Cosmic Microwave Background

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12/Sep/2011

Detectors
And Noise

Detectors

- A detector is a **transducer**, converting the incoming electromagnetic signal (E or W) into an electrical signal (V or i), in a controlled and repeatable way.
- The electrical signal can be amplified, digitized, stored, analyzed.
- Usually $V=\Re W$, where $\Re$ is the **responsivity** of the detector (units V/W): a constant which is measured following a calibration procedure.
- The calibration consists in observing a source producing a known power on the detector, and recording the output voltage produced by the detector.
- Other important characteristics of a detector are
  - Linearity and Dynamic Range
  - Time constant
  - Spectal response
  - Angular response
  - Noise Equivalent Power (NEP) or Noise Equivalent Temperature (NET)

Noise

- A noisy physical observable features random fluctuations of its value.
- The amplitude of these fluctuations can only be quantified statistically: their punctual behaviour cannot be predicted, it is not deterministic.
- The simplest statistical tool to characterize noise is the variance: for a given observable V(t) the variance is

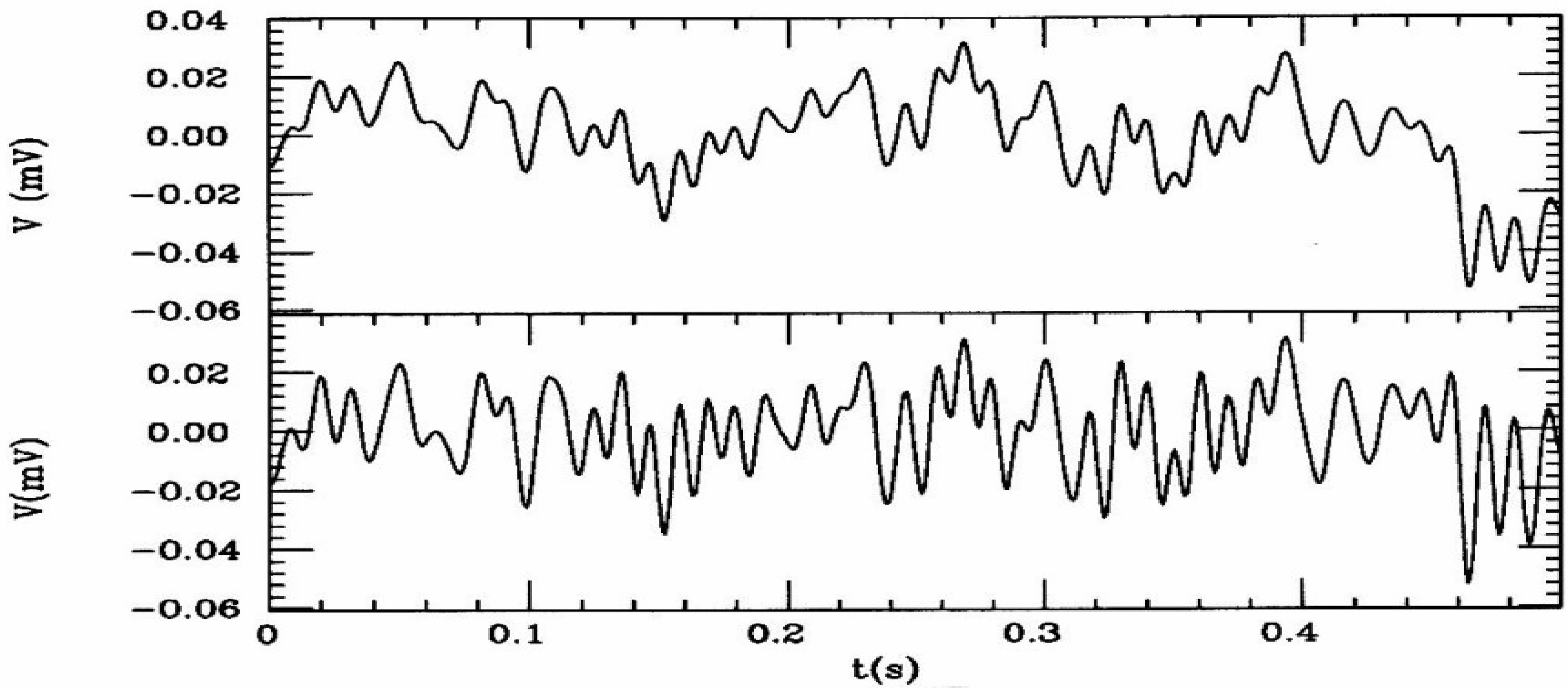
Continuous variable

$$\sigma_V^2 = \frac{1}{T}\int_0^T \left[V(t) - \langle V \rangle\right]^2 dt$$

Sampled variable

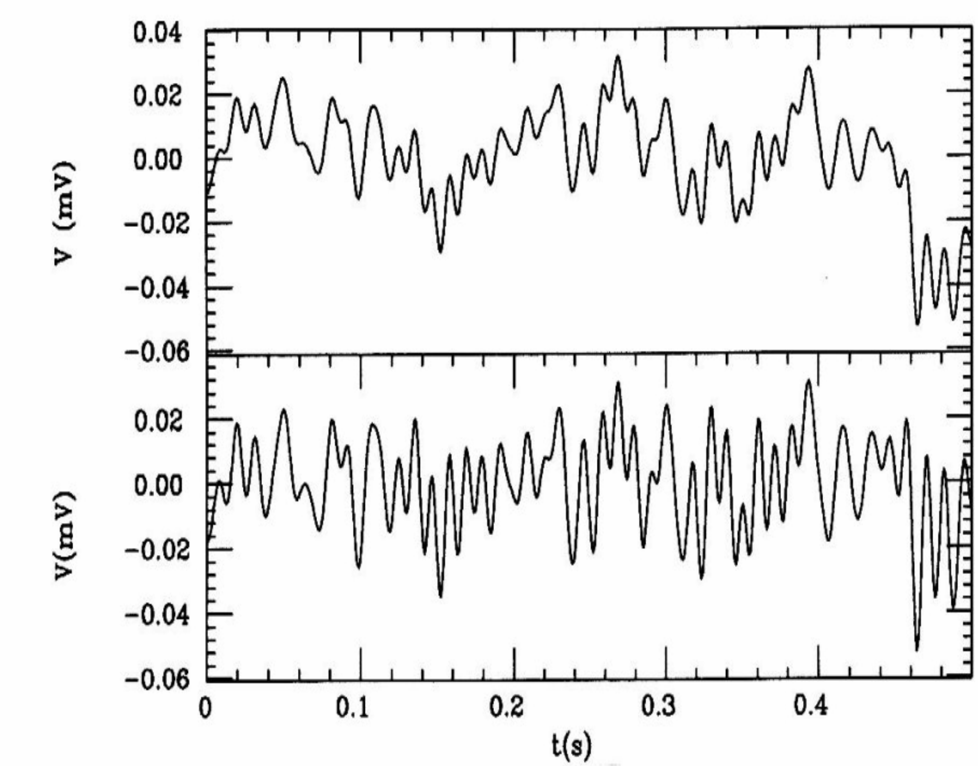
$$\sigma_V^2 = \frac{\sum_{i=1}^{N}\left[V(t_i) - \langle V \rangle\right]^2}{N-1}$$

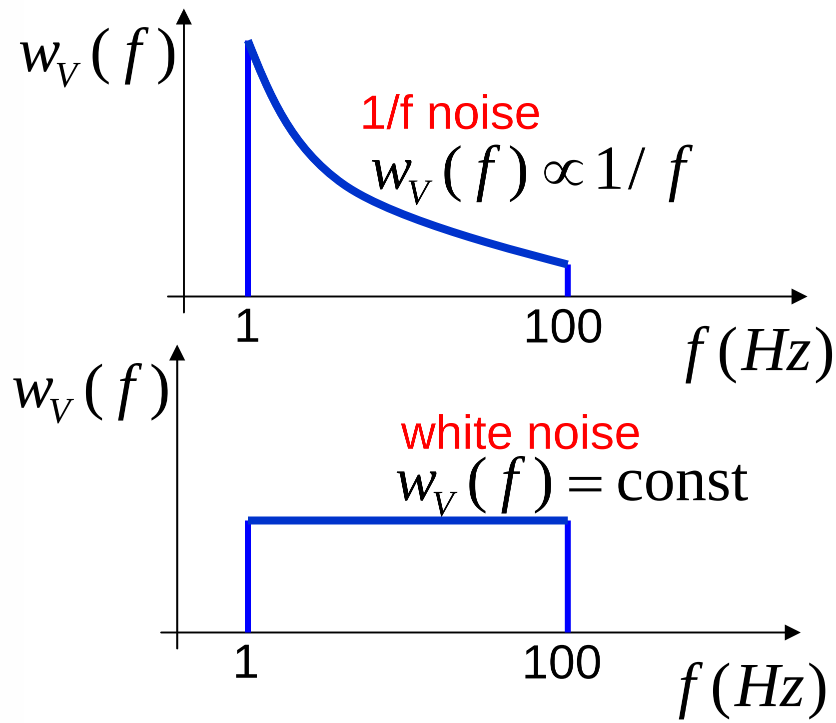
- Even if the noise is stationary (i.e. its statistical description does not change with time) and Gaussian (the histogram of the fluctuations is a Gaussian), the variance does not provide a full description of its characteristics.



- The two noise records above have the same variance but in the top record the signal deviates from the average for longer periods.
- To characterize this, we need a statistical estimator specifying the contributions to the variance coming from the different frequencies present in the noise.
- This estimator is the spectral density of the noise $w_V(f)$ (aka the power spectrum of the noise). Its integral over all frequencies is the variance.

$$\sigma_V^2 = \int_0^\infty w_V(f)\,df$$





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Noise and measurement errors

- The fluctuations due to the noise produce an error in the measurement of the observable:

$$\sigma_V^2 = \int_{f_1}^{f_2} w_V(f)\,df$$

- Where f_1 and f_2 are the minimum and maximum frequencies present in the observable.
- For a real measurement

$f_1 \cong 1/T$ where T is the duration of the measurement

$f_2 \cong 1/\tau$ where τ is the time constant of the detector

$$\sigma_V^2 = \int_{1/T}^{1/\tau} w_V(f)\,df \cong \frac{w_V}{\tau}$$ if the noise is white

Noise and measurement errors

- In order to reduce the error, data are integrated, so that the fluctuations due to the noise average out, reducing the variance of the observable.
- An integration for a time T is equivalent to filtering the data with a low-pass filter with cutoff frequency at f=1/T.
- So the variance of the observable will be

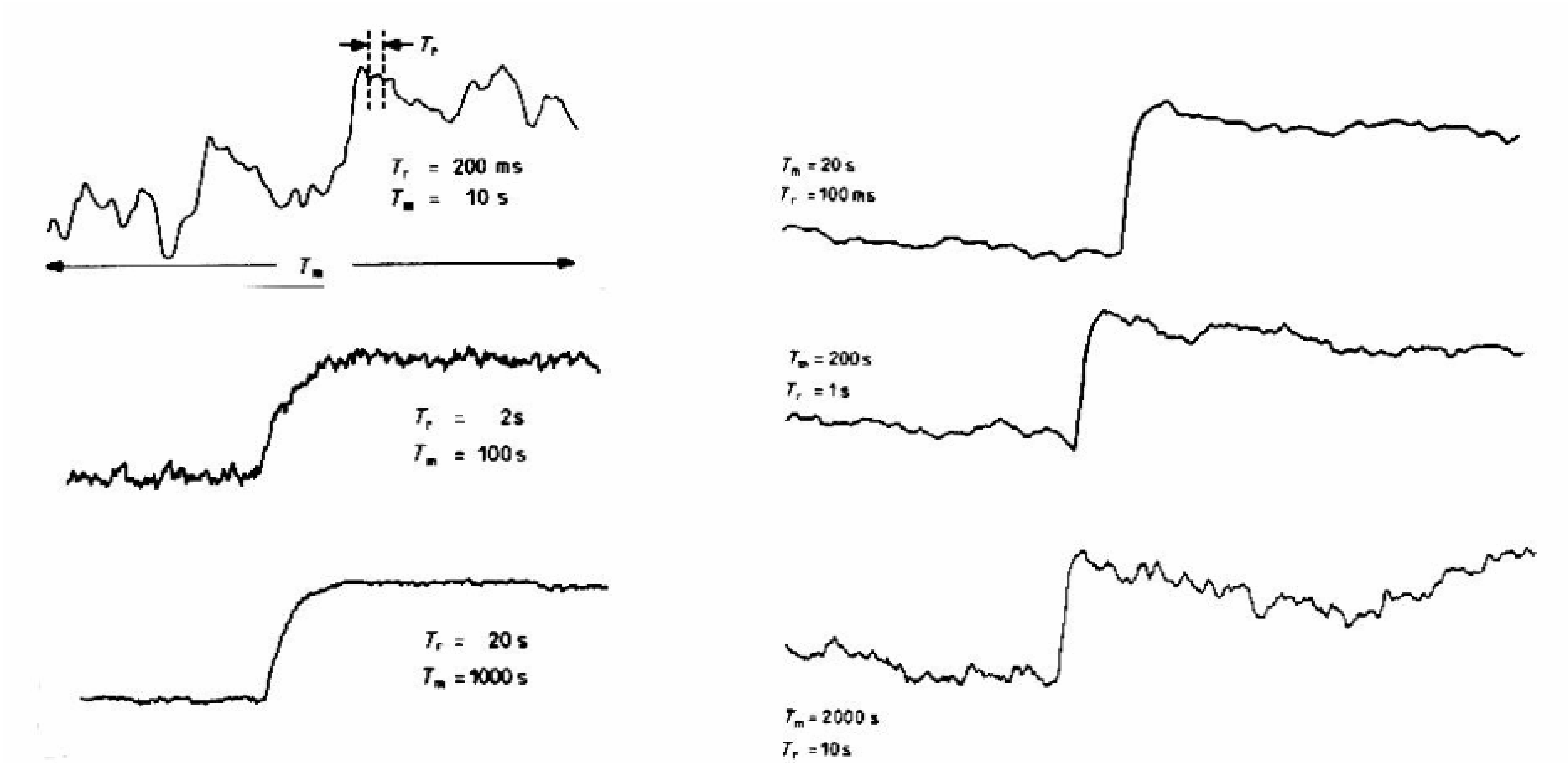
$$\sigma_V^2 \approx \int_0^{1/T} w_V(f)\,df = \frac{w_V}{T} \qquad \text{if the noise is white}$$

- The exact relationship depends on the actual shape of the filter (running average, low-pass filters with different orders…)
- Anyway, the relationship between error and integration time in the case of white noise is always

$$\sigma_V = a\frac{\sqrt{w_V}}{\sqrt{T}} \qquad \text{with } a \text{ of the order of unity}$$

Noise and measurement errors

- If the noise is not white, we have a problem…



White noise

1/f noise

- Noise with fundamental origin (thermal) is most often white

Noise and detectors

- To specify the noise of a detector, we need to specify how the voltage fluctuations produced by noise at the output of the detector compare to the voltage signal produced by the incoming radiative power.
- The Noise Equivalent Power (NEP) is the incoming radiative power which produces an output signal equal to the rms fluctuation due to the noise, in an integration time of 1 second.
- With this definition, it is evident that the noise equivalent power corresponds to the minimum power detectable in 1 s of integration.
- In formulas

$$NEP\,\Re = V = \sigma_{V,1s} = \frac{\sqrt{w_V}}{\sqrt{T(1s)}} = \quad \rightarrow \quad NEP = \frac{\sqrt{w_V}}{\Re}$$

- So the units of the *NEP* are $W/\sqrt{Hz}$
- If your detector has a specified *NEP*, the error in a measurement of power with an integration time T will be

$$\sigma_W = \frac{\sigma_V}{\Re} = \frac{\sqrt{w_V}}{\Re\sqrt{T}} = \frac{NEP}{\sqrt{T}}$$

Conversion from *NEP* to *NET*$_{CMB}$

$$\Delta W = A\Omega\Delta B = A\Omega\frac{dB}{dT}dT = A\Omega\frac{d}{dT}\left[\int_{\nu_1}^{\nu_2} B(T_{CMB},\nu)d\nu\right]dT =$$

$$= A\Omega\int_{\nu_1}^{\nu_2}\frac{xe^x}{e^x-1}B(T_{CMB},\nu)d\nu\,\frac{\Delta T_{CMB}}{T_{CMB}} \quad \rightarrow$$

$$\boxed{NET_{CMB} = T_{CMB}\frac{NEP}{A\Omega\int_{\nu_1}^{\nu_2}\frac{xe^x}{e^x-1}B(T_{CMB},\nu)d\nu}}$$

Radiation Noise

- The fundamental limit of any measurement.
- Photon noise reflects the particle-wave duality of photons.
- It is the sum of Poisson noise (particles) PLUS interference noise (waves)
- Poisson noise:

$$\left\langle \Delta E^2 \right\rangle = (h\nu)^2 \left\langle \Delta N^2 \right\rangle = (h\nu)^2 \left\langle N \right\rangle = (h\nu)^2 \frac{W}{h\nu} = h\nu W t$$

This is a typical random-walk process (variance prop.to time).
Using Einstein's generalization $\left\langle \Delta\theta^2 \right\rangle = 2kBTt \quad \Rightarrow \quad \left\langle \dot{\theta}_f^2 \right\rangle df = 4kBTdf$
we get the power spectrum and the variance
of radiative power fluctuations: $\boxed{\left\langle \Delta W_f^2 \right\rangle df = 2h\nu W df}$

Radiation Noise

- Orders of magnitude example: A He-Ne 1 mW laser beam has a perfect Poisson statistics, so

$$\sqrt{\left\langle \Delta W_f^{\,2} \right\rangle} = \sqrt{2h\nu W} = 2.5\times 10^{-11}\ \frac{W}{\sqrt{Hz}}$$

- Notice the power spectrum units (remember that the integral of the PS over frequency is the variance).
- In this case the intrinsic fluctuations per unit bandwidth are >7 orders of magnitude smaller than the signal.
- It is useless to build a complex detector with a noise of $10^{-15}\,W/\sqrt{Hz}$ for this measurement: the precision of the measurement will be limited at a level of $2.5\times 10^{-11}\ W/\sqrt{Hz}$

Radiation Noise

- Thermal radiation (like the CMB) has also wave interference noise: the correct statistics is Bose-Einstein.

$$\langle N \rangle = \frac{g}{e^{(E-\mu)/kT} - 1} \quad ; \quad \langle \Delta N^2 \rangle = T \left. \frac{d\langle N \rangle}{d\mu} \right|_{T,V}$$

$$\Rightarrow \boxed{\langle \Delta N^2 \rangle = \underbrace{\langle N \rangle}_{\text{Poisson noise}} + \underbrace{\frac{\langle N \rangle^2}{g}}_{\text{Wave interference noise}}}$$

Radiation Noise

- For a blackbody

$$\langle N \rangle = \frac{g}{e^{(E-\mu)/kT} - 1} \quad ; \quad g = 2\frac{\nu^2}{c^3} 4\pi V d\nu$$

$$\langle N \rangle = \frac{8\pi\nu^2}{c^3} \frac{1}{e^{h\nu/kT} - 1} V d\nu$$

$$\Rightarrow \boxed{\langle \Delta N^2 \rangle = \langle N \rangle \left[1 + \frac{1}{e^{h\nu/kT} - 1} \right]}$$

Poisson noise, important at short wavelengths

Wave interference noise, Important at low frequencies

Noise and integration time

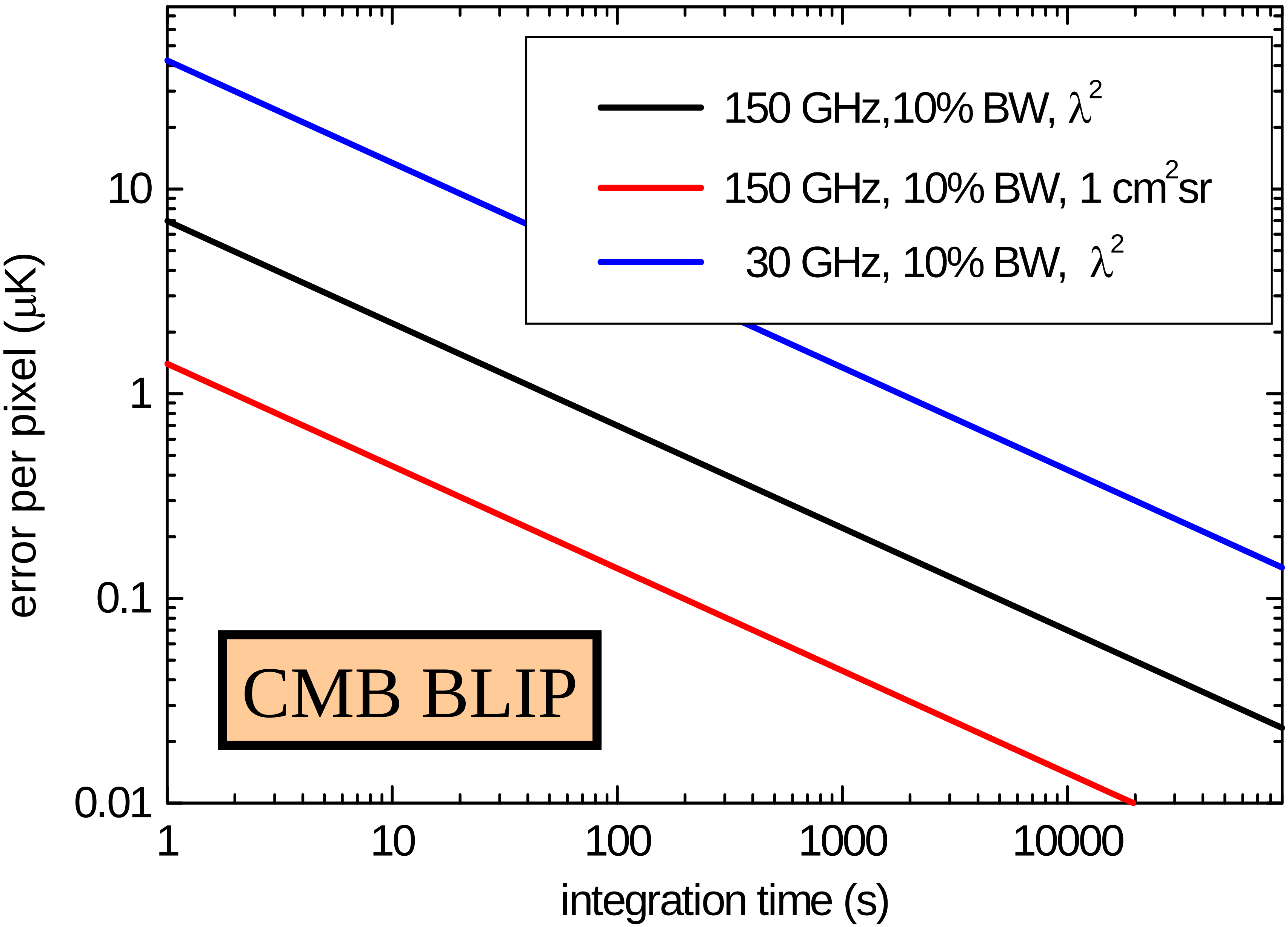
- Numerical example: CMB anisotropy (or polarization) measurement limited only by radiation noise:

$$\Delta B(\nu, T) = \frac{\Delta T}{T} \int_{x_1}^{x_2} \frac{xe^x}{e^x - 1} B(x, T)\, dx$$

$$\sigma\left(\frac{\Delta T}{T}\right) = \frac{\sigma(\Delta B)}{\int_{x_1}^{x_2} \frac{xe^x}{e^x - 1} B(x, T)\, dx}$$

$$\sigma\left(\frac{\Delta T}{T}\right) = \frac{\sqrt{\frac{4k^5T^5}{c^2h^3} A\Omega \int_{x_1}^{x_2} \frac{x^4 e^x}{\left(e^x - 1\right)^2} dx}}{\frac{2k^4T^4}{c^2h^3} A\Omega \int_{x_1}^{x_2} \frac{x^4 e^x}{(e^x - 1)^2} dx} \sqrt{\frac{1}{t}}$$

The ultimate sensitivity plot !!



Spider-Web Bolometers

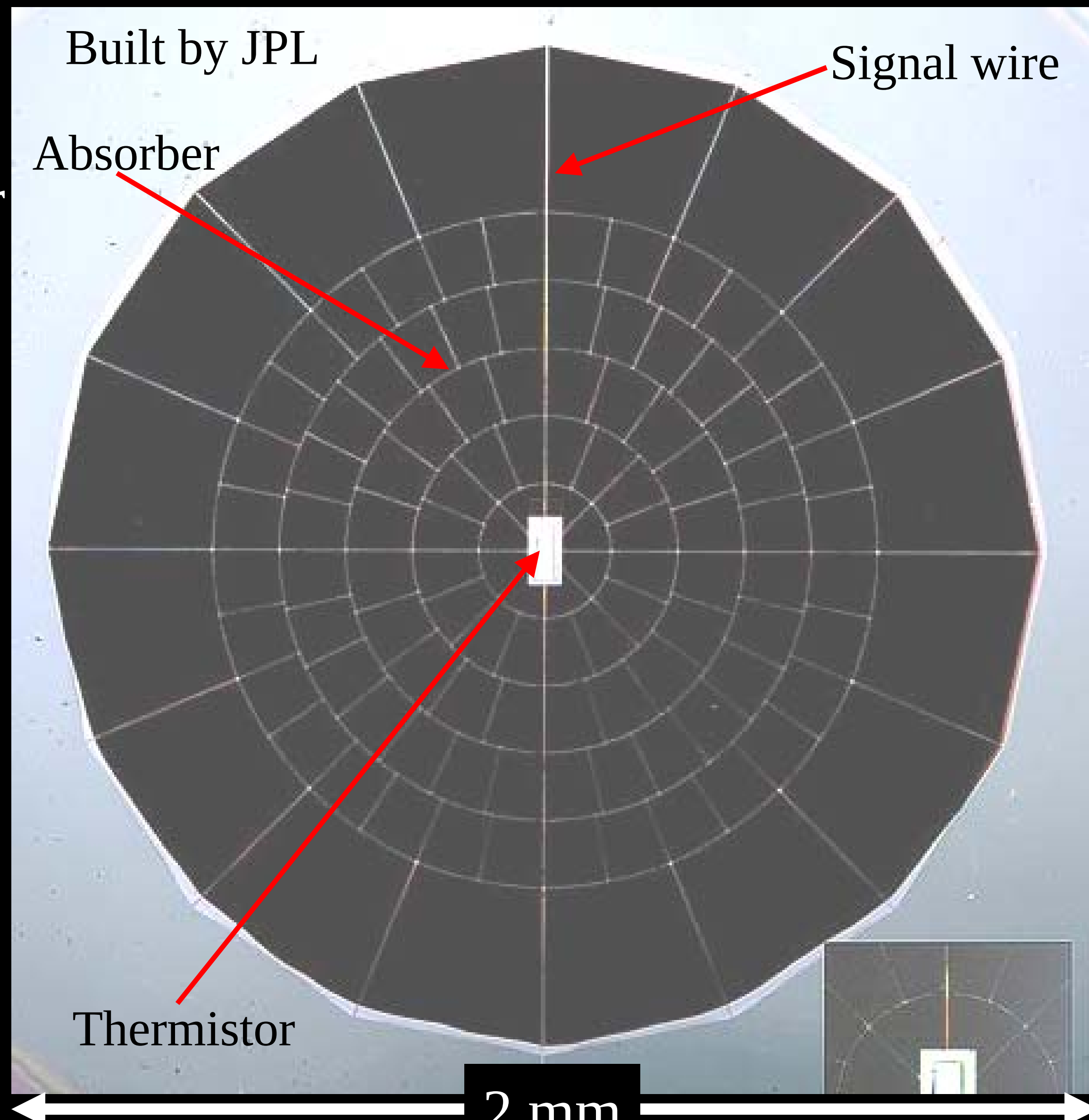
•The absorber is micro machined as a web of metallized Si_3N_4 wires, 2 μm thick, with 0.1 mm pitch.

•This is a good absorber for mm-wave photons and features a very low cross section for cosmic rays. Also, the heat capacity is reduced by a large factor with respect to the solid absorber.

•NEP ~ 2 10^{-17} W/Hz$^{0.5}$ is achieved @0.3K

•150μK_{CMB} in 1 s

•Mauskopf *et al.* Appl.Opt. **36**, 765-771, (1997)



Sensitivity to CMB anisotropy

- A map of CMB anisotropy is a sampled image $\Delta T_i = \Delta T(\ell_i, b_i)$ for $i=1,N_{pix}$, where $\Delta T(\ell_i, b_i)$ is the average of $\Delta T(\ell, b)$ over the pixel area, for the pixel centered in (ℓ_i, b_i).
- Knowing :
  - the instantaneous sensitivity (*NET*),
  - the instrument angular resolution θ,
  - the sky coverage of the survey Ω
- we can compute the standard error for the estimate of ΔT_i of each pixel, for a given total observation time *t*.
- Assuming uniform coverage and square pixels with side θ, we have simply

$$\sigma_{\Delta T} = \frac{NET}{\sqrt{t_{pix}}} = \frac{NET}{\theta}\sqrt{\frac{\Omega}{t}}$$

Sensitivity to CMB anisotropy

- Numerical example: assume

$$NET = 150\,\mu K\sqrt{s}$$

$$t = 5\,\text{days} = 4.3\times10^{5}\,s$$

$$\theta = 10'$$

$$\Omega = 20^{o}\times 20^{o} = 1200'\times1200'$$

- You get

$$\sigma_{\Delta T} = \frac{NET}{\sqrt{t_{pix}}} = \frac{NET}{\theta}\sqrt{\frac{\Omega}{t}} = 27\,\mu K$$

- Per pixel, over 14400 pixels: a large dataset, with a S/N ratio per pixel of the order of 3.

Sensitivity to CMB anisotropy

- An array of *n* detectors optimally used will simply multiply by *n* the observation time available for each pixel.
- So we get $$\sigma_{\Delta T} = \frac{NET}{\sqrt{n t_{pix}}} = \frac{NET}{\sqrt{t}} \frac{\sqrt{\Omega}}{\theta} \frac{1}{\sqrt{n}}$$
- The use of a large array can give more that just an improvement of sqrt(n). For ground based observations, atmospheric noise can be significantly reduced by exploiting the correlations of the noise over different pixels.

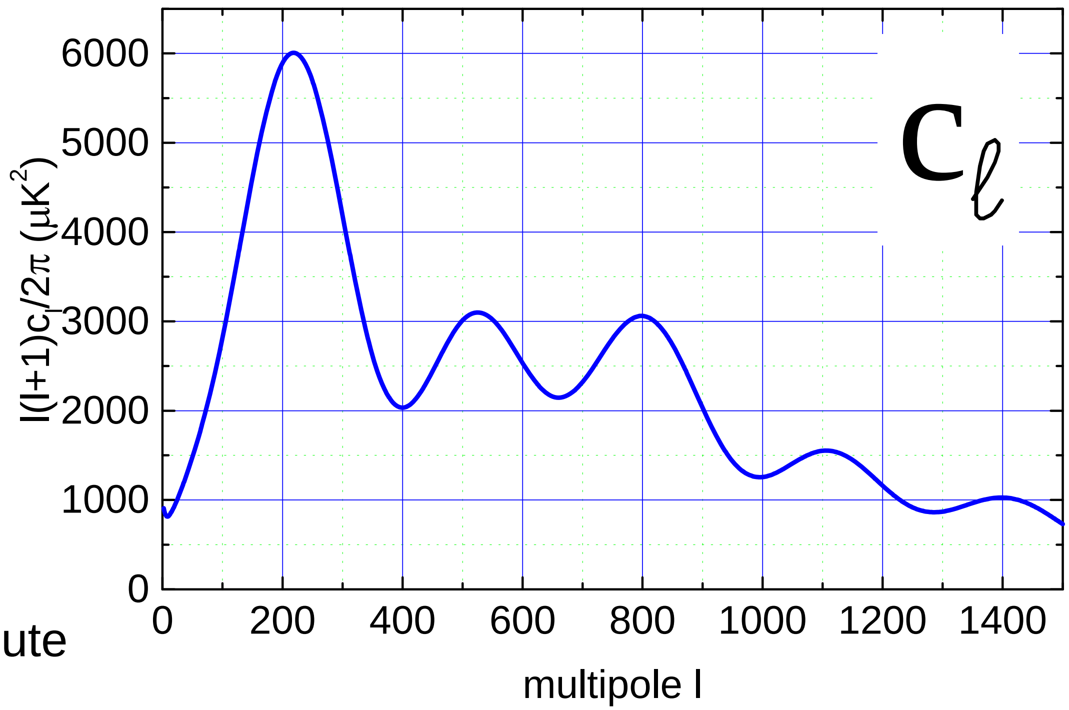
Expected power spectrum:

$$\Delta T(\theta,\varphi) = \sum_{\ell,m} a_{\ell m} Y_\ell^m(\theta,\varphi)$$

$$c_\ell = \langle a_{\ell m}^2 \rangle$$

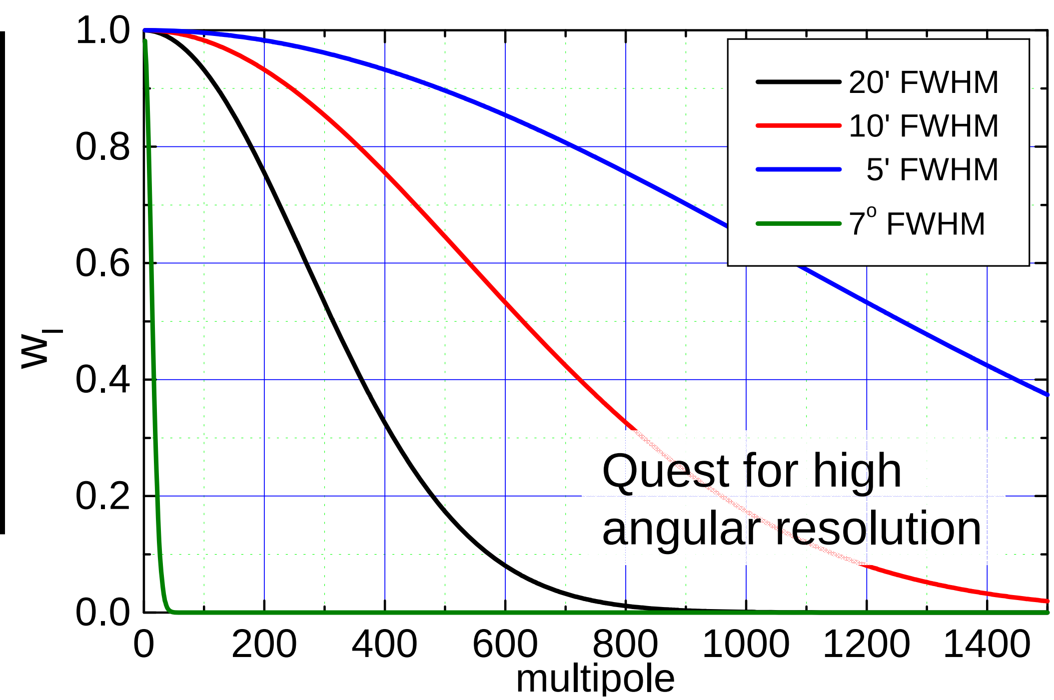
$$\langle \Delta T^2 \rangle = \frac{1}{4\pi} \sum_\ell (2\ell+1) c_\ell$$

See e.g. http://camb.info to compute c_ℓ for a given cosmological model



An instrument with finite angular resolution is not sensitive to the smallest scales (highest multipoles). For a gaussian beam with s.d. σ:

$$w_\ell^{LP} = e^{-\ell(\ell+1)\sigma^2}$$



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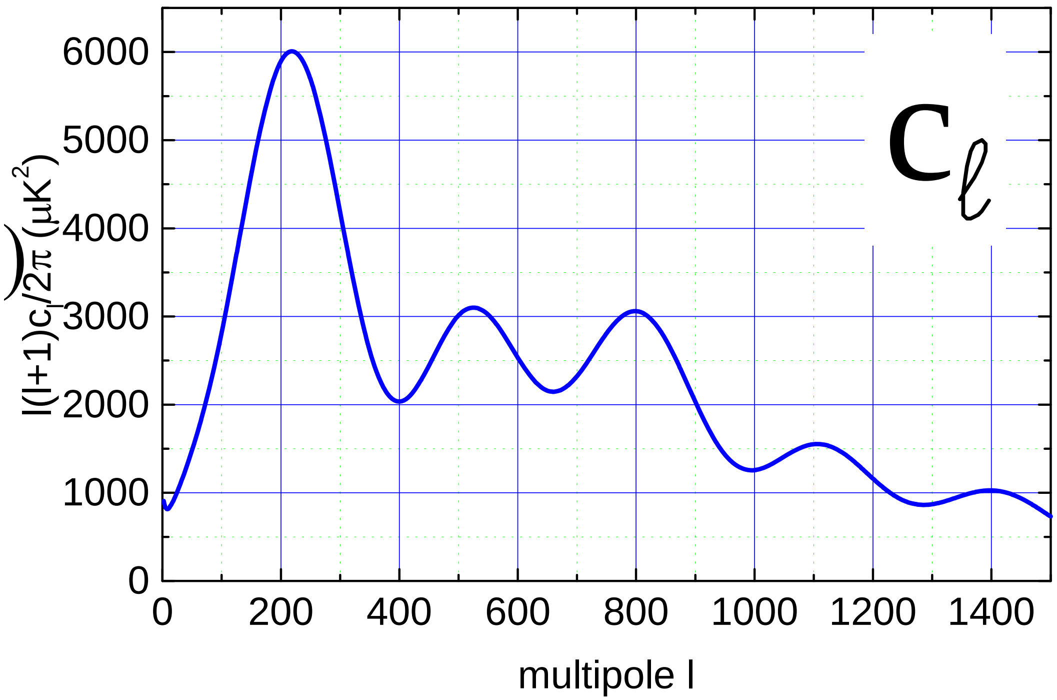
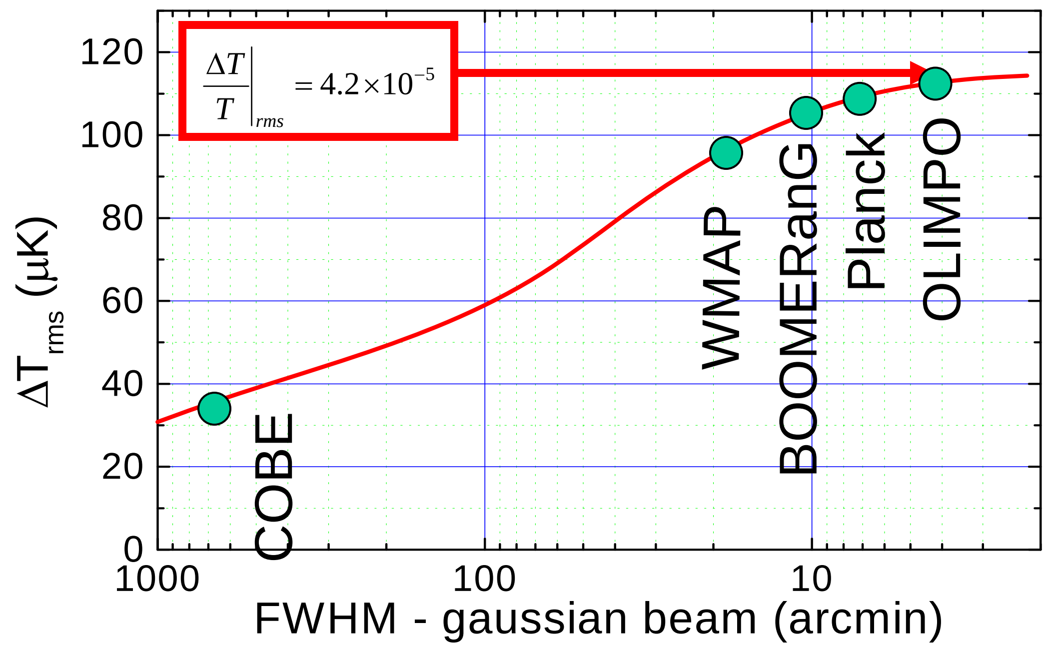


***rms* signal in an instrument with gaussian beam σ :**

$$\langle \Delta T^2 \rangle_{meas} = \frac{1}{4\pi} \sum_\ell (2\ell+1) w_\ell^{LP} c_\ell$$

$$w_\ell^{LP} = e^{-\ell(\ell+1)\sigma^2}$$



Sensitivity to the Power Spectrum

- Knowing :
  - the instantaneous sensitivity,
  - the angular resolution,
  - the sky coverage
- we can compute the sensitivity to the different multipoles of the power spectrum, for a given survey duration T.
- A first part of the fluctuation comes from the statistical nature of the observable c_ℓ .
- Since the $a_{\ell m}$ are gaussian, c_ℓ is distributed as a χ^2 with $2\ell+1$ *DOF*, so that

$$\Delta c_\ell = \sqrt{\frac{2}{2\ell+1}}\, c_\ell$$

Cosmic Variance

Sensitivity to the Power Spectrum

- If only a fraction f of the sky is surveyed, the cosmic variance becomes

$$\Delta c_\ell = \sqrt{\frac{2}{2\ell+1}}\sqrt{\frac{1}{f}}\, c_\ell$$

Cosmic / Sampling Variance

- The second contribution to the errors comes from detector noise.
- If a total of N pixels is observed, the error in the determination of the temperature in each pixel will be of the order of

$$\sigma = NET\sqrt{\frac{N}{T}}$$

Sensitivity to the Power Spectrum

- And the error on the c_ℓ becomes

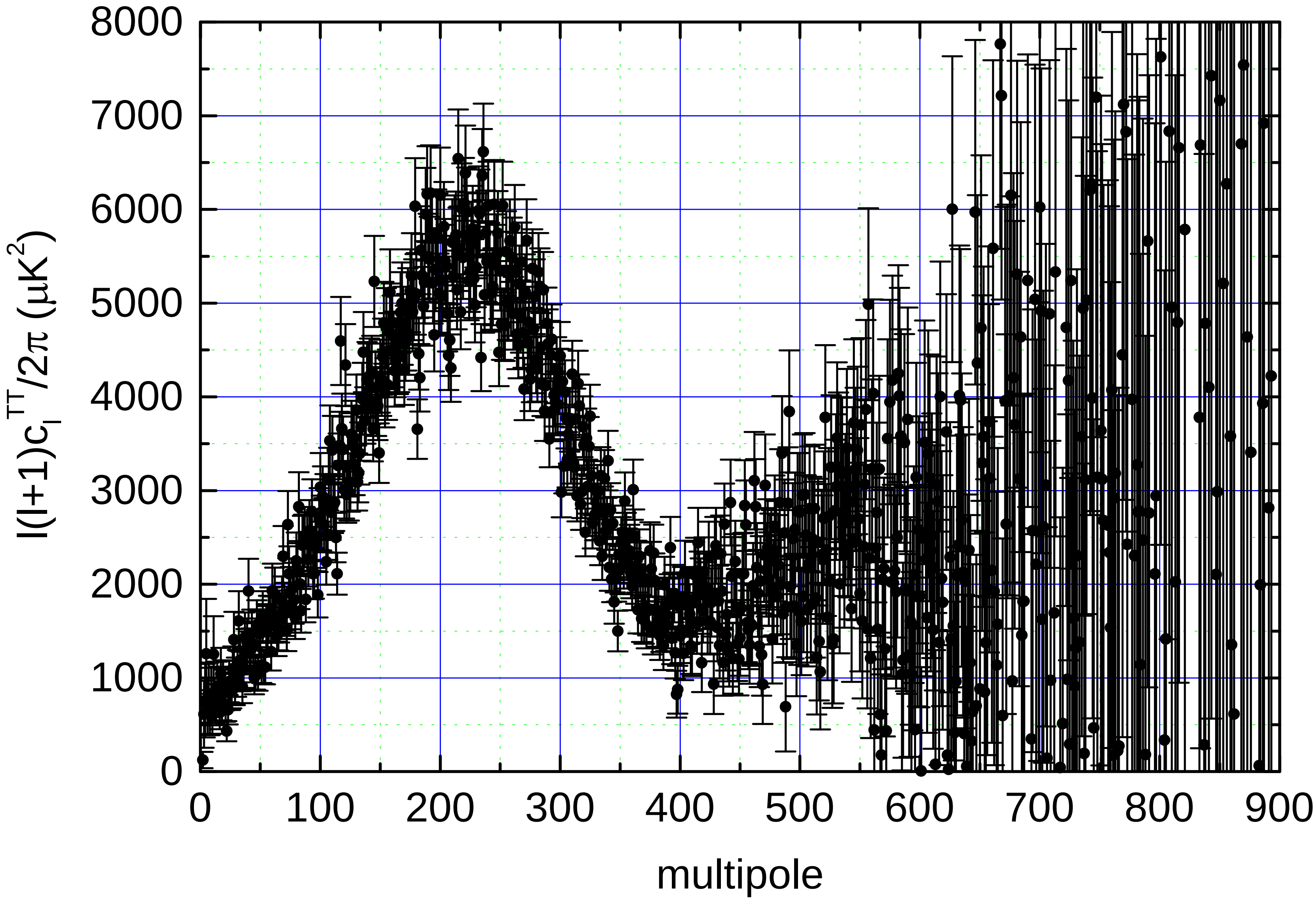
$$\Delta c_\ell = \sqrt{\frac{2}{2\ell+1}}\sqrt{\frac{1}{f}}\, c_\ell \left[1+\frac{A\sigma^2}{N c_\ell w_\ell}\right]$$

Knox's formula (1995)

- When several multipoles are binned in a band-power $< c_\ell >$ with bin-width $\Delta\ell$, we have roughly

$$\Delta\langle c_\ell\rangle = \frac{1}{\sqrt{\Delta\ell}}\sqrt{\frac{2}{2\ell+1}}\sqrt{\frac{1}{f}}\,\langle c_\ell\rangle\left[1+\frac{A\sigma^2}{N c_\ell w_\ell}\right]$$

- Since the power spectrum of CMB anisotropy and polarization is smooth, a binning with $\Delta\ell$ =20-30 is perfectly acceptable.



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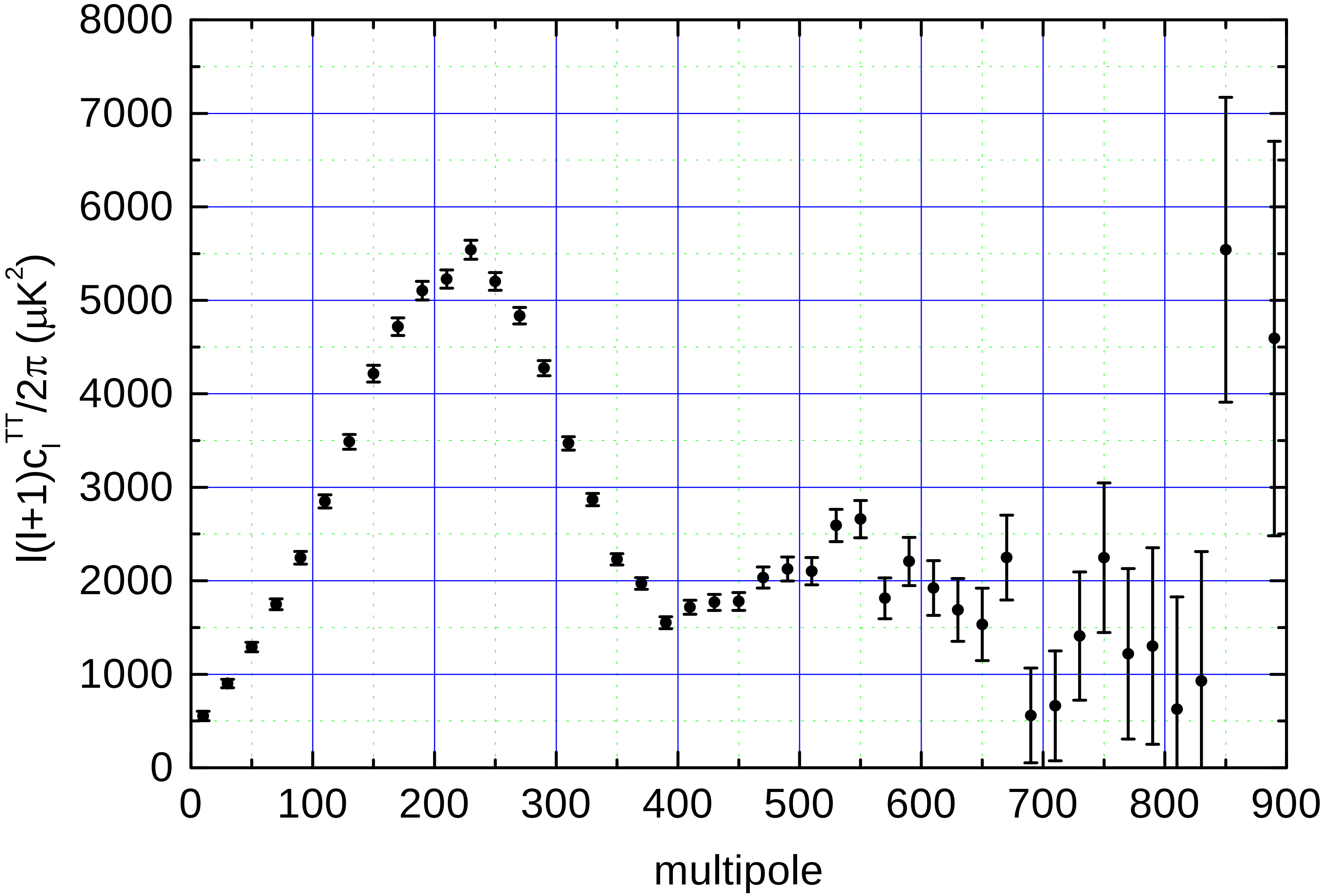
$l(l+1)c_l^{TT}/2\pi$ (μK^2)
multipole



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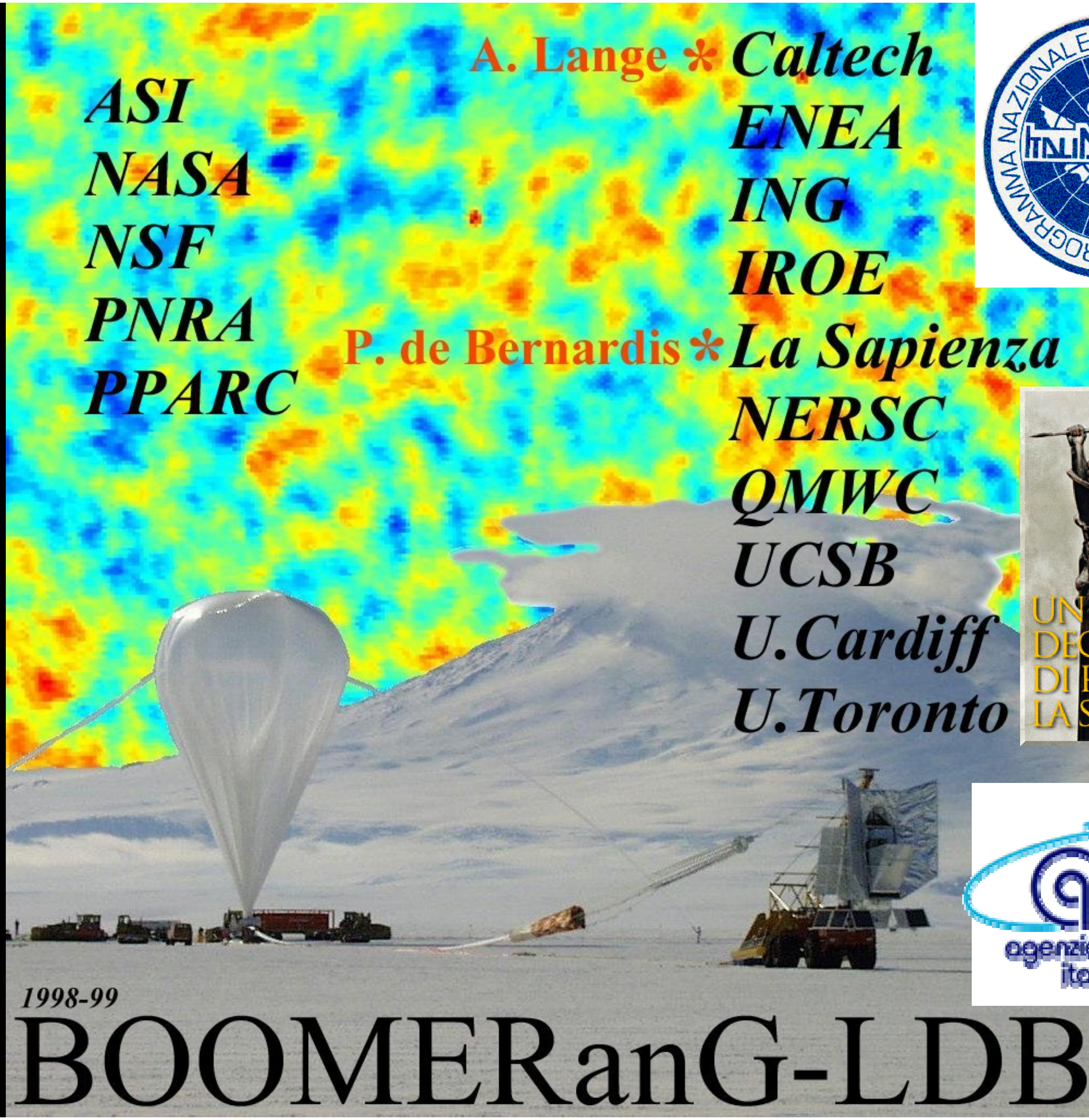
multipole
$l(l+1)c_l^{TT}/2\pi$ (μK^2)

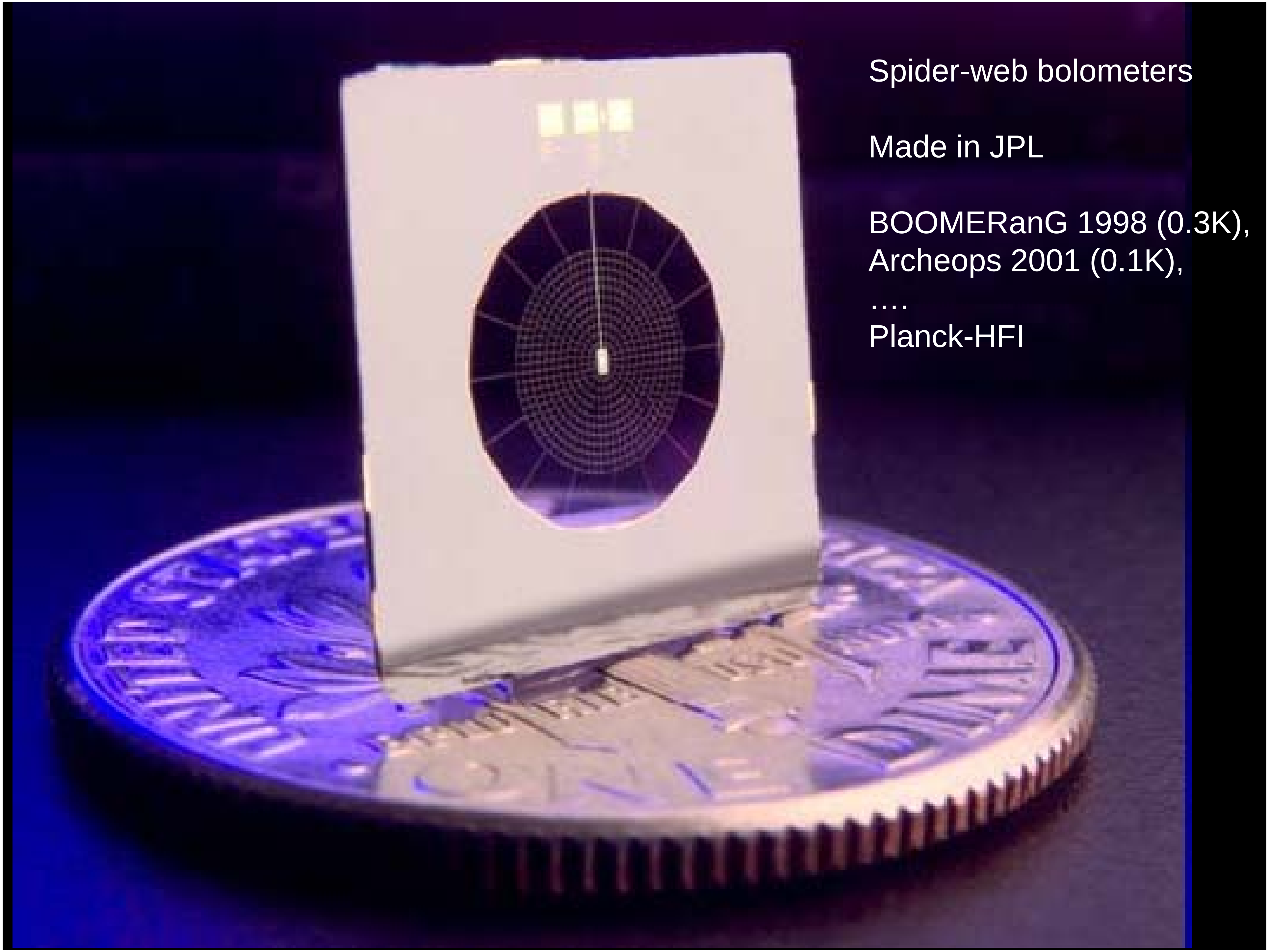
http: // oberon.roma1.infn.it / boomerang
http: // www.physics.ucsb.edu / ~boom



PROGRAMMA NAZIONALE DI RICERCHE IN ANTARTIDE

UNIVERSITÀ DEGLI STUDI DI ROMA LA SAPIENZA

agenzia spaziale italiana

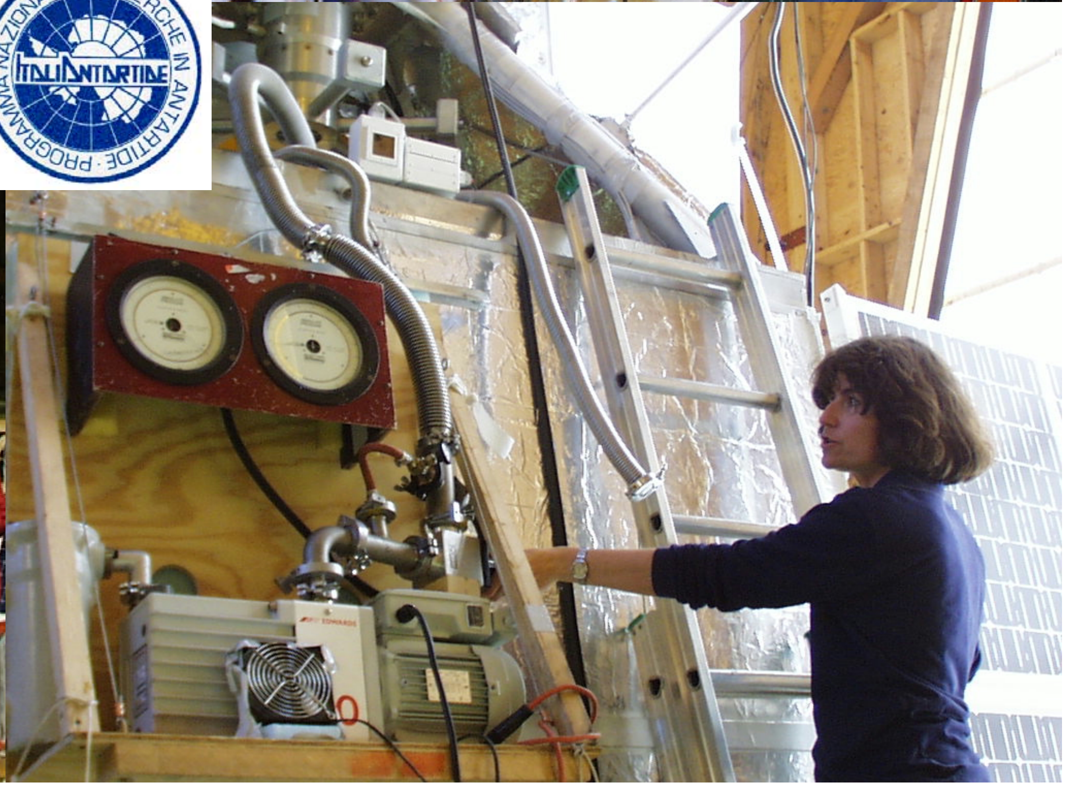


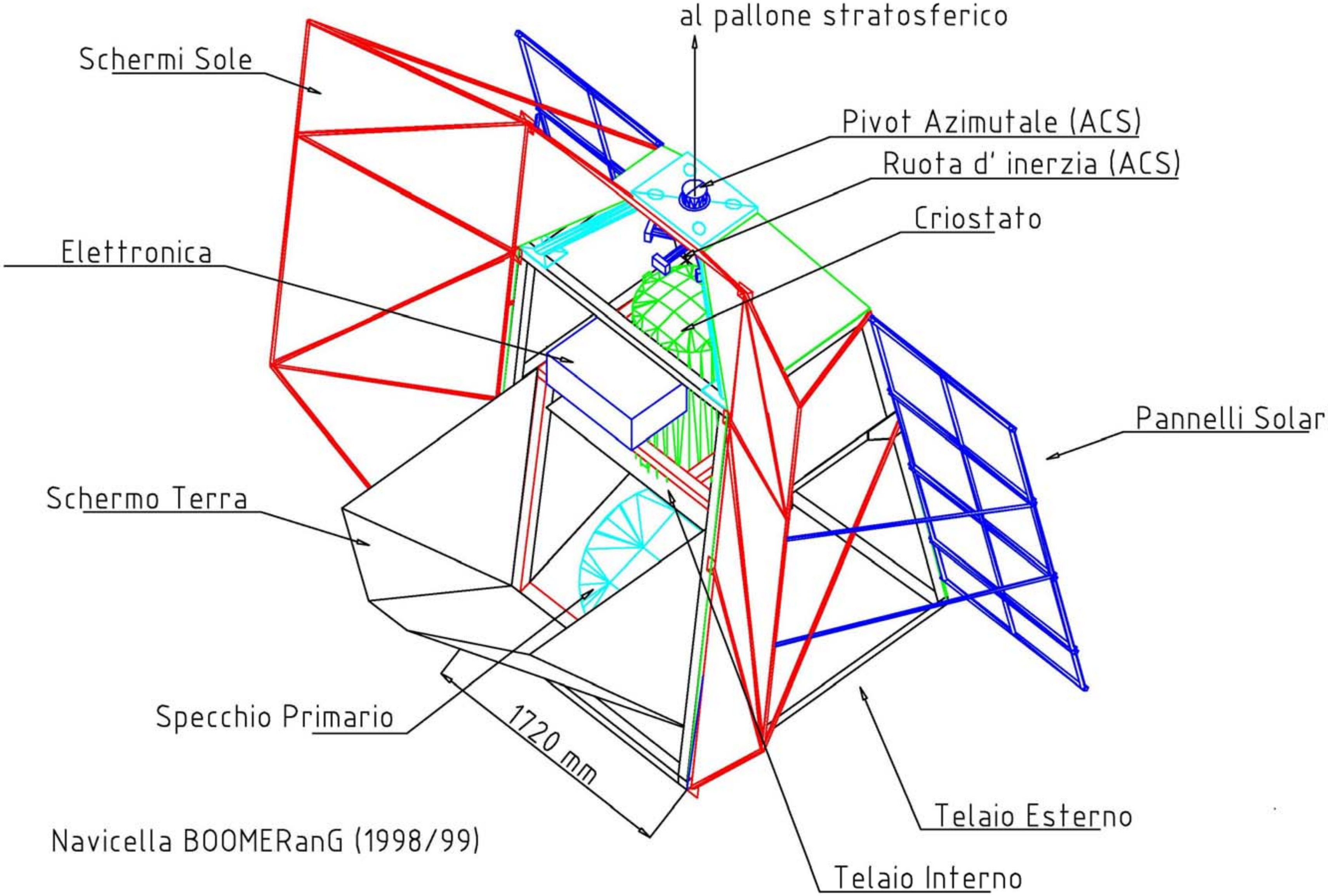
Spider-web bolometers

Made in JPL

BOOMERanG 1998 (0.3K),
Archeops 2001 (0.1K),
….
Planck-HFI

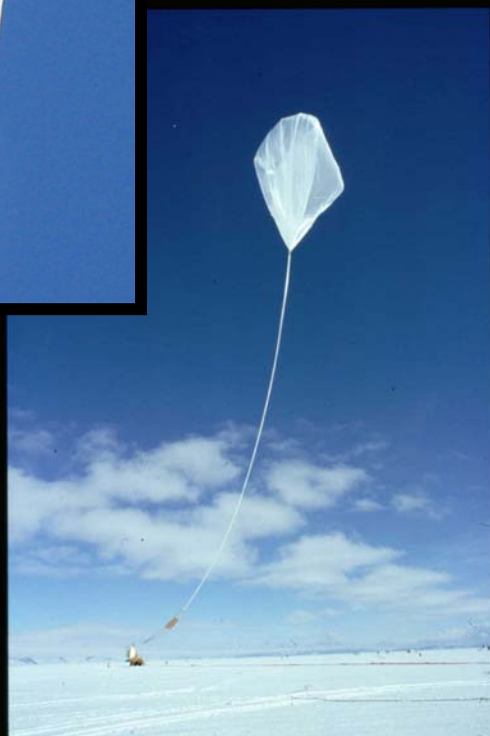
BOOMERanG (1998, 2003)





Navicella BOOMERanG (1998/99)

Il lancio



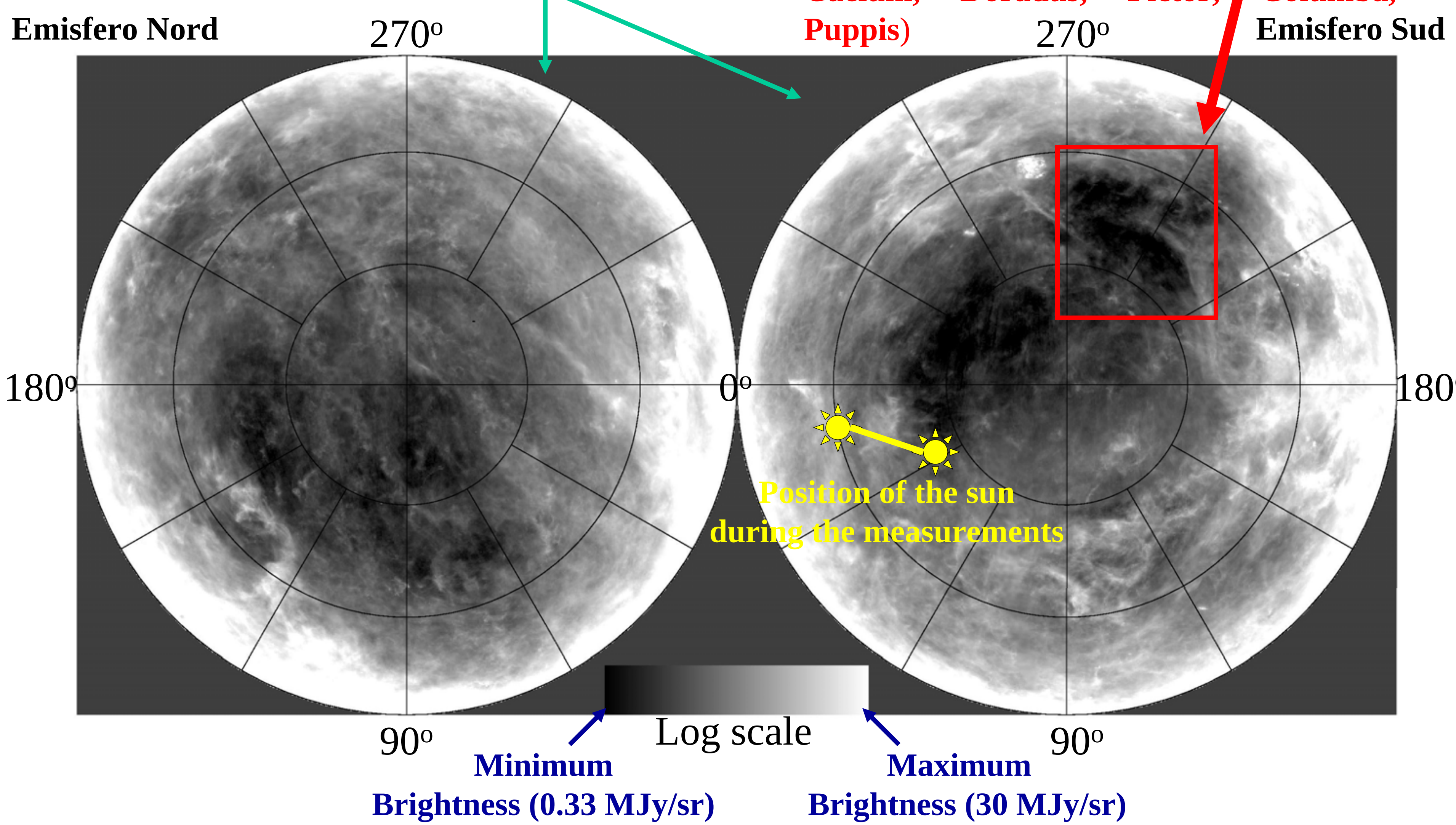
PROGRAMMA NAZIONALE DI RICERCHE IN ANTARTIDE

ITALIANTARTIDE

The target region

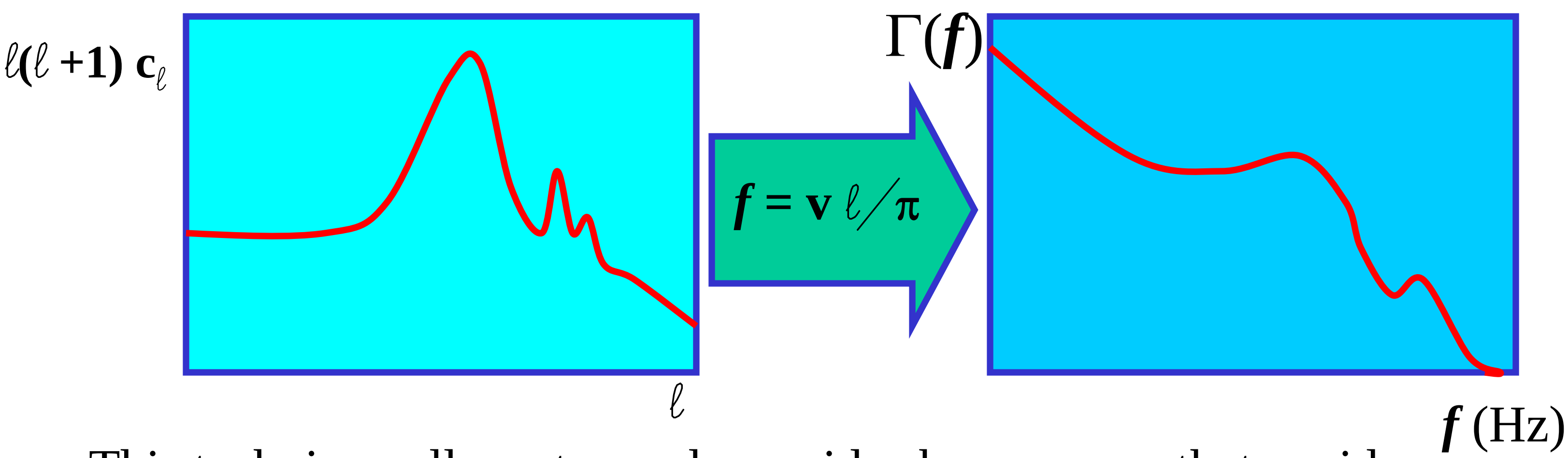
Map of mm-wave emission of dust in our galaxy as derived from IRAS and DIRBE measurements (Schlegel et al 1999)

The best (lowest contamination) area of all the sky is in the southern hemisphere, and happens to be far from the sun in the antarctic summer (constellations of Caelum, Doradus, Pictor, Columba, Puppis)



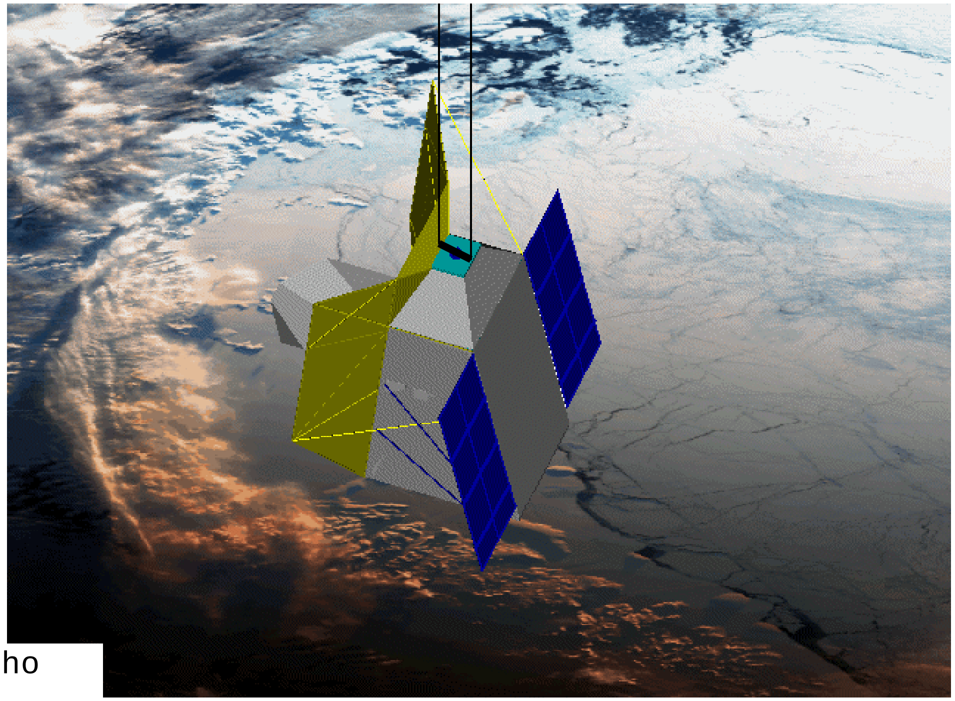
A scanning telescope

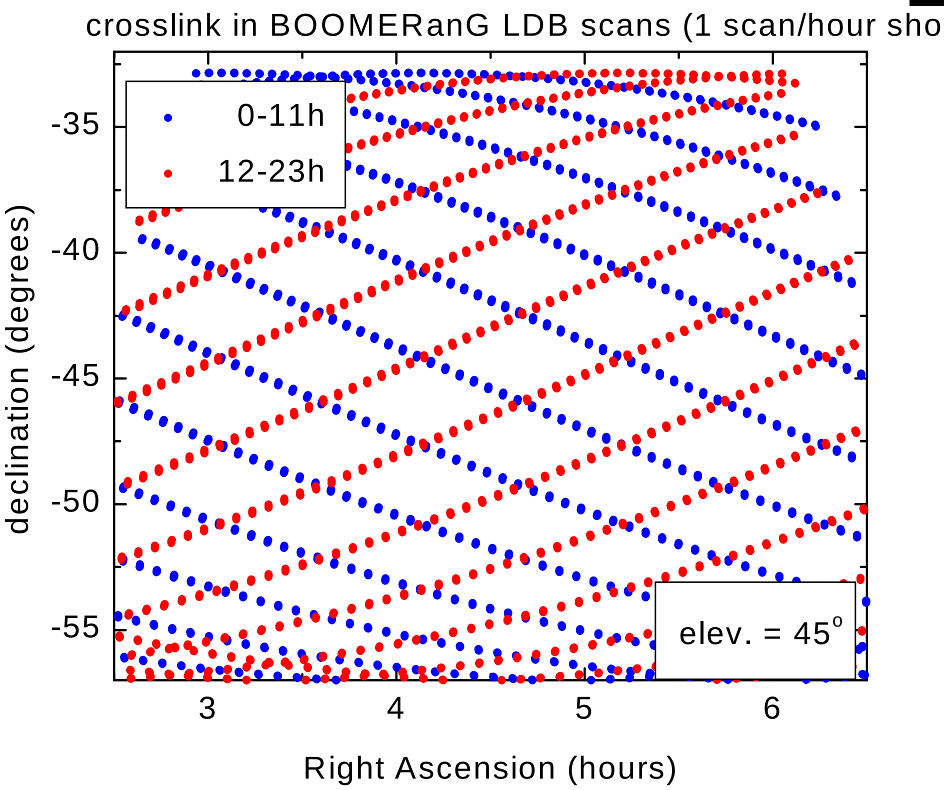
- BOOMERanG is a ***scanning*** experiment: the beam scans the sky at constant speed **v** (1 to 2^o/s), with 60^o wide scans.
- Different multipoles in the CMB temperature field produce different sub-audio frequencies in the detector (see e.g. astro-ph/9710349)



- This technique allows to produce wide sky maps, so that a wide multipoles coverage of the power spectrum can be obtained in a single experiment
- This approach requires extremely low detector noise, fast detectors, and a strategy allowing for repeated observations of the same sky pixel with different orientations of the scan.
- The full payload is rotated to scan the sky: no moving modulators in the optical path

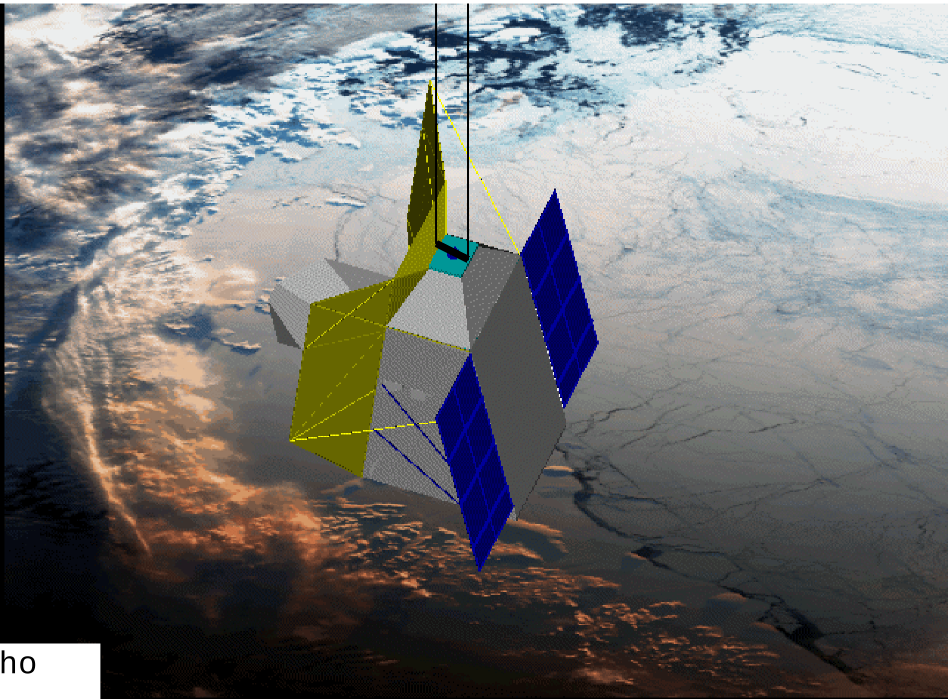
The sky scan

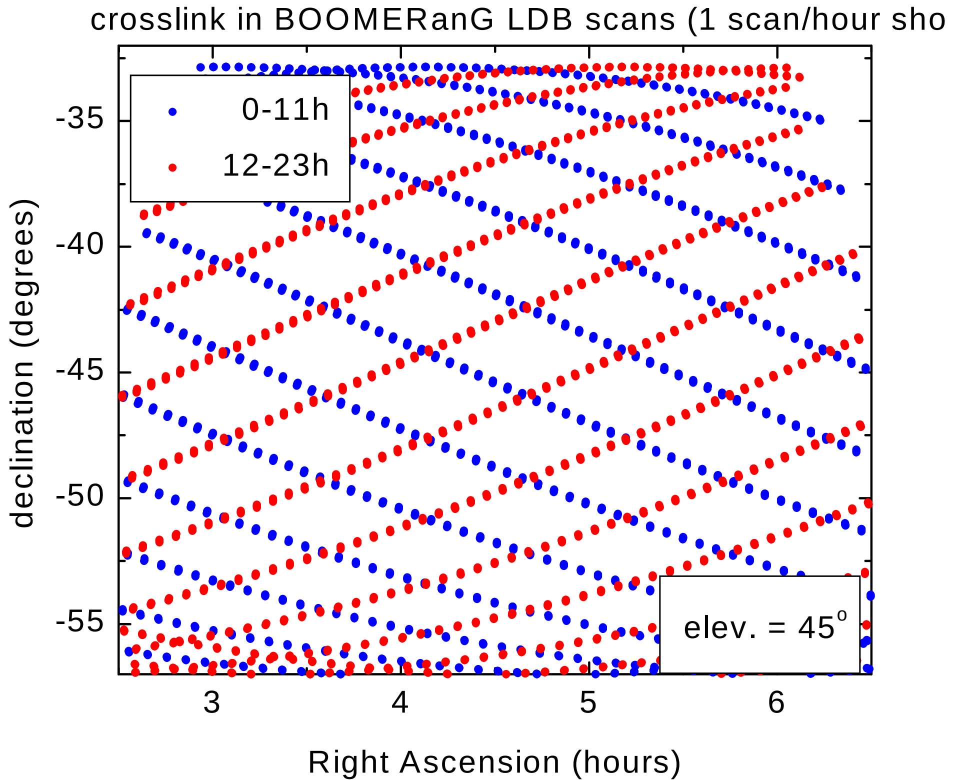



crosslink in BOOMERanG LDB scans (1 scan/hour sho
0-11h
12-23h
elev. = 45°
declination (degrees)
-35
-40
-45
-50
-55
3
4
5
6
Right Ascension (hours)

The sky scan

- The image of the sky is obtained by slowly scanning in azimuth ($\pm 30^o$) at constant elevation
- The optimal scan speed is between 1 and 2 deg/s in azimuth





- The scan center constantly tracks the azimuth of the lowest foreground region
- Every day we obtain a fully crosslinked map.

From time-ordered data to the map

- Pointing reconstruction (gyros + differential GPS + fine Sun Sensor)
- Bolometer data editing (cosmic rays hits and other instrumental events, <4% of the data removed)
- All our maps use HEALPIX pixelization (http://www.eso.org/~kgorski/healpix/)
- 1) "Naive" coadded maps (E.Hivon, B.Crill, F.Piacentini) with high pass ($\theta > 10^o$ removed)
- 2) "Rigorous" method: Maximum likelihood maps
- $$m = (A^T N^{-1} A)^{-1} A^T N^{-1} d$$

Time ordered data 57×10^6 samples

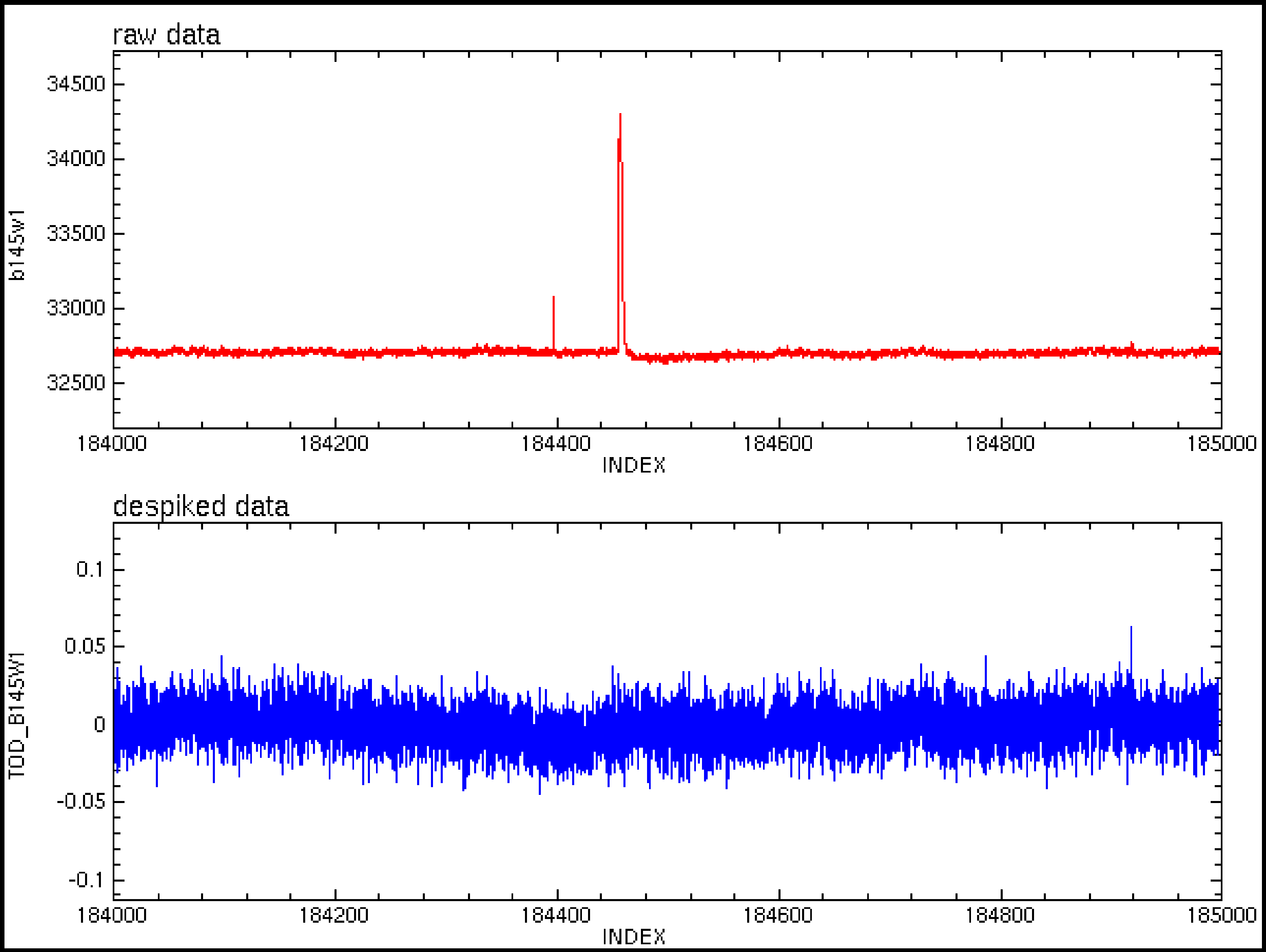
Map, 10^5 pixels

Time-time noise correlation matrix $57 \times 10^6 \times 57 \times 10^6$

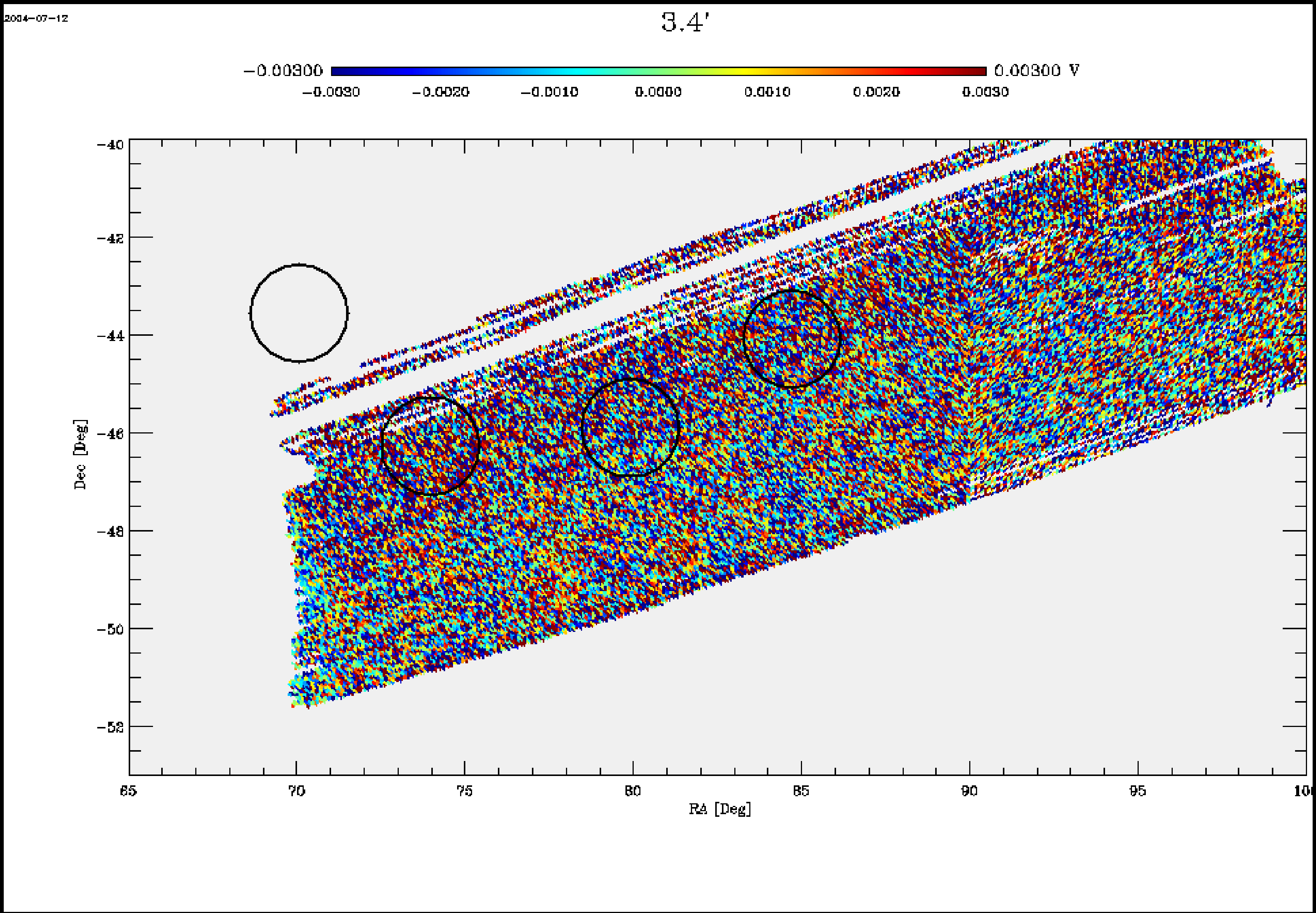
Pointing matrix $57 \times 10^6 \times 10^5$

- Needs:
  - estimate of noise N^{-1}: iterative method (Prunet et al. Astro-ph/0006052).
  - MADCAP(Borrill, astro-ph/9903204) http://cfpa.berkeley.edu/~borrill/cmb/madcap.html
- Outputs:
  - M=maximum likelihood map
  - $v = (A^T N^{-1} A)^{-1} A^T N^{-1} n$ $\gamma = \langle v v^T \rangle = (A^T N^{-1} A)^{-1}$ pixel-pixel noise covariance

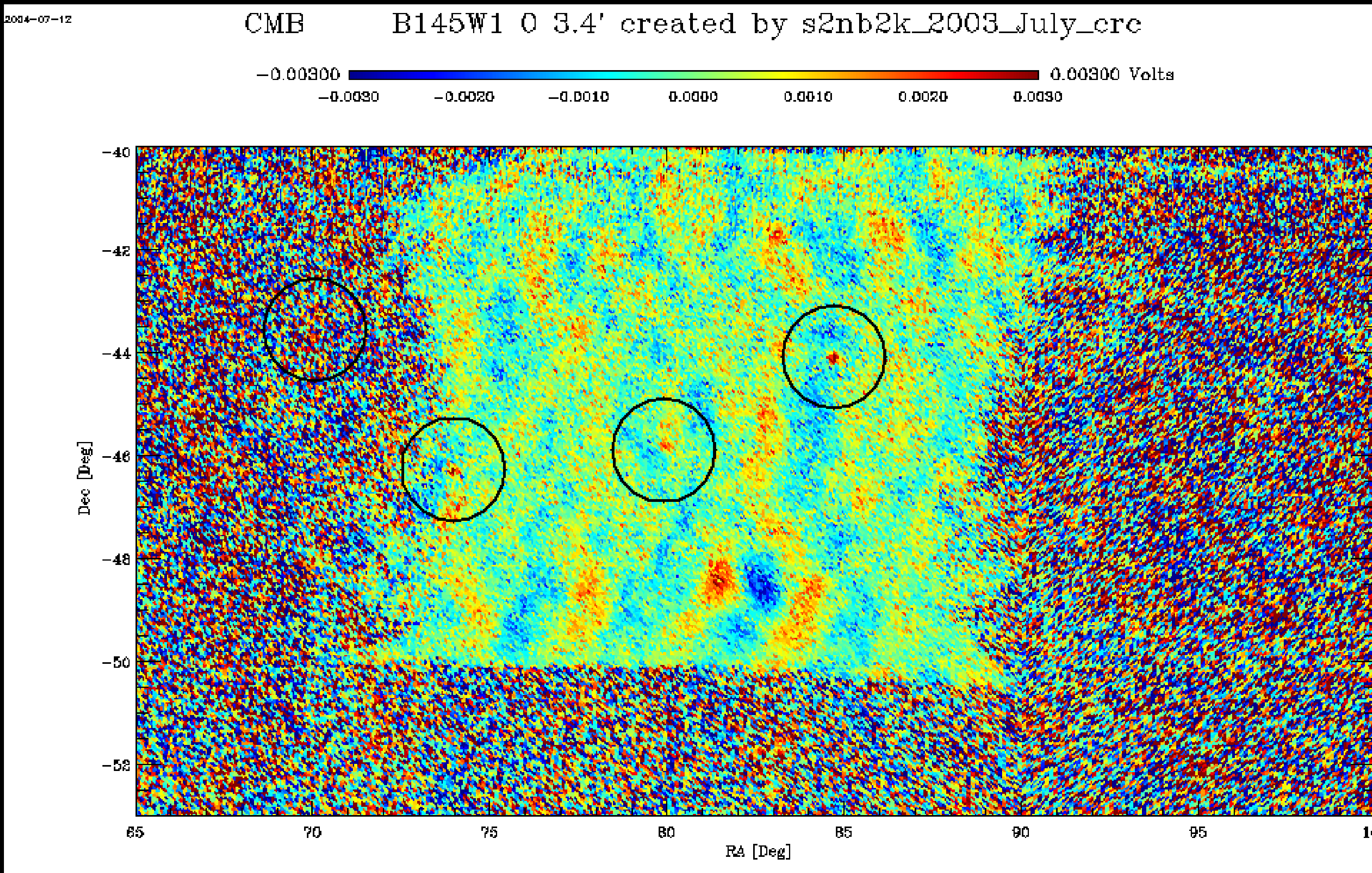
Data cleaning de-spiking



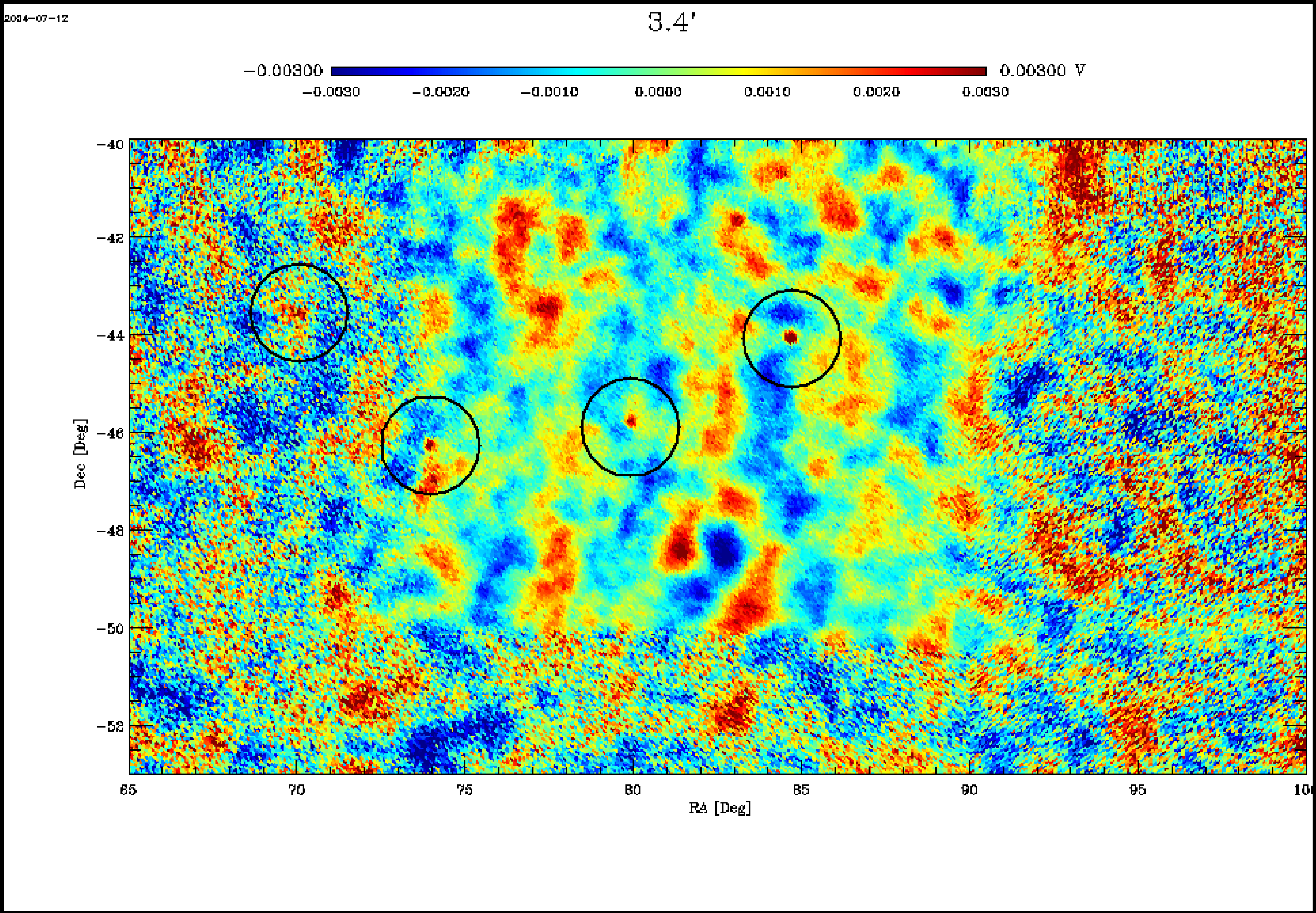
Data cleaning data slice

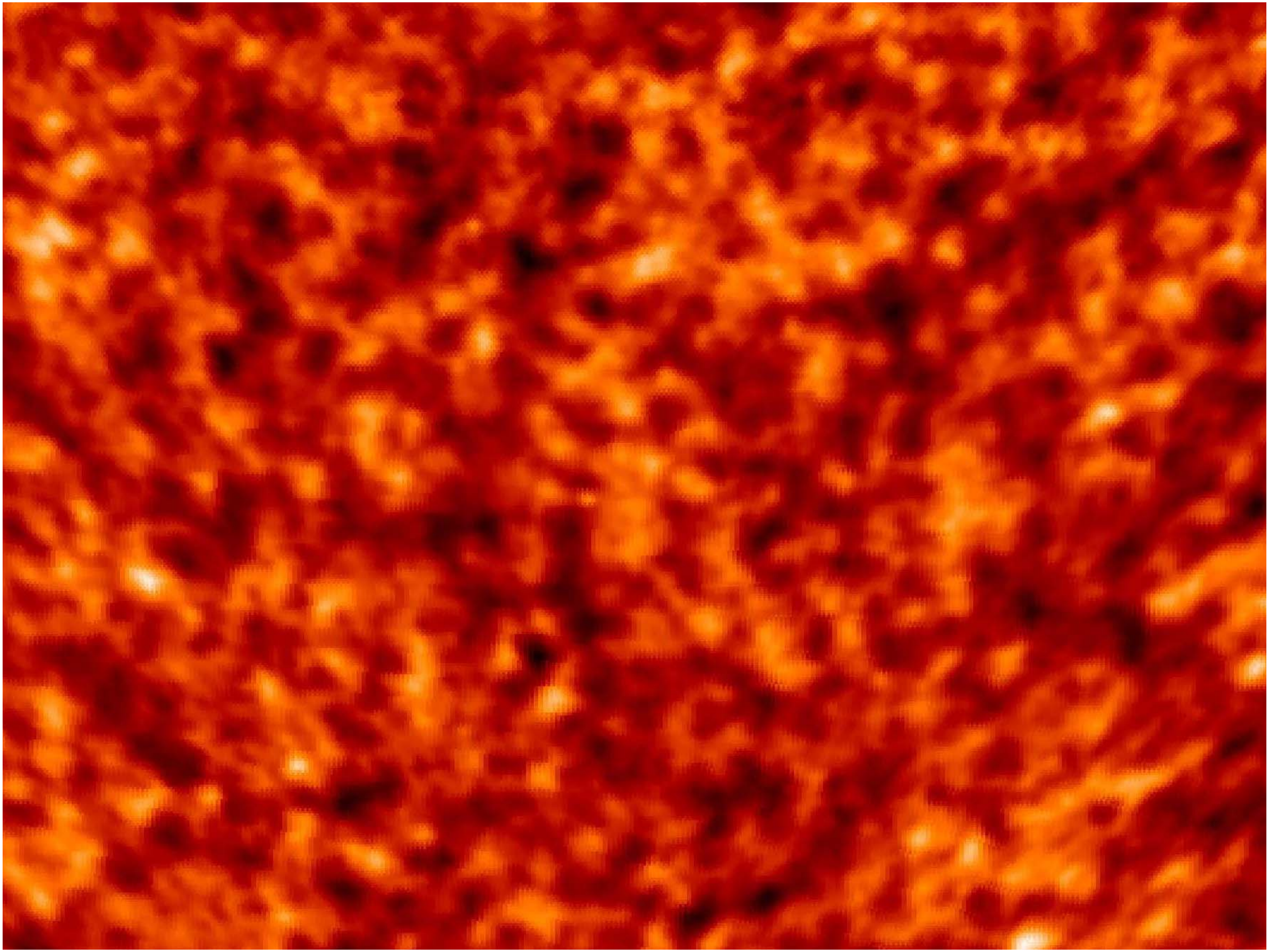


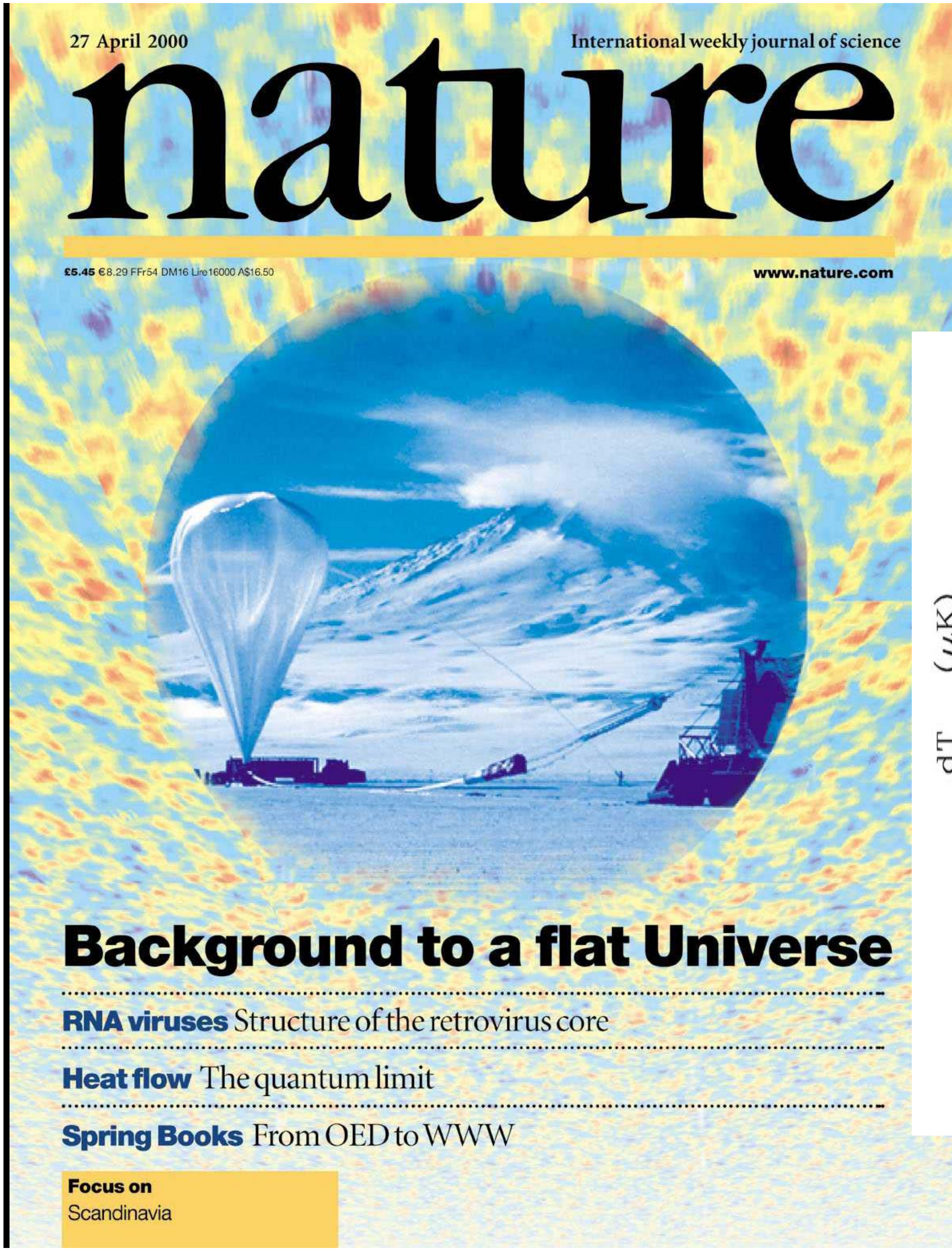
Data cleaning   naive combination



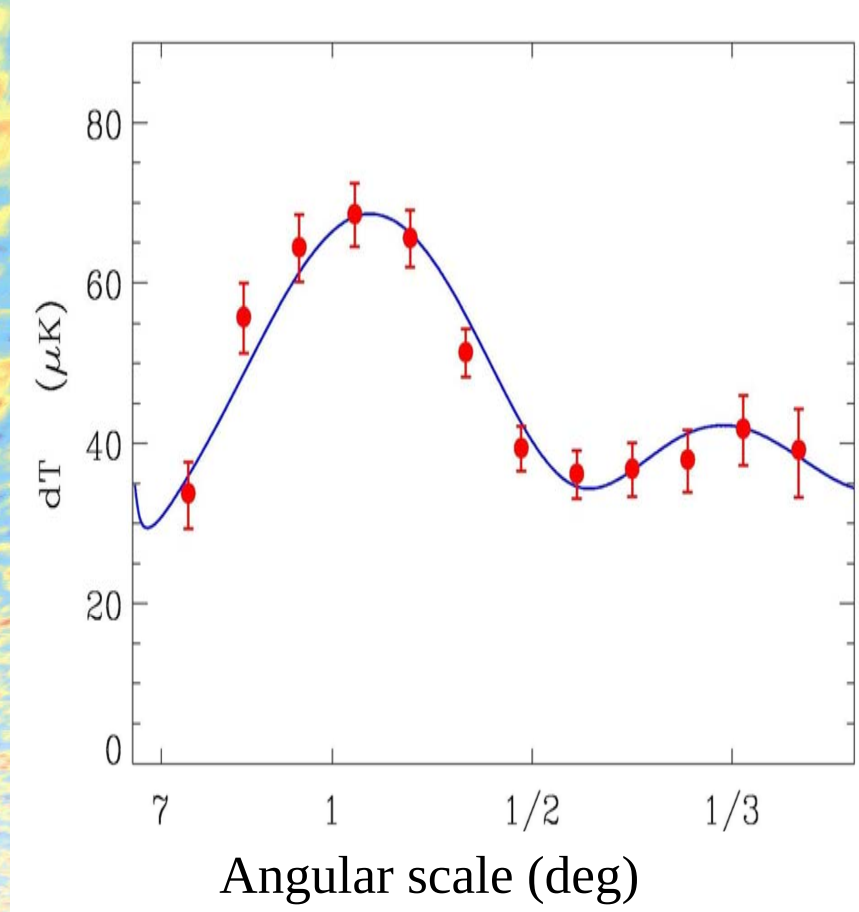
Data cleaning   optimal map-making



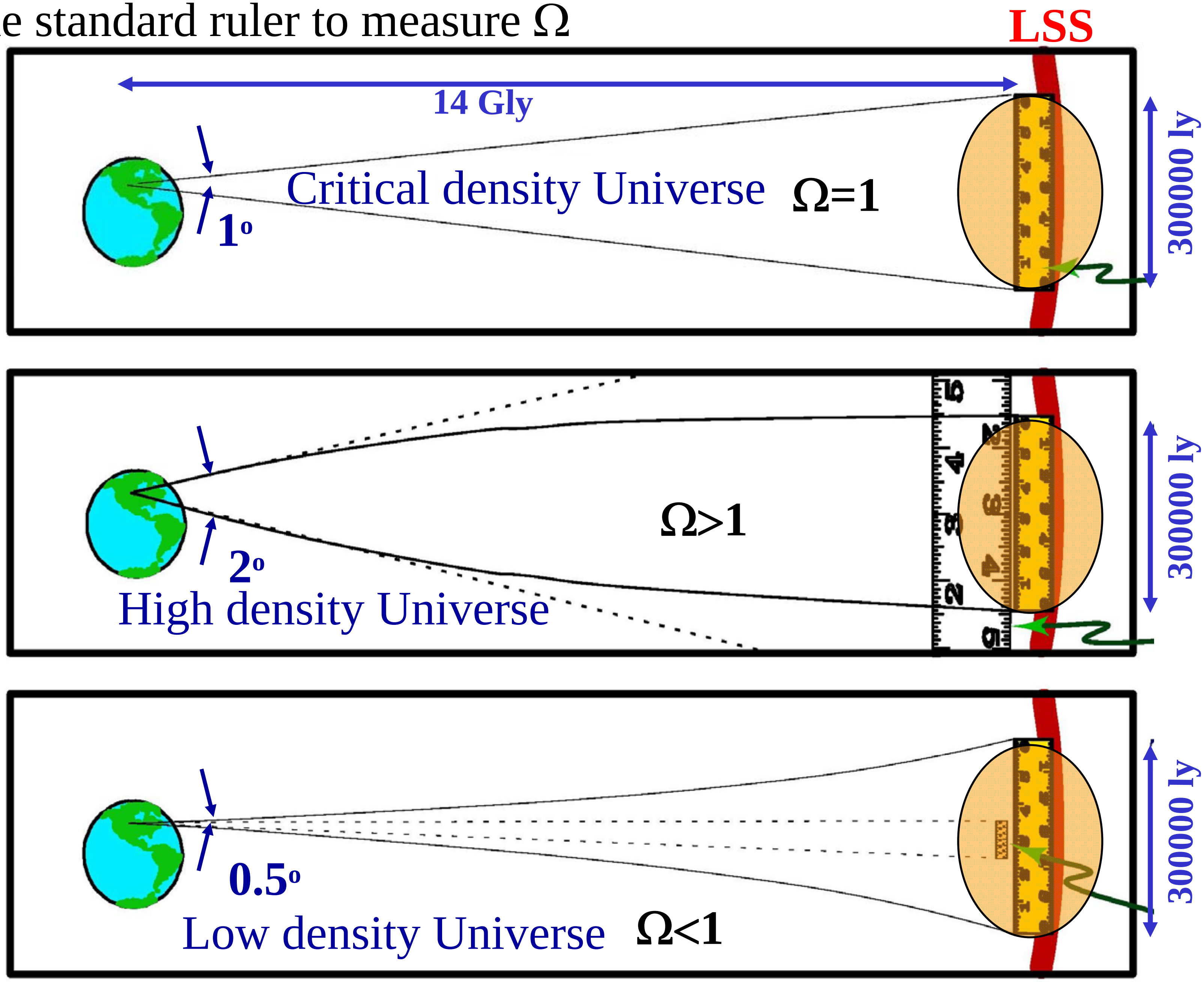




First evidence (2000) from BOOMERanG

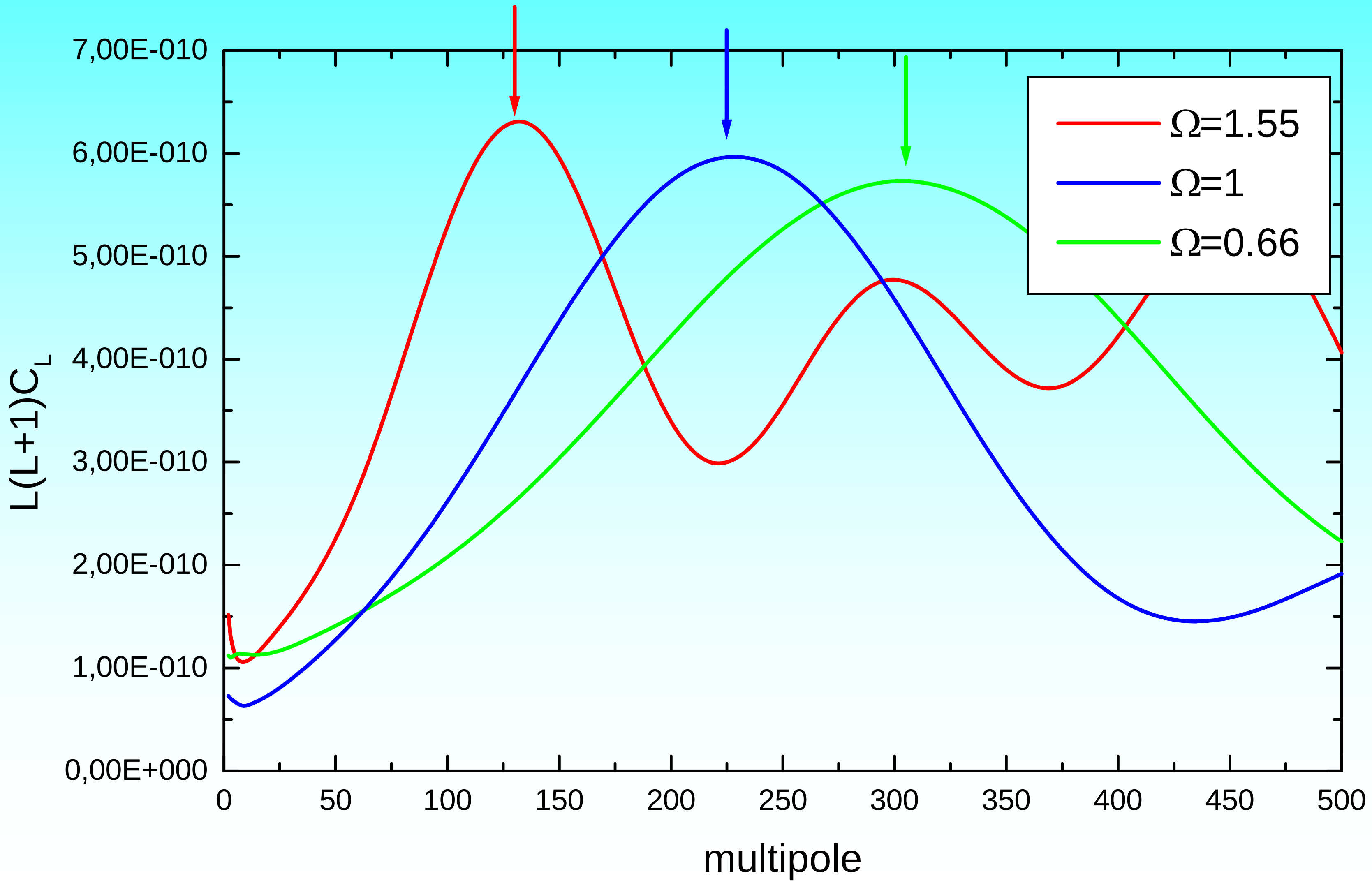


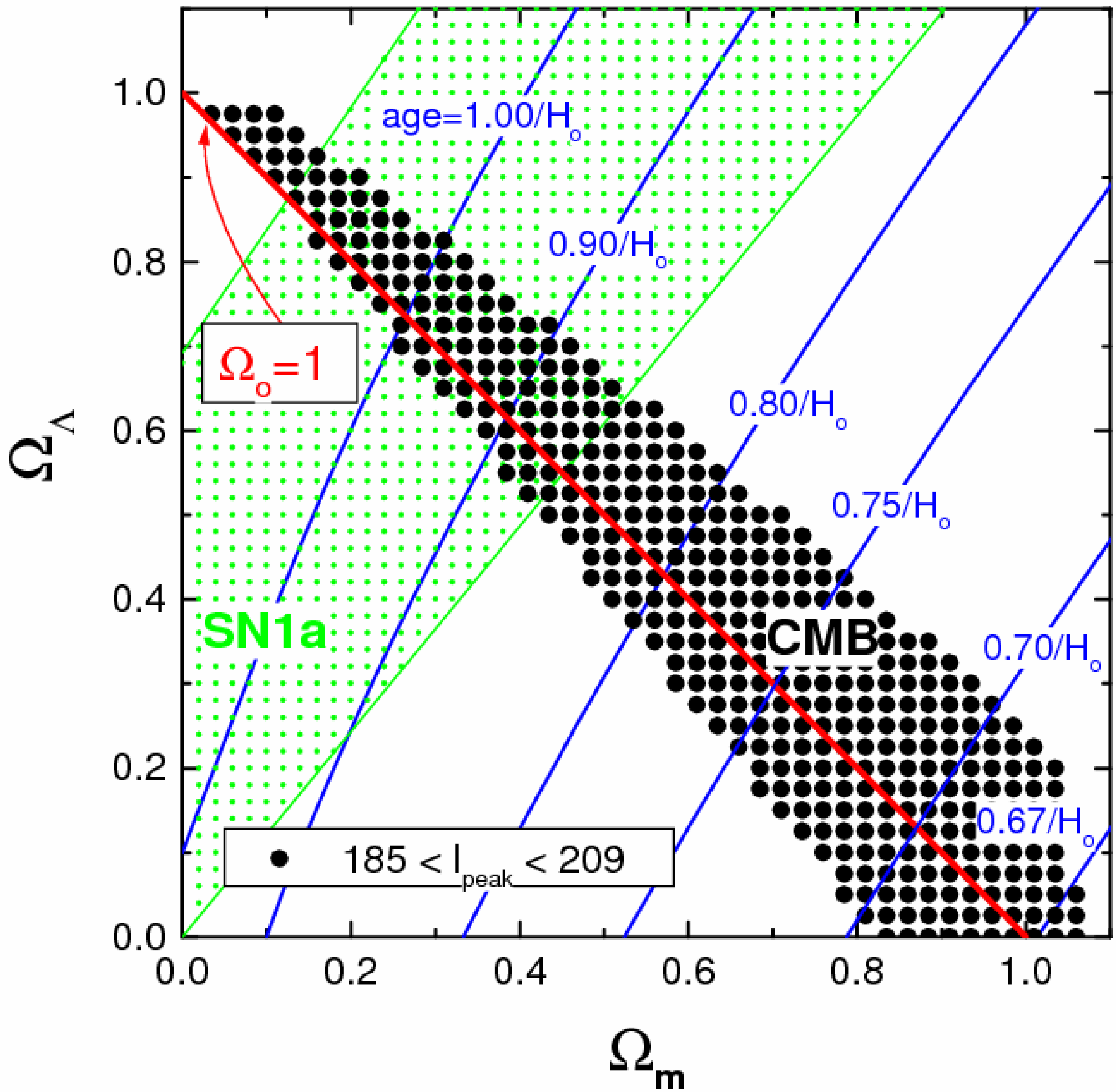
Use the standard ruler to measure Ω



Examples

Dependance on $\mathbf{\Omega}$ (curvature drives the location of first peak).
Not as simple as in these examples (see S.Weinberg, astro-ph/0006276)





age=1.00/H_o
0.90/H_o
0.80/H_o
0.75/H_o
0.70/H_o
0.67/H_o
Ω_o=1
SN1a
CMB
185 < l_{peak} < 209
Ω_Λ
Ω_m
0.0
0.2
0.4
0.6
0.8
1.0

THE ASTROPHYSICAL JOURNAL, 564:559–566, 2002 January 10

MULTIPLE PEAKS IN THE ANGULAR POWER SPECTRUM OF THE COSMIC MICROWAVE BACKGROUND: SIGNIFICANCE AND CONSEQUENCES FOR COSMOLOGY

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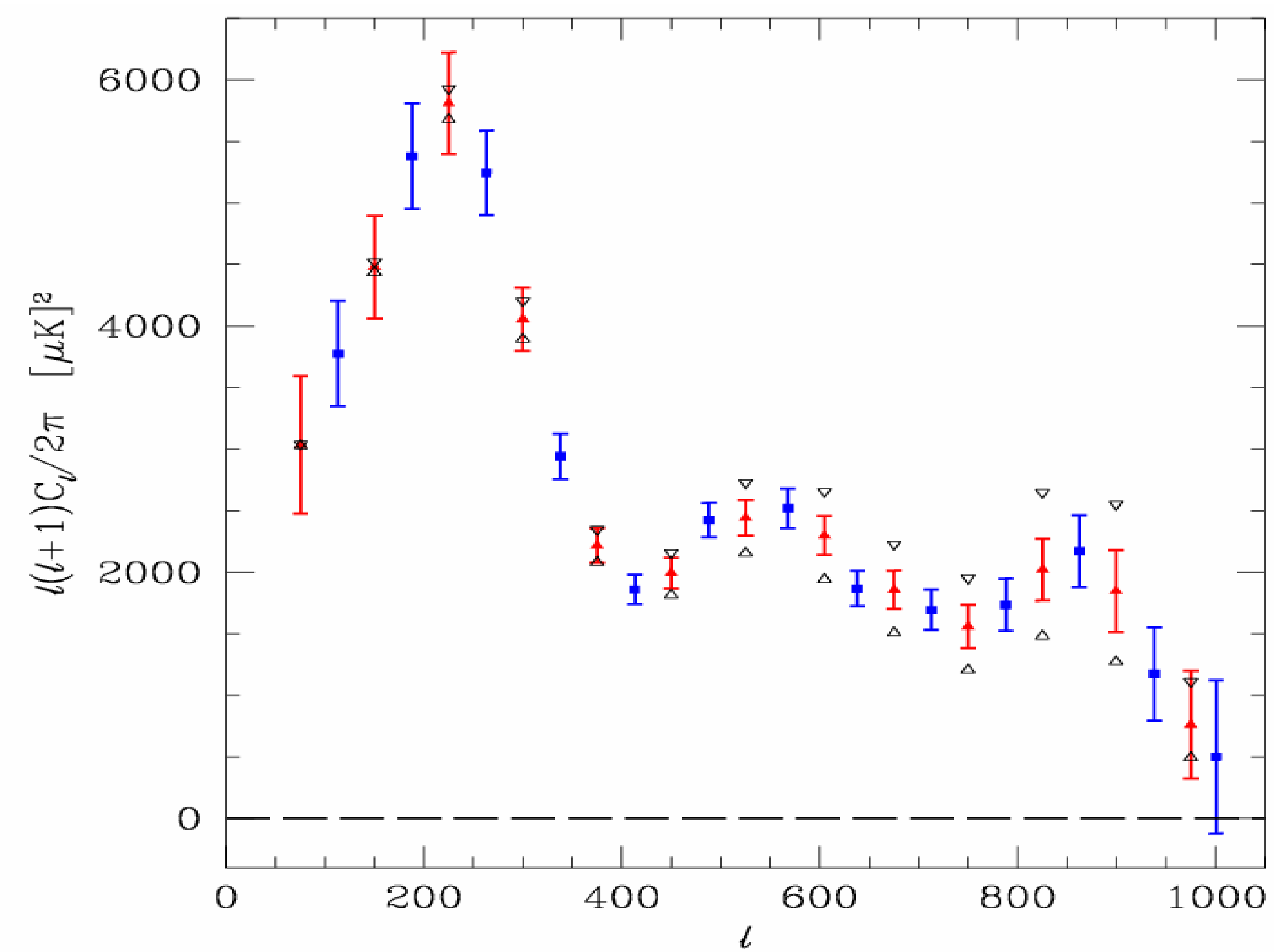
Received 2001 May 18; accepted 2001 September 11

ABSTRACT

Three peaks and two dips have been detected in the power spectrum of the cosmic microwave background by the BOOMERANG experiment, at $l = (213^{+10}_{-13})$, (541^{+20}_{-32}), (845^{+12}_{-25}) and $l = (416^{+22}_{-12})$, (750^{+20}_{-750}), respectively. Using model-independent analyses, we find that all five features are statistically significant, and we measure their location and amplitude. These are consistent with the adiabatic inflationary model. We also calculate the mean and variance of the peak and dip locations and amplitudes in a large seven-dimensional parameter space of such models, which gives good agreement with the model-independent estimates. We forecast where the next few peaks and dips should be found if the basic paradigm is correct. We test the robustness of our results by comparing Bayesian marginalization techniques on this space with likelihood maximization techniques applied to a second seven-dimensional cosmological parameter space, using an independent computational pipeline, and find excellent agreement: $\Omega_{\rm tot} = 1.02^{+0.05}_{-0.06}$ versus 1.04 ± 0.05, $\Omega_b h^2 = 0.022^{+0.004}_{-0.003}$ versus $0.019^{+0.005}_{-0.004}$, and $n_s = 0.96^{+0.09}_{-0.08}$ versus 0.90 ± 0.08. The determination of the best fit by the maximization procedure effectively ignores nonzero optical depth of reionization $\tau_C > 0$, and the difference in primordial spectral index n_s between the two methods is thus a consequence of the strong correlation of n_s with the τ_C.

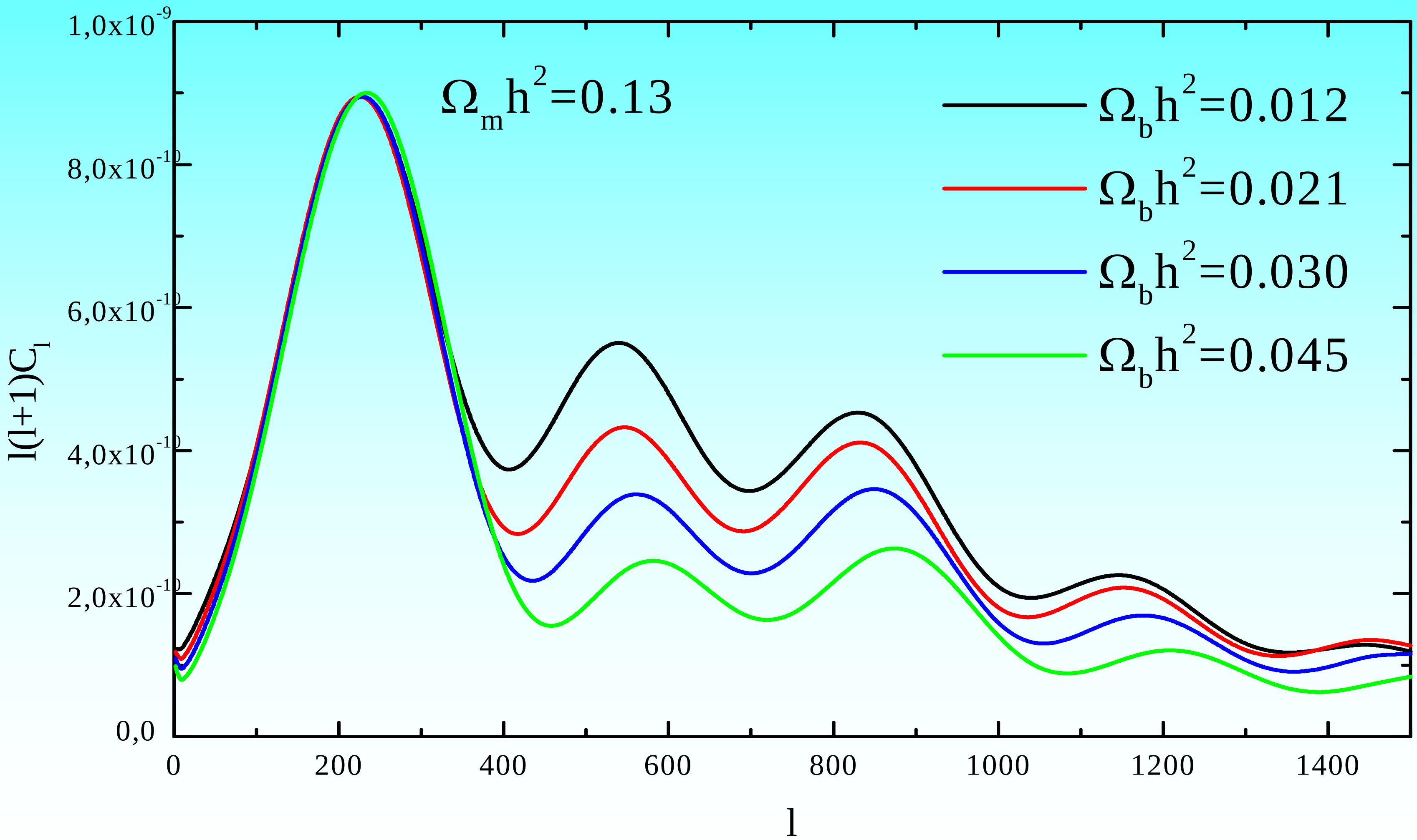
Subject headings: cosmic microwave background — cosmological parameters — cosmology: observations

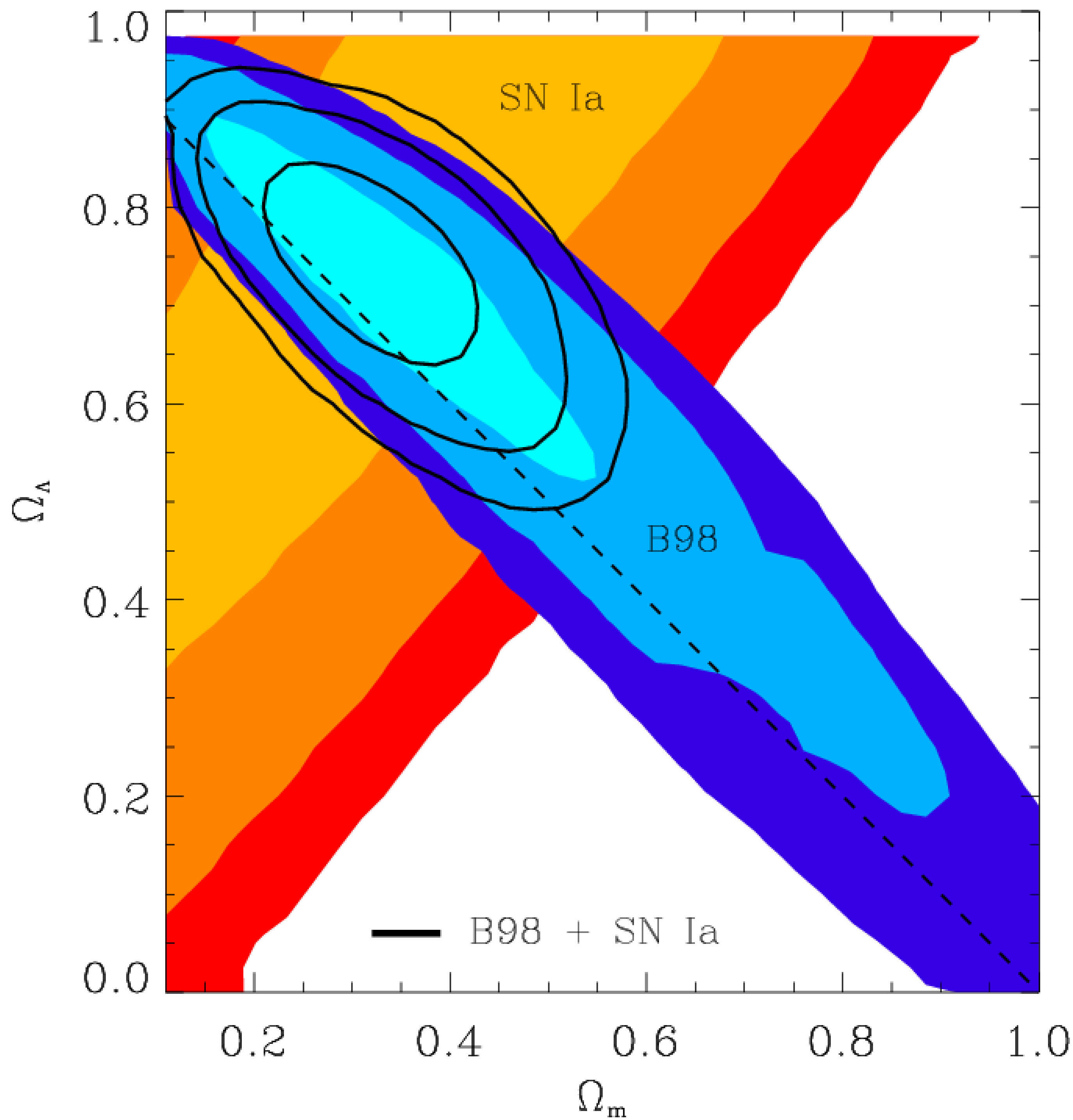


Netterfield et al. 2001, de Bernardis et al. 2002

Examples

Dependance on $\mathbf{\Omega_b}$ (Relative amplitudes second to first peak):
All the spectra are normalized to the first peak.



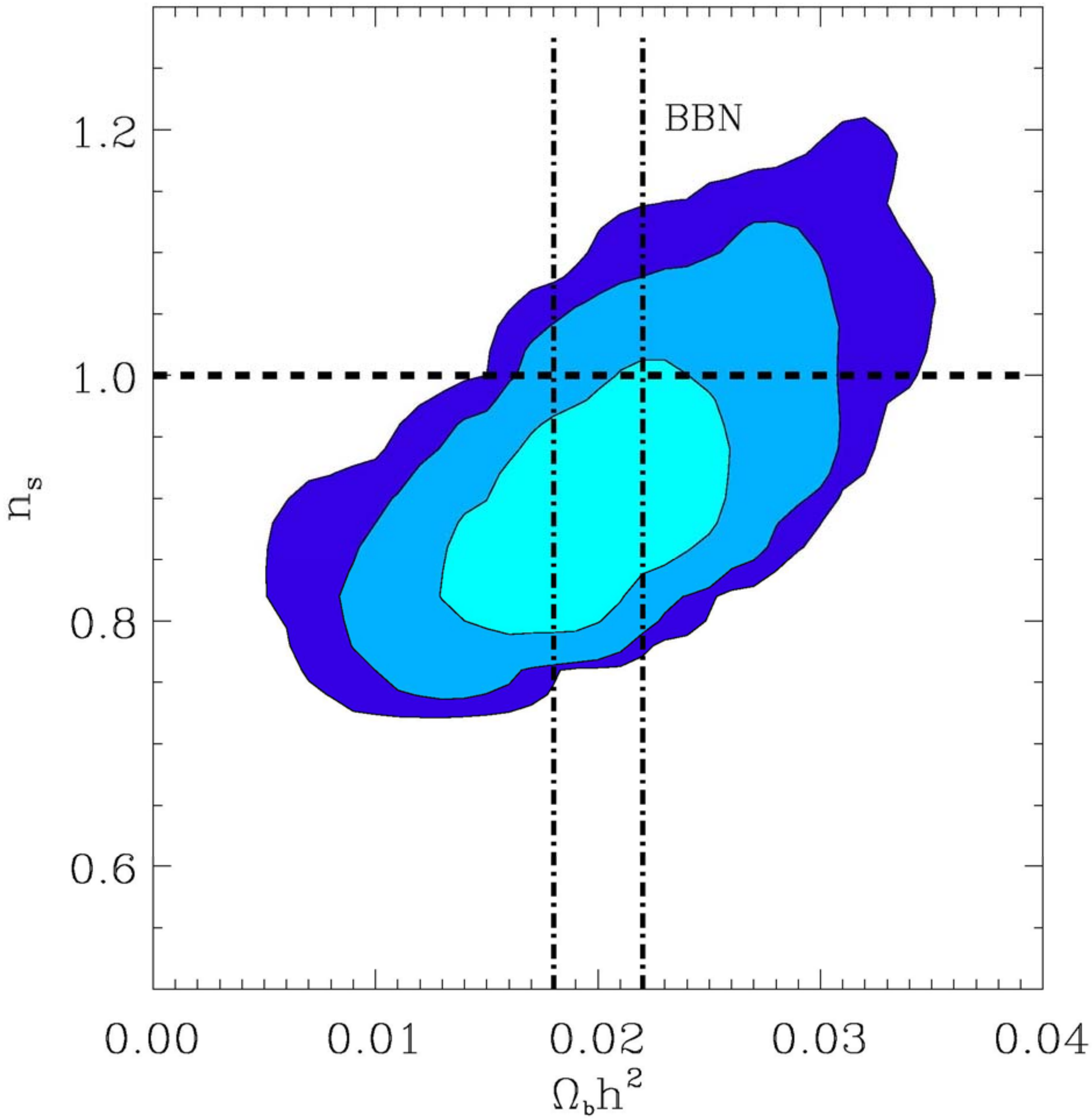


SN Ia
B98
B98 + SN Ia
1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0
Ω_Λ
Ω_m

Degeneracies are still present:
See e.g. n_s vs τ_c

This limits the precision of the determination of n_s (which is slightly sensitive to the ML vs BM method):

$n_s=(0.90\pm0.10)$ ML
$n_s=(0.96\pm0.08)$ BM



“The perfect universe”

TABLE 4

RESULTS OF PARAMETER EXTRACTION

Priors	$\Omega_{\rm tot}$	n_s	$\Omega_b h^2$	$\Omega_{\rm cdm} h^2$	Ω_Λ	Ω_m	Ω_b	τ_c	h	Age
Weak only	$1.03^{0.06}_{0.06}$	$0.93^{0.10}_{0.08}$	$0.021^{0.004}_{0.003}$	$0.12^{0.05}_{0.05}$	$(0.52^{0.24}_{0.19})$	$(0.50^{0.20}_{0.20})$	$0.07^{0.03}_{0.03}$	$0.10^{0.16}_{0.08}$	$(0.56^{0.11}_{0.11})$	$15.4^{2.1}_{2.1}$
LSS	$1.03^{0.05}_{0.05}$	$0.95^{0.09}_{0.07}$	$0.022^{0.004}_{0.003}$	$0.13^{0.03}_{0.02}$	$0.54^{0.09}_{0.09}$	$0.50^{0.11}_{0.11}$	$0.07^{0.02}_{0.02}$	$0.09^{0.13}_{0.07}$	$0.55^{0.09}_{0.09}$	$15.1^{1.3}_{1.3}$
SN1a	$1.02^{0.07}_{0.05}$	$0.96^{0.10}_{0.09}$	$0.023^{0.004}_{0.004}$	$0.09^{0.04}_{0.03}$	$0.74^{0.07}_{0.11}$	$0.31^{0.06}_{0.06}$	$0.06^{0.03}_{0.03}$	$0.12^{0.19}_{0.09}$	$0.60^{0.09}_{0.09}$	$16.2^{2.5}_{2.5}$
LSS & SN1a	$0.99^{0.03}_{0.04}$	$1.00^{0.09}_{0.08}$	$0.023^{0.003}_{0.003}$	$0.14^{0.03}_{0.02}$	$0.65^{0.05}_{0.06}$	$0.35^{0.07}_{0.07}$	$0.05^{0.02}_{0.02}$	$0.11^{0.15}_{0.08}$	$0.67^{0.09}_{0.09}$	$13.7^{1.3}_{1.3}$
$h = 0.71 \pm 0.08$	$0.98^{0.04}_{0.05}$	$0.94^{0.09}_{0.08}$	$0.021^{0.004}_{0.003}$	$0.14^{0.06}_{0.04}$	$0.62^{0.11}_{0.17}$	$0.39^{0.13}_{0.13}$	$0.05^{0.02}_{0.02}$	$0.09^{0.13}_{0.07}$	$(0.65^{0.08}_{0.08})$	$13.8^{1.7}_{1.7}$
Flat	(1.00)	$0.92^{0.08}_{0.08}$	$0.021^{0.003}_{0.003}$	$0.13^{0.04}_{0.04}$	$(0.57^{0.12}_{0.37})$	$(0.47^{0.25}_{0.25})$	$0.06^{0.02}_{0.02}$	$0.08^{0.11}_{0.06}$	$(0.62^{0.13}_{0.13})$	$14.3^{0.6}_{0.6}$
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NOTE.—Results of parameter extraction using successively more restrictive priors, following Lange et al. (2001). The confidence intervals re 1σ. The quoted values are reported after marginalizing over all other parameters. For the primary database parameters, 16% and 84% ntegrals are reported as $\pm 1\sigma$ errors. For Ω_m, Ω_b, h, and Age, which are functions of the other parameters, the mean and standard deviation ver the distribution are reported. All entries are subject to a weak prior in which only models with $0.45 < h < 0.90$ and age > 10 Gyr are onsidered. The LSS (Bond & Jaffe 1999) and SN1a supernovae (Riess et al. 1998; Perlmutter et al. 1999) priors are as described in Lange t al. (2001). The strong h prior is a Gaussian with the stated 1σ error. Parentheses are used to indicate parameters that did not shift more han 1-σ or have their errors reduced by a factor of two upon the inclusion of the CMB data, compared with an analysis using the priors only. Thus, in these cases the parameter range reflects the choice of prior, rather than a constraint by the CMB. The age column is in units of Gyr.

“The perfect universe”

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LSS & SN1a	$0.99^{0.03}_{0.04}$	$1.00^{0.09}_{0.08}$	$0.023^{0.003}_{0.003}$	$0.14^{0.03}_{0.02}$	$0.65^{0.05}_{0.06}$	$0.35^{0.07}_{0.07}$	$0.05^{0.02}_{0.02}$	$0.11^{0.15}_{0.08}$	$0.67^{0.09}_{0.09}$	$13.7^{1.3}_{1.3}$
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Flat	(1.00)	$0.92^{0.08}_{0.08}$	$0.021^{0.003}_{0.003}$	$0.13^{0.04}_{0.04}$	$(0.57^{0.12}_{0.37})$	$(0.47^{0.25}_{0.25})$	$0.06^{0.02}_{0.02}$	$0.08^{0.11}_{0.06}$	$(0.62^{0.13}_{0.13})$	$14.3^{0.6}_{0.6}$
Flat & LSS	(1.00)	$0.96^{0.09}_{0.07}$	$0.021^{0.003}_{0.003}$	$0.13^{0.02}_{0.01}$	$0.62^{0.06}_{0.07}$	$0.38^{0.07}_{0.07}$	$0.05^{0.01}_{0.01}$	$0.10^{0.13}_{0.07}$	$0.62^{0.06}_{0.06}$	$14.4^{0.7}_{0.7}$
Flat & SN1a	(1.00)	$0.94^{0.10}_{0.08}$	$0.022^{0.003}_{0.003}$	$0.12^{0.01}_{0.02}$	$0.68^{0.04}_{0.06}$	$0.33^{0.05}_{0.05}$	$0.05^{0.01}_{0.01}$	$0.08^{0.12}_{0.06}$	$0.66^{0.05}_{0.05}$	$14.0^{0.5}_{0.5}$
Flat, LSS & SN1a	(1.00)	$1.00^{0.09}_{0.08}$	$0.022^{0.003}_{0.003}$	$0.13^{0.01}_{0.01}$	$0.66^{0.04}_{0.06}$	$0.33^{0.05}_{0.05}$	$0.05^{0.01}_{0.01}$	$0.12^{0.15}_{0.08}$	$0.66^{0.05}_{0.05}$	$14.0^{0.6}_{0.6}$

NOTE.—Results of parameter extraction using successively more restrictive priors, following Lange et al. (2001). The confidence intervals are 1σ. The quoted values are reported after marginalizing over all other parameters. For the primary database parameters, 16% and 84% integrals are reported as $\pm 1\sigma$ errors. For Ω_m, Ω_b, h, and Age, which are functions of the other parameters, the mean and standard deviation over the distribution are reported. All entries are subject to a weak prior in which only models with $0.45 < h < 0.90$ and age > 10 Gyr are considered. The LSS (Bond & Jaffe 1999) and SN1a supernovae (Riess et al. 1998; Perlmutter et al. 1999) priors are as described in Lange et al. (2001). The strong h prior is a Gaussian with the stated 1σ error. Parentheses are used to indicate parameters that did not shift more than 1-σ or have their errors reduced by a factor of two upon the inclusion of the CMB data, compared with an analysis using the priors only. Thus, in these cases the parameter range reflects the choice of prior, rather than a constraint by the CMB. The age column is in units of Gyr.

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TABLE 4

RESULTS OF PARAMETER EXTRACTION

Priors	$\Omega_{\rm tot}$	n_s	$\Omega_b h^2$	$\Omega_{\rm cdm} h^2$	Ω_Λ	Ω_m	Ω_b	τ_c	h	Age
Weak only	$1.03^{0.06}_{0.06}$	$0.93^{0.10}_{0.08}$	$0.021^{0.004}_{0.003}$	$0.12^{0.05}_{0.05}$	$(0.52^{0.24}_{0.19})$	$(0.50^{0.20}_{0.20})$	$0.07^{0.03}_{0.03}$	$0.10^{0.16}_{0.08}$	$(0.56^{0.11}_{0.11})$	$15.4^{2.1}_{2.1}$
LSS	$1.03^{0.03}_{0.05}$	$0.95^{0.09}_{0.07}$	$0.022^{0.004}_{0.003}$	$0.13^{0.03}_{0.02}$	$0.54^{0.09}_{0.09}$	$0.50^{0.11}_{0.11}$	$0.07^{0.02}_{0.02}$	$0.09^{0.13}_{0.07}$	$0.55^{0.09}_{0.09}$	$15.1^{1.3}_{1.3}$
SN1a	$1.02^{0.07}_{0.05}$	$0.96^{0.10}_{0.09}$	$0.023^{0.004}_{0.004}$	$0.09^{0.04}_{0.03}$	$0.74^{0.07}_{0.11}$	$0.31^{0.06}_{0.06}$	$0.06^{0.03}_{0.03}$	$0.12^{0.19}_{0.09}$	$0.60^{0.09}_{0.09}$	$16.2^{2.5}_{2.5}$
LSS & SN1a	$0.99^{0.03}_{0.04}$	$1.00^{0.09}_{0.08}$	$0.023^{0.003}_{0.003}$	$0.14^{0.03}_{0.02}$	$0.65^{0.05}_{0.06}$	$0.35^{0.07}_{0.07}$	$0.05^{0.02}_{0.02}$	$0.11^{0.15}_{0.08}$	$0.67^{0.09}_{0.09}$	$13.7^{1.3}_{1.3}$
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Flat, LSS & SN1a	(1.00)	$1.00^{0.09}_{0.08}$	$0.022^{0.003}_{0.003}$	$0.13^{0.01}_{0.01}$	$0.66^{0.04}_{0.06}$	$0.33^{0.05}_{0.05}$	$0.05^{0.01}_{0.01}$	$0.12^{0.15}_{0.08}$	$0.66^{0.05}_{0.05}$	$14.0^{0.6}_{0.6}$

NOTE.—Results of parameter extraction using successively more restrictive priors, following Lange et al. (2001). The confidence intervals are 1σ. The quoted values are reported after marginalizing over all other parameters. For the primary database parameters, 16% and 84% integrals are reported as $\pm 1\sigma$ errors. For Ω_m, Ω_b, h, and Age, which are functions of the other parameters, the mean and standard deviation over the distribution are reported. All entries are subject to a weak prior in which only models with $0.45 < h < 0.90$ and age > 10 Gyr are considered. The LSS (Bond & Jaffe 1999) and SN1a supernovae (Riess et al. 1998; Perlmutter et al. 1999) priors are as described in Lange et al. (2001). The strong h prior is a Gaussian with the stated 1σ error. Parentheses are used to indicate parameters that did not shift more than 1-σ or have their errors reduced by a factor of two upon the inclusion of the CMB data, compared with an analysis using the priors only. Thus, in these cases the parameter range reflects the choice of prior, rather than a constraint by the CMB. The age column is in units of Gyr.

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Table 4

Results of Parameter Extraction

Priors	$\Omega_{\rm tot}$	n_s	$\Omega_b h^2$	$\Omega_{\rm cdm} h^2$	Ω_Λ	Ω_m	Ω_b	τ_c	h	Age
Weak only	$1.03^{0.06}_{0.06}$	$0.93^{0.10}_{0.08}$	$0.021^{0.004}_{0.003}$	$0.12^{0.05}_{0.05}$	$(0.52^{0.24}_{0.19})$	$(0.50^{0.20}_{0.20})$	$0.07^{0.03}_{0.03}$	$0.10^{0.16}_{0.08}$	$(0.56^{0.11}_{0.11})$	$15.4^{2.1}_{2.1}$
LSS	$1.03^{0.03}_{0.05}$	$0.95^{0.09}_{0.07}$	$0.022^{0.004}_{0.003}$	$0.13^{0.03}_{0.02}$	$0.54^{0.09}_{0.09}$	$0.50^{0.11}_{0.11}$	$0.07^{0.02}_{0.02}$	$0.09^{0.13}_{0.07}$	$0.55^{0.09}_{0.09}$	$15.1^{1.3}_{1.3}$
SN1a	$1.02^{0.07}_{0.05}$	$0.96^{0.10}_{0.09}$	$0.023^{0.004}_{0.004}$	$0.09^{0.04}_{0.03}$	$0.74^{0.07}_{0.11}$	$0.31^{0.06}_{0.06}$	$0.06^{0.03}_{0.03}$	$0.12^{0.19}_{0.09}$	$0.60^{0.09}_{0.09}$	$16.2^{2.5}_{2.5}$
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$h = 0.71 \pm 0.08$	$0.98^{0.04}_{0.05}$	$0.94^{0.09}_{0.08}$	$0.021^{0.004}_{0.003}$	$0.14^{0.06}_{0.04}$	$0.62^{0.11}_{0.17}$	$0.39^{0.13}_{0.13}$	$0.05^{0.02}_{0.02}$	$0.09^{0.13}_{0.07}$	$(0.65^{0.08}_{0.08})$	$13.8^{1.7}_{1.7}$
Flat	(1.00)	$0.92^{0.08}_{0.08}$	$0.021^{0.003}_{0.003}$	$0.13^{0.04}_{0.04}$	$(0.57^{0.12}_{0.37})$	$(0.47^{0.25}_{0.25})$	$0.06^{0.02}_{0.02}$	$0.08^{0.11}_{0.06}$	$(0.62^{0.13}_{0.13})$	$14.3^{0.6}_{0.6}$
Flat & LSS	(1.00)	$0.96^{0.09}_{0.07}$	$0.021^{0.003}_{0.003}$	$0.13^{0.02}_{0.01}$	$0.62^{0.06}_{0.07}$	$0.38^{0.07}_{0.07}$	$0.05^{0.01}_{0.01}$	$0.10^{0.13}_{0.07}$	$0.62^{0.06}_{0.06}$	$14.4^{0.7}_{0.7}$
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Flat, LSS & SN1a	(1.00)	$1.00^{0.09}_{0.08}$	$0.022^{0.003}_{0.003}$	$0.13^{0.01}_{0.01}$	$0.66^{0.04}_{0.06}$	$0.33^{0.05}_{0.05}$	$0.05^{0.01}_{0.01}$	$0.12^{0.15}_{0.08}$	$0.66^{0.05}_{0.05}$	$14.0^{0.6}_{0.6}$

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RESULTS OF PARAMETER EXTRACTION

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LSS	$1.03^{0.03}_{0.05}$	$0.95^{0.09}_{0.07}$	$0.022^{0.004}_{0.003}$	$0.13^{0.03}_{0.02}$	$0.54^{0.09}_{0.09}$	$0.50^{0.11}_{0.11}$	$0.07^{0.02}_{0.02}$	$0.09^{0.13}_{0.07}$	$0.55^{0.09}_{0.09}$	$15.1^{1.3}_{1.3}$
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LSS & SN1a	$0.99^{0.03}_{0.04}$	$1.00^{0.09}_{0.08}$	$0.023^{0.003}_{0.003}$	$0.14^{0.03}_{0.02}$	$0.65^{0.05}_{0.06}$	$0.35^{0.07}_{0.07}$	$0.05^{0.02}_{0.02}$	$0.11^{0.15}_{0.08}$	$0.67^{0.09}_{0.09}$	$13.7^{1.3}_{1.3}$
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Flat	(1.00)	$0.92^{0.08}_{0.08}$	$0.021^{0.003}_{0.003}$	$0.13^{0.04}_{0.04}$	$(0.57^{0.12}_{0.37})$	$(0.47^{0.25}_{0.25})$	$0.06^{0.02}_{0.02}$	$0.08^{0.11}_{0.06}$	$(0.62^{0.13}_{0.13})$	$14.3^{0.6}_{0.6}$
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LSS	$1.03^{0.03}_{0.05}$	$0.95^{0.09}_{0.07}$	$0.022^{0.004}_{0.003}$	$0.13^{0.03}_{0.02}$	$0.54^{0.09}_{0.09}$	$0.50^{0.11}_{0.11}$	$0.07^{0.02}_{0.02}$	$0.09^{0.13}_{0.07}$	$0.55^{0.09}_{0.09}$	$15.1^{1.3}_{1.3}$
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Flat, LSS & SN1a	(1.00)	$1.00^{0.09}_{0.08}$	$0.022^{0.003}_{0.003}$	$0.13^{0.01}_{0.01}$	$0.66^{0.04}_{0.06}$	$0.33^{0.05}_{0.05}$	$0.05^{0.01}_{0.01}$	$0.12^{0.15}_{0.08}$	$0.66^{0.05}_{0.05}$	$14.0^{0.6}_{0.6}$

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Weak only	$1.03^{0.06}_{0.06}$	$0.93^{0.10}_{0.08}$	$0.021^{0.004}_{0.003}$	$0.12^{0.05}_{0.05}$	$(0.52^{0.24}_{0.19})$	$(0.50^{0.20}_{0.20})$	$0.07^{0.03}_{0.03}$	$0.10^{0.16}_{0.08}$	$(0.56^{0.11}_{0.11})$	$15.4^{2.1}_{2.1}$
LSS	$1.03^{0.03}_{0.05}$	$0.95^{0.09}_{0.07}$	$0.022^{0.004}_{0.003}$	$0.13^{0.03}_{0.02}$	$0.54^{0.09}_{0.09}$	$0.50^{0.11}_{0.11}$	$0.07^{0.02}_{0.02}$	$0.09^{0.13}_{0.07}$	$0.55^{0.09}_{0.09}$	$15.1^{1.3}_{1.3}$
SN1a	$1.02^{0.07}_{0.05}$	$0.96^{0.10}_{0.09}$	$0.023^{0.004}_{0.004}$	$0.09^{0.04}_{0.03}$	$0.74^{0.07}_{0.11}$	$0.31^{0.06}_{0.06}$	$0.06^{0.03}_{0.03}$	$0.12^{0.19}_{0.09}$	$0.60^{0.09}_{0.09}$	$16.2^{2.5}_{2.5}$
LSS & SN1a	$0.99^{0.03}_{0.04}$	$1.00^{0.09}_{0.08}$	$0.023^{0.003}_{0.003}$	$0.14^{0.03}_{0.02}$	$0.65^{0.05}_{0.06}$	$0.35^{0.07}_{0.07}$	$0.05^{0.02}_{0.02}$	$0.11^{0.15}_{0.08}$	$0.67^{0.09}_{0.09}$	$13.7^{1.3}_{1.3}$
$h = 0.71 \pm 0.08$	$0.98^{0.04}_{0.05}$	$0.94^{0.09}_{0.08}$	$0.021^{0.004}_{0.003}$	$0.14^{0.06}_{0.04}$	$0.62^{0.11}_{0.17}$	$0.39^{0.13}_{0.13}$	$0.05^{0.02}_{0.02}$	$0.09^{0.13}_{0.07}$	$(0.65^{0.08}_{0.08})$	$13.8^{1.7}_{1.7}$
Flat	(1.00)	$0.92^{0.08}_{0.08}$	$0.021^{0.003}_{0.003}$	$0.13^{0.04}_{0.04}$	$(0.57^{0.12}_{0.37})$	$(0.47^{0.25}_{0.25})$	$0.06^{0.02}_{0.02}$	$0.08^{0.11}_{0.06}$	$(0.62^{0.13}_{0.13})$	$14.3^{0.6}_{0.6}$
Flat & LSS	(1.00)	$0.96^{0.09}_{0.07}$	$0.021^{0.003}_{0.003}$	$0.13^{0.02}_{0.01}$	$0.62^{0.06}_{0.07}$	$0.38^{0.07}_{0.07}$	$0.05^{0.01}_{0.01}$	$0.10^{0.13}_{0.07}$	$0.62^{0.06}_{0.06}$	$14.4^{0.7}_{0.7}$
Flat & SN1a	(1.00)	$0.94^{0.10}_{0.08}$	$0.022^{0.003}_{0.003}$	$0.12^{0.01}_{0.02}$	$0.68^{0.04}_{0.06}$	$0.33^{0.05}_{0.05}$	$0.05^{0.01}_{0.01}$	$0.08^{0.12}_{0.06}$	$0.66^{0.05}_{0.05}$	$14.0^{0.5}_{0.5}$
Flat, LSS & SN1a	(1.00)	$1.00^{0.09}_{0.08}$	$0.022^{0.003}_{0.003}$	$0.13^{0.01}_{0.01}$	$0.66^{0.04}_{0.06}$	$0.33^{0.05}_{0.05}$	$0.05^{0.01}_{0.01}$	$0.12^{0.15}_{0.08}$	$0.66^{0.05}_{0.05}$	$14.0^{0.6}_{0.6}$

NOTE.—Results of parameter extraction using successively more restrictive priors, following Lange et al. (2001). The confidence intervals re 1σ. The quoted values are reported after marginalizing over all other parameters. For the primary database parameters, 16% and 84% ntegrals are reported as $\pm 1\sigma$ errors. For Ω_m, Ω_b, h, and Age, which are functions of the other parameters, the mean and standard deviation ver the distribution are reported. All entries are subject to a weak prior in which only models with $0.45 < h < 0.90$ and age > 10 Gyr are onsidered. The LSS (Bond & Jaffe 1999) and SN1a supernovae (Riess et al. 1998; Perlmutter et al. 1999) priors are as described in Lange t al. (2001). The strong h prior is a Gaussian with the stated 1σ error. Parentheses are used to indicate parameters that did not shift more han 1-σ or have their errors reduced by a factor of two upon the inclusion of the CMB data, compared with an analysis using the priors only. Thus, in these cases the parameter range reflects the choice of prior, rather than a constraint by the CMB. The age column is in units of Gyr.

“The perfect universe”

TABLE 4

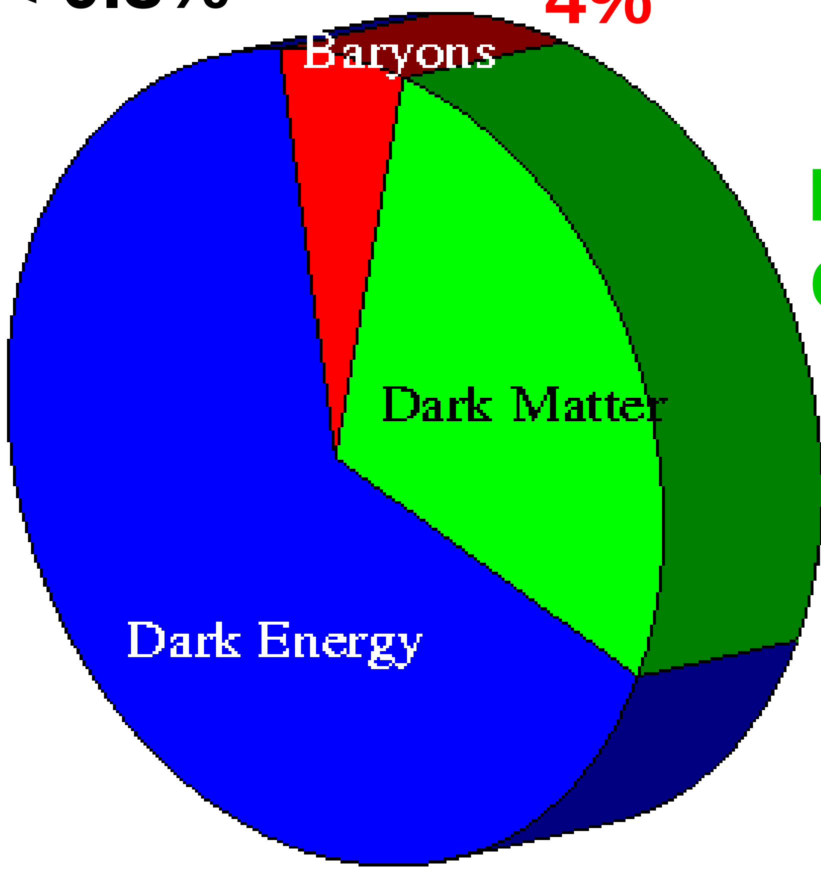
RESULTS OF PARAMETER EXTRACTION

Priors	Ω_{tot}	n_s	$\Omega_b h^2$	$\Omega_{cdm} h^2$	Ω_Λ	Ω_m	Ω_b	τ_c	h	Age
Weak only	$1.03^{0.06}_{0.06}$	$0.93^{0.10}_{0.08}$	$0.021^{0.004}_{0.003}$	$0.12^{0.05}_{0.05}$	$(0.52^{0.24}_{0.19})$	$(0.50^{0.20}_{0.20})$	$0.07^{0.03}_{0.03}$	$0.10^{0.16}_{0.08}$	$(0.56^{0.11}_{0.11})$	$15.4^{2.1}_{2.1}$
LSS	$1.03^{0.03}_{0.05}$	$0.95^{0.09}_{0.07}$	$0.022^{0.004}_{0.003}$	$0.13^{0.03}_{0.02}$	$0.54^{0.09}_{0.09}$	$0.50^{0.11}_{0.11}$	$0.07^{0.02}_{0.02}$	$0.09^{0.13}_{0.07}$	$0.55^{0.09}_{0.09}$	$15.1^{1.3}_{1.3}$
SN1a	$1.02^{0.07}_{0.05}$	$0.96^{0.10}_{0.09}$	$0.023^{0.004}_{0.004}$	$0.09^{0.04}_{0.03}$	$0.74^{0.07}_{0.11}$	$0.31^{0.06}_{0.06}$	$0.06^{0.03}_{0.03}$	$0.12^{0.19}_{0.09}$	$0.60^{0.09}_{0.09}$	$16.2^{2.5}_{2.5}$
LSS & SN1a	$0.99^{0.03}_{0.04}$	$1.00^{0.09}_{0.08}$	$0.023^{0.003}_{0.003}$	$0.14^{0.03}_{0.02}$	$0.65^{0.05}_{0.06}$	$0.35^{0.07}_{0.07}$	$0.05^{0.02}_{0.02}$	$0.11^{0.15}_{0.08}$	$0.67^{0.09}_{0.09}$	$13.7^{1.3}_{1.3}$
$h = 0.71 \pm 0.08$	$0.98^{0.04}_{0.05}$	$0.94^{0.09}_{0.08}$	$0.021^{0.004}_{0.003}$	$0.14^{0.06}_{0.04}$	$0.62^{0.11}_{0.17}$	$0.39^{0.13}_{0.13}$	$0.05^{0.02}_{0.02}$	$0.09^{0.13}_{0.07}$	$(0.65^{0.08}_{0.08})$	$13.8^{1.7}_{1.7}$
Flat	(1.00)	$0.92^{0.08}_{0.08}$	$0.021^{0.003}_{0.003}$	$0.13^{0.04}_{0.04}$	$(0.57^{0.12}_{0.37})$	$(0.47^{0.25}_{0.25})$	$0.06^{0.02}_{0.02}$	$0.08^{0.11}_{0.06}$	$(0.62^{0.13}_{0.13})$	$14.3^{0.6}_{0.6}$
Flat & LSS	(1.00)	$0.96^{0.09}_{0.07}$	$0.021^{0.003}_{0.003}$	$0.13^{0.02}_{0.01}$	$0.62^{0.06}_{0.07}$	$0.38^{0.07}_{0.07}$	$0.05^{0.01}_{0.01}$	$0.10^{0.13}_{0.07}$	$0.62^{0.06}_{0.06}$	$14.4^{0.7}_{0.7}$
Flat & SN1a	(1.00)	$0.94^{0.10}_{0.08}$	$0.022^{0.003}_{0.003}$	$0.12^{0.01}_{0.02}$	$0.68^{0.04}_{0.06}$	$0.33^{0.05}_{0.05}$	$0.05^{0.01}_{0.01}$	$0.08^{0.12}_{0.06}$	$0.66^{0.05}_{0.05}$	$14.0^{0.5}_{0.5}$
Flat, LSS & SN1a	(1.00)	$1.00^{0.09}_{0.08}$	$0.022^{0.003}_{0.003}$	$0.13^{0.01}_{0.01}$	$0.66^{0.04}_{0.06}$	$0.33^{0.05}_{0.05}$	$0.05^{0.01}_{0.01}$	$0.12^{0.15}_{0.08}$	$0.66^{0.05}_{0.05}$	$14.0^{0.6}_{0.6}$

NOTE.—Results of parameter extraction using successively more restrictive priors, following Lange et al. (2001). The confidence intervals ıre 1σ. The quoted values are reported after marginalizing over all other parameters. For the primary database parameters, 16% and 84% ntegrals are reported as $\pm 1\sigma$ errors. For Ω_m, Ω_b, h, and Age, which are functions of the other parameters, the mean and standard deviation ıver the distribution are reported. All entries are subject to a weak prior in which only models with $0.45 < h < 0.90$ and age > 10 Gyr are onsidered. The LSS (Bond & Jaffe 1999) and SN1a supernovae (Riess et al. 1998; Perlmutter et al. 1999) priors are as described in Lange t al. (2001). The strong h prior is a Gaussian with the stated 1σ error. Parentheses are used to indicate parameters that did not shift more han 1-σ or have their errors reduced by a factor of two upon the inclusion of the CMB data, compared with an analysis using the priors only. Thus, in these cases the parameter range reflects the choice of prior, rather than a constraint by the CMB. The age column is in units of Gyr.

Radiazione
< 0.3%

Materia
Normale
4%



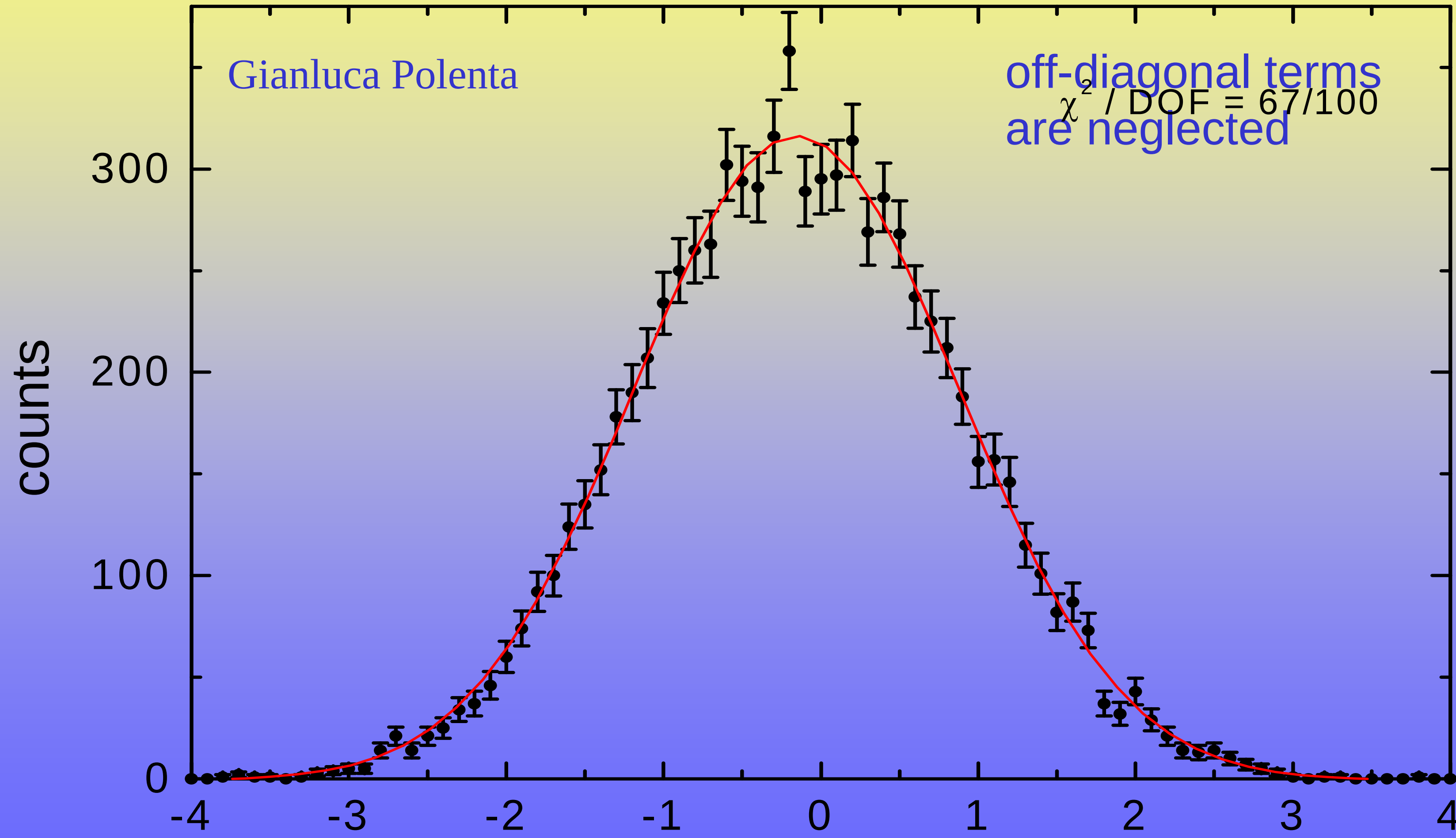
Materia
Oscura
22%

Energia
Oscura
74%

Are the BOOMERanG data Gaussian ?

150 GHz
In the 1% region
14’ Pixels

off-diagonal terms are neglected



$(\text{Measured } \Delta T_i)/\sigma_i$; $\sigma_i^2 = \sigma_{noise,i}^2 + \sigma_{sky}^2$

<u>Minkowski functionals</u>:

For a given threshold ν we compute the excursion set $Q(\nu)=\{\text{all } T_i: (T_i-\langle T\rangle)/\boldsymbol{\sigma} > \boldsymbol{\nu}\}$ and its contour δQ. If dA and ds are the differential elements of Q and δQ, and **k** is the normal to δQ, the Minkowski functional densities are:

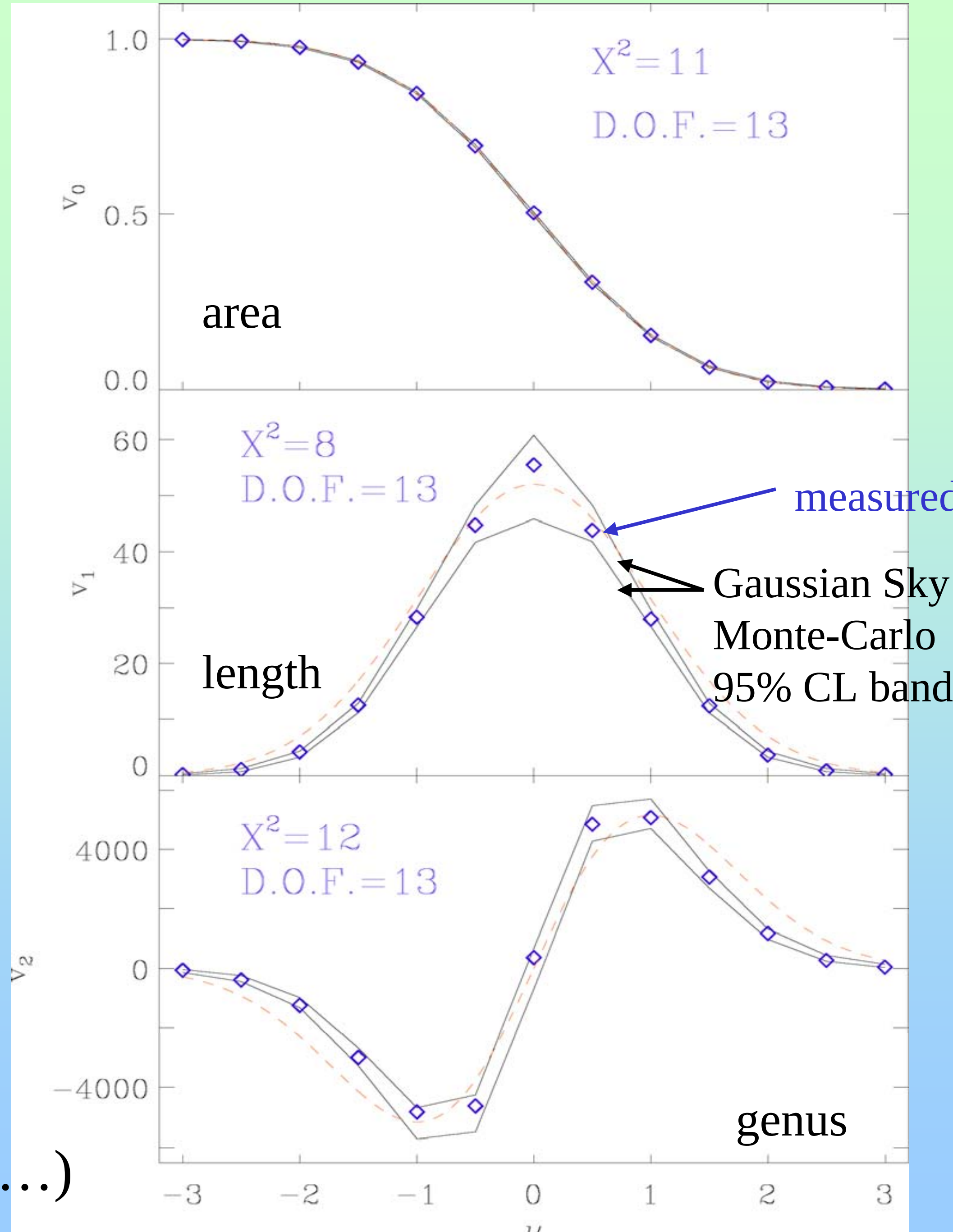
$v_o = [\int_Q dA\,]/A$ (area)

$v_1 = [\int_{\delta Q} ds\,]/A$ (length)

$v_2 = [\int_{\delta Q} \mathbf{k}\, ds\,]/A$ (genus)

(Polenta et al. 2001)

Other test being performed (Gurzadyan & Kashin, Contaldi, Ferreira, De Troia…)



Systematics

Systematics ARE there.

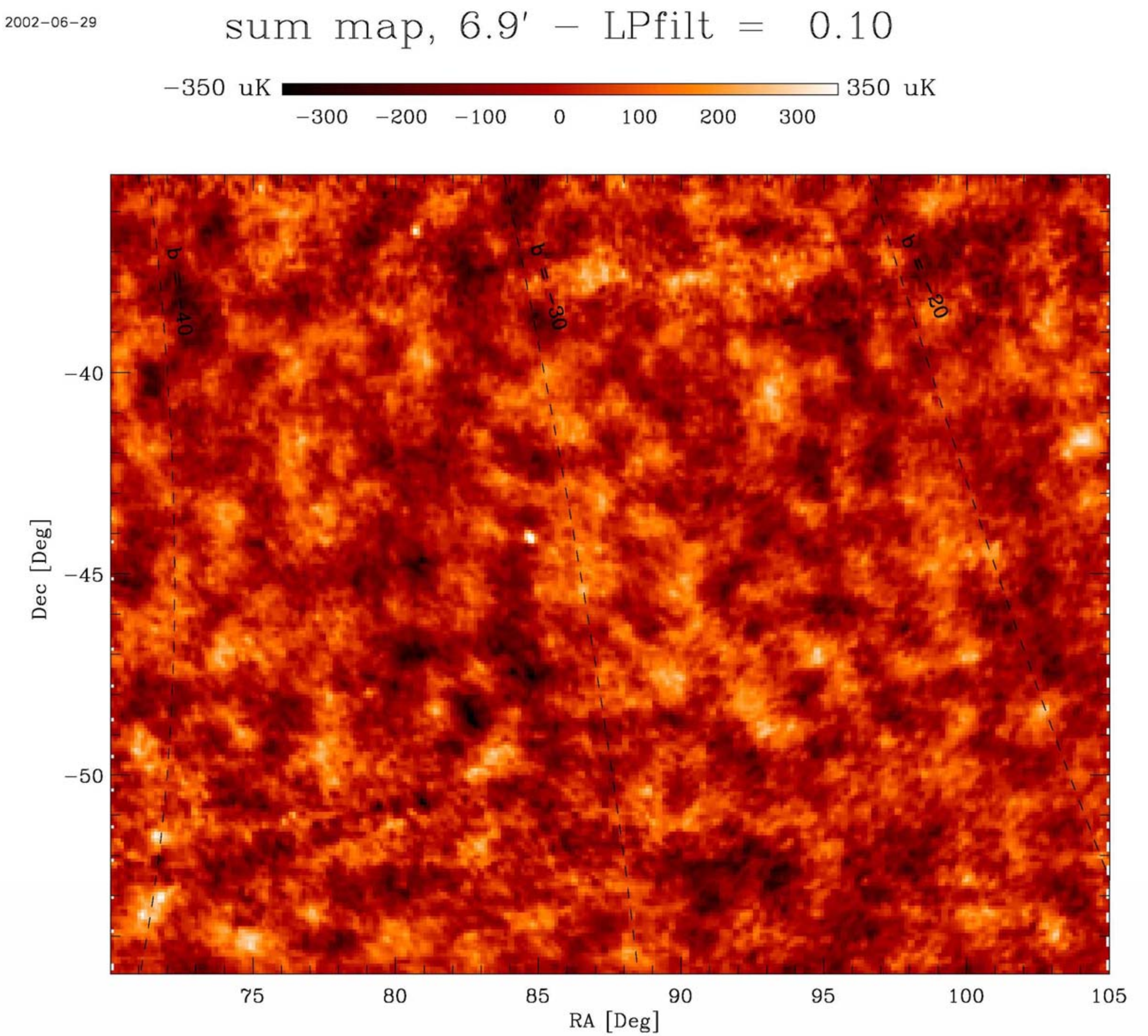
- Knox’s formula assumes simple white gaussian noise.
- In the real world noise is not gaussian and we have drifts, spikes, events of different kind in the raw data.
- Detectors characteristics (responsivity, noise) can change with time during the survey.
- Moreover, low-level local emission can contaminate the sky signal in a non gaussian way.
- Evident features are easily identified and rejected.
- Features smaller than the noise cannot be removed, and contaminate the results.
- The experiment needs to have internal redundancy in order to make tests for the presence of systematics.

Systematics ARE there.

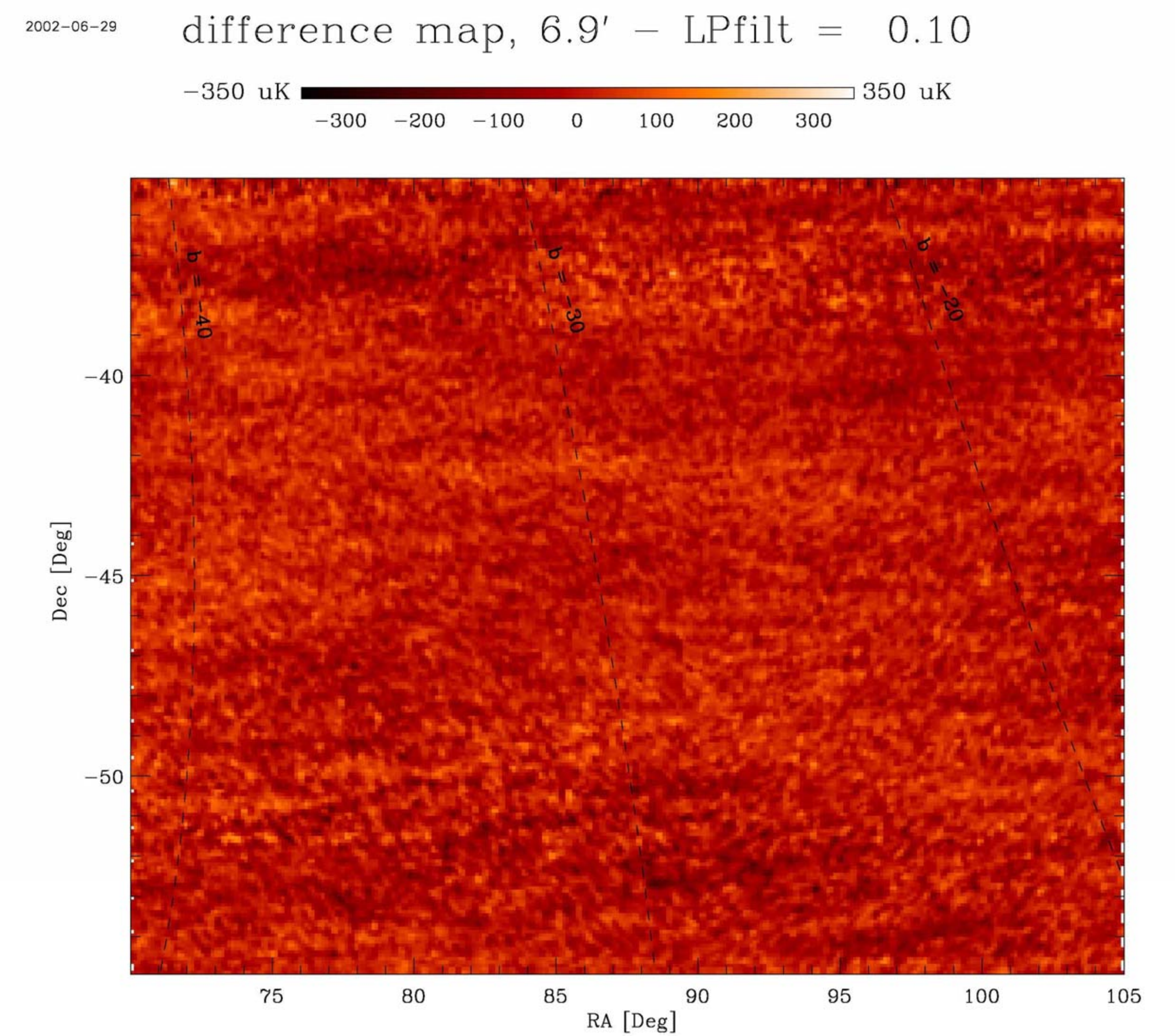
- The experiment needs to have internal redundancy in order to make tests for the presence of systematics.
  A. Several detectors at the same frequency
  B. Several different frequencies
- The experimental conditions must be changed, to check the reliability of the result
  C. Experiment different scan speeds
  D. Experiment different sidelobes conditions
  E. Experiment different locations of sun, moon, strong sources.
  F. Results must be compared to results of similar, independent experiments.
- Calibration should be carried out several times during the survey

Test A:

- Compare independent channels at the same frequency.
- Different bolometers have different noise performance.
- Two channels with similar performance are B150A and (B150A1+B150A2)/2
- Sum and difference maps:



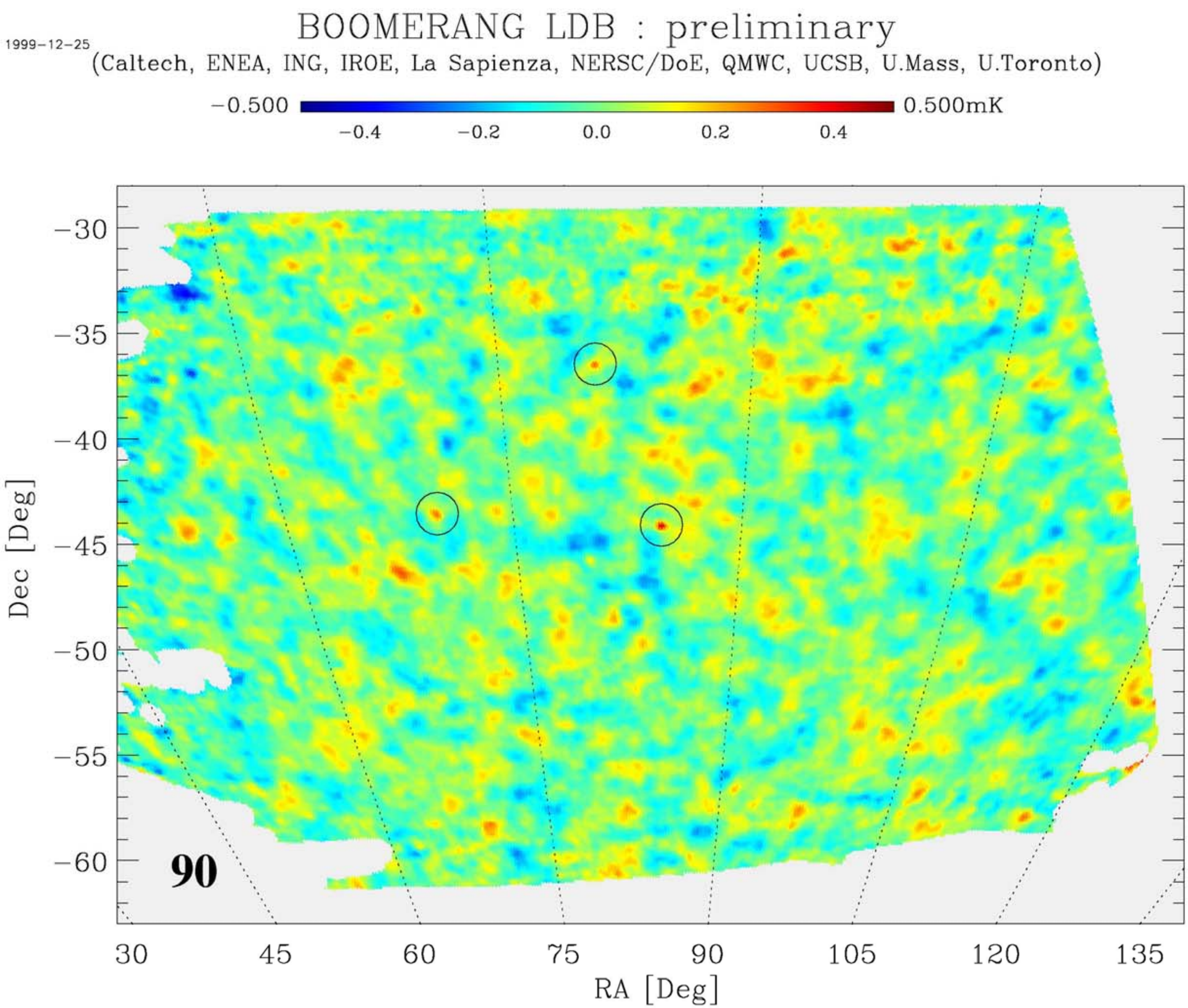
[B150A+(B150A1+B150A2)/2]/2

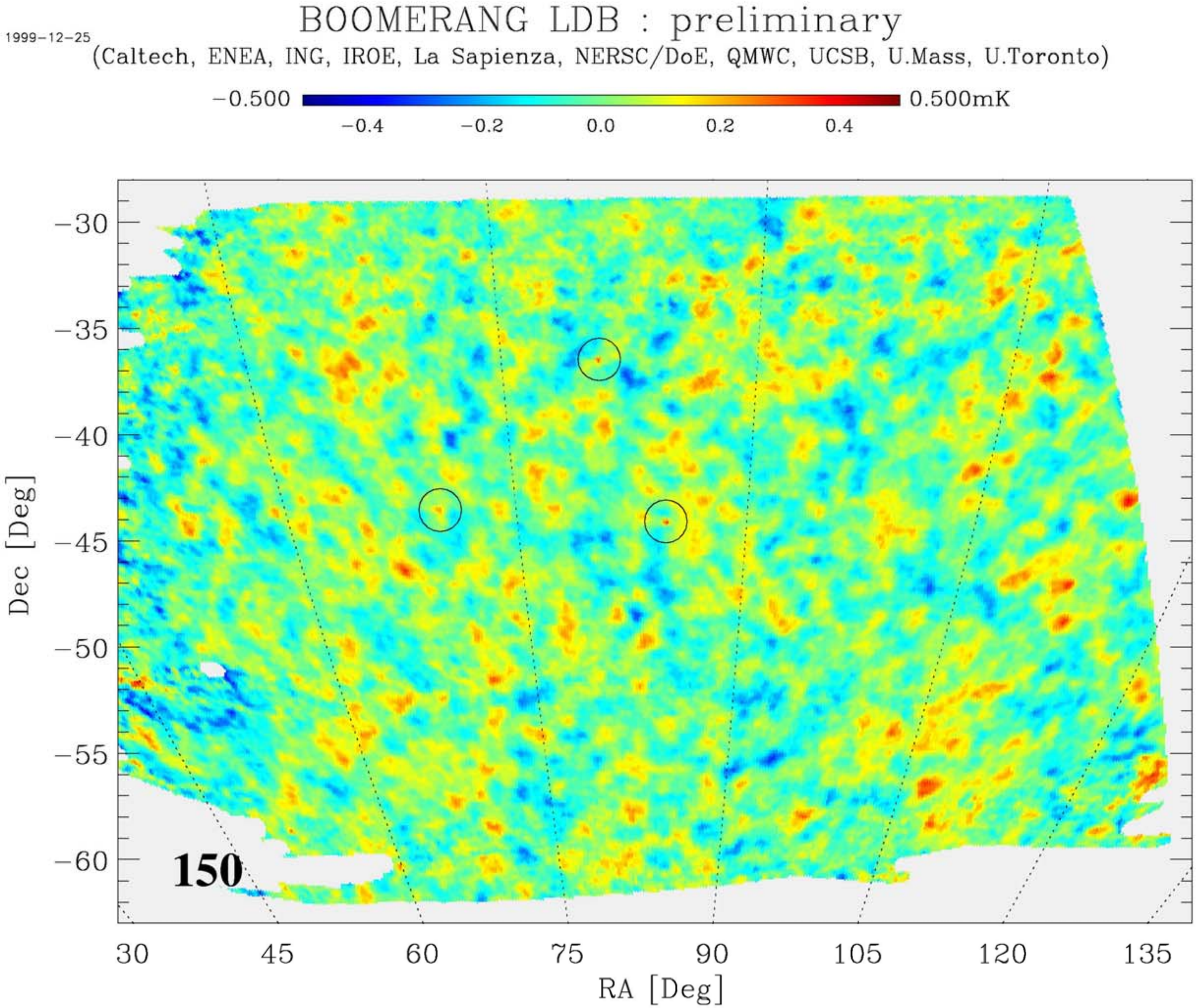


[B150A-(B150A1+B150A2)/2]/2

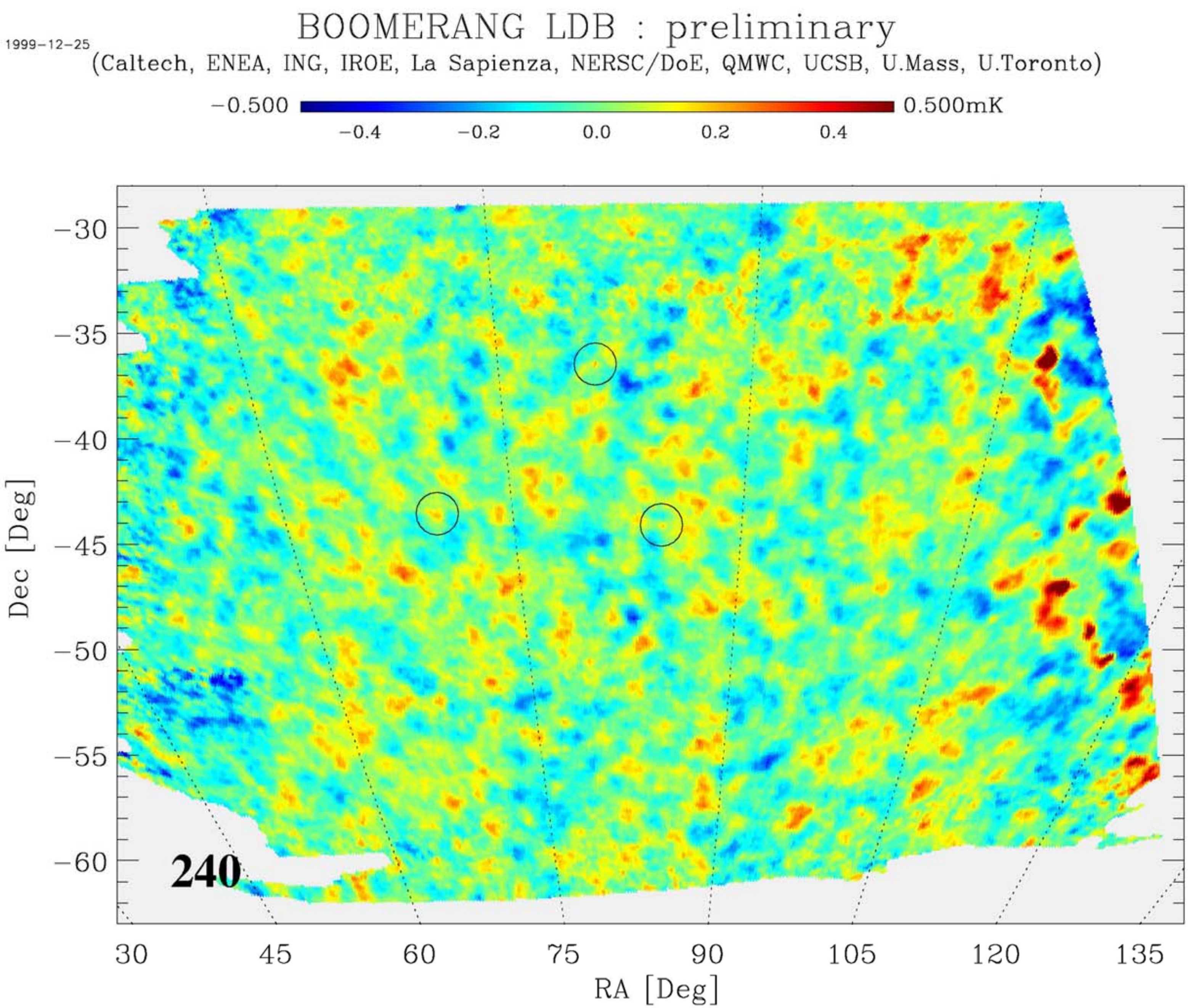
Test B:

- The spectral test shows that the structures present in the maps are CMB anisotropies. In fact:
- The maps at different frequencies are plotted in thermodynamic temperature units for the CMB (mK) so that structures with the spectrum of the CMB will appear the same at all frequencies.
- ***Structures with the spectrum of the CMB are evident in the maps and have high S/N at 90, 150, 240 GHz***. The dust monitor channel at 410 GHz shows no CMB and ***very little dust***.

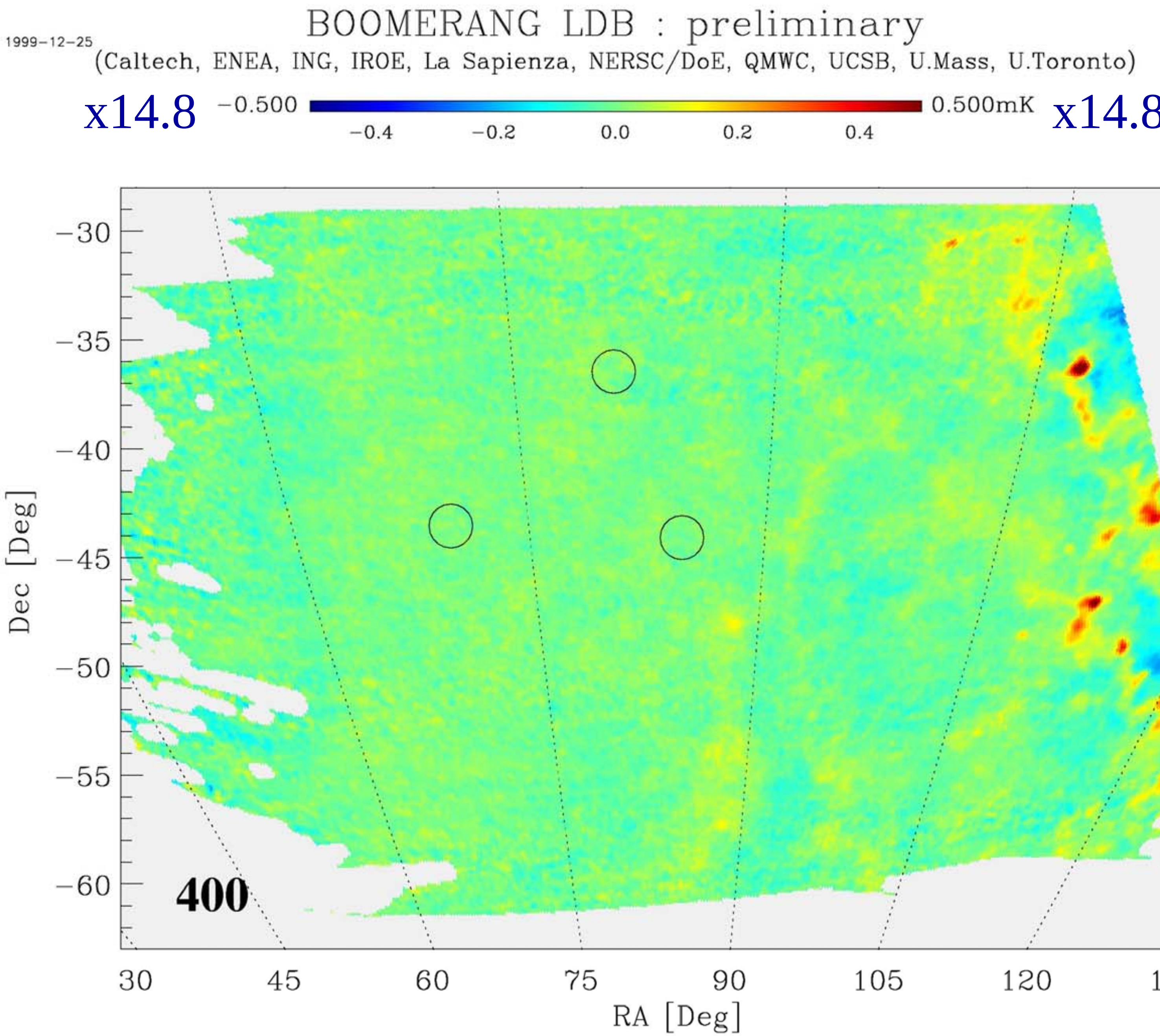

BOOMERANG LDB : preliminary
1999-12-25
(Caltech, ENEA, ING, IROE, La Sapienza, NERSC/DoE, QMWC, UCSB, U.Mass, U.Toronto)
−0.500
0.500mK
−0.4
−0.2
0.0
0.2
0.4
−30
−35
−40
−45
−50
−55
−60
Dec [Deg]
30
45
60
75
90
105
120
135
RA [Deg]
90



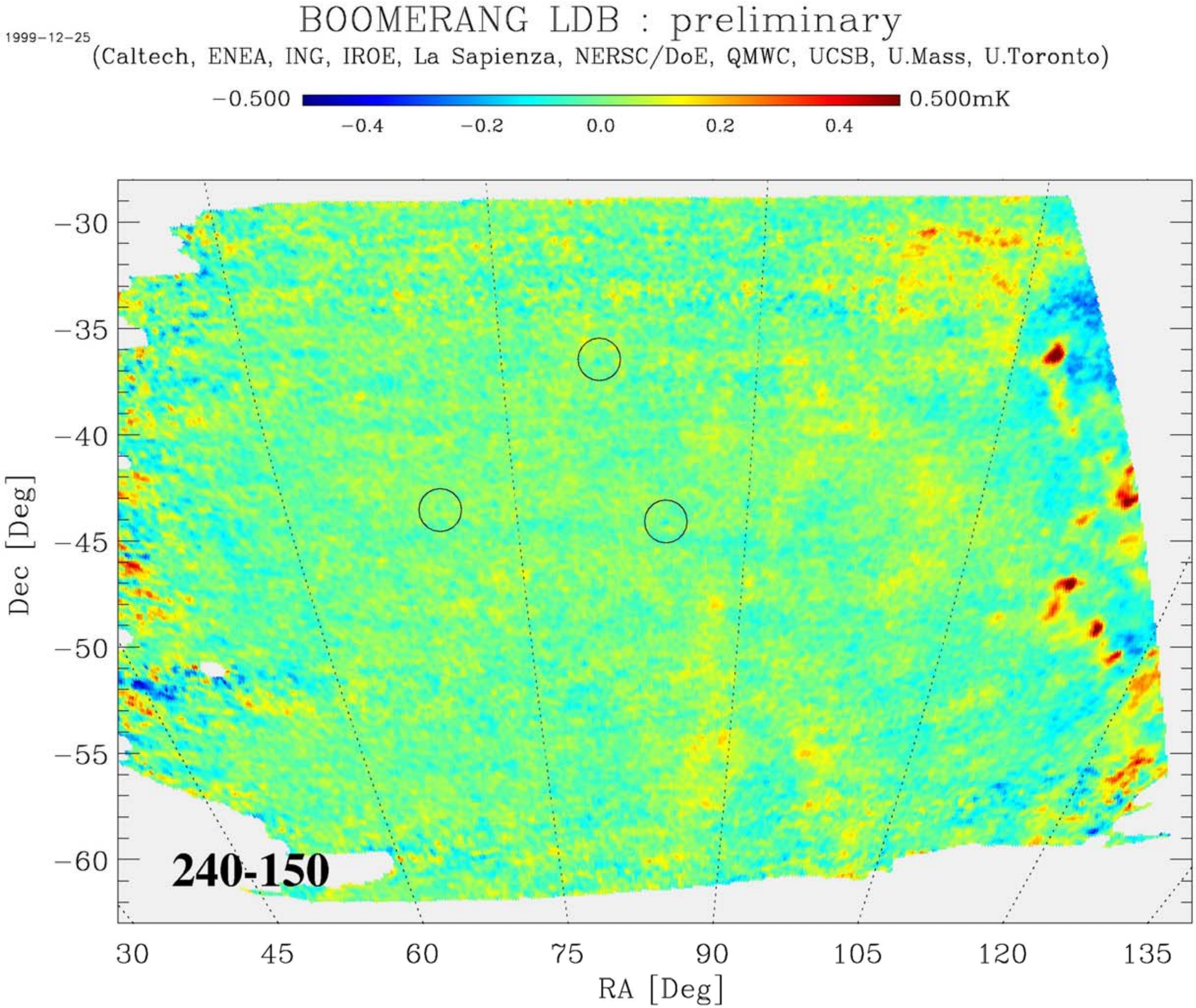
BOOMERANG LDB : preliminary
1999-12-25
(Caltech, ENEA, ING, IROE, La Sapienza, NERSC/DoE, QMWC, UCSB, U.Mass, U.Toronto)
−0.500
0.500mK
−0.4
−0.2
0.0
0.2
0.4
−30
−35
−40
−45
−50
−55
−60
Dec [Deg]
150
30
45
60
75
90
105
120
135
RA [Deg]

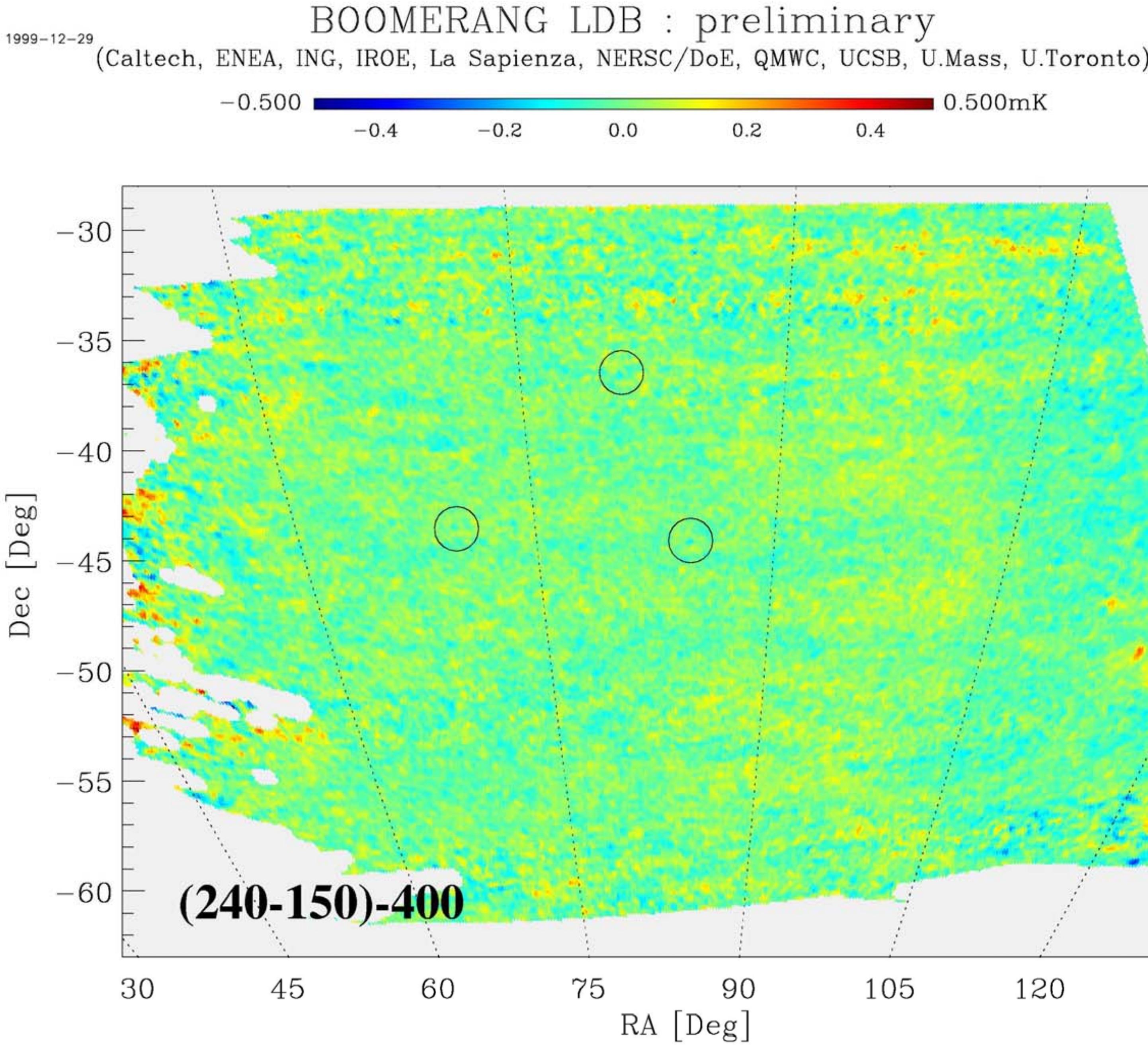


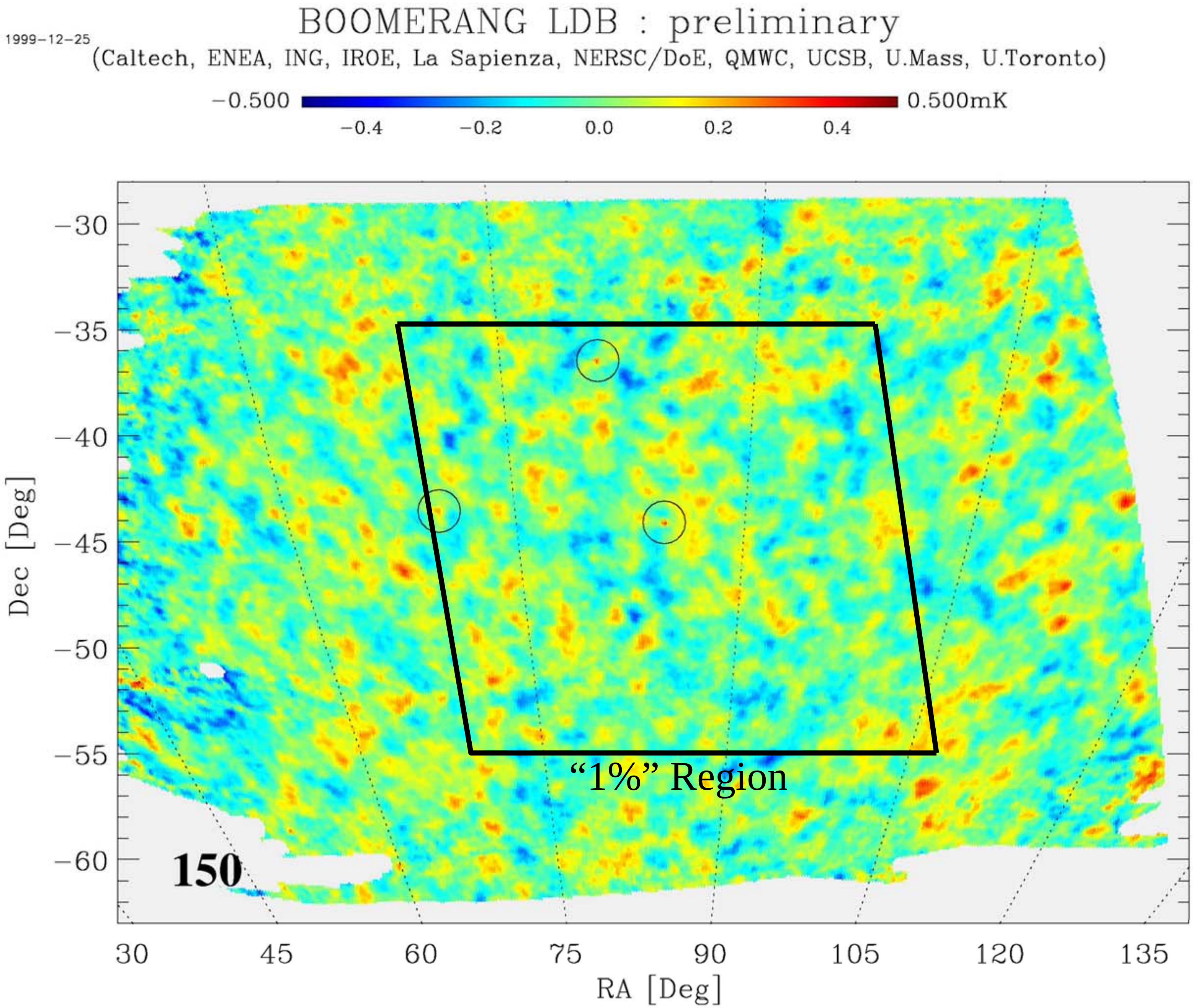
1999-12-25
BOOMERANG LDB : preliminary
(Caltech, ENEA, ING, IROE, La Sapienza, NERSC/DoE, QMWC, UCSB, U.Mass, U.Toronto)
−0.500
0.500mK
−0.4
−0.2
0.0
0.2
0.4
−30
−35
−40
−45
−50
−55
−60
Dec [Deg]
30
45
60
75
90
105
120
135
RA [Deg]
240



BOOMERANG LDB : preliminary
1999-12-25
(Caltech, ENEA, ING, IROE, La Sapienza, NERSC/DoE, QMWC, UCSB, U.Mass, U.Toronto)
x14.8
−0.500
0.500mK
x14.8
−0.4
−0.2
0.0
0.2
0.4
−30
−35
−40
−45
−50
−55
−60
Dec [Deg]
400
30
45
60
75
90
105
120
135
RA [Deg]


1999-12-25
BOOMERANG LDB : preliminary
(Caltech, ENEA, ING, IROE, La Sapienza, NERSC/DoE, QMWC, UCSB, U.Mass, U.Toronto)
−0.500
0.500mK
−0.4
−0.2
0.0
0.2
0.4
−30
−35
−40
−45
−50
−55
−60
Dec [Deg]
240-150
30
45
60
75
90
105
120
135
RA [Deg]


1999-12-29
BOOMERANG LDB : preliminary
(Caltech, ENEA, ING, IROE, La Sapienza, NERSC/DoE, QMWC, UCSB, U.Mass, U.Toronto)
−0.500
0.500mK
−0.4
−0.2
0.0
0.2
0.4
−30
−35
−40
−45
−50
−55
−60
Dec [Deg]
30
45
60
75
90
105
120
135
RA [Deg]
(240-150)-400


1999-12-25
BOOMERANG LDB : preliminary
(Caltech, ENEA, ING, IROE, La Sapienza, NERSC/DoE, QMWC, UCSB, U.Mass, U.Toronto)
−0.500
0.500mK
−0.4
−0.2
0.0
0.2
0.4
−30
−35
−40
−45
−50
−55
−60
Dec [Deg]
30
45
60
75
90
105
120
135
RA [Deg]
"1%" Region
150

Are these genuine CMB fluctuations ?

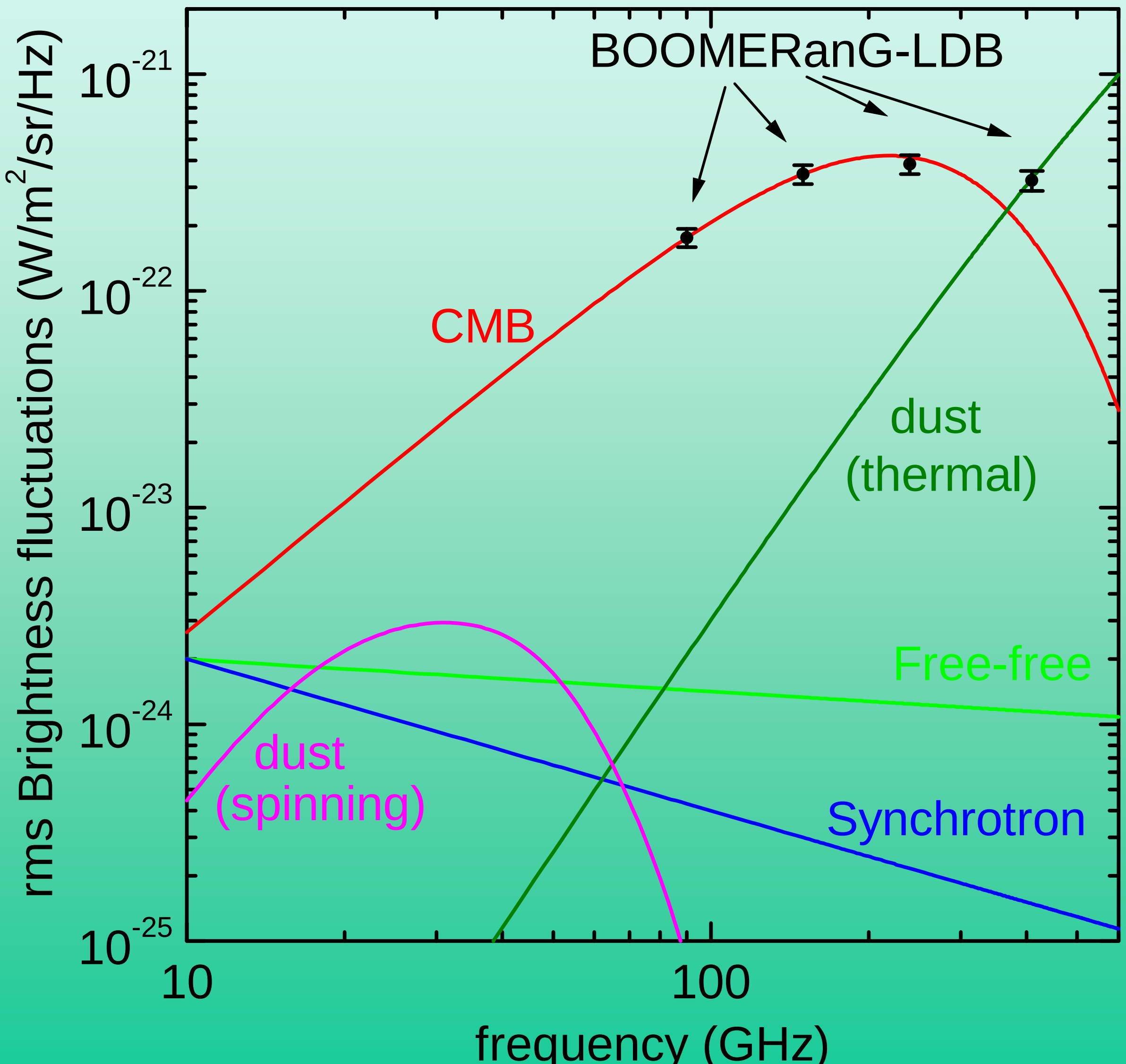
The rms fluctuations

$$\Delta T_{rms} = \{\Sigma_\ell (2\ell+1)\, \mathbf{c}_\ell\, w_\ell /4\pi\}^{1/2}$$

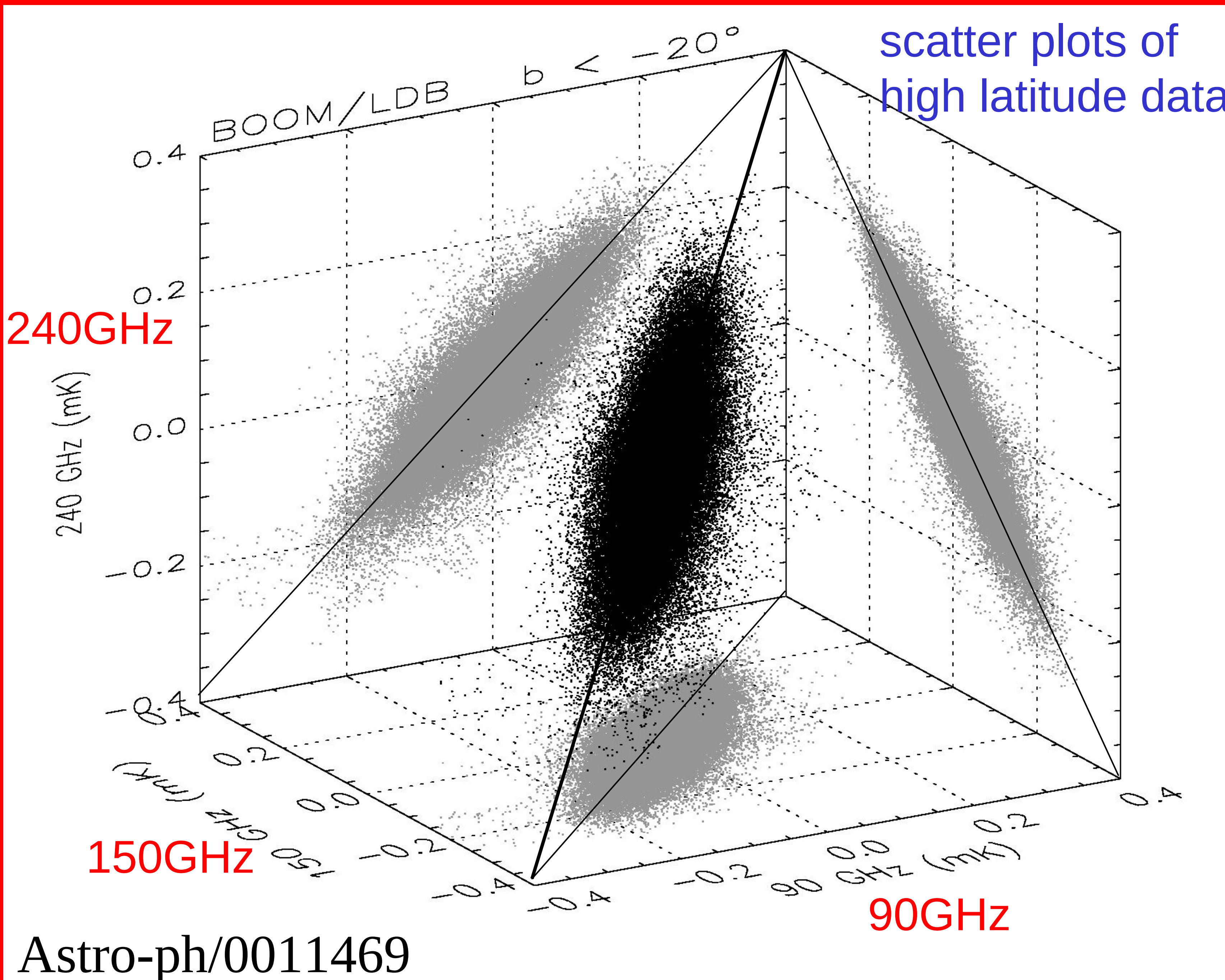
are spectrally distributed as the derivative of a 2.73K blackbody. All other astrophysical sources of confusion do not fit the data.

This means that **the bulk** of the observed fluctuations has a cosmological origin.

2-D and 3-D scatter plots confirm this conclusion



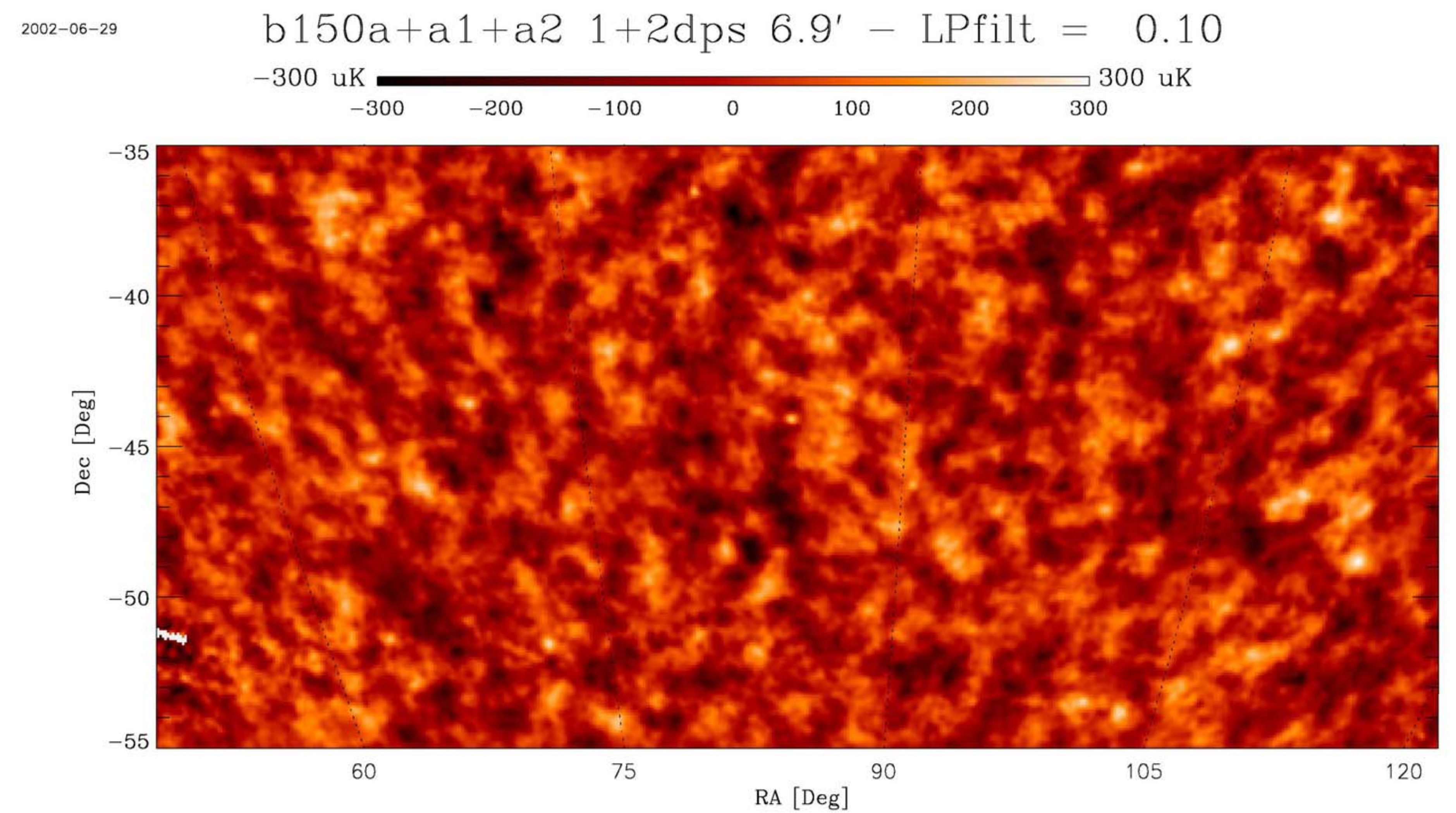
Astro-ph/0011469



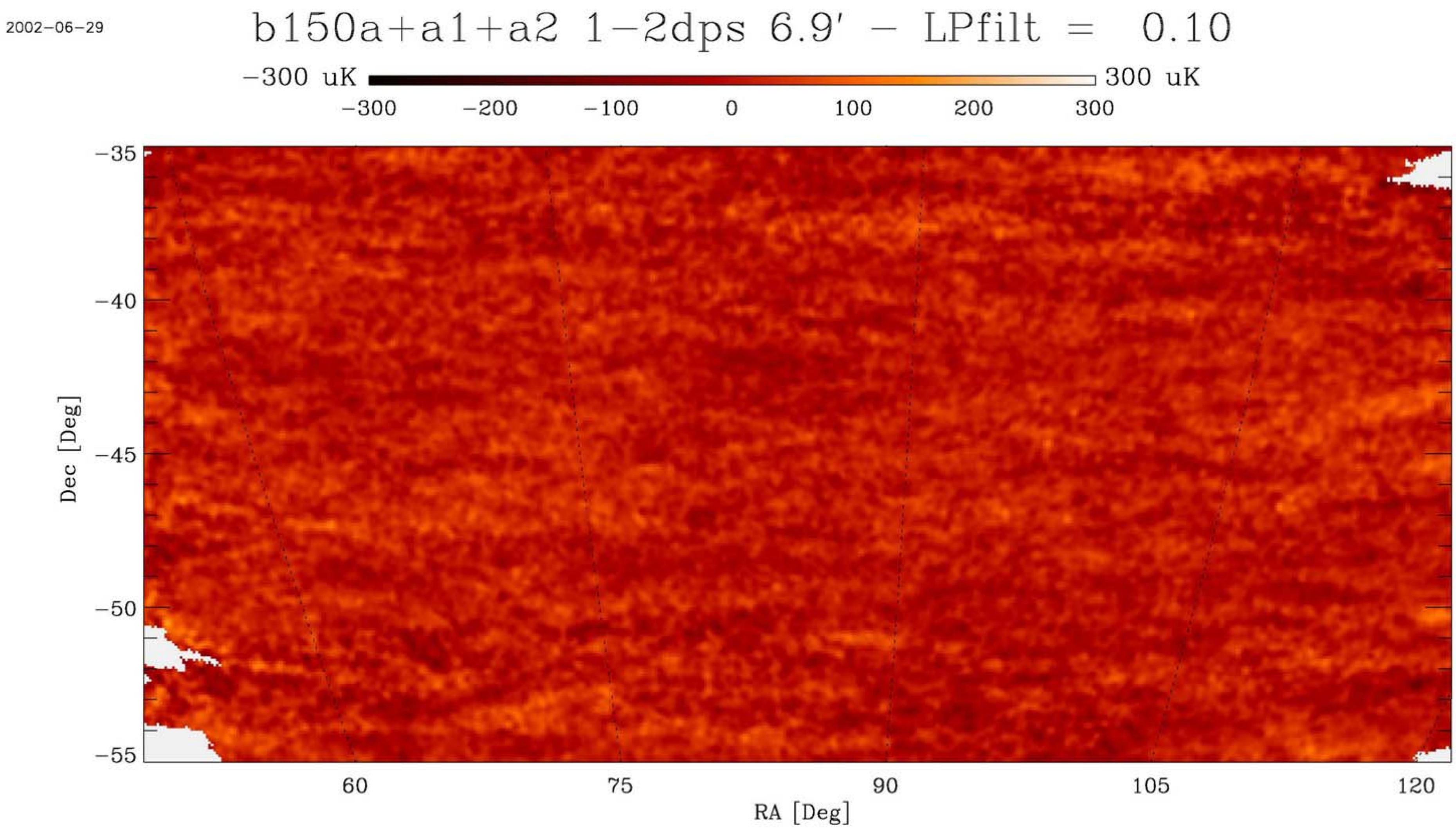
scatter plots of
high latitude data
BOOM/LDB b < −20°
240GHz
240 GHz (mK)
0.4
0.2
0.0
−0.2
−0.4
150GHz
150 GHz (mK)
0.4
0.2
0.0
−0.2
−0.4
90GHz
90 GHz (mK)
−0.4
−0.2
0.0
0.2
0.4
Astro-ph/0011469

Test C

- We have a powerful tool: data were taken at two different scan speeds: 1 dps and 2 dps.
- At 2dps the sky signal is converted into an electrical signal at twice the frequency, while instrument related effects (transfer function, 1/f noise, microphonic lines etc.) remain at the same frequency.
- For the same detectors compare maps from data taken at 1dps and from data taken at 2 dps



1 dps map + 2 dps map



1 dps map - 2 dps map

TABLE 2

INTERNAL CONSISTENCY TESTS.

Test	Reduced χ^2	$P_>$
B150A 1dps - 2dps	0.91	0.57
B150A1 1dps - 2dps	0.92	0.56
B150A2 1dps - 2dps	1.04	0.41
B150B1 1dps - 2dps	2.73	7×10^{-5}
B150B2 1dps - 2dps	0.60	0.91
4 Ch 1dps - 2dps	1.80	0.02
4 Ch Left - Right	1.21	0.24
(A+A1) - (A2+B2)	0.61	0.90

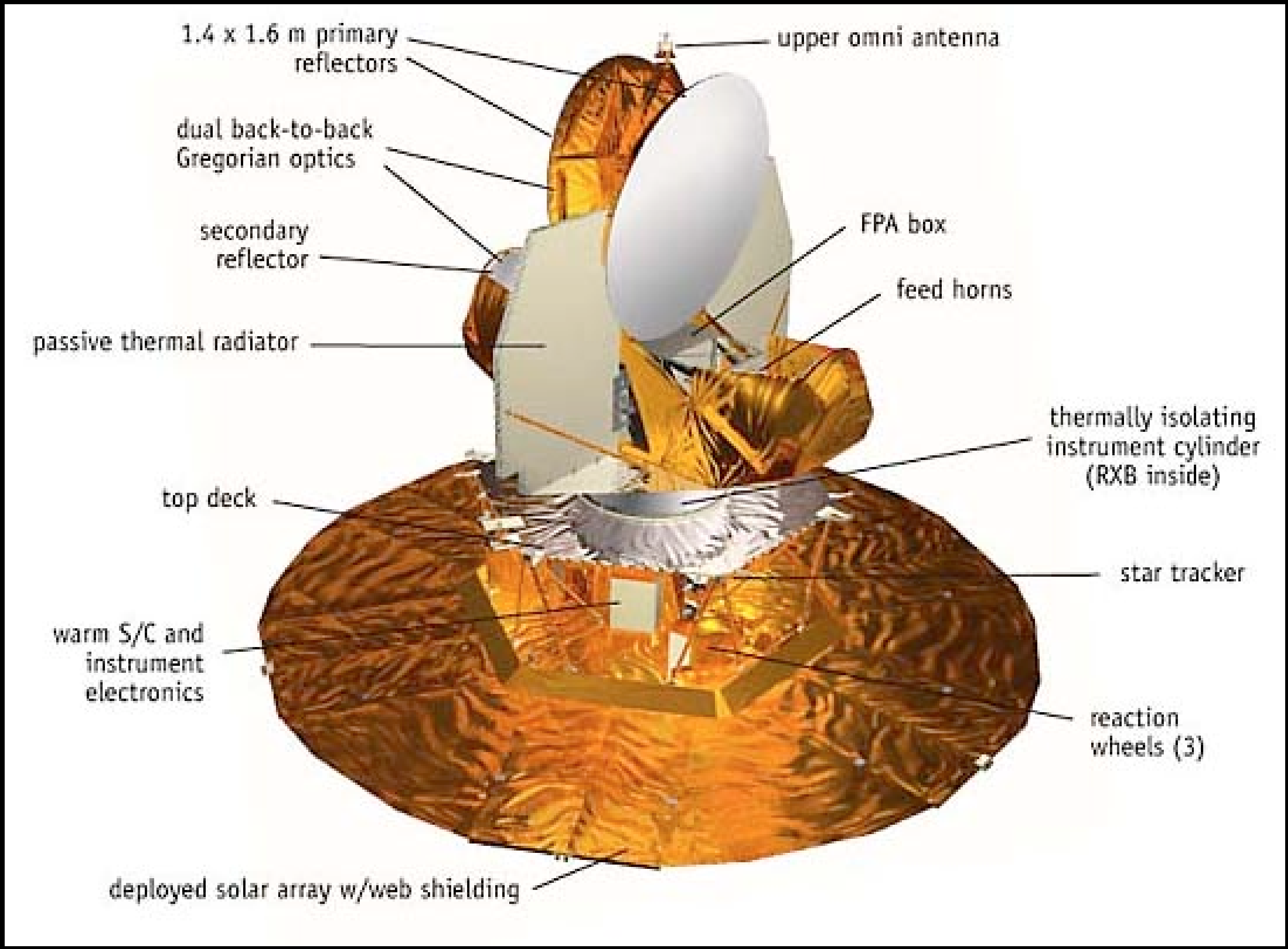
NOTE.—Reduced χ^2 with 19 degrees of freedom for internal symmetry tests for BOOMERANG. $P_>$ gives the probability of obtaining a χ^2 larger than the one reported. B150B1 fails the test, and is not used in the analysis. The '4 Ch' entries combine maps from B150A, B150A1, B150A2, and B150B2. The 1-2 dps 4 Ch spectrum fails marginally. This is dominated by 4 bins centered between $l = 150$ and $l = 300$. The mean signal of these 4 bins is $50\mu K^2$, compared to a signal over the same range of $\approx 4000\mu K^2$(see Table 3).

Test F:

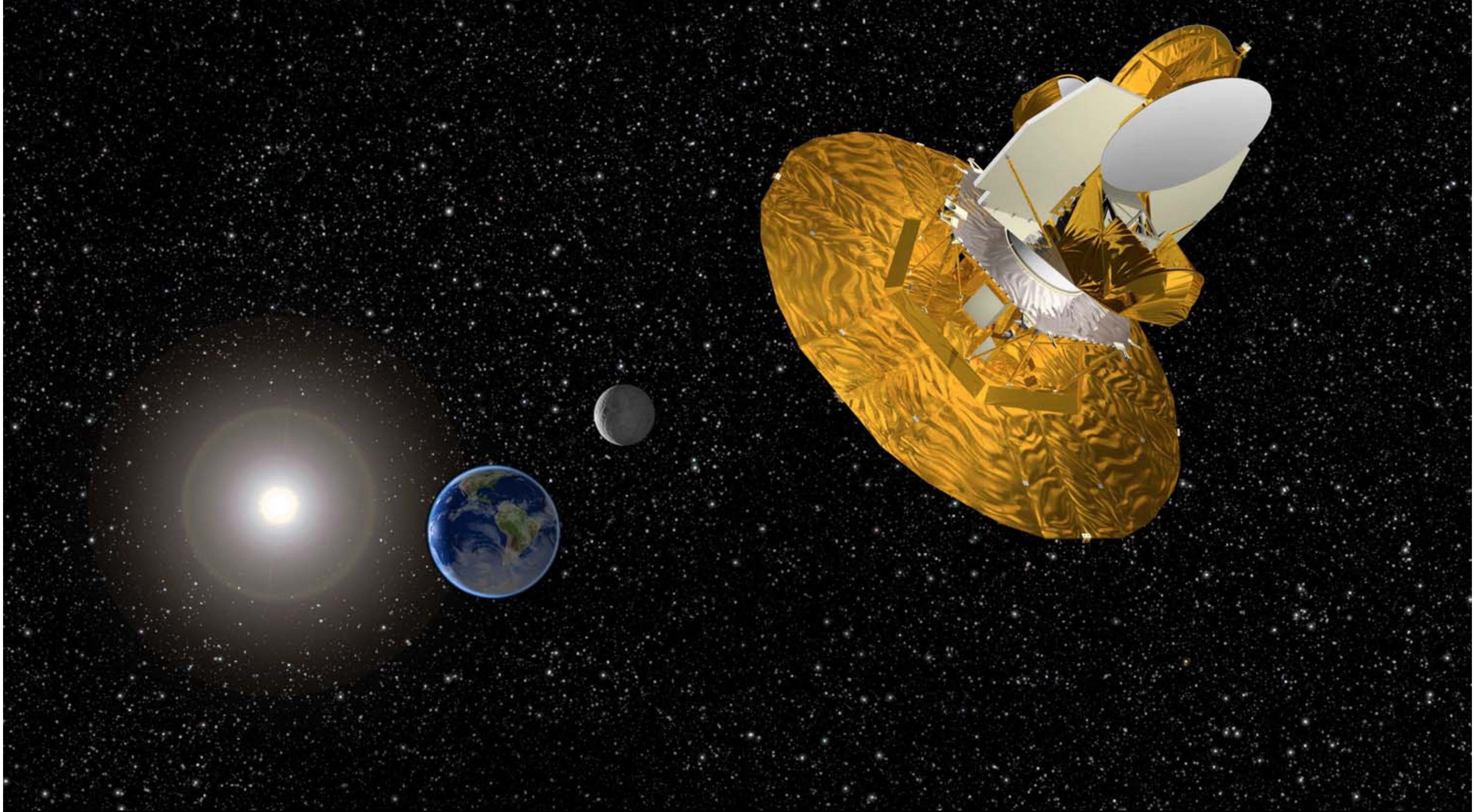
- BOOMERanG vs. WMAP

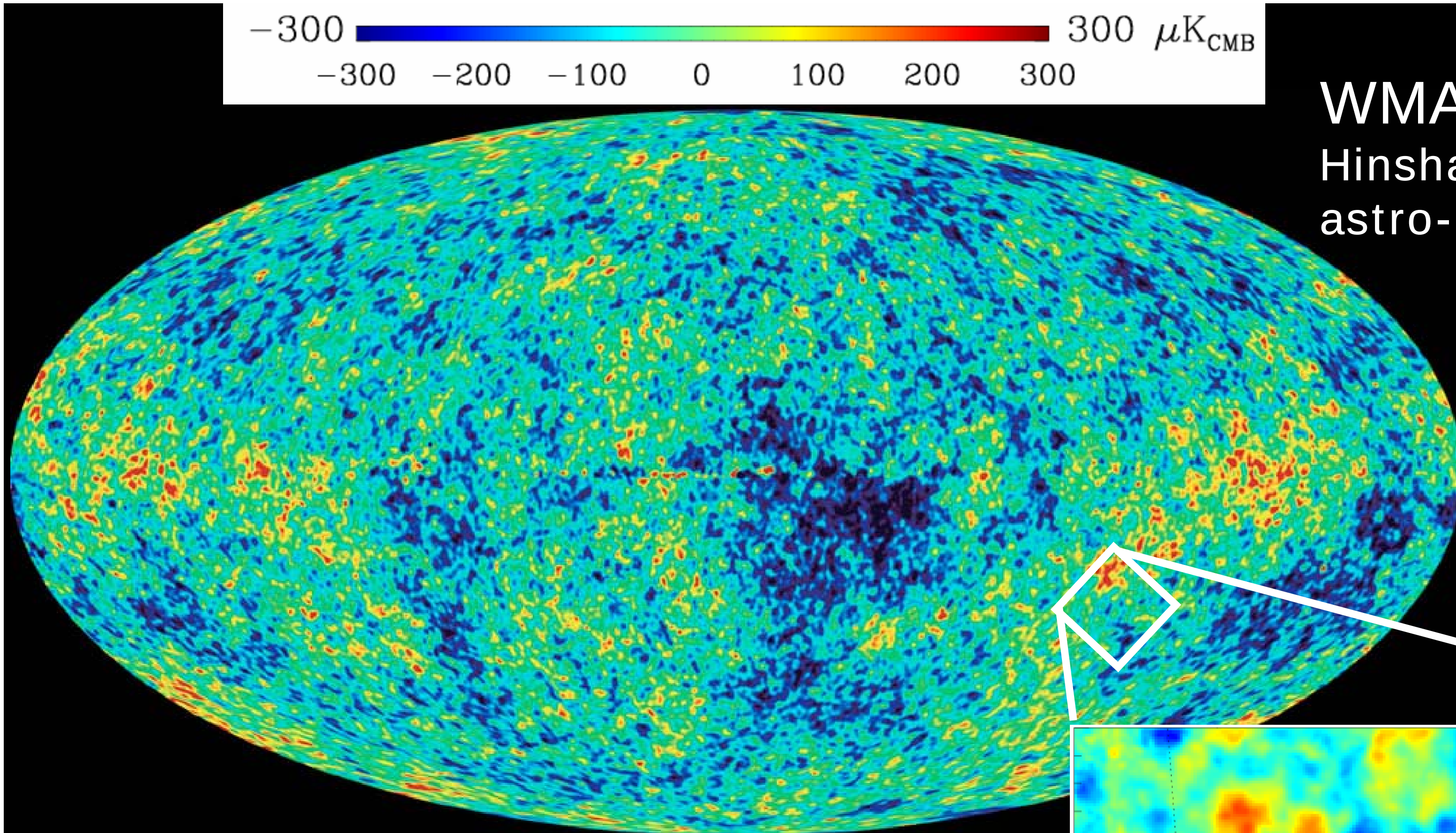
WMAP (2002)

Wilkinson Microwave Anisotropy Probe



WMAP in L_2 : sun, earth, moon are all well behind the solar shield.

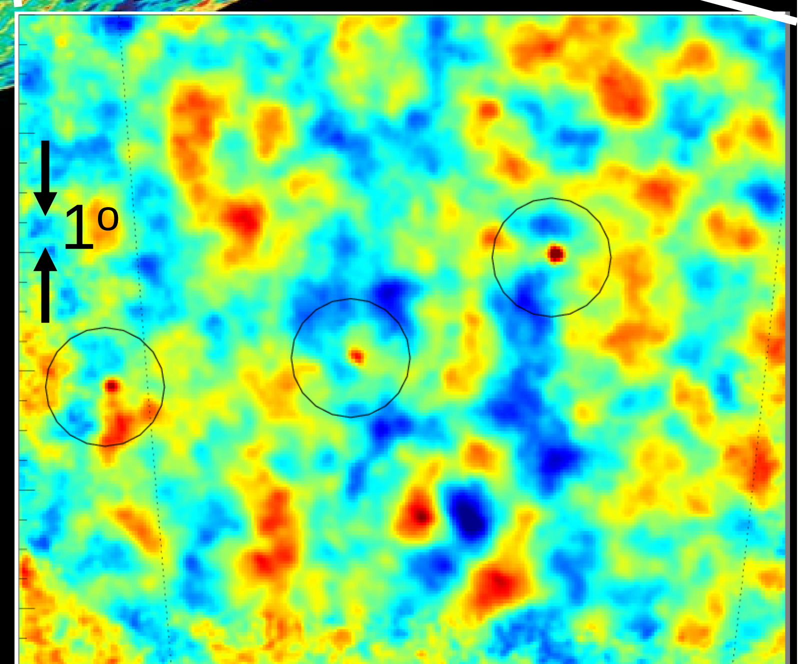


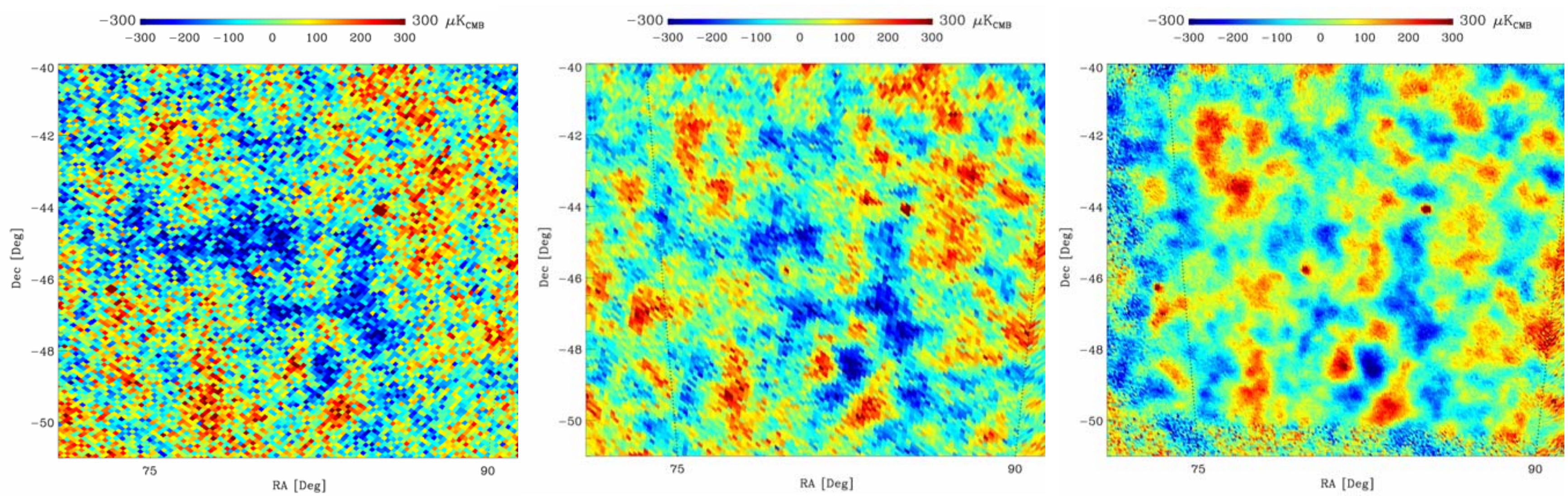


WMAP
Hinshaw et al. 2006
astro-ph/0603451

Detailed Views of the Recombination Epoch (z=1088, 13.7 Gyrs ago)

BOOMERanG
Masi et al. 2005
astro-ph/0507509





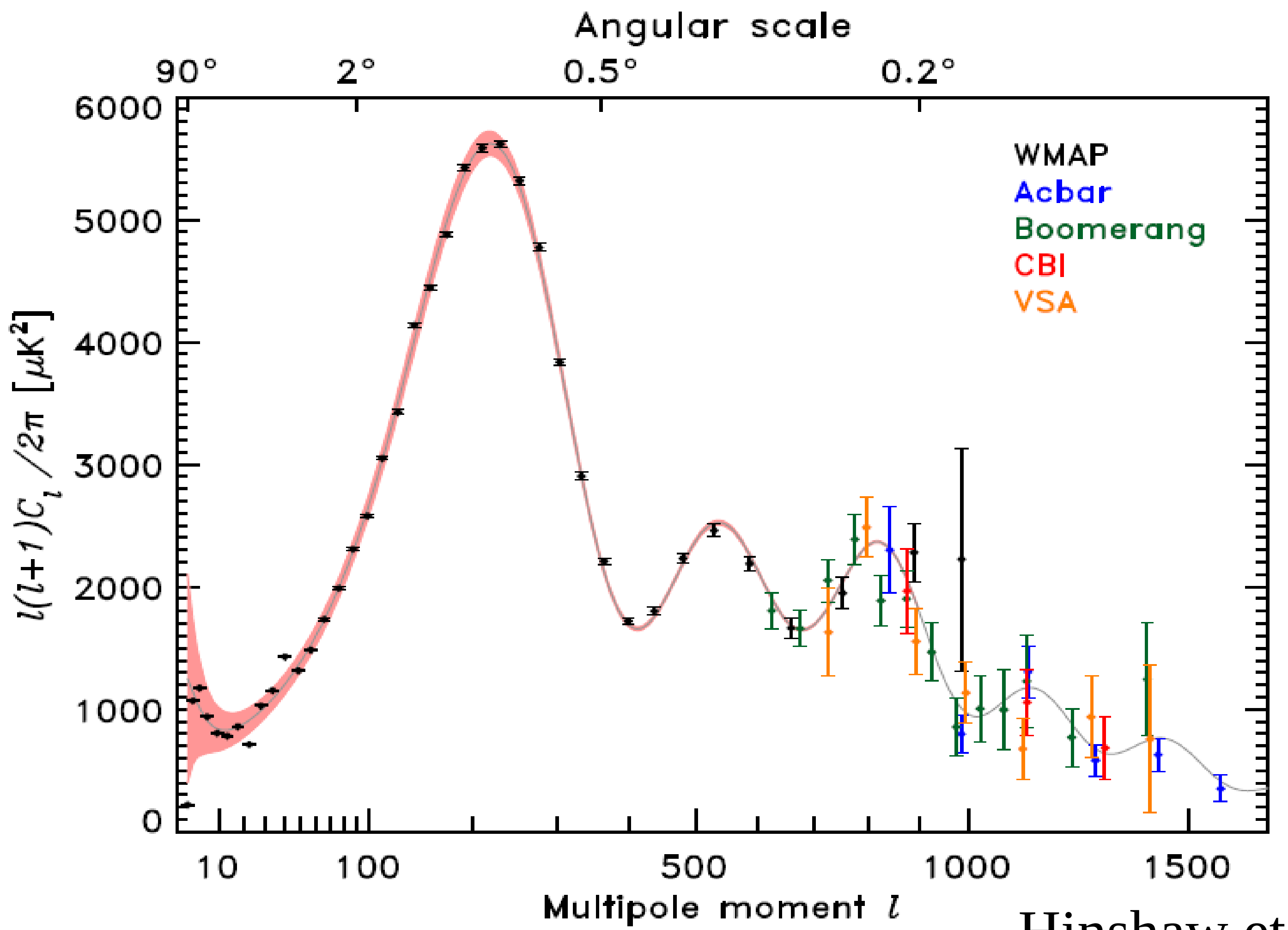
WMAP 3 years 23-94 GHz | BOOMERanG-98 145 GHz | BOOMERanG-03 145 GHz

The consistency of the maps from three *independent* experiments, working at very different frequencies and with very different mesurement methods, is the best evidence that the faint structure observed

•*is not due to instrumental artifacts*

•*has exactly the spectrum of CMB anisotropy, so it is not due to foreground emission*

•The comparison also shows the *extreme sensitivity of cryogenic bolometers* operated at balloon altitude (the B03 map is the result of 5 days of observation)



Hinshaw et al. 2006

Fig. 18.— The *WMAP* three-year power spectrum (in black) compared to other recent measurements of the CMB angular power spectrum, including Boomerang (Jones et al. 2005), Acbar (Kuo et al. 2004), CBI (Readhead et al. 2004), and VSA (Dickinson et al. 2004). For clarity, the $l < 600$ data from Boomerang and VSA are omitted; as the measurements are consistent with *WMAP*, but with lower weight. These data impressively confirm the turnover in the 3rd acoustic peak and probe the onset of Silk damping. With improved sensitivity on sub-degree scales, the *WMAP* data are becoming an increasingly important calibration source for high-resolution experiments.

CMB Polarization – Why ?

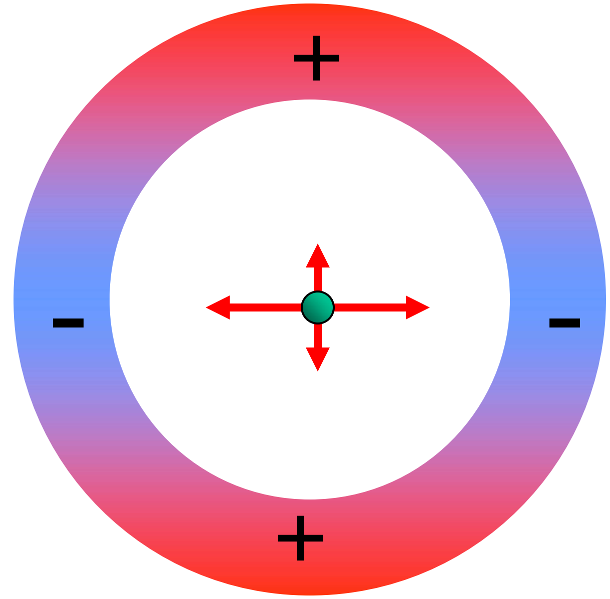
- An **inflation** phase at **E=10^{16}–10^{15} GeV** (t=10^{-36}-10^{-33} s) is currently the most popular scenario to explain
  - The origin of our universe
  - The geometry of our universe
  - The origin and morphology of structures in our universe
  - The lack of defects, and the smoothness of the CMB at super-horizon scales.
- Inflation is a **predictive** theory:
  1. Any initial curvature is flattened by the huge expansion: we expect an Euclidean universe.
  2. Adiabatic, gaussian density perturbations are produced from quantum fluctuations. This is the physical origin for structures in the Universe.
  3. The power spectrum of scalar perturbations is approximately scale invariant, $P(k)=Ak^{n-1}$ with n slightly less than 1.
  4. Tensor perturbations produce a background of primordial gravitational waves (PGW)
- 1.,2.,3. have been confirmed already by measurements of CMB anisotropy
- 4. can be tested measuring CMB polarization

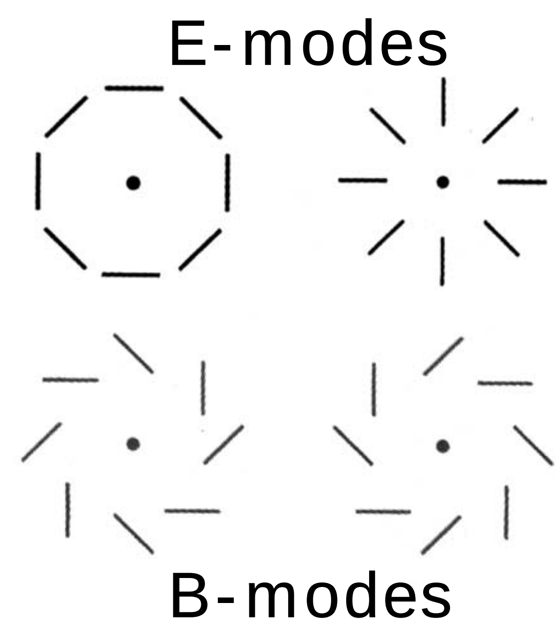
CMB Polarization – Why ?

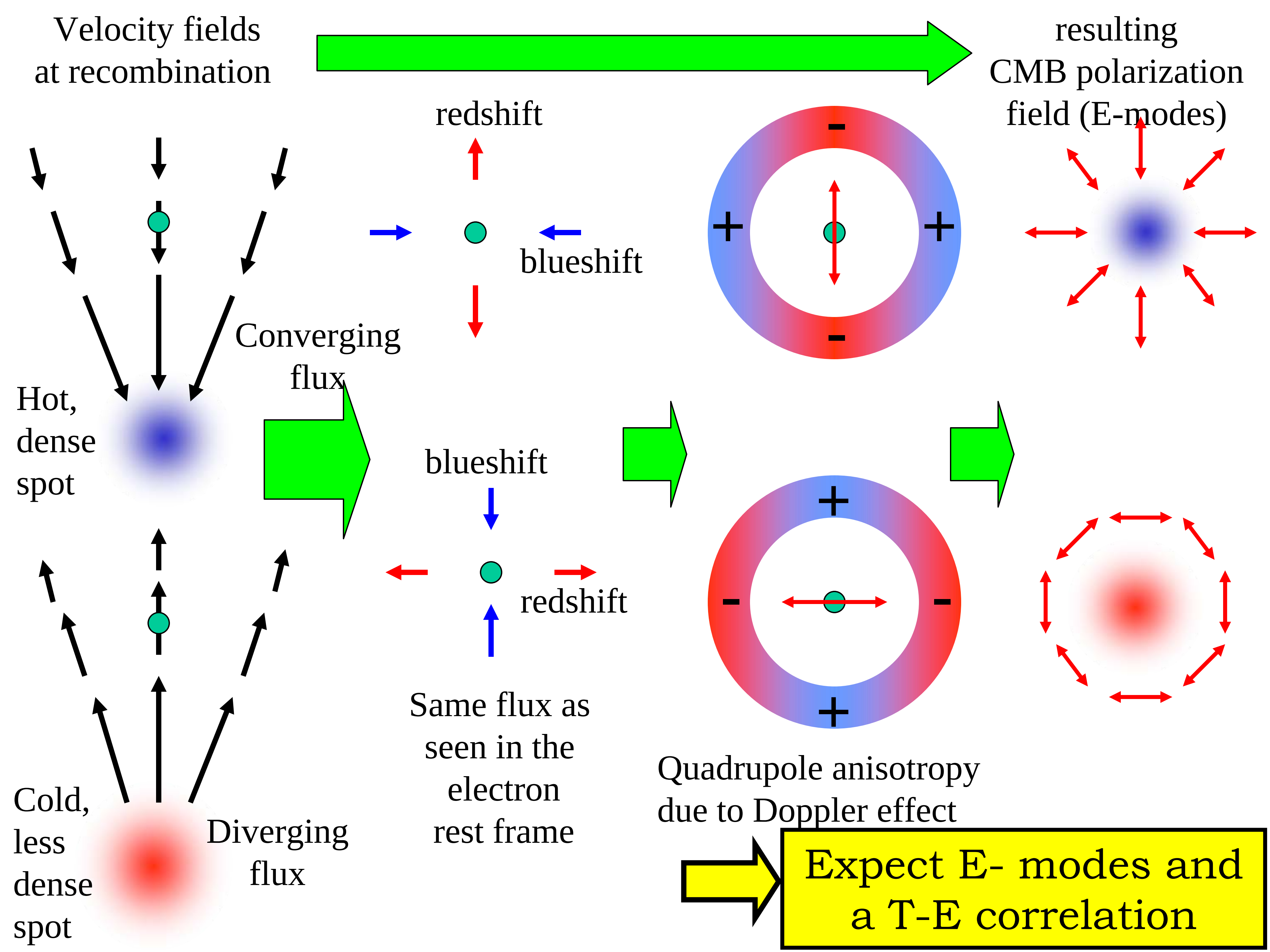
- Linear Polarization of CMB photons is induced via Thomson scattering by quadrupole anisotropy at recombination (z=1100, t =1.2x10^{13}s).

- In turn, quadrupole anisotropy is induced by
  - Density perturbations (*scalar* relics of inflation) producing a curl-free polarization vectors field (**E-modes**)
  - Gravitational waves (*tensor* relics of inflation) producing both curl-free and curl polarization fields (**B-modes**)

- No other sources for a curl polarization field of the CMB at large angular scales:
- **B-modes are a clear signature of inflation**.






Velocity fields
at recombination
resulting
CMB polarization
field (E-modes)
redshift
blueshift
Converging
flux
Hot,
dense
spot
blueshift
redshift
Same flux as
seen in the
electron
rest frame
Cold,
less
dense
spot
Diverging
flux
Quadrupole anisotropy
due to Doppler effect
Expect E- modes and
a T-E correlation

E-modes & B-modes

Spin-2 quantity Spin-2 basis

$$(Q \pm iU)(\vec{n}) = \sum_{\ell,m} \left(a^E_{\ell m} \pm i a^B_{\ell m}\right) {}_{\pm 2}Y_{\ell m}(\vec{n})$$

- From the measurements of the Stokes Parameters $\boldsymbol{Q}$ and $\boldsymbol{U}$ of the linear polarization field we can recover both irrotational and rotational a_{lm} by means of modified Legendre transforms:

E-modes produced by scalar and tensor perturbations

$$a^E_{\ell m} = \frac{1}{2} \int d\Omega W(\vec{n}) \left[(Q + iU)(\vec{n})_{+2} Y_{\ell m}(\vec{n}) + (Q - iU)(\vec{n})_{-2} Y_{\ell m}(\vec{n}) \right]$$

B-modes produced **only** by tensor perturbations

$$a^B_{\ell m} = \frac{1}{2i} \int d\Omega W(\vec{n}) \left[(Q + iU)(\vec{n})_{+2} Y_{\ell m}(\vec{n}) - (Q - iU)(\vec{n})_{-2} Y_{\ell m}(\vec{n}) \right]$$

B-modes from P.G.W.

- The amplitude of this effect is very small, but depends on the Energy scale of inflation. In fact the amplitude of tensor modes normalized to the scalar ones is:

$$R = \left(\frac{T}{S}\right)^{1/4} \equiv \left(\frac{C_2^{GW}}{C_2^{Scalar}}\right)^{1/4} \cong \frac{V^{1/4}}{3.7\times 10^{16}\,\mathrm{GeV}}$$

Inflation potential

- and

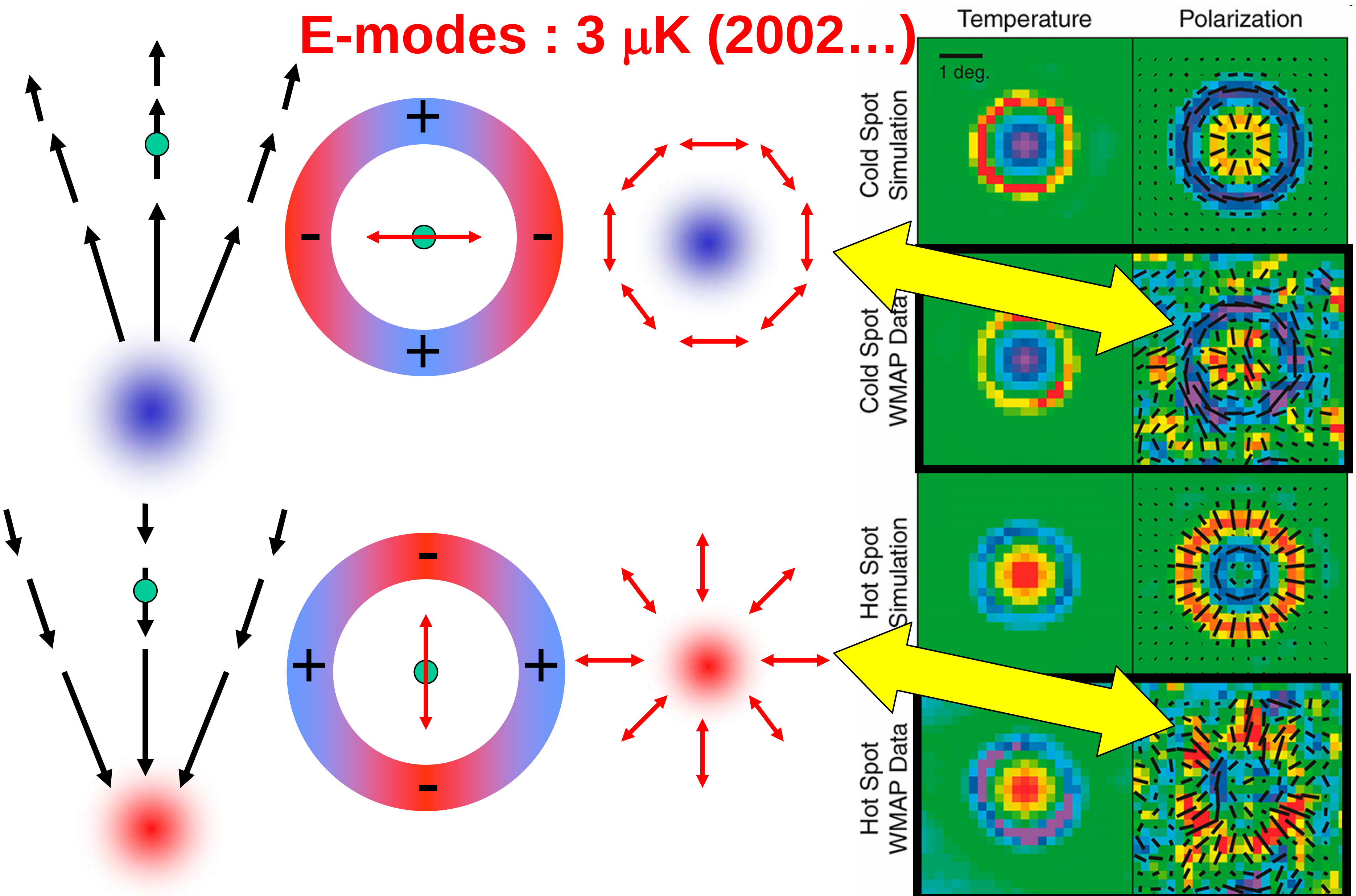
$$\sqrt{\frac{\ell(\ell+1)}{2\pi} c_{\ell\max}^{B}} \cong 0.1\mu K\left[\frac{V^{1/4}}{2\times 10^{16}\,\mathrm{GeV}}\right]$$

- There are theoretical arguments to expect that the energy scale of inflation is close to the scale of GUT i.e. around 10^{16} GeV.
- The measurement of B-modes is a good way to investigate fundamental physics at extremely high energies.

The signal is extremely weak

- The current upper limit on anisotropy at large scales gives R<0.5 (at 2σ)
- A competing effect is lensing of E-modes, which is important at large multipoles.
- Nobody really knows how to detect this.
  - Pathfinder experiments are needed
- Whatever smart, ambitious experiment we design to detect the B-modes:
  - It needs to be extremely sensitive
  - It needs an extremely careful control of systematic effects
  - It needs careful control of foregrounds
  - It will need independent experiments with orthogonal systematic effects.
- A lot has been done, but there is still a long way to go: ...

E-modes : 3 μK (2002…)



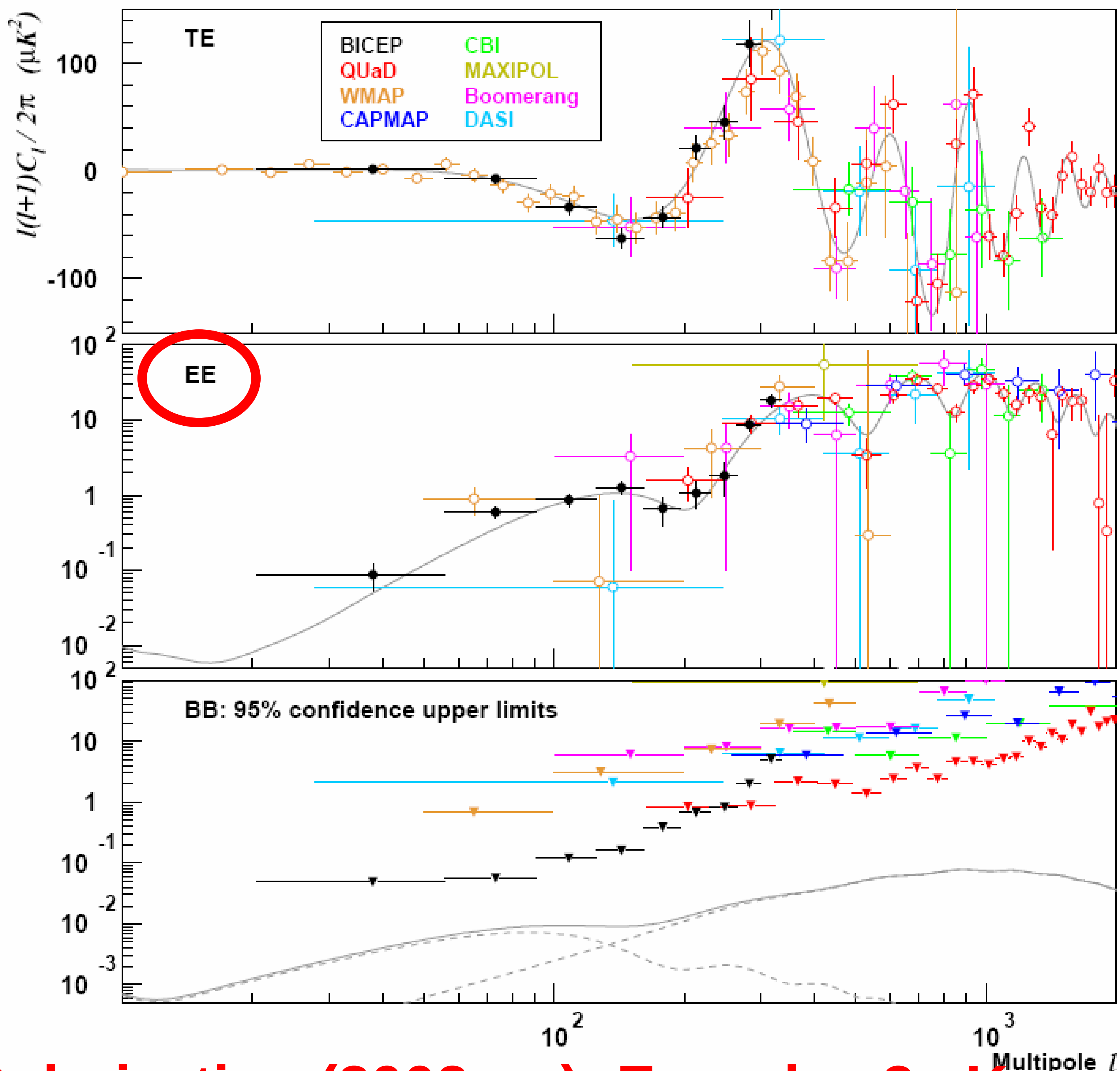
Velocity field near density fluctuation

Resulting anisotropy seen by e-

Resulting polarization pattern

Komatsu et al. 2010 – astro-ph/1001.4538

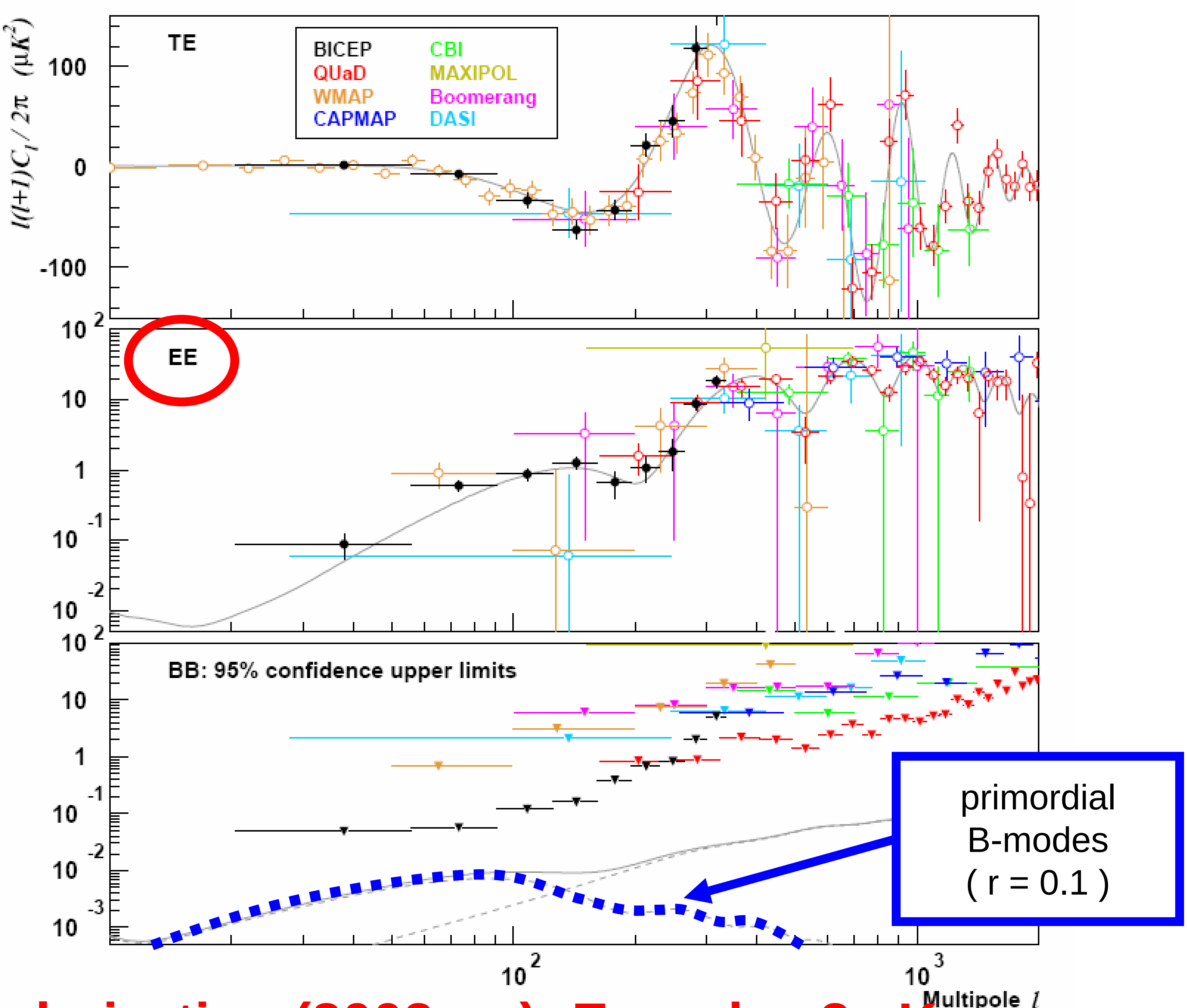
WMAP7 measured data (stacked)



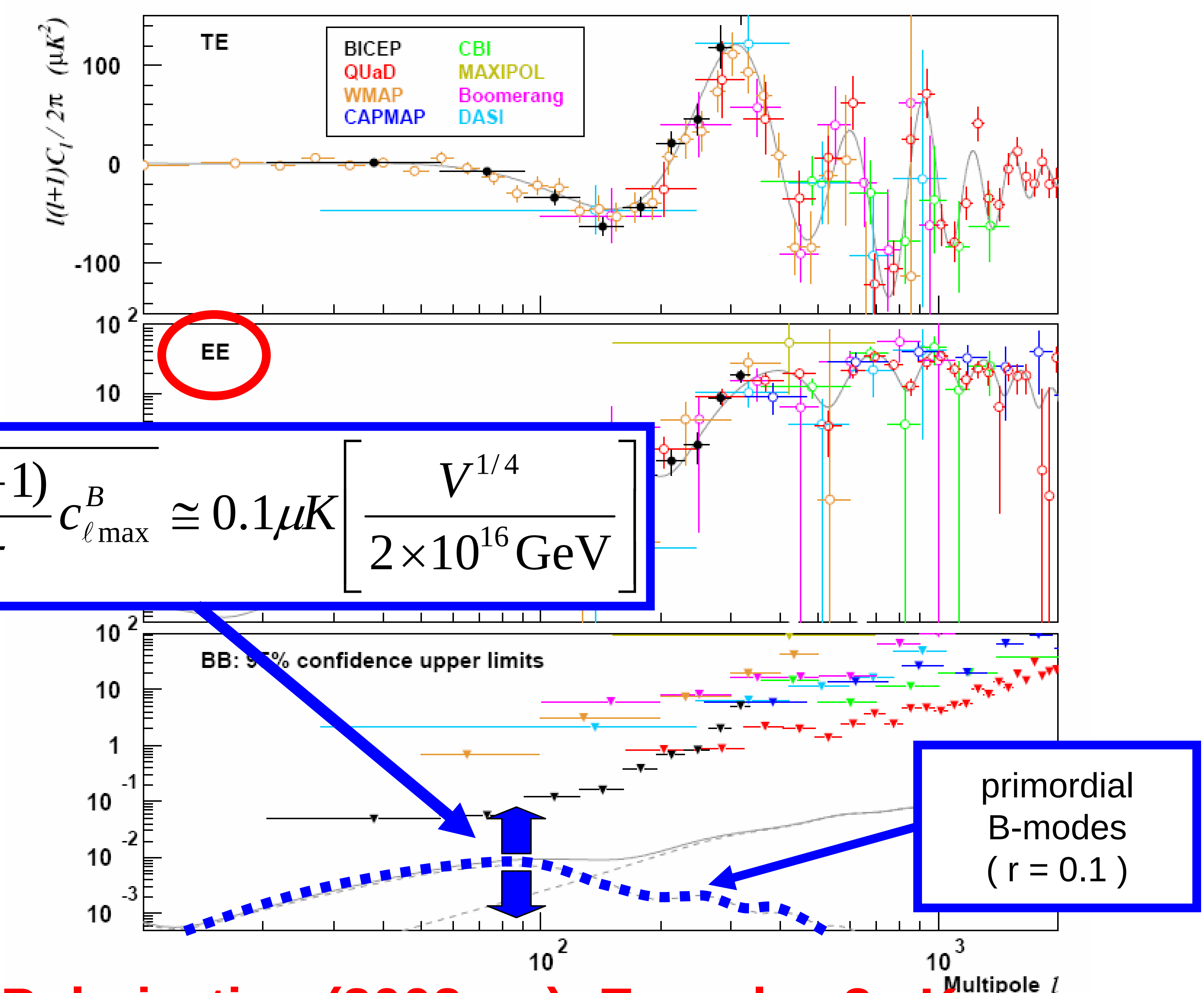
Chiang et al. 2010

CMB Polarization (2002 …): E-modes 3 μK

Chiang et al. 2010

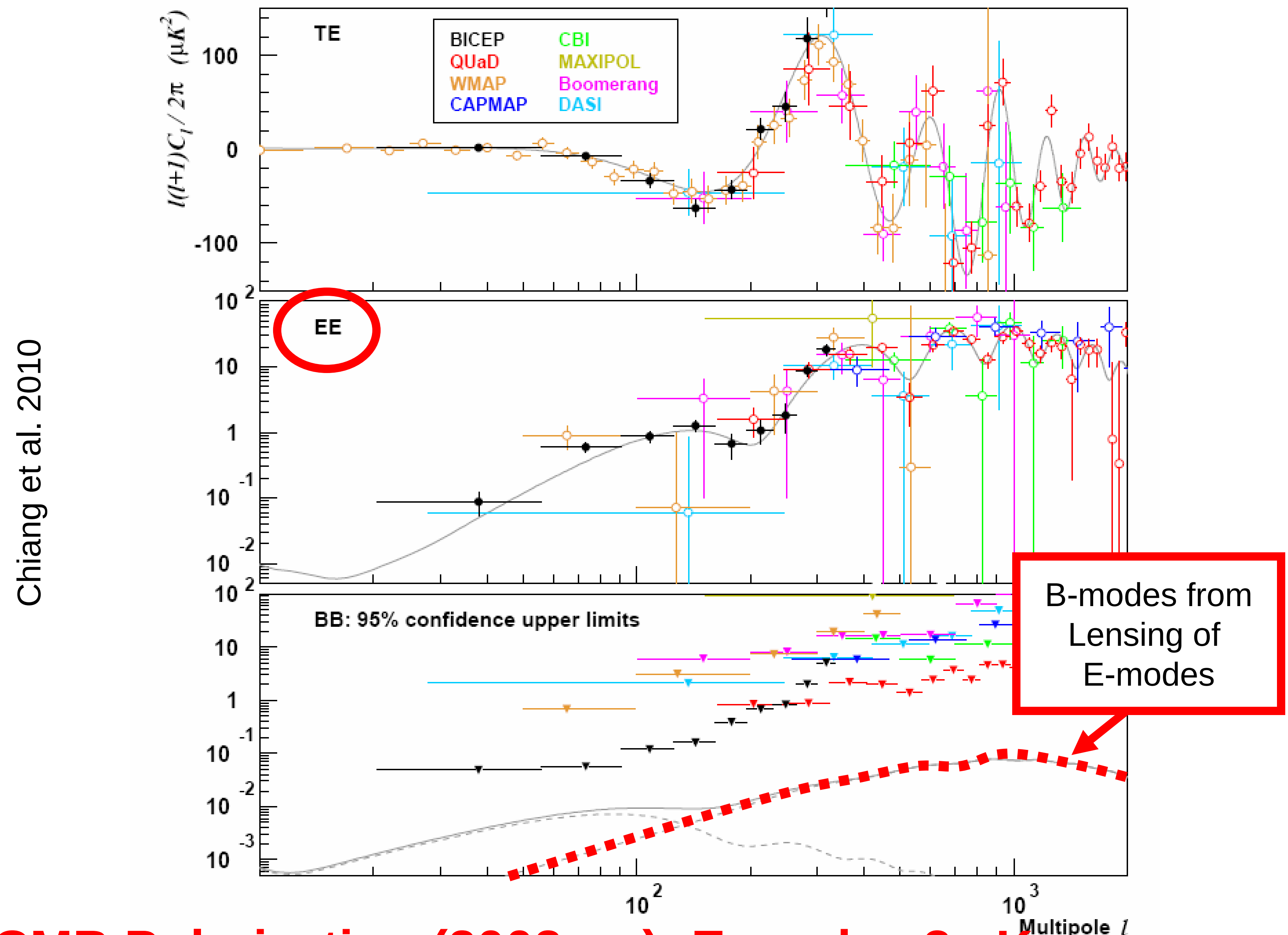


CMB Polarization (2002 …): E-modes 3 μK



$$\sqrt{\frac{\ell(\ell+1)}{2\pi} c^B_{\ell\,\max}} \cong 0.1\mu K \left[\frac{V^{1/4}}{2\times 10^{16}\,\mathrm{GeV}} \right]$$

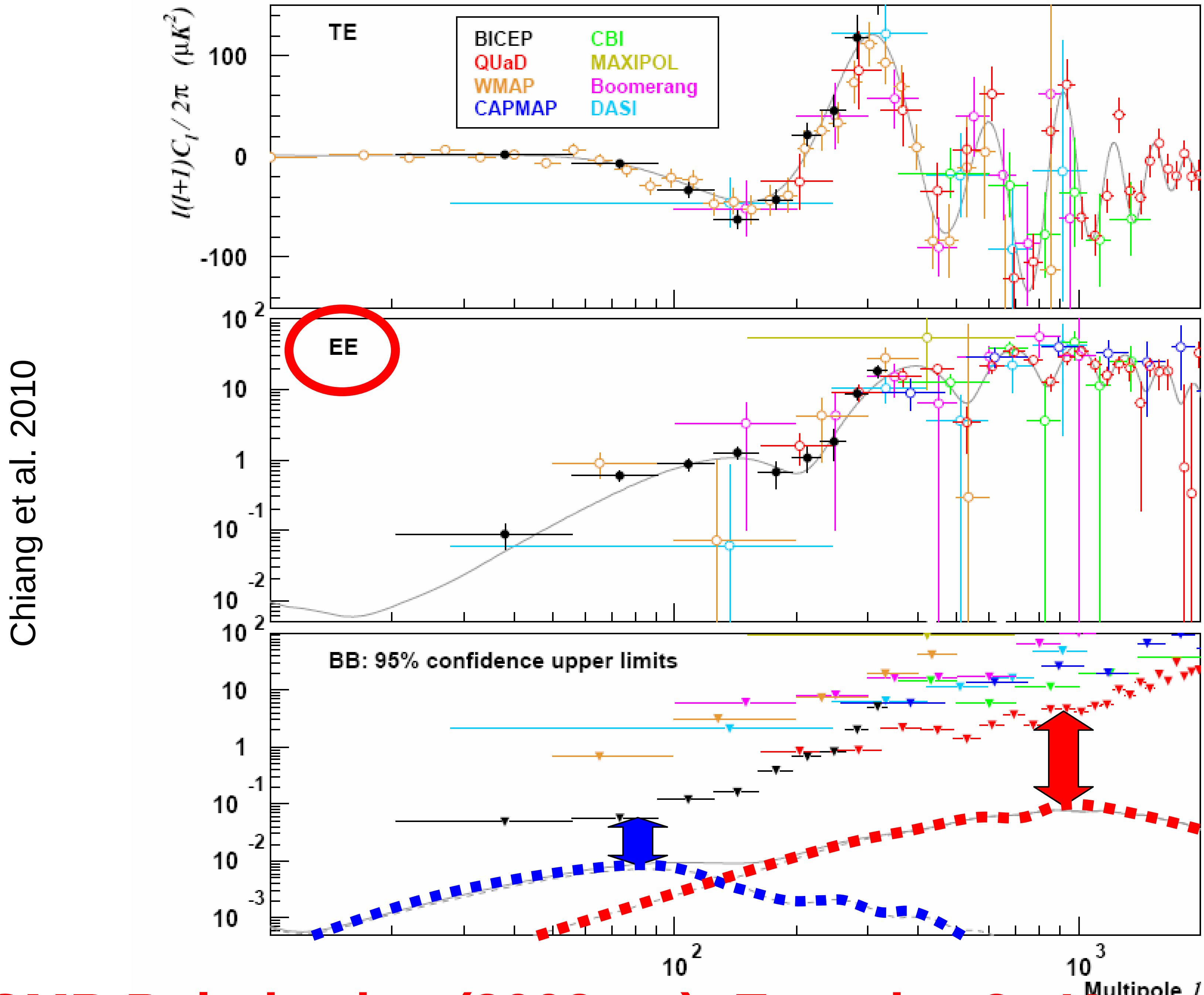
CMB Polarization (2002 …): E-modes 3 μK



CMB Polarization (2002 …): E-modes 3 μK

Lensing of E-modes

- E-modes have been measured already with good accuracy, and will be measured with exquisite accuracy by Planck and other experiments.
- They depend on the distribution of mass (mainly dark matter) so their study can shed light on the nature of dark matter (including massive neutrinos).
- While the primordial B-mode is maximum at multipoles around 100 ($\theta=2°$), the lensed B-mode is maximum at multipoles around 1000 ($\theta=0.2°$), requiring high angular resolution polarization experiments



CMB Polarization (2002 …): E-modes 3 μK