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The ΛCDM Cosmological Model

Lecture 1

XXVII Special Courses at the Observatório Nacional – Rio de Janeiro, Brazil

Why am I here?

4. **Brazil** is awesome! ❤ BR

3. **Rio** is awesome! ❤

2. **Clarissa** (who is awesome!) invited me

1. I hope to **meet you all** and to **work with you**!

   you are the generation that will discover the **Dark Matter**!

Who am I?

- ✓ MS **Scuola Normal Superiore** (2001)

- ✓ PhD **Theoretical Particle Physics** (2004)
  International School for Advanced Studies (SISSA-ISAS), Trieste, Italy

- ✓ Postdoc, FSU and California Institute of Technology (2005-2007)
  Theoretical Astrophysics and Particle Physics

- ✓ Joined **UCSC Physics** Faculty (Assistant Professor, 2007-2011,
  Associate Professor, July 2011-2015
  Full Professor, July 2015-)

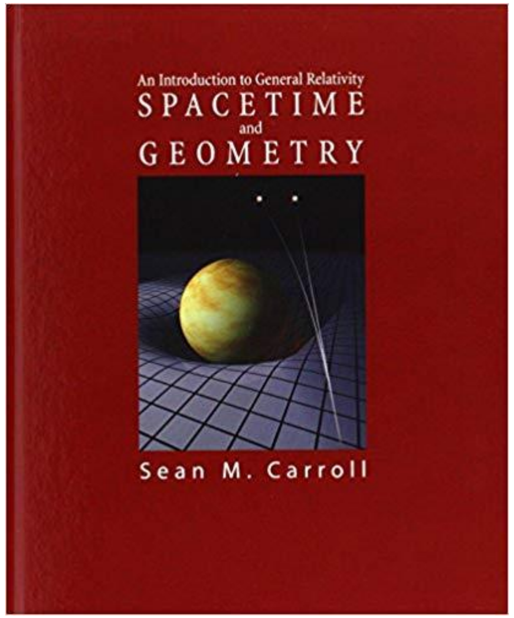
- ✓ Associate Dean of **Graduate Studies**, Faculty of Science (2024-)

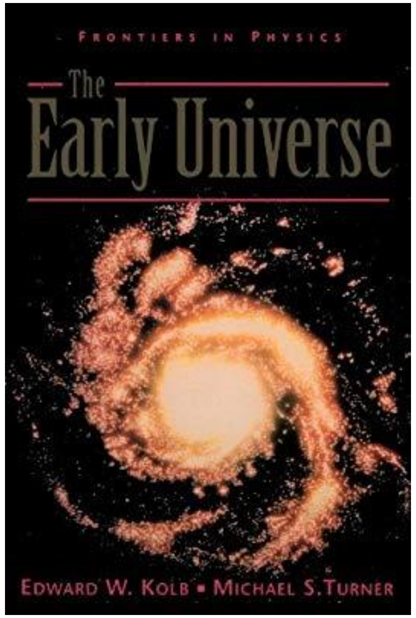
Please come **introduce yourselves**!

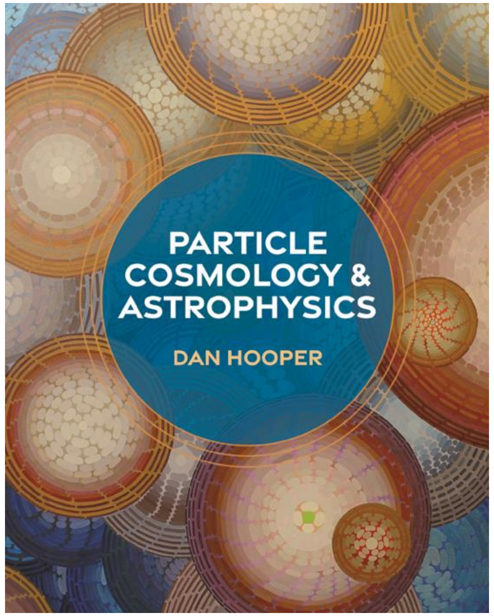
If you are ever on the **US West Coast** please let me know!

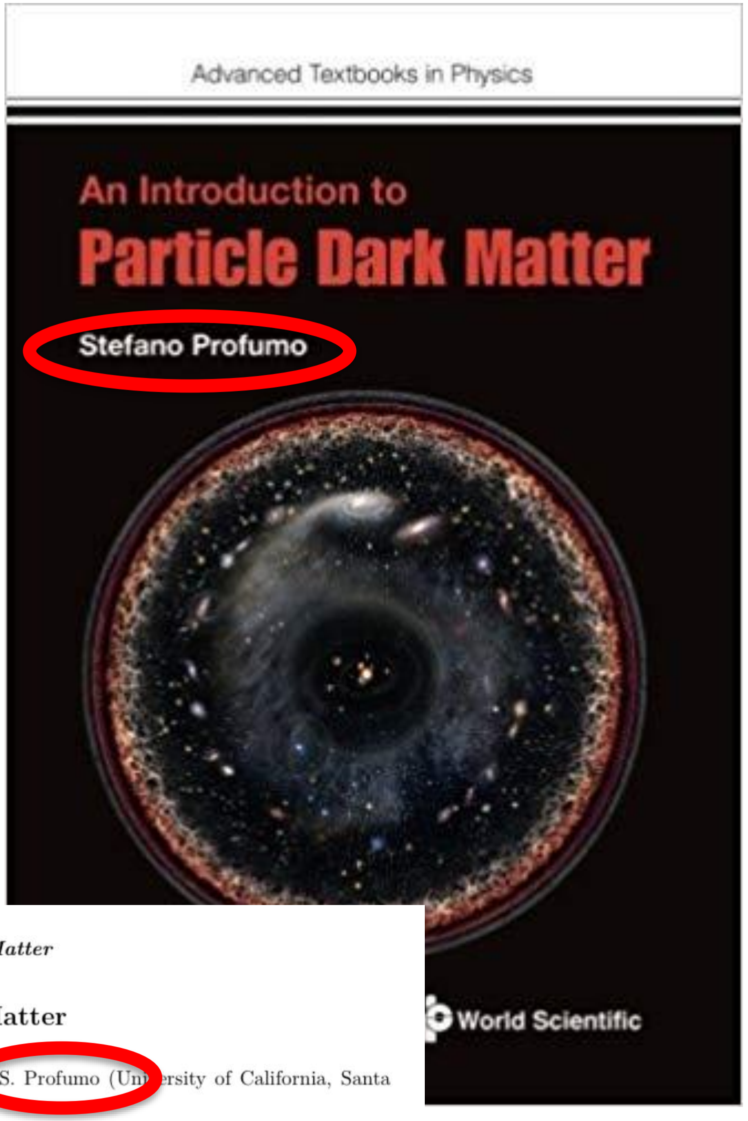
We are humans
…therefore **social** animals
…and so is **science**:
a **social enterprise**!

References









21. Big-Bang cosmology 1

21. BIG-BANG COSMOLOGY

Revised September 2011 by K.A. Olive (University of Minnesota) and J.A. Peacock (University of Edinburgh).

21.1. Introduction to Standard Big-Bang Model

1 *26. Dark Matter*

26. Dark Matter

Revised August 2018 by L. Baudis (Zürich U.) and S. Profumo (University of California, Santa Cruz).

26.1 The case for dark matter

Lambda-CDM: A Timeline

🪐 1910s–1930s: Theoretical Foundations and Expanding Universe

1915–1917:

Einstein formulates General Relativity (1915).
He introduces the **cosmological constant Λ** in 1917 to maintain a static universe—later called his "biggest blunder."

1922:

Alexander Friedmann derives **dynamical cosmological solutions** to Einstein's equations, predicting expanding (or contracting) universes.

1927:

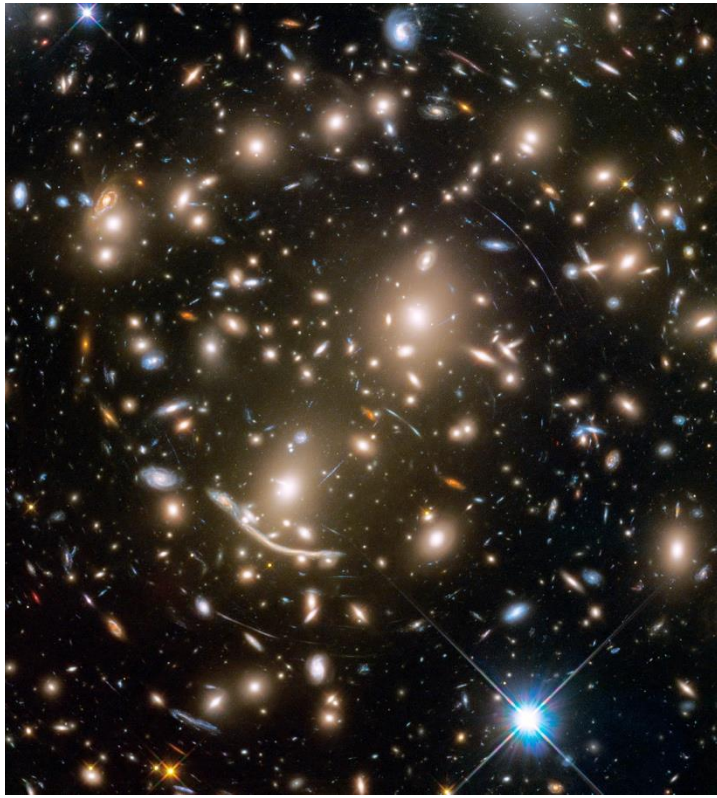
Georges Lemaître independently rediscovers Friedmann's solutions and interprets **redshift** observations as evidence for expansion.

1929:

Edwin Hubble discovers the **expansion of the universe** (Hubble's Law), in agreement with Friedmann–Lemaître models.

1933:

Fritz Zwicky infers the presence of **dark matter** from the virial mass discrepancy in the Coma galaxy cluster.



1940s–1960s: Early Universe Physics and the Hot Big Bang

1948:

Alpher, Herman, and Gamow propose the **Hot Big Bang model**, predict **Big Bang nucleosynthesis** **(BBN)** and predict a residual radiation: the **CMB** (~5 K).

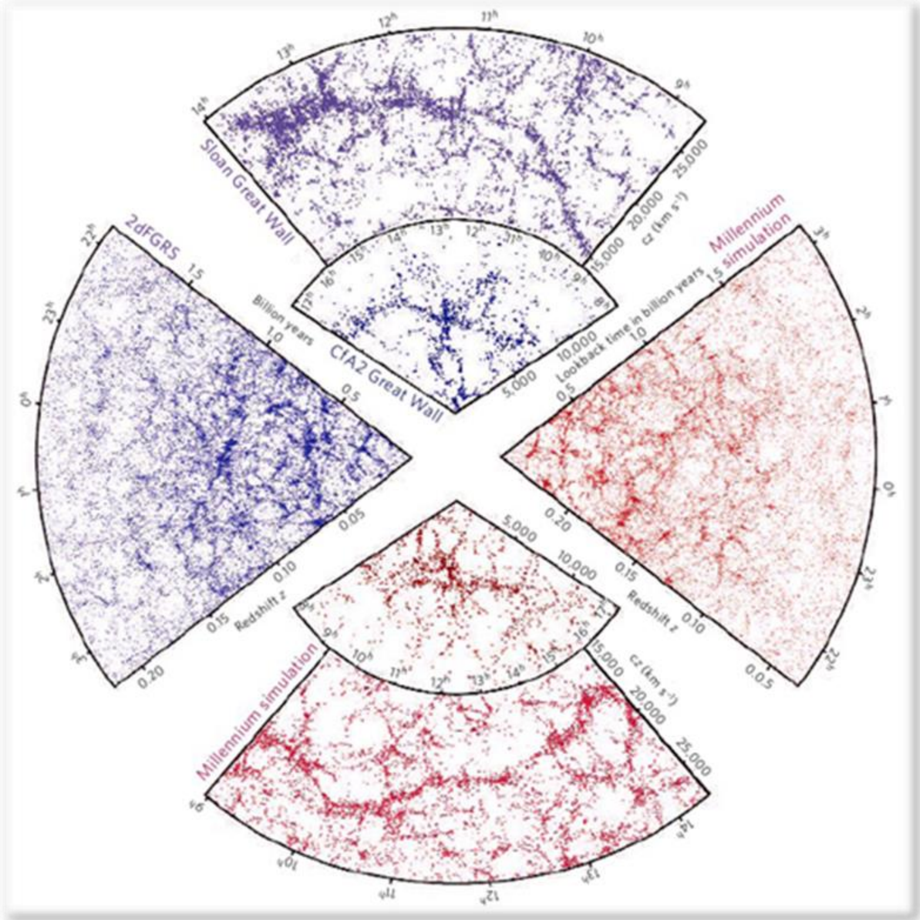
1950s–60s:

Dennis Sciama, **Jim Peebles**, and others develop the theory of **structure formation** in an expanding universe with growing perturbations

1965:

Penzias and Wilson *accidentally* discover the **cosmic microwave background** (CMB).

Dicke and Peebles immediately identify it as the **relic radiation** from the Big Bang.

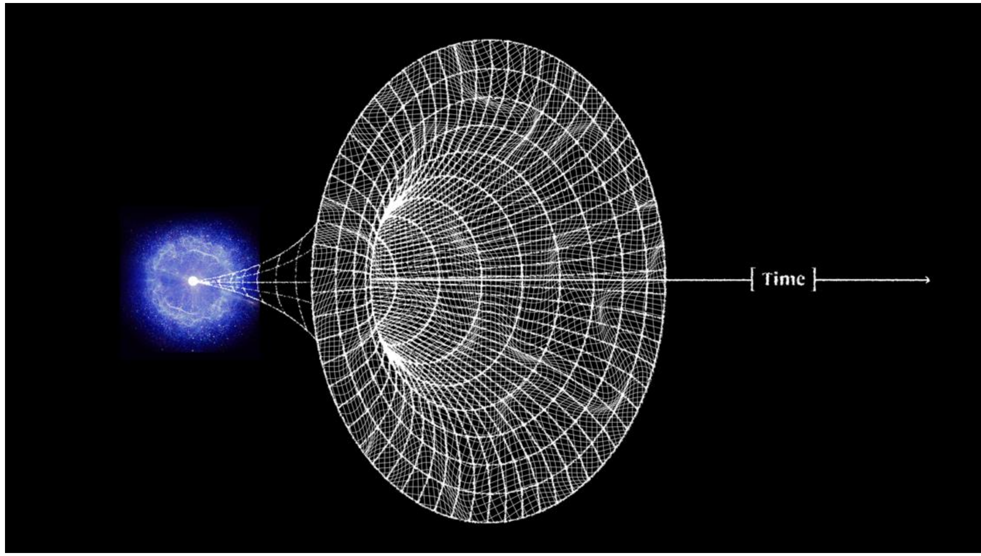


1970s–1980s: Inflation, Dark Matter, and Structure Formation

1970s:

Vera Rubin and **Kent Ford** measure galaxy rotation curves, implying **non-luminous, cold dark matter** in galactic halos.

Early ***N*-body simulations** suggest that **cold** dark matter (CDM), not hot (like neutrinos), better explains structure growth.



1980–1981:

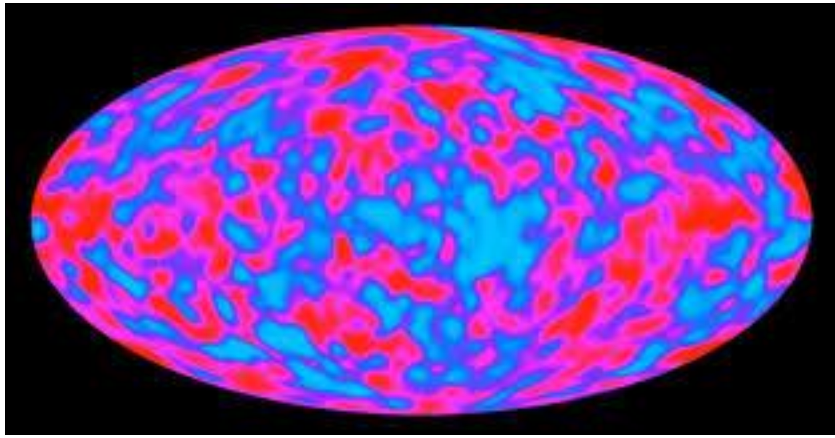
Alan Guth proposes **inflation**, a rapid early expansion that solves the flatness, horizon, and relics problems.
Inflation also predicts a **nearly scale-invariant** spectrum of **initial fluctuations**.

1984–1986:

Blumenthal et al. (**UC Santa Cruz**!) formally propose the **Cold Dark Matter model (CDM)** as the backbone of structure formation.

UC SANTA CRUZ

1990s: The Birth of the ΛCDM Model



1992:

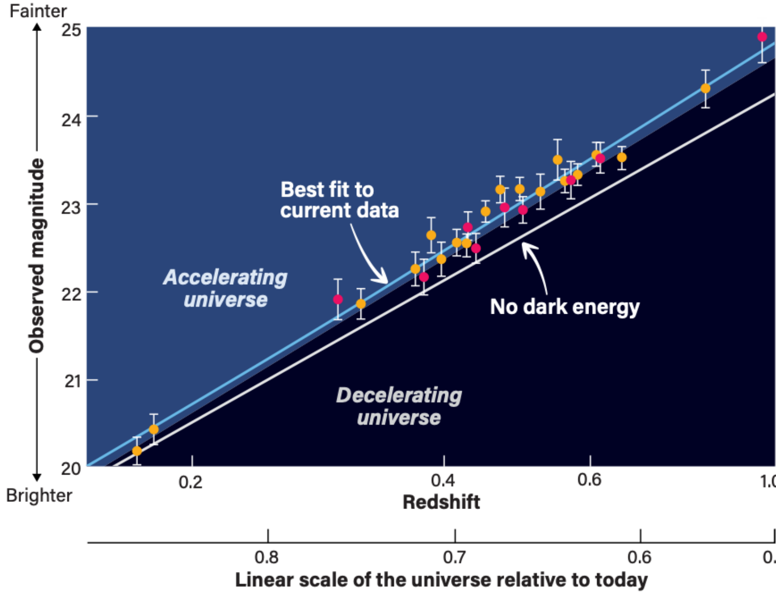
COBE detects **temperature anisotropies** in the CMB—validating inflationary predictions of structure seeds.

1995–1998:

Observations increasingly favor a **flat universe** with insufficient matter ($\Omega_m \sim 0.3$), implying **missing energy**.

1998–1999:

Supernova observations (Perlmutter, Riess, Schmidt) show the universe's expansion is **accelerating**, reviving **Einstein's Λ** as **dark energy**.

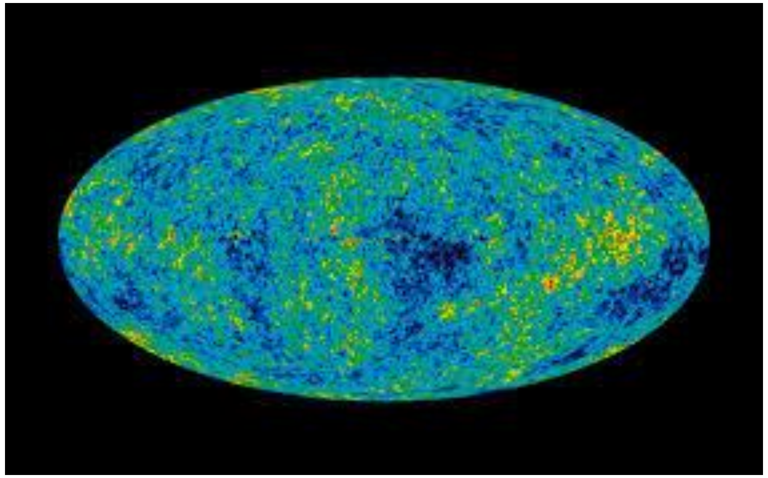


ΛCDM is born: A model with

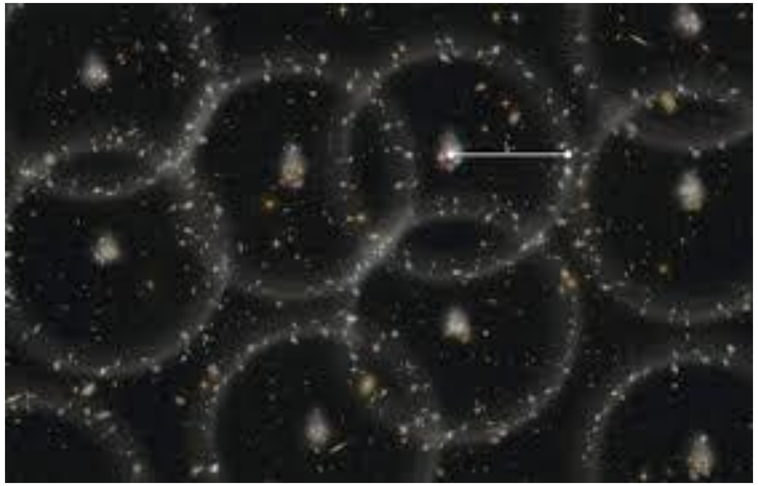
70% dark energy (Λ),
25% cold dark matter,
5% baryonic matter.

📈 2000s: Precision Cosmology and Concordance

WMAP confirms flatness, the scale-invariant spectrum, and CDM dominance, tightly **constraining** ΛCDM parameters.



BAO (Baryon Acoustic Oscillations) detected in large-scale structure surveys (e.g., SDSS), providing another **geometric ruler**.



📉 2010s–2020s: Cracks in the Model—Tensions Emerge

2013–2018:

Planck provides the most precise CMB data to date—ΛCDM fits extremely well **but**…

Tensions emerge:

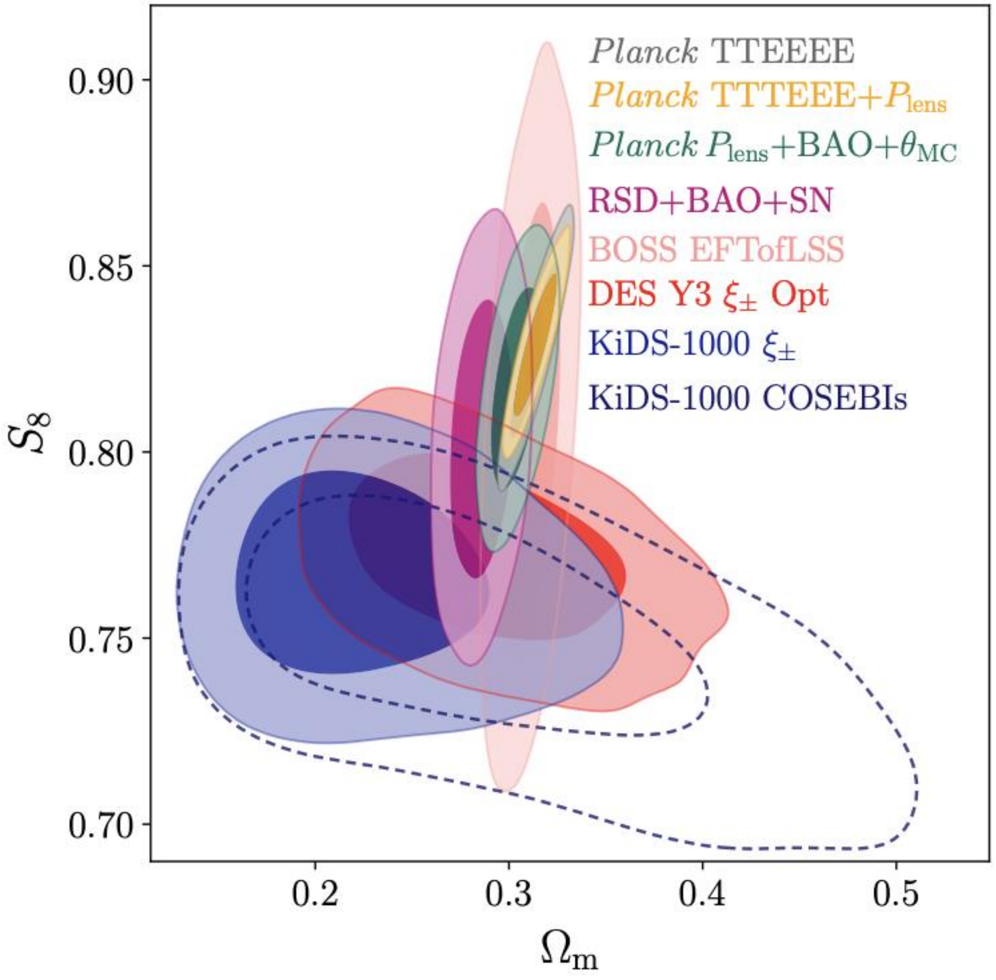
H_0 tension: Planck (H_0 ~ 67) vs local distance ladder (H_0 ~ 73).

S_8 tension: Large-scale structure (e.g., weak lensing) predicts lower clustering than Planck.

2020–2025:

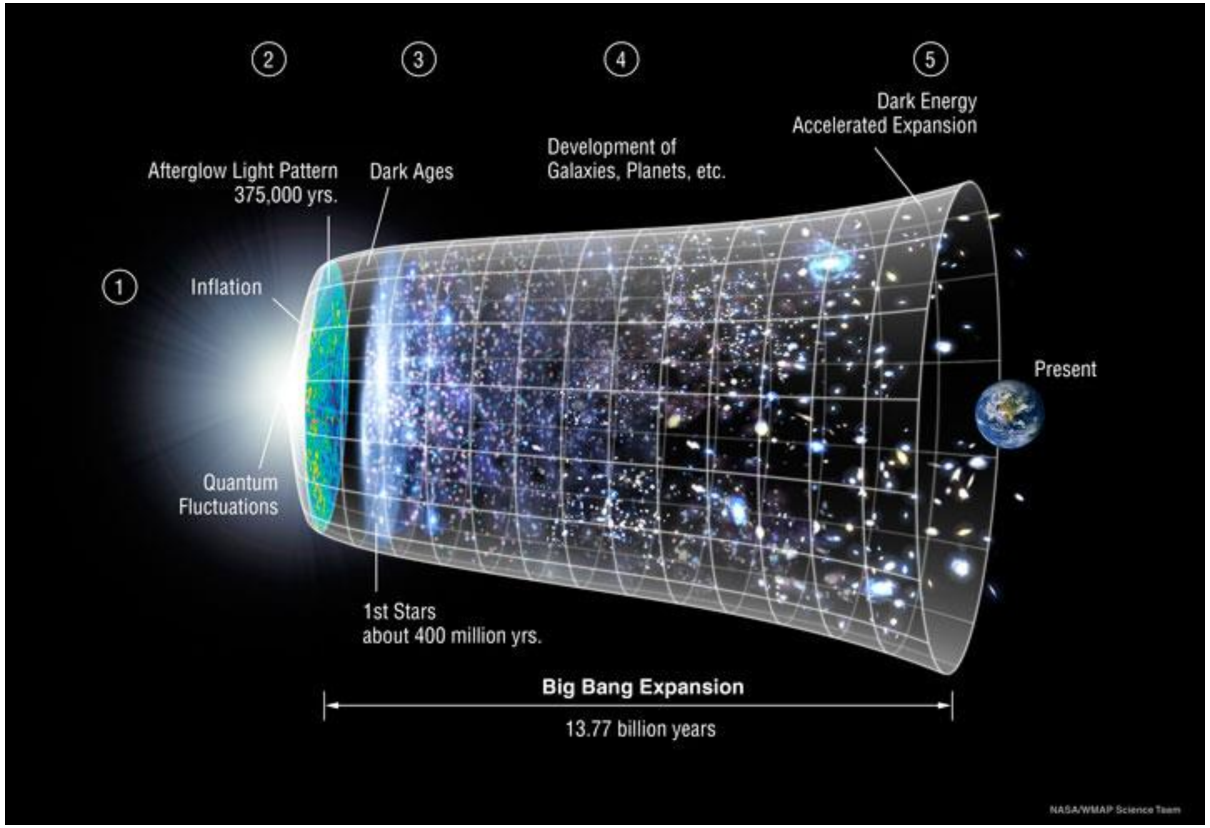
DESI, **Euclid**, **Rubin Observatory** (LSST), and other surveys begin high-precision tests of ΛCDM and its alternatives.

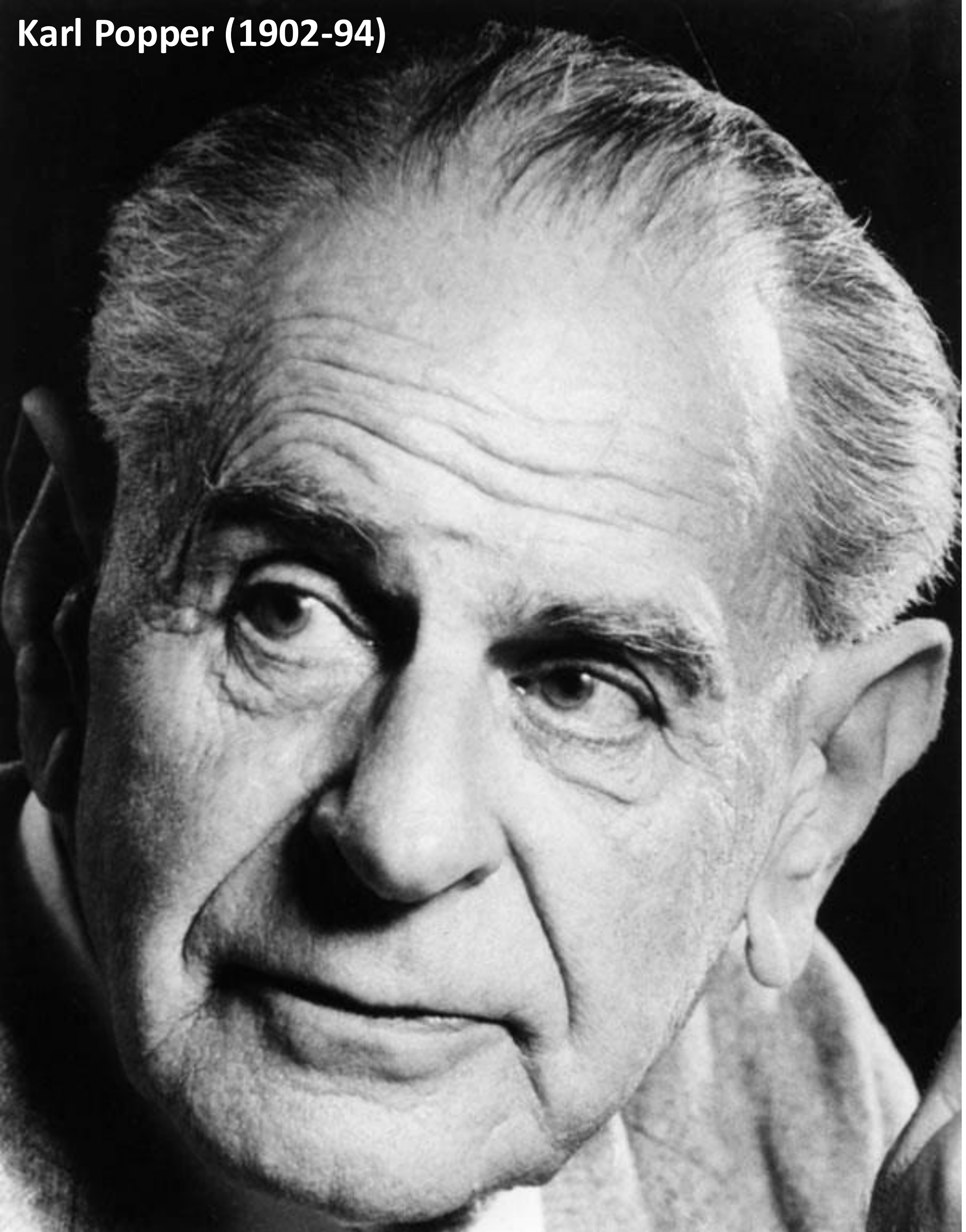
DESI++ may imply **evolving DE equation of state**



ΛCDM: ingredients of the "Standard Model of Cosmology"

- **General relativity**
- A **homogeneous**, **isotropic**, **flat universe**
- Three **massless neutrinos**, plus SM of P.P.
- **Cold**, **collision-less dark matter**
- A **cosmological constant (Λ)** as dark energy
- **Adiabatic**, **nearly scale-invariant** initial fluctuations, as seeded from **inflation**.

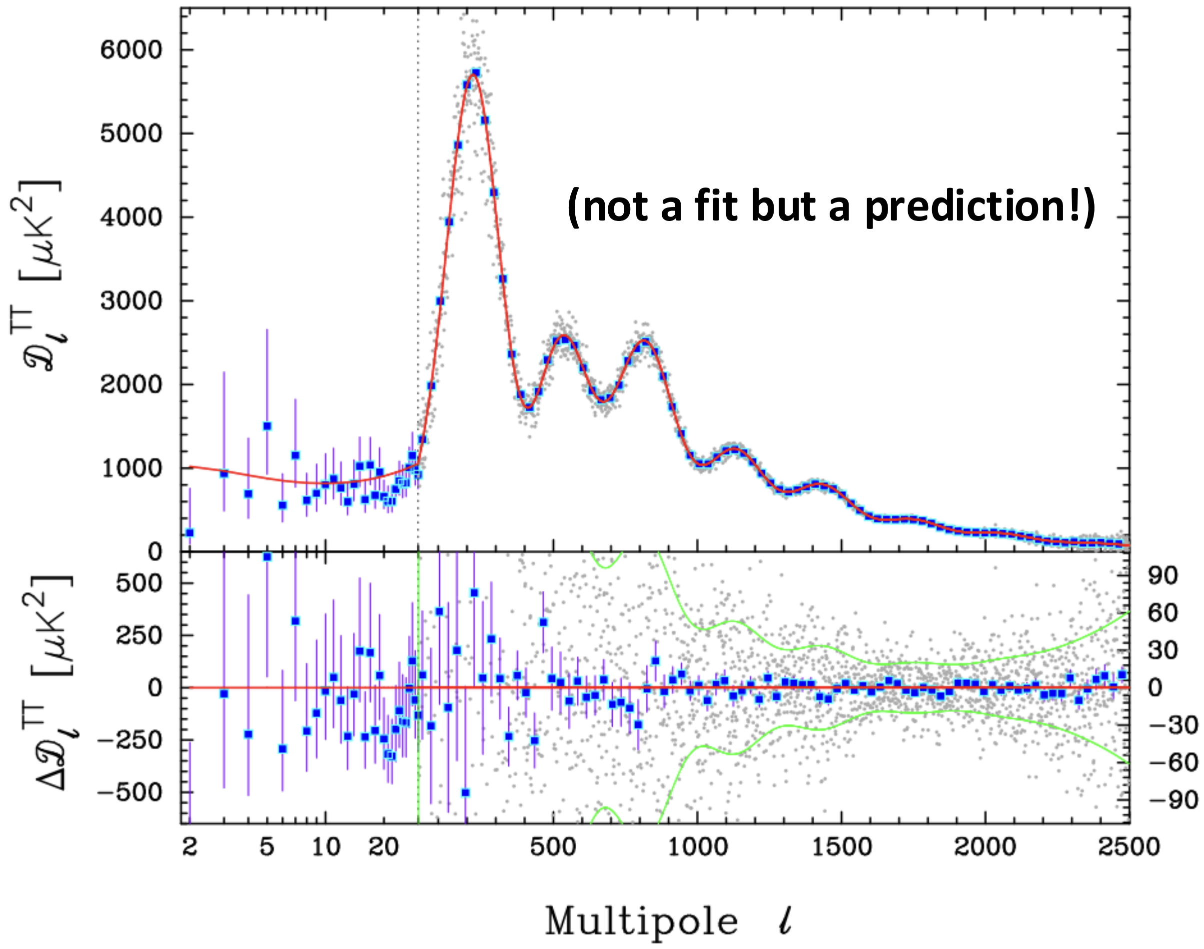




In so far as a scientific theory speaks about reality, it must be falsifiable: and in so far as it is not falsifiable, it does not speak about reality

The Logic of Scientific Discovery
(original German: *Logik der Forschung*, 1934)

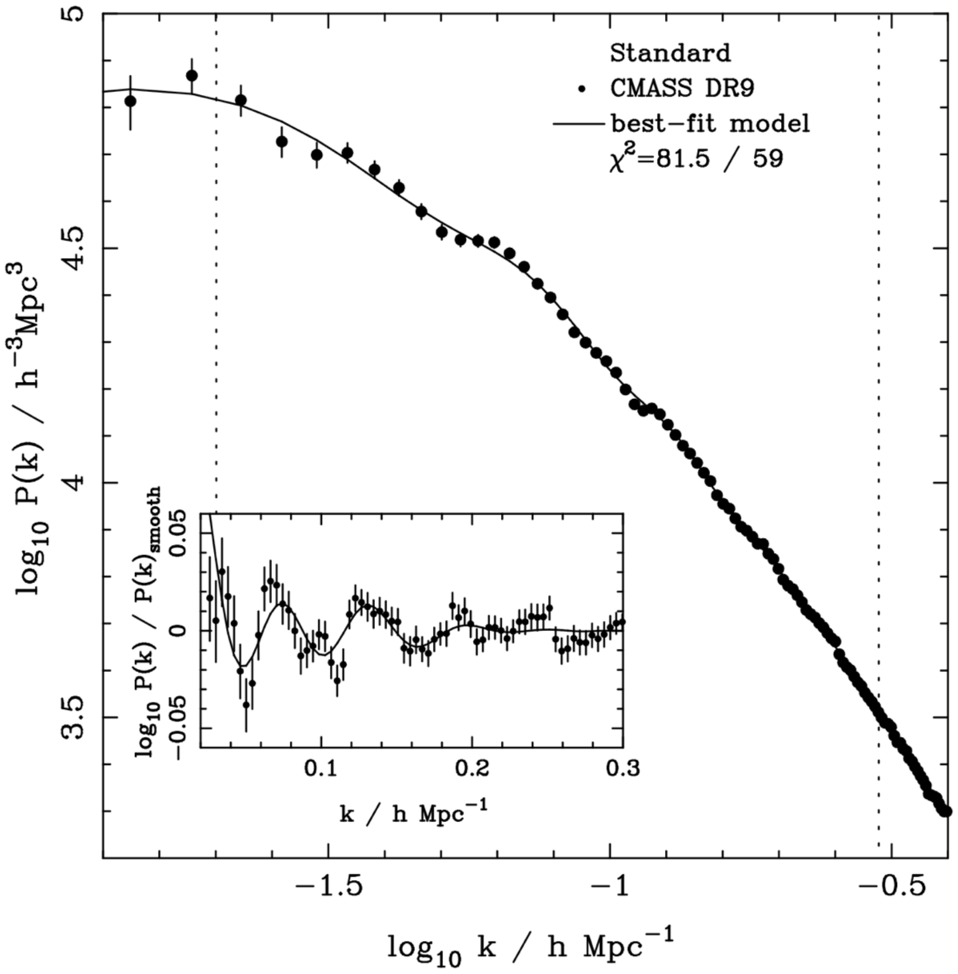
ΛCDM: ingredients of the "Standard Model of Cosmology"



6-parameter model ("vanilla" ΛCDM)

Baryon and CDM abundance, Hubble rate, tilt and amplitude of primordial fluctuations, optical depth

ΛCDM: ingredients of the "Standard Model of Cosmology"



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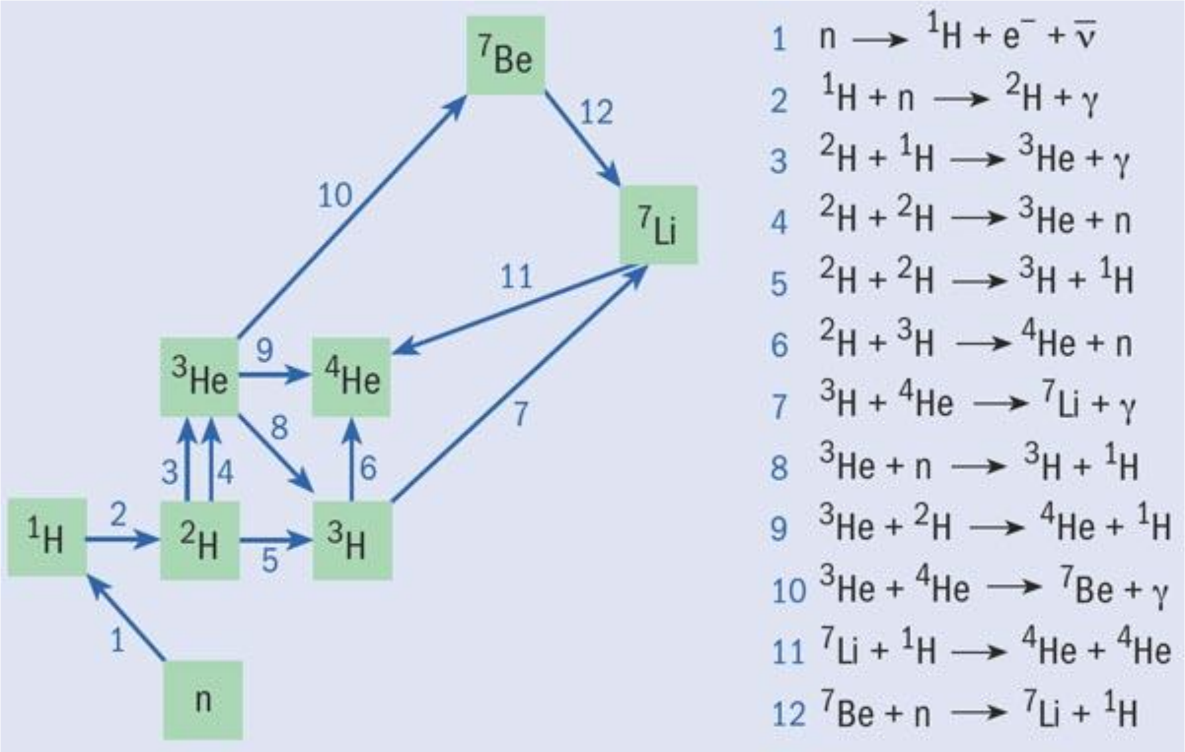
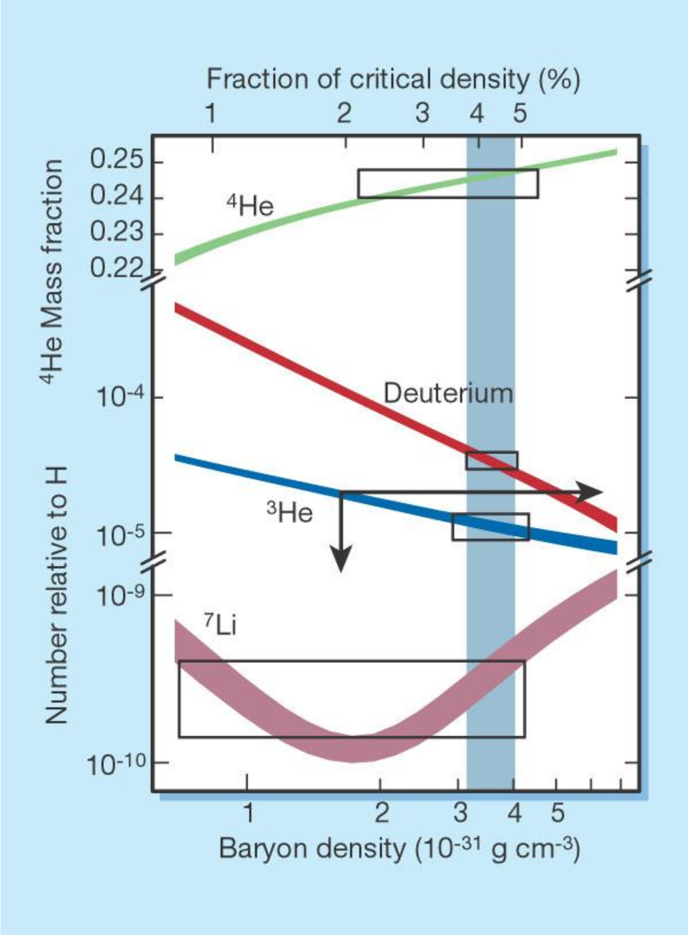




6-parameter model ("vanilla" ΛCDM)

Baryon and CDM abundance, Hubble rate, tilt and amplitude of primordial fluctuations, optical depth

The Geometry of Lambda CDM

ΛCDM relies on the assumption of the
"**Copernican Principle**":
the universe is (pretty much) the **same** everywhere

(are we in a totally **random place** in the universe?
...or are we the **center** of the universe?)

The Copernican Principle is clearly **false** locally...
but quite true on **large scales**:
galaxy surveys, X-ray, γ-ray backgrounds, CMB background

Mathematically: Copernican Principle is related to
properties of a **manifold**: **isotropy** and **homogeneity**

Modern **cosmology** assumes isotropy and homogeneity ***in space***.

However, the observation that distance galaxies are receding indicates that the **universe is changing in time**!

In **differential geometry** terms: the universe can be **foliated** into **homogeneous** and **isotropic space-like slices**

$$M = R \times \Sigma$$

if Σ is a 3D homogeneous and isotropic manifold, differential geometry demands it must be **maximally symmetric,** and provides us with the **general form** for the **metric**

$$ds^2 = -dt^2 + a^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)\right]$$

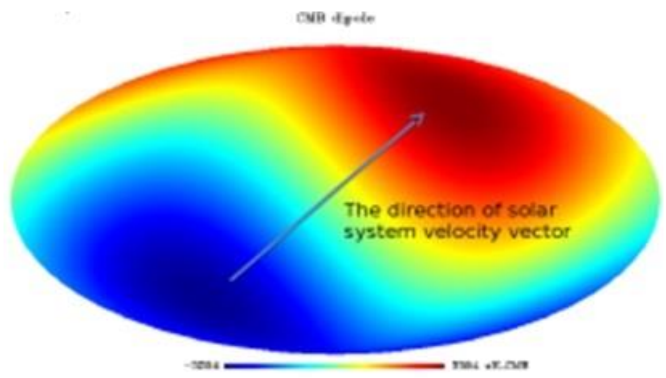
time-like coordinate

scale factor (only dep. on t)

maximally-symmetric 3D metric on Σ

This choice of metric, such that $dtdu^i$ are absent and there's a universal $a(t)$: "**comoving**" coordinates

Only a **comoving observer** (observer at fixed *u* coordinates) will see the universe as **isotropic** (**we are not** quite **comoving** – in fact we see a CMB dipole anisotropy...)



$$ds^2 = -dt^2 + a^2(t)\left[\frac{\mathrm{d}r^2}{1 - kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)\right]$$

Notice that the metric is left **unchanged** by simultaneously

$$\begin{cases} k \rightarrow \frac{k}{|k|} \\ r \rightarrow \sqrt{|k|}\,r \\ a \rightarrow \frac{a}{\sqrt{|k|}} \end{cases}$$

…and thus it **only** depends on ***k/|k|***,
leaving three distinct possibilities: ***k = -1, 0, 1***
*…open, **flat**, closed*

Einstein's equations explain how the **metric** derives from the **stress-energy tensor**

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu}$$

Assuming the universe is filled by **perfect fluids** with given equation of state $p = w\rho$

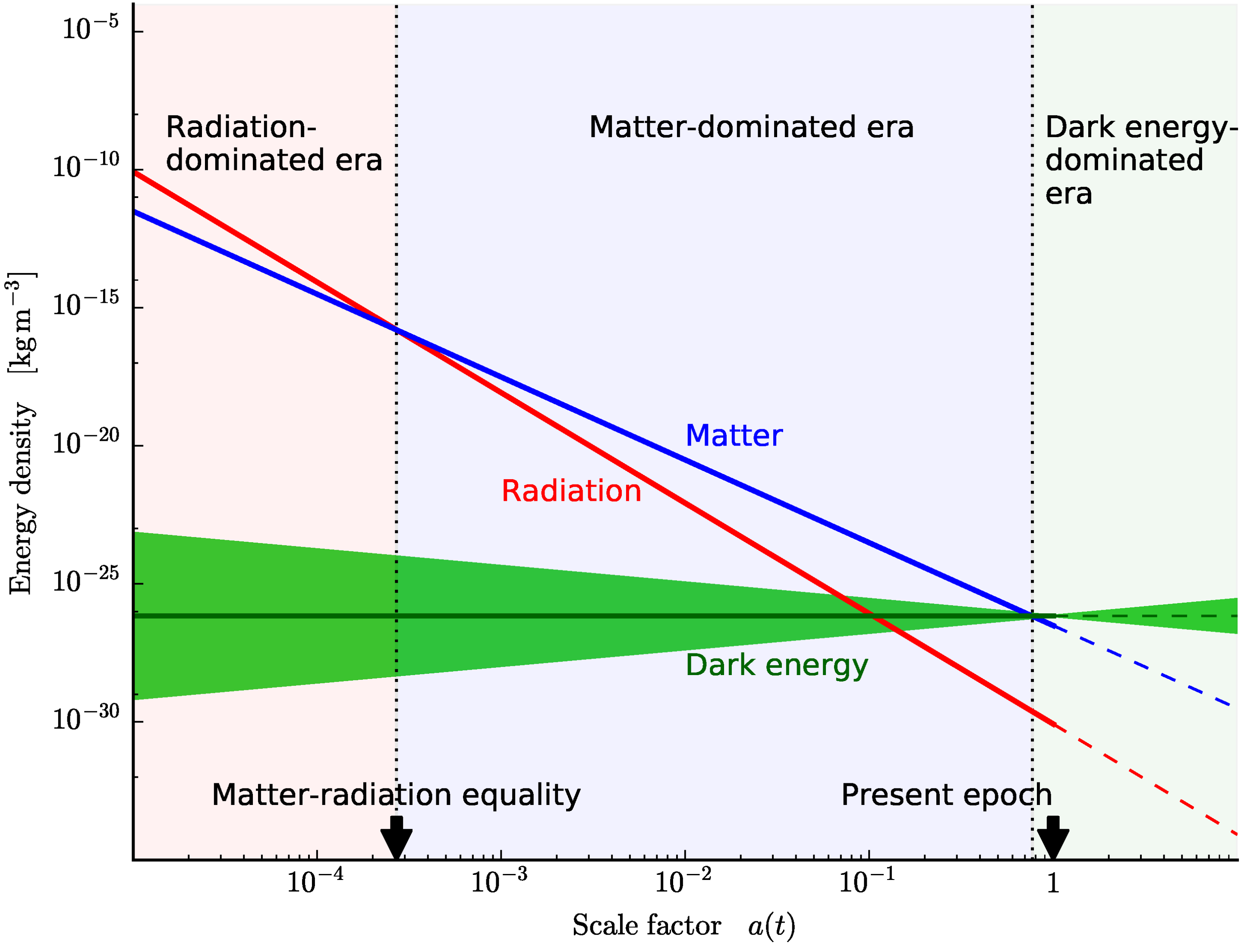
$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}$$

…the 00 component of Einstein's equation produces **Friedman's equation** connecting the **scale factor *a*** with the **energy-matter content**

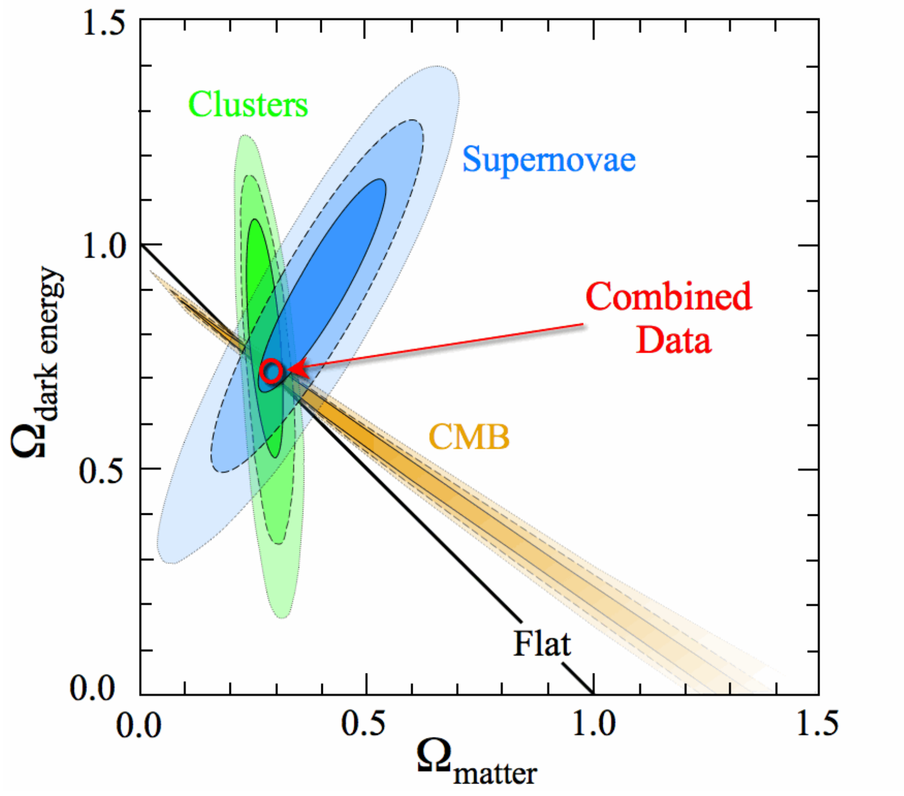
$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

H: inverse **age of the universe**, ~time derivative of the expansion scale factor *a*

Measurements today of the abundance of each fluid imply **which fluid** dominated the energy density, and **when**



key **parameters** for the **geometry** of the universe are **measured** in a variety of ways…



Executive Summary:

data indicate that universe today is compatible with being **flat**, with ~30% pressureless **matter**, 70% **cosmological-constant**- a ~perfect fluid, negligible relativistic matter

Lambda CDM and the hot Big Bang

General Relativity ❤️ Statistical Mechanics

$$(\hbar = c = 1 = k_B = 1)$$

$$f(\vec{p}) = \left[\exp\left(\frac{E-\mu}{T}\right) \pm 1\right]^{-1}$$

$$E(p) = (p^2 + m^2)^{1/2}$$

Particle species' **distribution function** (occupation number) in **thermal equilibrium**, e.g. Maxwell-Boltzmann, Fermi-Dirac, Bose-Einstein…

$$n = \frac{g}{(2\pi)^3} \int f(\vec{p})\, d^3p,$$

$$\rho = \frac{g}{(2\pi)^3} \int f(\vec{p})\, E(\vec{p})\, d^3p$$

$$P = \frac{g}{(2\pi)^3} \int f(\vec{p})\, \frac{|\vec{p}|^2}{3E}\, d^3p$$

Important regimes: high- and low-temperature

$T \gg m$

- Energy density $\rho \propto T^4$,
- Number density $n \propto T^3$,
- Pressure $p = \frac{1}{3}\rho$ (relativistic equation of state).

$T \ll m$

- Number density: $n \propto \left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T}$,
- Energy density: $\rho \approx nm \propto m \left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T}$,
- Pressure: $p \ll \rho$, and equation of state is approximately **dust-like**: $p \approx 0$.

$$H = \sqrt{\frac{\pi^2 g_*}{3 \cdot 30}} \frac{T^2}{M_P} \simeq 3.4 \frac{T^2}{M_P}$$

(radiation domination)

Inverse Hubble: age of the universe

The Big Bang Theory: Paradigm of Thermal Decoupling

A reaction is in **thermal equilibrium** ←→ **more than one** reaction occurs

→ The reaction **time scale** is **shorter** than the **age of the universe**

→ The **reaction rate** Γ is larger than the **Hubble rate** H

Condition for Equilibrium

$$\Gamma \gg H \quad \Rightarrow \quad \text{Interactions fast enough to maintain equilibrium.}$$

Condition for Decoupling / Freeze-Out

$$\Gamma \lesssim H \quad \Rightarrow \quad \text{Interactions too slow to keep up with expansion.}$$

Event	time t
Inflation	10^{-34} s (?)
Baryogenesis	?
EW phase transition	20 ps
QCD phase transition	20 μs
Dark matter freeze-out	?
Neutrino decoupling	1 s
Electron-positron annihilation	6 s
Big Bang nucleosynthesis	3 min
Matter-radiation equality	60 kyr
Recombination	260–380 kyr
Photon decoupling	380 kyr
Reionization	100–400 Myr
Dark energy-matter equality	9 Gyr
Present	13.8 Gyr

Example 1: Neutrino Decoupling (also: Hot Dark Matter)

🔶 Interactions Involved:

Neutrinos interact via **weak interactions** with electrons and positrons:

$$\nu + e^- \leftrightarrow \nu + e^-$$
$$\nu + \bar{\nu} \leftrightarrow e^- + e^+$$

🔶 Reaction Rate Γ_ν:

The weak interaction cross section at energy $E \sim T$ is:

$$\sigma_{\text{weak}} \sim G_F^2 T^2$$

Number density of relativistic species: $n \sim T^3$

So the interaction rate per neutrino:

$$\Gamma_\nu \sim n_e \langle \sigma v \rangle \sim G_F^2 T^5$$

🔶 Compare to Hubble Rate:

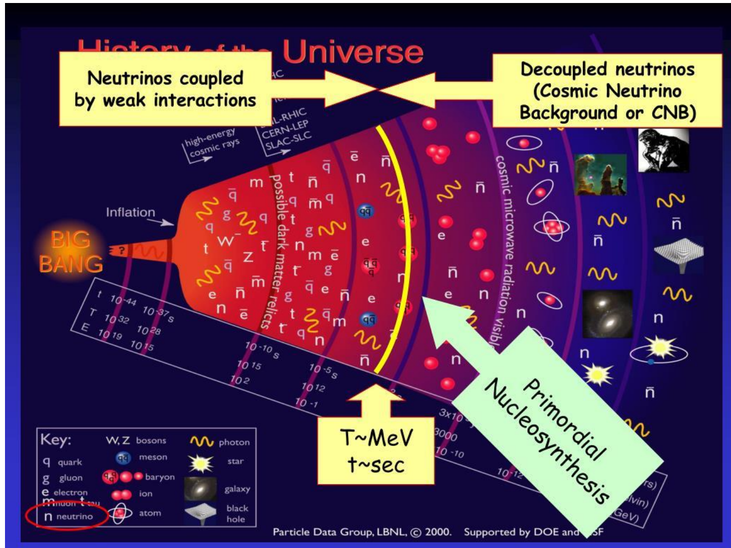
During radiation domination:

$$H \sim \frac{T^2}{M_{\text{Pl}}}$$

🔶 Decoupling Condition:

$$\Gamma_\nu \sim H \quad \Rightarrow \quad G_F^2 T^5 \sim \frac{T^2}{M_{\text{Pl}}} \Rightarrow T_{\nu,\text{dec}} \sim \left(\frac{1}{G_F^2 M_{\text{Pl}}} \right)^{1/3} \sim 1\,\text{MeV}$$

So neutrinos decouple when $T \sim 1\,\text{MeV}$, just before electron–positron annihilation.



Example 1: Neutrino Decoupling (also: Hot Dark Matter)

Let's slow down and think about our **assumptions** so far:
[T_ν~ 1 MeV]

1. **hot** relic assumption works! $T_\nu \gg m_\nu$
2. **Fermi** effective theory OK! $T_\nu \ll m_W$

now, how do we calculate the **relic** thermal **abundance** of this prototypical hot relic?

Introduce ***Y=n/s*** (ratio of number and entropy **density**, $V=a^3$)

If universe is iso-entropic (i.e. adiabatic), ***s x a³=S*** is conserved: ***1/s ~ a³***

Y ~ n a³ is thus ~ **comoving number density**, and (without entropy injection)

Example 1: Neutrino Decoupling (also: Hot Dark Matter)

$$Y_{\text{today}} = Y_{\text{freeze-out}} = Y(T_\nu)$$ [by definition of "**frozen out**"]

$$Y_{\text{freeze-out}} = \frac{n(T_\nu)}{s(T_\nu)} = \frac{\rho_\nu(T_\nu)}{m_\nu \cdot s(T_\nu)}$$

$$n_{\text{today}} = s_{\text{today}} \times Y_{\text{today}} = s_{\text{today}} \times Y_{\text{freeze-out}}$$

$$\rho_{\nu,\text{today}} = m_\nu \times Y_{\text{freeze-out}} \times s_{\text{today}}$$

$$\Omega_\nu h^2 = \frac{\rho_\nu}{\rho_{\text{crit}}} h^2 \simeq \frac{m_\nu}{91.5 \text{ eV}}$$

1. **Cowsik-Mc-Clelland** limit
2. Constraints on **neutrinos** as **DM components**
3. (vice-versa) **constraints** on **neutrino masses**

Example 2: Photon Decoupling

When:

- Around $T \sim 0.26\,\mathrm{eV}$
- Redshift $z \sim 1100$

Interactions Involved:

Photon decoupling is tied to **electron–photon scattering**, i.e., **Thomson scattering**:

Reaction Rate Γ_γ:

The Thomson cross section is nearly constant:

$$\sigma_T \approx 6.65 \times 10^{-25}\,\mathrm{cm}^2$$

The number density of **free electrons** before recombination is $n_e \sim x_e n_b \sim x_e \eta T^3$, with:

- x_e: ionization fraction,
- $\eta \sim 10^{-9}$: baryon-to-photon ratio.

So:

$$\Gamma_\gamma \sim n_e \sigma_T$$

Compare to Hubble Rate:

Still in radiation/matter transition:

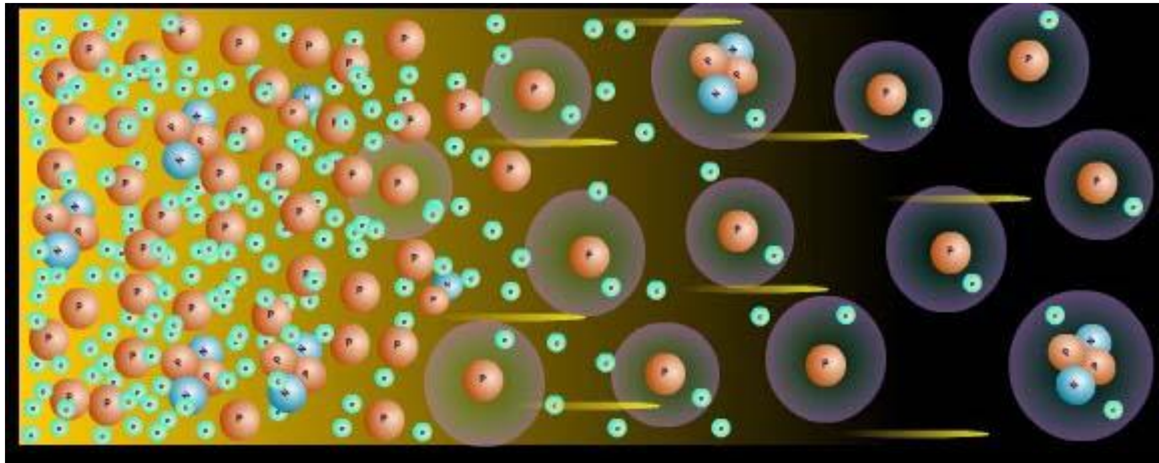
$$H \sim \frac{T^{3/2}}{M_{\mathrm{Pl}}^{1/2}} \quad \text{(during matter era)}$$

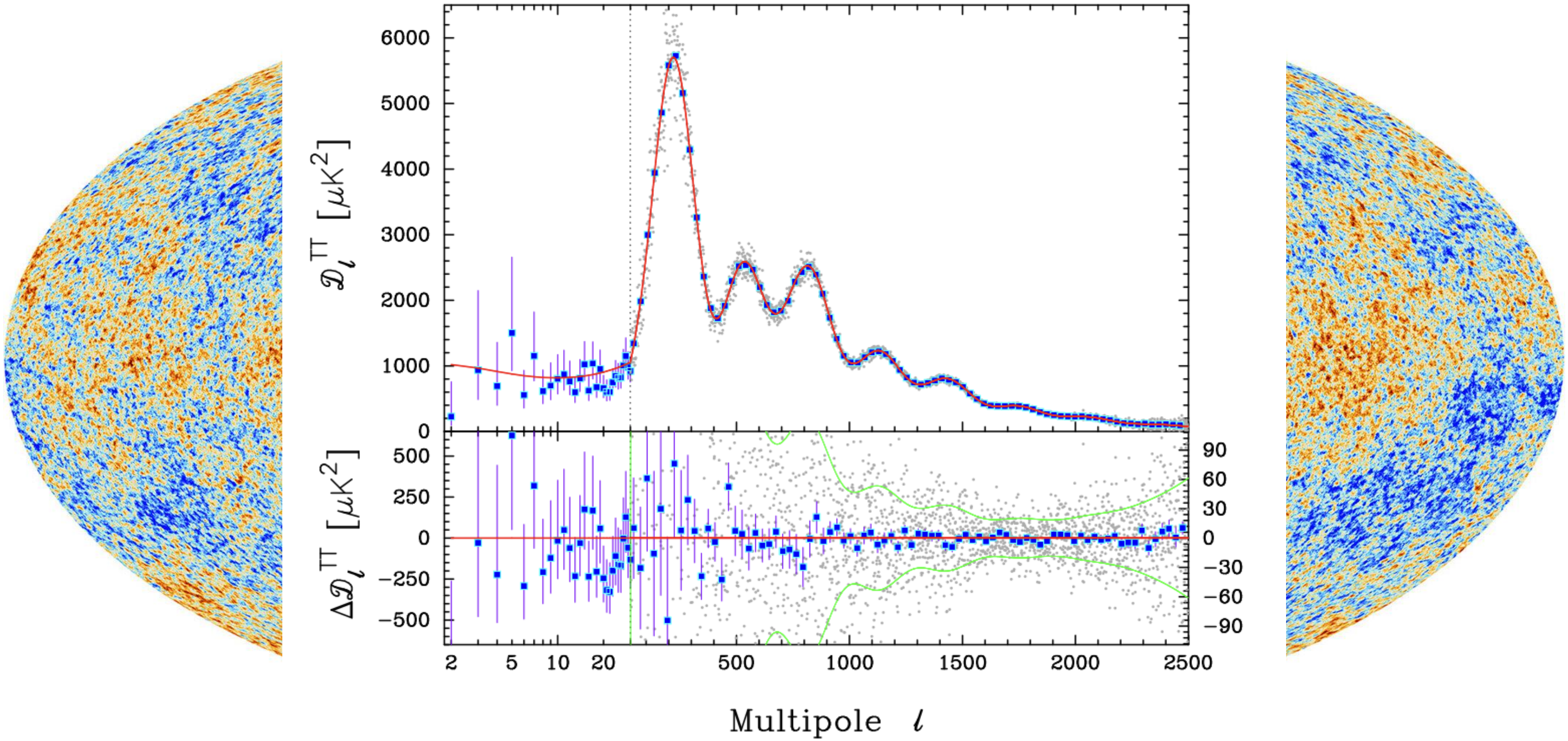
Decoupling Condition:

$$\Gamma_\gamma \sim H \quad \Rightarrow \quad \text{Occurs when } x_e \text{ drops sharply (i.e., recombination)}$$

Photon decoupling thus occurs when:

- The ionization fraction $x_e \ll 1$,
- Free electrons are too rare to efficiently scatter photons.

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Multipole ℓ

Example 3: Cold Relics

Try **proton-antiproton** freeze-out:
what's the **relic** matter **abundance** in a baryon-symmetric Universe?

$$\sigma \sim \Lambda_{\rm QCD}^{-2}$$

$$n\,\sigma = H \rightarrow T^3\,\Lambda^{-2} = T^2/M_P \rightarrow T = \Lambda^2/M_P$$

doesn't quite work, we're way **outside**
the regime of validity for **hot relics**, since $T \lll\lll m_p$ ...

Need to work out the case of **cold relics**, which looks nastier by eye

$$n \sim (m_\chi T)^{3/2} \exp\left(-\frac{m_\chi}{T}\right)$$