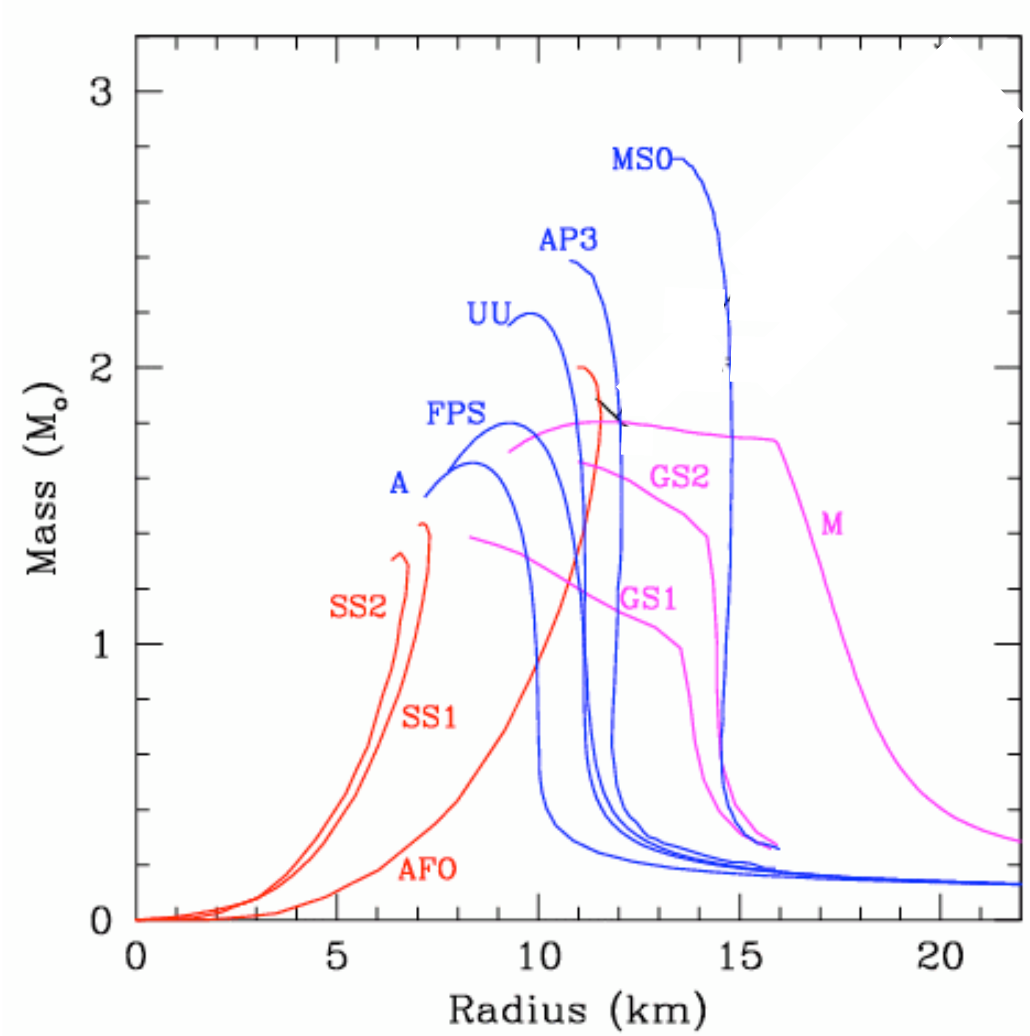
INPE Advanced Course on Compact Objects

Course III--Lecture 3

Masses and Radii
of
Neutron Stars

Mass-Radius Relation for Neutron Stars



Baryonic vs. Gravitational Mass

Important point about what we mean by NS mass:

We measure "gravitational" mass from astrophysical observations: the quantity that determines the curvature of its spacetime. This is different than "baryonic" mass: the sum of the masses of the constituents of the NS.

Remember the equation of structure for the NS:

$$\frac{dM_{grav}(r)}{dr} = 4\pi r^2 \rho(r) \quad \Rightarrow \quad M_{grav} = 4\pi \int_0^R r^2 \rho(r)\, dr$$

Here, "**r**" is not the proper radius (the one a local observer would measure) but the Schwarzschild radius (which is smaller)

The baryonic mass can be calculated from

$$M_b = 4\pi \int_0^R \left(1 - \frac{2GM(r)}{rc^2}\right) r^2 \rho(r)\, dr$$

And is larger than M_{grav}.

Why is $M_{grav} < M_b$?

Classically, the total energy in the volume of the NS is

$$E_{tot} = M_b c^2 + E_{pot}$$

$E_{pot} < 0$

The mass seen by a test particle outside the neutron star is related to the total energy,

$$M_{grav} \approx \frac{E_{tot}}{c^2} \approx M_b - \frac{|E_{pot}|}{c^2} < M_b$$

This potential energy is released during the formation of the neutron star and is converted into heat. The heat escapes (mostly) in the form of neutrinos and (a small fraction) as photons.

Methods of Determining NS Mass and/or Radius

- **Dynamical mass measurements (very important but mass only)**
- Neutron star cooling (provides --fairly uncertain-- limits)
- Quasi Periodic Oscillations
- Glitches (provides limits)
- Maximum spin measurements

Dynamical Mass Measurements

Use the general relativistic decay of a binary orbit containing a NS

$$(\dot{P}_b)_{GR} = f(m_1, m_2, \sin(i))$$

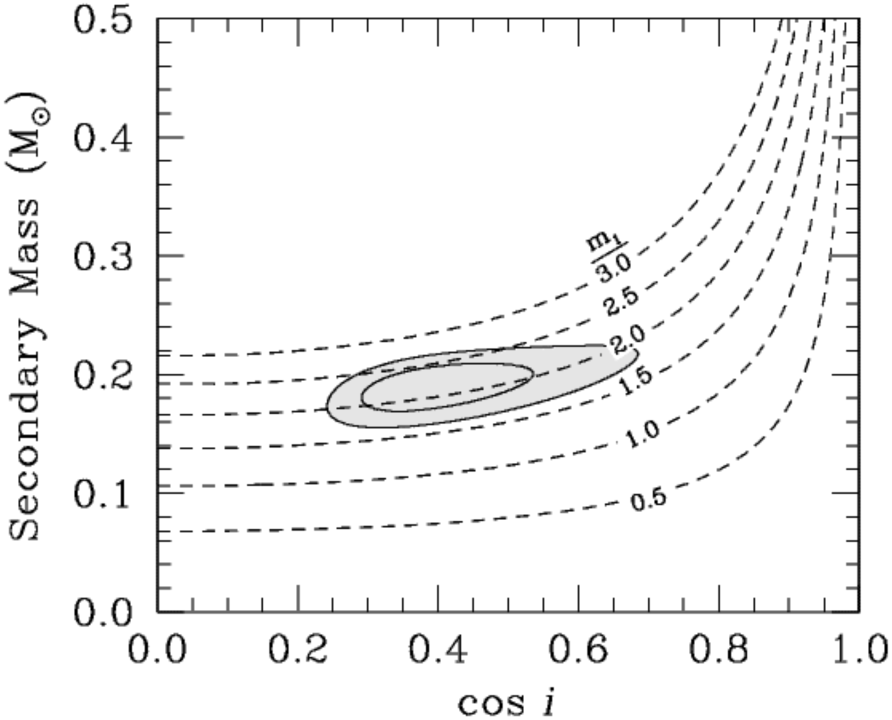
The observed binary period derivative can be expressed in terms of the binary mass function.
Need a short binary period, preferably a fast pulsar, a long baseline to get accurate timing parameters.

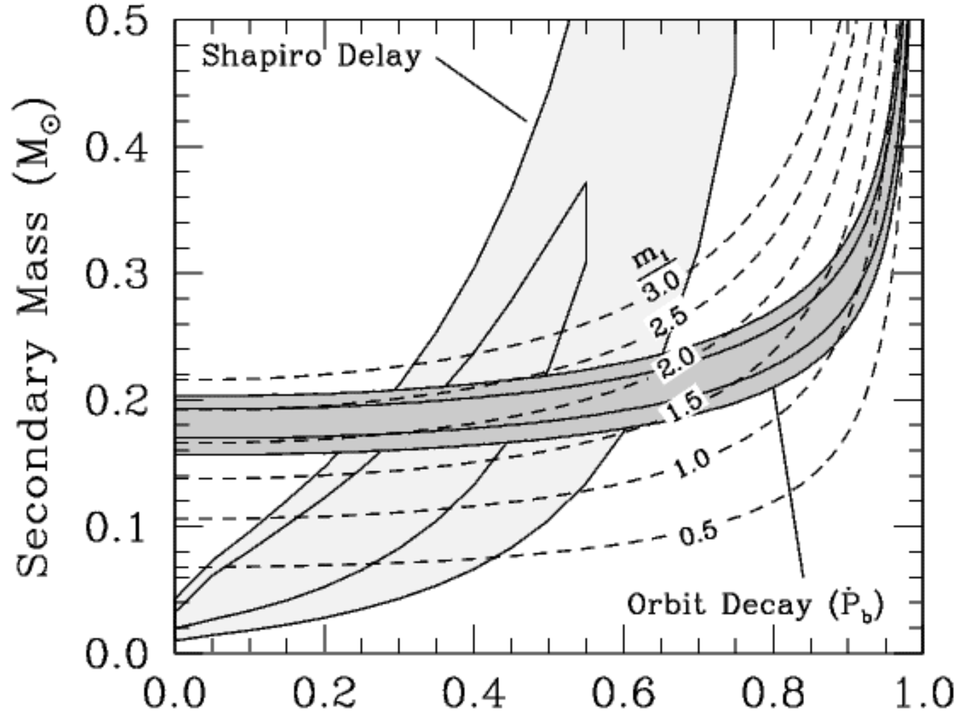
Also use Shapiro delay,

$$\Delta t = f(m_2, \sin(i))$$

(For black holes, measurements are more approximate and rely on the binary mass function)

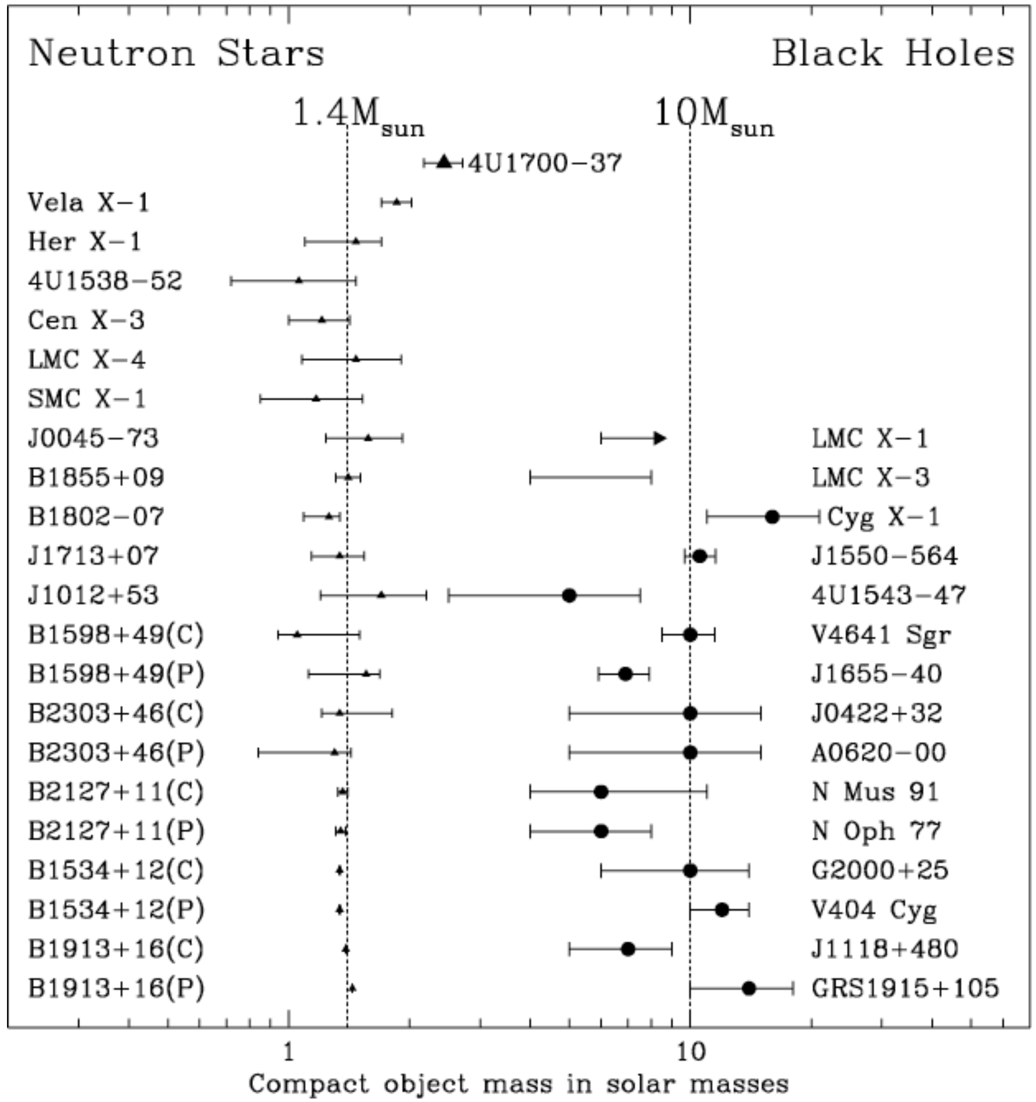
Limits on PSR J0751+1807





from Nice et al. 05

M = 2.1 $M_\odot$


Neutron Stars
Black Holes
1.4M$_{sun}$
10M$_{sun}$
4U1700−37
Vela X−1
Her X−1
4U1538−52
Cen X−3
LMC X−4
SMC X−1
J0045−73
B1855+09
B1802−07
J1713+07
J1012+53
B1598+49(C)
B1598+49(P)
B2303+46(C)
B2303+46(P)
B2127+11(C)
B2127+11(P)
B1534+12(C)
B1534+12(P)
B1913+16(C)
B1913+16(P)
LMC X−1
LMC X−3
Cyg X−1
J1550−564
4U1543−47
V4641 Sgr
J1655−40
J0422+32
A0620−00
N Mus 91
N Oph 77
G2000+25
V404 Cyg
J1118+480
GRS1915+105
1
10
Compact object mass in solar masses

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Neutron Star Cooling

Why is cooling sensitive to the neutron star interior?

The interior of a proto-neutron star loses energy at a rapid rate by neutrino emission.

Within ~10 to 100 years, the thermal evolution time of the crust, heat transported by electron conduction into the interior, where it is radiated away by neutrinos, creates an isothermal core.

The star continuously emits photons, dominantly in X-rays,
with an effective temperature T_{eff} that tracks the interior temperature.

The energy loss from photons is swamped by neutrino emission from
the interior until the star becomes about 3×10^5 years old.

The overall time that a neutron star will remain visible to terrestrial observers is not yet known, but there are two possibilities: the standard and enhanced cooling scenarios. The dominant neutrino cooling reactions are of a general type, known as Urca processes, in which thermally excited particles alternately undergo β- and inverse- β decays. Each reaction produces a neutrino or antineutrino, and thermal energy is thus continuously lost.

Neutron Star Cooling

The most efficient Urca process is the direct Urca process.

This process is only permitted if energy and momentum can be simultaneously conserved. This requires that the proton to neutron ratio exceeds 1/8, or the proton fraction $x \geq 1/9$.

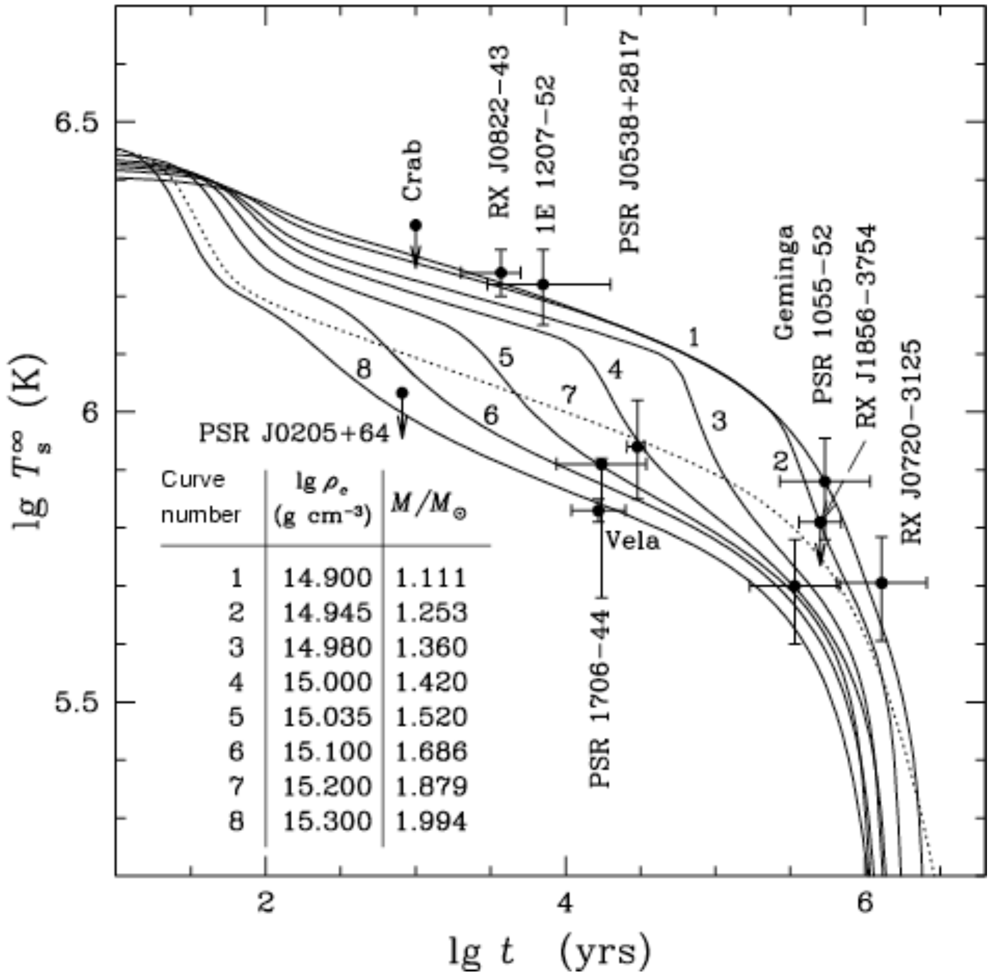
If the direct process is not possible, neutrino cooling must occur by the modified Urca process

$$n + (n, p) \rightarrow p + (n, p) + e^- + \nu_e$$

$$p + (n, p) \rightarrow n + (n, p) + e^+ + \nu_e$$

Which of these processes take place, and where in the interior, depend sensitively on the composition of the interior.

Neutron Star Cooling



Caveats: Very difficult to determine ages and distances

Magnetic fields change cooling rates significantly

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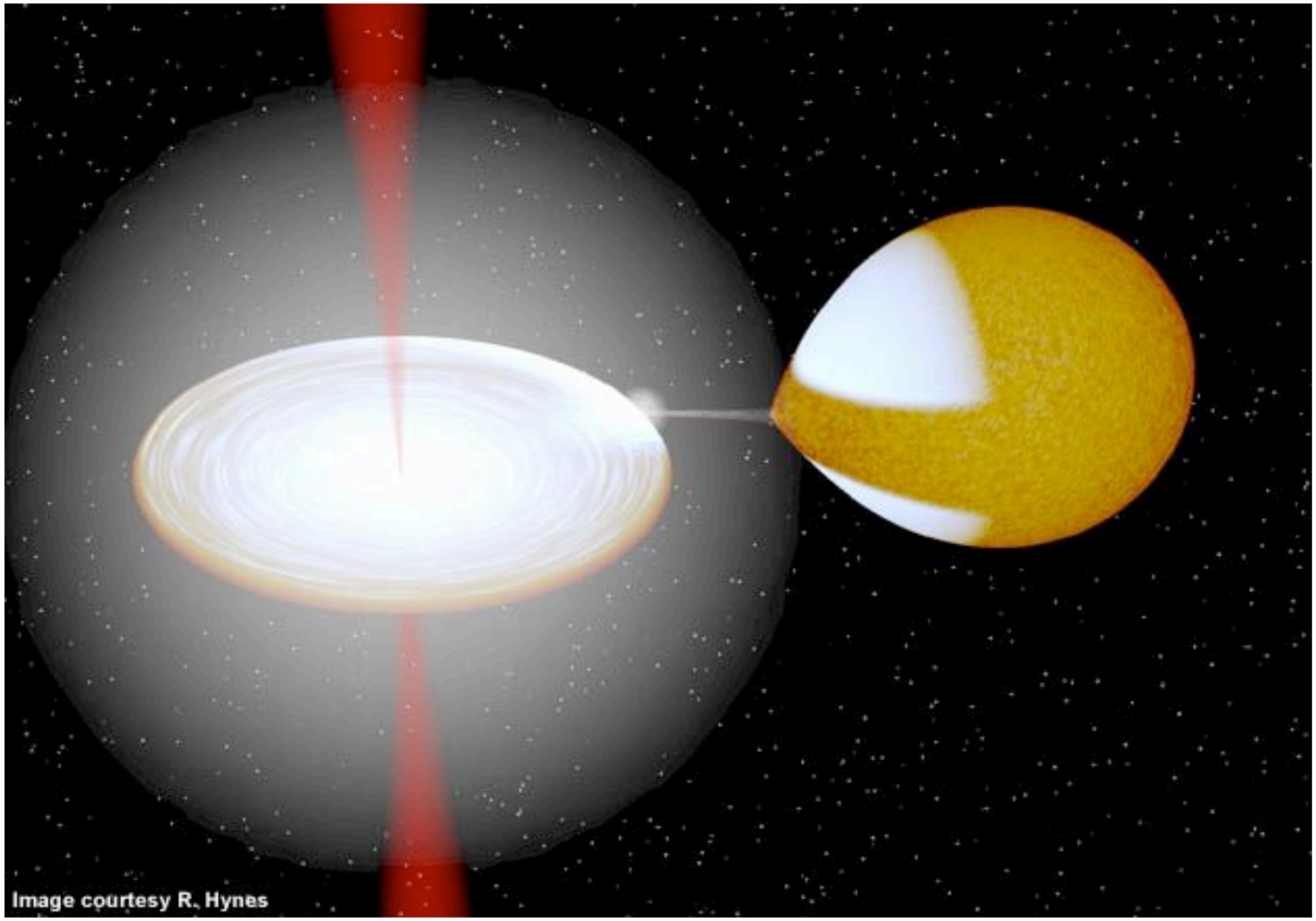
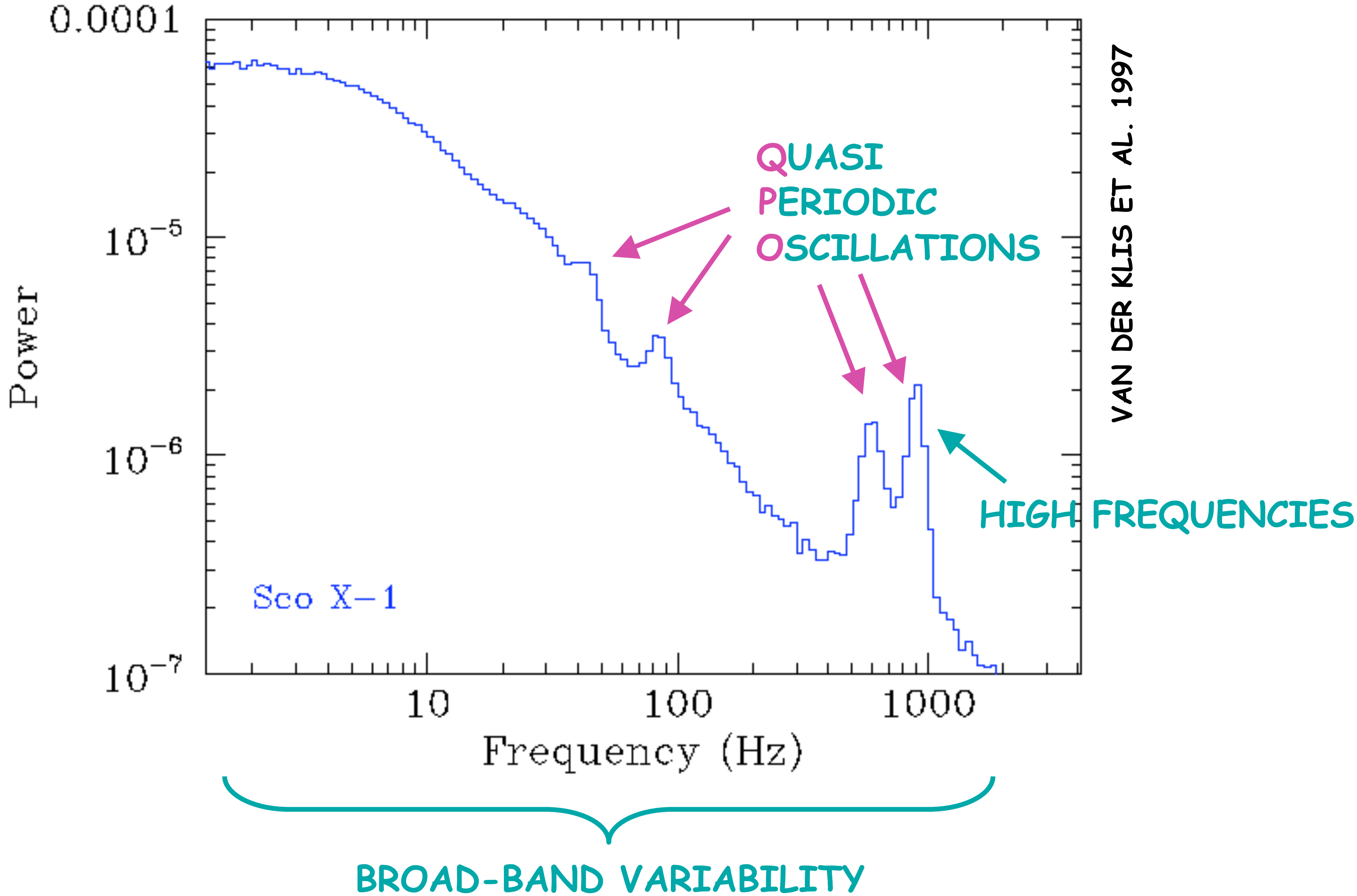
Quasi-periodic Oscillations

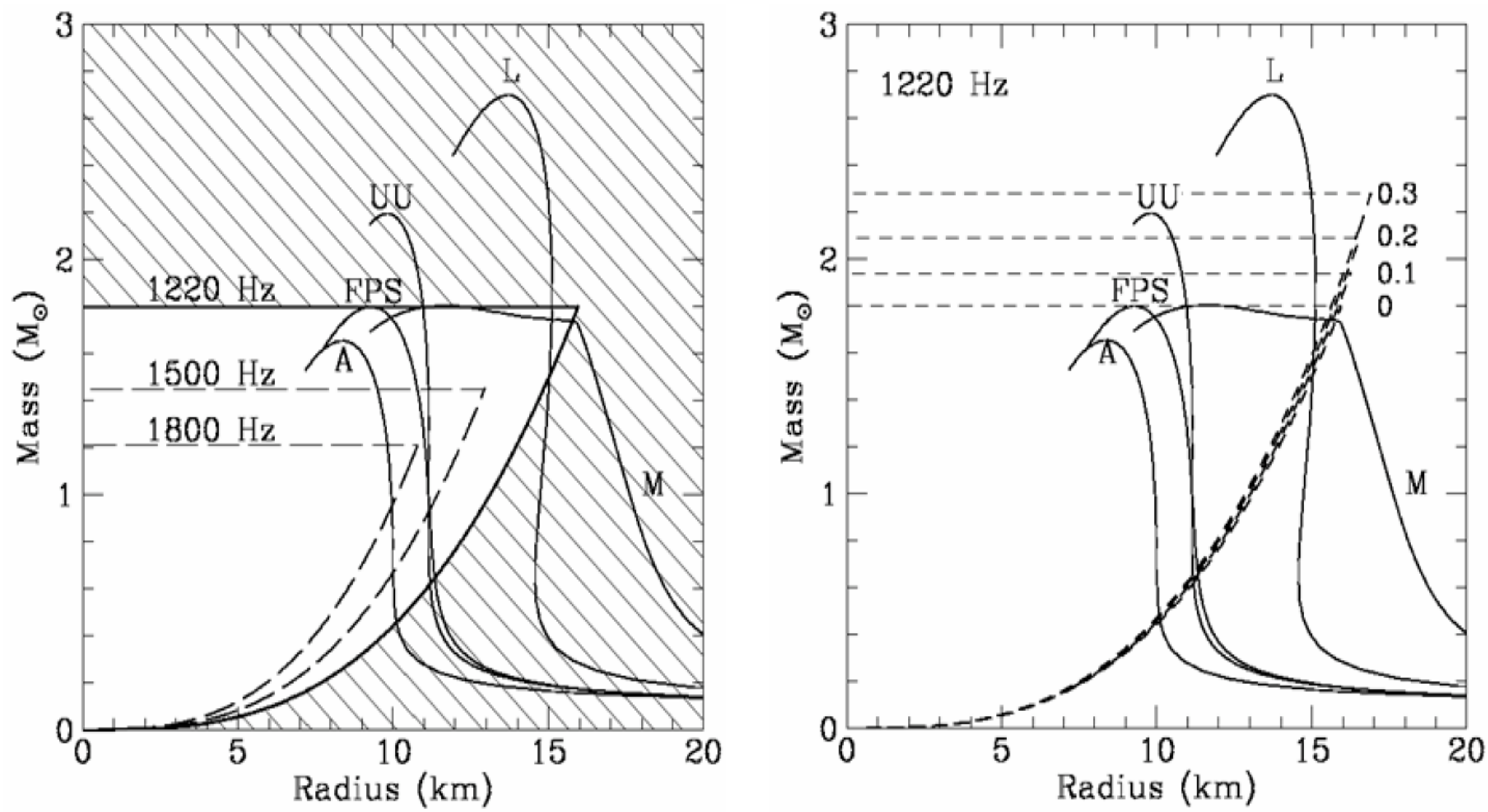


Accretion flows are very variable, with timescales ranging from 1ms to 100 days!

Power Spectra of Variability:



Quasi-periodic Oscillations



from Miller, Lamb, & Psaltis 1998

Methods of Determining NS Mass and/or Radius

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- Glitches (provides limits)
- **Maximum spin measurements**

Limits from Maximum Neutron Star Spin

The mass-shedding limit for a rigid Newtonian sphere is the Keplerian rate:

$$P_{\min}^{N} = 2\pi\left(\frac{R^3}{GM}\right)^{1/2} = 0.545\left(\frac{M_\circ}{M}\right)^{1/2}\left(\frac{R}{10km}\right)^{3/2} ms$$

Fully relativistic calculations yield a similar result:

$$P_{\min} = 0.83\left(\frac{M_\circ}{M}\right)^{1/2}\left(\frac{R}{10km}\right)^{3/2} ms$$

for the maximum mass, minimum radius configuration.

Depending on the actual values of M and R in each equation of state, the obtainable maximum spin frequency changes.

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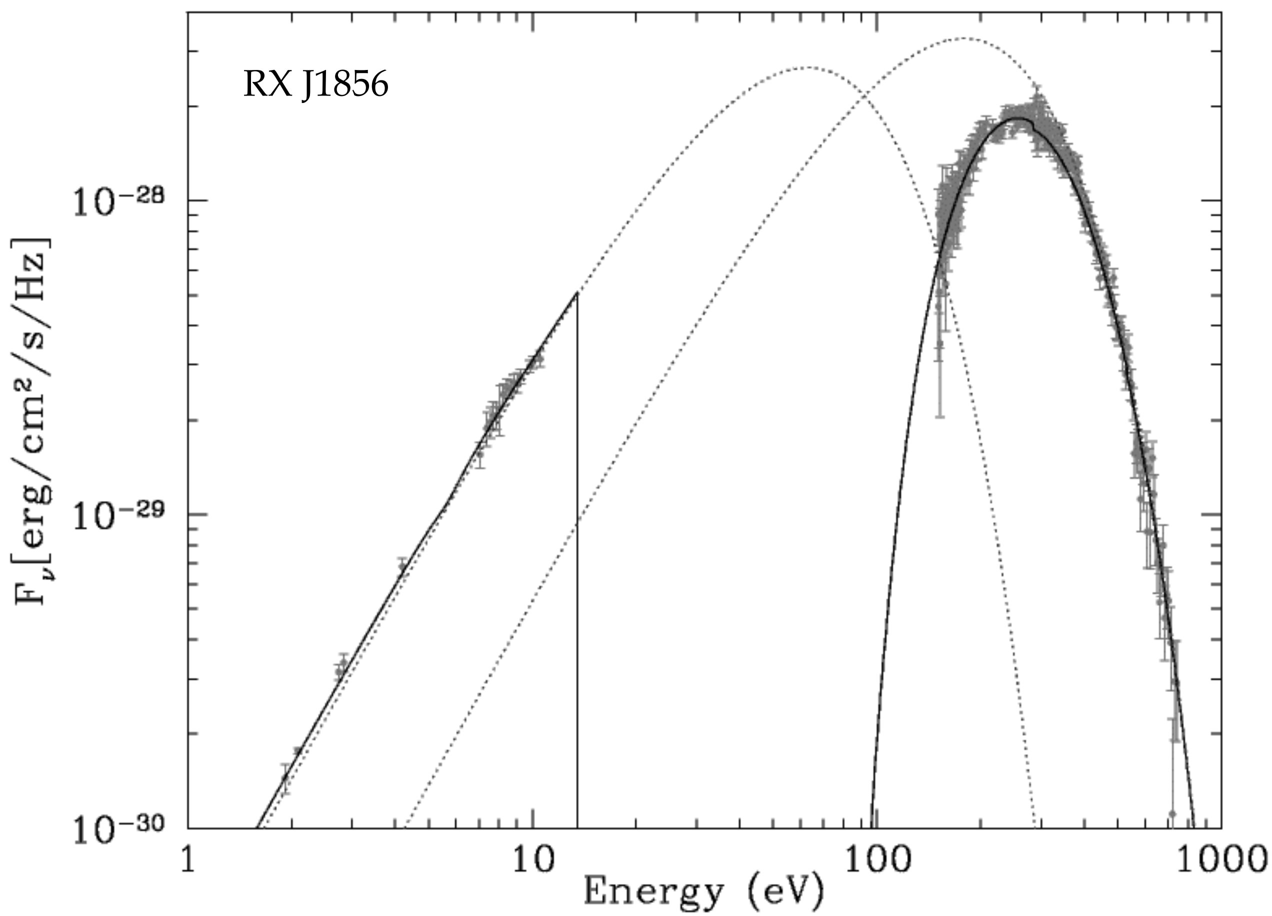
More promising methods (entirely in my opinion):

- Thermal Emission from Neutron Star Surface
- Eddington-limited Phenomena
- Spectral Features

Methods to Determine M and/or R

Radius for a thermally emitting object from continuum spectra:

$$R^2 = \frac{F\,D^2}{\sigma\,T^4}$$



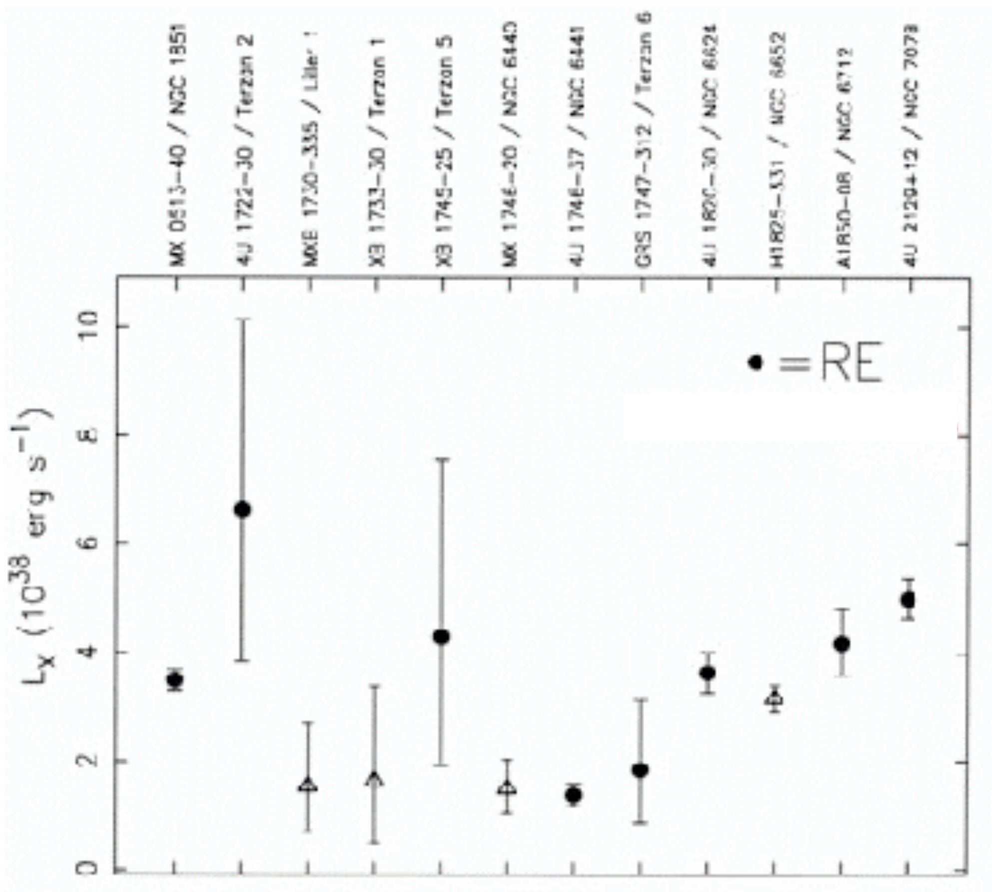
Methods to Determine M and/or R

Mass from the Eddington limit:

$$L_{Edd} = \frac{4 \pi G c M}{\sigma \, (1+X)}$$

At the Eddington Limit, radiation pressure provides support against gravity

Methods to Determine M and/or R



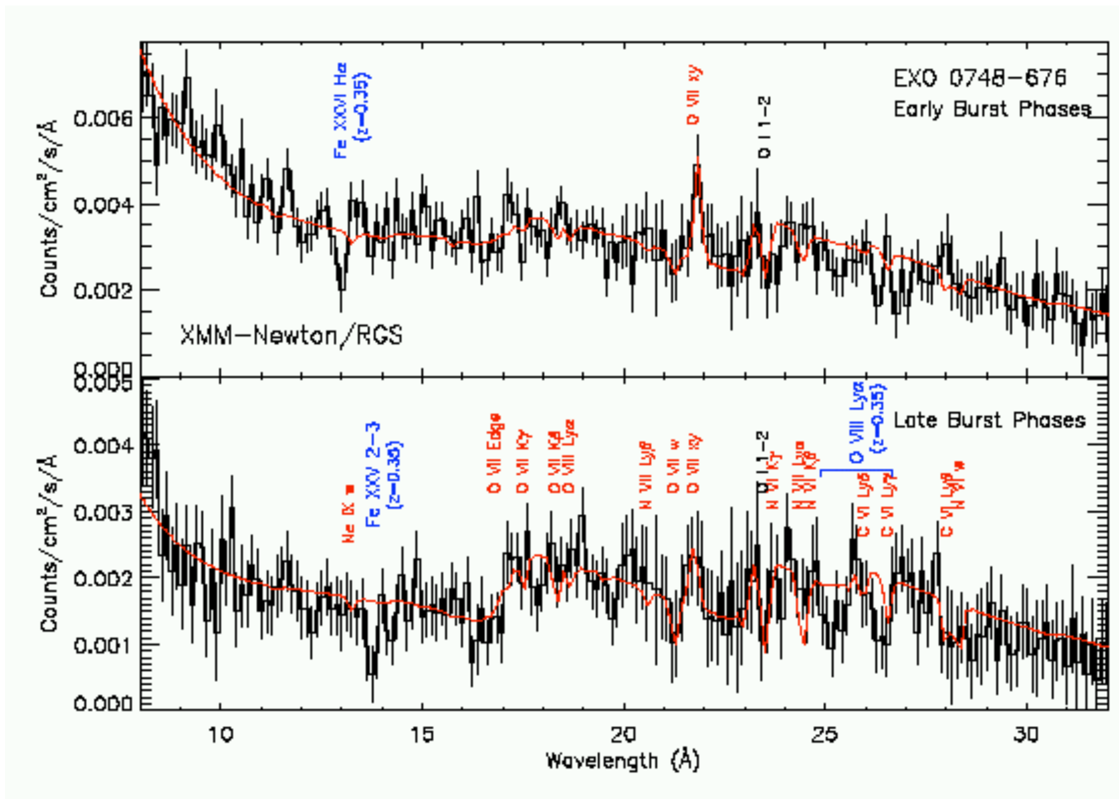
Globular Cluster Burster

Kuulkers et al. 2003

Methods to Determine M and/or R

M/R from spectral lines:

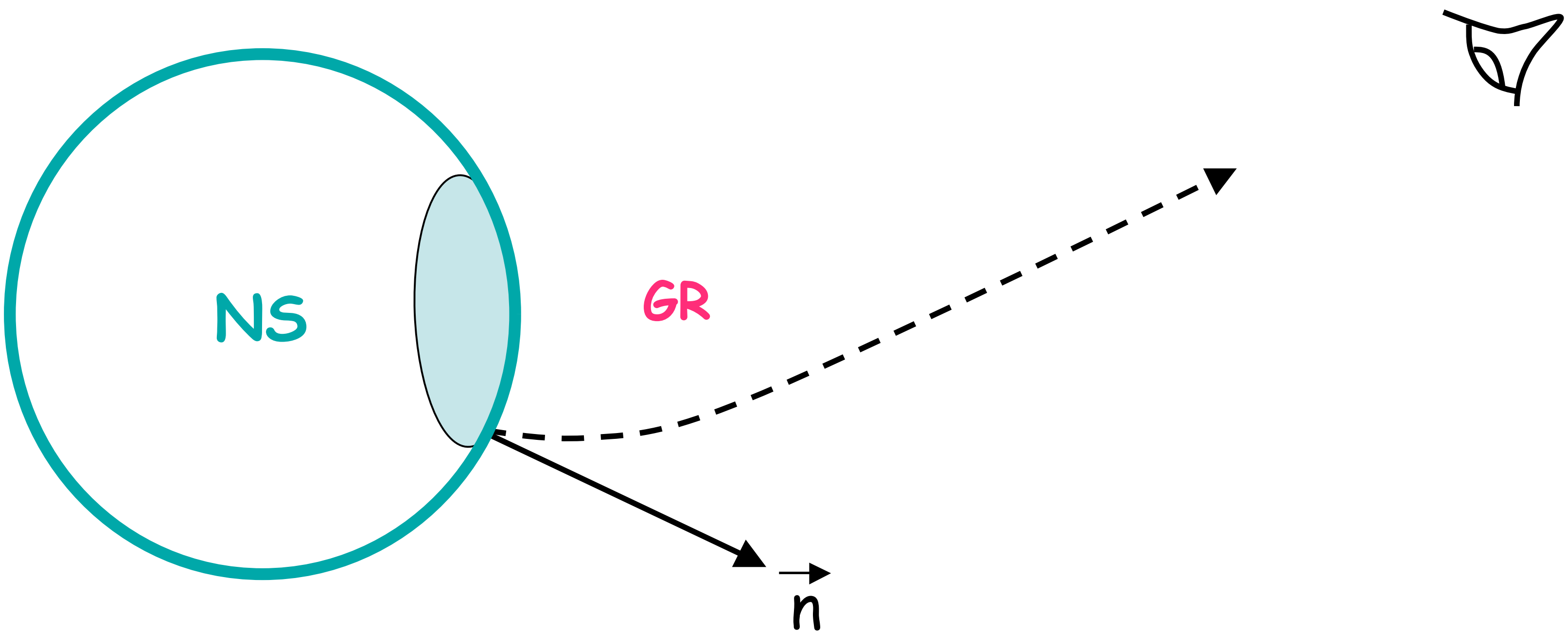
$$E = E_0 \left(1 - \frac{2M}{R}\right)$$



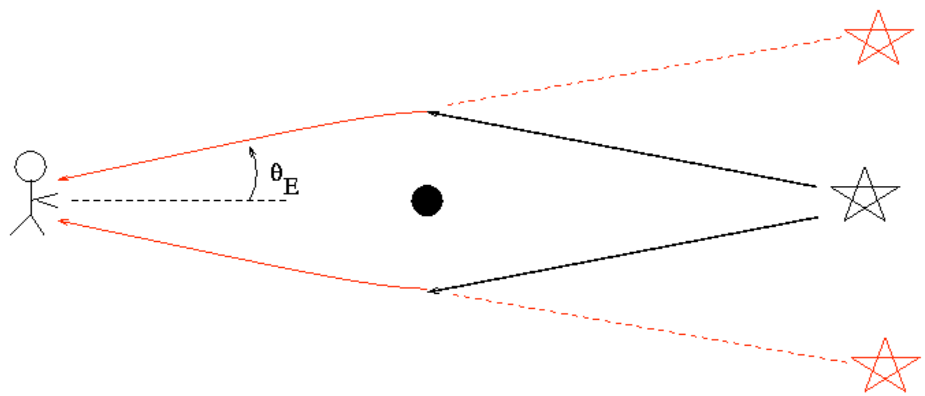
Cottam et al. 2003

In reality, Mass and Radius are always coupled because

neutron stars lens their own surface radiation due to their strong gravity



Gravitational Lensing

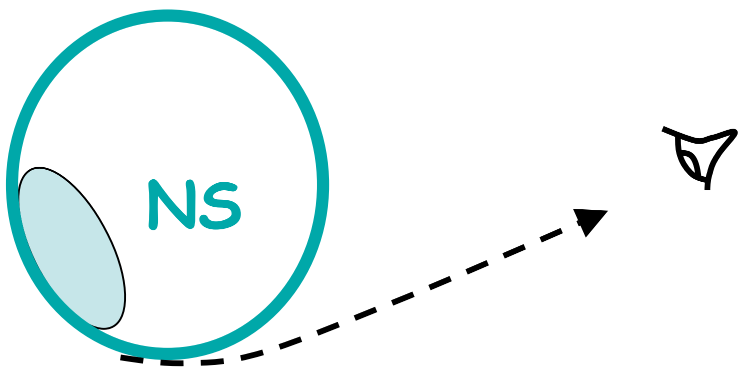


$$\vartheta = \frac{4GM}{c^2 b}$$

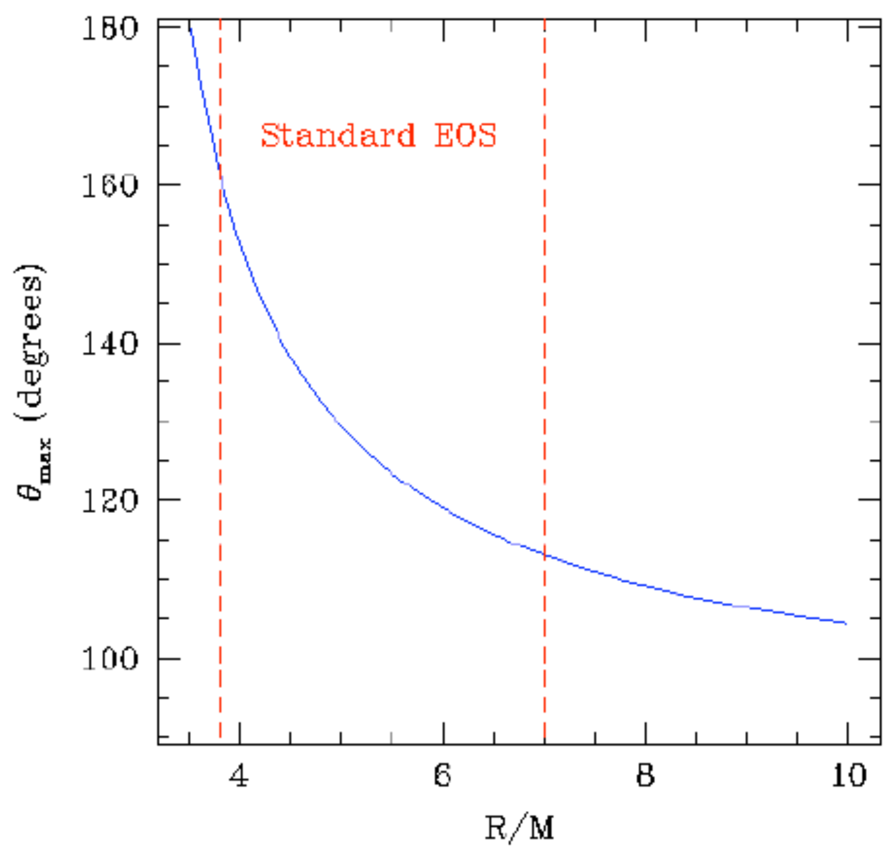
θ → deflection angle

b → impact parameter

Gravitational Self-Lensing



θ_{max}= 90^0+deflection angle



A perfect ring of radiation:
➔ R/M = 3.52

Self-Lensing

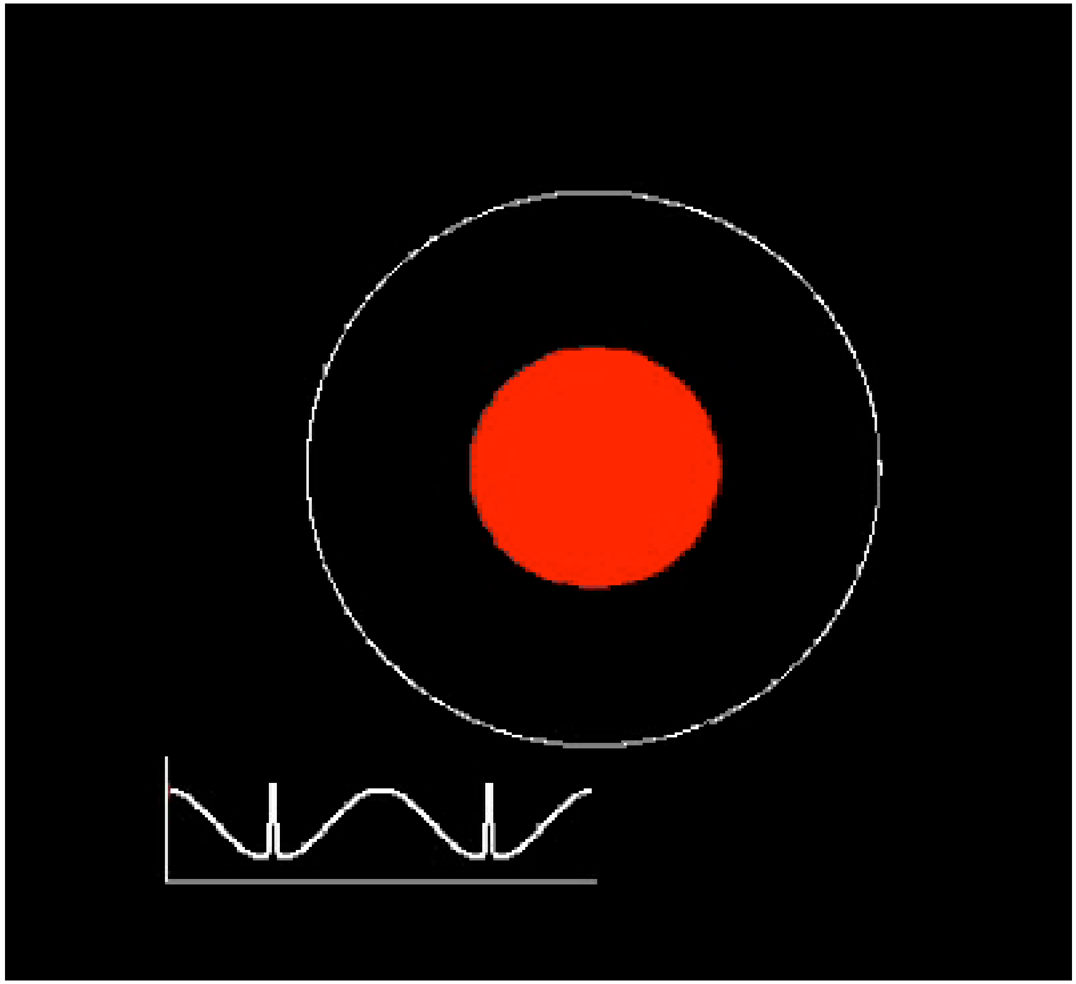
The Schwarzschild metric:

$$ds^2 = dt^2\left(1-\frac{2M}{R}\right) - dr^2\left(1-\frac{2M}{R}\right)^{-1} - f(\vartheta,\phi)$$

Photons with impact parameters $b<b_{max}$ can reach the observer:

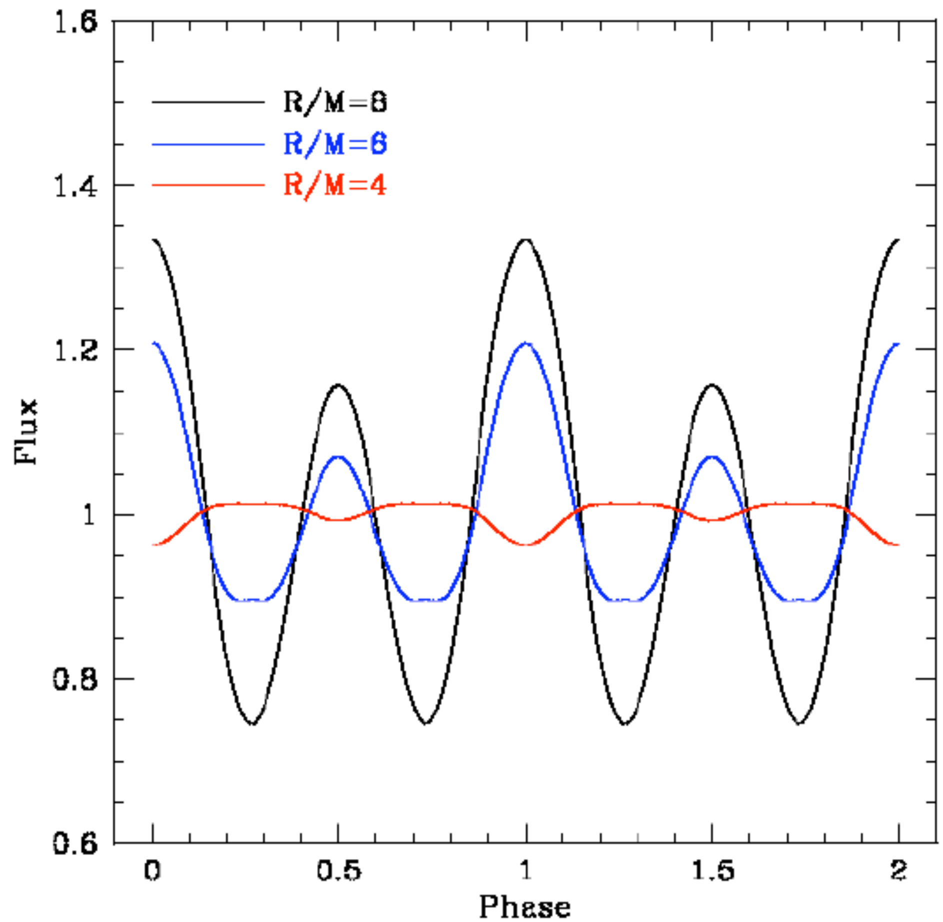
$$b_{\max} = R(1-2\frac{M}{R})^{-1/2}$$

General Relativistic Effects



Lensing of a hot spot on the neutron star surface

Pulse Amplitudes



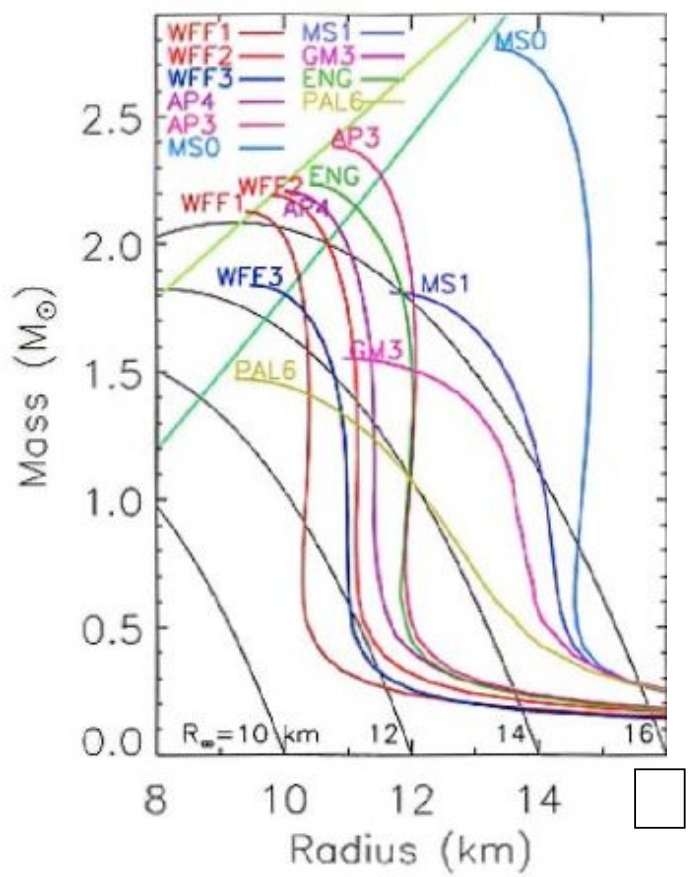
* Normalized to DC

Two antipodal hot spots at a 45 degree angle from the rotation axis

Apparent Radius of a Neutron Star

$$b_{\max} = R(1 - 2\frac{M}{R})^{-1/2}$$

Because of lensing, the apparent radius of neutron stars changes



GR Modifications

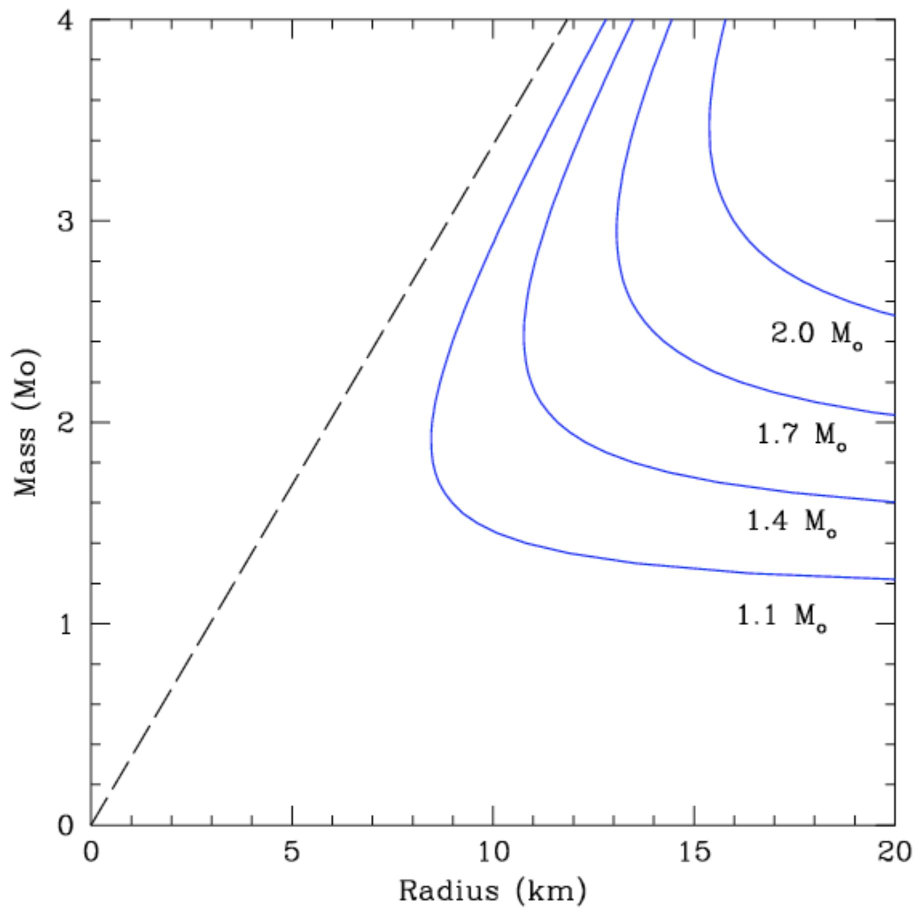
The correct expressions (lowest order)

$$R^2 = \frac{F\,D^2}{\sigma\,T^4}\left(1 - \frac{2M}{R}\right)^{-1}$$

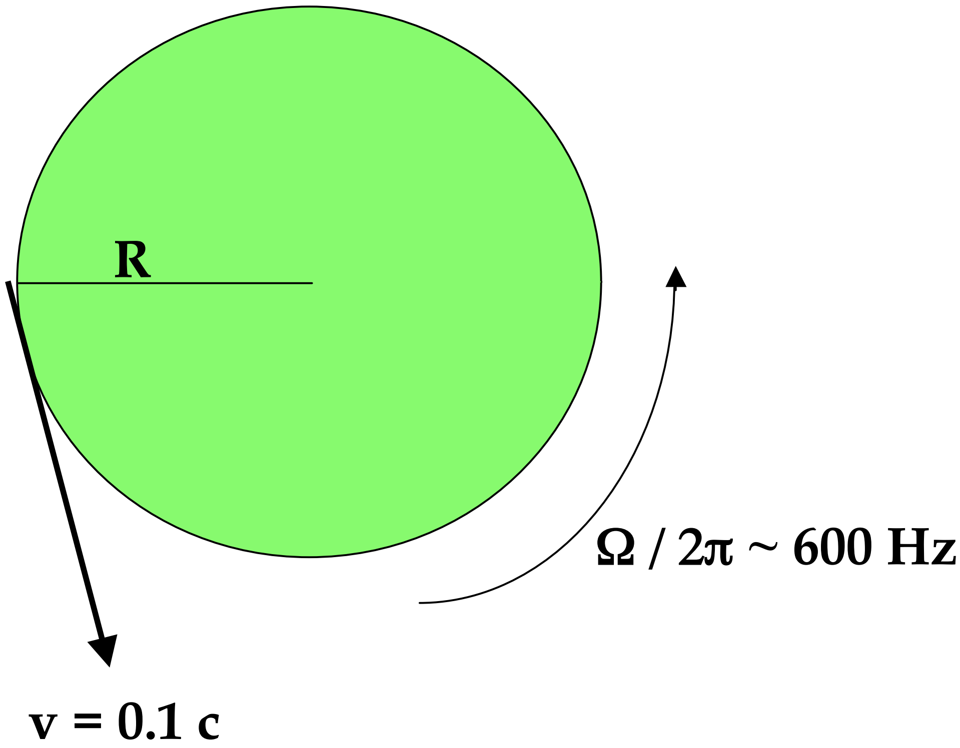
$$L_{Edd} = \frac{4\,\pi\,G\,c\,M}{\sigma\,(1+X)}\left(1 - \frac{2M}{R}\right)^{1/2}$$

Effects of GR

Modifications to the Eddington limit



What if the NS is rotating rapidly?



$$E_\infty = E_0 \gamma (1+\Omega R/c)$$

Doppler Boosts

$$\delta t = \pi/\Omega \sim \pi R/c$$

Time delays

<u>Other effects:</u>

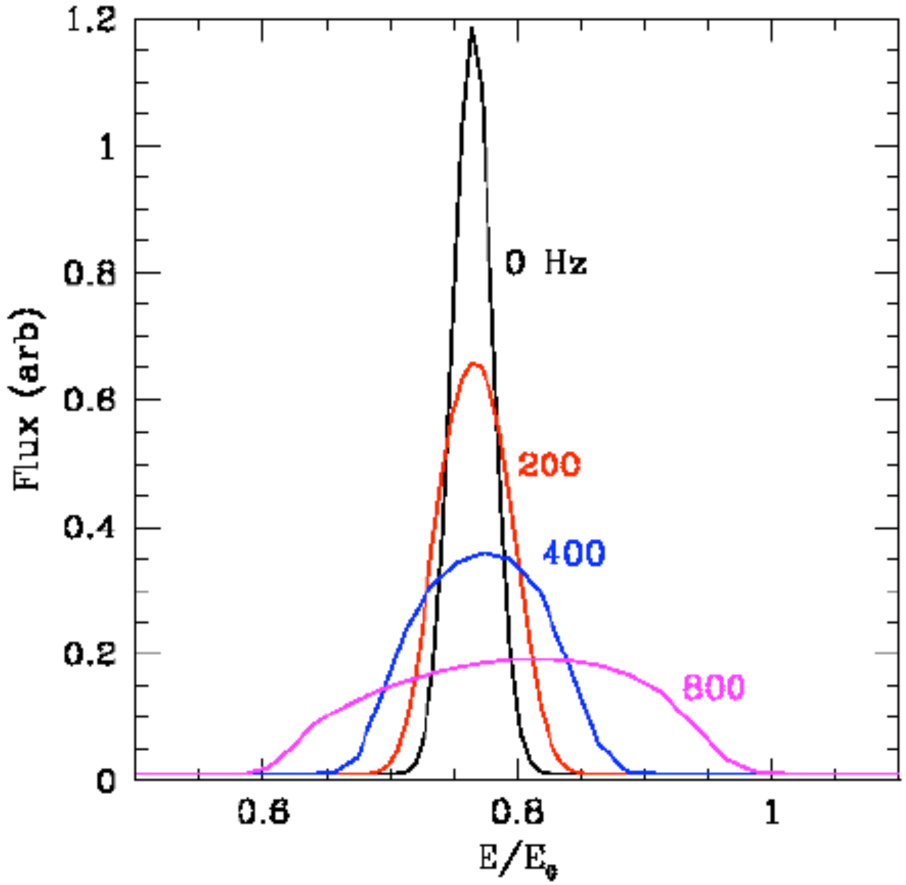
Frame dragging

Oblateness

Equation of State

(Stergioulas, Morsink,Cook)

Effect of Rotation on Line Widths



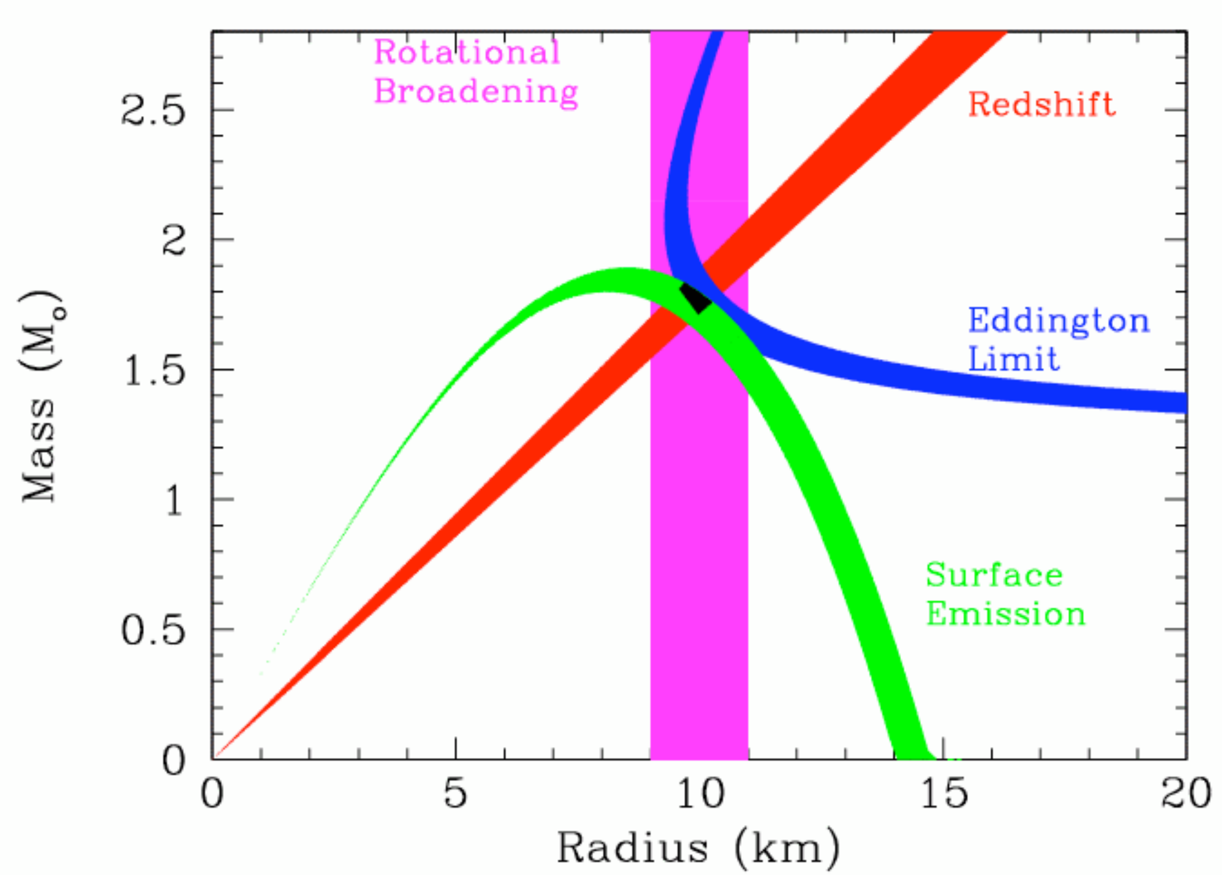
Özel & Psaltis 03

May affect the inferred redshift and detectability BUT

E/E_o ➔ M/R	FWHM ➔ R

A fourth method!

Combining the Methods



Özel 2006

1. The methods have different M-R dependences: they are complementary!
2. Surface emission gives a maximum NS mass!!
3. Eddington limit gives a minimum radius!!

➡ gravity effects can be undone