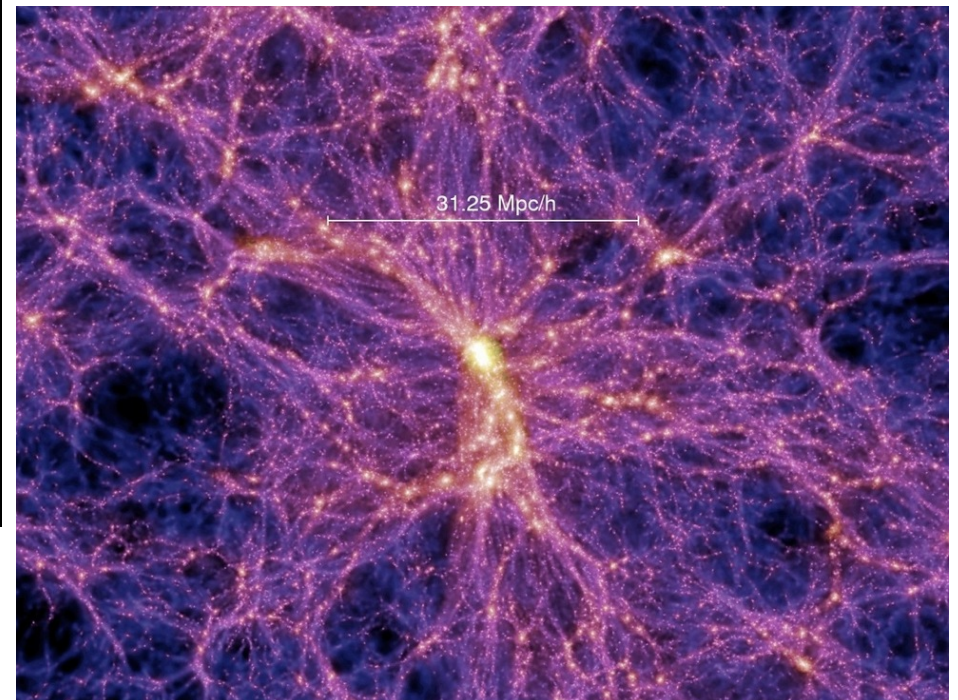
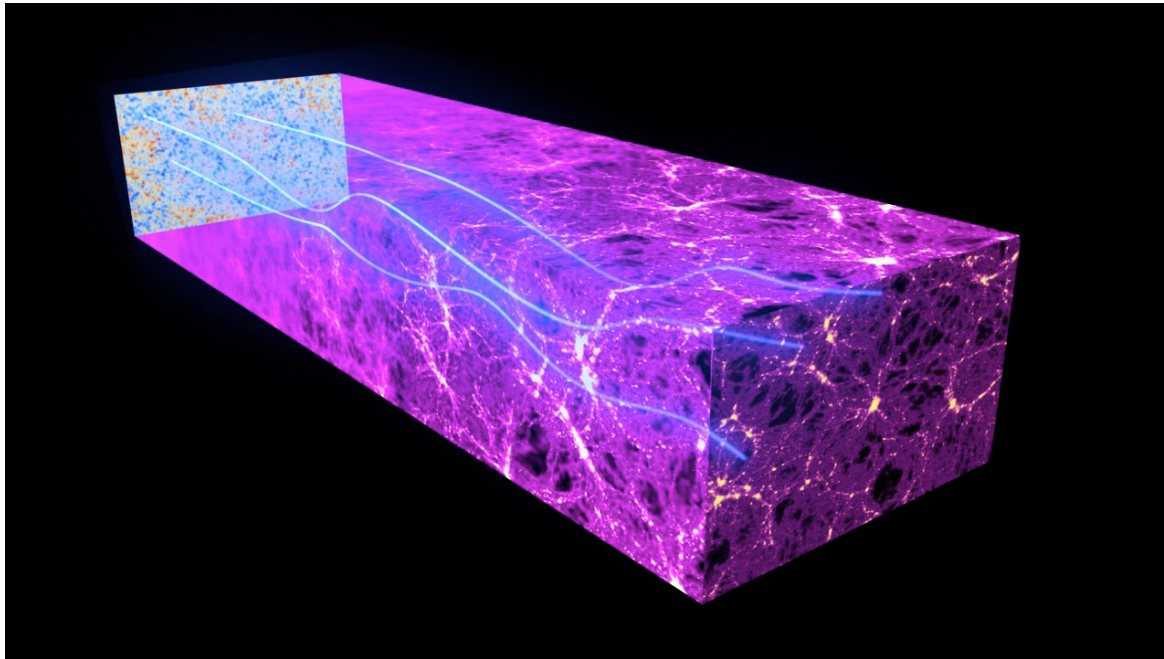


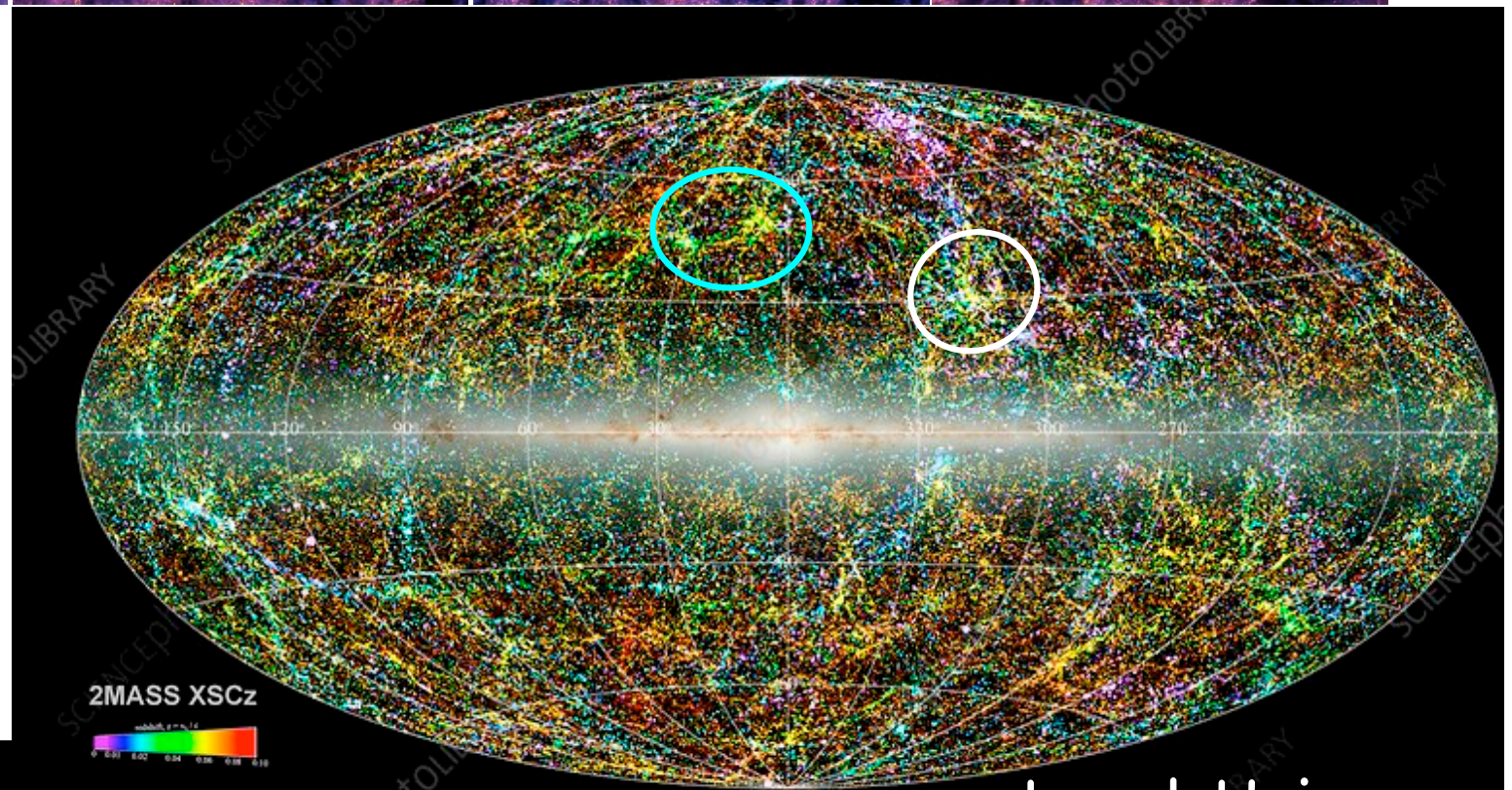
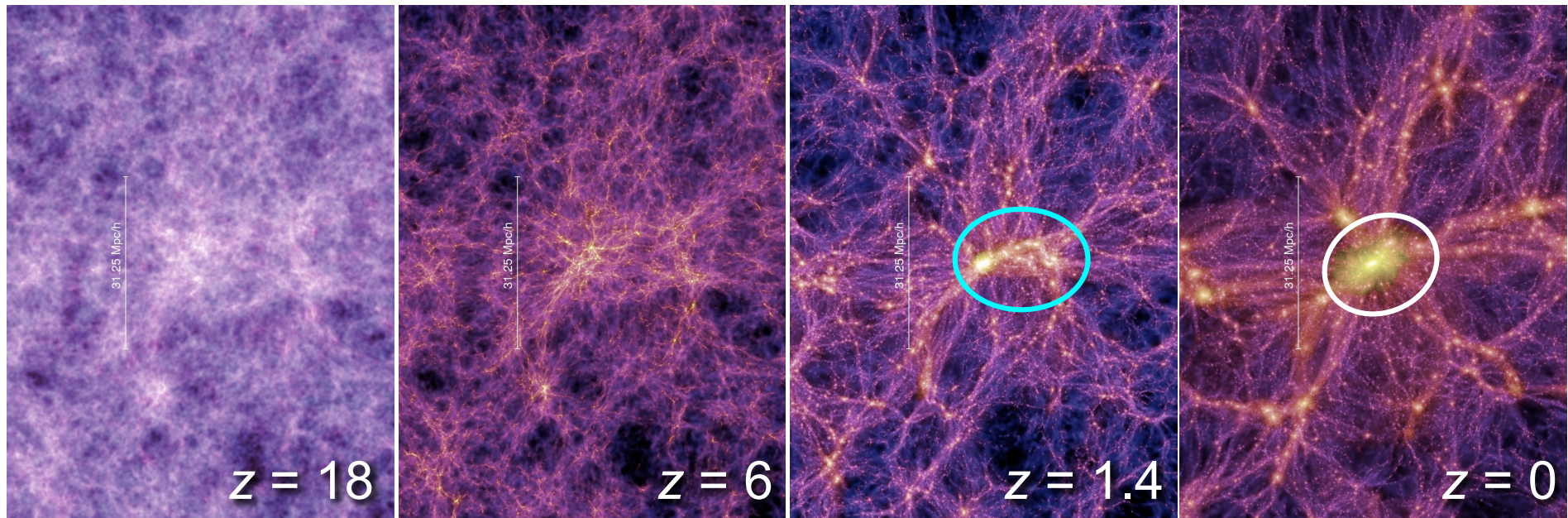


O Universo em grande escala: das flutuações primordiais à formação das estruturas

Armando Bernui



The challenge:



Hercules

Shapley

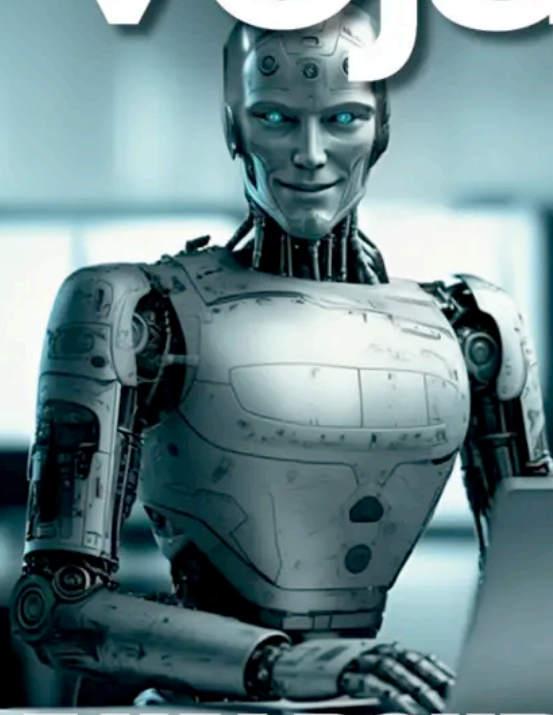
Local Universe

TARIFAS AS BOAS E AS MÁS NOTÍCIAS PARA O BRASIL NA GUERRA ENTRE ESTADOS UNIDOS E CHINA



veja
www.veja.com

Editora ABRIL
edição 2940 - ano 58 - nº 16
17 de abril de 2025



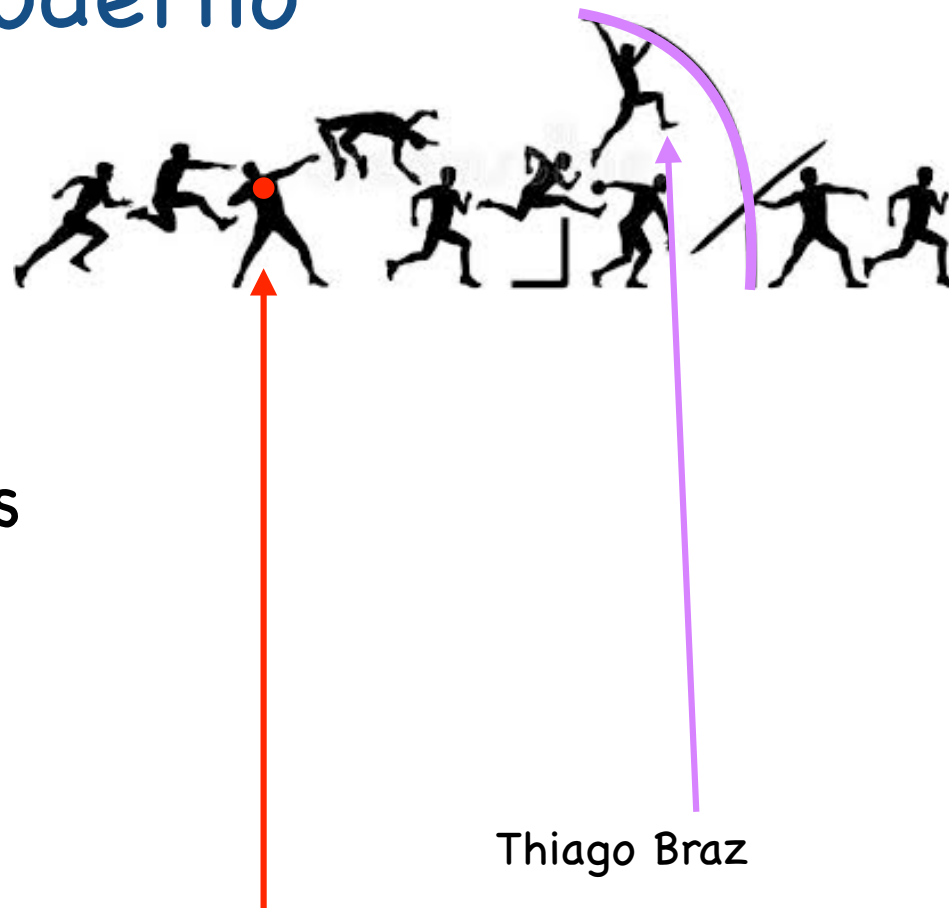
ELE VAI ROUBAR O SEU EMPREGO

Os robôs ocupam espaço cada vez maior, mas muitas vagas estão sendo criadas e a chave de sobrevivência no mercado será a capacidade de adaptação à era da inteligência artificial



Decatlo moderno

- 100 mt. rasos
- 400 mt. rasos
- 1500 mt. rasos
- 110 mt. com barreiras
- salto em distância
- salto em altura
- salto com vara
- arremesso do peso
- lançamento do disco
- lançamento do dardo



Darlan Romani

Thiago Braz



“É difícil passar a quem não sabe matemática
a sensação da beleza profunda da natureza"

Richard Feynman

Para estas aulinhas eu assumi que vocês sabem:
cálculo dif. & integral; física geral, mecânica,
electromagnetismo, termodinâmica básica, ondas,...

LESSONS

1. The Minkowski and the curved spacetimes
2. The background and the clumpy spacetimes
3. The observed Universe: modern cosmological probes

LESSON 1

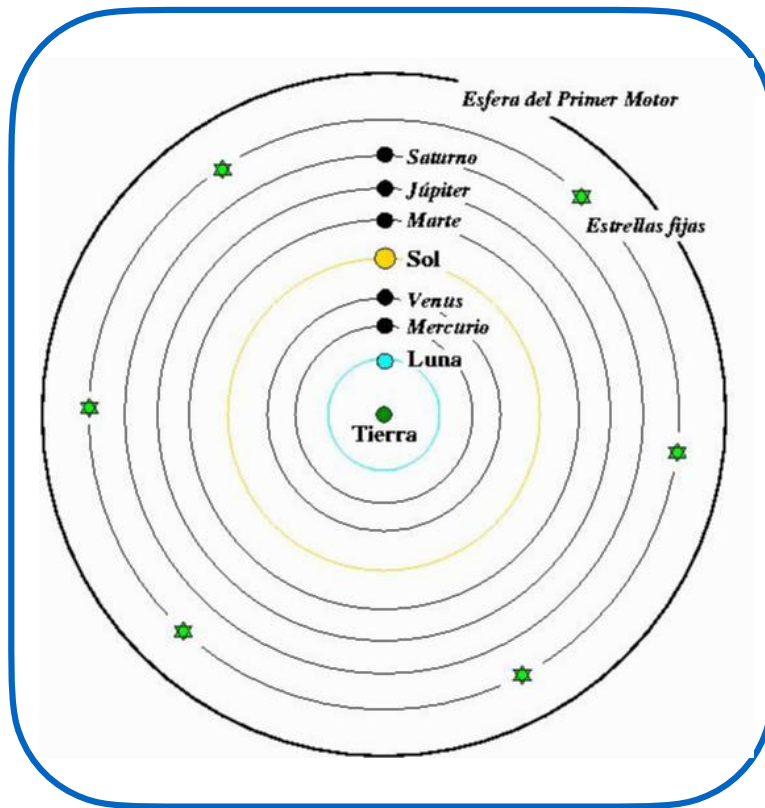
The Minkowski and the curved spacetimes



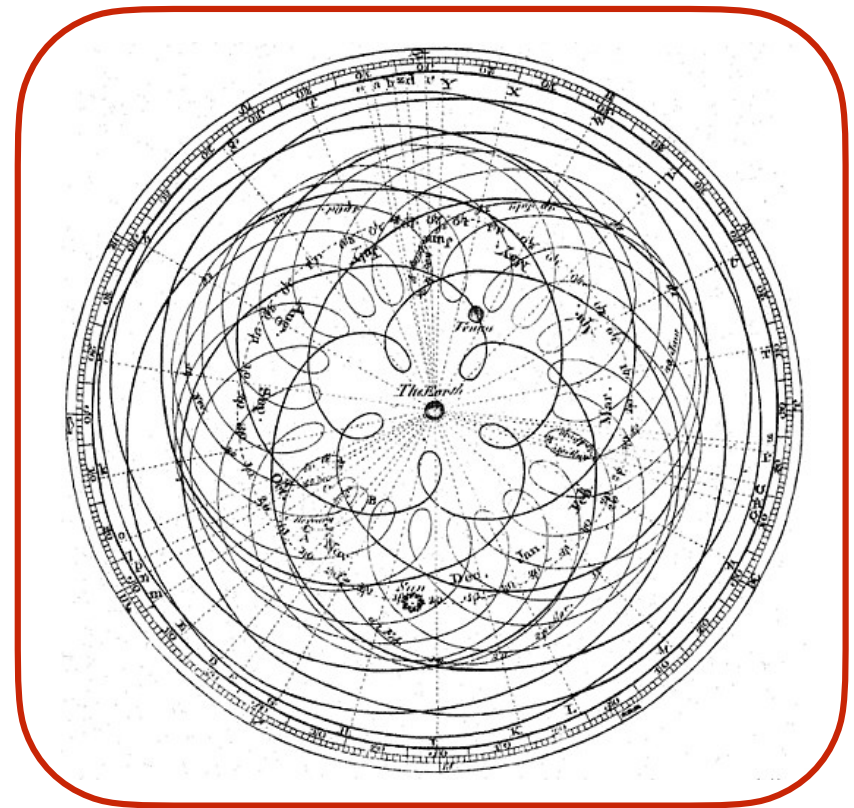
Theory (model)

vs.

Observations (data)



Circular motion



Retrograde motion



- Theory (model) versus Observations (data):

exemplo: uma xícara c/café quente



Por que a asa está fria
enquanto a xícara está
quente?



Modelando a vida através das eq. diferenciais

ou, o por que as xícaras tem alça?

Seja uma xícara com café a temperatura T ,

θ é a temperatura do ar

A é a área da xícara

l é a largura da parede da xícara

η é a condutibilidade térmica do material da xícara

cantidad de calor ganho pelo ar no tempo dt :

$$dQ = \eta A \frac{T - \theta}{l} dt$$

c é o calor especifico do café

m é a massa do café

cantidad de calor perdida pelo café no tempo dt :

$$dQ = -c m dT$$



* Estamos assumindo que o café esfria pelas paredes laterais, não pela parte superior/inferior



Modelando a vida através das eq. diferenciais

ou, o por que as xícaras tem alça?

Seja uma xícara com café a temperatura T ,

θ é a temperatura do ar

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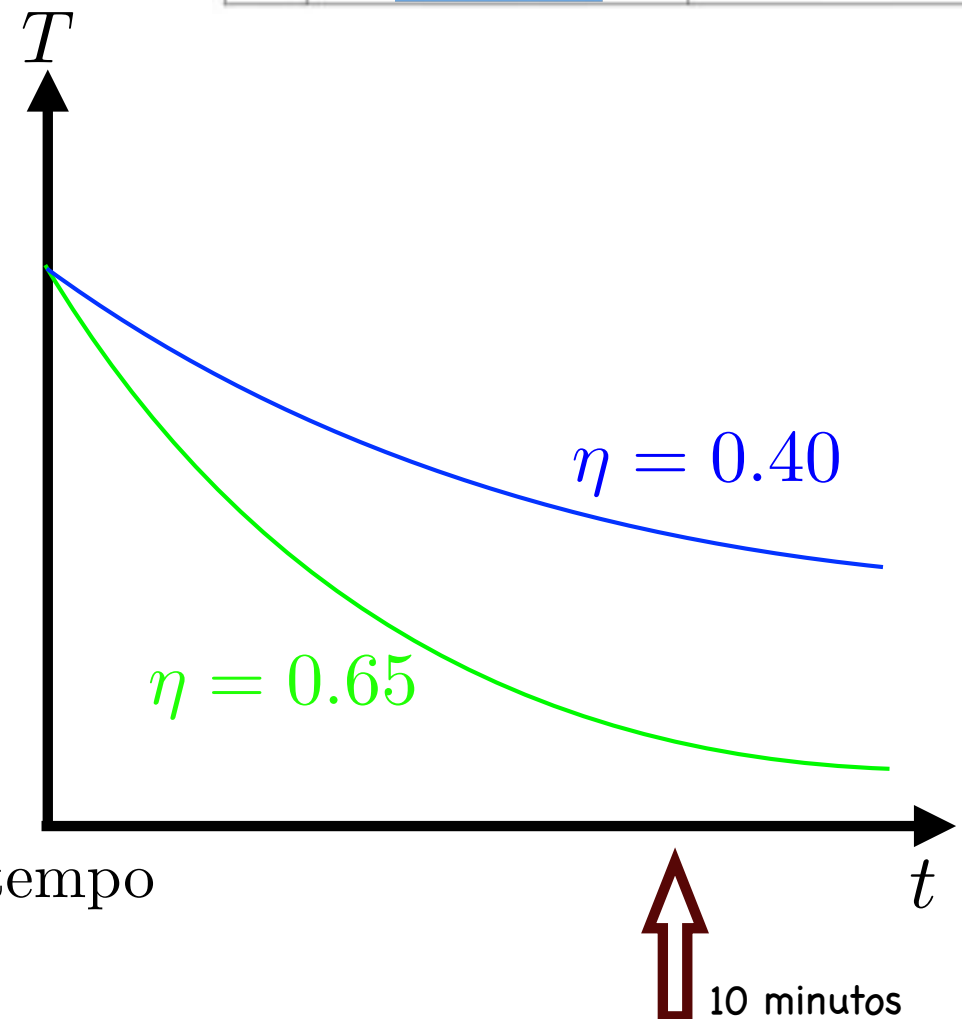
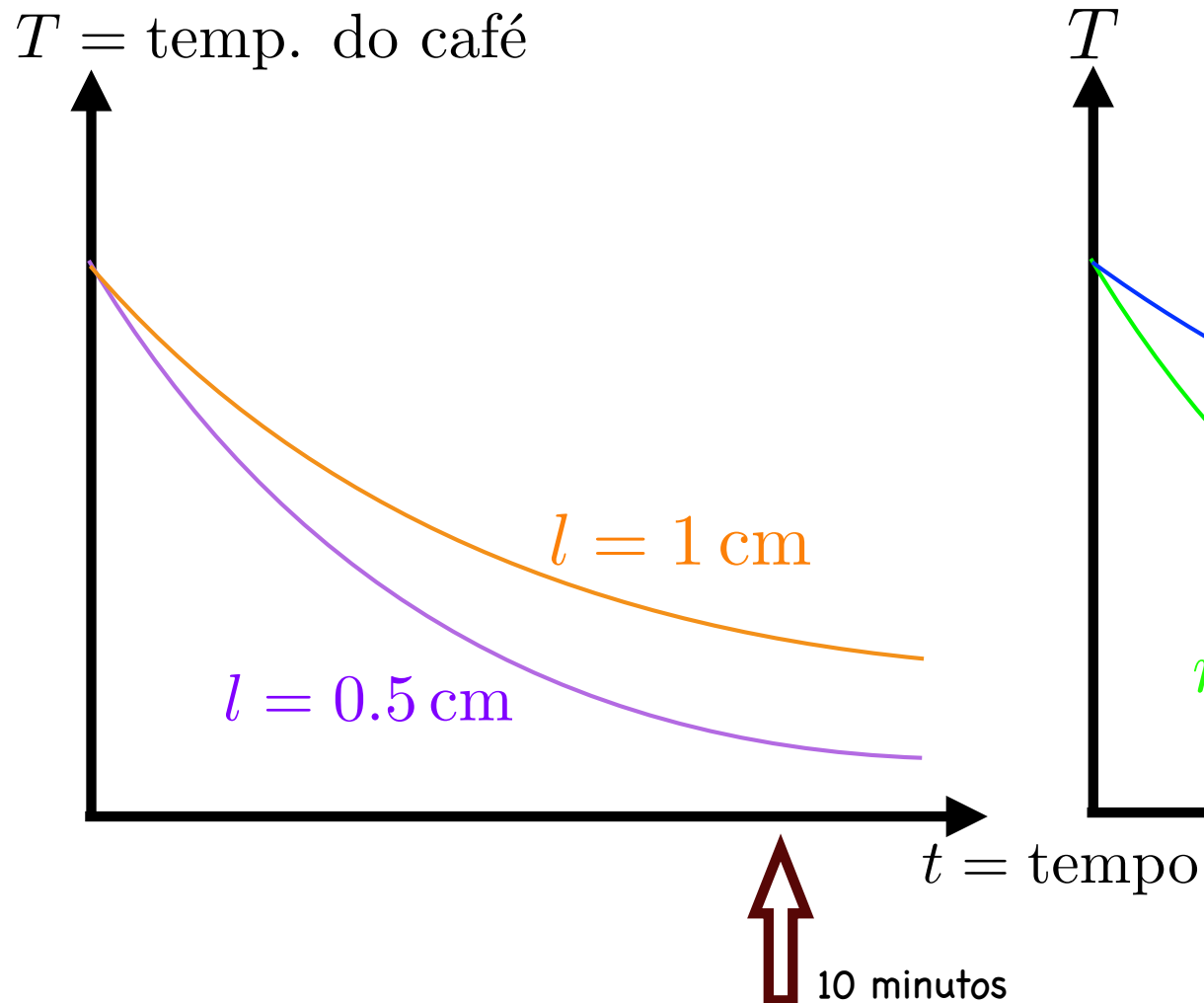
cantidad de calor perdida
pelo café no tempo dt :

$$dQ = -c m dT$$

A solução é: $\Rightarrow T = \theta + (T_0 - \theta) \exp^{-\frac{\eta A}{l c m} t}$

$$T = \theta + (T_0 - \theta) \exp^{-\frac{\eta A}{l c m} t}$$

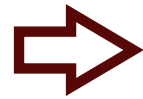
Condutividade térmica de materiais		
	Material	k(kcal/h m °C)
I	Cobre	332,0
II	Alumínio	175,0
III	Ferro	40,0
IV	Vidro	0,65
V	Cerâmica	0,40



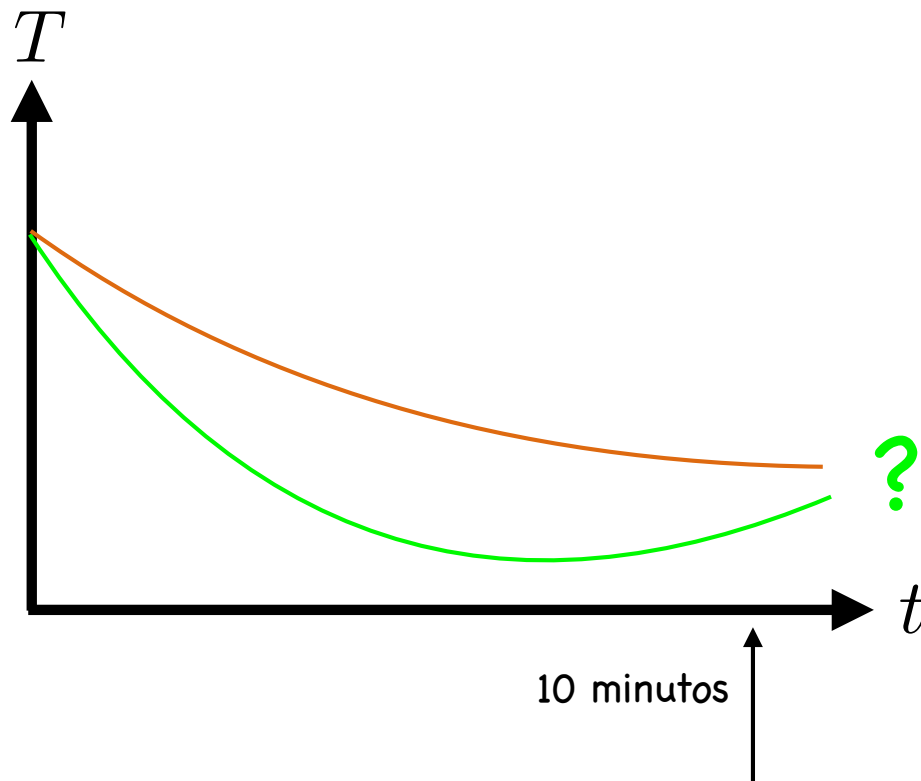
Resumindo este **toy-model** de método científico:

cantidad de calor ganho
pelo ar no tempo dt = cantidad de calor perdida
pelo café no tempo dt } ?

eq. diferencial
de 1er. ordem



$$T = \theta + (T_0 - \theta) \exp^{-\frac{\eta A}{l c m} t} \quad \text{?}$$

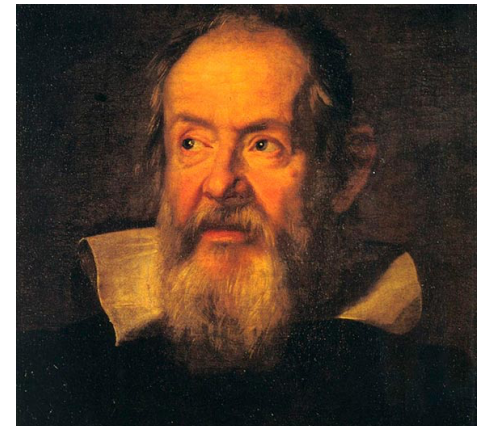


- Dados ajustam parâmetros (de modelos) e validam teorias
- Teorias permitem fazer previsões
- Previsões podem ser contrastadas





“Nelle questioni naturali ... la cognizione degli effetti é quella che ci conduce all’investigazione e ritrovamento delle cause”



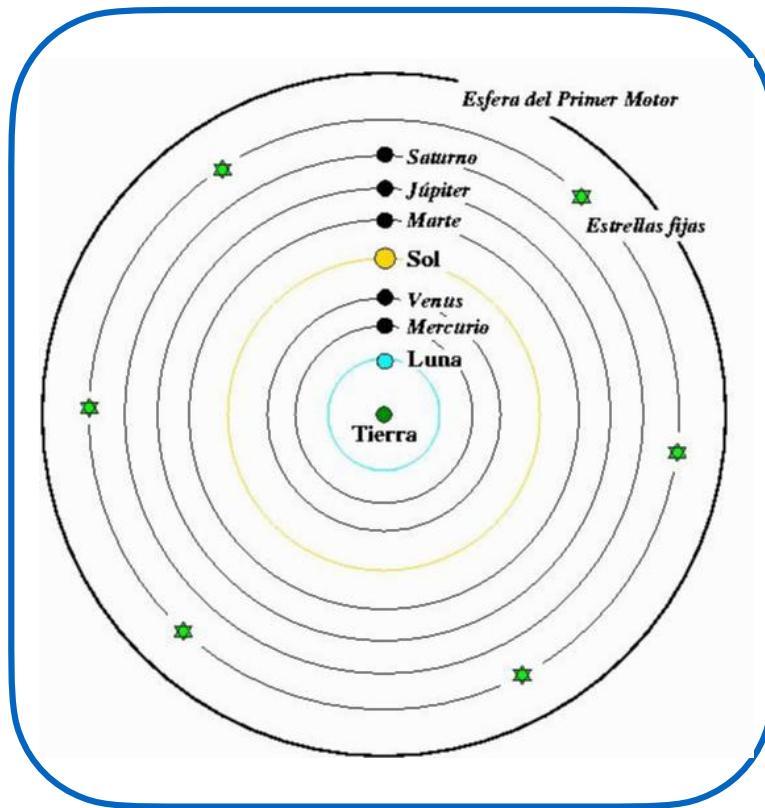


Once upon a time...

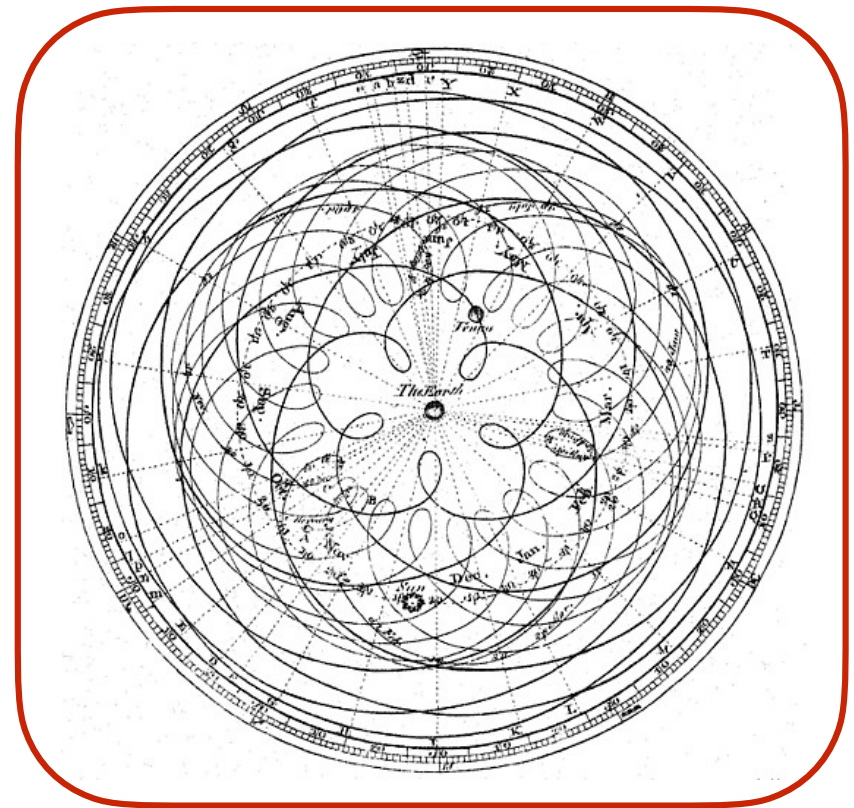
Theory (model)

vs.

Observations (data)



Circular motion



Retrograde motion



- Theory (model) versus Observations (data):



A first example

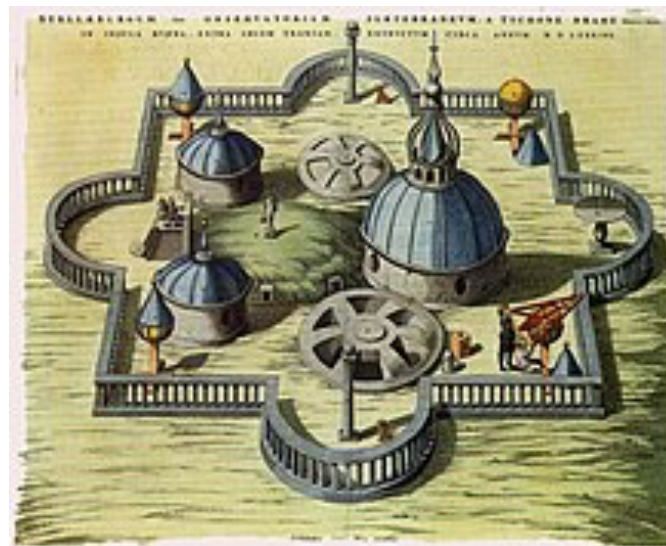
Johannes Kepler: assistant of Tycho Brahe from 1600 to 1601



Mathematician (1571–1630)

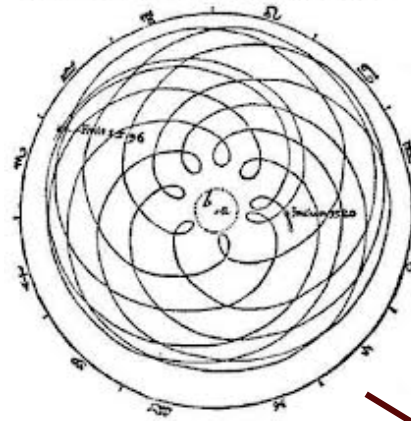


Royal Astronomer (1546–1601)

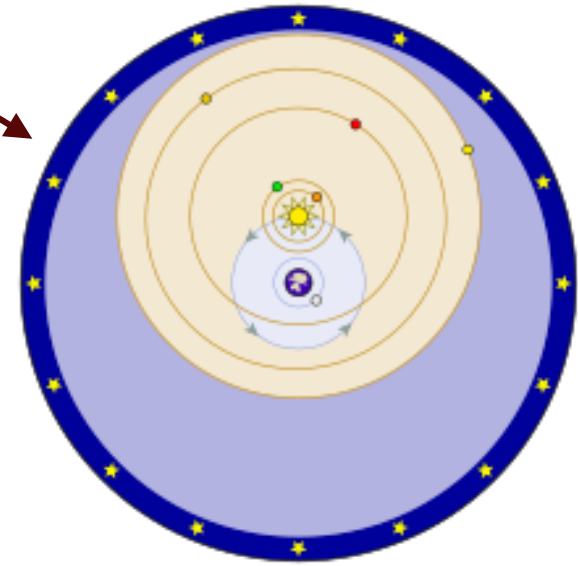


Performed regular and
precise astronomical
observations of Mars orbit

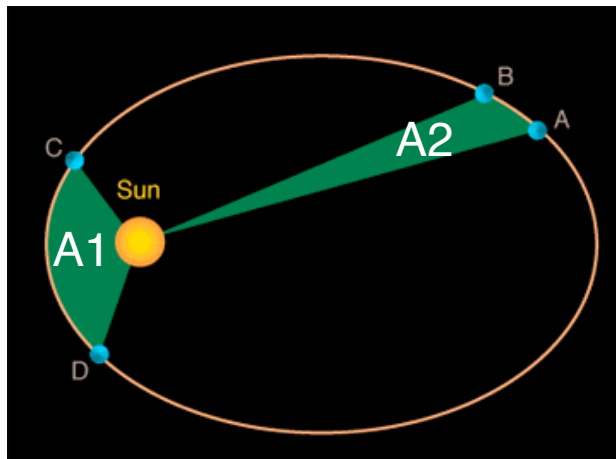
Castle of Uraniborg,
Dinamarca



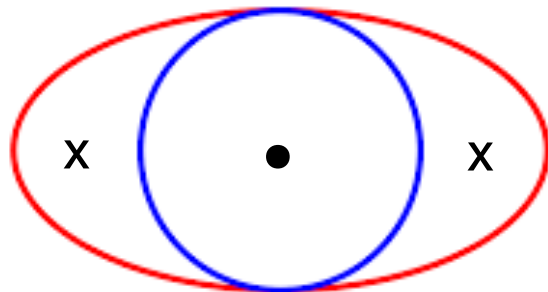
Tycho Brahe



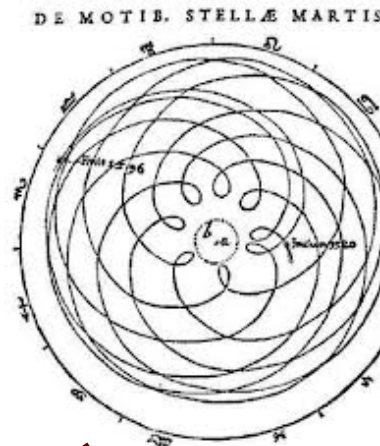
Johannes Kepler



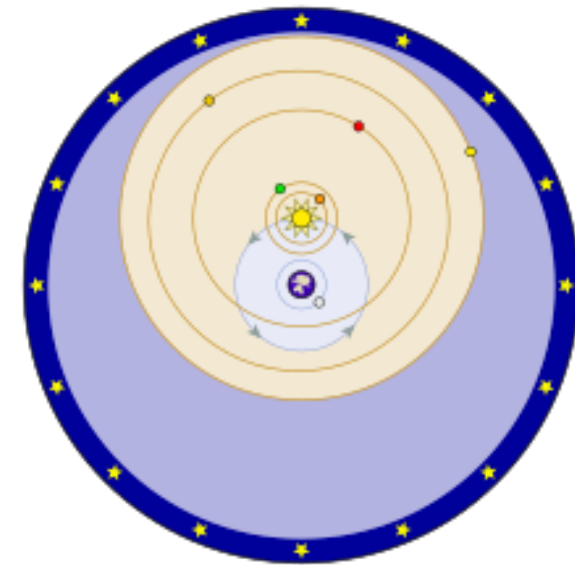
1. Orbits are ellipses
2. $A1 = A2$
3. $T^2 \propto \bar{R}^3$



Corollary: The circle is a particular case of an ellipse.



Tycho Brahe





A second example

- Theory (model) versus Observations (data):



- Studied the stability of the Solar system
- Investigating, in detail, the orbit of Uranus, he (and John C. Adams) predicted the existence of Neptune!

Urbain Le Verrier (1811-1877)

Mathematician and Astronomer



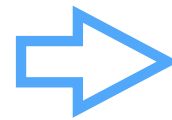
A second example

- Theory (model) versus Observations (data):



Urbain Le Verrier (1811-1877)
Mathematician and Astronomer

- Studied the stability of the Solar system
- Investigating, in detail, the orbit of Urano, he (and [John C. Adams](#)) predicted the existence of Neptune!
- To explain the Mercury perihelion precession he postulated the existence of "Vulcano"



Dark Matter
was invented
for the 1st time

Séculos de avanço do conhecimento científico nos mostram que quando uma teoria ou modelo entra em crise, uma destas abordagens é adoptada:

- (i) novas hipóteses são propostas dentro da teoria em crise;
- (ii) uma nova teoria é proposta

O modelo geocêntrico entra em crise por **causa** dodo que?

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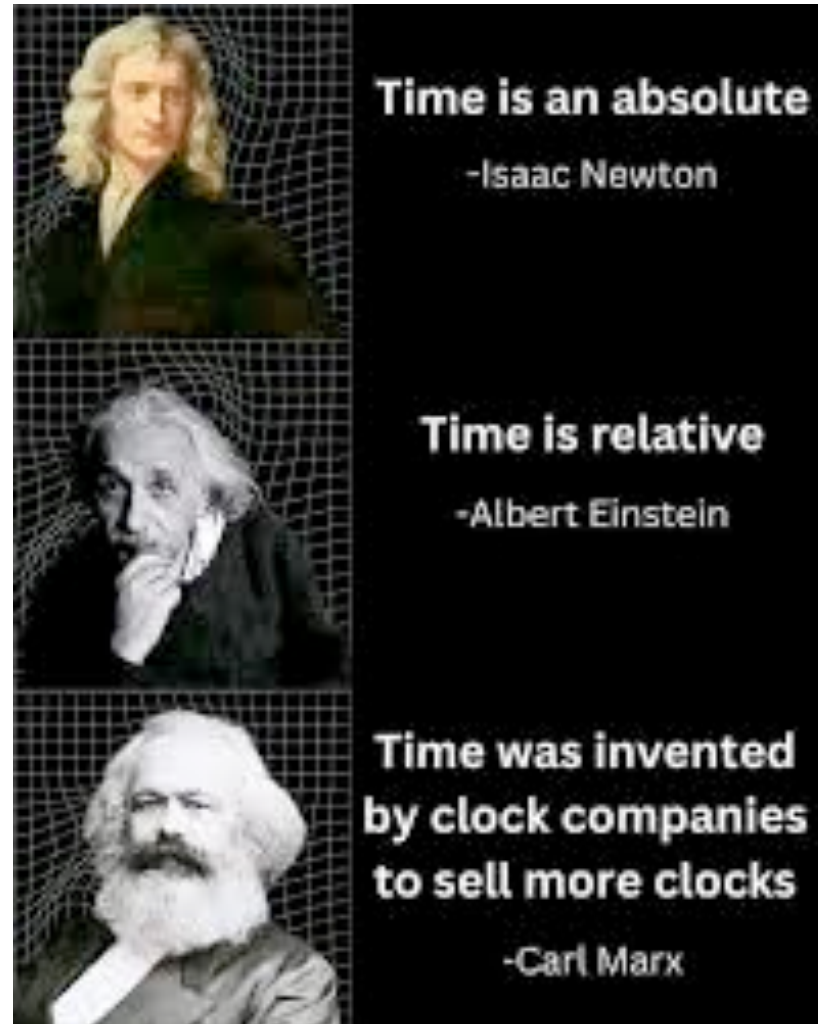
O modelo heliocêntrico+Kepler não explica **por que** as orbitas são elípticas; Newton **sim**: nasce o conceito de potencial gravitacional

Mas, a mecânica Newtoniana não explica corretamente:

- a precessão do perihelio de Mercúrio
 - o lenteamento gravitacional da luz
 - o redshift gravitacional dos fótons
- } GR?



SR versus GR



General Relativity (GR)

In general relativity space and time are related variables of a unique entity:
the spacetime

In GR there is no distance between 2 points, as in Newtonian theory, there is a distance between 2 spacetime events; such distance is calculated using
the spacetime metric

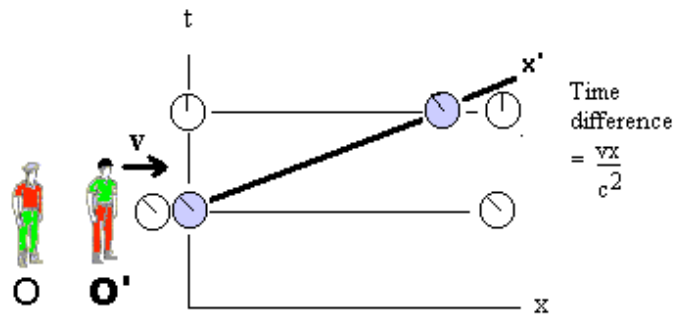
Introduction to General Relativity

General relativity is a relativistic theory of gravity

Newton

-space and time are separate entities

-interactions at infy speed: $t=\text{const.}$ means place of simultaneity



Phase describes how events that one observer measures to be simultaneous are not simultaneous for another observer.

SR

-space and time are part of an entity: the spacetime

-interactions at light-speed c : $t=\text{const.}$ means place of simultaneity for one observer

-only relative motions at constant speed are well described (i.e., no accelerations!): local & **global** inertial frames

GR

-space and time are part of an entity: the spacetime

-interactions at light-speed c

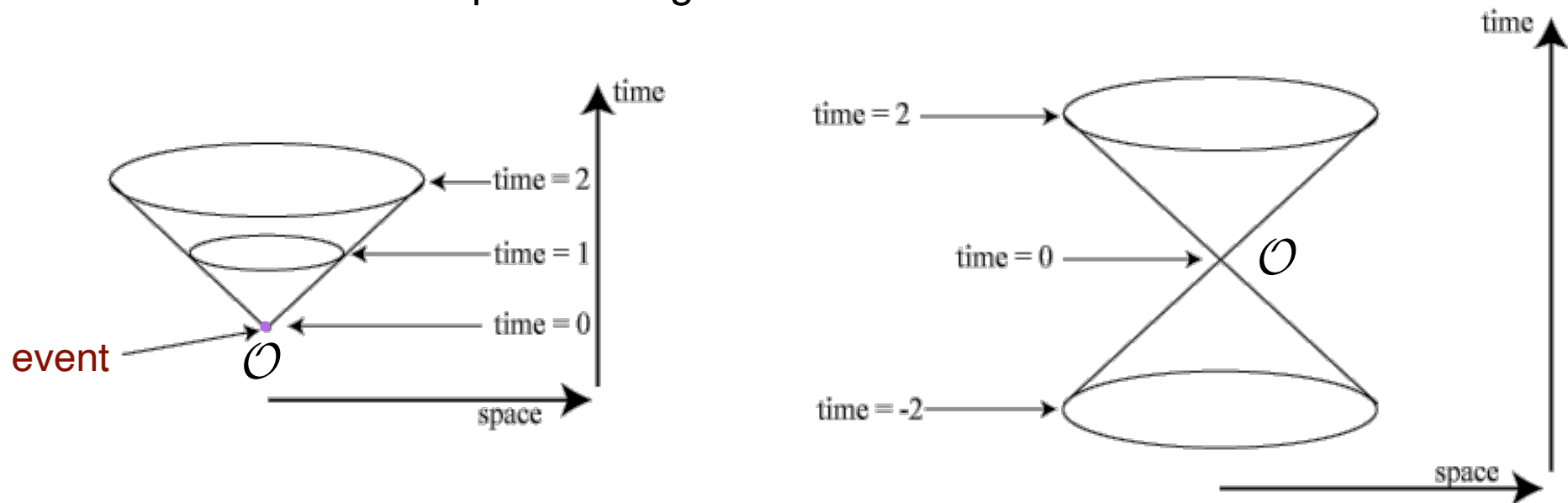
-gravity introduces accelerated motions; gravity avoids **global** inertial frames

Minkowski spacetime*

The Minkowski spacetime **metric** is

$$ds^2 = -c^2 dt^2 + \delta_{ij} dx^i dx^j = -c^2 dt^2 + d\mathbf{x}^2$$

The structure of a spacetime, given by the **metric**, determines how geodesics move, and introduces the concept/use of light-cones.



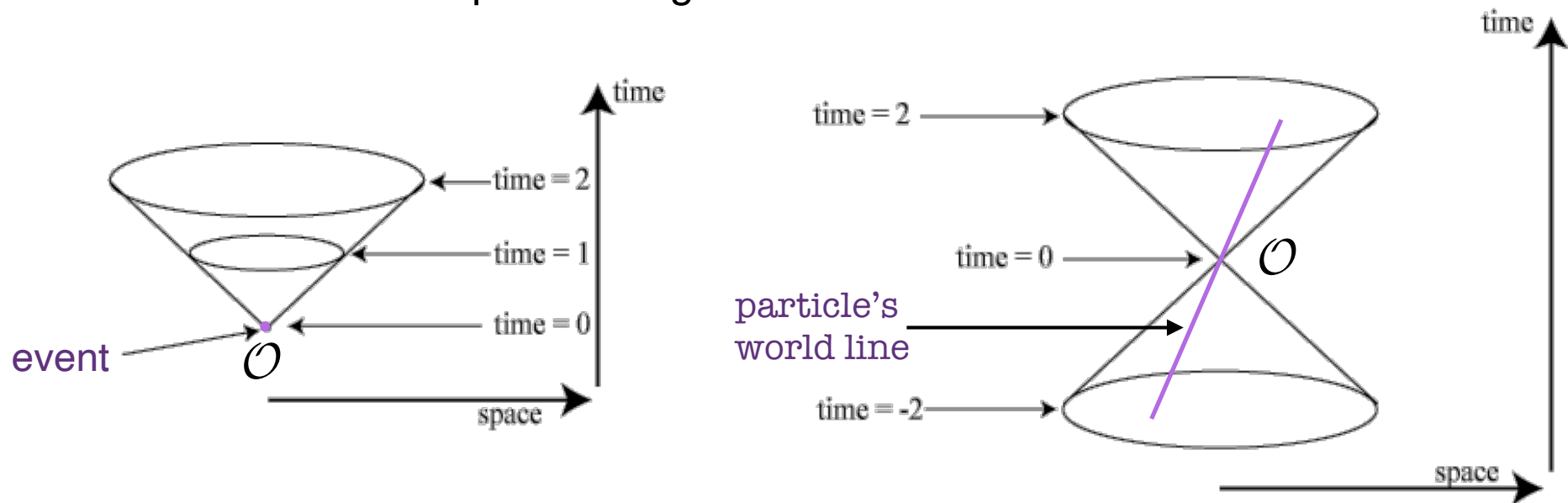
- * Spacetime (the set of all events) is a four-dimensional manifold with a metric.

Minkowski spacetime

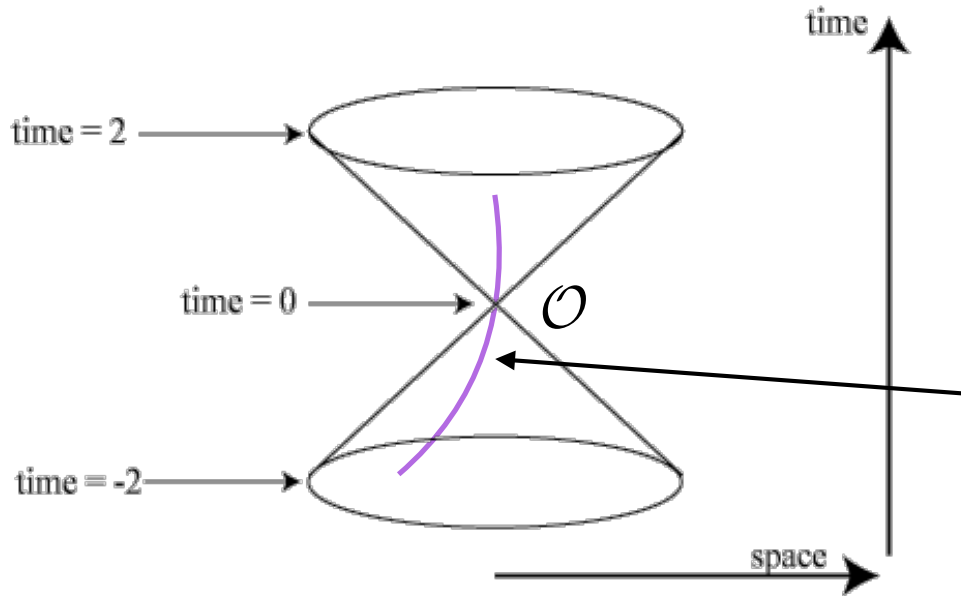
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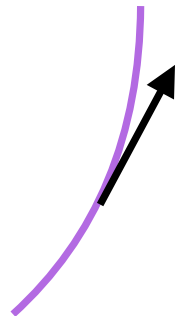
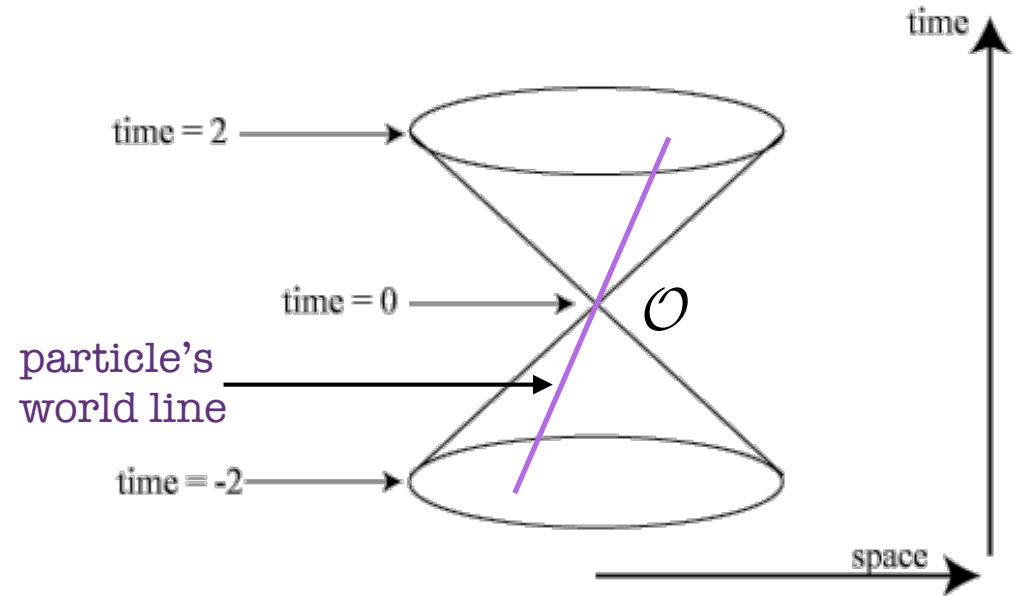
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Particle's world line:
accelerated motion

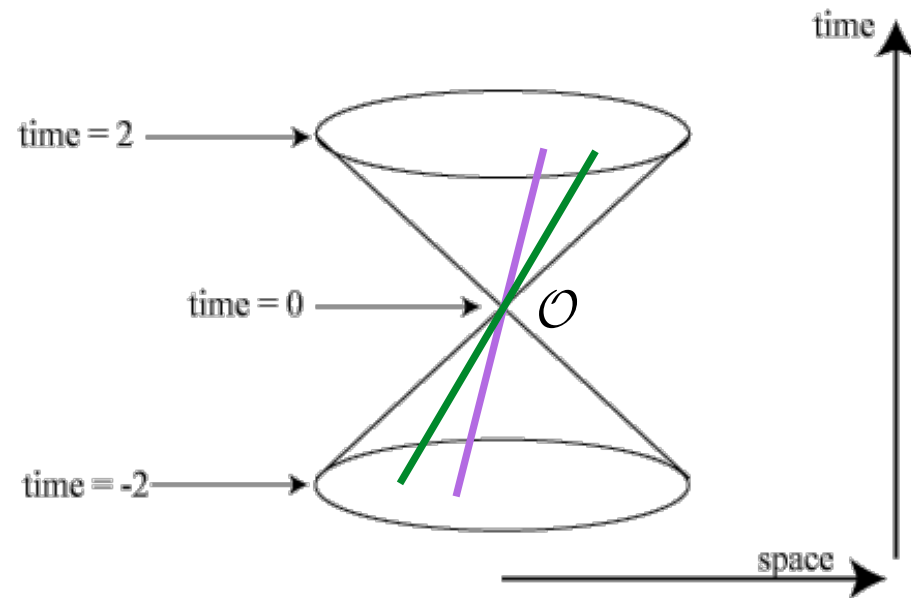


Particle's world line:
constant velocity



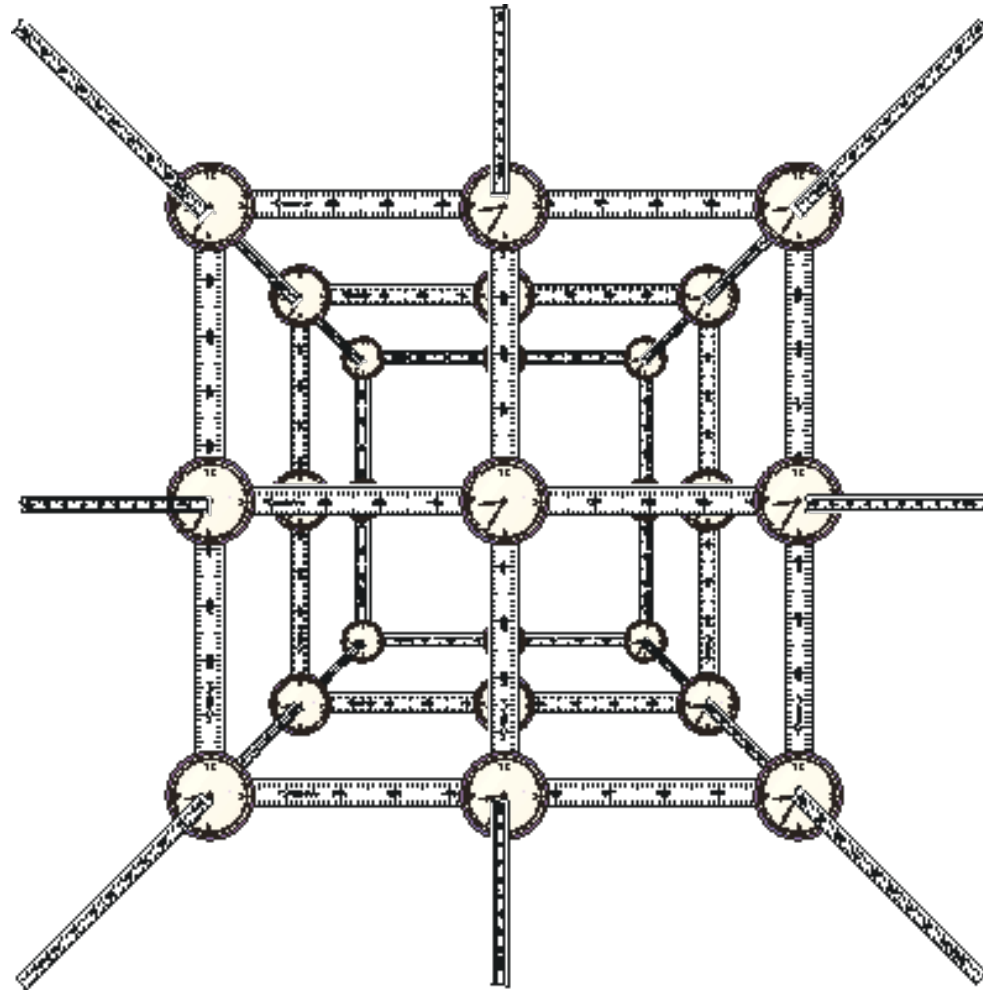
The particle's velocity is given by
the tangent line to the world line

Pergunta: qual partícula se move a maior velocidade (wrt $\mathcal{O}$): a lilás ou a verde?



Minkowski spacetime

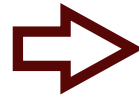
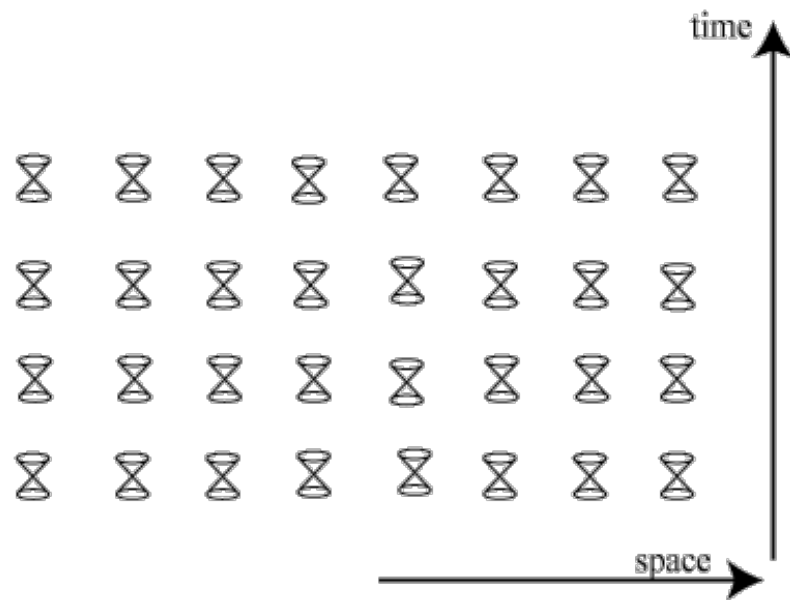
Minkowski spacetime **metric** $ds^2 = -c^2 dt^2 + d\mathbf{x}^2$



Continuando ...

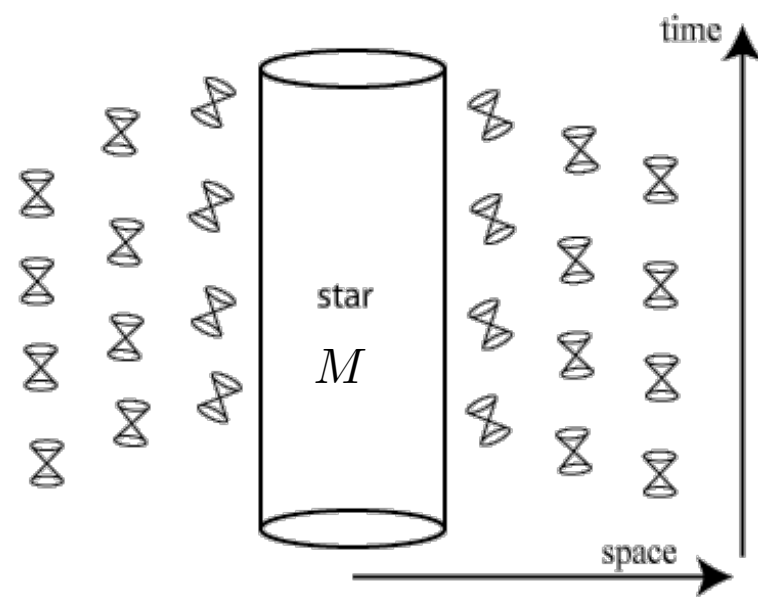
$$ds^2 = -c^2 dt^2 + d\mathbf{x}^2$$

Minkowski spacetime **metric**



?

Curved spacetime **metric**



Precisamos de conteúdo
para entender essa passagem!

Minkowski spacetime, or special relativity theory,
can describe gravitational fields?

Minkowski spacetime, or special relativity theory,
can describe gravitational fields?

Is gravity a special force?

Inertial Frame (IF)

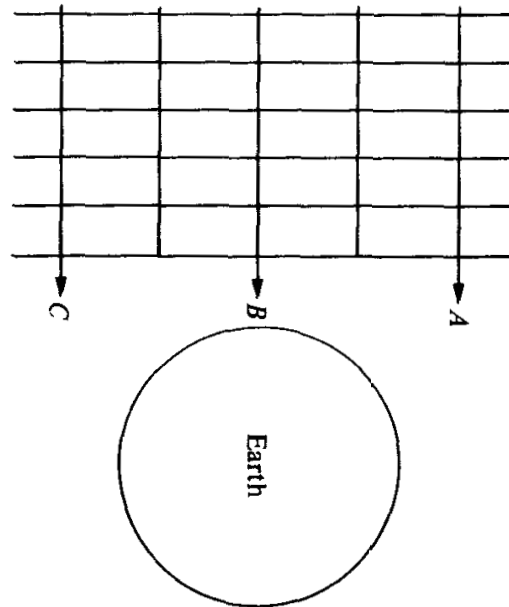
To define an IF we need test particles that remain at rest there; this is possible for 3 known forces, **but not for gravity**, why?

Attempting to define an IF **at rest on Earth** is vacuous, since no free particle (not even a photon) could be a physical tracer for an IF.

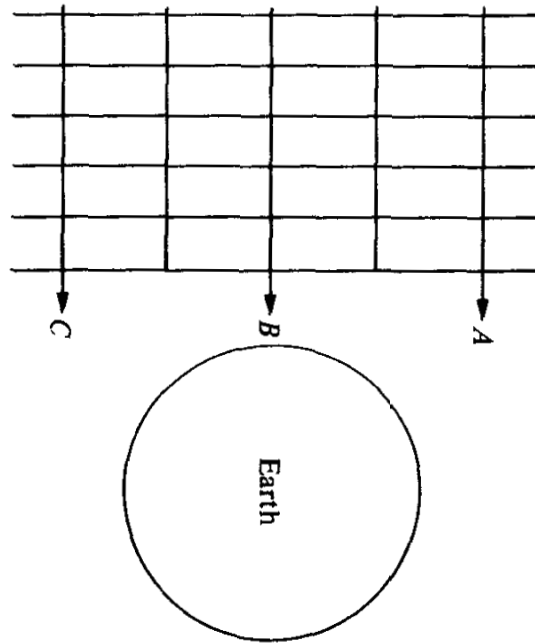
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Attempting to define an IF **at rest on Earth** is vacuous, since no free particle (not even a photon) could be a physical tracer for an IF.



A rigid frame cannot fall freely in the Earth's field and still remain rigid



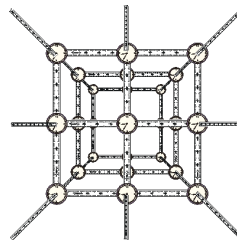
Conclusion:

There is no single freely falling **global** frame which is everywhere freely falling in Earth's gravitational field and which is still **rigid***

P.S.1: Pound-Rebka-Snider experiment shows that **local inertial frames** are possible!

P.S.2: Einstein built GR by taking the hypothesis that local freely falling frames are good IF!

* rigid means Minkowskian



Is gravity a special force? ➡ Yes!

Gravity is **distinguished** from all other forces: all bodies given the same initial velocity follow the same trajectory in a gravitational field, regardless of their internal composition



Conclusion I:

Neither Minkowski spacetime nor the theory of special relativity works to describe (accelerated particles in) gravitational fields!

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Conclusion I:

Neither Minkowski spacetime nor the theory of special relativity works to describe (accelerated particles in) gravitational fields!

Conclusion II:

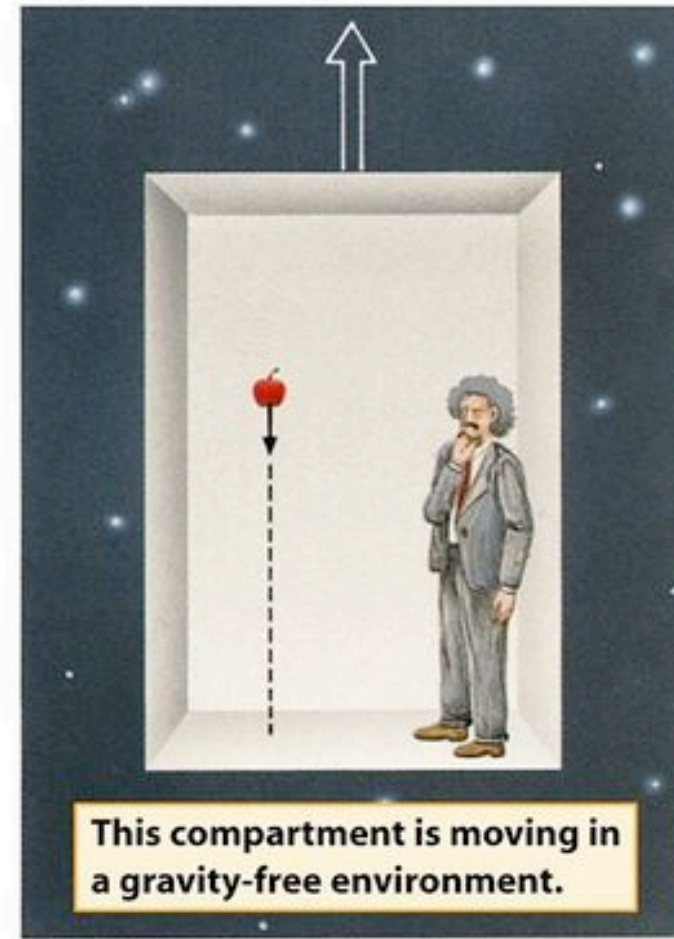
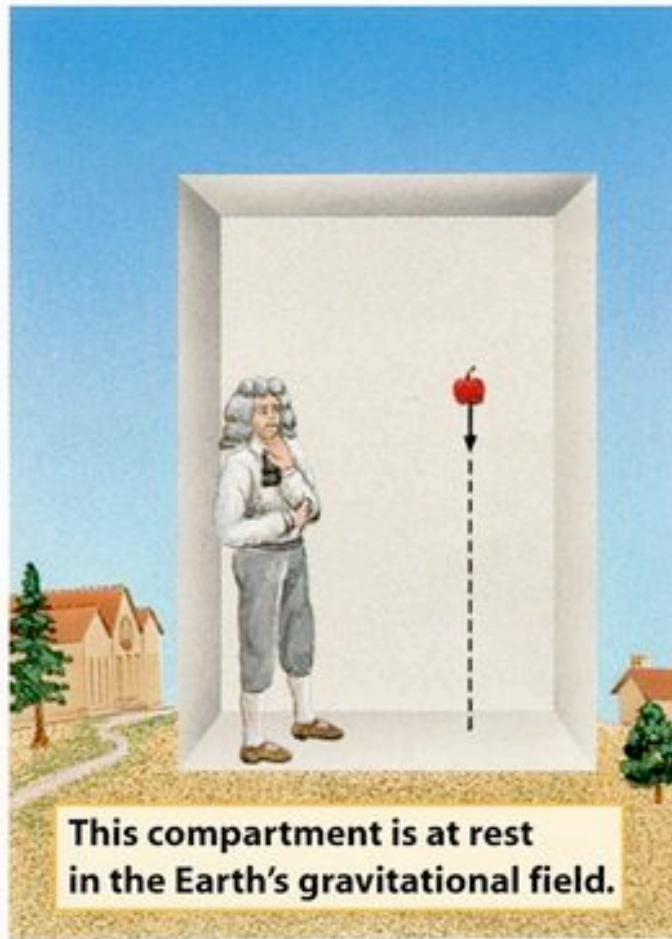
We need to construct a curved spacetime to describe gravitational fields!

Who helps with this transition
(from Minkowski to curved spacetimes)?

The equivalence principles

assim nasce:

The equivalence principle (weak: only gravitation)



The weak equivalence principle allows for the transition from SR to GR (where spacetime is curved by gravity, but locally it remains flat)

por completeza: **The equivalence principle** (strong: grav.+other forces)

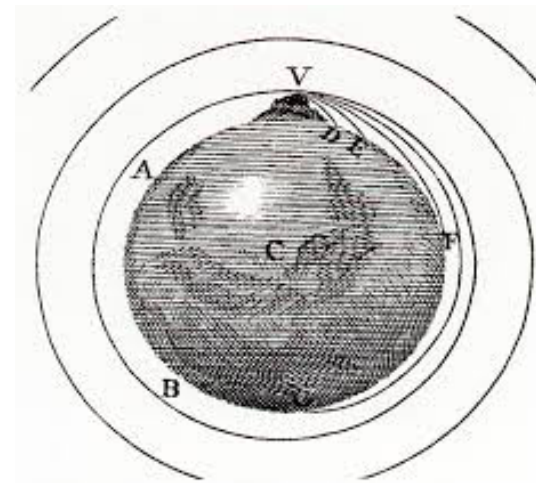
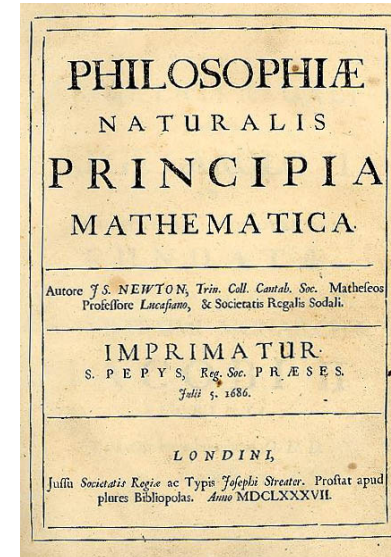
- ◆ (IV') Any physical law which can be expressed in tensor notation in SR has exactly the same form in a locally inertial frame of a curved spacetime.

The equivalence principle (the old weak)

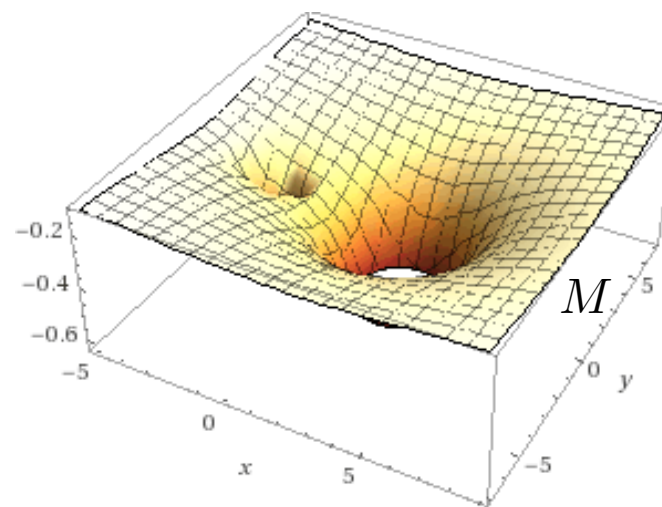
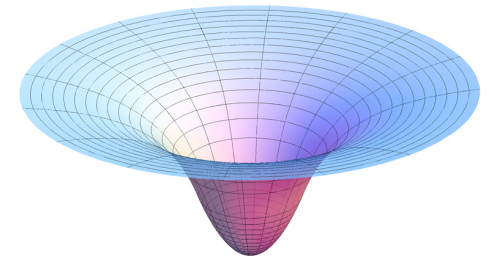
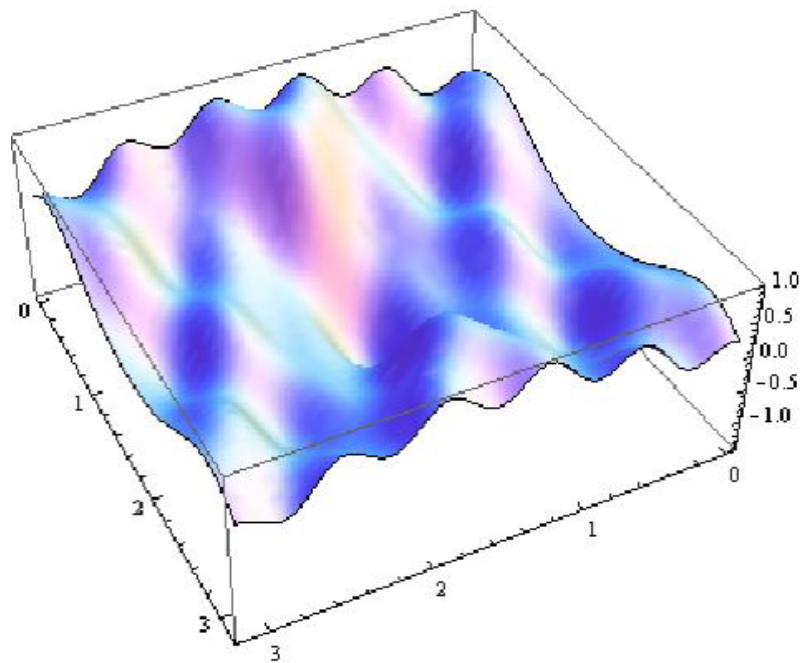
$$F = ma = m_i a$$

$$F = G \frac{Mm}{d^2} = G \frac{Mm_g}{d^2}$$

$$\frac{m_g}{m_i} = 1 \pm 10^{-10}$$

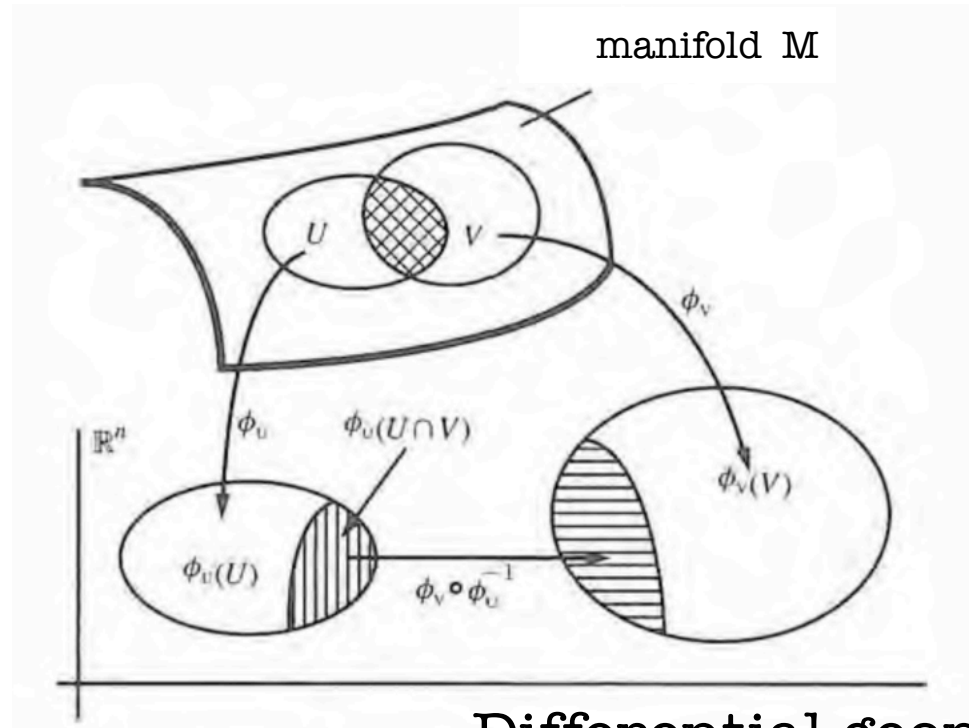


I. Constructing the Curved Spacetime Theory



Curved spacetime *

- ◆ (1) Spacetime (the set of all events) is a four-dimensional manifold with a metric.

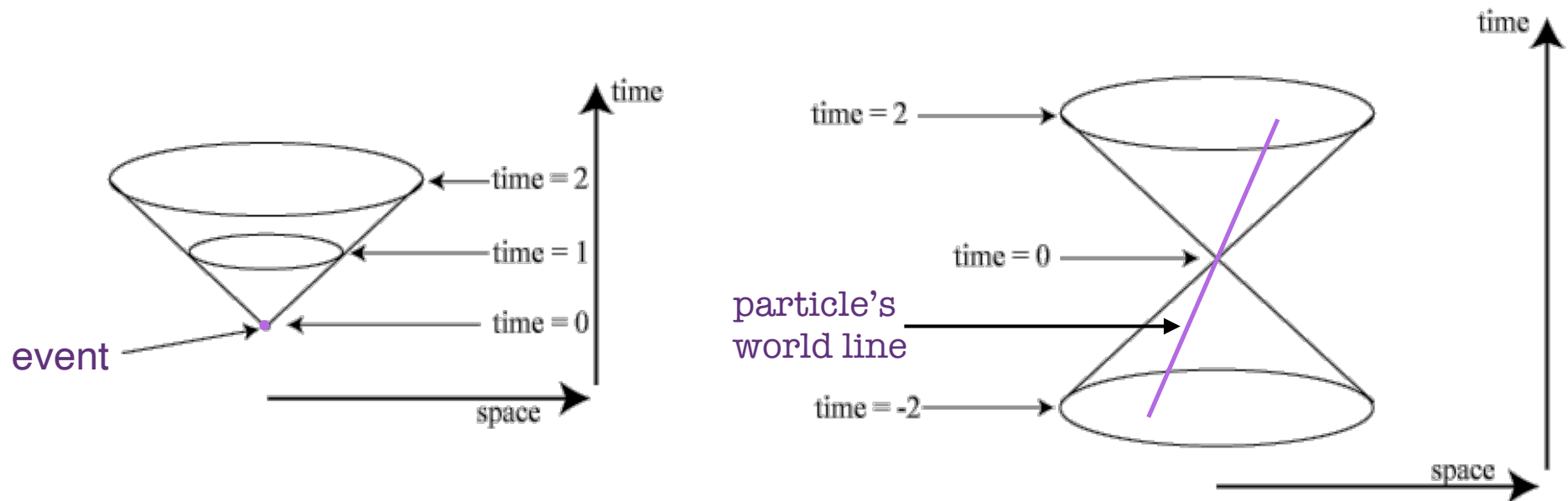


Differential geometry

Lembrando:

Minkowski spacetime

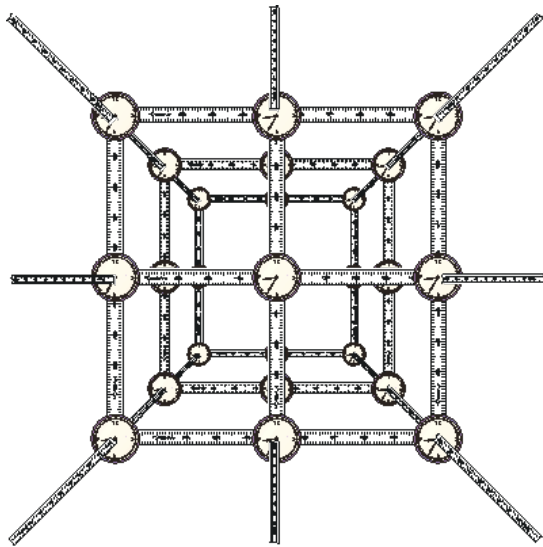
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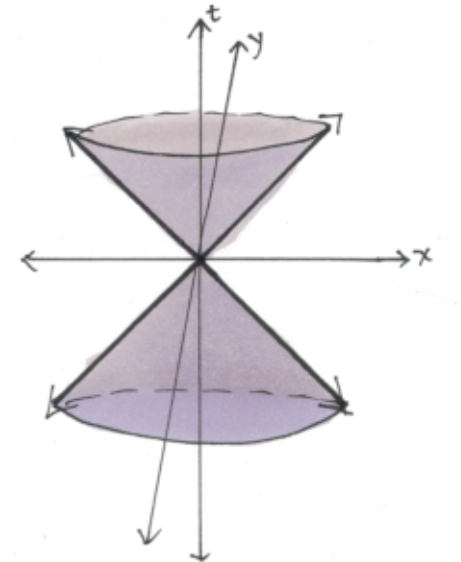
And where is encrypted the information regarding the spacetime?

Curved spacetime

- ◆ (I) Spacetime (the set of all events) is a four-dimensional manifold with a metric.
- ◆ (II) The **metric** is measurable by rods and clocks. The distance along a rod between two nearby points is $|\mathrm{d}\vec{x} \cdot \mathrm{d}\vec{x}|^{1/2}$ and the time measured by a clock that experiences two events closely separated in time is $|- \mathrm{d}\vec{x} \cdot \mathrm{d}\vec{x}|^{1/2}$.



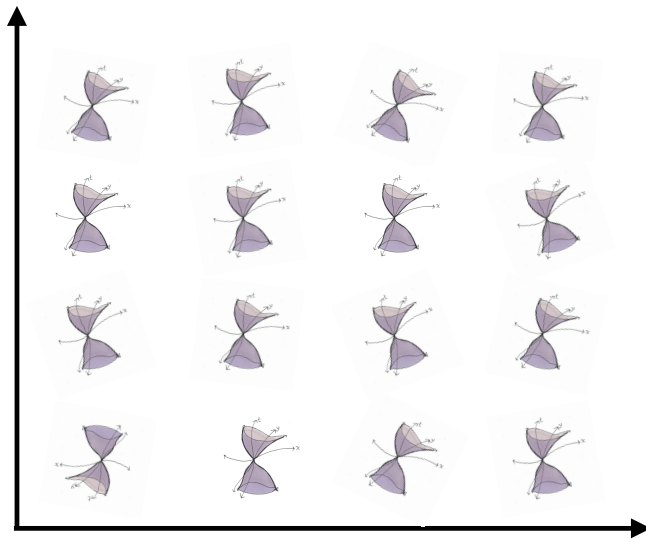
Minkowski spacetime



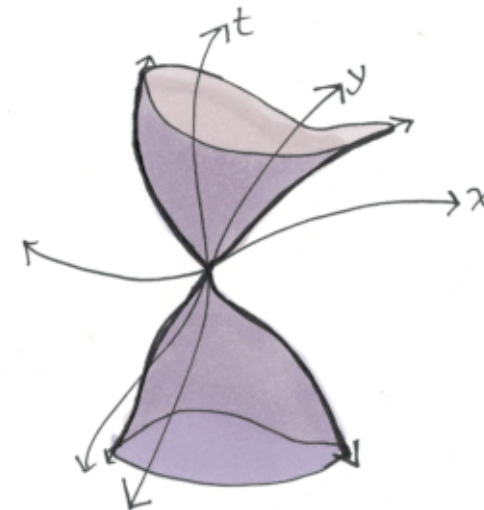
SpaceTime diagrams

Curved spacetime

- ◆ (I) Spacetime (the set of all events) is a four-dimensional manifold with a metric.
- ◆ (II) The metric is measurable by rods and clocks. The distance along a rod between two nearby points is $|\mathrm{d}\vec{x} \cdot \mathrm{d}\vec{x}|^{1/2}$ and the time measured by a clock that experiences two events closely separated in time is $|- \mathrm{d}\vec{x} \cdot \mathrm{d}\vec{x}|^{1/2}$.



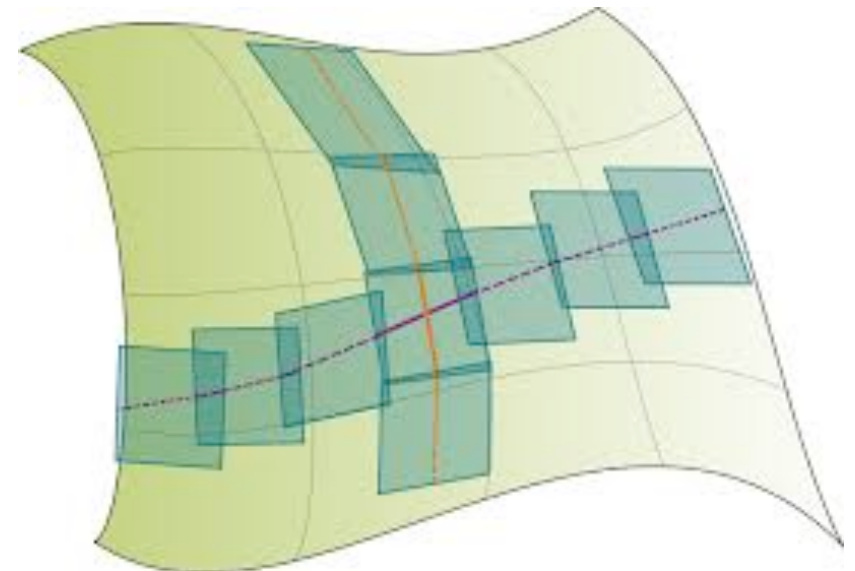
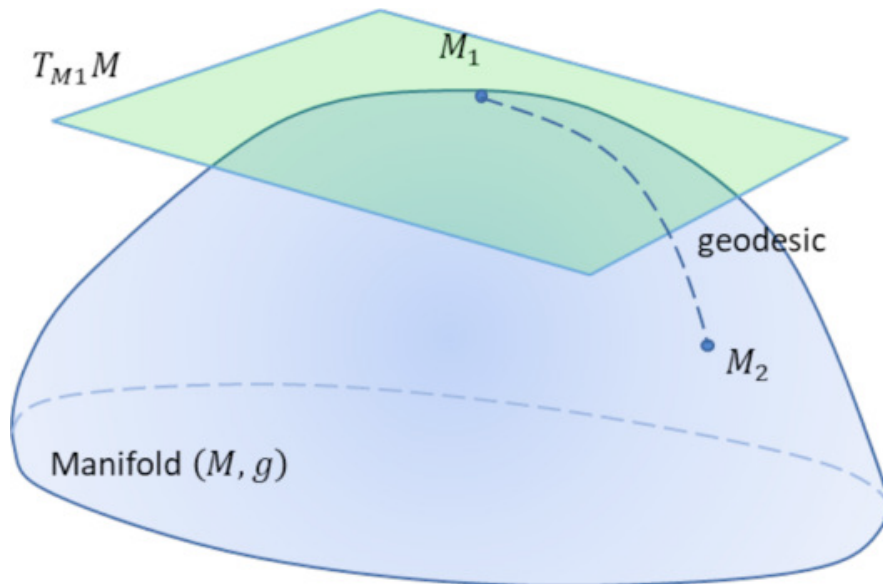
Curved spacetime



SpaceTime diagrams

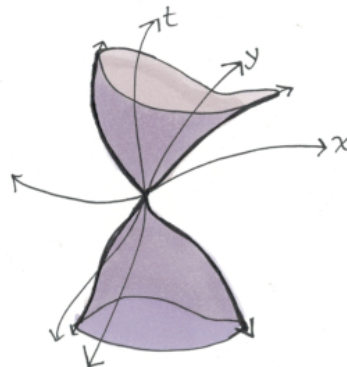
Curved spacetime

- ◆ (III) The metric of spacetime can be put in the Lorentz form $\eta_{\alpha\beta}$ at any particular event by an appropriate choice of coordinates.



Curved spacetime

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- ◆ (III) The metric of spacetime can be put in the Lorentz form $\eta_{\alpha\beta}$ at any particular event by an appropriate choice of coordinates.
- ◆ (IV) Freely falling particles move on timelike geodesics of the spacetime.

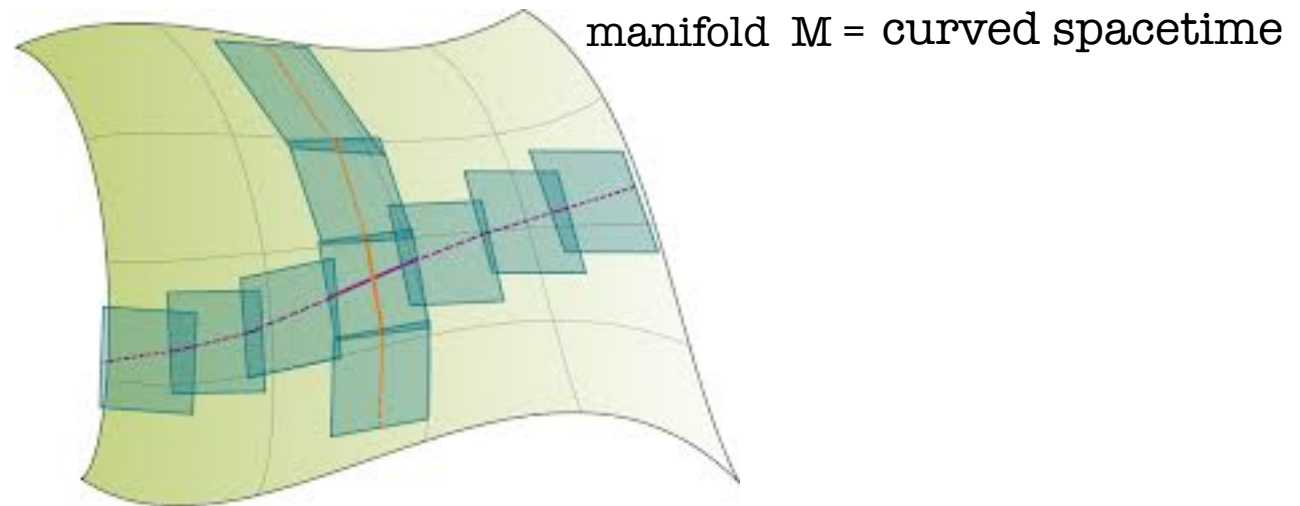


Curved spacetime

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- ◆ (III) The metric of spacetime can be put in the Lorentz form $\eta_{\alpha\beta}$ at any particular event by an appropriate choice of coordinates.
- ◆ (IV') Any physical law which can be expressed in tensor notation in SR has exactly the same form in a locally inertial frame of a curved spacetime.

called the [strong equivalence principle](#)

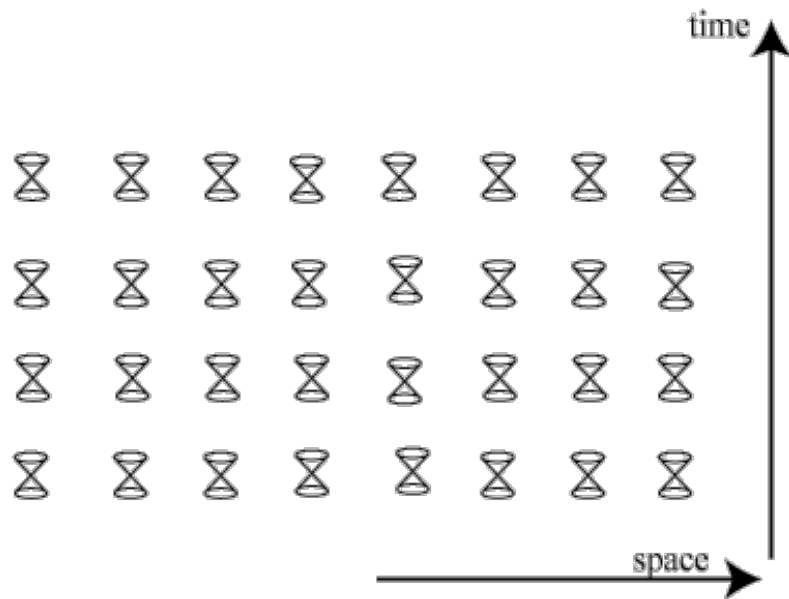
Pergunta: Por qué esta figura faz sentido?



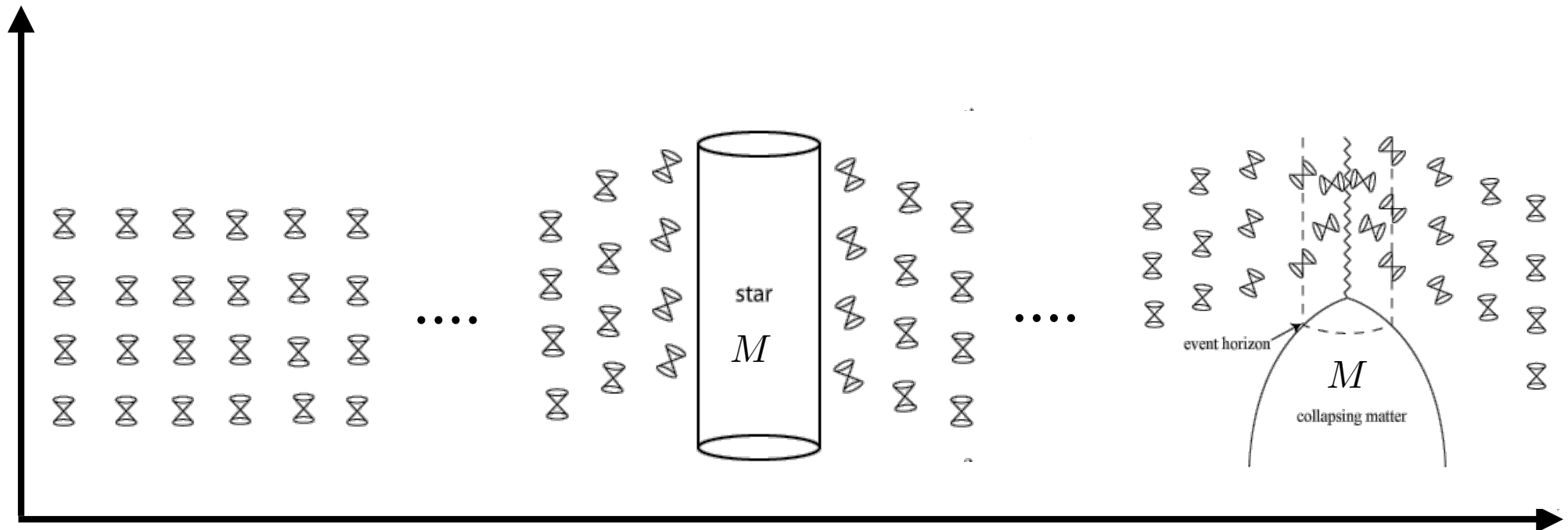
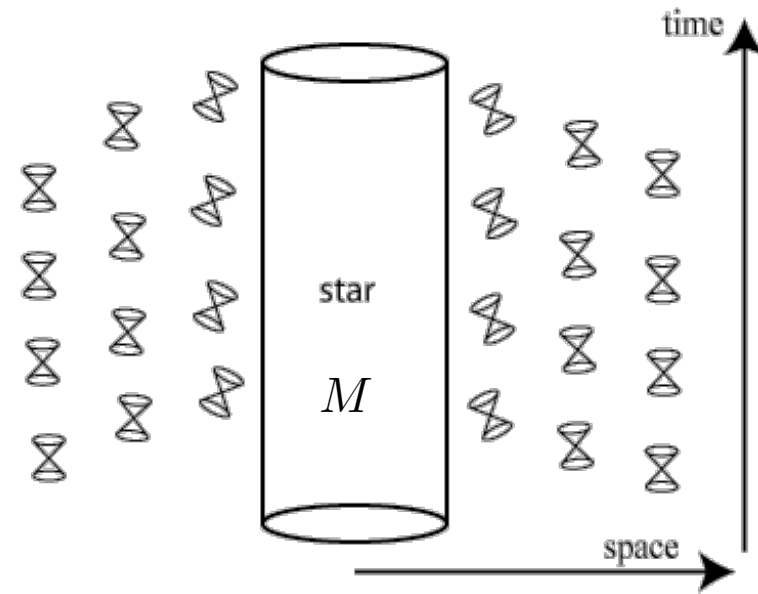
$$ds^2 = -c^2 dt^2 + a^2(t) d\mathbf{x}^2$$

$$ds^2 = -c^2 dt^2 + d\mathbf{x}^2$$

Minkowski spacetime



Curved spacetime



II. Constructing the

Curved Spacetime

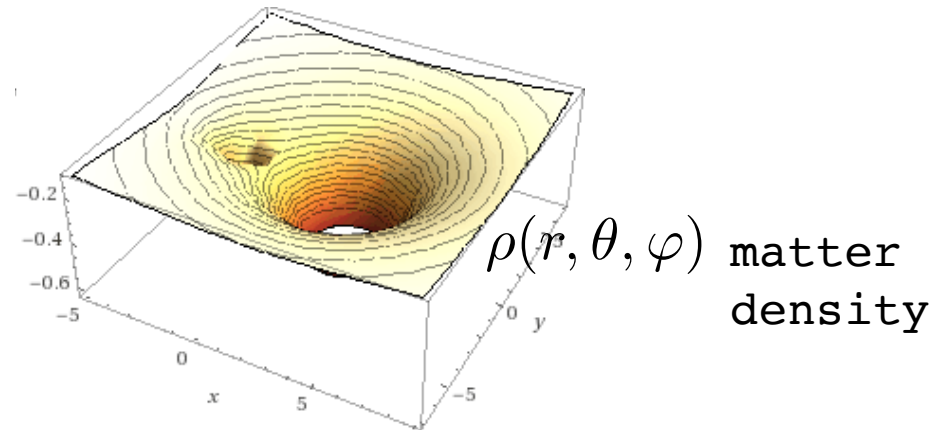
Newtonian theory of gravitation

We know that the source of a gravitational field in a certain region is due to the presence of a mass body in the neighborhood.

$$\vec{g} \equiv \frac{1}{m} \vec{F}_g = -\nabla\phi$$

$$\nabla \cdot \vec{g} = -4\pi G\rho$$

$$\nabla^2\phi = 4\pi G\rho \implies \text{Poisson eq.}$$



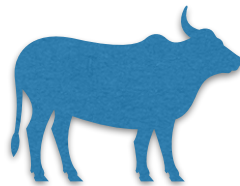
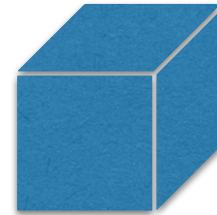
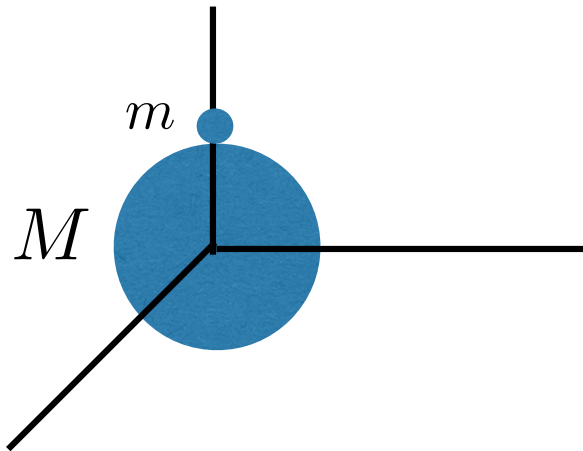
Knowing the matter density distribution $\rho(\vec{r})$, one solves this eq. to find the gravitational potential ϕ

If $\rho(r)$ has spherical symmetry, then $\phi = \phi(r)$ has.

Miscelânea:

Poisson equation for mass distributions

Given a mass distribution $\rho(\vec{r})$, find $\phi = \phi(\vec{r})$



.....

General Relativity theory (phenomenologically)

$$\nabla^2 \phi = 4\pi G \rho \quad \text{Poisson eq.}^*$$

* Does everyone know how to solve it?

$$\nabla^2 \phi = 4\pi G \rho \quad \text{Poisson eq.}$$

its solution for a point-like mass m , or very-very far from any mass distr., is

$$\phi = -\frac{G m}{r}$$

Exercise: what are the physical units of ϕ ?

In Newtonian theory, **mass** is the source of the gravitational field.

In GR, the source of gravity must be a relativistically meaningful concept:
invariant under spacetime coordinate transformations.

$$\nabla^2 \phi = 4\pi G \rho \quad \text{Poisson eq.}$$

Its solution for a point-like mass m , or very-very far from any mass distr., is

$$\phi = -\frac{G m}{r} \quad \text{Exercise: what are the physical units of } \phi ?$$

In Newtonian theory, **mass** is the source of the gravitational field.

In GR, the source of gravity must be a relativistically meaningful concept: **invariant under spacetime coordinate transformations**.

An invariant theory avoids introducing preferred coordinate systems by using the whole of the stress-energy tensor $\mathbf{T}$ as the source of the gravitational field. The generalization of the Poisson eq. is

$$\mathbf{O}(\mathbf{g}) = \kappa \mathbf{T}$$

where κ is a constant, and $\mathbf{O}$ is a differential operator of the **metric tensor** $\mathbf{g}$, which is the generalization of ϕ .

By analogy with Poisson eq., $\mathbf{O}$ should be a second order differential operator that produces a tensor of rank $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$, because in the eq. $\mathbf{O}(g) = \kappa \mathbf{T}$ $\mathbf{O}$ is associated to the $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ tensor $\mathbf{T}$.

In other words, $\{O^{\alpha\beta}\}$ must be the components of a $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ tensor and must be combinations of the **metric** $g_{\mu\nu}$, and its derivatives $g_{\mu\nu,\alpha}$, $g_{\mu\nu,\alpha\beta}$

From the field of Riemannian geometry, the Ricci tensor $R^{\alpha\beta}$ satisfies these conditions; more generally, any tensor of the form satisfies

$$O^{\alpha\beta} = R^{\alpha\beta} + A R g^{\alpha\beta} + \Lambda g^{\alpha\beta}$$

where A and Λ are constants. To determine the constant A we use the property of local conservation of energy and momentum (based on the strong equivalence principle)

$$T^{\alpha\beta}_{;\beta} = 0 \Rightarrow O^{\alpha\beta}_{;\beta} = 0$$

where both eqs. must be valid for any metric tensor. Because $g^{\alpha\beta}_{;\mu} = 0$

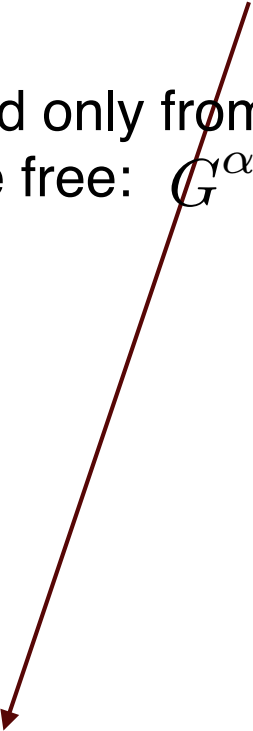
$$(R^{\alpha\beta} + A R g^{\alpha\beta})_{;\beta} = 0$$

Due to the Bianchi identities, the combination $R^{\alpha\beta} - \frac{1}{2} R g^{\alpha\beta}$

satisfies $(R^{\alpha\beta} - \frac{1}{2} R g^{\alpha\beta})_{;\beta} = 0$, then $A = -1/2$

Usually, one defines $G^{\alpha\beta} \equiv R^{\alpha\beta} - \frac{1}{2} R g^{\alpha\beta}$

tensor constructed only from the Riemann tensor and the **metric**,
and is divergence free: $G^{\alpha\beta}_{;\beta} = 0$



4 spacetime geometry:
metric, metric derivatives,
curvature (intrinsic!)

Geometry

Due to the Bianchi identities, the combination $R^{\alpha\beta} - \frac{1}{2} R g^{\alpha\beta}$

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tensor constructed only from the Riemann tensor and the **metric**,
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$G^{\alpha\beta}$ is the Einstein tensor, since its importance for gravity was first noticed by him. Because of the local conservation of energy and momentum $T^{\alpha\beta}_{;\beta} = 0$.

Einstein's idea:

4 spacetime geometry:
metric, metric derivatives,
curvature (intrinsic!)

Geometry



matter-energy content:
physical prop.,
conservation laws

Dynamics

Due to the Bianchi identities, the combination $R^{\alpha\beta} - \frac{1}{2} R g^{\alpha\beta}$

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tensor constructed only from the Riemann tensor and the **metric**,
and is divergence free: $G^{\alpha\beta}_{;\beta} = 0$

$G^{\alpha\beta}$ is the Einstein tensor, since its importance for gravity was first noticed by him. Because of the local conservation of energy and momentum $T^{\alpha\beta}_{;\beta} = 0$, Einstein postulates

$$G^{\alpha\beta} = \frac{8\pi}{c^4} T^{\alpha\beta}$$

4 spacetime geometry:
metric, metric derivatives,
curvature (intrinsic!)

Geometry

matter-energy content:
physical prop.,
conservation laws

Dynamics

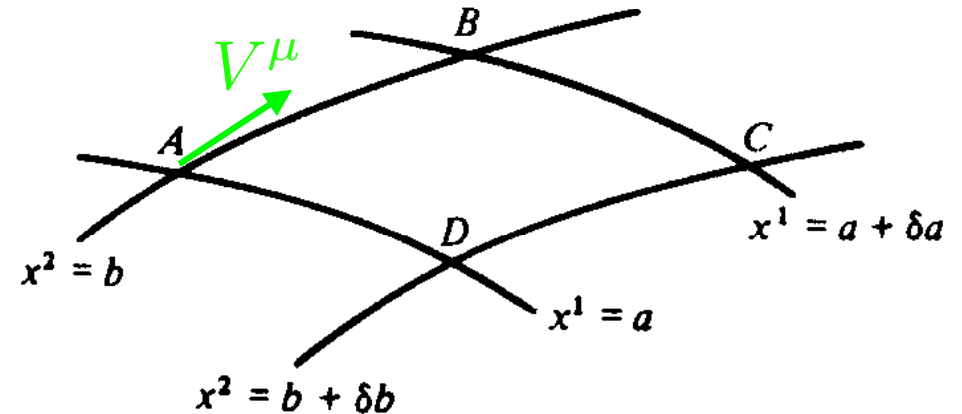
G.F.B. Riemann (1826–1866)



Differential geometry

$$R^{\alpha}_{\beta\mu\nu}$$

How can I understand curvature, in practice?



Small section of a coordinate grid.

$$\delta V^{\alpha} = \delta a \delta b [\Gamma^{\alpha}_{\mu 1, 2} - \Gamma^{\alpha}_{\mu 2, 1} + \Gamma^{\alpha}_{\nu 2} \Gamma^{\nu}_{\mu 1} - \Gamma^{\alpha}_{\nu 1} \Gamma^{\nu}_{\mu 2}] V^{\mu}$$

$$R^{\alpha}_{\beta\mu\nu} \equiv \Gamma^{\alpha}_{\beta\nu, \mu} - \Gamma^{\alpha}_{\beta\mu, \nu} + \Gamma^{\alpha}_{\sigma\mu} \Gamma^{\sigma}_{\beta\nu} - \Gamma^{\alpha}_{\sigma\nu} \Gamma^{\sigma}_{\beta\mu}$$

G.F.B. Riemann (1826–1866)



Differential geometry

$$R^{\alpha}_{\beta\mu\nu}$$

How can I understand curvature, in practice?

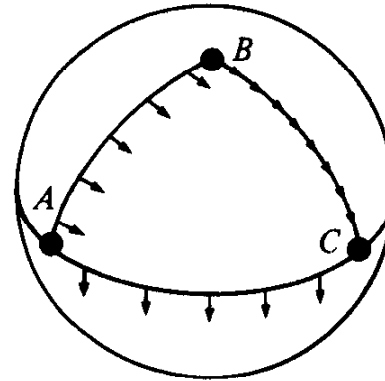
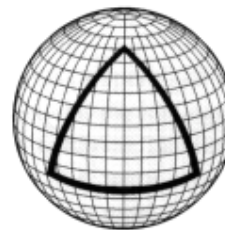


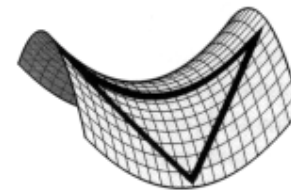
Fig. 6.3 Parallel transport around a spherical triangle.



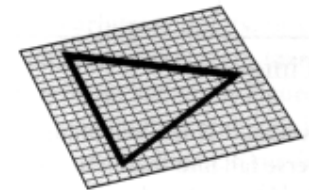
$$R^{\alpha}_{\beta\mu\nu} = 0 \Leftrightarrow \text{flat manifold.}$$
$$\forall \alpha, \beta, \mu, \nu$$



Positive Curvature



Negative Curvature



Flat Curvature

Attention: these are 2D exms.!

G.F.B. Riemann (1826–1866)



Differential geometry

$$R^{\alpha}_{\beta\mu\nu}$$

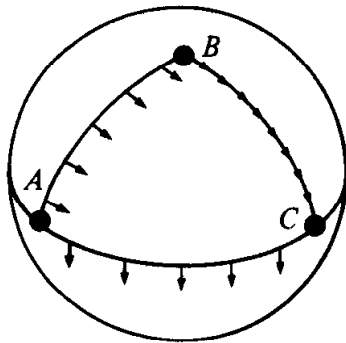
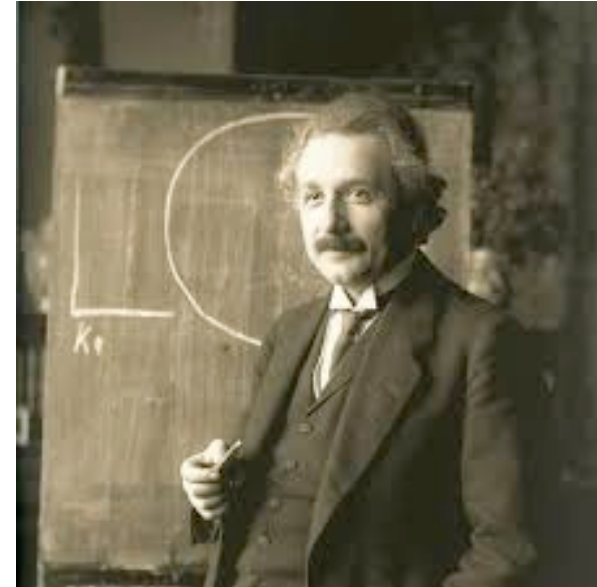


Fig. 6.3 Parallel transport around a spherical triangle.

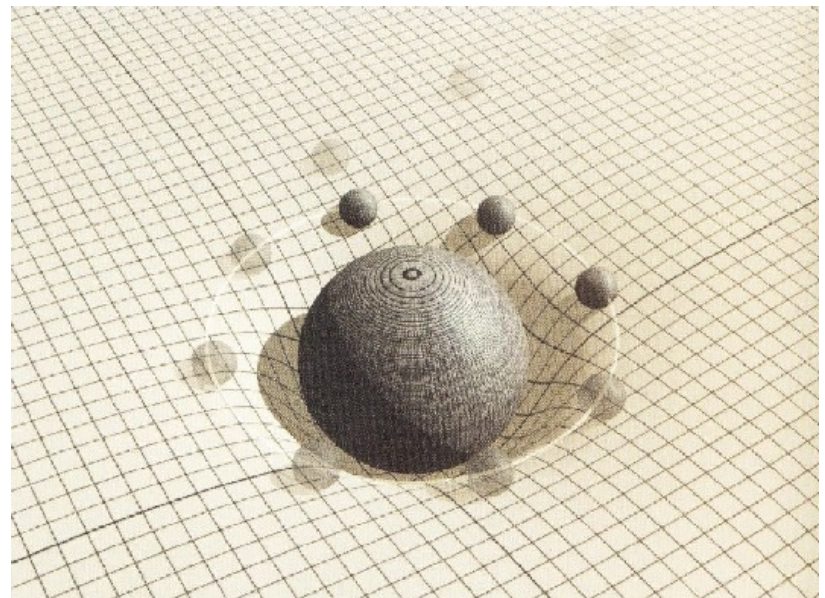
$$R^{\alpha}_{\beta\mu\nu} \equiv \Gamma^{\alpha}_{\beta\nu,\mu} - \Gamma^{\alpha}_{\beta\mu,\nu} + \Gamma^{\alpha}_{\sigma\mu}\Gamma^{\sigma}_{\beta\nu} - \Gamma^{\alpha}_{\sigma\nu}\Gamma^{\sigma}_{\beta\mu}$$

A. Einstein (1879–1955)



General Relativity

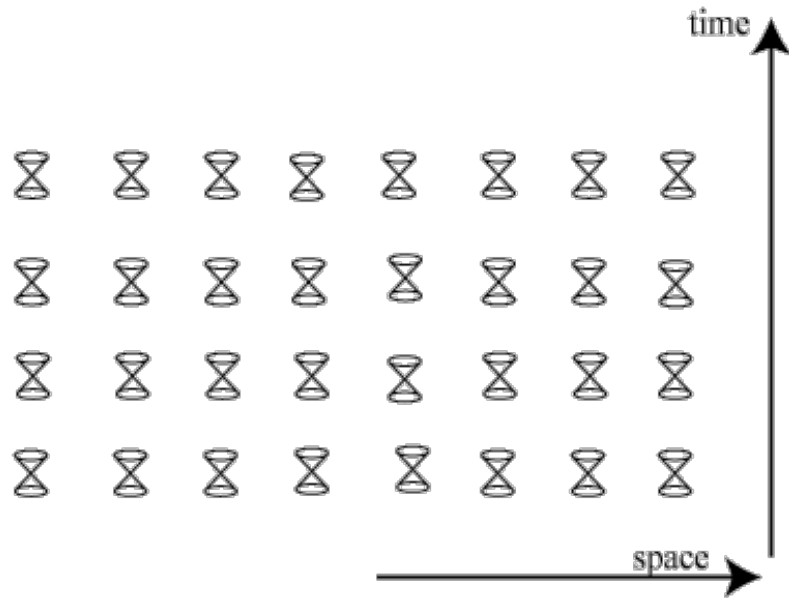
$$G^{\alpha\beta} \equiv R^{\alpha\beta} - \frac{1}{2} R g^{\alpha\beta} = \frac{8\pi}{c^4} T^{\alpha\beta}$$



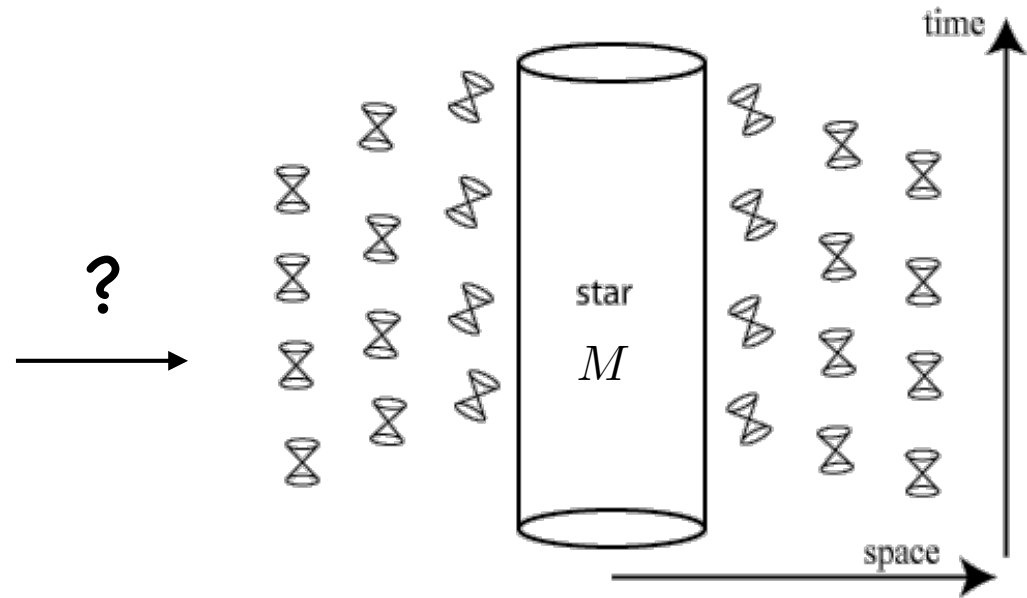
Now, we have the basics to (try to)
describe the observed Universe

From Minkowski to a curved spacetime

Minkowski spacetime



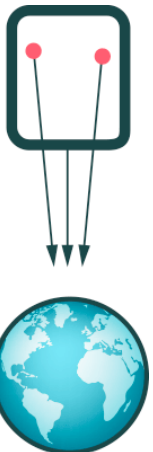
Curved spacetime



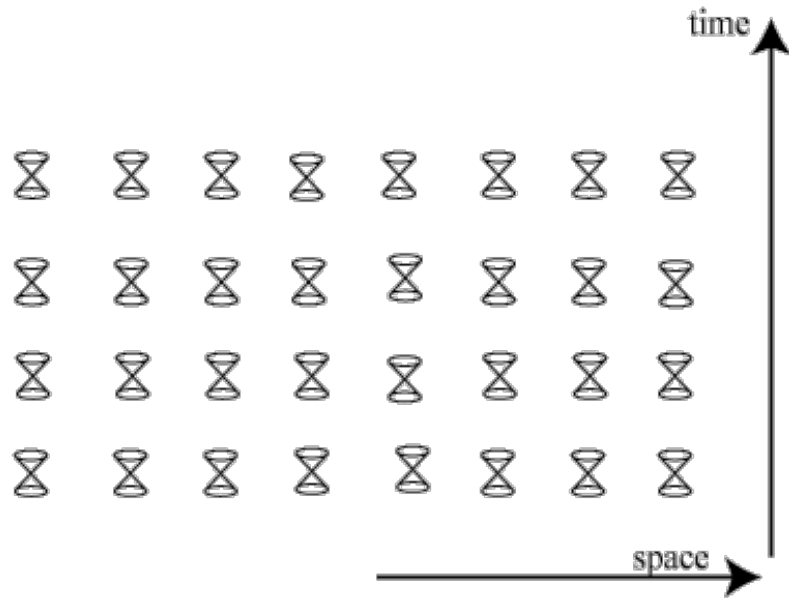
?

$$ds^2 = -c^2 dt^2 + d\mathbf{x}^2 \quad \xrightarrow{?} \quad ds^2 = -c^2 dt^2 + \underbrace{\left(1 + \frac{2GM}{c^2 r}\right)}_{\text{the gravitational field affects timelike geodesics}} d\mathbf{x}^2$$

the gravitational field
affects timelike geodesics

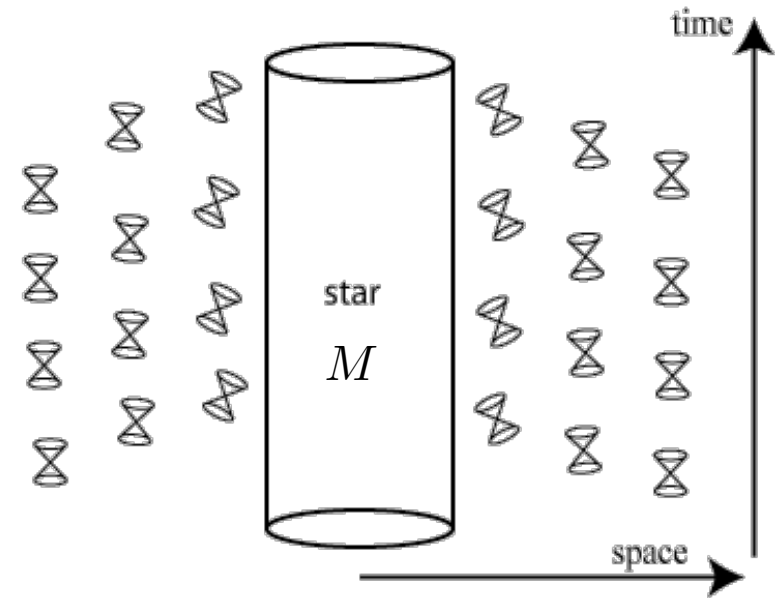


Minkowski spacetime



?

Curved spacetime



$$ds^2 = -c^2 dt^2 + d\mathbf{x}^2$$

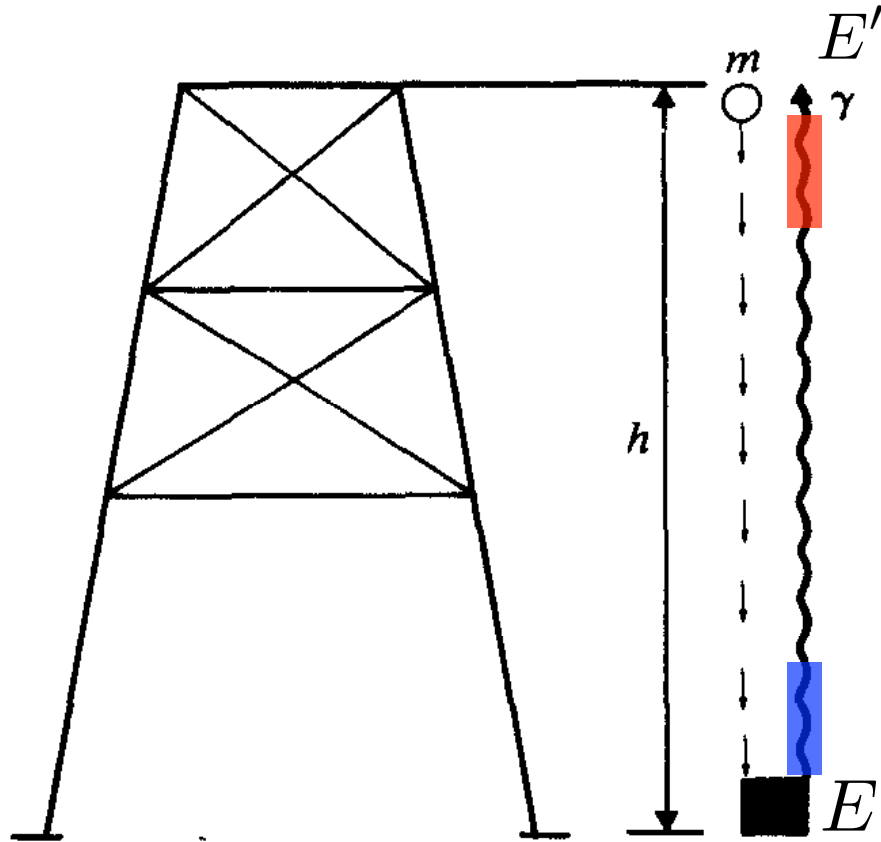
Why?

$$ds^2 = -c^2 dt^2 + \left(1 + \frac{2GM}{c^2 r}\right) d\mathbf{x}^2 \quad \times$$

$$ds^2 = -c^2 \underbrace{\left(1 - \frac{2GM}{c^2 r}\right)}_{\text{Por que?}} dt^2 + \underbrace{\left(1 + \frac{2GM}{c^2 r}\right)}_{\text{the gravitational field affects timelike geodesics}} d\mathbf{x}^2 \quad \checkmark$$

Gravitational Redshift

Gravitational Redshift

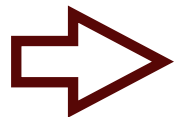


First suggested by Einstein (1911?),
experimentally proved by **Pound & Rebka** (1960)

Pound, R. V. & Rebka, G. A. (1960) *Phys. Rev. Lett.*, **4**, 337.

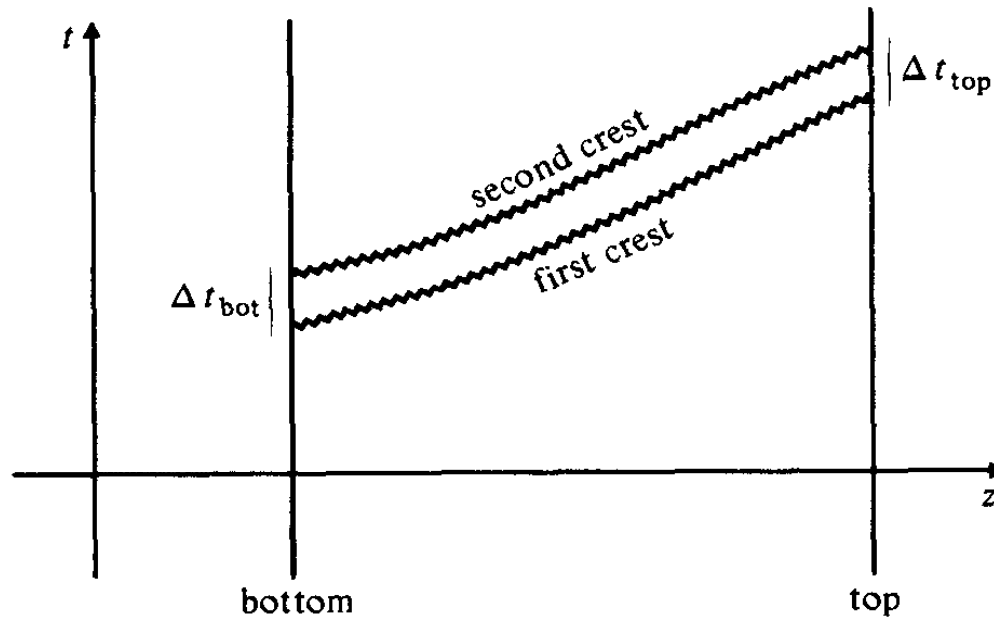
Misner, C. W., Thorne, K. S. & Wheeler, J. A. (1973) *Gravitation*

◆
$$\frac{\text{top}}{\text{bottom}} \frac{E'}{E} = \frac{h\nu'}{h\nu} = \frac{m}{m + mgh + \mathcal{O}(v^4)} = 1 - gh + \mathcal{O}(v^4)$$



A photon climbing in Earth's gravitational field will lose energy
and will consequently be redshifted

Pound-Rebka-Snider experiment



$$\nu_{\text{bottom}} \equiv \nu > \nu' \equiv \nu_{\text{top}}$$

Since: $\Delta t = 1/\nu$

⇒ $\Delta t_{\text{top}} > \Delta t_{\text{bottom}}$

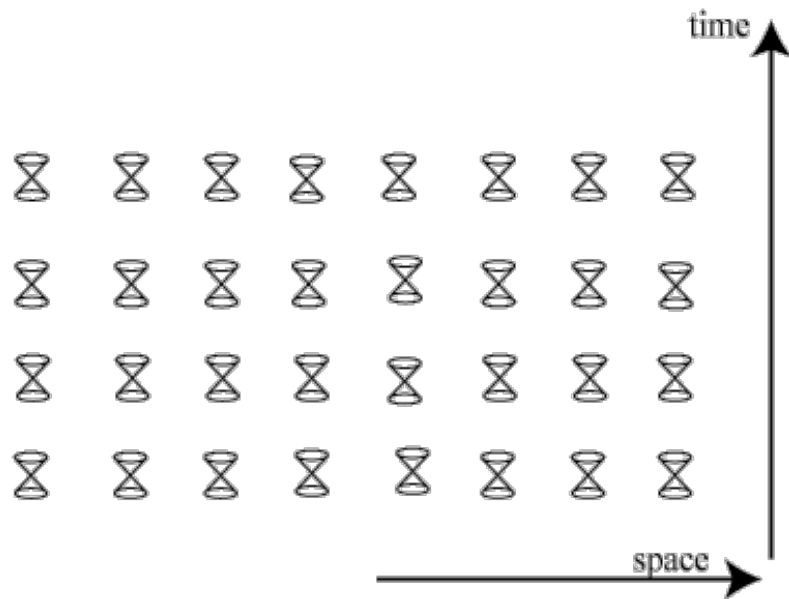
$$\frac{\nu'_{\text{(apparatus at top)}}}{\nu_{\text{(freely falling/bottom)}}} = 1 - gh + \mathcal{O}(v^4)$$

⇒ $\frac{\Delta \nu}{\nu} = -\frac{gh}{c^2}$ **Gravitational redshift**

Corollary: A gravitational field induces a time dilation in the spacetime metric

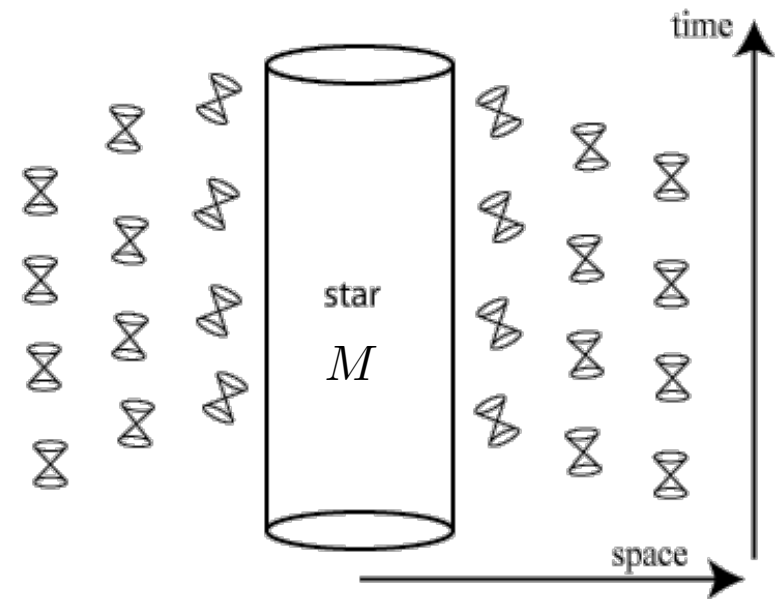
$$\frac{\delta t}{t} = \frac{gh}{c^2}$$

Minkowski spacetime



?

Curved spacetime



$$ds^2 = -c^2 dt^2 + d\mathbf{x}^2$$

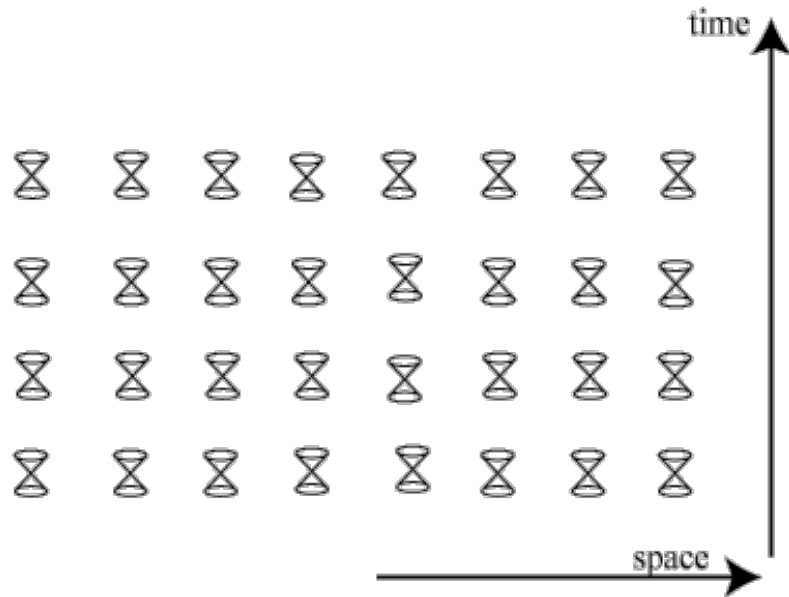
✗

$$ds^2 = -c^2 dt^2 + \left(1 + \frac{2GM}{c^2 r}\right) d\mathbf{x}^2$$

✓

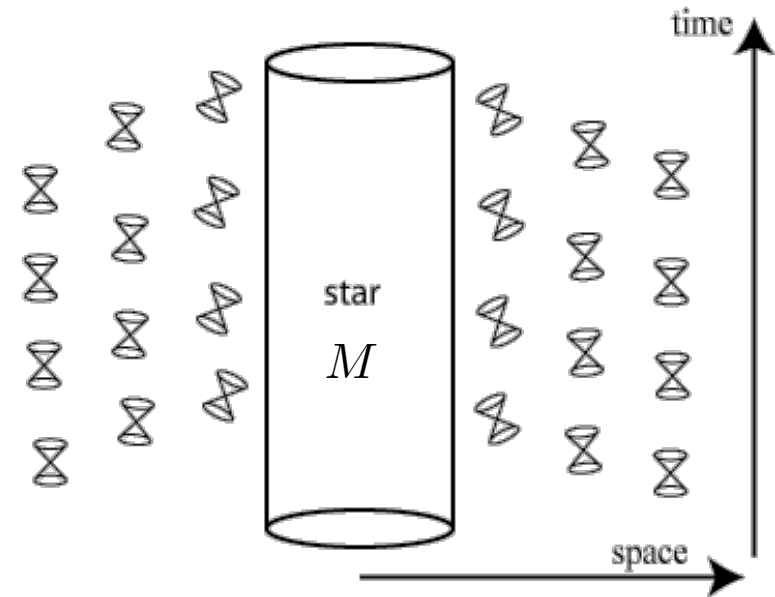
$$ds^2 = -c^2 \underbrace{\left(1 - \frac{2GM}{c^2 r}\right)}_{\text{time dilation due to a gravitational field; produces gravitational redshift}} dt^2 + \underbrace{\left(1 + \frac{2GM}{c^2 r}\right)}_{\text{the gravitational field affects timelike geodesics}} d\mathbf{x}^2$$

Minkowski spacetime



?

Curved spacetime

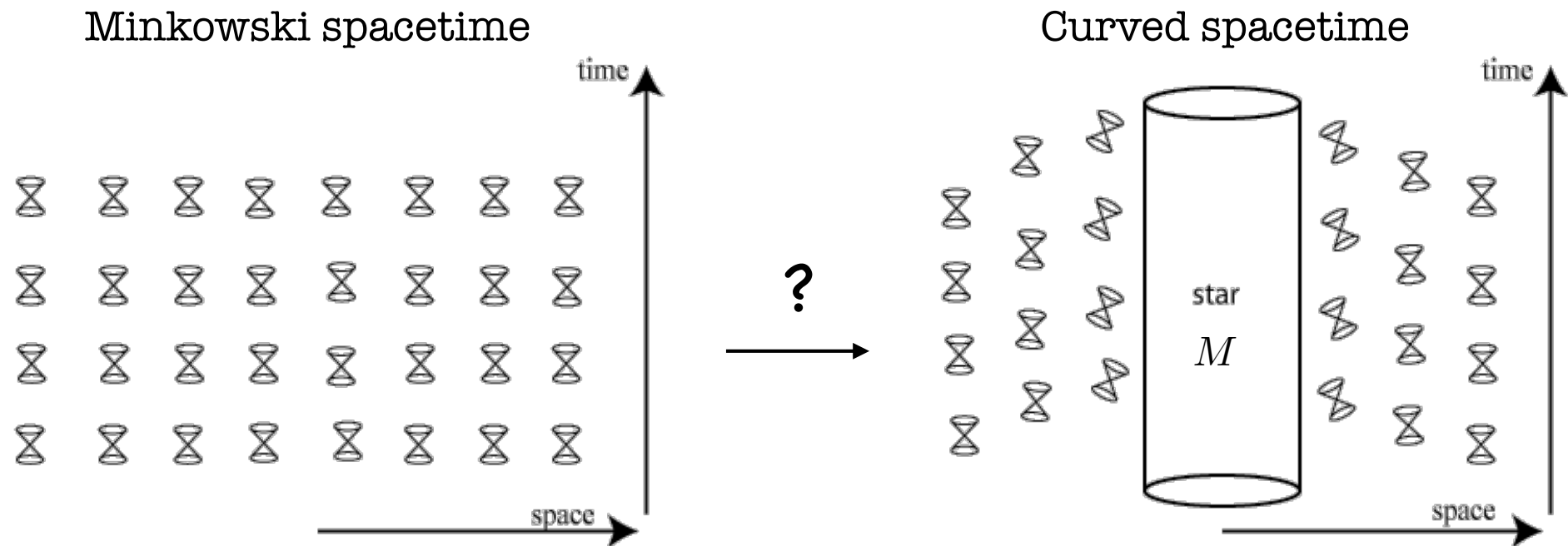


More generally, the perturbed **metric** describes the clumpy spacetime:

$$ds^2 = -c^2 dt^2 + d\mathbf{x}^2 \longrightarrow ds^2 = -c^2 \left(1 + \frac{2\phi}{c^2}\right) dt^2 + \left(1 - \frac{2\phi}{c^2}\right) d\mathbf{x}^2$$

where $\phi = \phi(\mathbf{x})$, is the gravitational potential of a mass-density fluctuation $\delta\rho(\mathbf{x})$

As a particular case $\phi = -GM/r$, is the gravitational potential of a mass M.



More generally, the perturbed **metric** describes the clumpy spacetime:

$$ds^2 = -c^2 dt^2 + d\mathbf{x}^2 \longrightarrow ds^2 = -c^2 \left(1 + \frac{2\phi}{c^2}\right) dt^2 + \left(1 - \frac{2\phi}{c^2}\right) d\mathbf{x}^2$$

where $\phi = \phi(\mathbf{x})$, is the gravitational potential of a mass-density fluctuation $\delta\rho(\mathbf{x})$

Exercise: Calcule o valor de ϕ/c^2 na superfície da Terra, do Sol, de uma estrela de neutrons.

Physical interpretation:

$$ds^2 = 0$$

$$|\phi/c^2| \ll 1$$



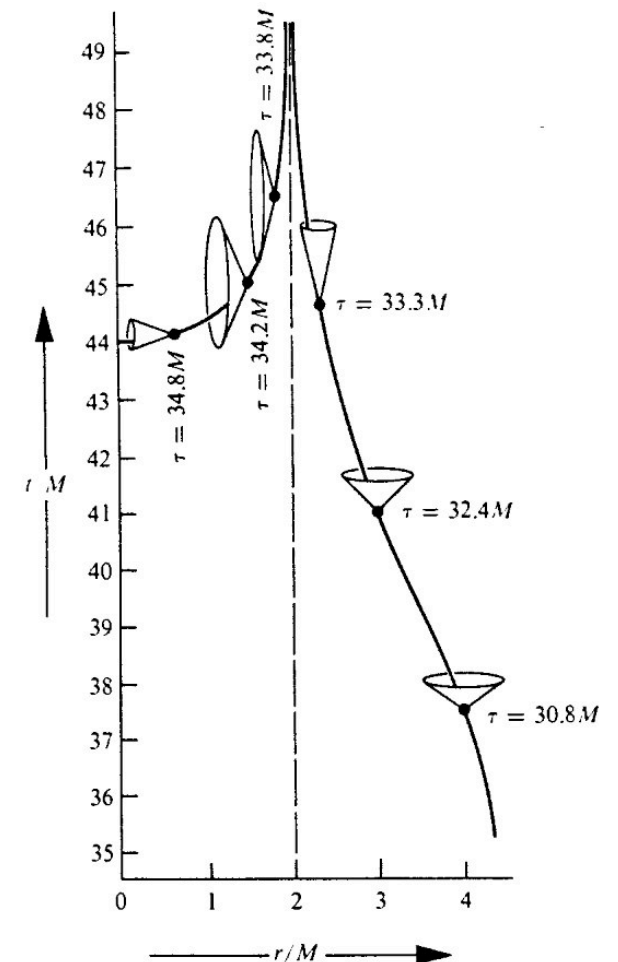
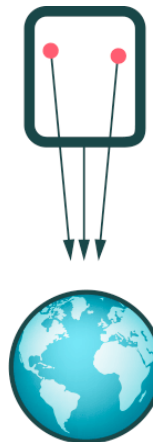
$$\frac{dx^2}{dt^2} = c^2 \frac{(1 + \frac{2\phi}{c^2})}{(1 - \frac{2\phi}{c^2})} = c^2 \left(1 + \frac{2\phi}{c^2}\right)^2$$

$$\frac{dx}{dt} = c \left(1 + \frac{2\phi}{c^2}\right) < c, \text{ because } \phi < 0$$

For $\frac{\phi}{c^2} \rightarrow -\frac{1}{2} \Rightarrow$

$$\frac{dx}{dt} \rightarrow 0$$

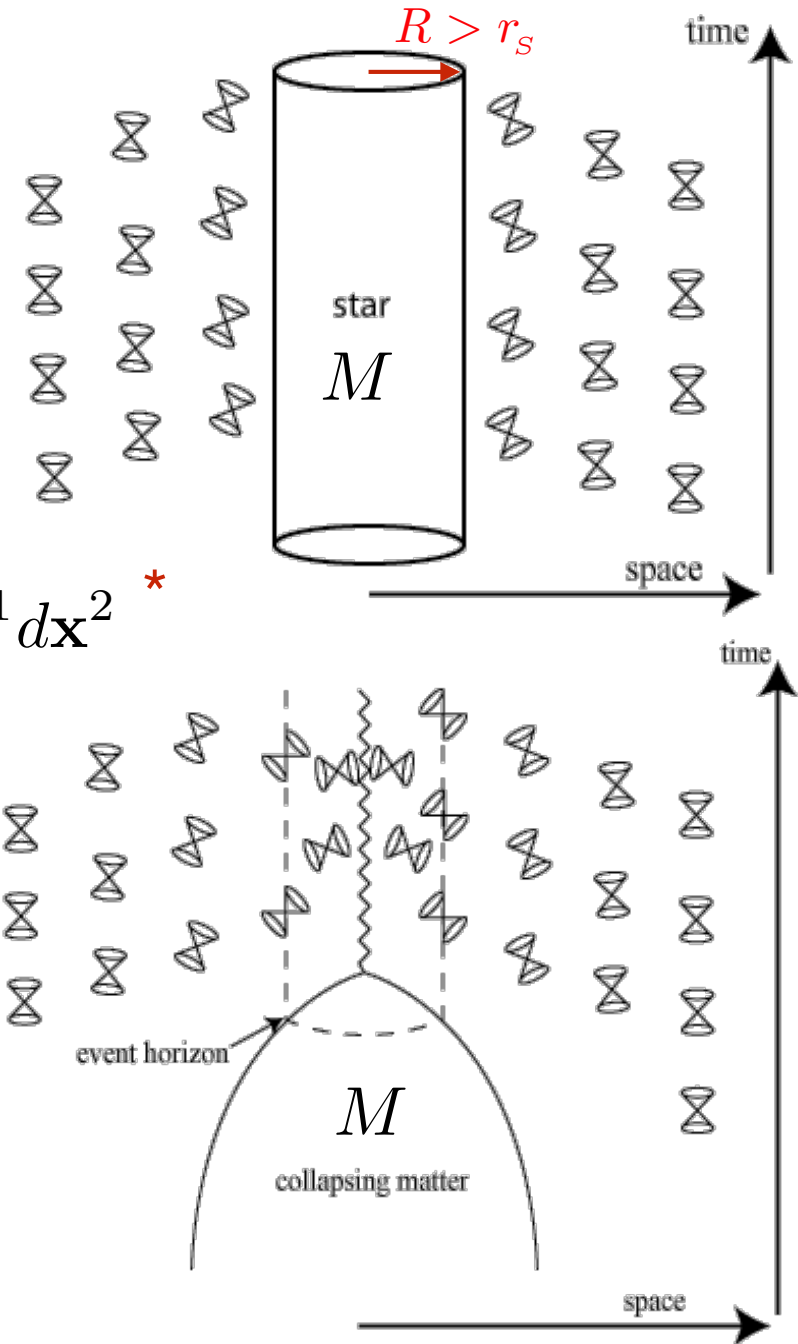
i.e., light-cone closes
and timelike geodesics
come together more
and more!



Curved spacetime

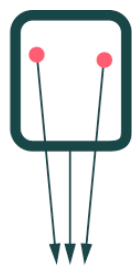
$$r > \frac{2GM}{c^2} \equiv r_s$$

$$ds^2 = -c^2\left(1 - \frac{2GM}{c^2 r}\right)dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1}d\mathbf{x}^2$$



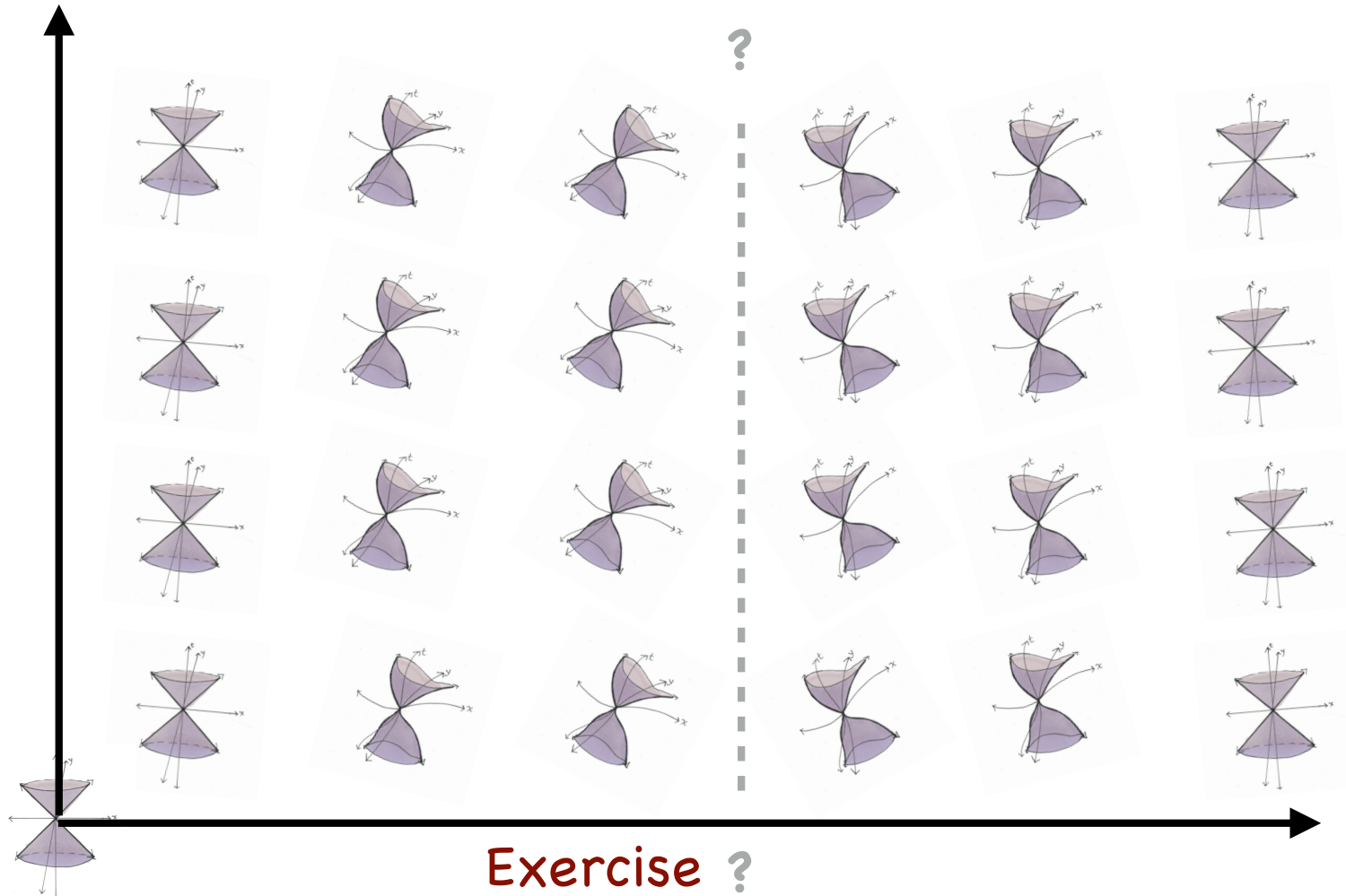
* Schwarzschild metric

Exercise



?

Question: What is the source of the gravitational field?



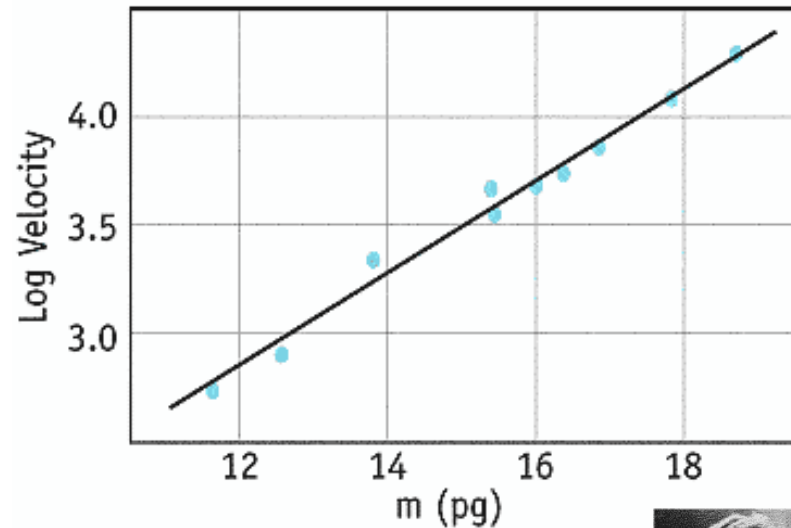
Exercise ?

The expanding spacetime

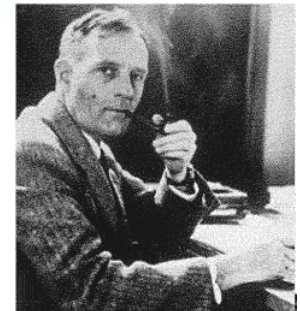
- Theory (model) versus Observations:



?



Edwin Hubble, 1929



Astronomical observations

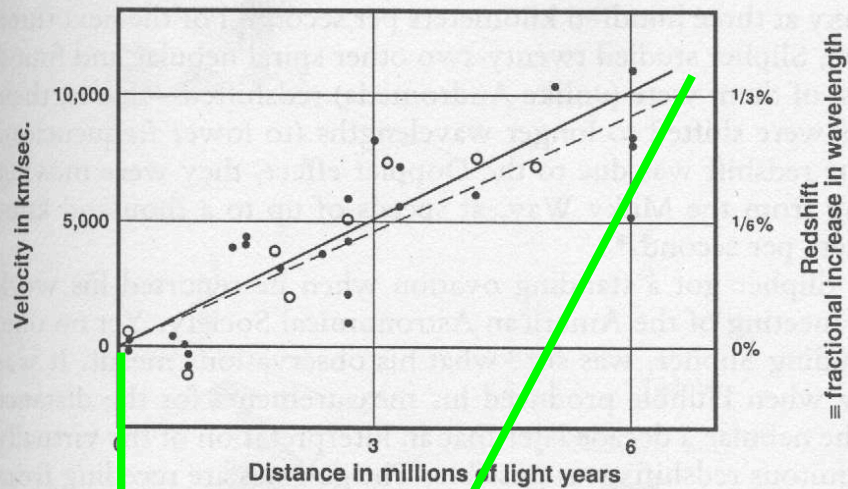
Hubble, 1929

$$v = H_0 d$$

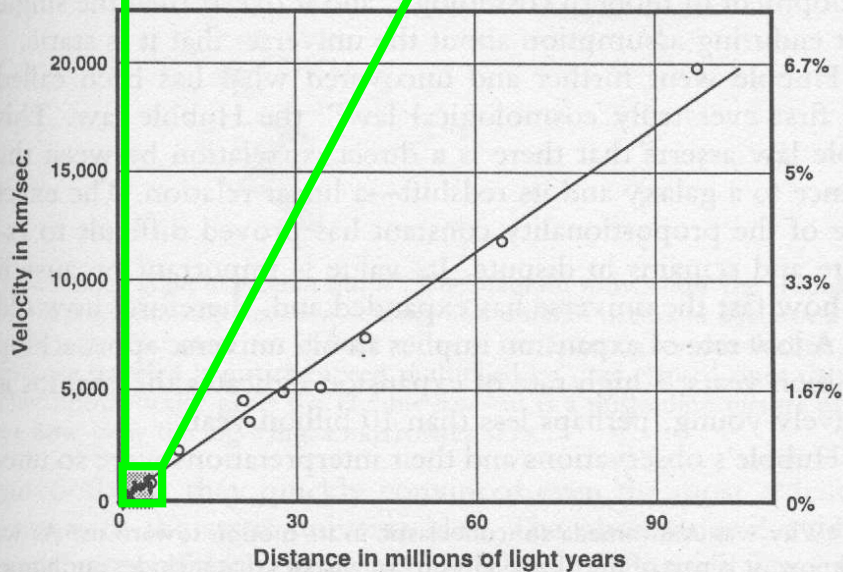
Hubble &
Humason, 1931

Hubble's velocity vs. distance plot of 1929.

The black discs and full line represent the Hubble law fitted to the nebulae individually. The circles and broken line represent the solution combining the nebulae into groups.

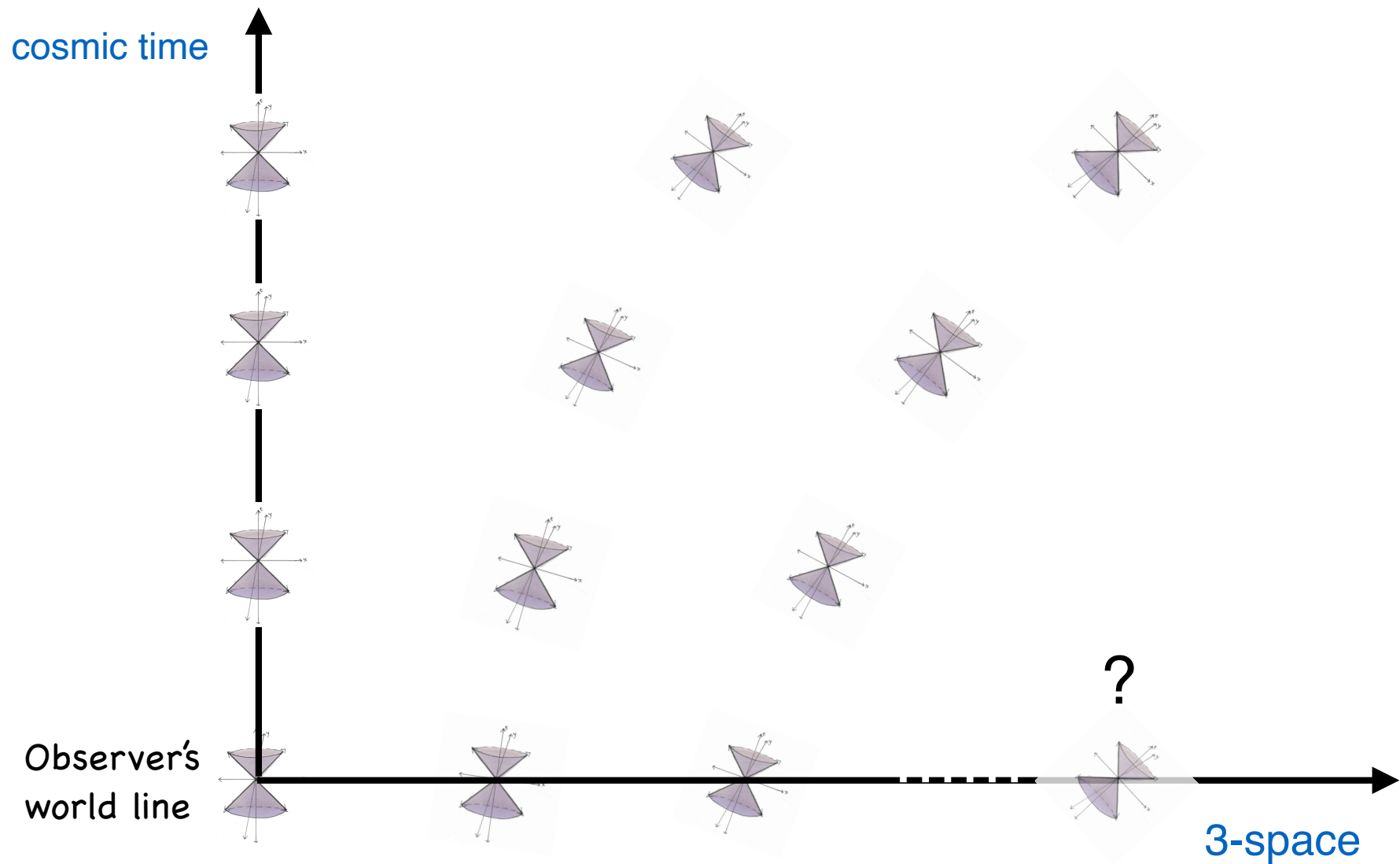


Hubble and Humason's 1931 velocity vs. distance diagram



Hubble law data. The upper plot shows Hubble's original data on distance to nebulae (galaxies) and their red shift (or velocity using the Doppler effect relation). The lower plot shows Hubble's original data in the shaded region and the new data collected with Milton Humason. The new data provide a convincing argument for the linear relationship between distance and red shift.

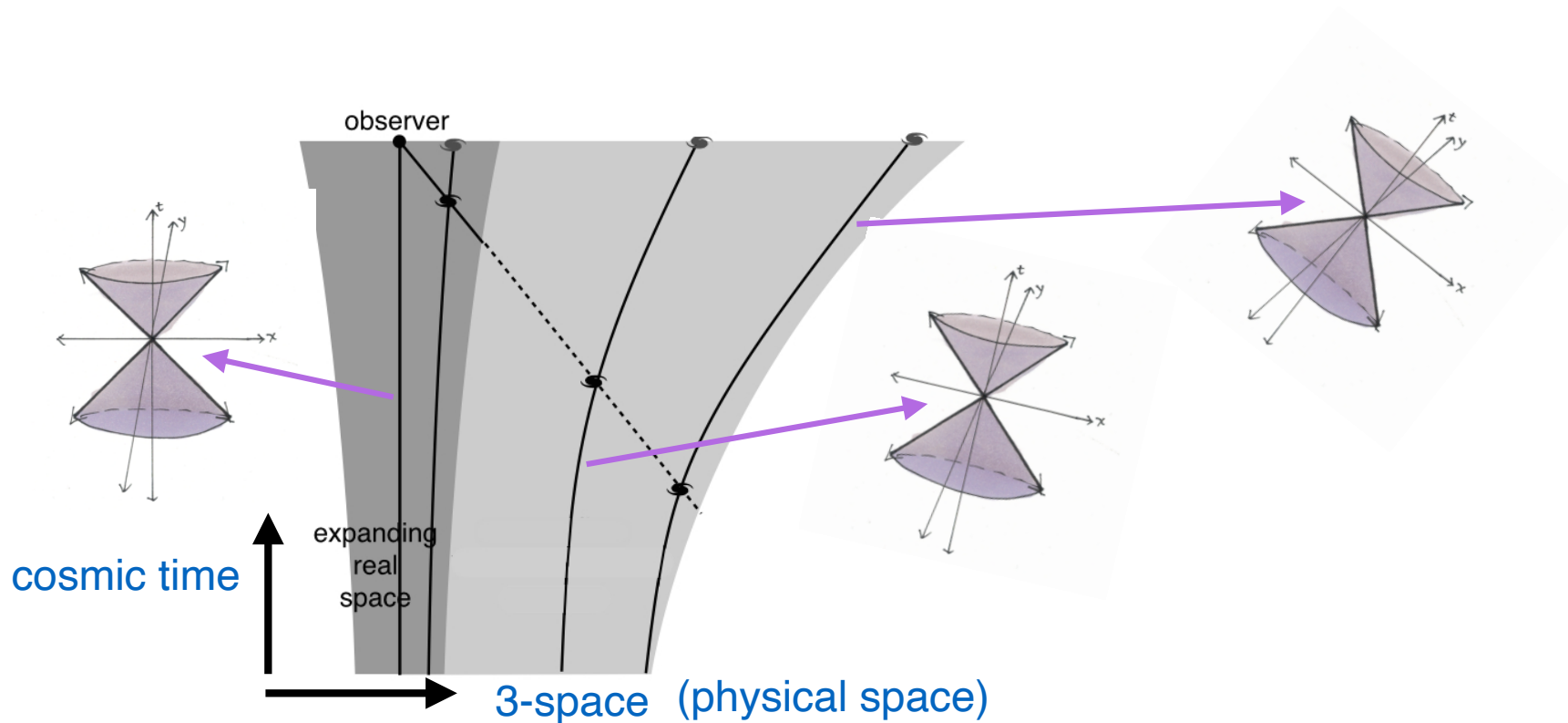
CHRISTOPHER SLYE



World lines of test particles: The particles velocity is given by the tangent line to the world line

Expanding spacetime: the Hubble flow

$$ds^2 = -c^2 dt^2 + a^2(t)[dr^2 + r^2 d\Omega^2]$$

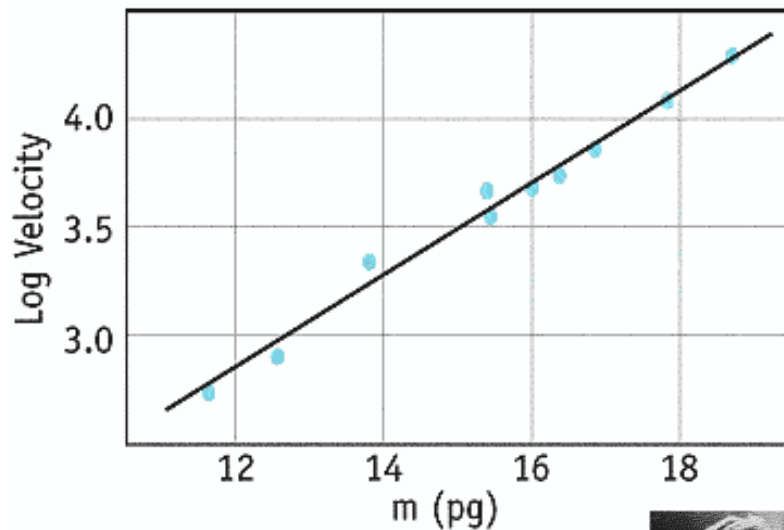
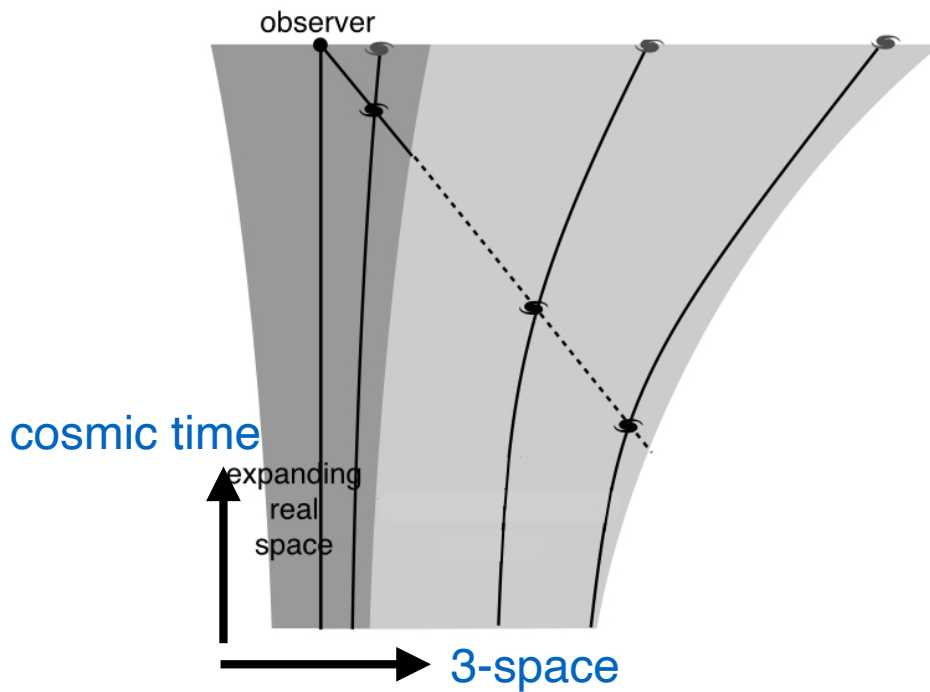


- ◆ (iv) Freely falling particles move on timelike geodesics of the spacetime.

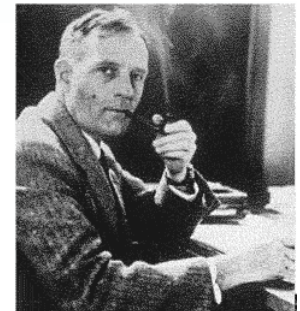
The observed Universe

- Theory (model) versus Observation:

$$ds^2 = -c^2 dt^2 + a^2(t)[dr^2 + r^2 d\Omega^2]$$



Edwin Hubble, 1929

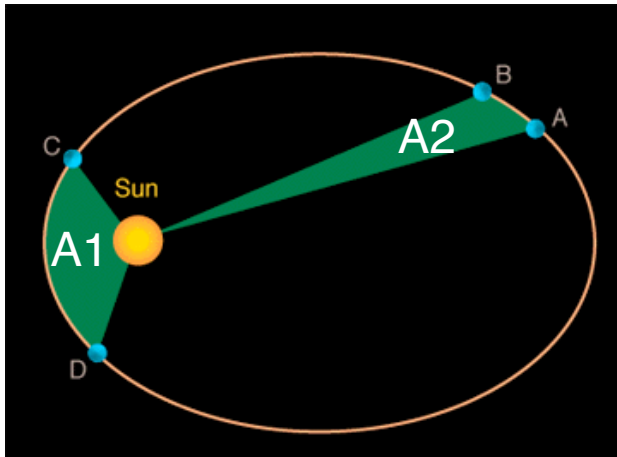
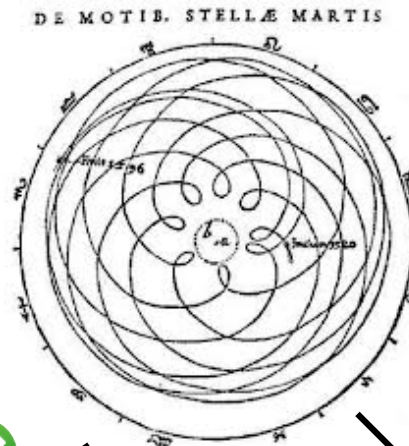


O que aprendemos hoje?

(60 minutos em 5+1 slides)

0

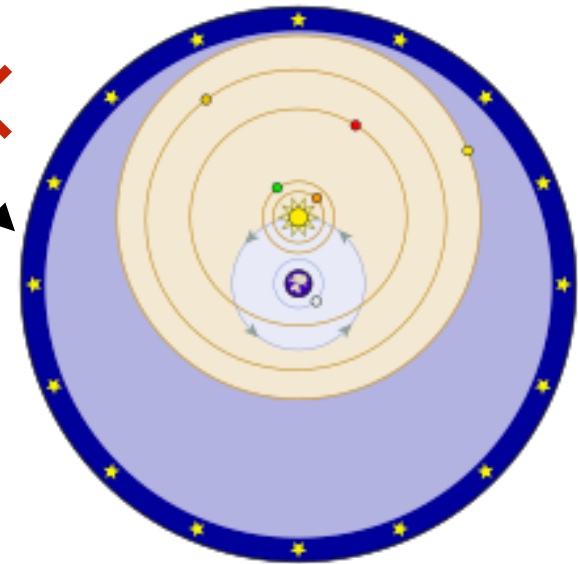
Observations



Model



1. Orbits are ellipses
2. $A_1 = A_2$
3. $T^2 \propto \bar{R}^3$



Johannes Kepler

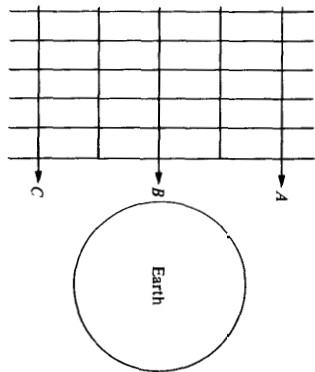
Tycho Brahe

Observations determine the correct Model!

1

Is gravity a special force? ➡ Yes!

Gravity is **distinguished** from all other forces: all bodies given the same initial velocity follow the same trajectory in a gravitational field, regardless of their internal composition



Conclusion 0:

There is no single freely falling **global** frame which is everywhere freely falling in Earth's gravitational field and which is still **rigid**

Conclusion I:

Neither Minkowski spacetime nor the theory of special relativity works to describe (accelerated particles in) gravitational fields!



Conclusion II:

We need to construct a curved spacetime to describe gravitational fields!

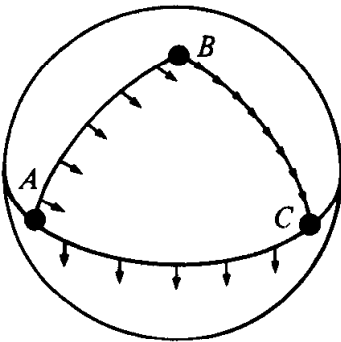
2

G.F.B. Riemann (1826–1866)



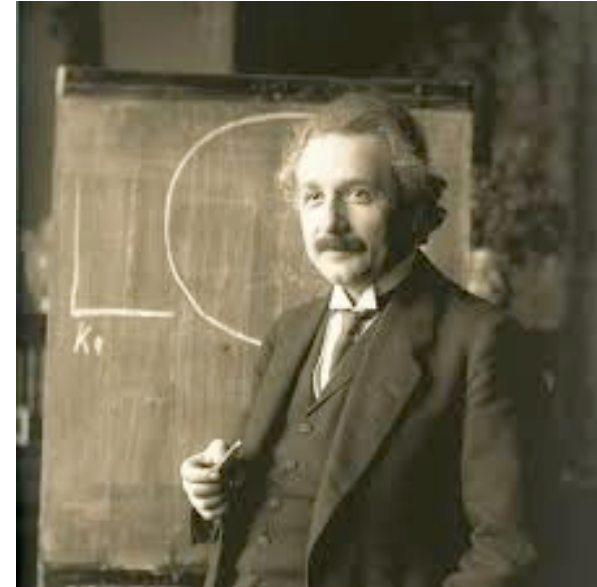
Differential geometry

$$R^{\alpha}_{\beta\mu\nu}$$



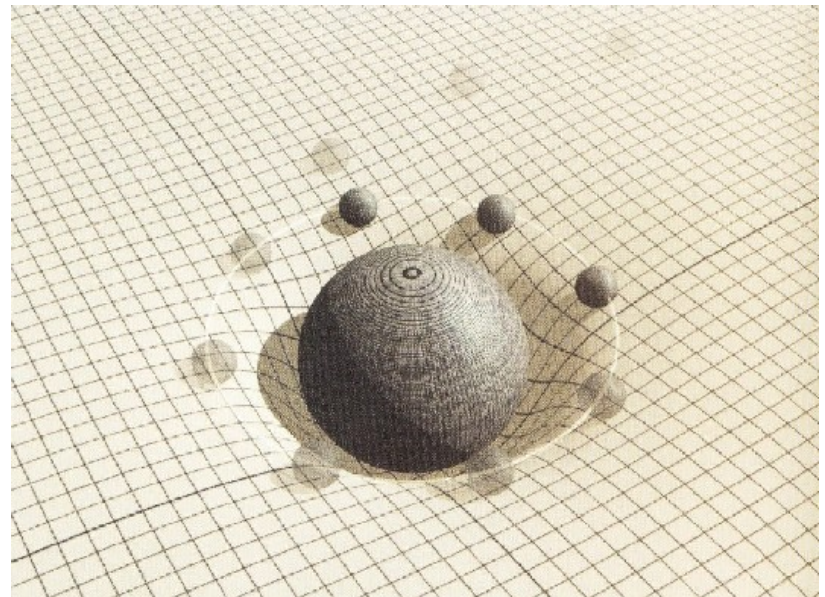
$$R^{\alpha}_{\beta\mu\nu} \equiv \Gamma^{\alpha}_{\beta\nu,\mu} - \Gamma^{\alpha}_{\beta\mu,\nu} + \Gamma^{\alpha}_{\sigma\mu}\Gamma^{\sigma}_{\beta\nu} - \Gamma^{\alpha}_{\sigma\nu}\Gamma^{\sigma}_{\beta\mu}$$

A. Einstein (1879–1955)



General Relativity

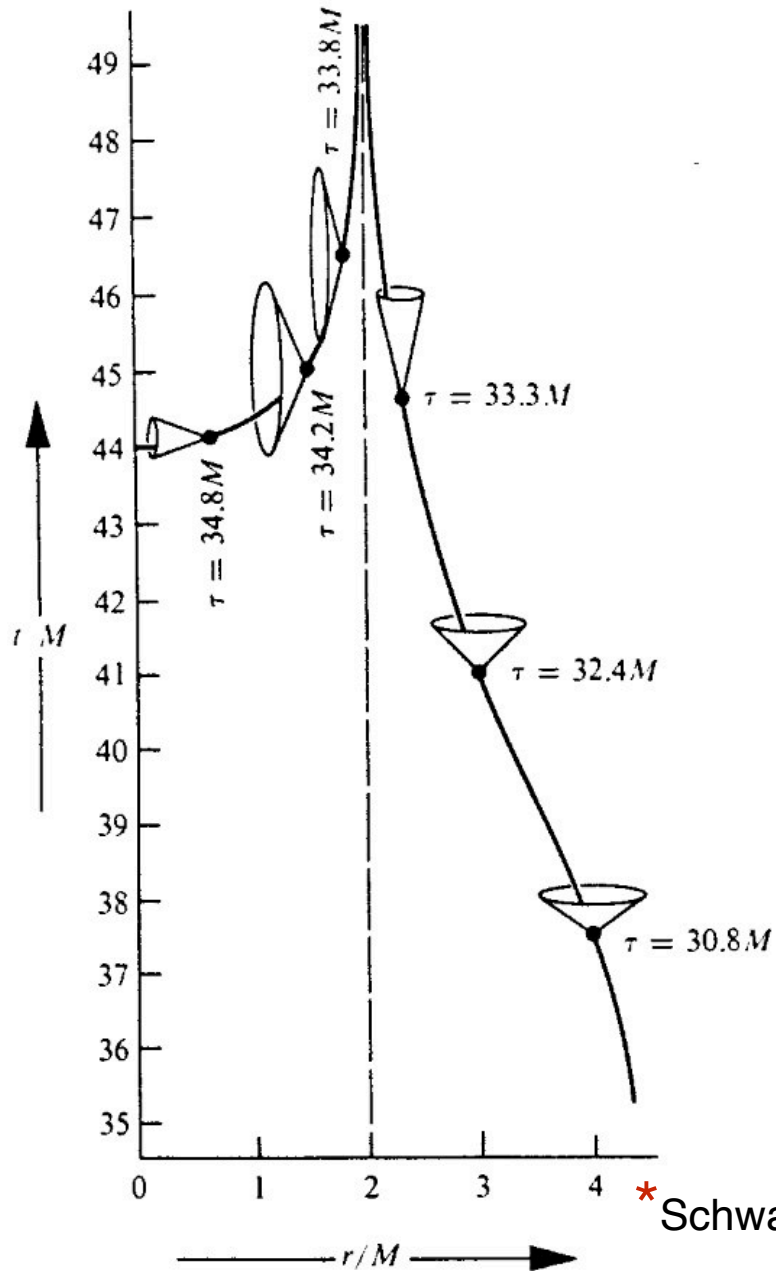
$$G^{\alpha\beta} \equiv R^{\alpha\beta} - \frac{1}{2} R g^{\alpha\beta} = \frac{8\pi}{c^4} T^{\alpha\beta}$$



3

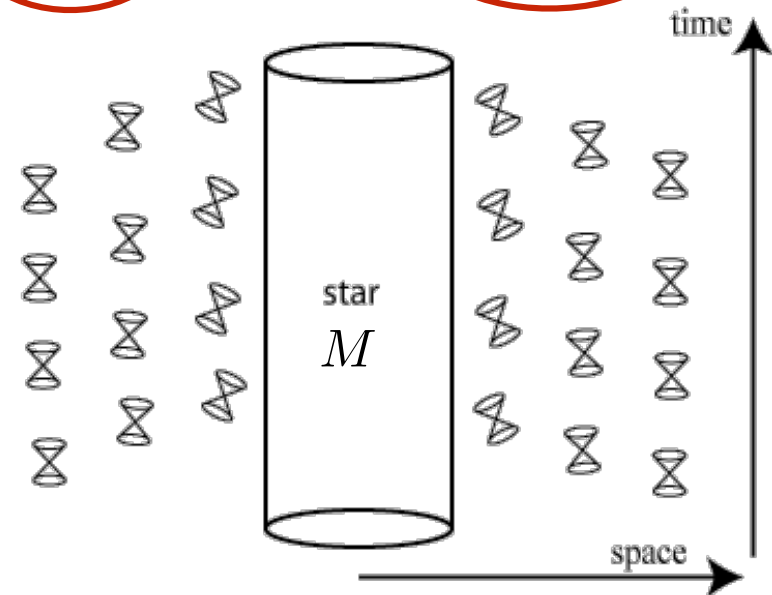
Minkowski spacetime

$$ds^2 = -c^2 dt^2 + d\mathbf{x}^2$$



$$ds^2 = -c^2 \left(1 - \frac{2GM}{c^2 r}\right) dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} d\mathbf{x}^2 *$$

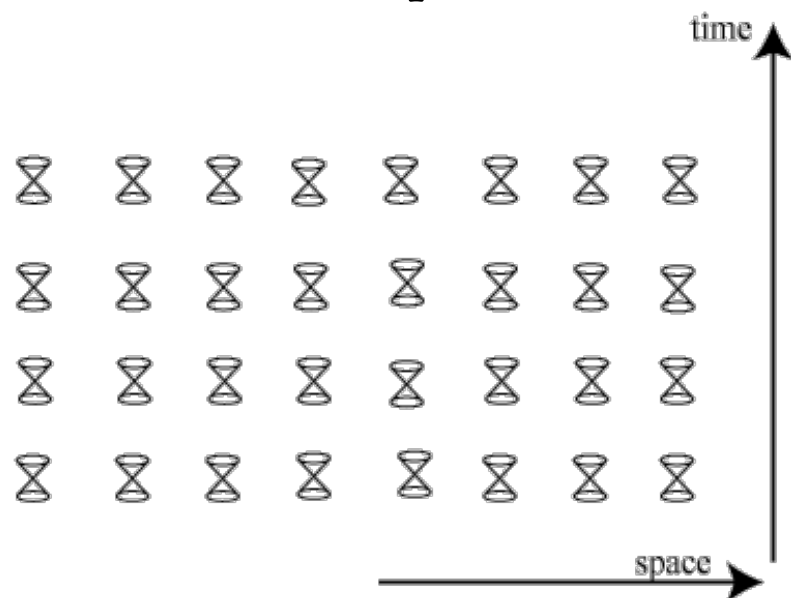
Curved
spacetime
of a mass M



* Schwarzschild metric

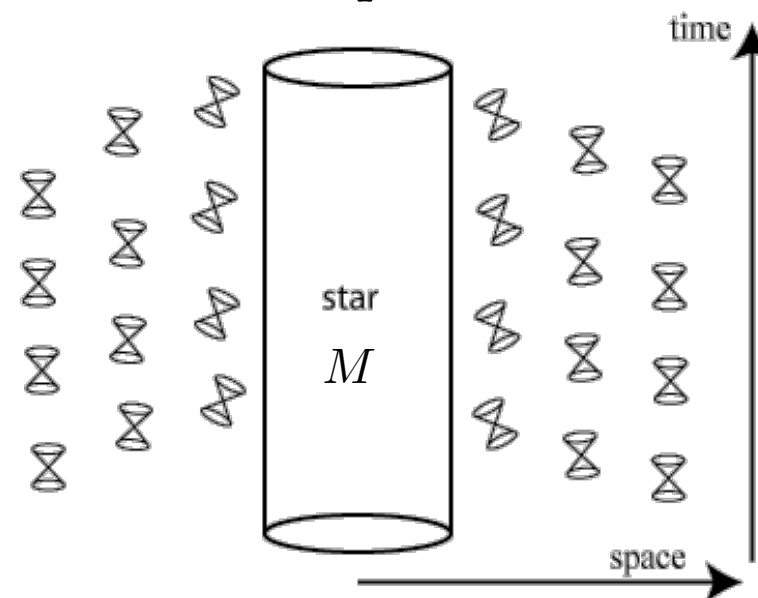
4

Minkowski spacetime



?

Curved spacetime



More generally, the perturbed **metric** describes the clumpy spacetime:

$$ds^2 = -c^2 dt^2 + d\mathbf{x}^2 \longrightarrow ds^2 = -c^2 \left(1 - \frac{2\phi}{c^2}\right) dt^2 + \left(1 + \frac{2\phi}{c^2}\right) d\mathbf{x}^2$$

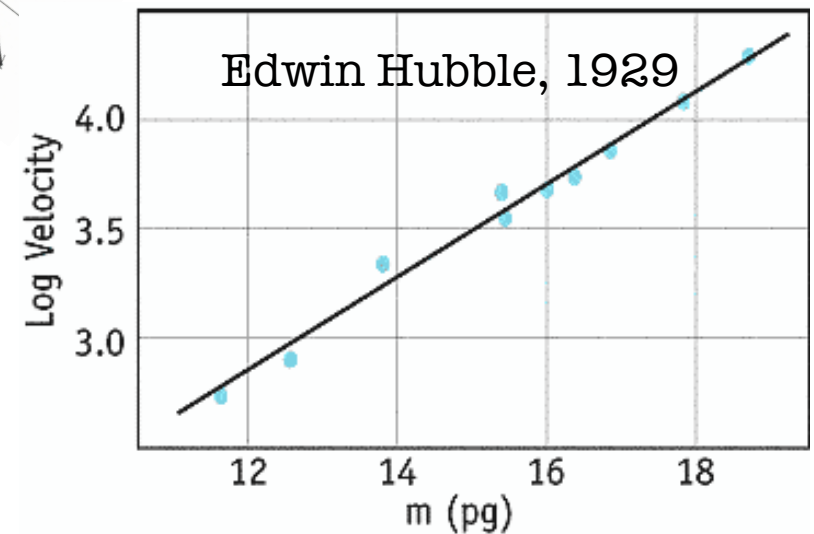
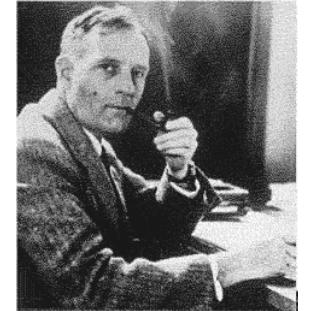
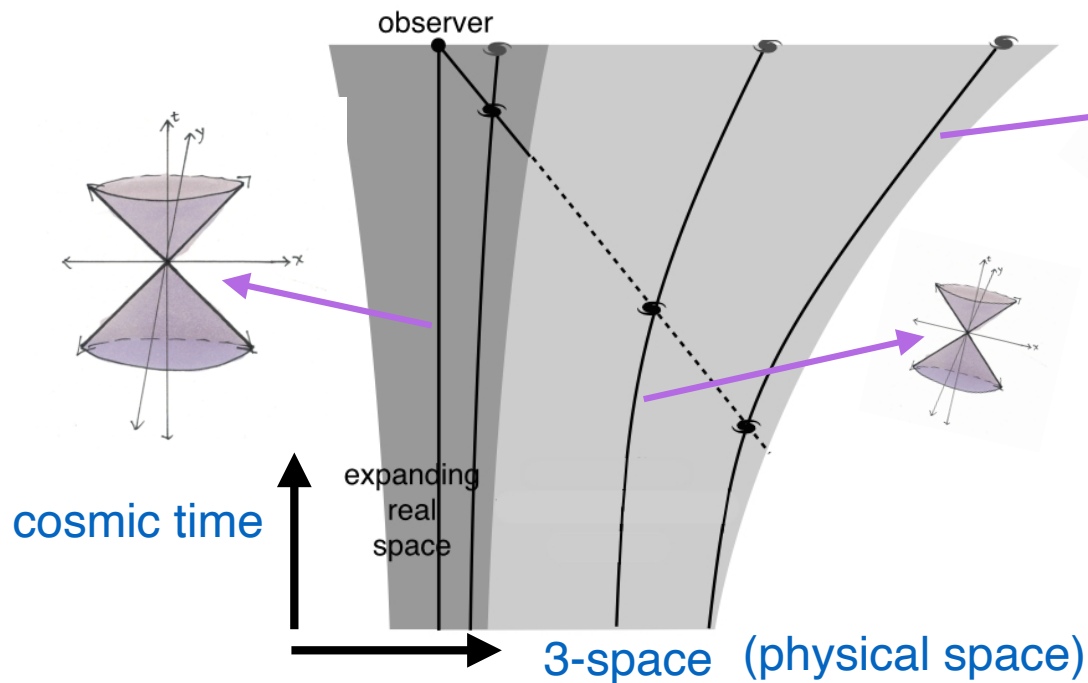
where $\phi = \phi(\mathbf{x})$, is the gravitational potential of a mass-density fluctuation $\delta\rho(\mathbf{x})$

As a particular case $\phi = -GM/r$, is the gravitational potential of a mass M .

5

Expanding spacetime: the Hubble flow

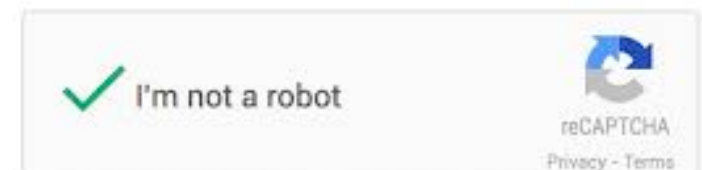
$$ds^2 = -c^2 dt^2 + a^2(t)[dr^2 + r^2 d\Omega^2]$$



- ◆ (iv) Freely falling particles move on timelike geodesics of the spacetime.

Ficamos por
aqui, a gente se
vê na quarta
.... cedo!!

não deixem de se inscrever no meu canal
(se gostou, deixa uns likes 👍)



Não sou um



Selecione as infos sugestivas de um espaçotempo plano

1	2	3 $\frac{\Delta \nu}{\nu} = -\frac{gh}{c^2}$
4	5 $R^\alpha_{\beta\mu\nu} = 0$ $\forall \alpha, \beta, \mu, \nu$	6
7 $\nabla^2 \phi = 4\pi G \rho$	8	9 $ds^2 = -c^2 dt^2 + d\mathbf{x}^2$

Não sou um

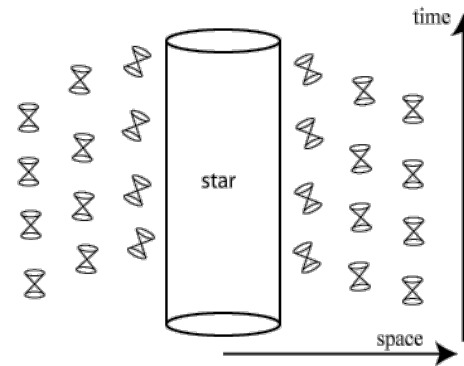


Selecione as infos sugestivas
de um espaçotempo curvo

1

$$ds^2 = -c^2 dt^2 + d\mathbf{x}^2$$

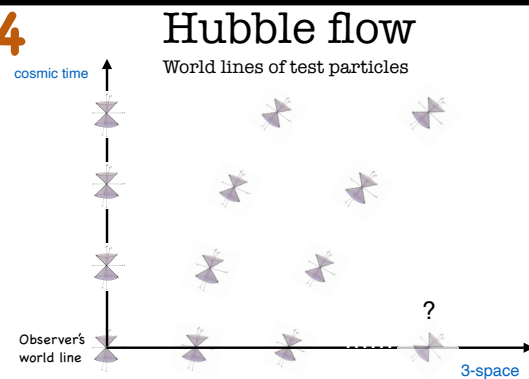
2



3

Minkowski
spacetime

4

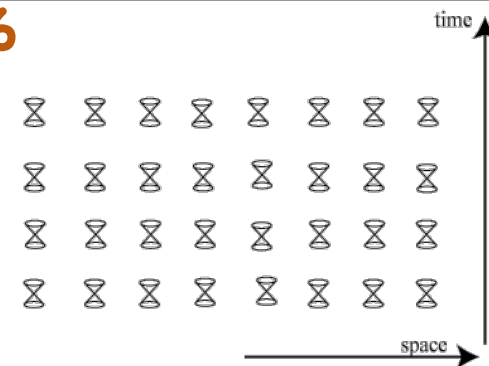


5

$$R^{\alpha}_{\beta\mu\nu} = 0$$

$$\nexists \alpha, \beta, \mu, \nu$$

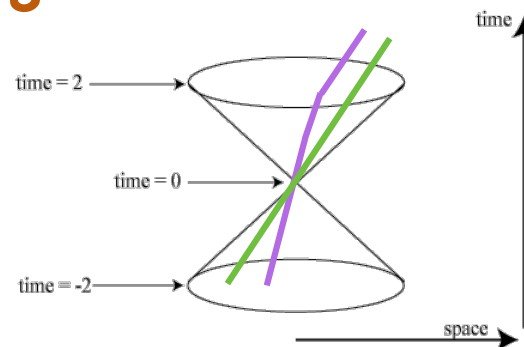
6



7

$$ds^2 = c^2 dt^2 + a(t)^2 d\mathbf{x}^2$$

8



9

$$\nabla^2 \phi = 0$$

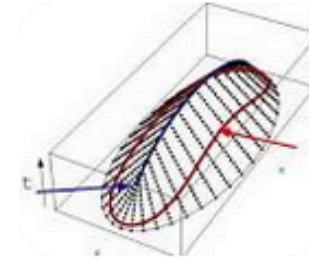
Bibliografia

- Abundante
- Especializada ou +/-
- Gratuita e paga
- Diferentes medios: PDF, livros, videos/youtube, paginas web, etc.

Is there a theorem saying that "if a spacetime is homogeneous and isotropic, then it is necessarily a Robertson-Walker spacetime?"

◆ AI Overview

Yes, there is a theorem stating that a spacetime which is both spatially homogeneous and isotropic must have a Robertson-Walker (also known as Friedmann-Lemaître-Robertson-Walker or FLRW) metric. This theorem is a cornerstone of modern cosmological models, as it provides a geometric framework for describing a universe that appears the same in all locations and directions on large scales. 



Here's a more detailed explanation:

Homogeneity and Isotropy:

In cosmology, these terms describe the large-scale structure of the universe. Homogeneity means the universe looks the same at all locations, while isotropy means it looks the same in all directions. 

Robertson-Walker Metric:

This metric is a mathematical description of spacetime that is spatially homogeneous and isotropic. It's characterized by a scale factor that determines the expansion or contraction of the universe, and a curvature parameter that defines the spatial geometry (flat, spherical, or hyperbolic). 

The Theorem:

The theorem essentially states that if you assume the universe is homogeneous and isotropic, then the geometry of spacetime must be described by the Robertson-Walker metric. 

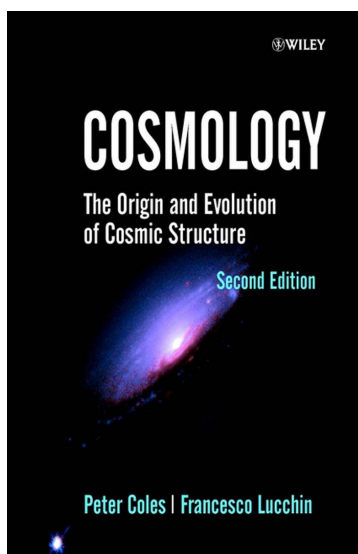
Original references

Friedmann, A., 1922. On the curvature of space. *Z. Phys.* **10**, 377

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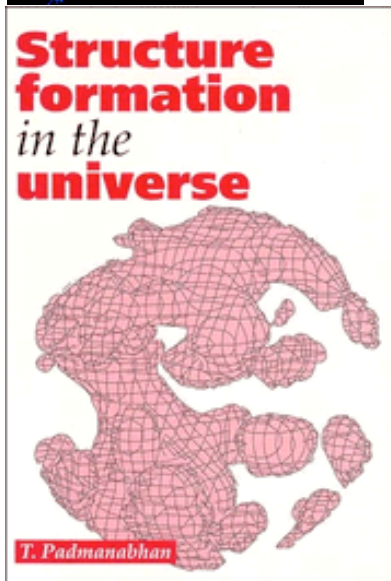
Lemaître, G., 1927. Un Univers homogène de masse constante et de rayon croissant rendant compte de la vitesse radiale des nébuleuses extra-galactiques.

Annales de la Société Scientifique de Bruxelles **47**, 49–59



https://fma.if.usp.br/~mlima/teaching/PGF5292_2021/Coles_COECS.pdf

Peter Coles &
Francesco Lucchin



Thanu Padmanabhan



Contemporary Physics

Publication details, including instructions for authors and subscription information:

<http://www.tandfonline.com/loi/tcph20>

The large-scale structure of the universe

Peter Coles^a

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Published online: 20 Aug 2006.

Large-scale surveys and cosmic structure

By J.A. PEACOCK

Institute for Astronomy, University of Edinburgh,
Royal Observatory, Edinburgh EH9 3HJ, UK

These lectures deal with our current knowledge of the matter distribution in the universe, focusing on how this is studied via the large-scale structure seen in galaxy surveys. We first assemble the necessary basics needed to understand the development of density fluctuations in an expanding universe, and discuss how galaxies are located within the dark-matter density field. Results from the 2dF Galaxy Redshift Survey are presented and contrasted with theoretical models. We show that the combination of large-scale structure and data on microwave-background anisotropies can eliminate almost all degeneracies, and yield a completely specified cosmological model. This is the ‘concordance’ universe: a geometrically flat combination of vacuum energy and cold dark matter. The study of cosmic structure is able to establish this in a manner independent of external information, such as the Hubble diagram; this extra information can however be used to limit non-standard alternatives, such as a variable equation of state for the vacuum.

1. Preamble

1.1. *The perturbed universe*

It has been clear since the 1930s that galaxies are not distributed at random in the universe (Hubble 1934). For decades, our understanding of this fact was limited by the lack of a three-dimensional picture, although some impressive progress was made: the dedication of pioneers such as Shane & Wirtanen in compiling galaxy catalogues by eye is humbling to consider. However, studies of the galaxy distribution came of age in the 1980s, via redshift surveys, in which Hubble’s $v = Hd$ law is used to turn spectroscopic redshifts into estimates of distance (e.g. Davis & Peebles 1983; de Lapparent, Geller & Huchra 1986; Saunders et al. 1991). We were then able to see clearly (e.g. figure 1) a wealth of large-scale structures of size exceeding 100 Mpc. The existence of these cosmological structures must be telling us something important about the initial conditions of the big bang, and about the physical processes that have operated subsequently. These lectures cover some of what we have learned in this regard.

Throughout, it will be convenient to adopt a notation in which the density (of mass, light, or any property) is expressed in terms of a dimensionless density perturbation δ :

$$1 + \delta(\mathbf{x}) \equiv \rho(\mathbf{x})/\langle\rho\rangle, \quad (1)$$

J. Peacock astro-ph/0309240

Astronomia Extragaláctica e Cosmologia

Notas de Aula

03/02/2025

Gastão Bierrenbach Lima Neto

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Universidade de São Paulo (USP)

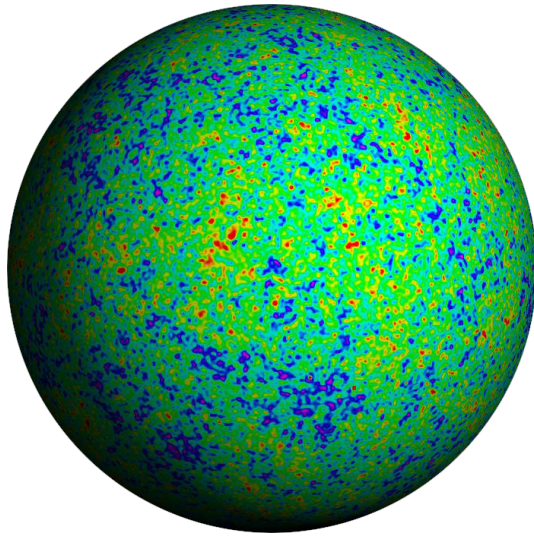
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TASI Lectures on Primordial Cosmology

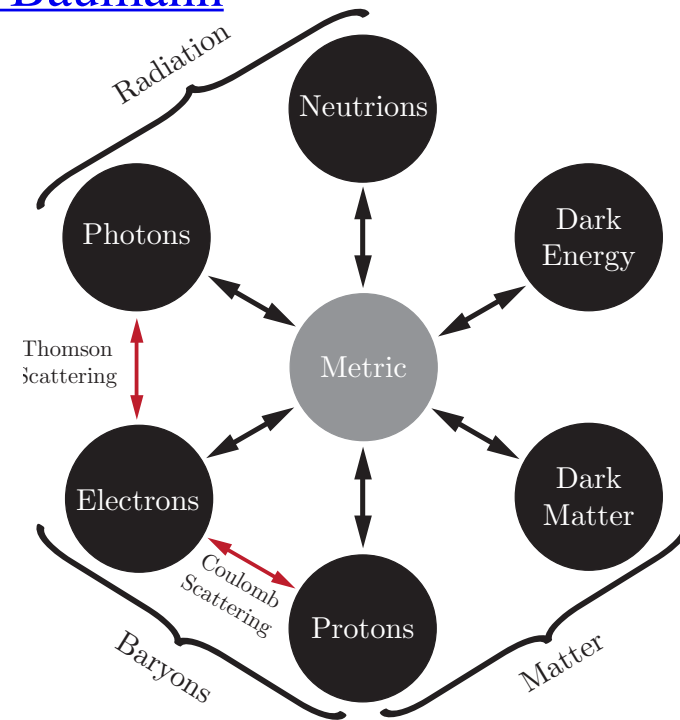
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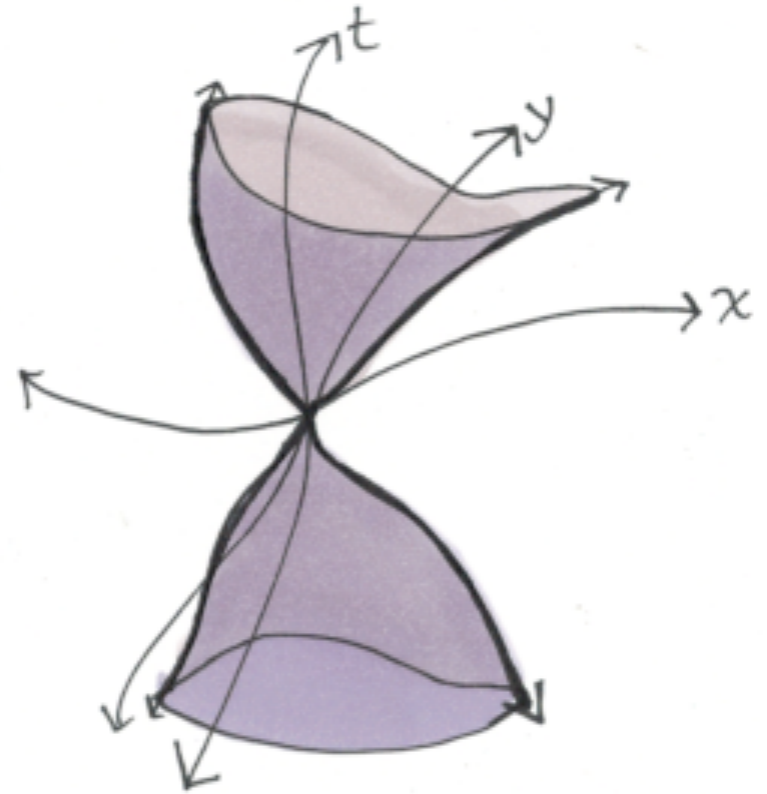
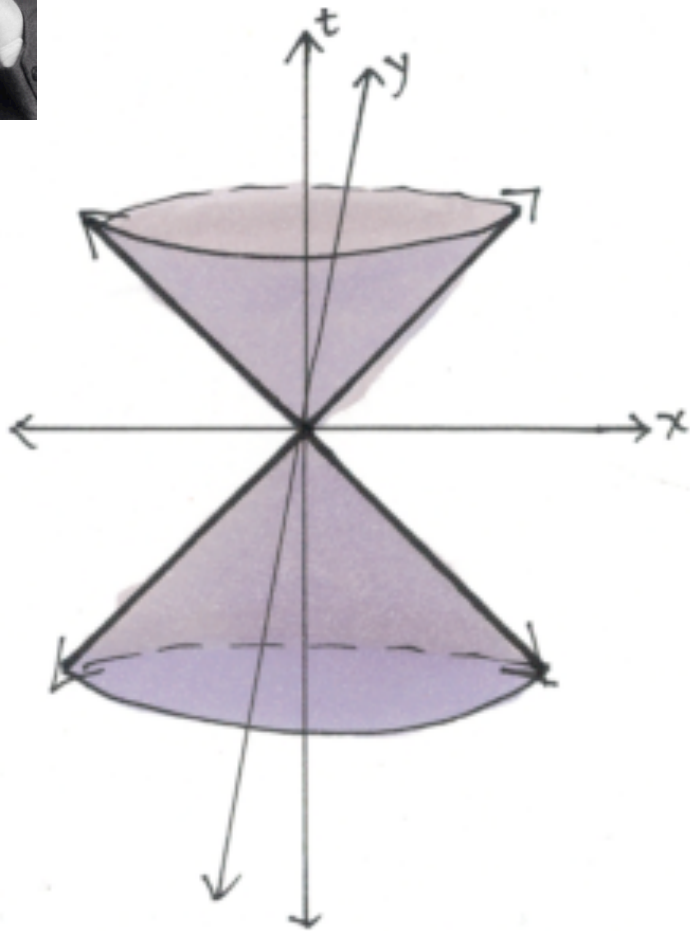
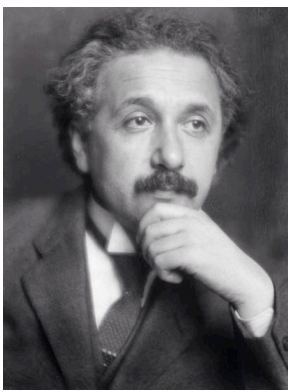


TASI Lectures on Primordial Cosmology

[Daniel Baumann](#)



<https://arxiv.org/pdf/1807.03098.pdf>



<https://notes.invertedpassion.com/Universe/Demystifying+general+relativity>