Post-Graduation in Space Geophysics
Mathematical Examination for Applicants

Name of candidate:

Date:

Maximum duration: 1 hour and 30 minutes

All sheets used for solving issues should be identified

READ THE INSTRUCTIONS CAREFULLY

In the set of problems enrolled in this examination, it is adopted the following conventions:

- $\mathbf{A}$ vector or matrix **A**;
- $\mathbf{A}\cdot\mathbf{B}$ scalar product between **A** and **B**;
- $\mathbf{A}\times\mathbf{B}$ vectorial product between **A** and **B**;
- $|x|$ absolute value of the real variable x
- $\mathbf{x} \triangleq (x, y, z)$ location vector in the real Euclidian space, $\mathbb{R}^3$;
- $\|\mathbf{x}\| = \sqrt{\mathbf{x}\cdot\mathbf{x}}$ Euclidian norm of the **x** in $\mathbb{R}^3$;
- i imaginary unit, defined by $i\,i = i^2 = -1$;
- z complex variable, equal to $\mathrm{Re}(z) + i\,\mathrm{Im}(z)$ $(z \in \mathbb{C})$;
- $\overline{z}$ conjugate complex of the z, equal to $\mathrm{Re}(z) - i\,\mathrm{Im}(z)$ $(z \in \mathbb{C})$;
- $|z| = \sqrt{z\,\overline{z}}$ module of the z $(z \in \mathbb{C})$.

Moreover, we state that all the rule for derivative, partial derivative, divergence, rotational, matrix operations, complex operations, among others, follows the usual notations for mathematical representation.

Question 1 [Spatial Vectors] (25)

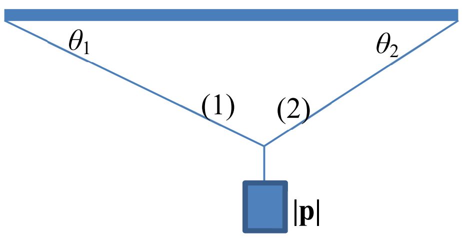
A moving kinematic particle is described by the following position vector:

$$\mathbf{r}(t) = (a\cos\omega t,\ a\sin\omega t,\ h(t)).$$

Determine what the general form of the function $h(t)$ should be such that the velocity vector of this motion is identically equal to the cross product between the angular velocity vector, $\boldsymbol{\omega} = (0, 0, \omega)$, and the position vector, $\mathbf{r}(t)$.

Question 2 [Vector Algebra] (25)

The material of weight $|\mathbf{p}|$ (magnitude of the weight force) is held suspended by two ideal wires (1) and (2) in static equilibrium, as shown in the drawing below.



(a) Determine, in function of the angles θ_1, θ_2 and $|\mathbf{p}|$, the expressions of the magnitudes of the tensions in the wires (1) and (2).

(b) If $\theta_1 = \theta_2 = \theta$, and for very small angles, show that $|\mathbf{T}_1| = |\mathbf{T}_2| \simeq \dfrac{|\mathbf{p}|}{2\theta}$.

Question 3 [Complex Algebra] (25)

Consider that $z, w \in \mathbb{C}$, $(z, w \neq 0)$ are complex variables such that form the following system of equations:

$$\bar{z} - \bar{w} = \alpha$$
$$w^2 - z^2 = \beta,$$

where $\alpha, \beta \in \mathbb{R}\,(\neq 0)$. Using the properties of conjugate complexes ($\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$ and $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$), show that the sum of the variables $z + w = -\dfrac{\beta}{\bar{\alpha}}$.

Question 4. [Optimization] (25)

Two isotropic point sources (isotropic radiators) of sound are placed on the same straight line, both separated by a distance a. One of these sound sources has three times the power compared to another. To **minimize** the hearing effect (sound intensity) of a listener located in the same straight line between the sources, evaluate, according to the separation distance of the sources, how this listener should position himself.

Note 1: The sound intensity of an isotropic radiator is proportional to the power of the source and inversely proportional to the square of the source distance.

Note 2: The sound intensity is an additive quantity.