Post-Graduation in Space Geophysics
Mathematical Examination for Applicants

Name of candidate:

Date:

Maximum duration: 1 hour and 30 minutes

All sheets used for solving issues should be identified

READ THE INSTRUCTIONS CAREFULLY

In the set of problems enrolled in this examination, it is adopted the following conventions:

$\mathbf{A}$	vector or matrix $\mathbf{A}$;
$\mathbf{A}\cdot\mathbf{B}$	scalar product between $\mathbf{A}$ and $\mathbf{B}$;
$\mathbf{A}\times\mathbf{B}$	vectorial product between $\mathbf{A}$ and $\mathbf{B}$;
$\lvert x\rvert$	absolute value of the real variable x
$\mathbf{x}\triangleq(x,y,z)$	location vector in the real Euclidian space, $\mathbb{R}^3$;
$\lVert\mathbf{x}\rVert=\sqrt{\mathbf{x}\cdot\mathbf{x}}$	Euclidian norm of the $\mathbf{x}$ in $\mathbb{R}^3$;
i	imaginary unit, defined by $i\,i=i^2=-1$;
z	complex variable, equal to $\mathrm{Re}(z)+i\,\mathrm{Im}(z)$ $\left(z\in\mathbb{C}\right)$;
$\overline{z}$	conjugate complex of the z, equal to $\mathrm{Re}(z)-i\,\mathrm{Im}(z)$ $\left(z\in\mathbb{C}\right)$;
$\lvert z\rvert=\sqrt{z\,\overline{z}}$	module of the $z\left(z\in\mathbb{C}\right)$.

Moreover, we state that all the rule for derivative, partial derivative, divergence, rotational, matrix operations, complex operations, among others, follows the usual notations for mathematical representation.

Question 1 [Functions, Curves] (30)

Let the plane curve-(x,y) represented by the parameter t, $\gamma(t): I\to\mathbb{R}^2$, where $t\in I$, the domain defined in terms of the parametrization.

(a) Find a regular parametrization of the plane curve $\gamma(t)$, and the domain I, defined by the equation,

$$\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}=1,\quad a,b\in\mathbb{R}_+.$$

(b) Find the Radius of Curvature of $\gamma(t)$.

(c) Show that if $a=b=r$, the Radius of Curvature of $\gamma(t)$ is r.

[Suggestion: The Radius of Curvature, R_γ, can be calculated by the follow vector expression,

$$\frac{1}{R_\gamma}=\frac{\lVert\boldsymbol{\gamma}'(t)\times\boldsymbol{\gamma}''(t)\rVert}{\lVert\boldsymbol{\gamma}'(t)\rVert^3}$$, where $\boldsymbol{\gamma}(t)=(x(t),y(t))$; $\boldsymbol{\gamma}'(t)=\frac{\partial}{\partial t}\boldsymbol{\gamma}(t)$; $\boldsymbol{\gamma}''(t)=\frac{\partial}{\partial t}\boldsymbol{\gamma}'(t)$.]

Question 2 [Vector Algebra] (15)

Let two vectors represented by the rectangular components, $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$:

$$\mathbf{x} = \mathbf{e}_1 + 2\mathbf{e}_2 - \mathbf{e}_3,$$
$$\mathbf{y} = 2\mathbf{e}_1 + \mathbf{e}_2 + \mathbf{e}_3.$$

Find the Unit Normal vector relative to the $\mathbf{x}$ and $\mathbf{y}$ vectors.

Question 3 [Complex Algebra] (15)

The sum of the complex number $z \in \mathbb{C},\ (z \neq 0)$ with the triple of your conjugate results the unitary complex number. Therefore, find the complex modulus of z.

Question 4. [Optimization] (20)

A very thin wire of length l is cutted into two pieces. With one of them, a circle is formed; with the remaining piece, a regular hexagon is formed. Find how to cut the wire (the lengths of the two pieces), so that the sum of the two areas covered by the circle and the hexagon is minimal.

Question 5. [Matrix Algebra] (20)

Discuss (consistent / inconsistent) and solve the following set of linear equations- (x, y), in terms of α and β parameters:

$$\begin{cases} x - y = \beta \\ 2x + \alpha y = 4 \end{cases}.$$