Stefano Profumo

Santa Cruz Institute for Particle Physics
University of California, Santa Cruz

Dark Energy and Dark Matter

Lecture 2

XXVII Special Courses at the Observatório Nacional – Rio de Janeiro, Brazil

Key ideas from last lecture

- **General Relativity** and the **metric** of a homogeneous and isotropic **universe**
- **Thermodynamics** of the early universe – the **thermal decoupling paradigm** (ν,γ)
- **Cold** Relic Freeze out

Cold Relics

Try **proton-antiproton** freeze-out:
what's the **relic** matter **abundance** in a baryon-symmetric Universe?

$$\sigma \sim \Lambda_{\rm QCD}^{-2}$$

$$n\,\sigma = H \rightarrow T^3 \Lambda^{-2} = T^2/M_P \rightarrow T = \Lambda^2/M_P$$

doesn't quite work, we're way **outside**
the regime of validity for **hot relics**, ($T \gg m_p$), since we found $T \lll\lll m_p$ ...

Need to work out the case of **cold relics**, which looks nastier by eye

$$n \sim (m_\chi T)^{3/2} \exp\left(-\frac{m_\chi}{T}\right)$$

Cold Relics

$$n \sim (m_\chi T)^{3/2} \exp\left(-\frac{m_\chi}{T}\right)$$

$n\ \sigma \sim H \sim T^2/M_P$

[freeze out condition]

$$n_{\rm f.o.} \sim \frac{T^2_{\rm f.o.}}{M_P \cdot \sigma}$$

now define $m_\chi/T \equiv x$ (cold relic: **x>>1**)

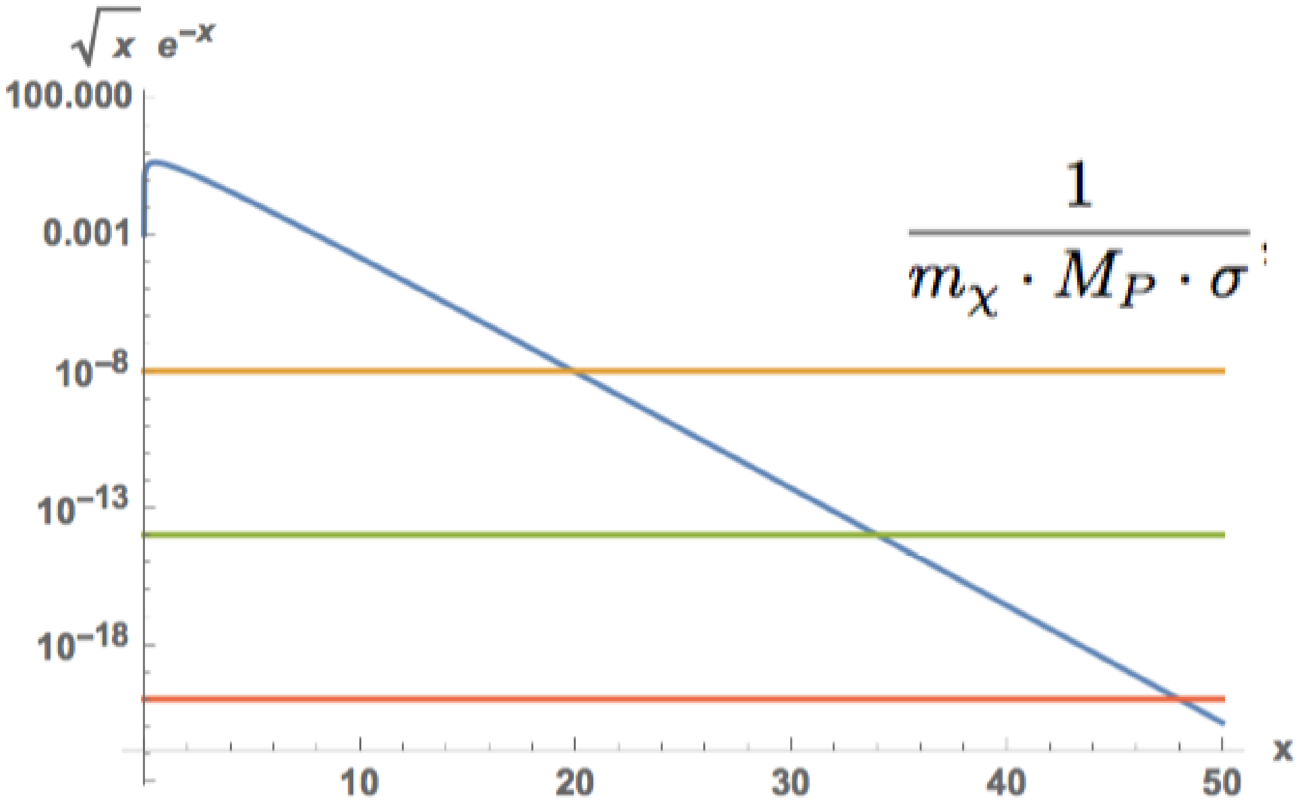
Freeze-out condition (x) now reads

$$\frac{m_\chi^3}{x^{3/2}} e^{-x} = \frac{m_\chi^2}{x^2 \cdot M_P \cdot \sigma}$$

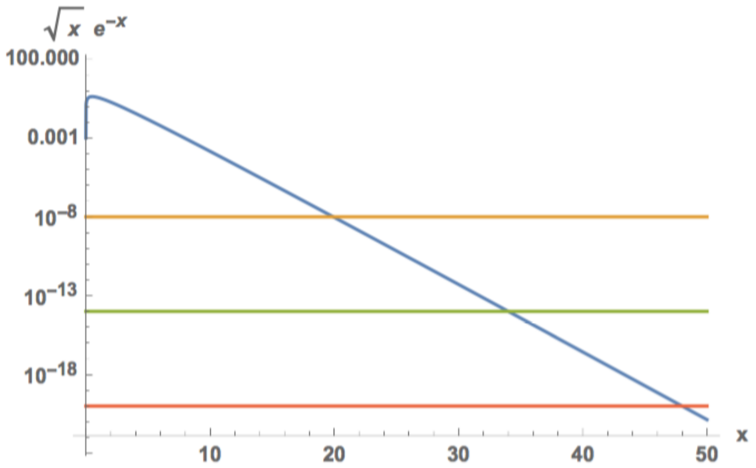
…so we must **solve** $$\sqrt{x} \cdot e^{-x} = \frac{1}{m_\chi \cdot M_P \cdot \sigma}$$

Cold Relics

$$\sqrt{x} \cdot e^{-x} = \frac{1}{m_\chi \cdot M_P \cdot \sigma};$$



Cold Relics



$$\sqrt{x} \cdot e^{-x} = \frac{1}{m_\chi \cdot M_P \cdot \sigma}$$

Take e.g. a "**weakly interacting massive particle**"

$$\sigma \sim G_F^2 m_\chi^2$$

$$m_\chi \sim 10^2 \text{ GeV}.$$

$$\sqrt{x} \cdot e^{-x} = \frac{1}{m_\chi \cdot M_P \cdot \sigma} \sim \frac{1}{10^2 \cdot 10^{18} \cdot 10^{-6}} \sim 10^{-14}.$$

$x = m_\chi / T \sim 35$

proton-antiproton: $1/(1 \text{ GeV } 10^{18} \text{ GeV } 10^{-2} \text{ GeV}^{-2}) \sim 10^{-16}$

$x = m_p / T \sim 37$

Cold Relics

Off to calculating the **thermal relic density**

$$\Omega_\chi = \frac{m_\chi \cdot n_\chi(T = T_0)}{\rho_c} = \frac{m_\chi\, T_0^3}{\rho_c} \frac{n_0}{T_0^3}$$

iso-entropic universe $aT \sim$const

$$\frac{n_0}{T_0^3} \simeq \frac{n_{\rm f.o.}}{T_{\rm f.o.}^3}$$

$$\Omega_\chi = \frac{m_\chi\, T_0^3}{\rho_c} \frac{n_{\rm f.o.}}{T_{\rm f.o.}^3} = \frac{T_0^3}{\rho_c} x_{\rm f.o.} \left(\frac{n_{\rm f.o.}}{T_{\rm f.o.}^2}\right) = \left(\frac{T_0^3}{\rho_c\, M_P}\right) \frac{x_{\rm f.o.}}{\sigma}$$

$$\left(\frac{\Omega_\chi}{0.2}\right) \simeq \frac{x_{\rm f.o.}}{20} \left(\frac{10^{-8}\ {\rm GeV}^{-2}}{\sigma}\right)$$

Cold Relics

$$\left(\frac{\Omega_\chi}{0.2}\right) \simeq \frac{x_{\rm f.o.}}{20}\left(\frac{10^{-8}\ {\rm GeV}^{-2}}{\sigma}\right)$$

If you have a **WIMP**,
defined by a cross section

$$\sigma \sim G_F^2\, m_\chi^2$$

$$\Omega_\chi h^2 \sim 0.1\,\frac{10^{-8}\ {\rm GeV}^{-2}}{G_F^2\, m_\chi^2} \sim 0.1\left(\frac{10\ {\rm GeV}}{m_\chi}\right)^2$$

For **protons**:

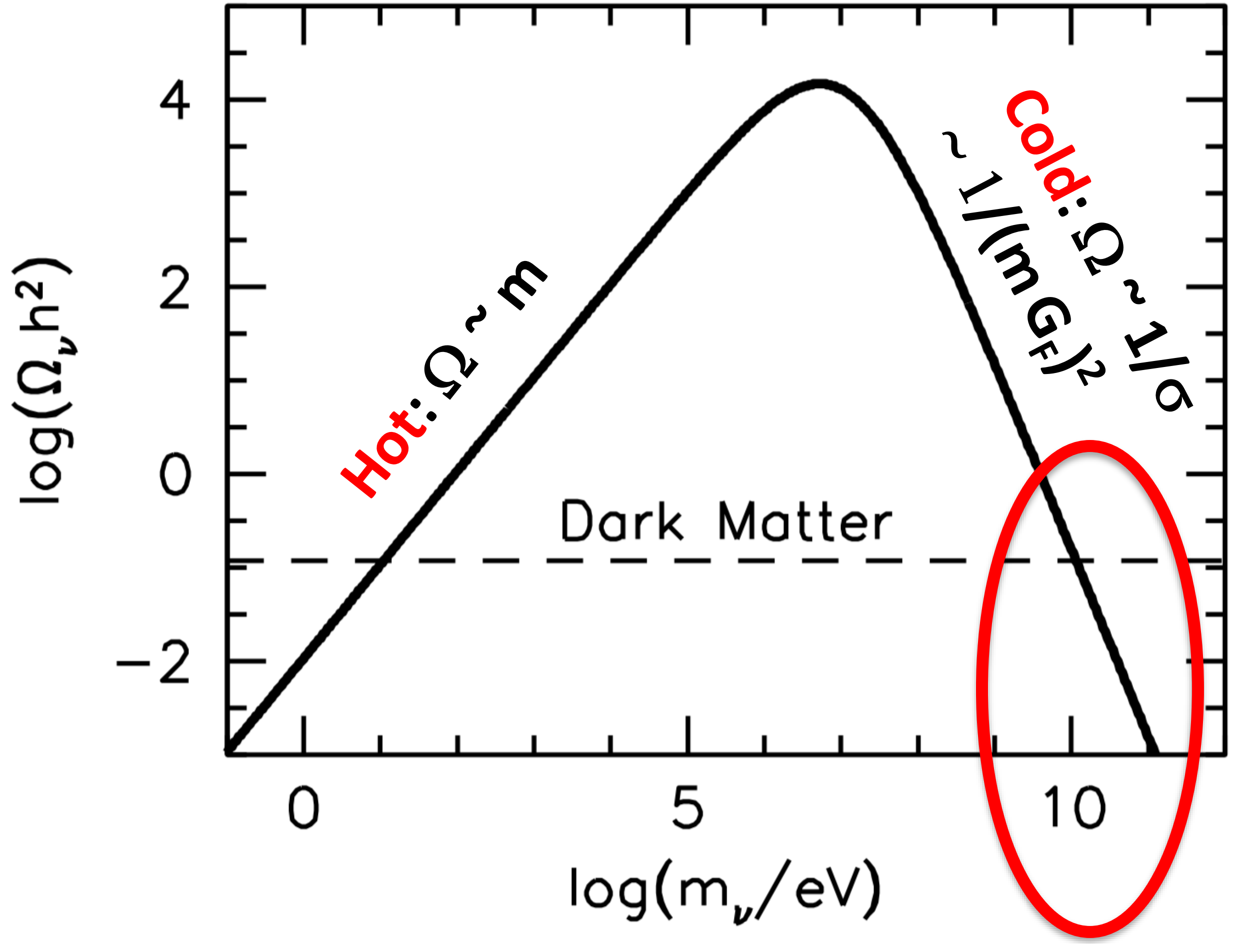
$\sigma \sim 1/(0.1\ \mathrm{GeV})^2$

$\Omega_B \sim 10^{-10}$

$\Omega_{B,obs} \sim 10^{-1}$

➔ Need **BARYOGENESIS**!

WIMP's **thermal** relic density



"**Lee-Weinberg**" limit

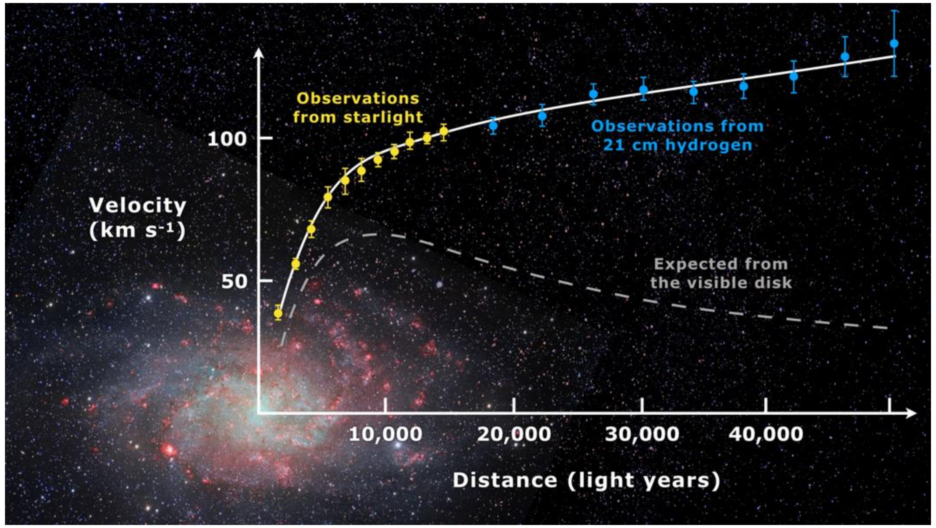
"Classical" **WIMPs cannot be lighter** than ~ 1 GeV

The CDM in Lambda CDM

Astrophysical Evidence

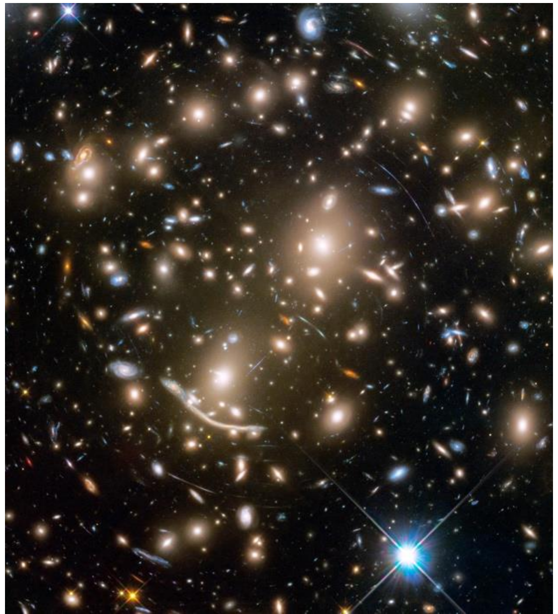
1. Galaxy Rotation Curves

Observation: The rotational velocity of stars in spiral galaxies remains approximately constant at large radii, rather than decreasing as expected from the visible mass.
Implication: Indicates the presence of an extended, unseen mass component—interpreted as a dark matter halo.



2. Galaxy Clusters (Zwicky & the Virial Theorem)

Observation: The velocity dispersion of galaxies within clusters suggests much more mass than observed in luminous matter.



🔭 Astrophysical Evidence

3. Gravitational Lensing

Observation: The deflection of light from background objects by foreground mass often exceeds what is expected from visible matter alone.

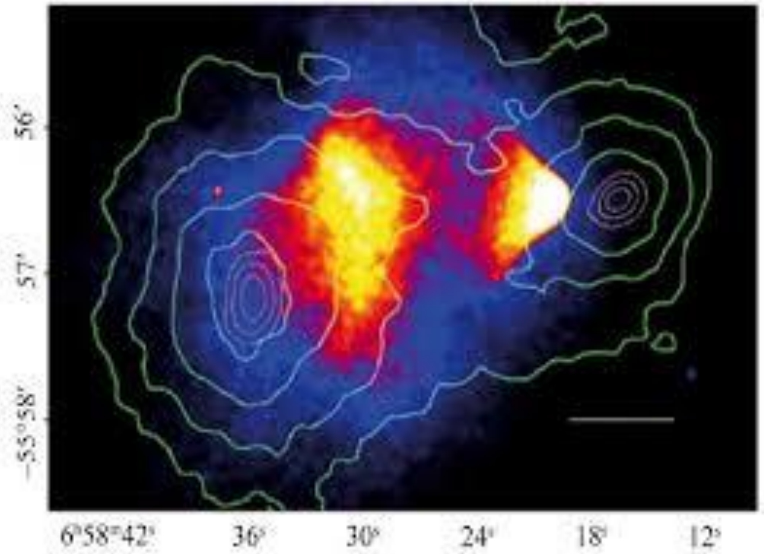
Examples:

- Strong lensing (Einstein rings, arcs)
- Weak lensing (statistical distortions of background galaxies)

4. Bullet Cluster (1E 0657-56)

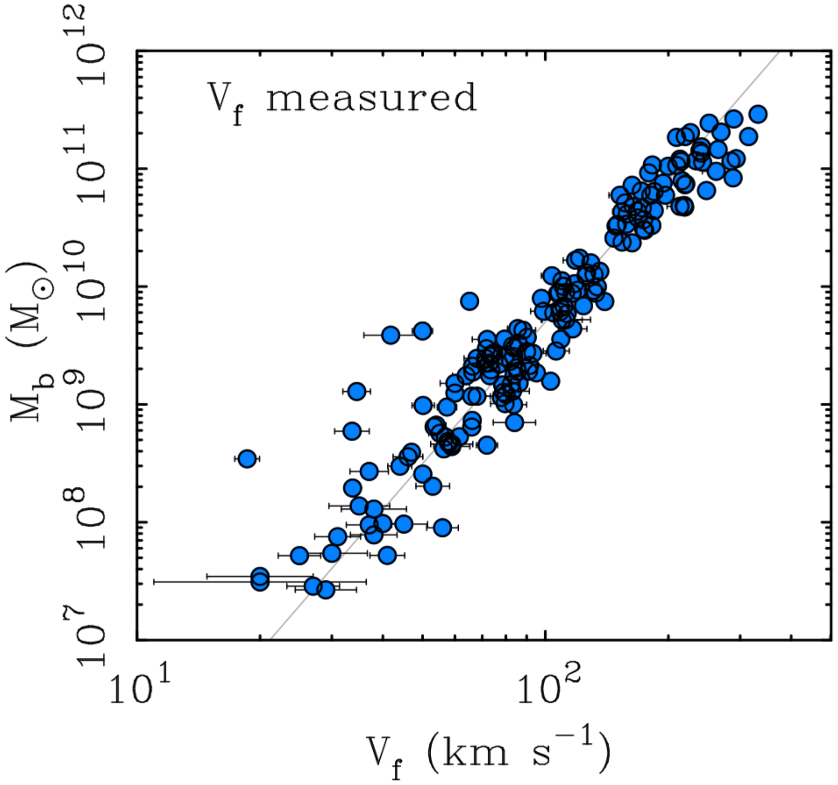
Observation: In a collision of two galaxy clusters, the hot X-ray emitting gas (which contains most of the baryonic mass) is spatially separated from the gravitational potential inferred from lensing.

Implication: Demonstrates that most of the mass is collisionless—supporting a non-baryonic dark matter component.



Astrophysical Evidence

5. Galactic and Sub-galactic Dynamics
Observation: Dark matter is necessary to explain stability and properties of dwarf spheroidal galaxies, satellite galaxies, and the Tully–Fisher relation.



Cosmological Evidence

6. Cosmic Microwave Background (CMB) Anisotropies
Observation: Precision measurements (e.g., by Planck and WMAP) of the temperature power spectrum reveal the relative densities of baryons, dark matter, and dark energy.
Implication: The height and spacing of acoustic peaks are best explained by a Universe composed of ~5% baryons, ~25% dark matter, and ~70% dark energy.

7. Baryon Acoustic Oscillations (BAO)
Observation: The imprint of primordial sound waves in the early Universe manifests as a characteristic scale in the distribution of galaxies.
Implication: BAO measurements agree with ΛCDM cosmology, requiring a substantial dark matter component.

8. Big Bang Nucleosynthesis (BBN)
Observation: The abundances of light elements (H, He, Li) match theoretical predictions only if the baryon density is much lower than the total matter density.
Implication: Reinforces that most matter is non-baryonic (i.e., dark).

9. Large-Scale Structure Formation
Observation: The distribution and growth of galaxies over time match simulations only if dark matter is present.
Key Point: Cold dark matter (CDM) is essential for early structure formation; baryons alone cannot collapse early enough.

9. Large-Scale Structure Formation

CMB sky is very **boring** – *T* fluctuations very **small**!

T fluctuations proportional to (baryonic) **density** fluctuations,

$$\delta\rho/\rho \lesssim 10^{-4}$$

Matter **over-densities** in linear regime

grow **linearly** with scale factor

But the scale factor since CMB decoupling grew by z_{rec}***~1,100***

Not enough time (since recombination) for structures to go **non-linear**!

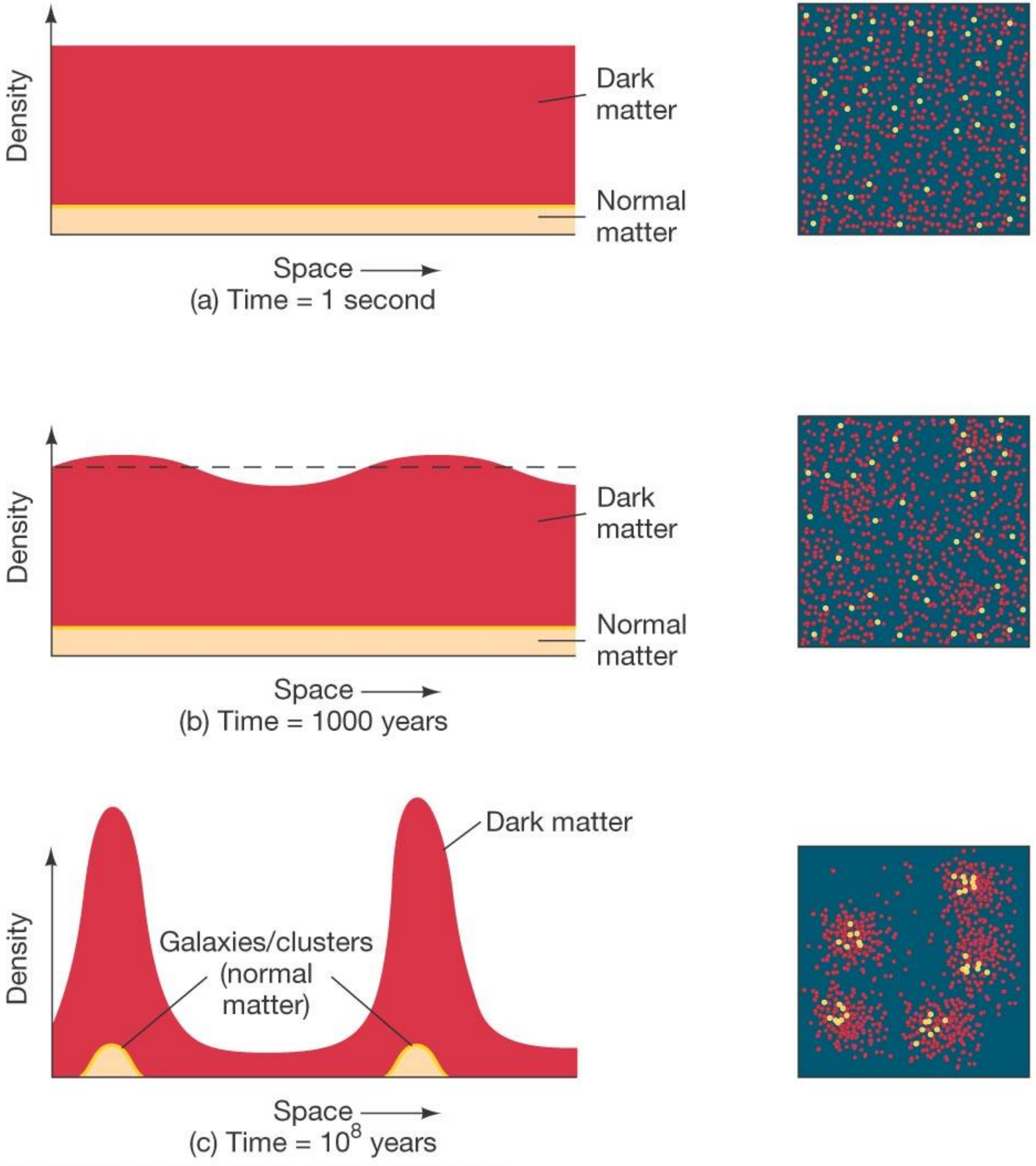
$\delta\rho/\rho * z_{rec} \sim 0.1$, but today $\delta\rho/\rho >> 1$

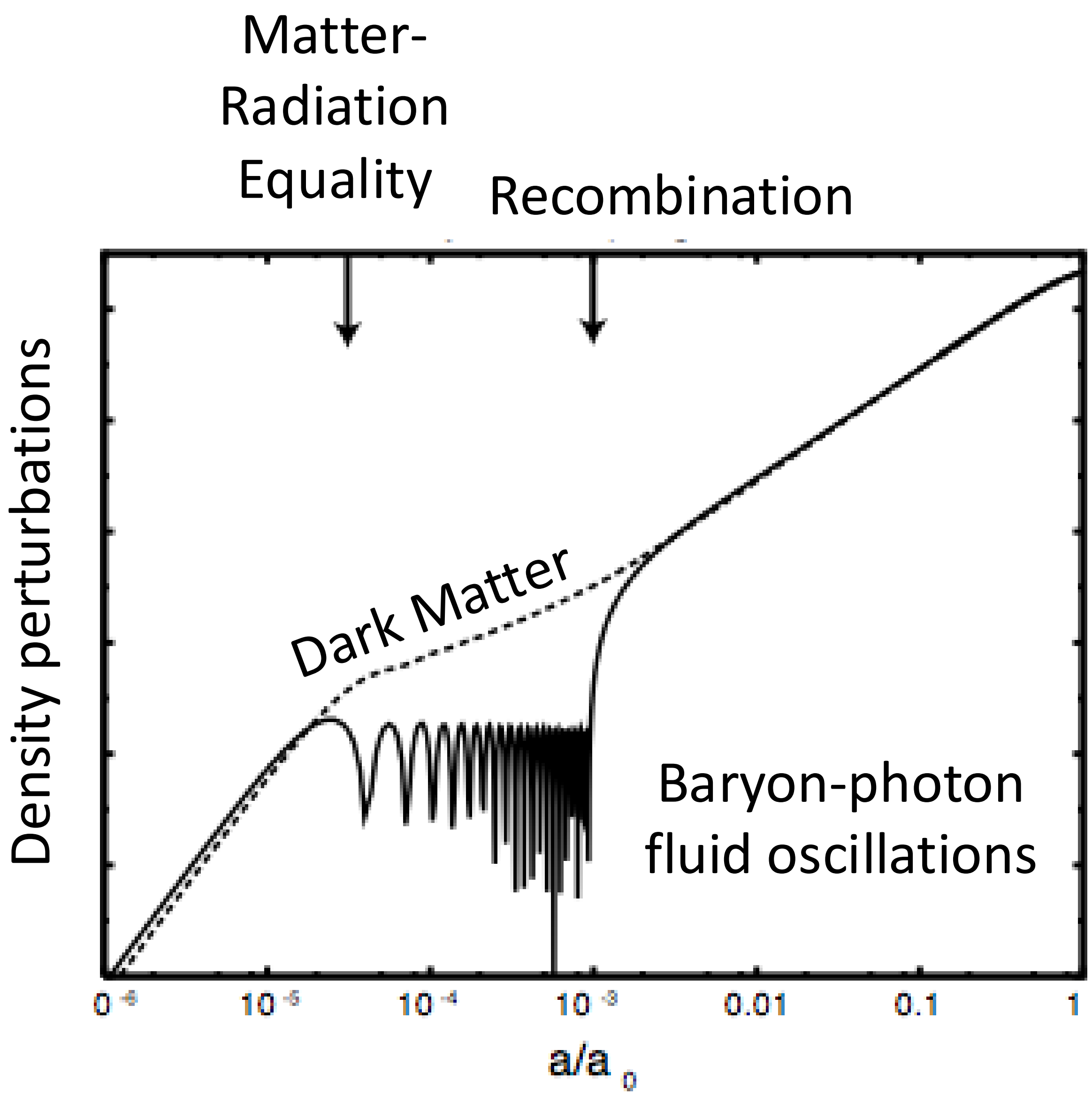
9. Large-Scale Structure Formation

We need a **species** that has **decoupled** from photons much earlier (**Dark Matter**) so that its density **perturbations** are much **larger** at recombination!

$$(\delta\rho/\rho)_{\rm DM} \gg 10^{-4}$$

Dark matter **seeds** timely structure formation!

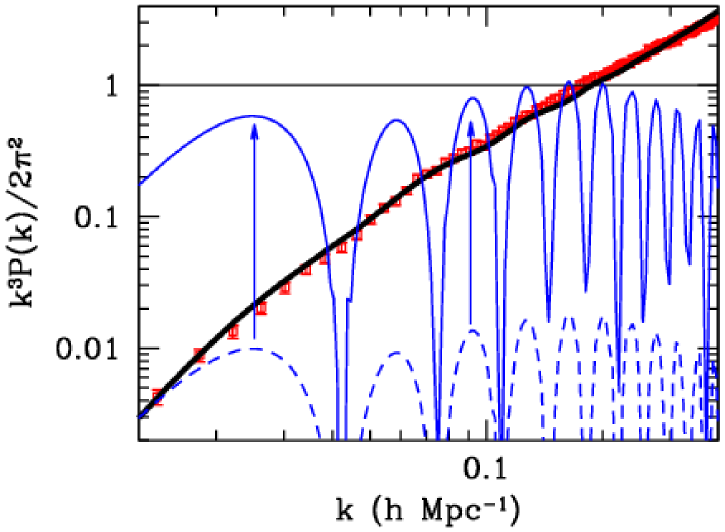

Density
Dark matter
Normal matter
Space
(a) Time = 1 second
Density
Dark matter
Normal matter
Space
(b) Time = 1000 years
Dark matter
Density
Galaxies/clusters (normal matter)
Space
(c) Time = 10^8 years



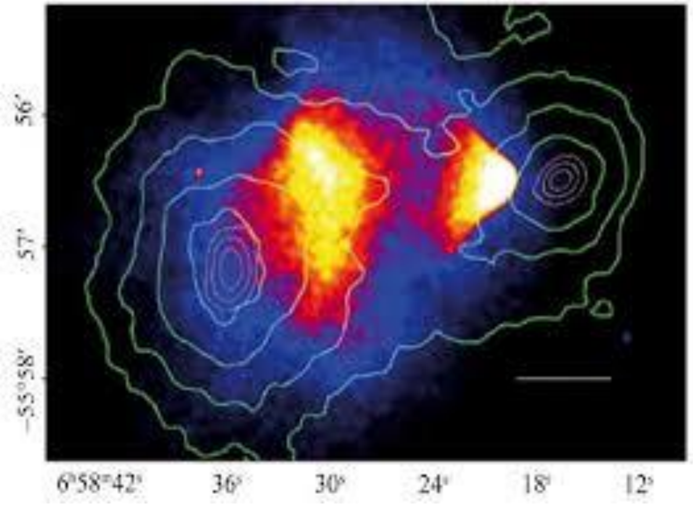
Matter-
Radiation
Equality
Recombination
Density perturbations
Dark Matter
Baryon-photon
fluid oscillations
10⁻⁵
10⁻⁴
10⁻³
0.01
0.1
1
a/a₀

Stefano Profumo
University of California Santa Cruz

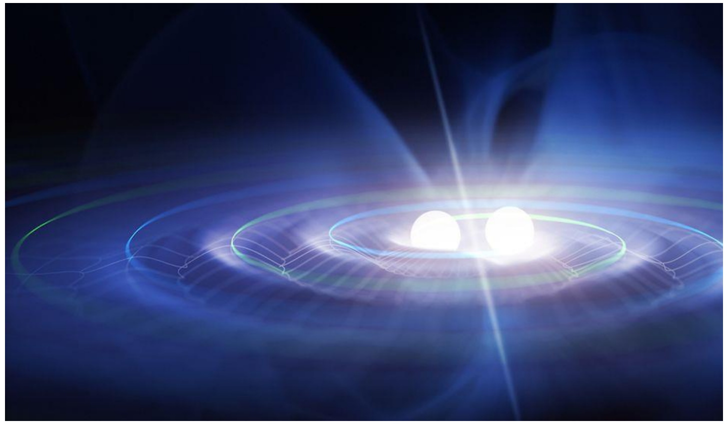
…but what if gravity works differently at large scales?



The Fourier Transform of Matter Perturbations is completely wrong



Bullet cluster observations are impossible to reproduce



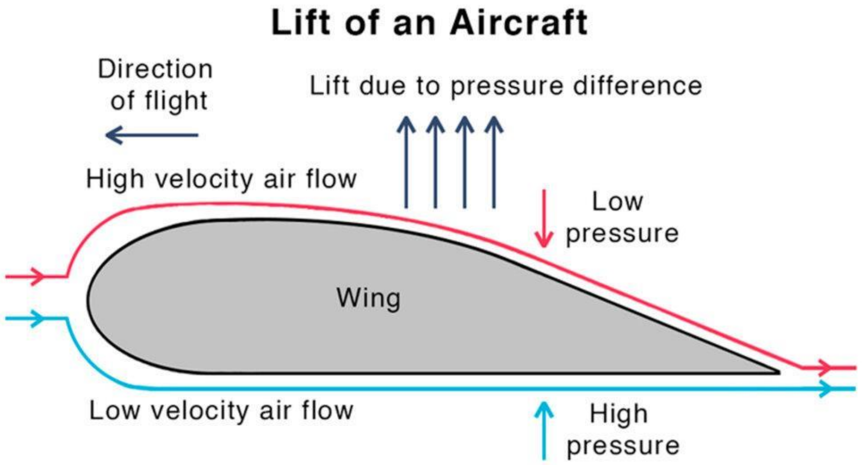
Gravitational waves do not propagate at the speed of light
(they do to 1 part in 10^{15})

Stefano Profumo
University of California Santa Cruz

…but what if gravity works differently at large scales?

“Maybe we have not formulated yet the right modified theory of gravity”

“Maybe we have not formulated yet the right theory of flying gnomes lifting aircrafts”



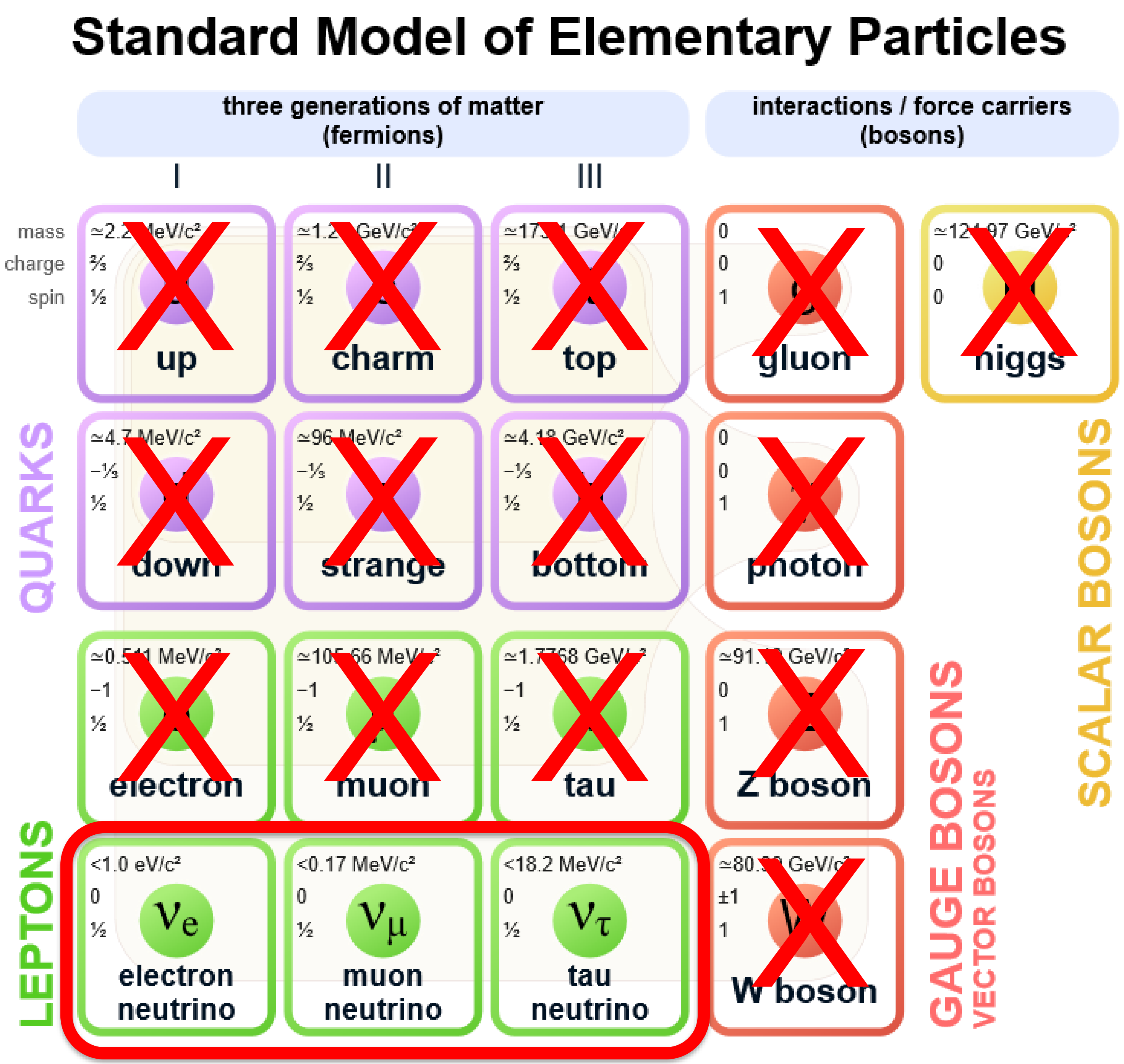
You can’t “prove” a scientific theory, nor can you rule out all alternatives
A scientific theory is useful if it is consistent with all data and if it makes testable, falsifiable predictions

What do we know about the Dark Matter?

- **Dark** (no EM charge)
- **Long-lived**
- **No color charge**

(and **B=0**)

Cosmology + Neutrino Osc.:
Neutrinos are 1% of the dark matter



Stefano Profumo
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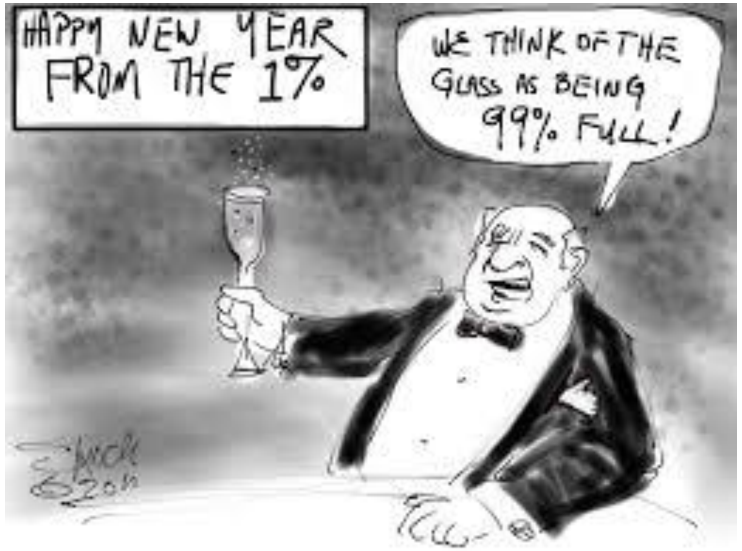


What about the **99%**?

Why Cold and not Hot Relics?

Hot relics decouple when **T >> m_ν**

Structures can only collapse when **T ~ m_ν**

(i.e. when things slow down enough for gravitational collapse!)

Structures are cutoff to the **horizon size** at that temperature

$$d_\nu \sim H^{-1}(T \sim m_\nu)$$

$$d_\nu \sim \frac{M_P}{m_\nu^2}$$

$$H = \sqrt{\frac{\pi^2 g_*}{3 \cdot 30}}\frac{T^2}{M_P} \simeq 3.4\frac{T^2}{M_P}$$

$$\frac{M_P^3}{m_\nu^2} \sim 10^{15}\ M_\odot \left(\frac{m_\nu}{30\ \mathrm{eV}}\right)^{-2} \sim 10^{12}\ M_\odot \left(\frac{m_\nu}{1\ \mathrm{keV}}\right)^{-2}$$

Why Cold and not Hot Relics?

Observational **constraints** give

$$M_{\rm cutoff} \ll M_{\rm Ly-\alpha} \simeq 10^{10}\ M_\odot$$

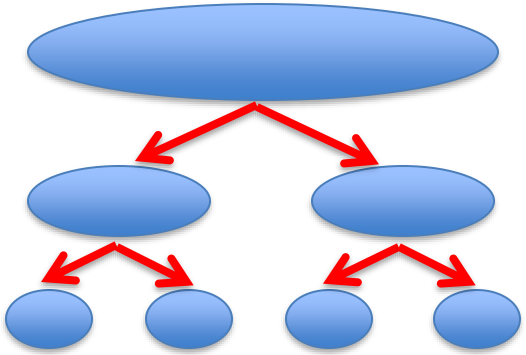
So at best dark matter can be **keV** scale, if produced thermally
Cannot be **SM neutrinos**!!

$$\frac{M_P^3}{m_\nu^2} \sim 10^{15}\ M_\odot \left(\frac{m_\nu}{30\ {\rm eV}}\right)^{-2} \sim 10^{12}\ M_\odot \left(\frac{m_\nu}{1\ {\rm keV}}\right)^{-2}$$

Why Cold and not Hot Relics?

Structure formation looks strikingly different

for **hot** and **cold** dark matter

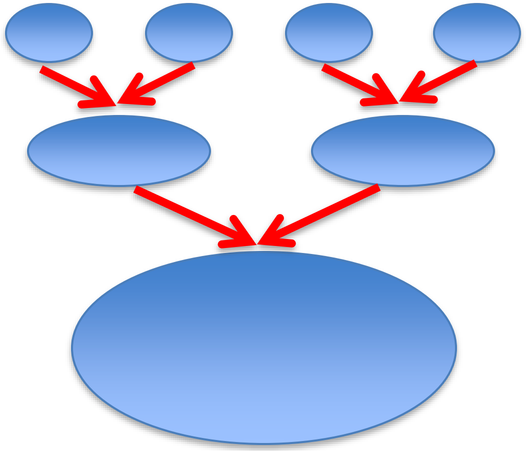


Hot Dark Matter

Top-Down

[doesn’t work!

Small structure is oldest!]



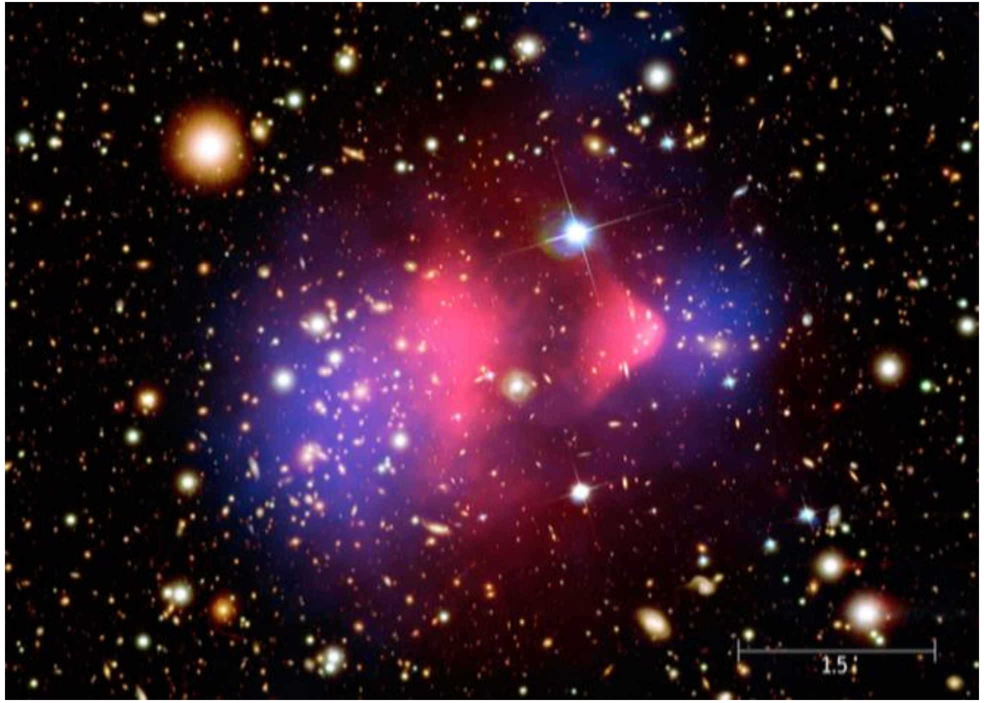
Cold Dark Matter

Bottom-Up

[Yeah!]

What else do we know about the
microscopic nature of (**cold**) dark matter
from its **macroscopic** features?

- "**Dark**" (**Milli-charge** allowed, **plasma** effects possible...)
- **~Collisionless**... really? Let's calculate the relevant constraints!

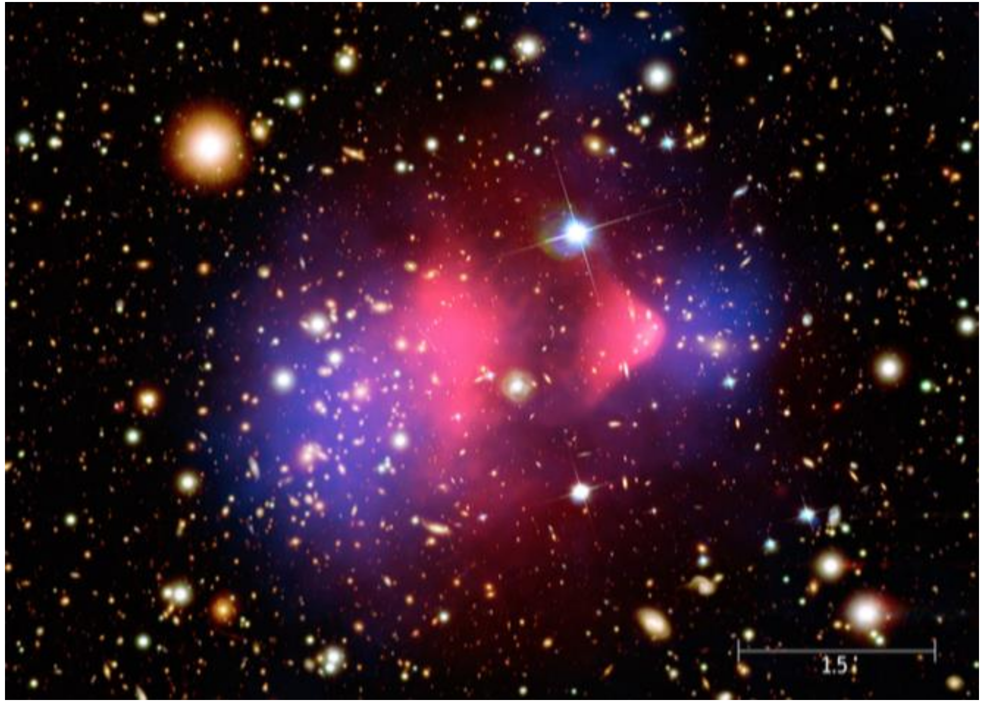


mean free path λ larger than cluster size, ~ 1 Mpc

cluster **density**: ρ ~ 1 GeV/cm^3, thus…

$\lambda = 1/(\sigma\,(\rho/m)) > 1$ Mpc → $\sigma/m < 1$ Mpc / 1 GeV/cm^3

→ ***$\sigma/m < 1$ cm^2/g***, or ***1 barn/GeV***



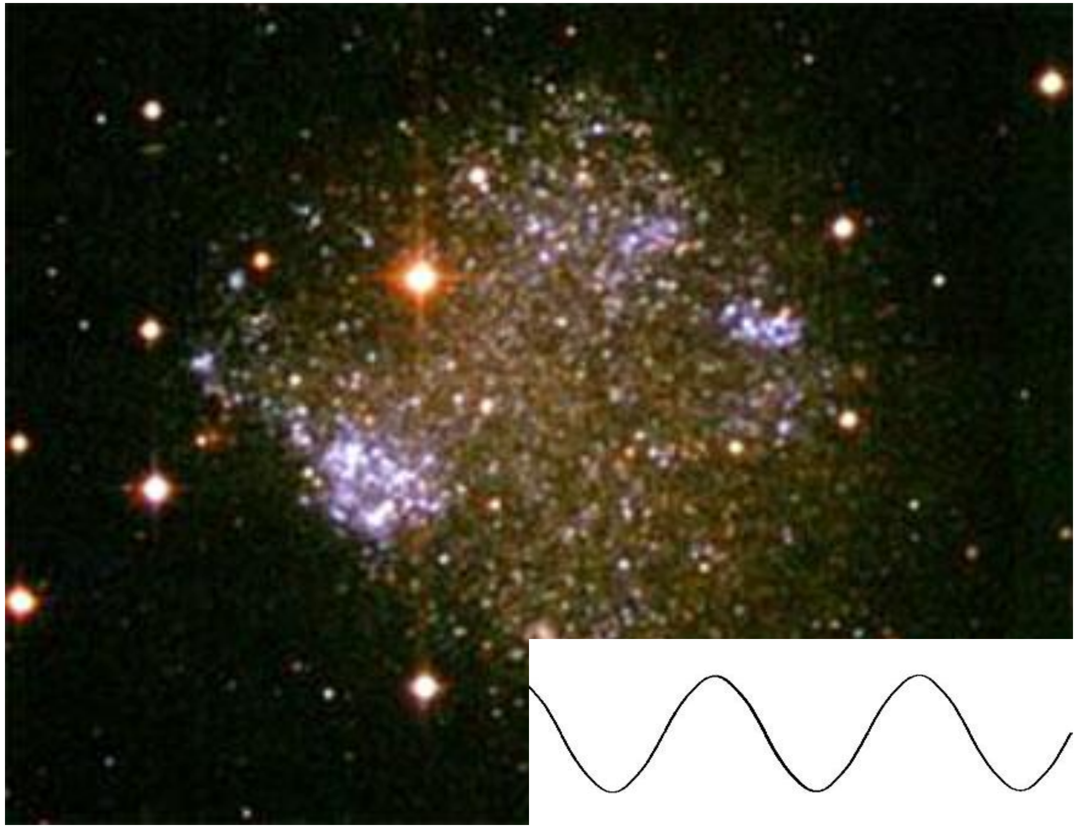
1 barn/GeV… which is **strong interaction**-size...

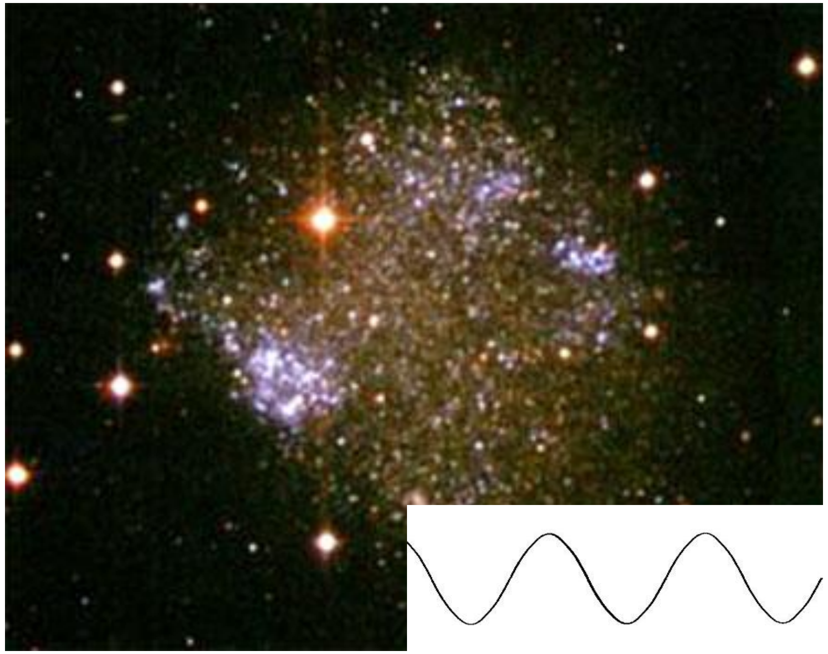
is this **small**?

Also, if cross section is **slightly smaller**, no **visible effect**...
if cross section **slightly larger**, **disaster**...

Begs the question: is “collisional” **self-interacting** dark matter a “**natural**” possibility??

- **Classical**: quantum effects (deBroglie wavelength) must be **smaller** than **smallest** observed collapsed **structure**





little **exercise**: consider *v ~ 10 km/s*, show that Compton wavelength: ***λ = h/p***

$$\lambda \sim 3\ \mathrm{mm}\ \left(\frac{1\ \mathrm{eV}}{m}\right)$$

which means that to have λ << kpc ~ 3×10^{21} cm, **$m>10^{-22}$ eV**

Much, much **better constraints** if the DM is a fermion – we know that the **phase space** density is bounded (Pauli blocking): $f = g h^{-3}$

Using **observed** density and velocity dispersion of dSph, **Tremaine-Gunn** limit (1979): observed phase space density cannot exceed upper bound! (Liouville theorem) Exercise!

$$\sigma \sim 150 \text{ km/s} \qquad \rho \gtrsim 1 \text{ GeV/cm}^3$$

$$\left(\frac{\rho}{m}\right)\left(\frac{1}{(2\pi m^2\sigma^2)^{3/2}}\right) < g h^{-3} \quad \Rightarrow \quad m^4 > \frac{\rho h^3}{g\,(2\pi\sigma^2)^{3/2}} \sim (25 \text{ eV})^4$$

- **Fluid**: don't want to **disrupt** pretty (and old!) **clusters** of stars

State of the art: Koulen, Profumo & Smyth 2403.1901

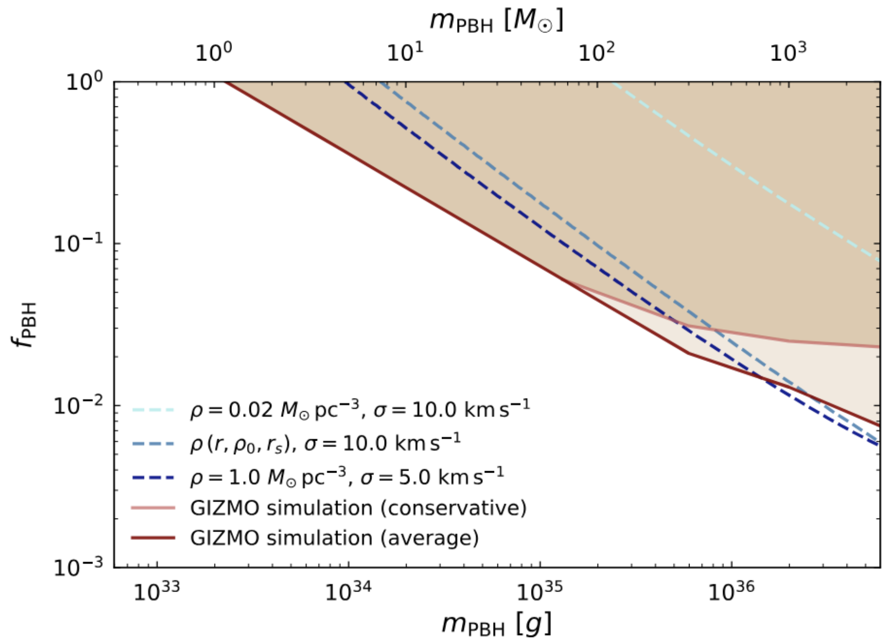


Figure 5. Constraints on f_{PBH} as a function of m_{PBH}. Limits are derived using the average r_h over 50 simulations for each mass. Also shown is a conservative case using r_h one standard deviation below the mean. The dashed lines are the semi-analytically derived constraints.

exercise to estimate the
exchanged by encounters
and BH, in the impulse
mation, demand that that
be smaller than binding
get **maximal mass** for BH

bility ("heating")

Bottom line: *m < 10^3* solar masses *~ 10^{70} eV*