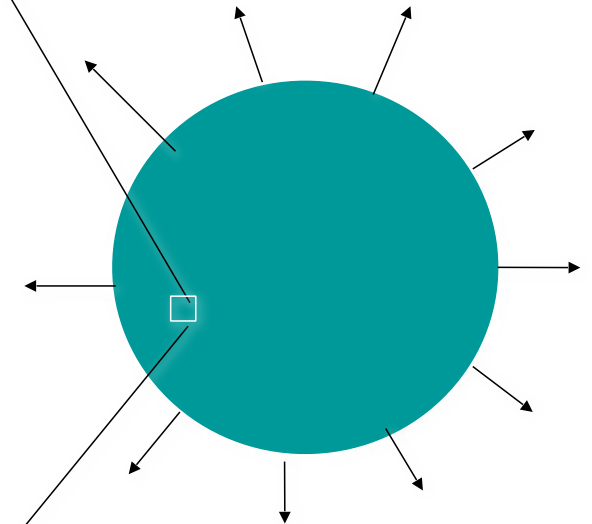
Evolution of small inhomogeneities due to gravitational instability

comoving
volume

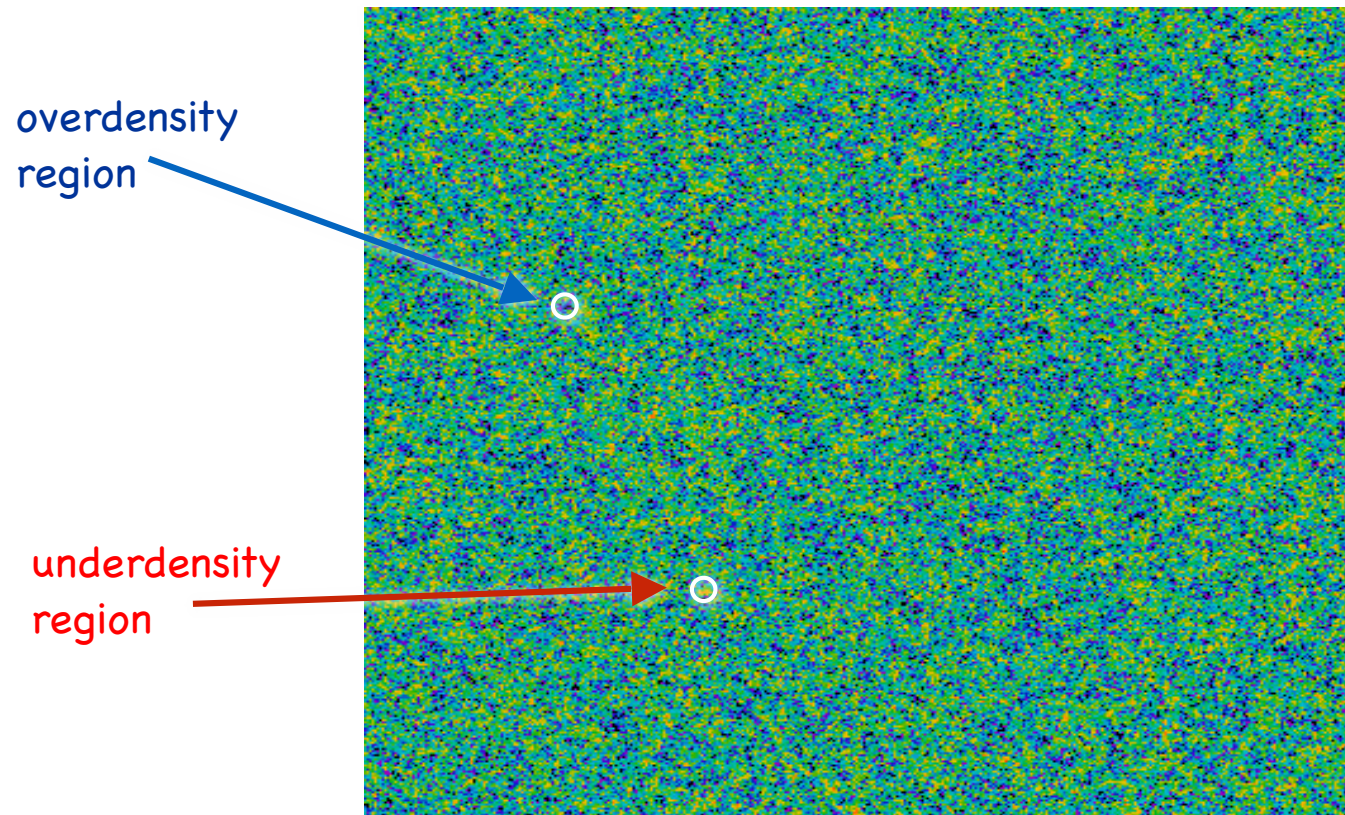


primordial density perturbations

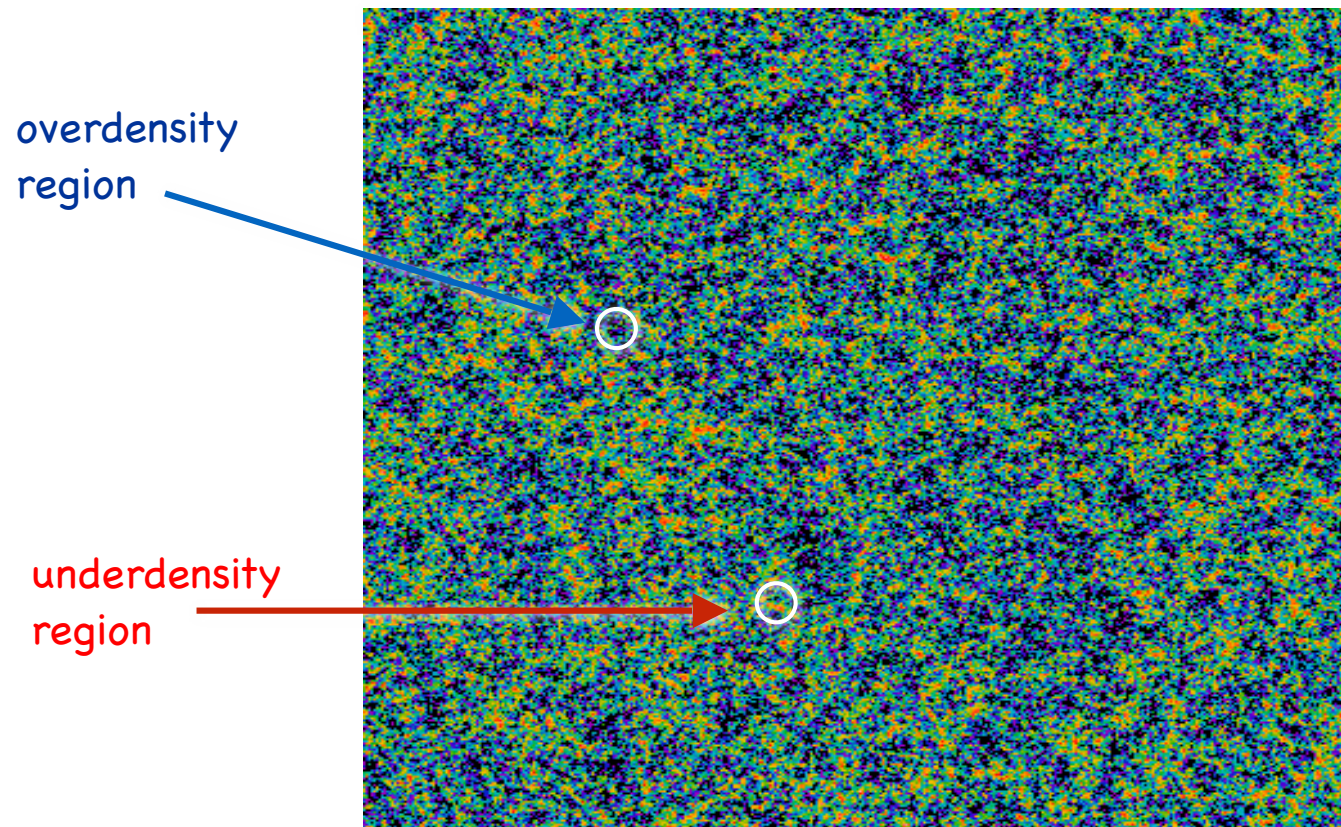
Primordial Universe (smooth
with tiny homogeneities)



Evolution of small inhomogeneities due to gravitational instability



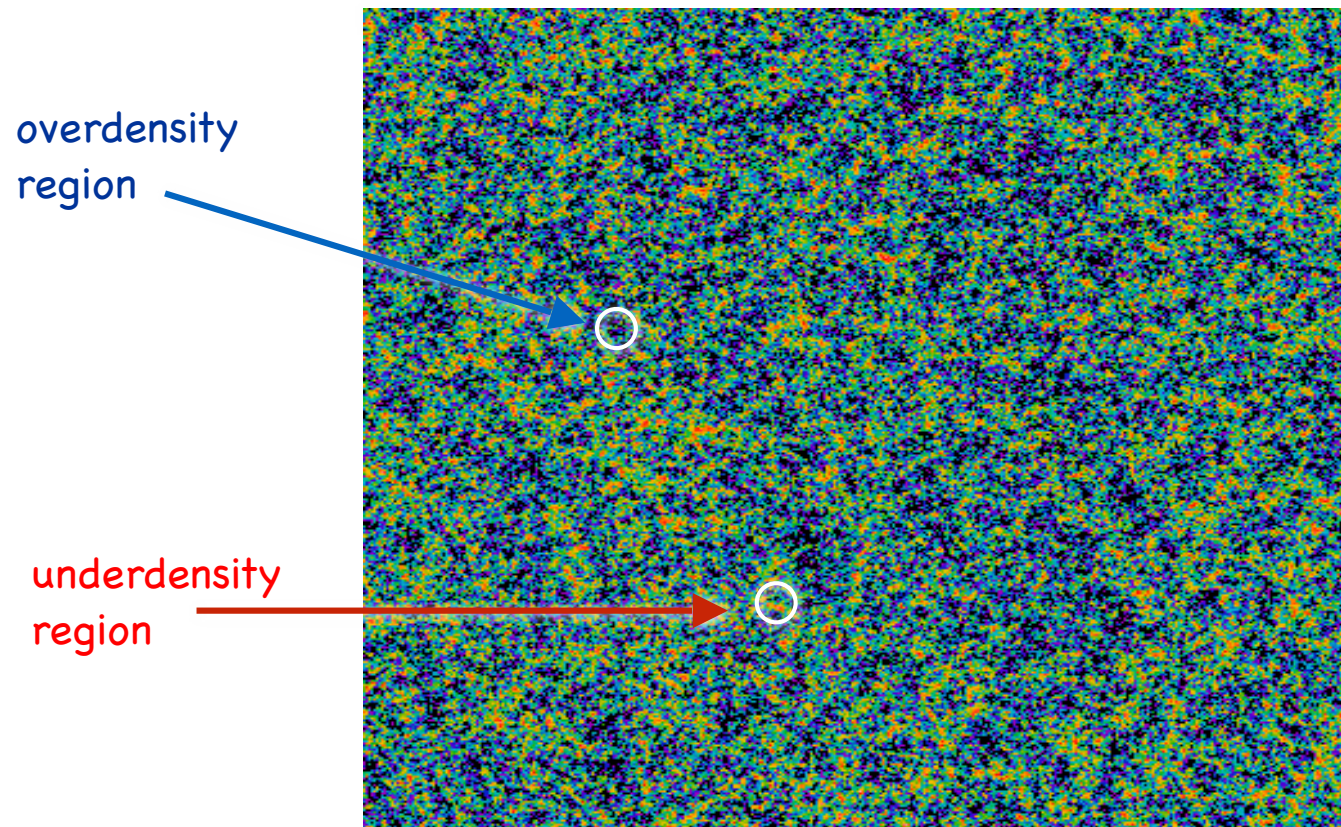
Evolution of small inhomogeneities due to gravitational instability



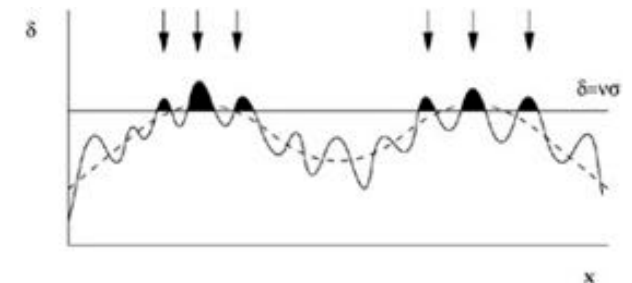
Note that this illustrative exm. only considers gravitational instability (i.e., it holds for DM, but...)

Conclusion: the nature of temp. fluct. is ...

Evolution of small inhomogeneities due to gravitational instability



$$\delta(x, t) \equiv \frac{\rho(x, t) - \bar{\rho}(t)}{\bar{\rho}(t)}$$

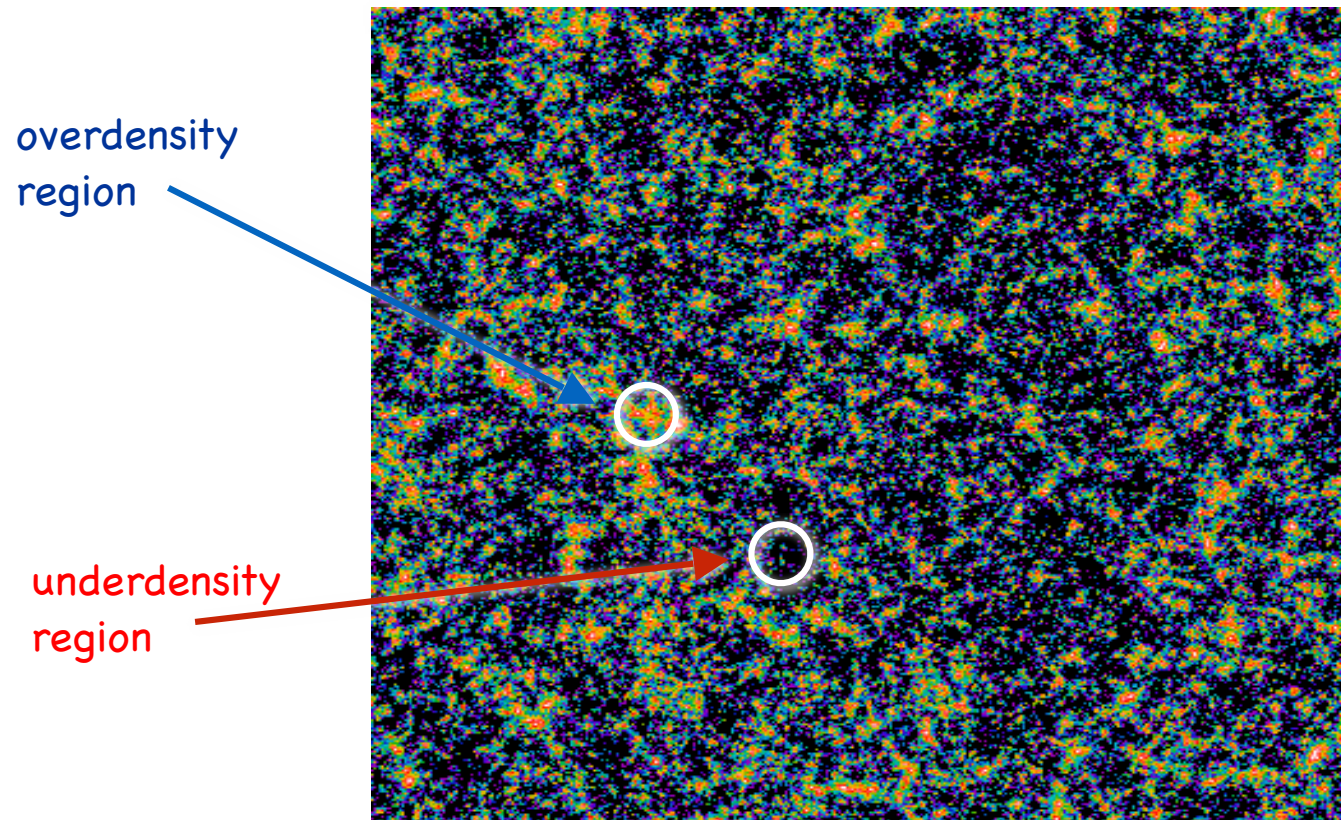


Note that this illustrative exm. only considers gravitational instability (i.e., it holds for DM, but...)

Conclusion: the nature of temp. fluct. is ... $\delta\rho_{\gamma_b}(t, \mathbf{r}) \sim \delta T(t, \mathbf{r})$

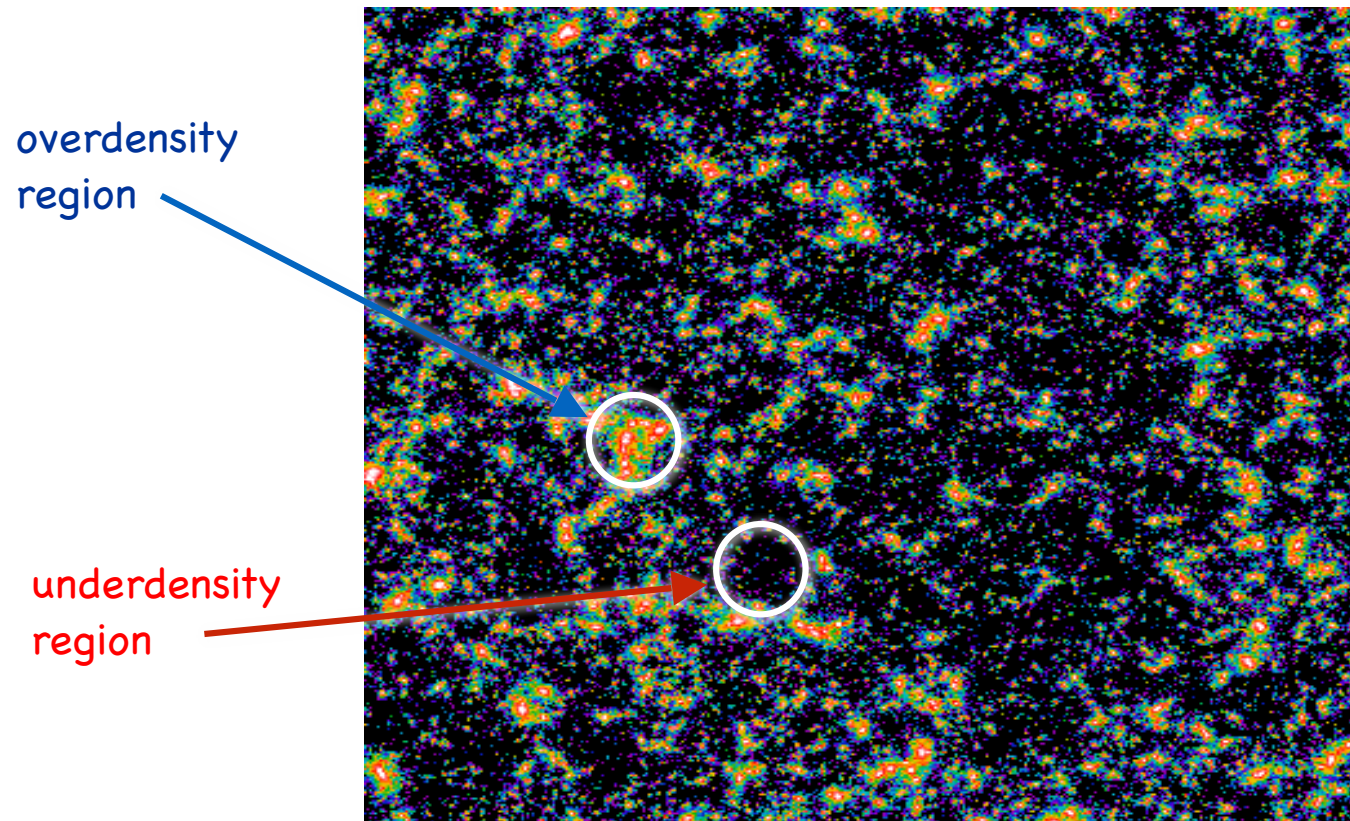
Galaxy assembling

Evolution of small inhomogeneities due to gravitational instability



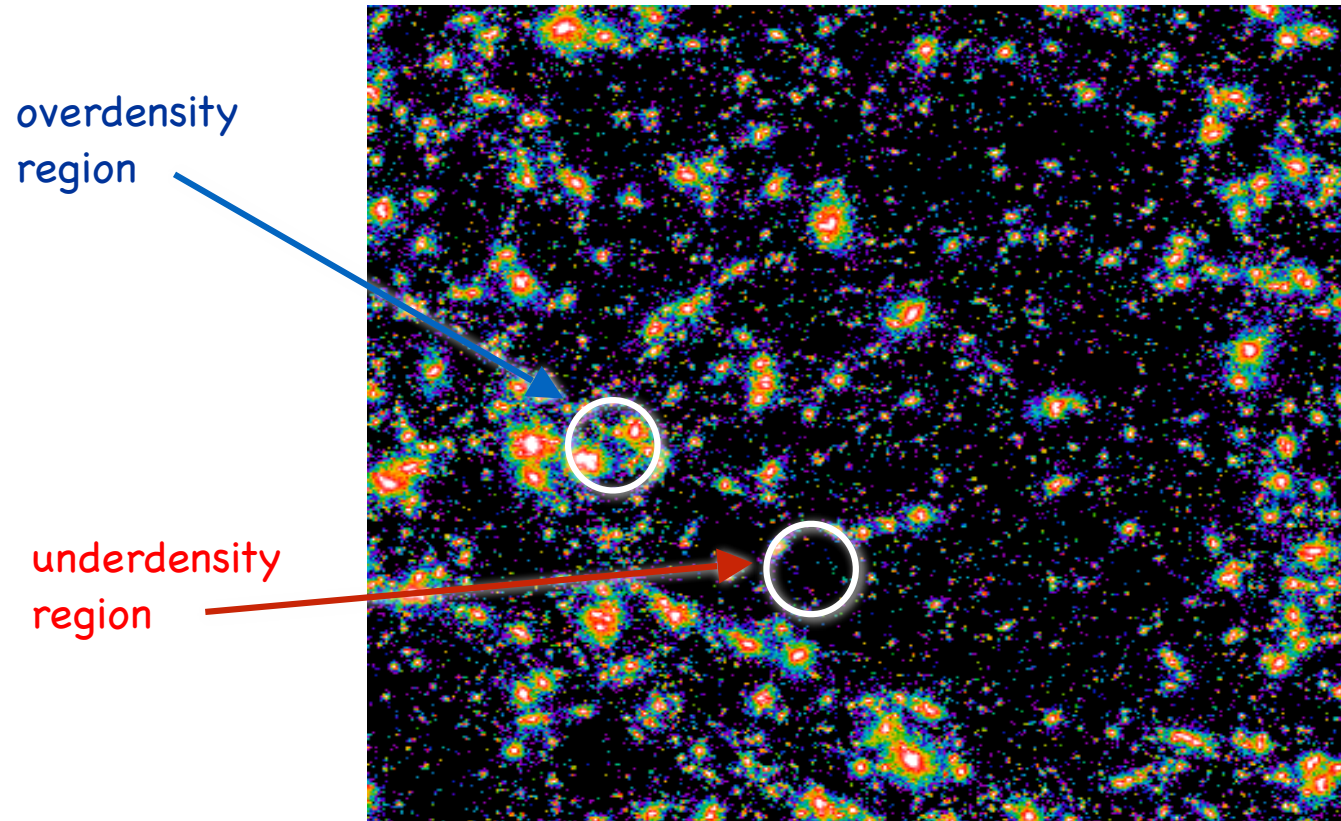
Galaxy assembling

Evolution of small inhomogeneities due to gravitational instability



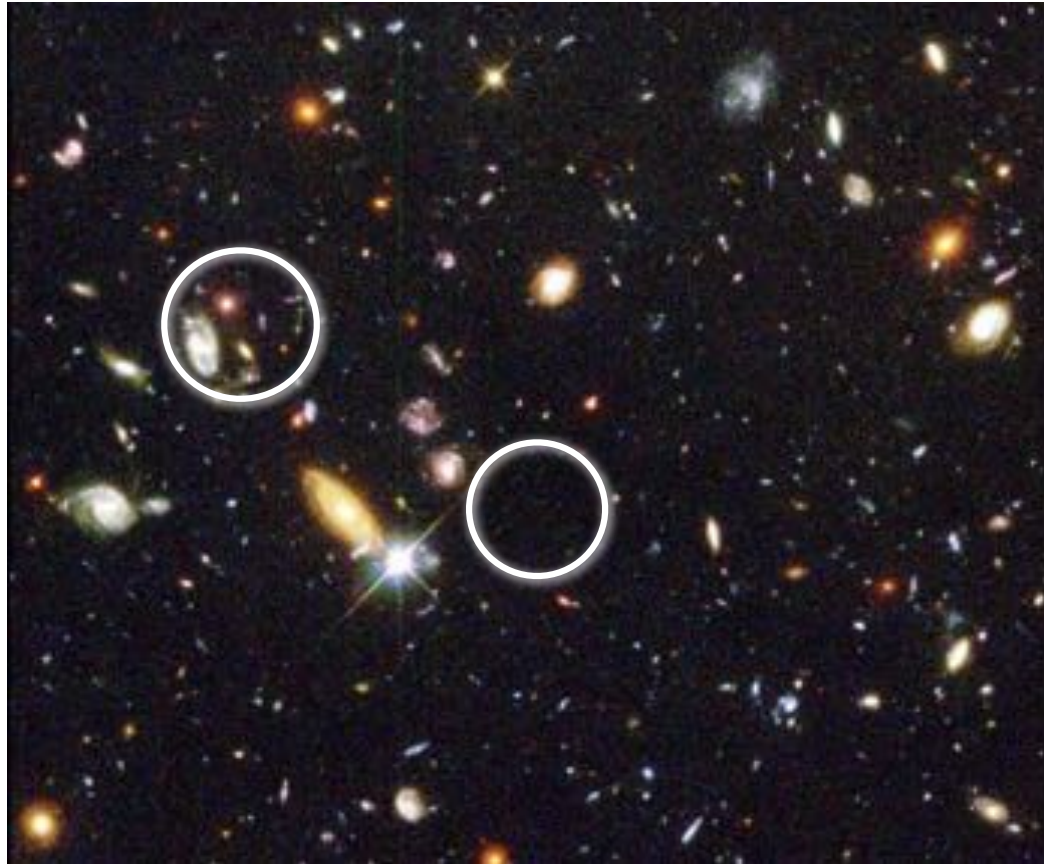
Galaxy assembling

Evolution of small inhomogeneities due to gravitational instability



Galaxy assembling

Evolution of small inhomogeneities due to gravitational instability



Perturbation theory

From the theory of cosmological perturbations, one obtains the hydrodynamical equations (the fluid evolution eqs.):

- Continuity, Euler, Poisson equations

Fluid evolution equations

Continuity, Euler, Poisson eqs.

$$\left\{ \begin{array}{l} \dot{\delta} + \frac{1}{a} \nabla \cdot \vec{v} = 0 \\ \frac{\partial \vec{v}}{\partial t} + \frac{\dot{a}}{a} \vec{v} = -\frac{1}{a} \nabla \phi - \frac{c_s^2}{a} \nabla \delta \\ \nabla^2 \Phi = 4\pi G \bar{\rho} a^2 \delta \end{array} \right. \quad \begin{array}{l} \dot{\delta} \equiv \frac{\partial \delta}{\partial t}; \quad \vec{v} \equiv \frac{d\vec{x}}{dt} \\ c_s^2 \equiv c^2 \frac{\partial p}{\partial \rho} \end{array}$$

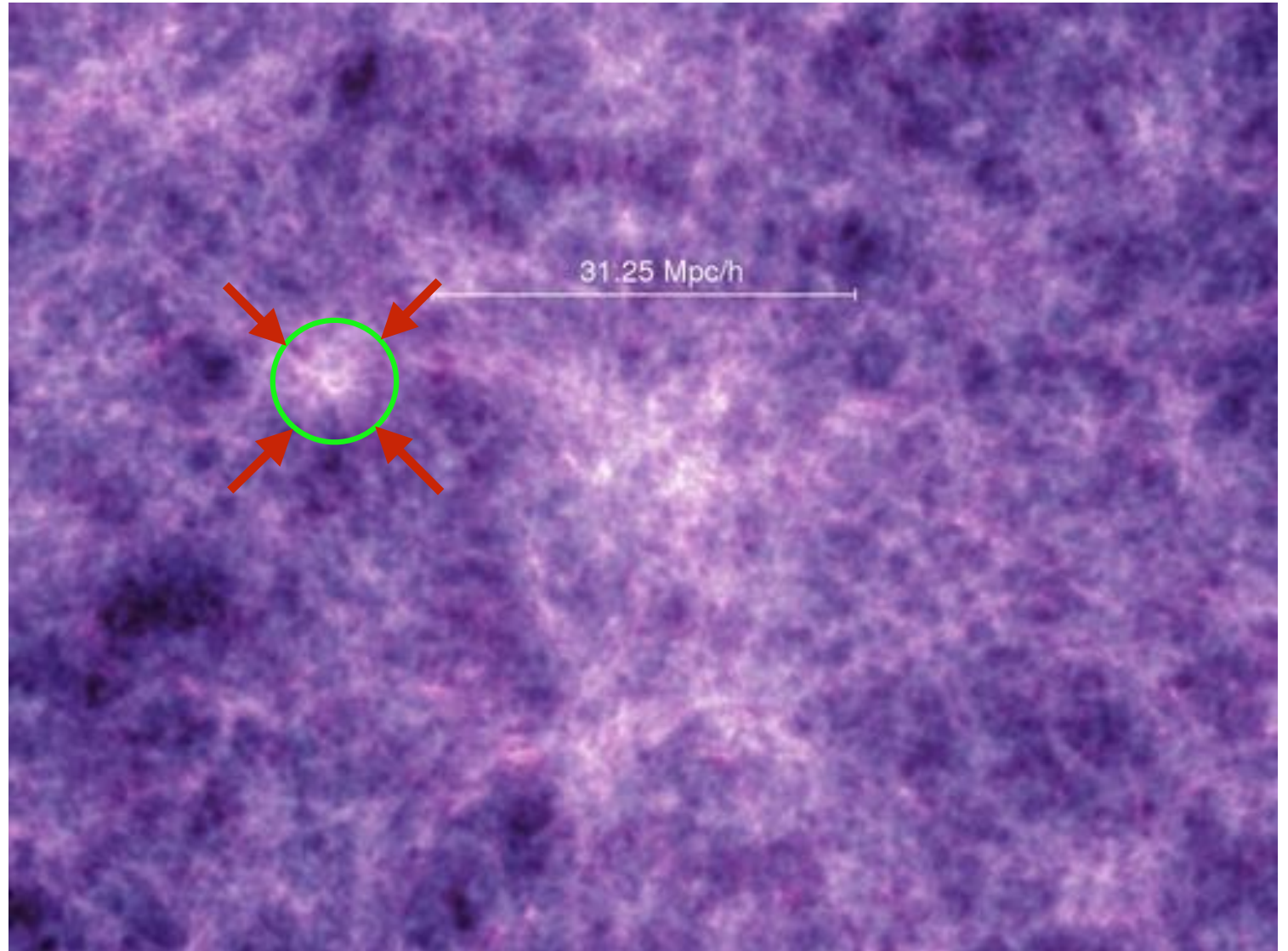
$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} - \frac{c_s^2}{a^2}\nabla^2\delta - 4\pi G\bar{\rho}\delta = 0$$

Hypothesis:

$$\delta(t, \vec{x}) = \delta_k(t) e^{i\vec{k}\cdot\vec{x}} \Rightarrow \ddot{\delta}_k + 2\frac{\dot{a}}{a}\dot{\delta}_k + \underbrace{\left(\frac{c_s^2 k^2}{a^2}\right)}_{\text{pressure repulsion}} \underbrace{- 4\pi G\bar{\rho}}_{\text{gravitational attraction}} \delta_k = 0$$

density fluctuations field: $\delta(t_0, \mathbf{r})$

$$-4\pi G \bar{\rho}(t) \delta(t, \mathbf{r})$$

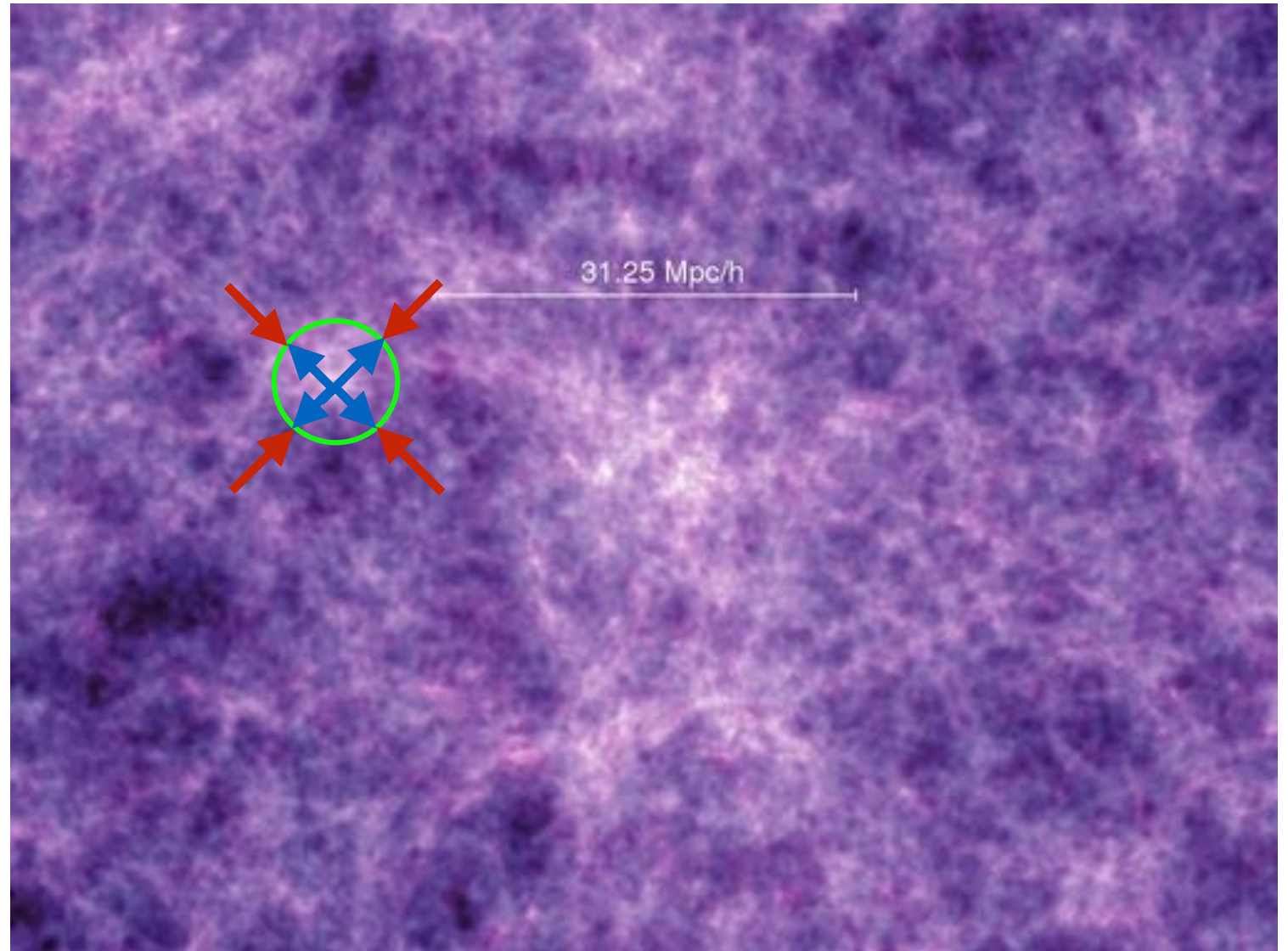


density fluctuations field: $\delta(t_0, \mathbf{r})$

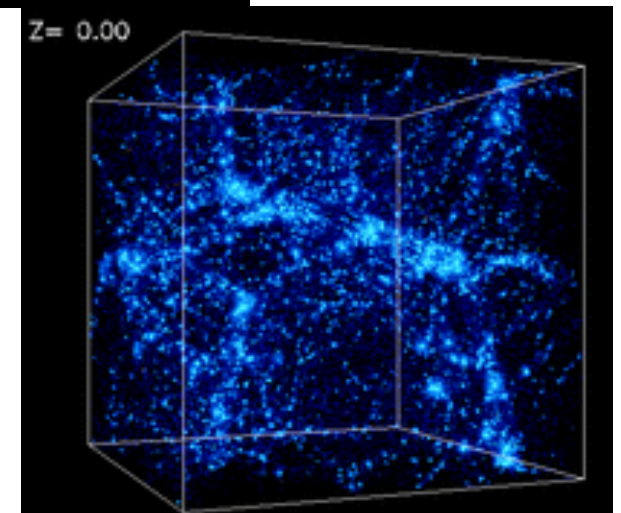
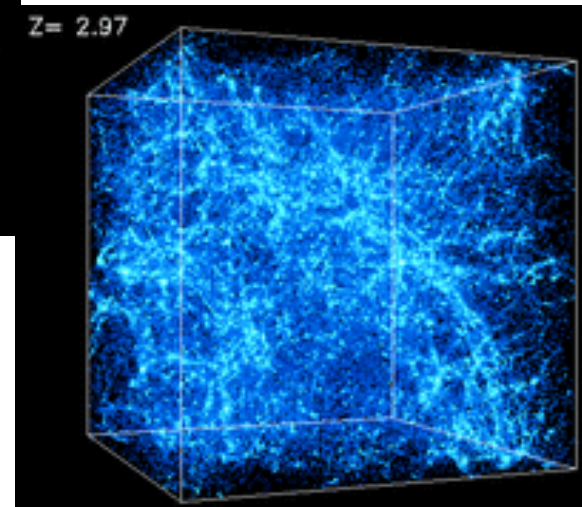
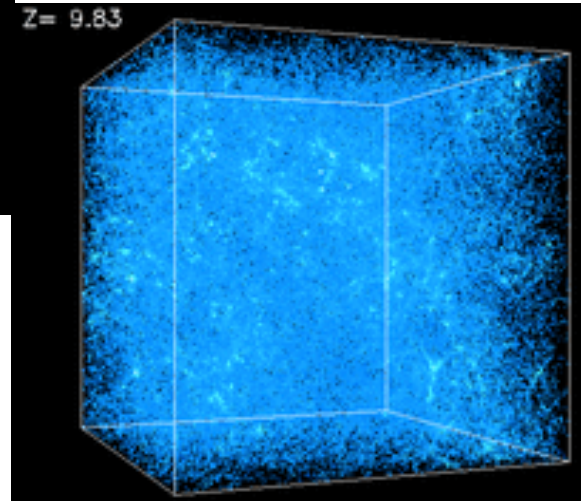
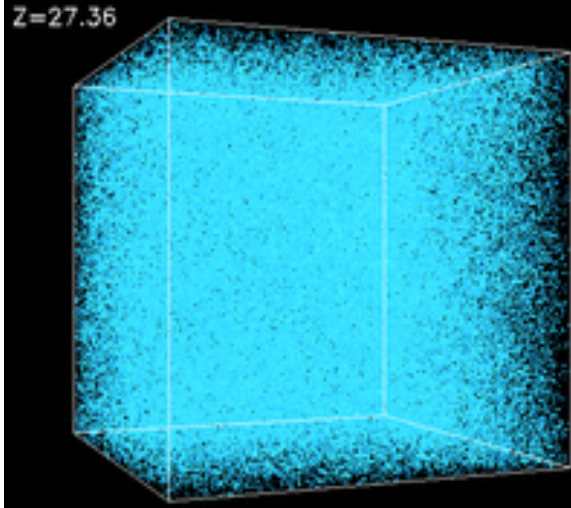
$$-4\pi G \bar{\rho}(t) \delta(t, \mathbf{r})$$

$$> = <$$

$$- \frac{c_s^2 \nabla^2 \delta(t, \mathbf{r})}{a(t)^2}$$



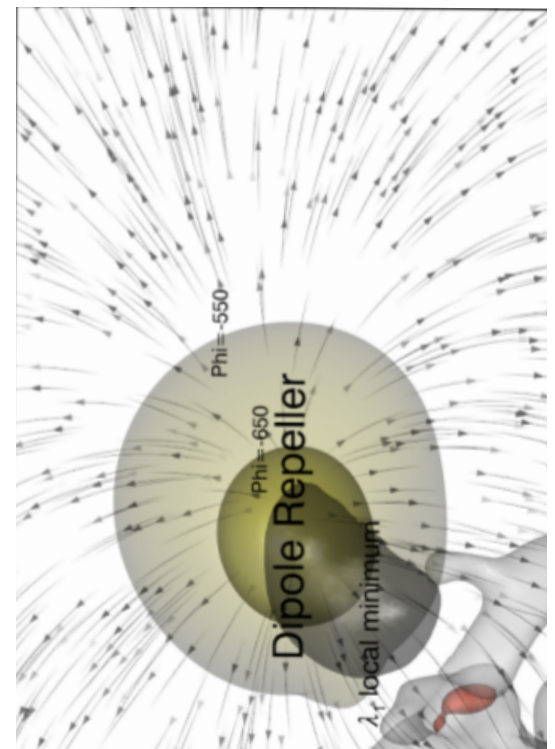
Structures formation: gravitational instability (first proposed by Sir James Jeans, 1902)



desafiando eqs-CEP

Continuity eq. $\dot{\delta} + \frac{1}{a} \nabla \cdot \vec{v} = 0$

Density contrast δ changes with cosmic time due to flows into and out a given 3D region



desfiando eqs-CEP

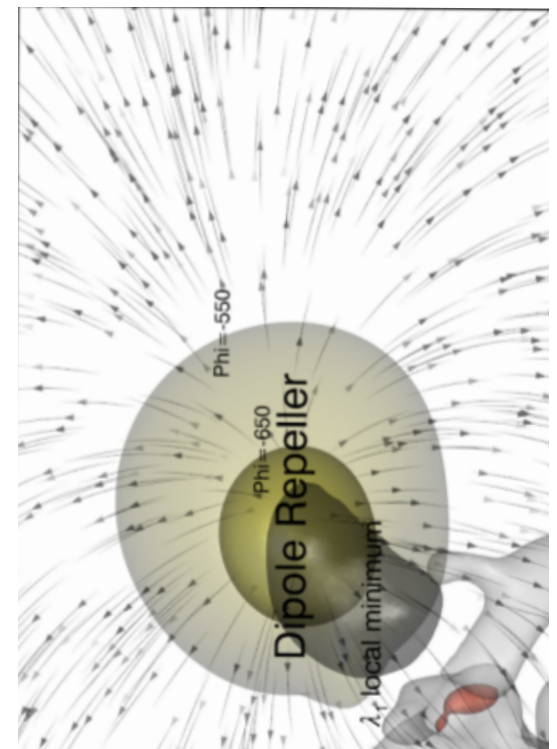
Continuity eq. $\dot{\delta} + \frac{1}{a} \nabla \cdot \vec{v} = 0$

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Euler eq. $\frac{\partial \vec{v}}{\partial t} + \frac{\dot{a}}{a} \vec{v} = -\frac{1}{a} \nabla \phi - \frac{c_s^2}{a} \nabla \delta$

Newton's second law for a fluid

generalized force + expansion effect force due to a grav. field force due to radiation pressure (photon-baryon fluid)



desfiando eqs-CEP

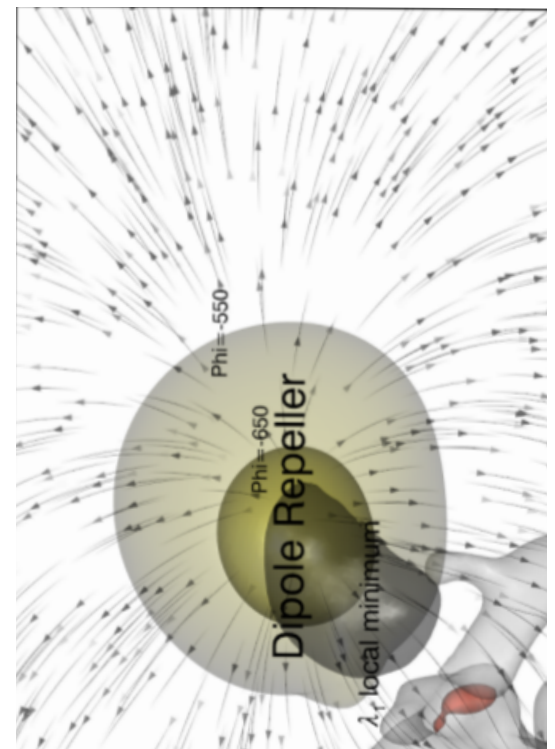
Continuity eq. $\dot{\delta} + \frac{1}{a} \nabla \cdot \vec{v} = 0$

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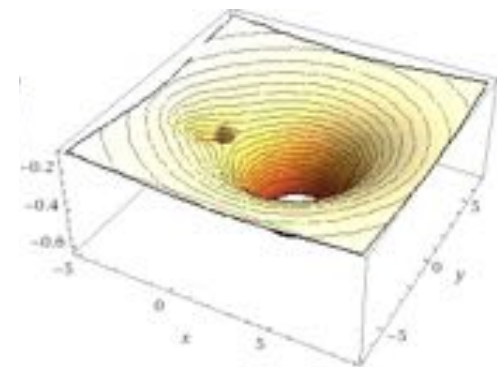


Poisson eq. $\nabla^2 \Phi = 4\pi G \bar{\rho} a^2 \delta$

↑
gravitational field

source of the gravitational field

?



Structures formation: evolution of the perturbations (gravitational instability)

desfiando a eq.
do contraste

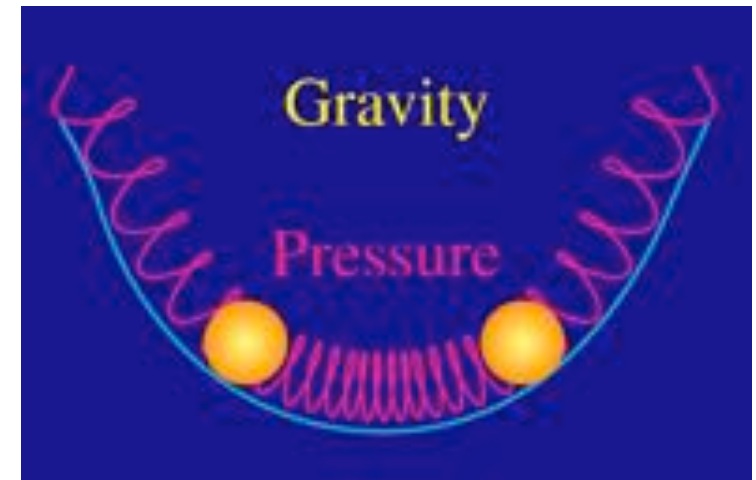
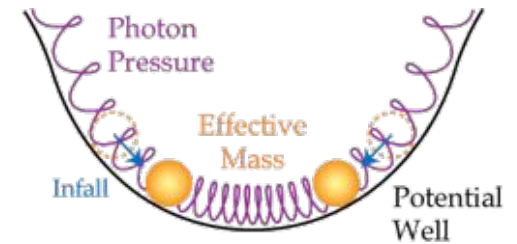
$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} - \frac{c_s^2}{a^2}\nabla^2\delta - 4\pi G\bar{\rho}\delta = 0$$

acceleration

?

?

action of two effects



Structures formation: evolution of the perturbations (gravitational instability)

desafiando a eq.
do contraste

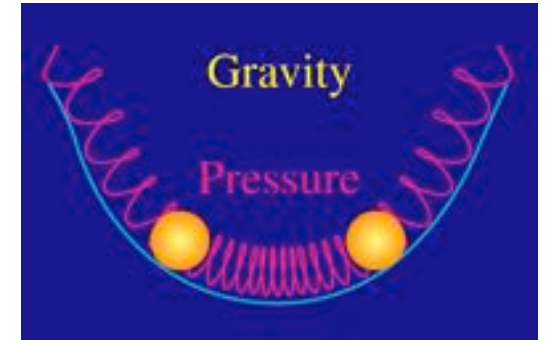
$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} - \frac{c_s^2}{a^2}\nabla^2\delta - 4\pi G\bar{\rho}\delta = 0$$

acceleration

coupling with expansion?

damping term

action of two effects



MD: After matter-radiation decoupling*, $z < 1100$, this eq. has two solutions: δ^+ , δ^- , the second one decreases!

$$\delta^+(t) = \delta_0 t^{2/3} \propto a(t)$$

this solution form gravitational potential wells that are the seeds to form structures (by gravitational instability)!

* In this case, the term $-\frac{c_s^2}{a^2}\nabla^2\delta$ is absent because

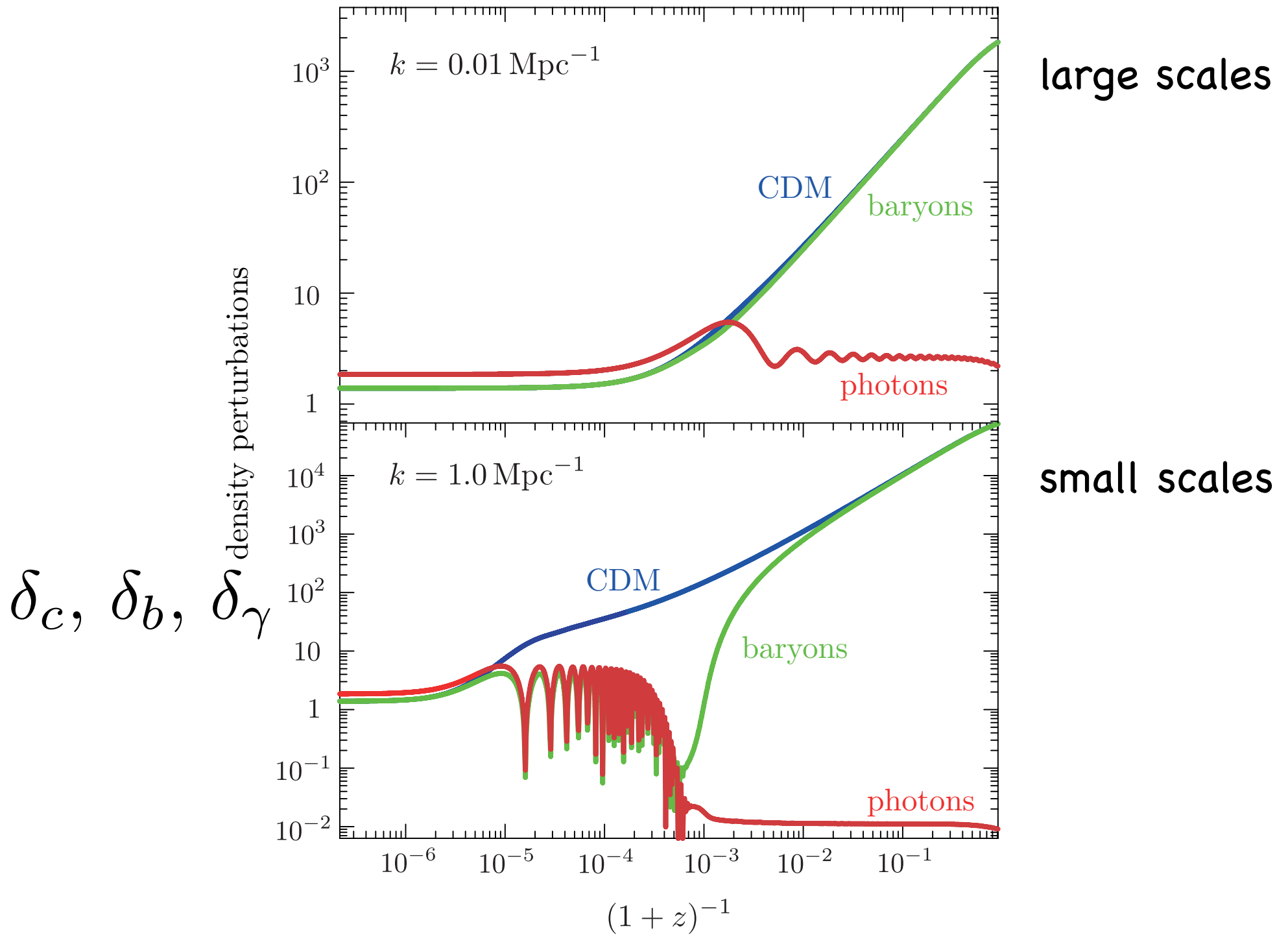


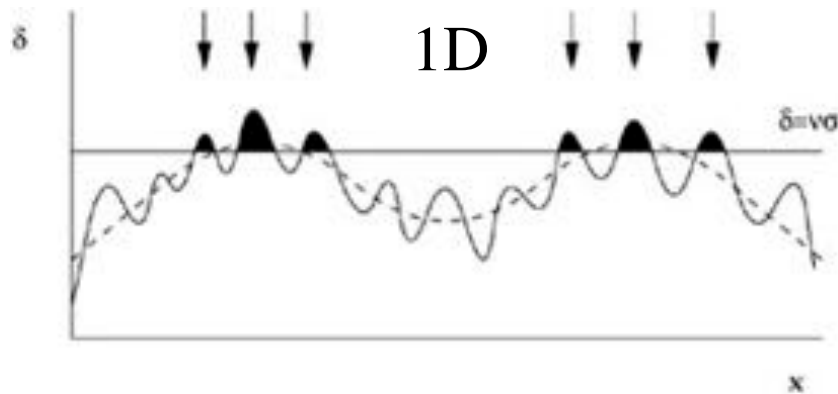
Figure 5.5: *Evolution of photons, baryons and dark matter.*

density contrast:

$$\delta_{LSS} = ? \quad \begin{array}{l} \nearrow \delta_{b/LSS} = ? \\ \searrow \delta_{cdm/LSS} = ? \end{array}$$

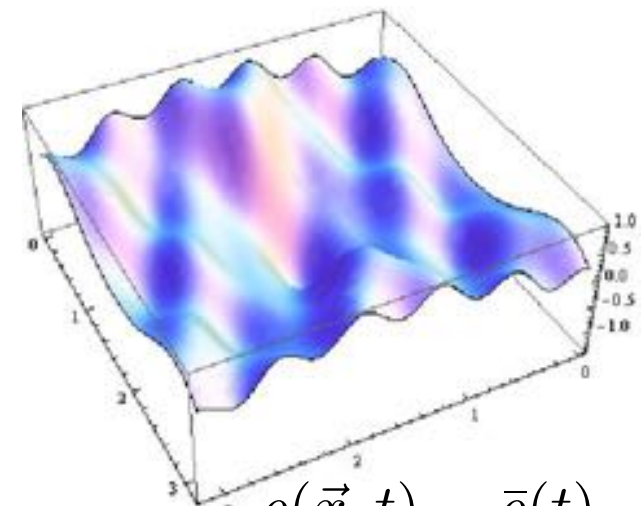
$$\delta_{cdm/LSS} \sim a(z) = \frac{1}{1+z}$$

$$\left. \begin{array}{l} \text{z=1100} \\ \delta_{b/LSS} \simeq 10^{-5} \\ + \\ \delta_{cdm/LSS} \simeq 10^{-2} - 10^{-3} \end{array} \right\} \text{z=0} \quad \delta_{b+cdm/today} \simeq 1$$



$$\delta(x, t) \equiv \frac{\rho(x, t) - \bar{\rho}(t)}{\bar{\rho}(t)}$$

3D



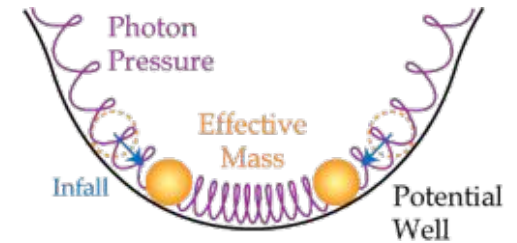
$$\delta(\vec{x}, t) \equiv \frac{\rho(\vec{x}, t) - \bar{\rho}(t)}{\bar{\rho}(t)}$$

Structures formation: evolution of the perturbations (gravitational instability)

desfiando a eq.
do contraste

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} - \frac{c_s^2}{a^2}\nabla^2\delta - 4\pi G\bar{\rho}\delta = 0$$

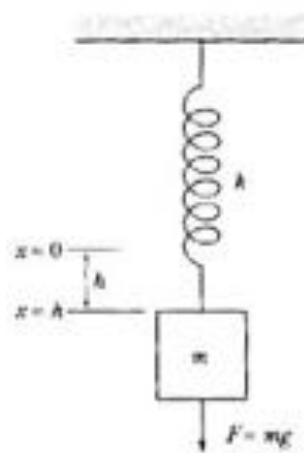
acceleration
↑
coupling
with expansion?
damping
term
action of
two effects



desafiando a eq.
do contraste

$$\frac{dv}{dt} + \textcolor{red}{k}v - g = 0$$

$$m \frac{d^2 x}{dt^2} + \textcolor{red}{2\Gamma} \frac{dx}{dt} + kx = 0$$



miscelânea:

2-point angular correlation function

How to measure matter clustering?

2-point correlation function

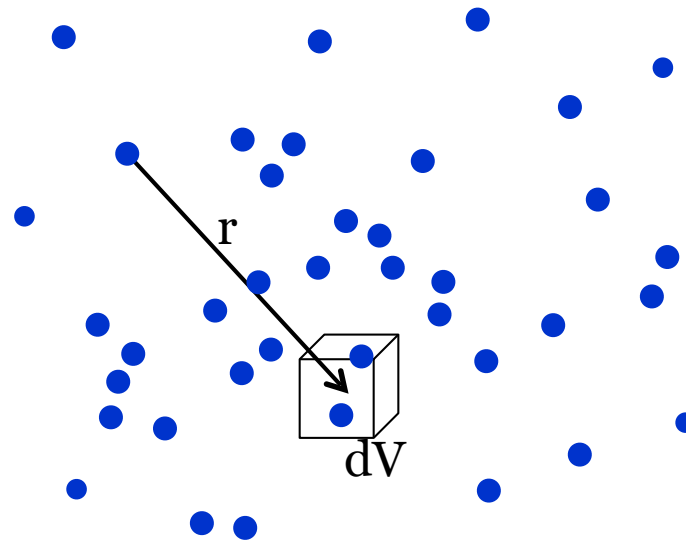
The 2-point statistics $(P(k), \xi(r))$ provide a complete statistical description of the density field under Gaussian initial conditions and linear growth of fluctuations

2-p correlation function (2PCF): $\xi(r)$

If you are sitting on a galaxy, the probability dP that you will find another galaxy in a volume dV a distance r away from you is given by

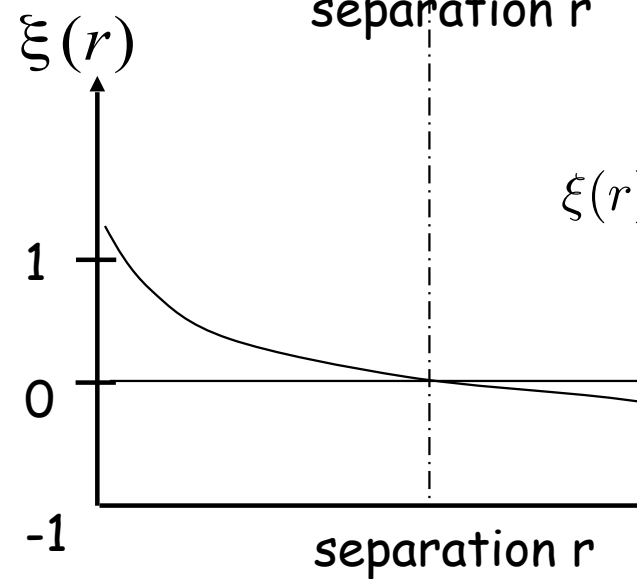
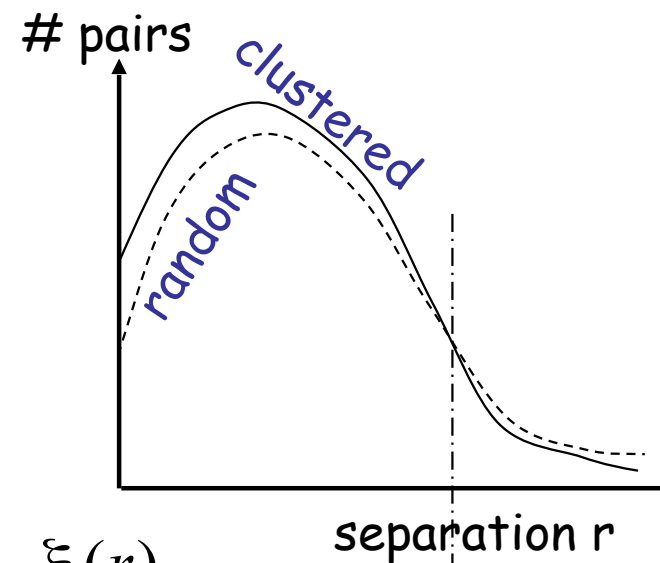
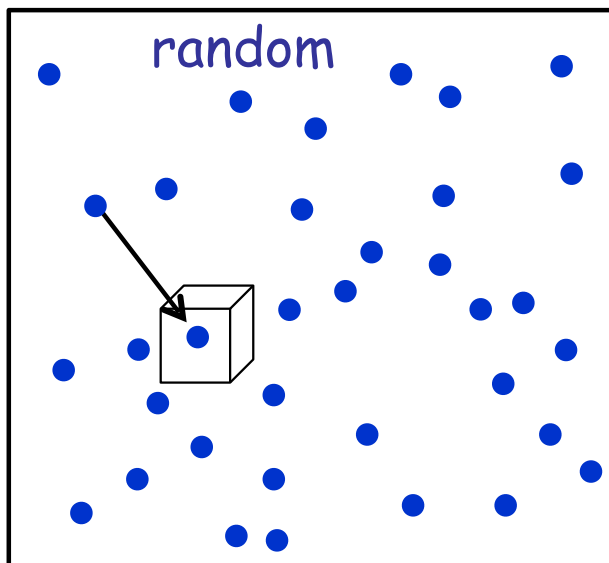
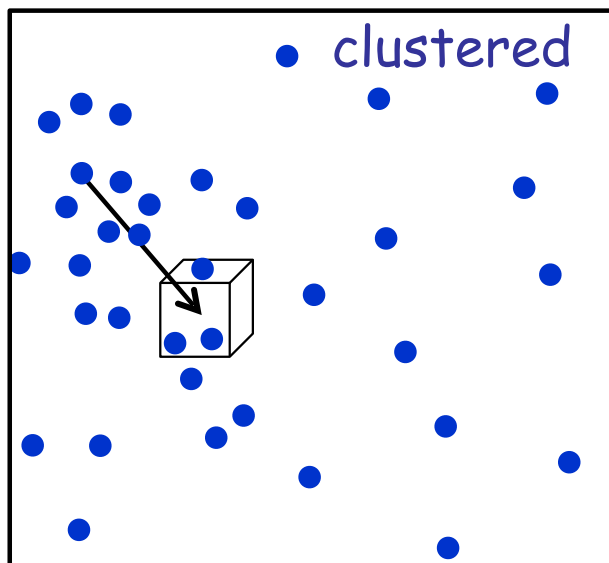
$$dP = n dV [1 + \xi(r)]$$

where n is the average number density of galaxies.



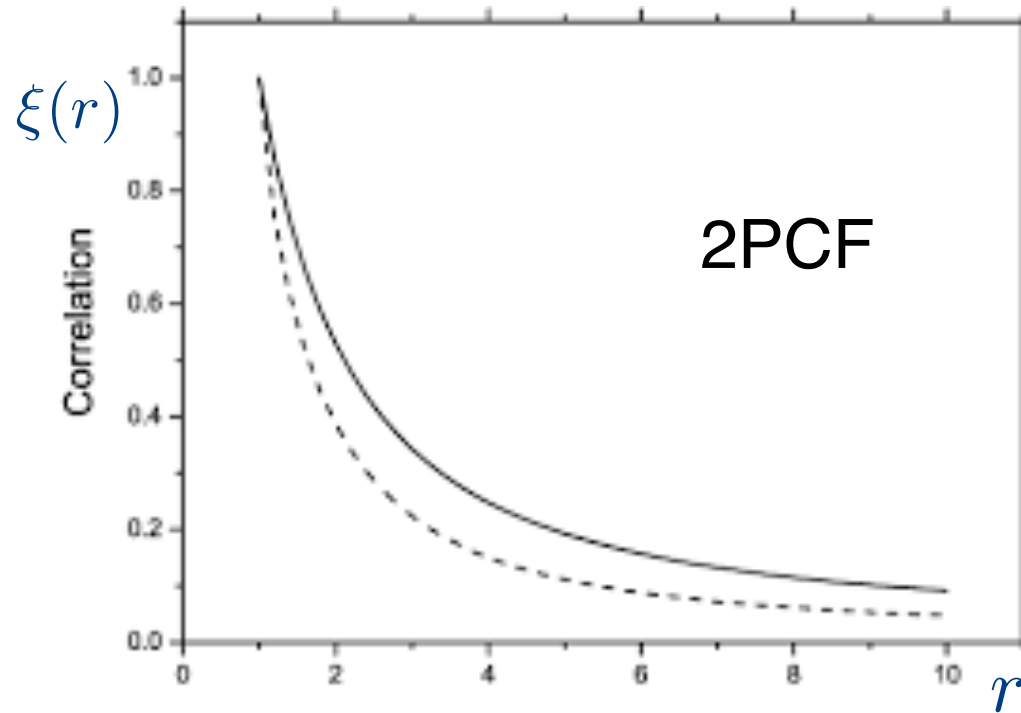
dP is the number of galaxies you expect to find in a volume dV .

What does the 2PCF measures?



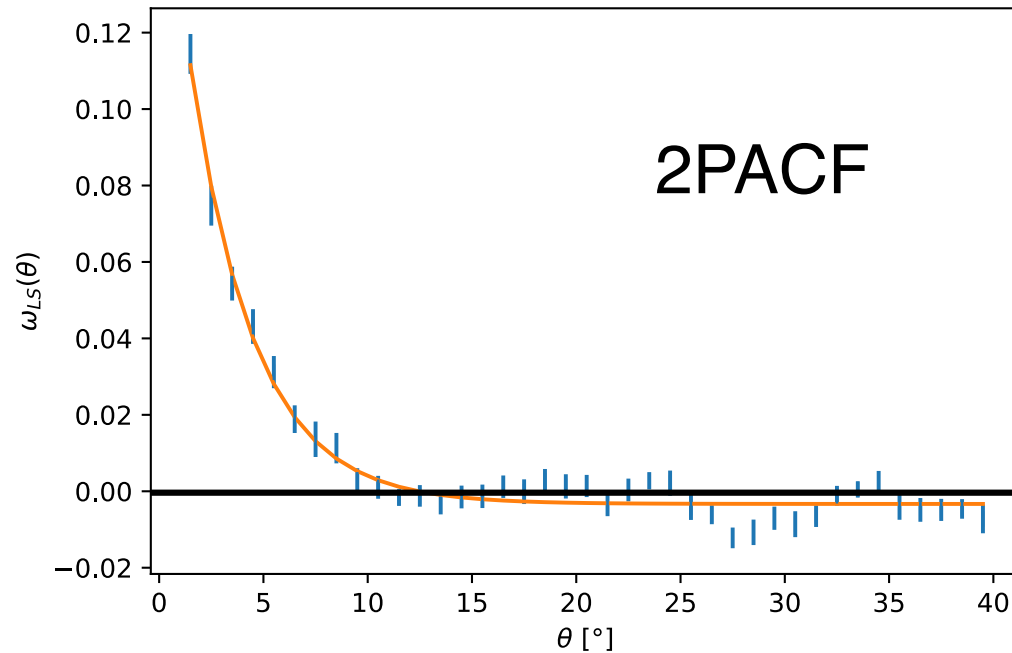
$$\xi(r) = \frac{DD(r)}{RR(r)} - 1$$

2-point correlation function



$$\xi(r) \propto r^{-\alpha}$$

$$\xi(r) = \left(\frac{r_0}{r} \right)^\alpha$$



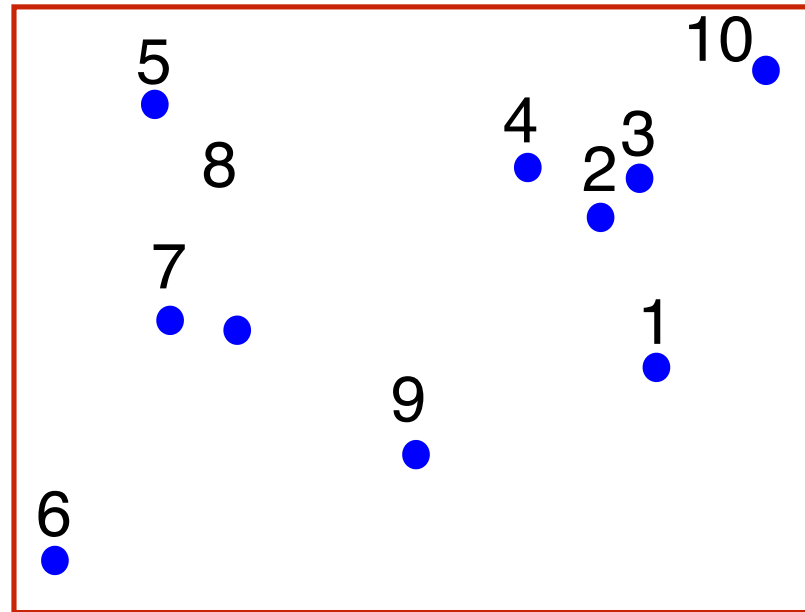
$$\omega(\theta) \propto \theta^{-\beta}$$

$$\omega(\theta) = \left(\frac{\theta_0}{\theta} \right)^\beta$$

How to count pairs in a data catalogue?

D

pairs distances
D_{12}
D_{13}
$\dots$
$D_{1,10}$
$D_{21}?$ X
D_{23}
$\dots$
$D_{2,10}$
D_{34}
$\dots$
$D_{9,10}$



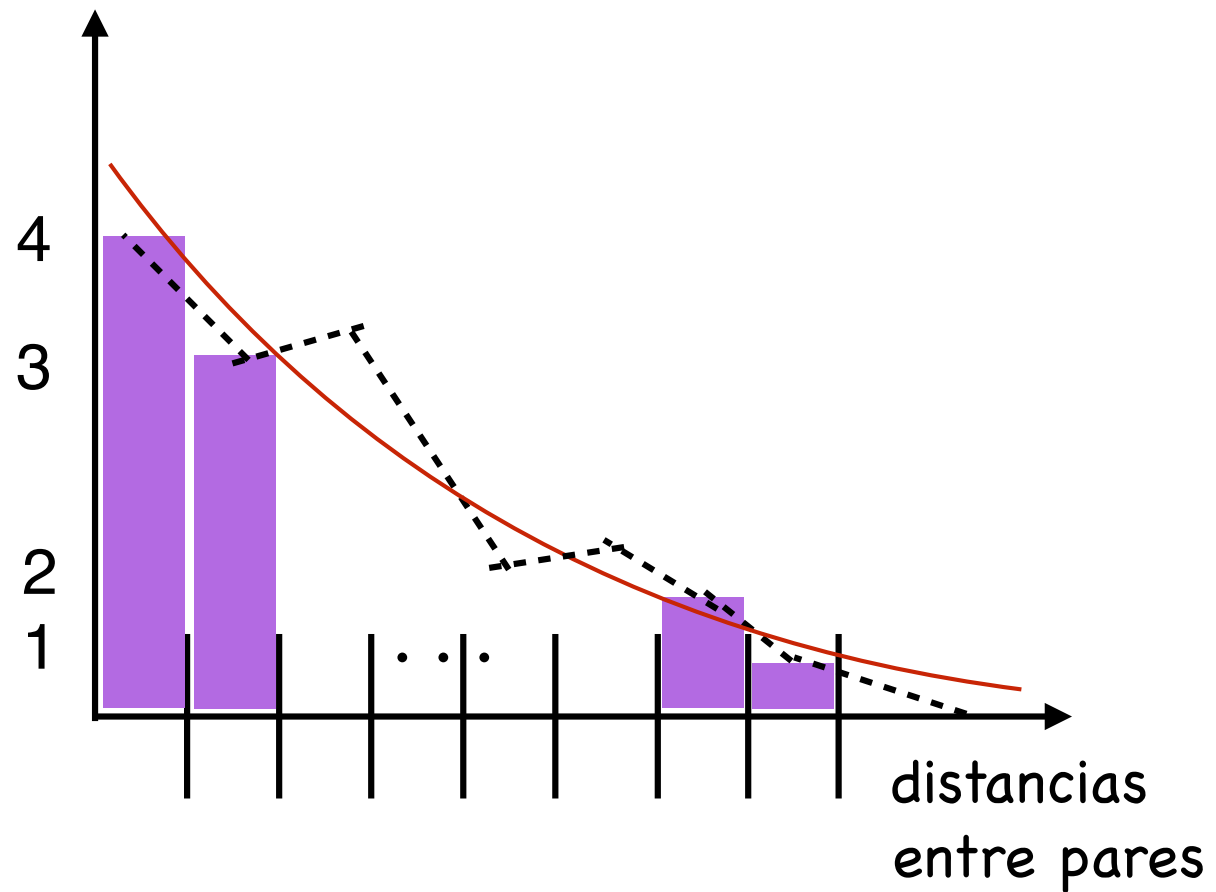
$$\{D_{23}, D_{24}, D_{34}, D_{58}\} \in \text{bin}_1$$

$$\{D_{12}, D_{39}, D_{78}\} \in \text{bin}_2$$

$$\{D_{26}, D_{36}\} \in \text{bin}_{n-1} \cdots \{D_{6,10}\} \in \text{bin}_n$$

How many data pairs did we get?

pairs distances
D_{12}
D_{13}
...
$D_{1,10}$
D_{21} ? X
D_{23}
...
$D_{2,10}$
D_{34}
...
$D_{9,10}$



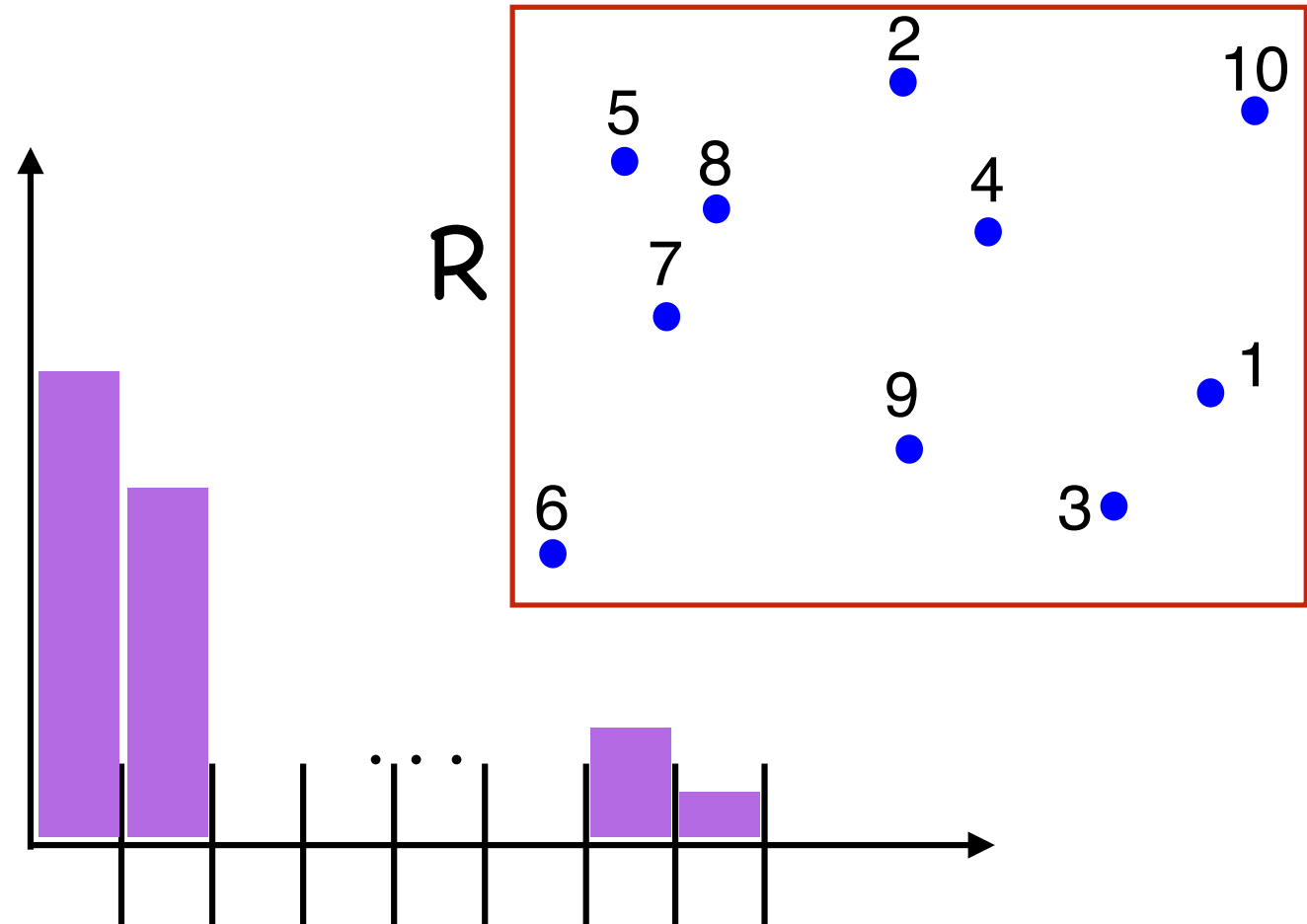
**Histograma de contagem de
distancias entre pares $\mathbf{D}$**

$$= DD(r)$$

$$\neq \xi(r)$$

Now, we generate a random catalogue
in the same region of the data sample

pairs distances
R_{12}
R_{13}
...
$R_{1,10}$
$R_{21}?$ X
R_{23}
...
$R_{2,10}$
R_{34}
...
$R_{9,10}$



**Histograma de contagem de
distancias entre pares R** $= RR(r)$

2-point correlation function

Landy-Szalay

$$\xi(r) = \frac{DD(r) - 2DR(r) + RR(r)}{RR(r)}$$

Peebles-Hauser

$$\xi(r) = \frac{DD(r)}{RR(r)} - 1$$

$$\xi(r) = \frac{1}{N} \sum_{i=1}^N \frac{DD(r) - RR_i(r)}{RR_i(r)}$$

Homework

2-point angular correlation function

Landy-Szalay

$$\omega(\gamma) = \frac{DD(\gamma) - 2DR(\gamma) + RR(\gamma)}{RR(\gamma)}$$

Peebles-Hauser

$$\omega(\gamma) = \frac{DD(\gamma) - RR(\gamma)}{RR(\gamma)}$$

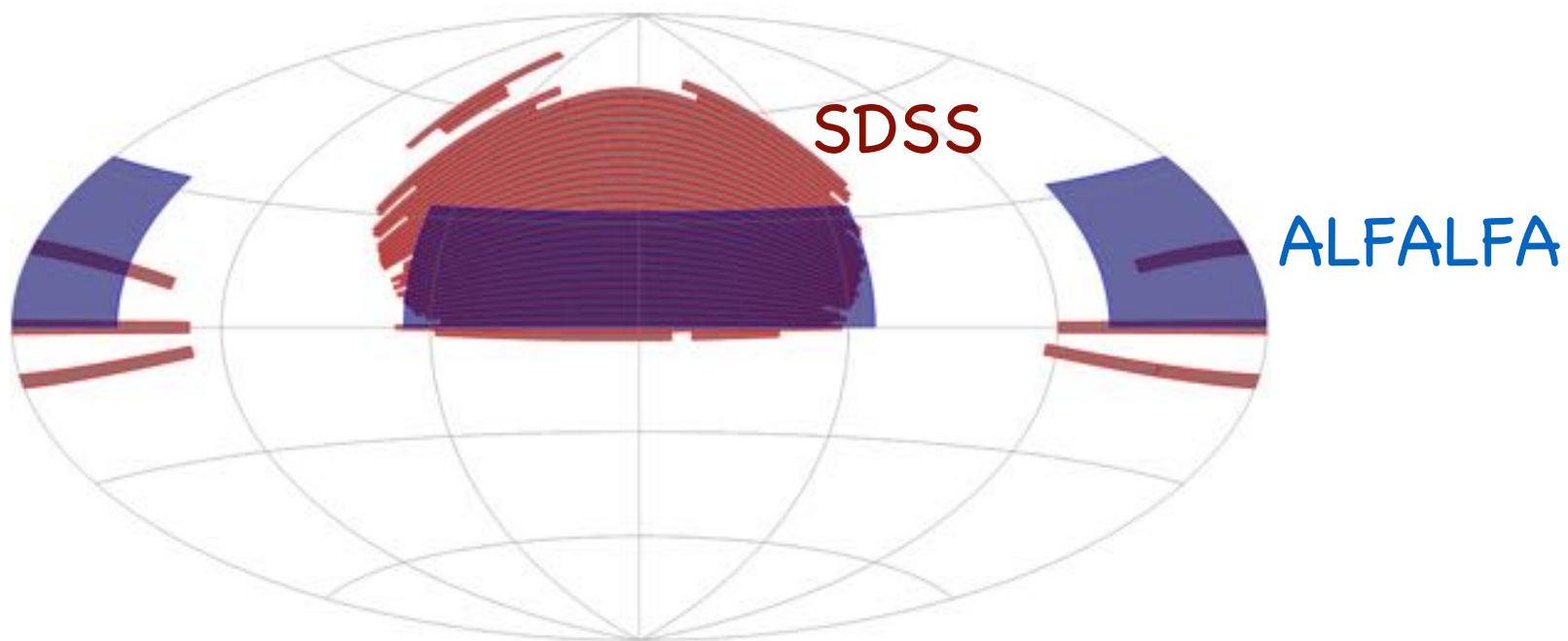
$$\omega(\gamma) = \frac{1}{N} \sum_{i=1}^N \frac{DD(\gamma) - RR_i(\gamma)}{RR_i(\gamma)}$$

Compute $\omega(\gamma)$ for a sample of HI sources from ALFALFA survey

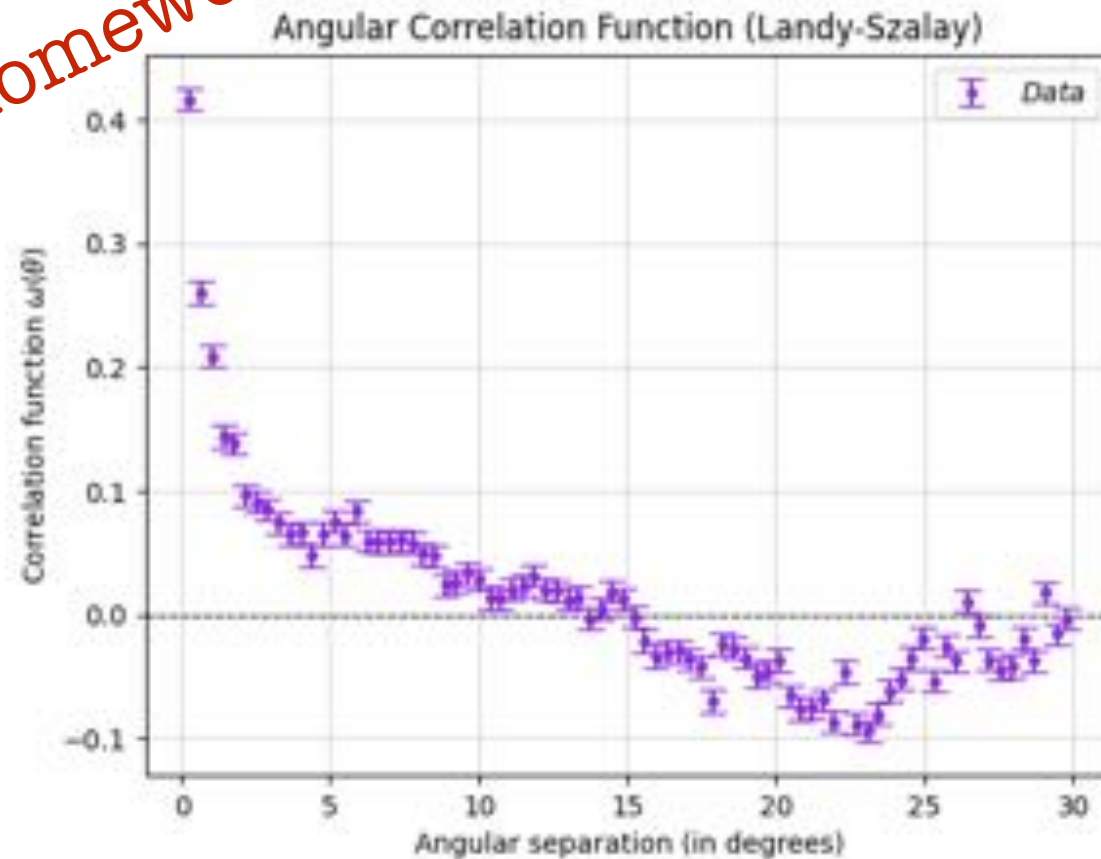
- 2D with data on a sphere with: $\theta \in [\theta_1, \theta_2]$, $\phi \in [\phi_1, \phi_2]$

$$\gamma = \cos^{-1}[\cos(\theta_1) \cos(\theta_2) + \sin(\theta_1) \sin(\theta_2) \cos(\phi_1 - \phi_2)]$$

Survey ALFALFA



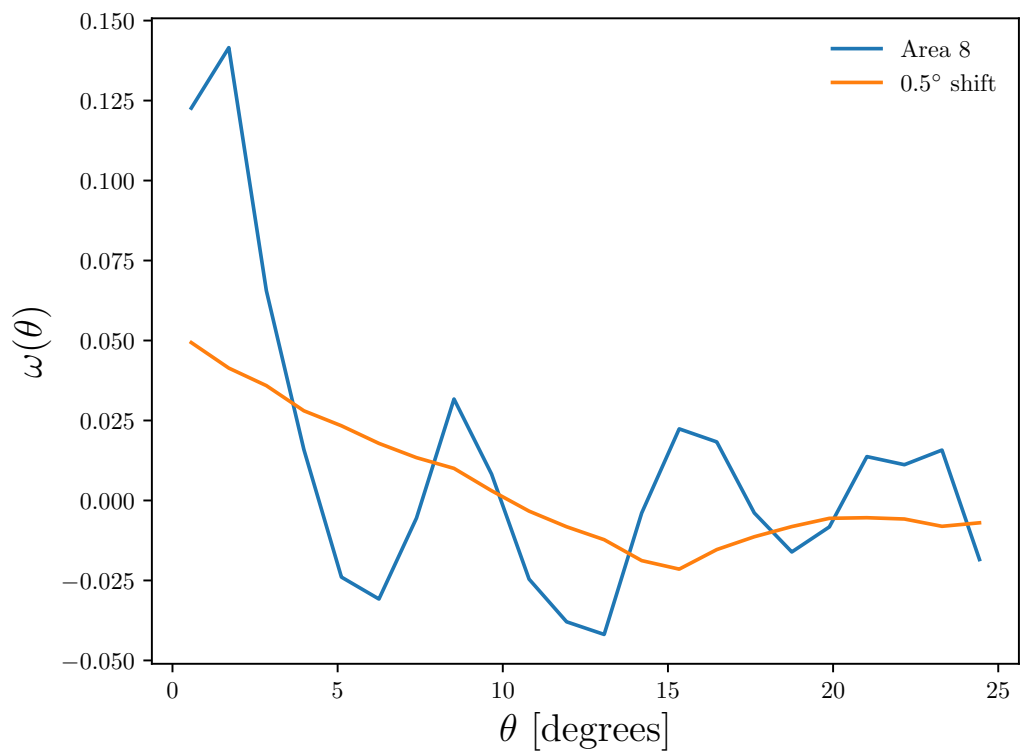
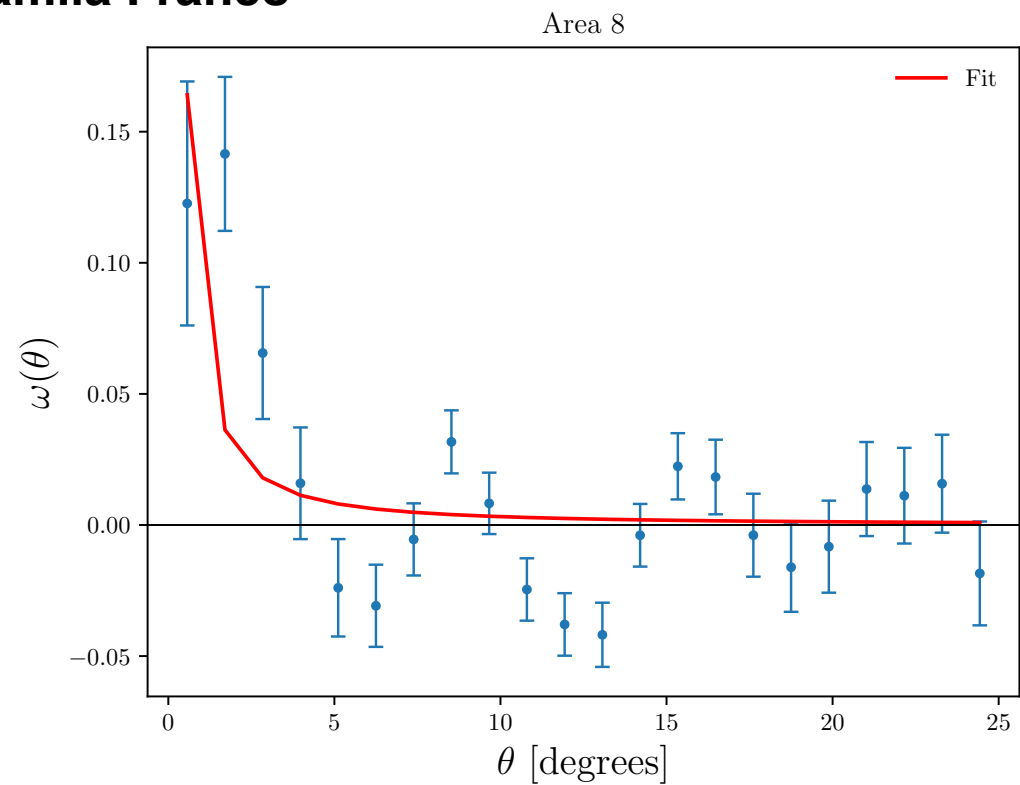
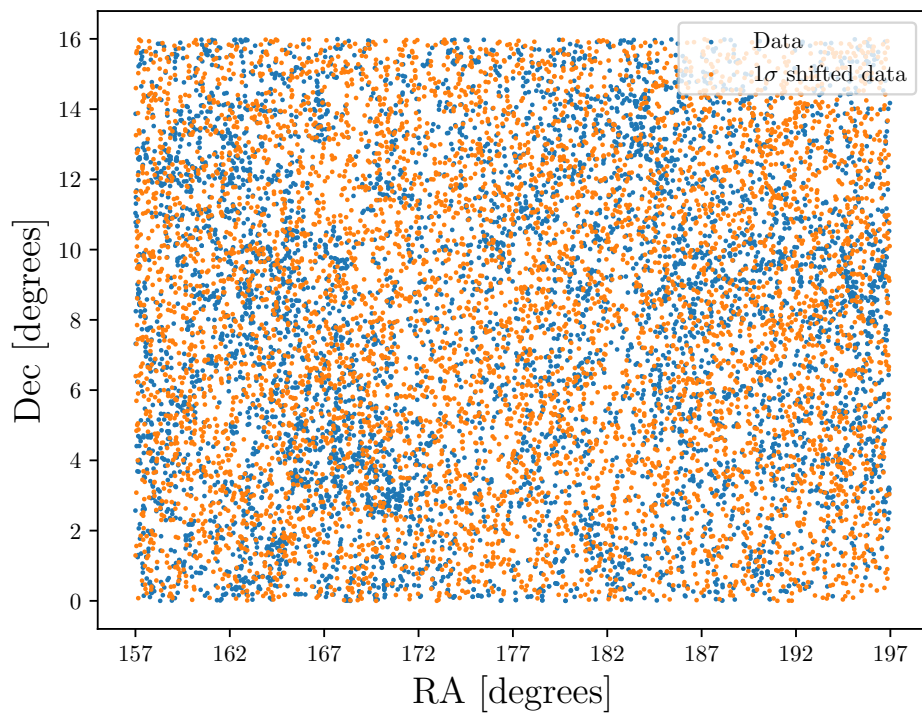
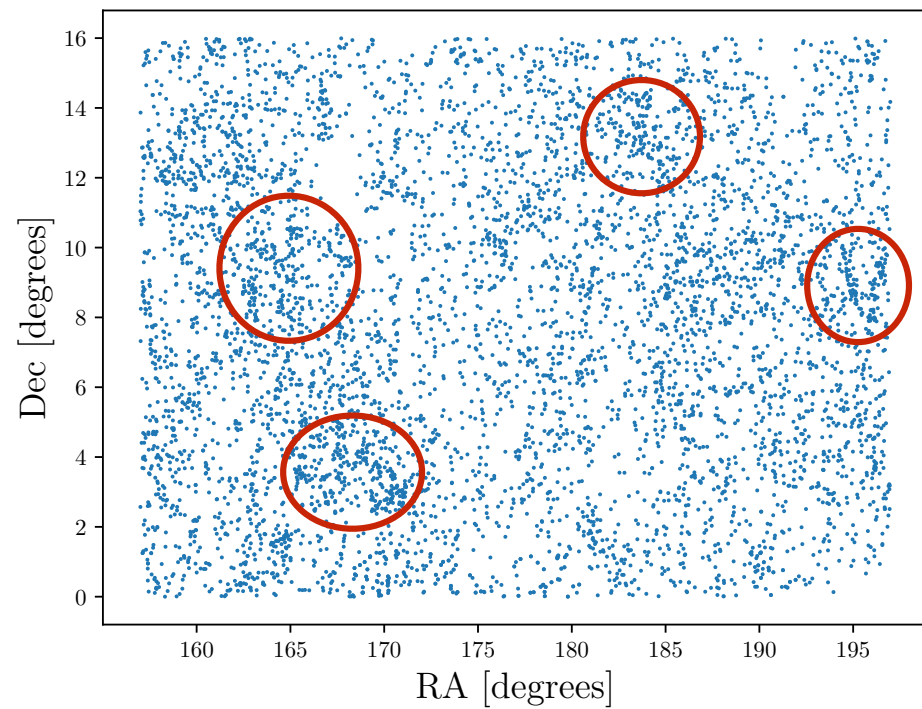
Homework



GitHub: 2PACF

https://github.com/mavipoppi/wsON_correlationfunction

Gentileza de: **Maria Vitoria Lazarin**





Camila Franco

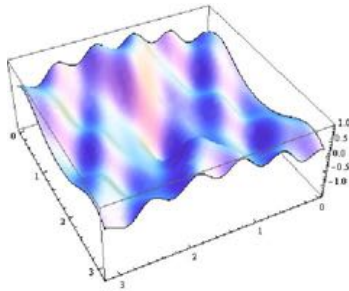
Maria Vitoria Lazarin

O que aprendemos hoje?

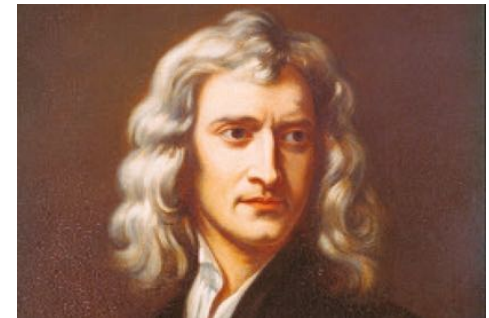
(60 minutos em 7+1 slides)

0

From Newtonian theory of gravitation we learned:
(i) the universality of conservation laws;
(ii) to develop perturbative calculus of Newton eqs.
(iii) the (concept of) gravitational potential is more useful than grav. force, ...

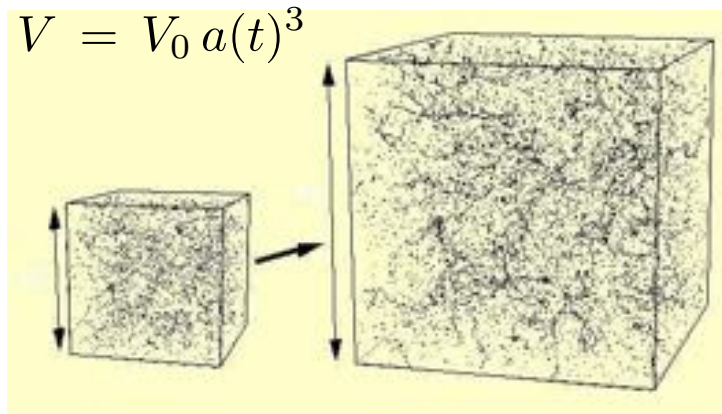
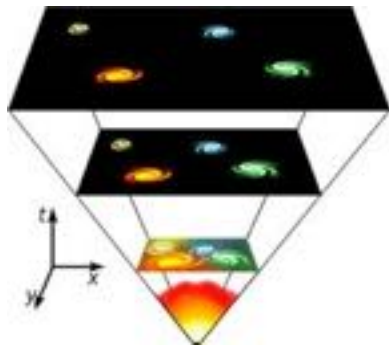


the concept of field $\phi(\vec{r})$ is born!



1

The expanding Universe cools:



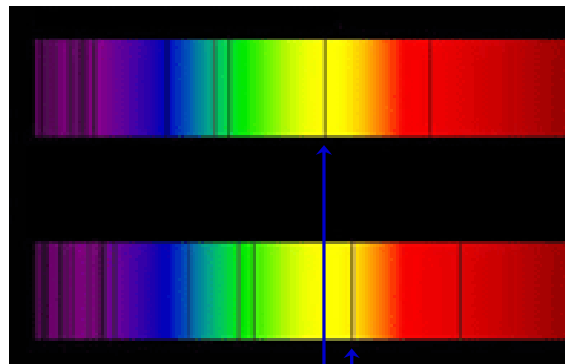
$a = a(t)$ is the scale factor

$$T(t) = \frac{a_0 T_0}{a(t)}$$

O desvio para o vermelho

Espectro “de referência”

Espectro observado



$$z = \frac{\lambda_r - \lambda_e}{\lambda_e}$$

Cosmological redshift:

$$a(z) = \frac{a_0}{1 + z}$$

2

Assuming that the Universe is filled with a perfect fluid, one solves the Einstein equations and find the Friedmann eq.

$$\frac{\dot{a}(t)^2}{a(t)^2} = \frac{8\pi G}{3} \rho(t) - \frac{k c^2}{R_0^2 a(t)^2}$$

defining: $\dot{a} \equiv \frac{da}{dt}$ and $H(t) \equiv \frac{\dot{a}(t)}{a(t)}$ $a = a(t)$ is the scale factor

$$H(t)^2 = \frac{8\pi G}{3} \rho(t) - \frac{k c^2}{R_0^2 a(t)^2}$$

Friedmann equation

matter-energy
conservation equation

$$\dot{\rho} = -3 \frac{\dot{a}}{a} \left(\rho + \frac{p}{c^2} \right)$$



3

 ρ

Universe content:

- photons
- matter (barions + CDM)
- lambda (cosmolog. const.)

$$\rho_r \sim a^{-4}$$

$$\rho_m \sim a^{-3}$$

$$\rho_\Lambda \sim \text{const}$$

$$a(t_{m\Lambda})$$

0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5

$$a \sim t^{1/2}$$

$$a(t_{\text{eq}})$$

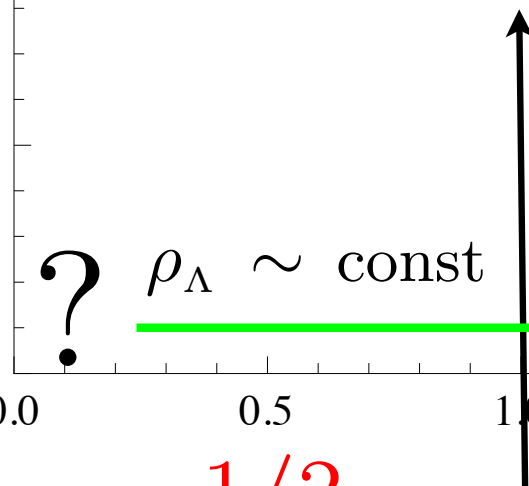
$$a \sim t^{2/3}$$

 a

RD universe

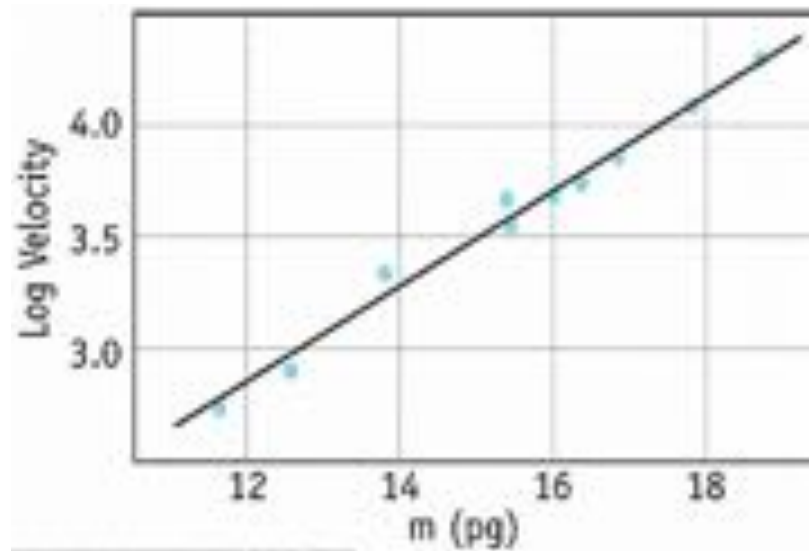
$$a(t_{\text{rm}})$$

MD universe

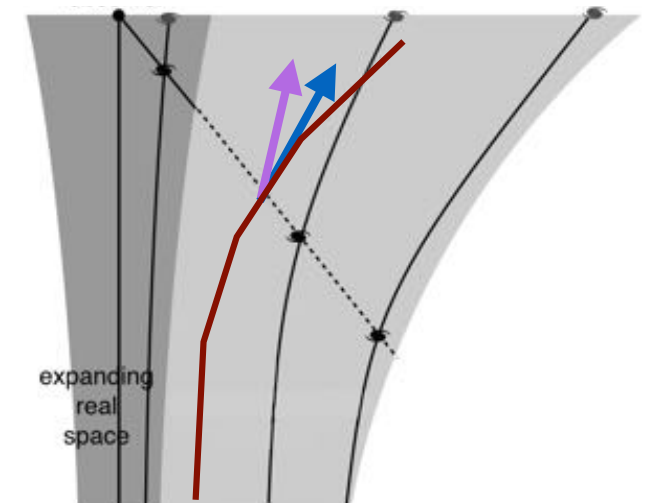
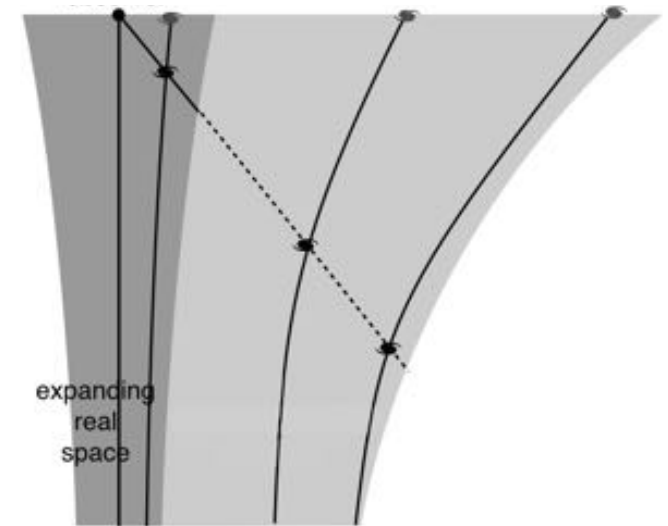
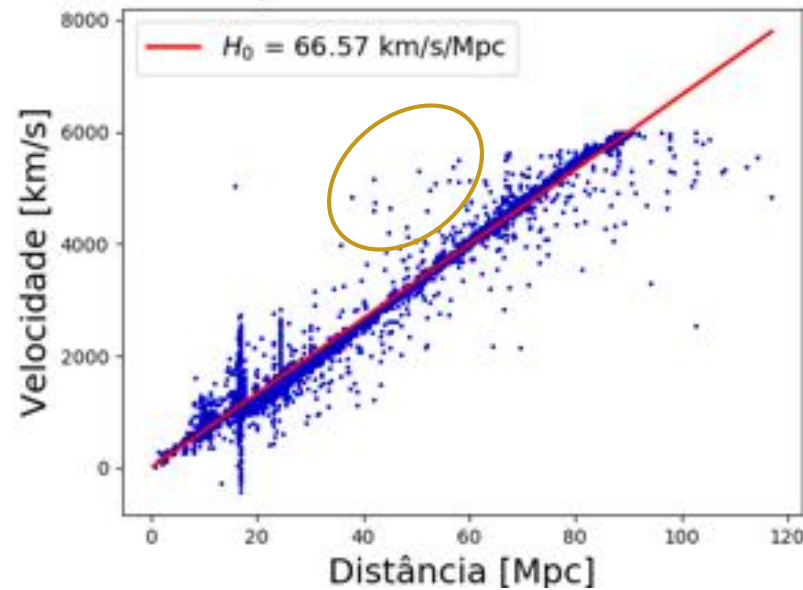


4 The problem:

expected



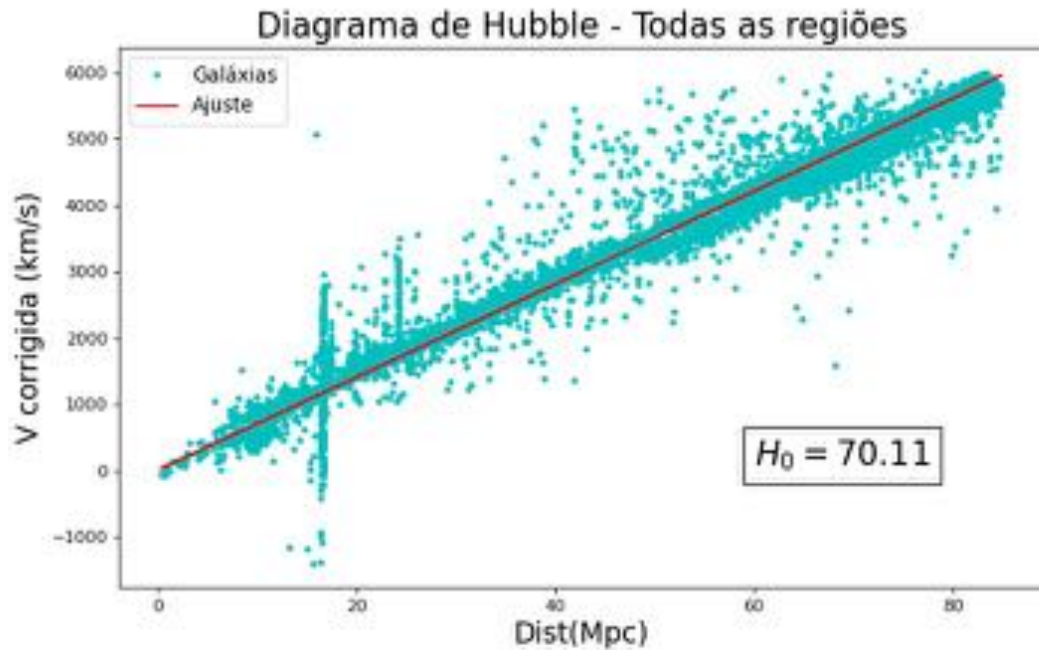
observed



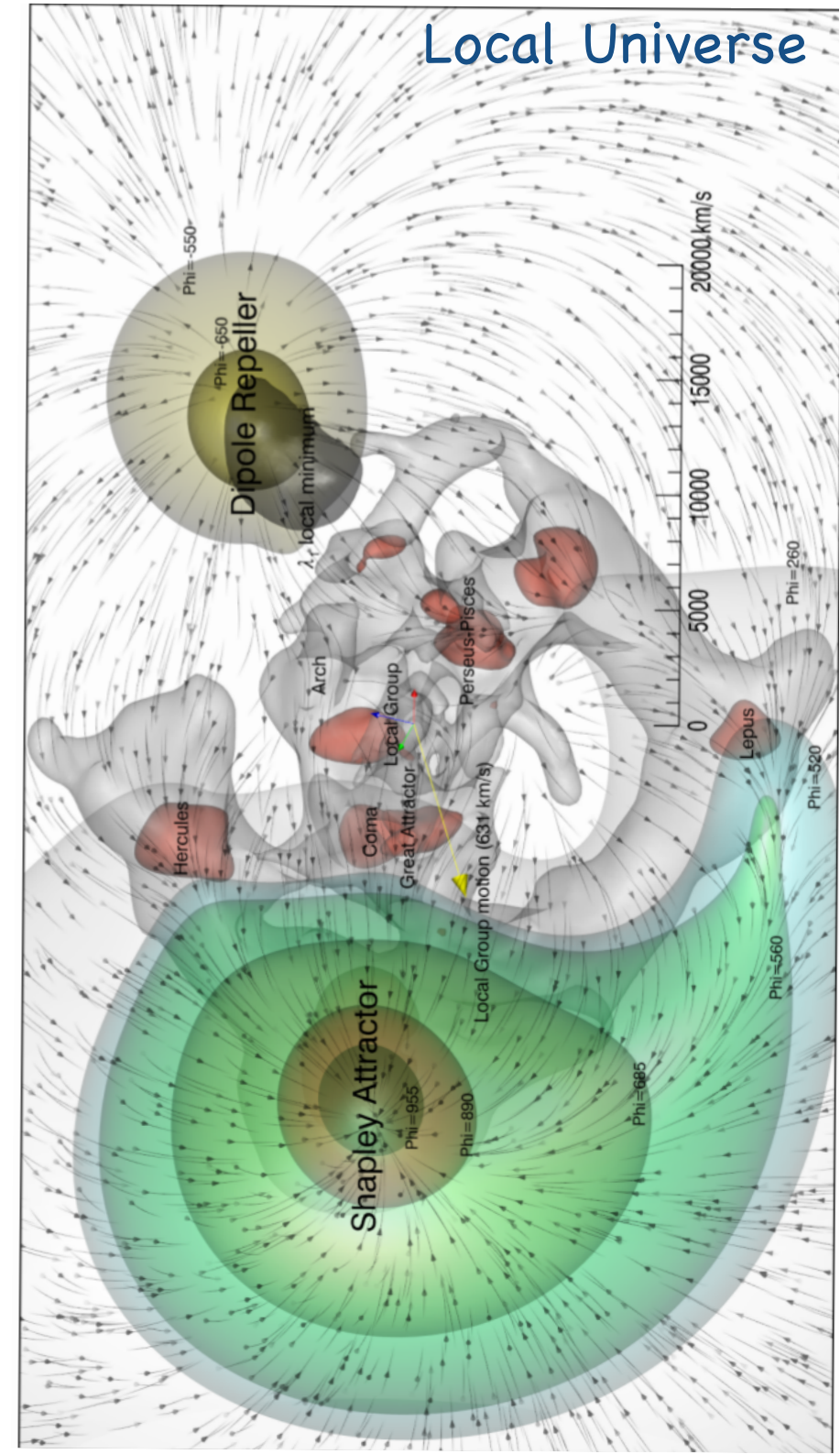
velocidade esperada  velocidade observada 

5

Conclusion: If deviations from the Hubble flow are due to clustered matter, **then** peculiar velocities traces the clumpy Universe!



Still, we need a **model** to interpret these **data**



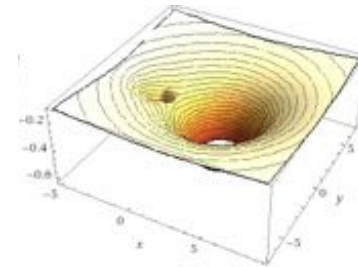
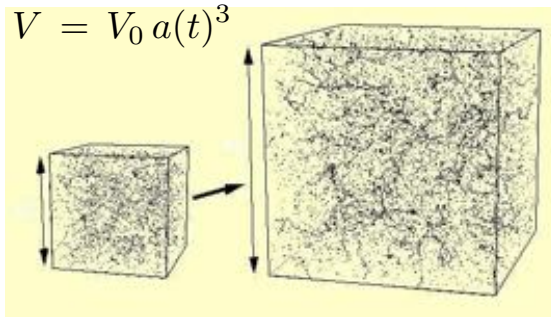
6

The proposal:

spacetime
metric

$$ds^2 = c^2 dt^2 + a(t)^2 d\mathbf{x}^2 \quad + \quad ds^2 = -c^2 \left(1 + \frac{2\phi}{c^2}\right) dt^2 + \left(1 - \frac{2\phi}{c^2}\right) d\mathbf{x}^2$$

3D-space

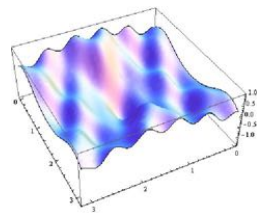


The solution:

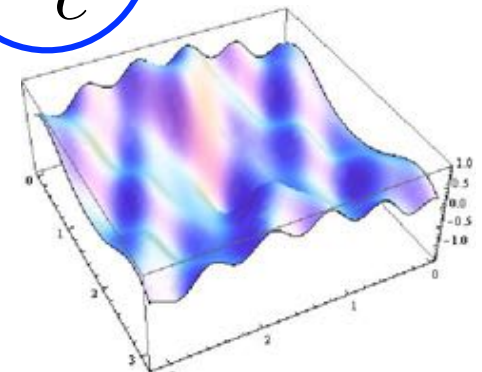
spacetime
metric

$$ds^2 = -\left(1 + \frac{2\Phi}{c^2}\right) c^2 dt^2 + a(t)^2 \left(1 - \frac{2\Phi}{c^2}\right) \gamma_{ij} dx^i dx^j$$

3D-space

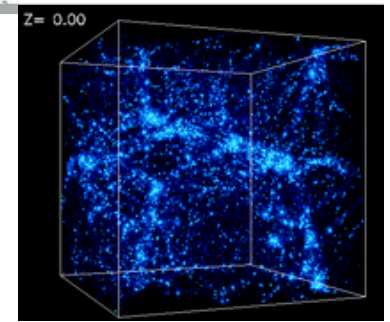
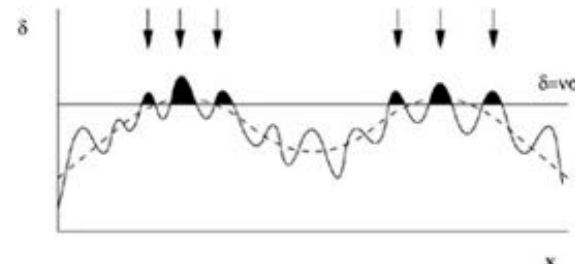


$$V = V_0 a(t)^3$$



density contrast

$$\delta(x, t) \equiv \frac{\rho(x, t) - \bar{\rho}(t)}{\bar{\rho}(t)}$$



7

Fluid evolution equations

Continuity, Euler, Poisson eqs.

$$\left\{ \begin{array}{l} \dot{\delta} + \frac{1}{a} \nabla \cdot \vec{v} = 0 \\ \frac{\partial \vec{v}}{\partial t} + \frac{\dot{a}}{a} \vec{v} = -\frac{1}{a} \nabla \phi - \frac{c_s^2}{a} \nabla \delta \\ \nabla^2 \Phi = 4\pi G \bar{\rho} a^2 \delta \end{array} \right. \quad \begin{array}{l} \dot{\delta} \equiv \frac{\partial \delta}{\partial t}; \quad \vec{v} \equiv \frac{d\vec{x}}{dt} \\ c_s^2 \equiv c^2 \frac{\partial p}{\partial \rho} \end{array}$$

Hypothesis:

$$\delta(t, \vec{x}) = \delta_k(t) e^{i\vec{k} \cdot \vec{x}} \Rightarrow \ddot{\delta}_k + \underbrace{2\frac{\dot{a}}{a}\dot{\delta}_k}_{\text{damping due to coupling w/expanding universe}} + \underbrace{\left(\frac{c_s^2 k^2}{a^2} - 4\pi G \bar{\rho}\right)}_{\substack{\text{pressure repulsion} \\ \text{gravitational attraction}}} \delta_k = 0$$

MD: $z < 1100$

$$\delta^+(t) = \delta_0 t^{2/3} \propto a(t)$$

