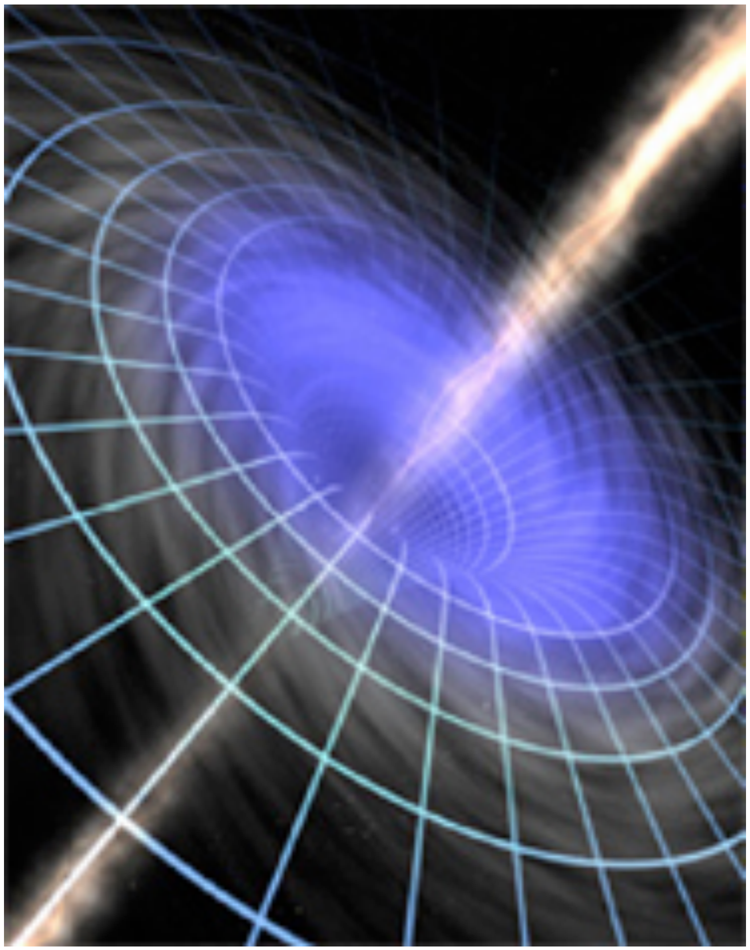
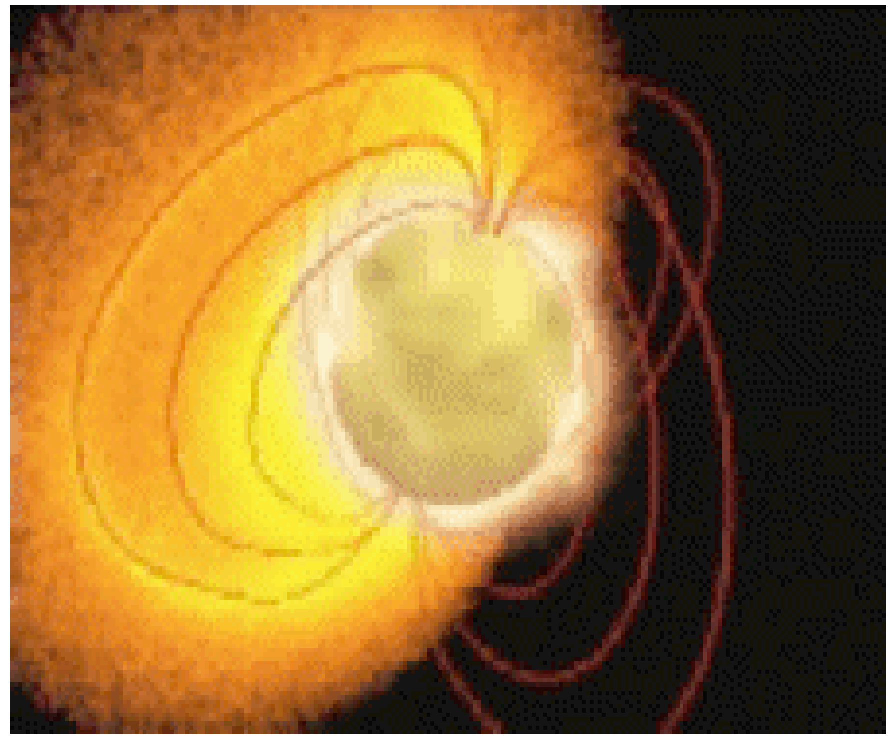
White Dwarfs
Neutron Stars
Black Holes





Some Warnings

- I will assume no background in fluid dynamics, general relativity, statistical mechanics, or radiative processes. (If you've seen them, some of this will be easy for you).

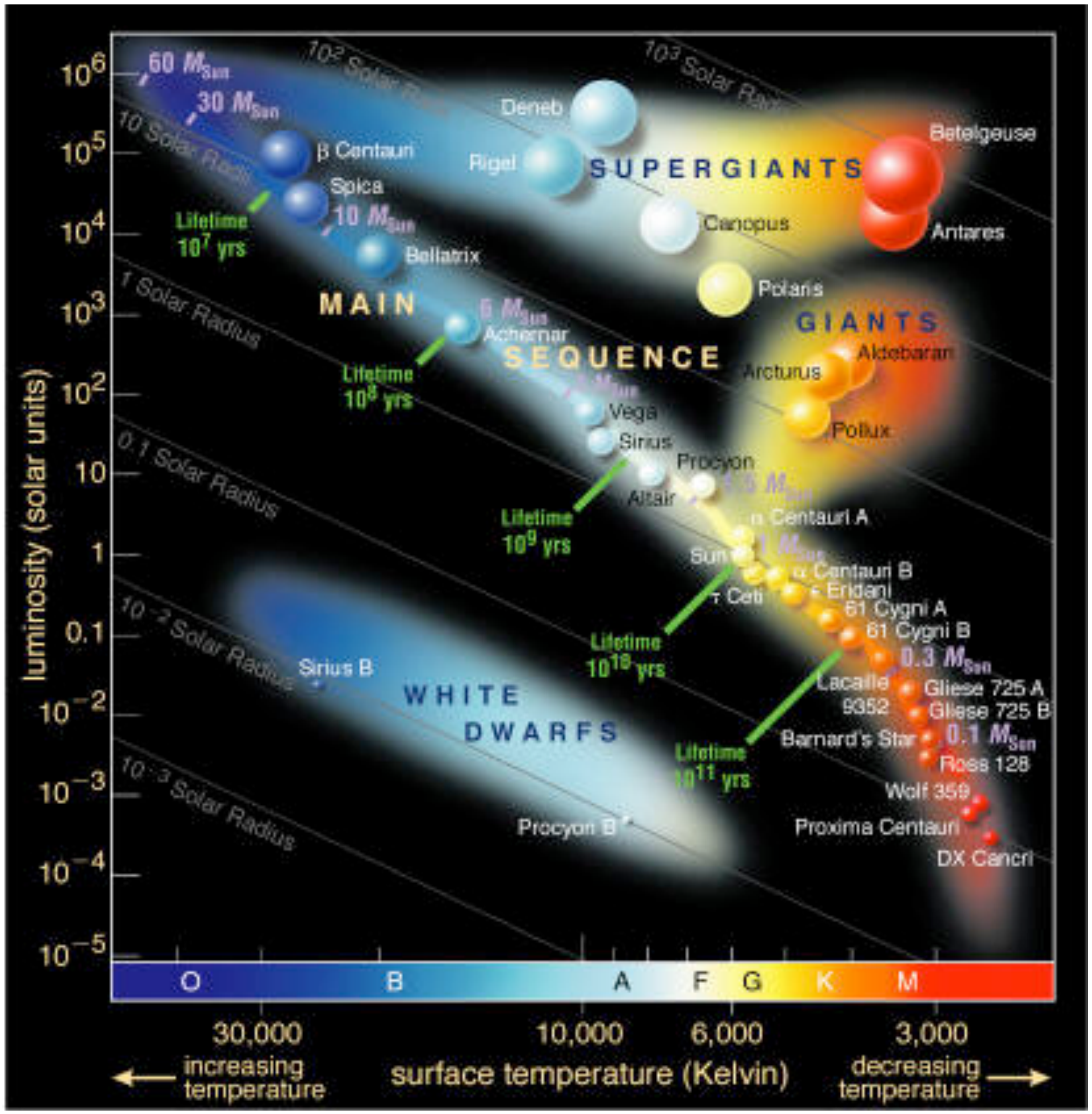
- Because I'm charged with covering a wide range of topics, I made some choices based on personal preferences. (Really, neutron stars ARE very interesting).

- Still, I am leaving out a lot. You can find more background material in e.g., "Black Holes, White Dwarfs, and Neutron Stars: The Physics of Compact Objects" by Shapiro & Teukolsky. For current research on individual subjects, I'll try to give references as we go, and you're welcome to ask for more after the lectures.

- I'm going to focus on their structure, their interiors, and their appearance as it relates to determining the properties of their interiors.

- Please ask questions, interrupt, ask for more explanation, etc.

Lives of Stars



End Stages of Stellar Evolution

Main Sequence stars: H burning in the core, synthesizing light elements

Heavier elements form in the later stages, after H in the core is exhausted and core contracts, central T rises to ignite "triple-α" reaction

$$3\ He^4 \dashrightarrow C^{12}$$

Which stars can ignite He? If they cannot, what happens during the contraction phase?

The stellar mass determines if there is sufficient contraction (and thus heating) to ignite further nuclear reactions or if matter becomes degenerate (at very high densities) *before* nuclear reactions set in.

Let's first look at equation of state of degenerate Fermions.

Kinetic Theory Preliminaries

Let's start with the distribution function and define number density:

$$n = \int \frac{df}{d^3x\, d^3p} d^3p$$

All averages, such as energy density are given by

$$\varepsilon = \int E \frac{df}{d^3x\, d^3p} d^3p \qquad ; \qquad E = (p^2c^2 + m^2c^4)^{1/2}$$ includes particle rest mass

For an ideal fermion/boson gas in equilibrium,

$$f(E) = \frac{1}{\exp[(E-\mu)/kT] \pm 1}$$

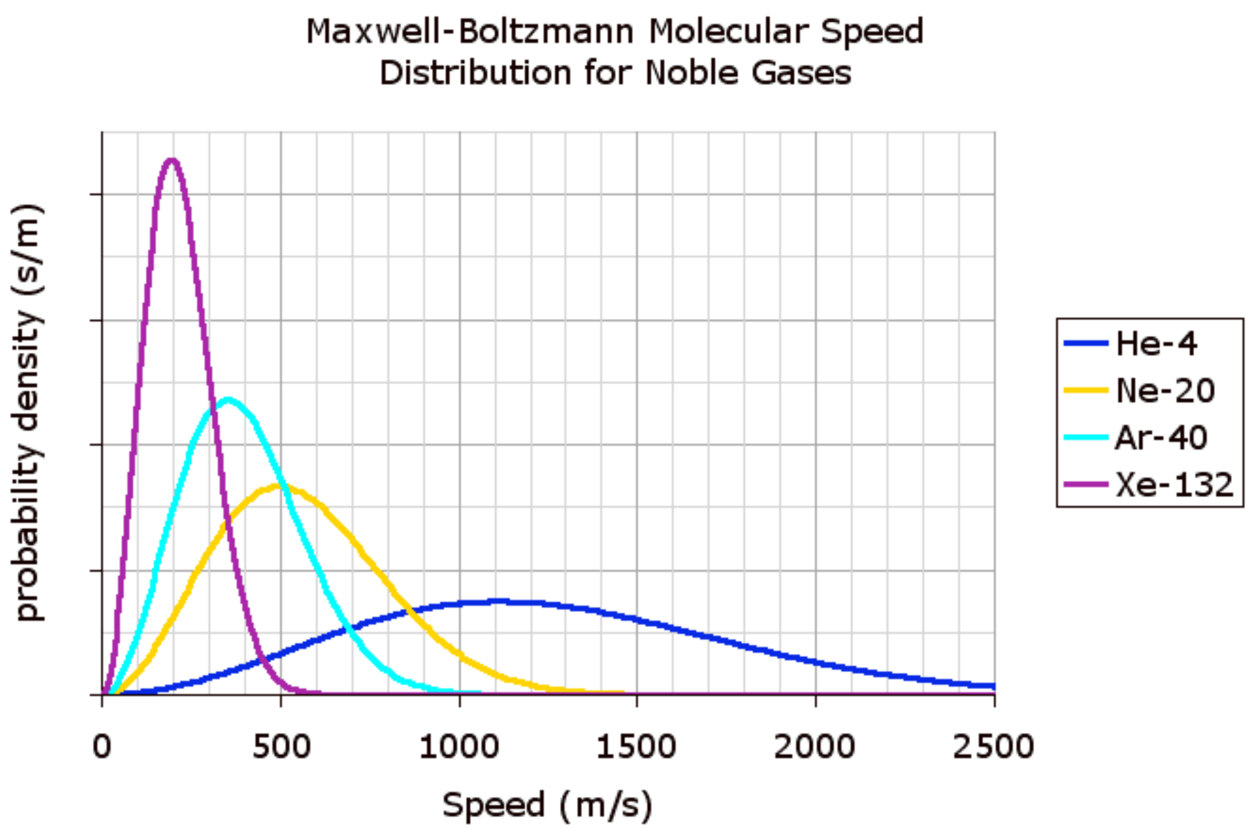
Fermion (half-integer spin particles)

Boson (integer-spin particles)

Some limits of **f(E)**:

High temperature, low density:

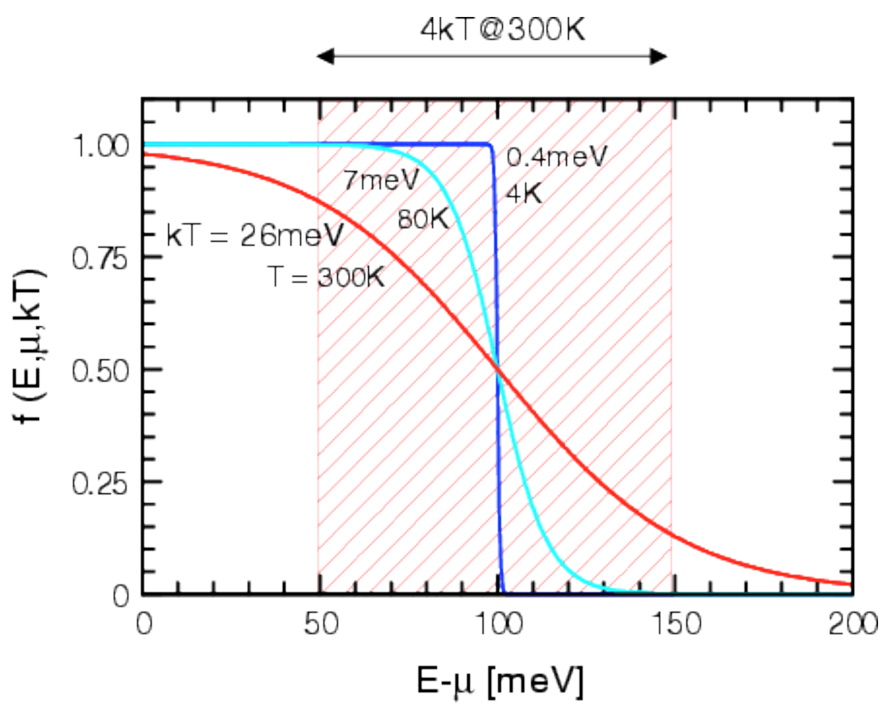
$$f(E) \approx \exp(\frac{\mu - E}{kT})$$



For fermions, chemical potential (energy cost of adding one particle) is the Fermi energy

$$f(E) = \frac{1}{\exp[(E - E_F)/kT] + 1}$$

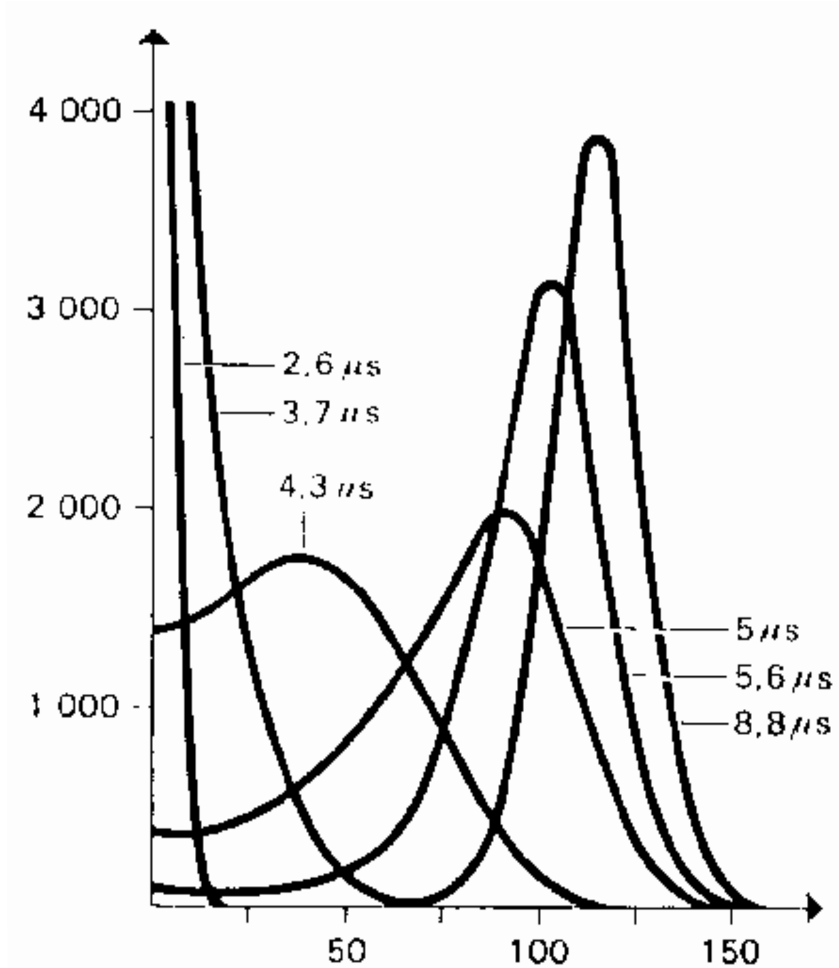
where Fermi energy E_F is defined such that $f(E_F) = \frac{1}{2}$



Fermions at zero temperature (complete degeneracy):

$$\mathbf{f(E)} \sim \begin{cases} 1 & \mathbf{(E \leq E_F)} \\ 0 & \mathbf{(E > E_F)} \end{cases}$$

For comparison, let's look at bosons:



Statistical distributions of photons detected at different times following the startup of the laser oscillation.
At short times the source is chaotic and the distribution is of Bose-Einstein type.
At longer times the source is a laser and the distribution becomes Poissonian.

Unlike Fermions, as T--> 0, an unlimited number of bosons condense to the ground state.

- We can write the available number of cells in terms momentum:

$$N(p)dp = 2 \times \frac{V}{h^3} 4\pi p^2 dp$$

or in terms of energy by using $E=p^2/2m$

$$N(E)dE = \frac{8\pi V}{h^3}(2m^3)^{1/2} E^{1/2} dE$$

Thus, at a given E and for fixed V, the phase space available to the system of particles decreases with the particle mass, and electrons can fill the phase space much more easily than protons.

- The pressure associated with the degenerate electron gas is given by

$$P_e = \frac{1}{3}\int_0^\infty N_e(p) f(p) v\, p\, dp \longrightarrow P_e = \frac{1}{3}\int_0^{p_F} N_e(p) v\, p\, dp$$

Which can be evaluated for the nonrelativistic case $v<<c$ ($v=p/m_e$) and for extreme relativistic case ($v \sim c$) to give

$$P_e = \frac{(3\pi^2)^{2/3}}{5}\frac{\hbar^2}{m_e} n_e^{5/3}, \quad v << c$$

$$P_e = \frac{(3\pi^2)^{1/3}}{4}\hbar c\, n_e^{4/3}, \quad v \sim c$$

Notice there is no dependence on m_e.

Note: P-ρ relations of the type $P=K\rho^{\Gamma}$ are called polytropic equations of state.

We saw that this is exact for degenerate matter (e.g., inside a white dwarf) and a good approximation for some normal stars.

Now back to the fate of the evolving stars:

During the contraction of a star, nuclear reactions must start when $P \geq P_e$. (otherwise, pressure due to degenerate electrons stops the contraction before necessary T for the reactions is reached)

Because stellar temperatures scale with mass, this condition equates to a minimum mass for the onset of reactions:

(i) For H-burning at 10^6 K:

$M \geq 0.031\ M_\odot$ $\longrightarrow$ Smaller masses do not become MS

(ii) For He-burning at $2\text{x}10^8$ K:

$M \geq 0.439\ M_\odot$ $\longrightarrow$ Just the mass if the He core; smaller mass stars become degenerate before triple-α kicks in.

Polytropes

Polytropes are simple stellar models applicable to degenerate stars as well as normal stars.

To obtain the structures of stars, we need to solve hydrodynamic equations along with an "equation of state" relating pressure P to density ρ.

When we use a P-ρ relation of the type $P=K\rho^{\Gamma}$ (i.e., a polytropic equation of state), we can obtain a simple polytropic stellar model.

Hydrodynamics Preliminaries:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho u_i) = 0$$

Continuity equation

$$\frac{\partial u_j}{\partial t} + u_i \frac{\partial u_j}{\partial x_i} = a_j - \frac{1}{\rho}\frac{\partial \Psi_{ij}}{\partial x_i}$$

Euler (momentum) equation--a fancy F=ma for pliable materials)

What is Ψ_{ij}?

$$\Psi_{ij} = P\delta_{ij} - \sigma_{ij}$$

diagonal terms:
Pressure

off-diagonal terms:
Viscosity-stress

So Euler equation becomes

$$\frac{\partial u_j}{\partial t} + u_i \frac{\partial u_j}{\partial x_i} = a_j - \underbrace{\frac{1}{\rho}\frac{\partial P}{\partial x_j}}_{\text{pressure gradient}} + \underbrace{\frac{1}{\rho}\frac{\partial \sigma_{ij}}{\partial x_i}}_{\text{viscosity}}$$

To study the structures of compact objects, we assume hydrostatic equilibrium (as is the case for stars in general)

In steady state: $\dfrac{\partial}{\partial t} = 0$

Static: $u_i = 0$

So the momentum eqn reads: (with $a_j = \dfrac{\partial \psi}{\partial x_j}$)

$$-\frac{\partial \psi}{\partial x_j} - \frac{1}{\rho}\frac{\partial P}{\partial x_j} = 0$$

Which in spherical coordinates is

$$\frac{1}{\rho}\frac{\partial P}{\partial r} = -\frac{GM(r)}{r^2}$$

and

$$M(r) = \int_0^r 4\pi r^2 \rho(r) dr$$

Now we're ready to solve our set of equations for the structure of a polytrope:

$$\frac{1}{\rho}\frac{\partial P}{\partial r} = -\frac{GM(r)}{r^2}$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$$

$$P = K\rho^{\Gamma} \qquad \text{(alternatively} \quad P = K\rho^{1+\frac{1}{n}} \quad)$$

3 equations for 3 unknowns P, M, and ρ. We can specify boundary conditions and solve this set of equations. Typically the density at the center

$$\rho(r=0) = \rho_c$$

and the density derivative at the center dρ/dr=0 are specified as the two boundary conditions.

It is common to express radius in terms of a unit length

$$a \equiv \left[\frac{(n+1)K\rho_c^{\frac{1}{n}-1}}{4\pi G}\right]^{1/2}$$

$$\xi = \frac{r}{a}$$

So that the solution for R and M become

$$R = \xi_1 \left[\frac{(n+1)K}{4\pi G} \right]^{1/2} \rho_c^{\frac{1-n}{2n}}$$

where ξ_1 is the radial point at which density becomes zero (the "surface" of the star) and depends only on the polytropic index n.

Note that for 0<n<1, R increases with increasing central density, while for N>1, R decreases.

Similarly,

$$M = f(\xi_1)\, 4\pi \left[\frac{(n+1)K}{4\pi G} \right]^{3/2} \rho_c^{\frac{3-n}{2n}}$$

We can also write a relation for M and R:

$$M \propto R^{\frac{3-n}{1-n}}$$

Whether M increases or decreases with R depends on the polytropic index n!

Note that for n=3 (Γ = 4/3), M=constant! (independent of R)

Possible Outcomes of Stellar Deaths

The possible end states of stellar evolution are
(i) White dwarfs
(ii) Neutron stars
(iii) Black holes

Which compact object will form depends on whether electron degeneracy is achieved at high or low Temperature (which in turn depends on the stellar mass).

$M \leq 1.4\ M_\odot$: Electron degeneracy is reached at a relatively low T. Consequently, advanced nuclear burning is not reached. Support against gravity is provided by P_e.

$1.4\ M_\odot \leq M \leq 4\ M_\odot$: At the red giant phase, H burns in a shell, and He in another shell. P_{rad} supports against gravity. Mass loss at this stage. Subsequent evolution to a white dwarf.

$M > 8\ M_\odot$: C^{12} ignites prior to the development of a degenerate core. Advanced burning stages can be reached. The core eventually collapses to form a compact object.

How many of each form??

A LARGE NUMBER OF UNCERTAINTIES:

- late stages of evolution (especially some mass regimes)
- mass loss during evolution
- differential rotation of the star as the core collapses
- explosion energies for supernovae
- fallback during supernovae
- whether dynamo and/or flux freezing play a role in generating magnetic fields
- theoretical uncertainties in maximum NS mass

Inverse β-decay

One other reaction we should briefly talk about in the evolution of stars into compact objects is the inverse β-decay

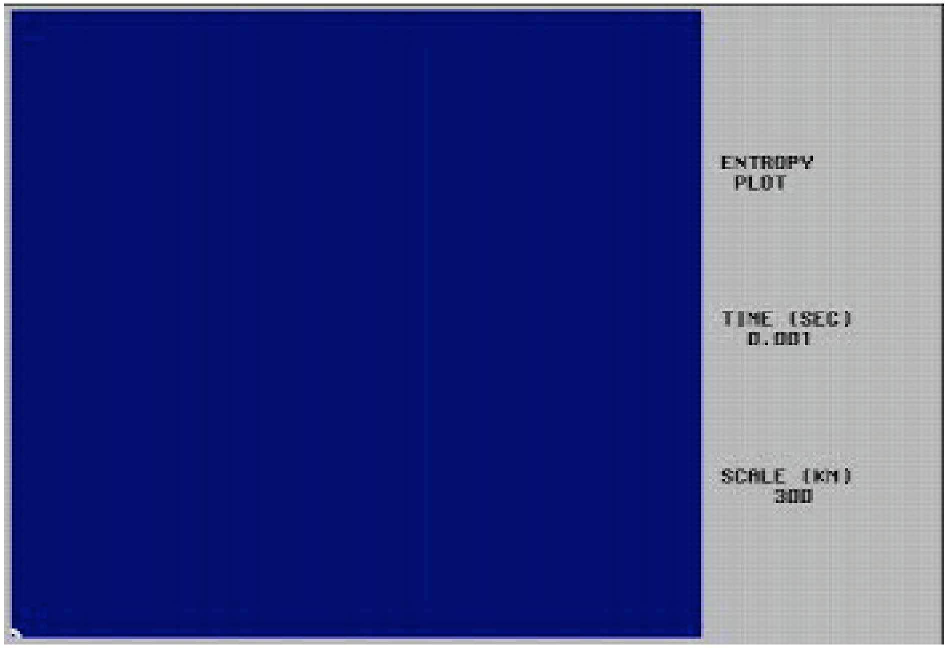
$$\mathbf{p + e \rightarrow n + \nu_e}$$

In "ordinary" environments, β-decay

$$\mathbf{n \rightarrow p + e + \bar{\nu}_e}$$

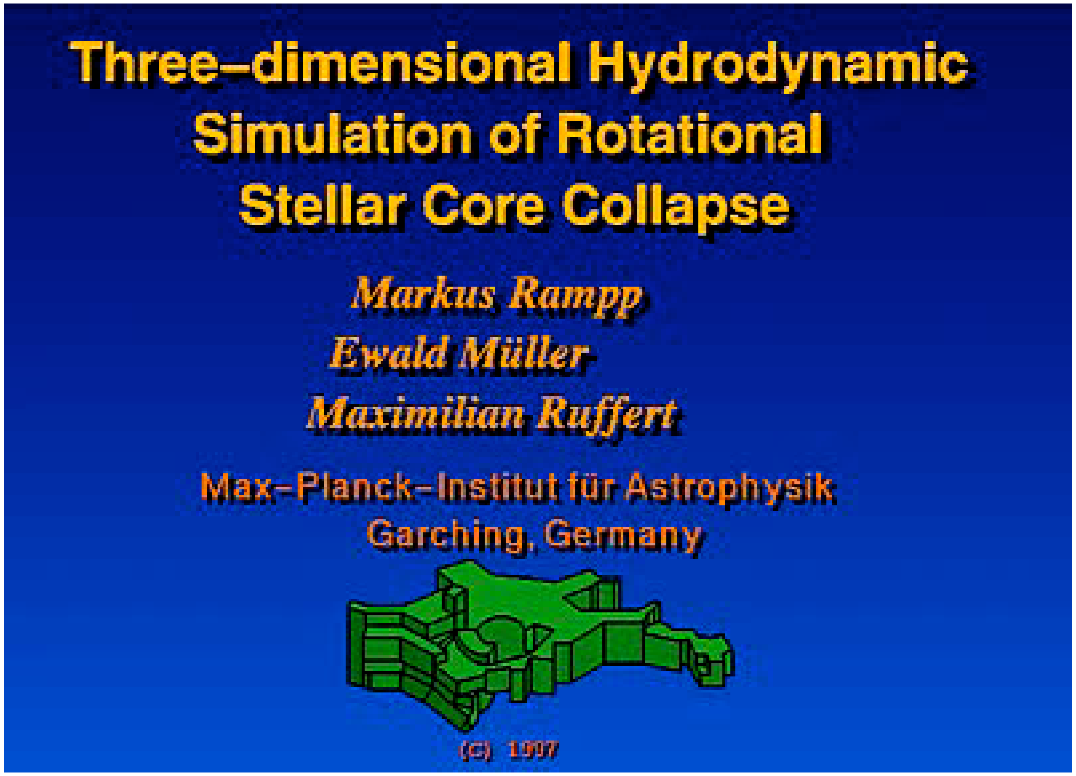
also proceeds efficiently and enables an equilibrium between electrons, protons, and neutrons. But at high densities, when electron Fermi energy is high and the electron produced by β-decay does not have sufficient energy, the inverse decay proceeds to primarily create more neutrons.

Formation of Neutron Stars

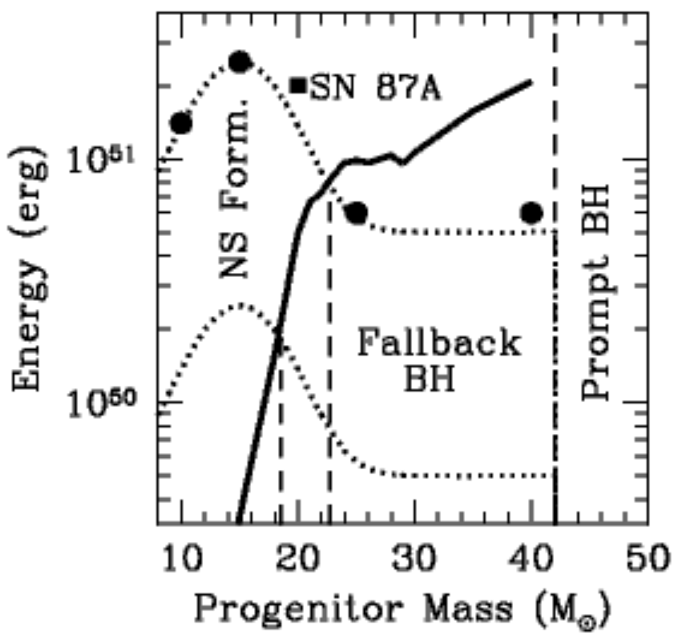


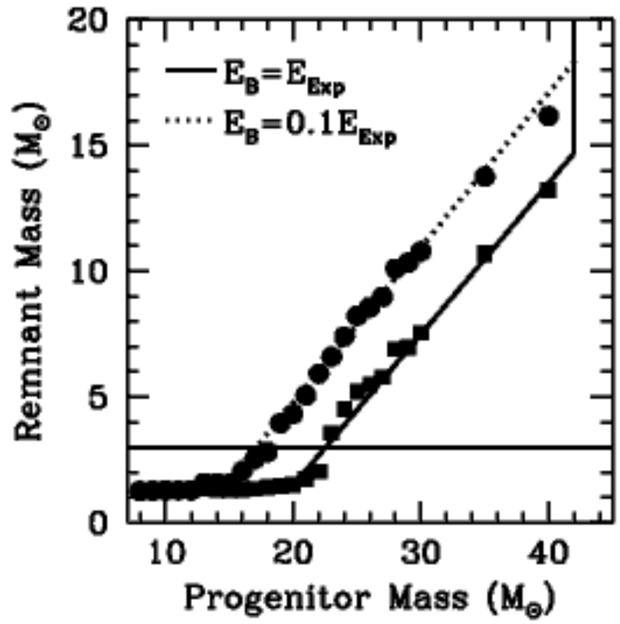
A supernova simulation from Burrows et al.

Formation of Neutron Stars



Dividing Lines





From Fryer et al. 99

N.B. This is mostly to show uncertainties and possibilities than hard numbers!

How do Compact Objects Appear?

The most important factor that determines the observable properties of a compact object is **whether or not it is in a (interacting) binary.**

Isolated White Dwarfs:

Cooling white dwarfs

Isolated Neutron Stars:

Radio pulsars, millisecond radio pulsars, magnetars (AXPs and SGRs), CCOs, nearby dim isolated stars

Isolated Black Holes:

Not visible!!
(except in gravity waves)

Accreting White Dwarfs:

Cataclysmic variables, dwarf novae, …..

Accreting Neutron Stars:

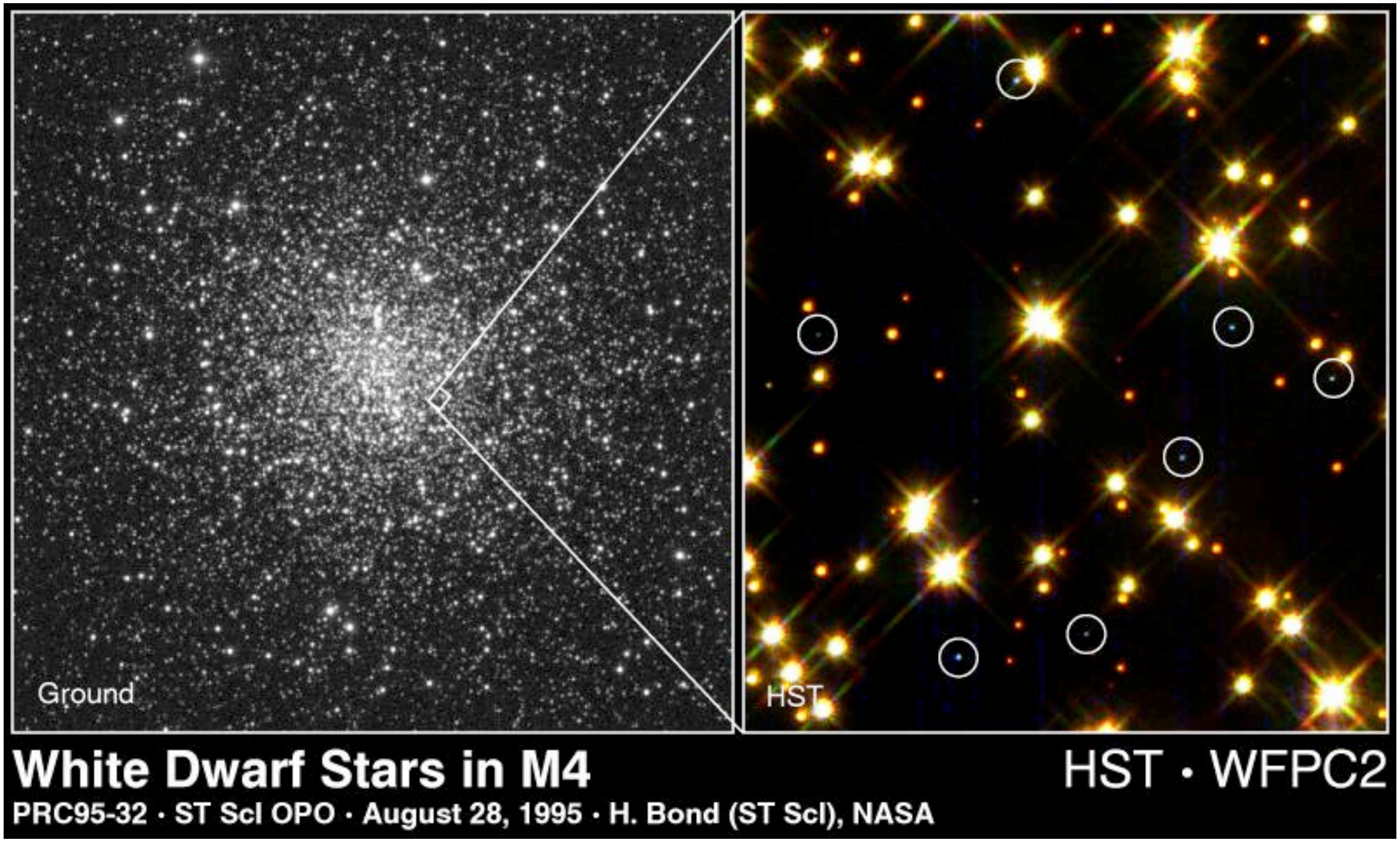
low-mass X-ray binaries,
high-mass X-ray binaries,
bursters

Accreting Black Holes:

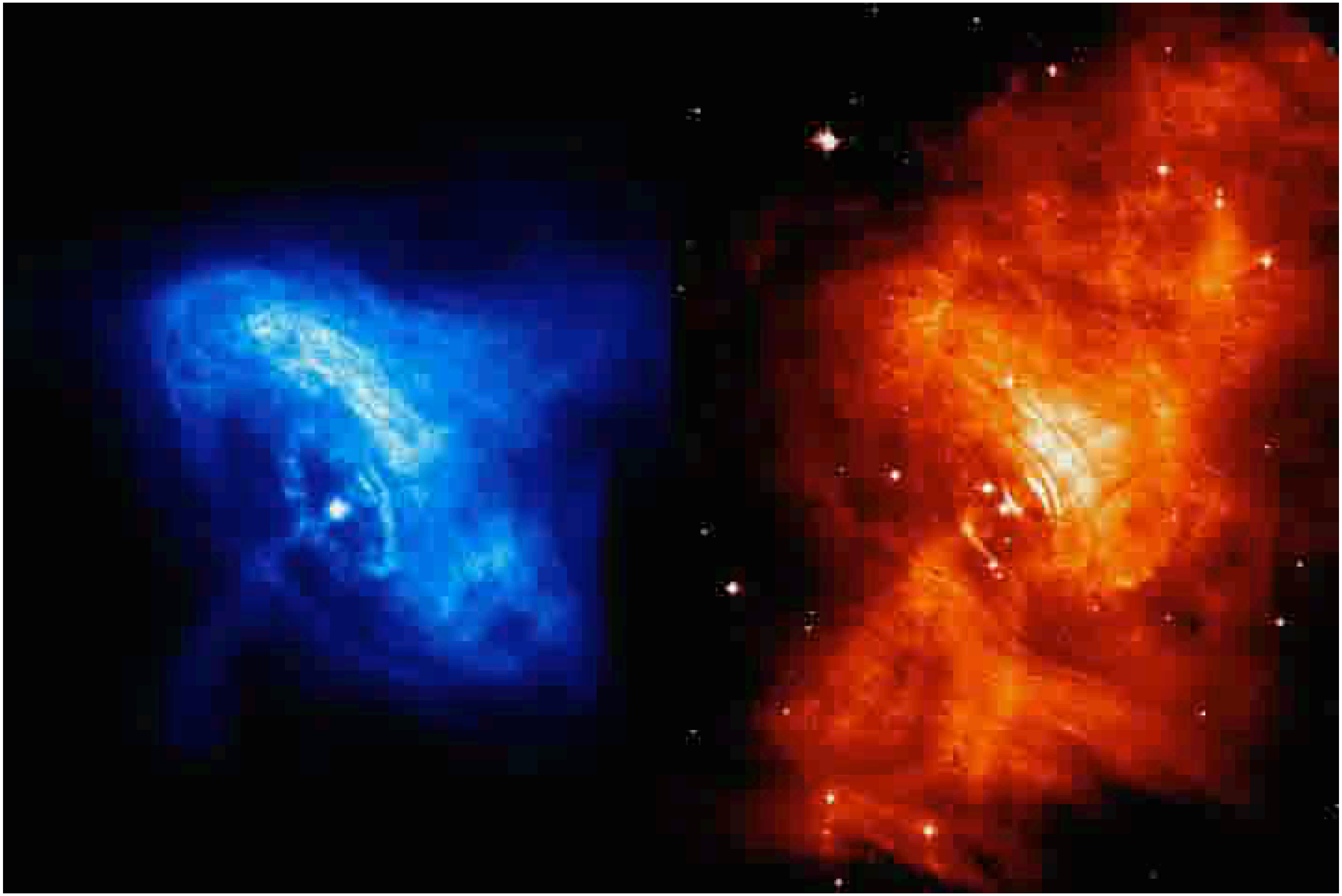
X-ray binaries
AGN, low-luminosity AGN

This is a non-trivial statement because accretion seems to alter the properties of the compact object permanently.

Cooling white dwarfs in globular cluster M4

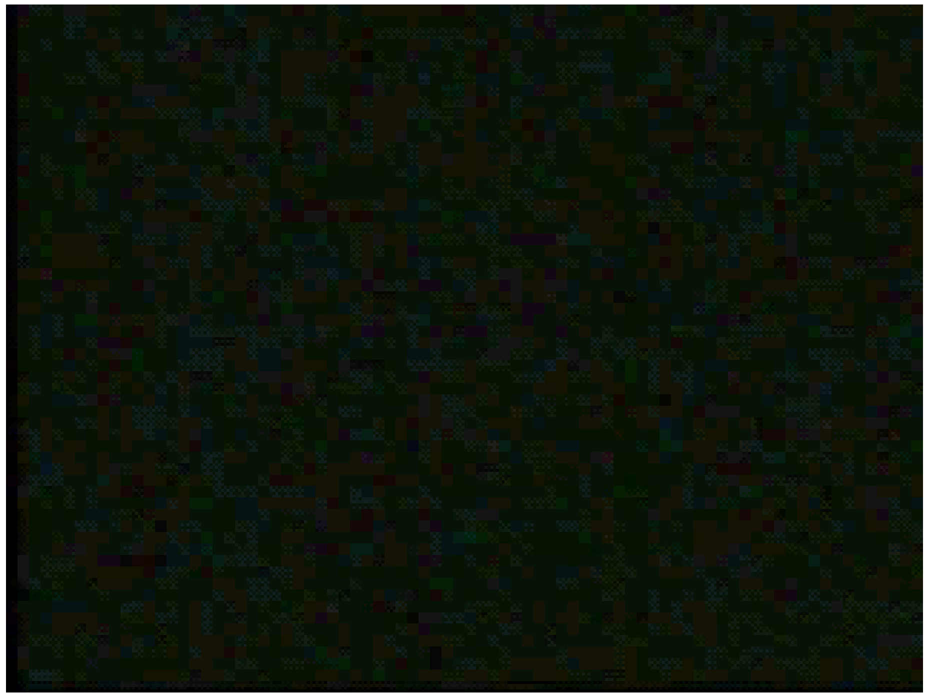


The Crab Pulsar -- A "Prototypical" Rotation Powered Pulsar



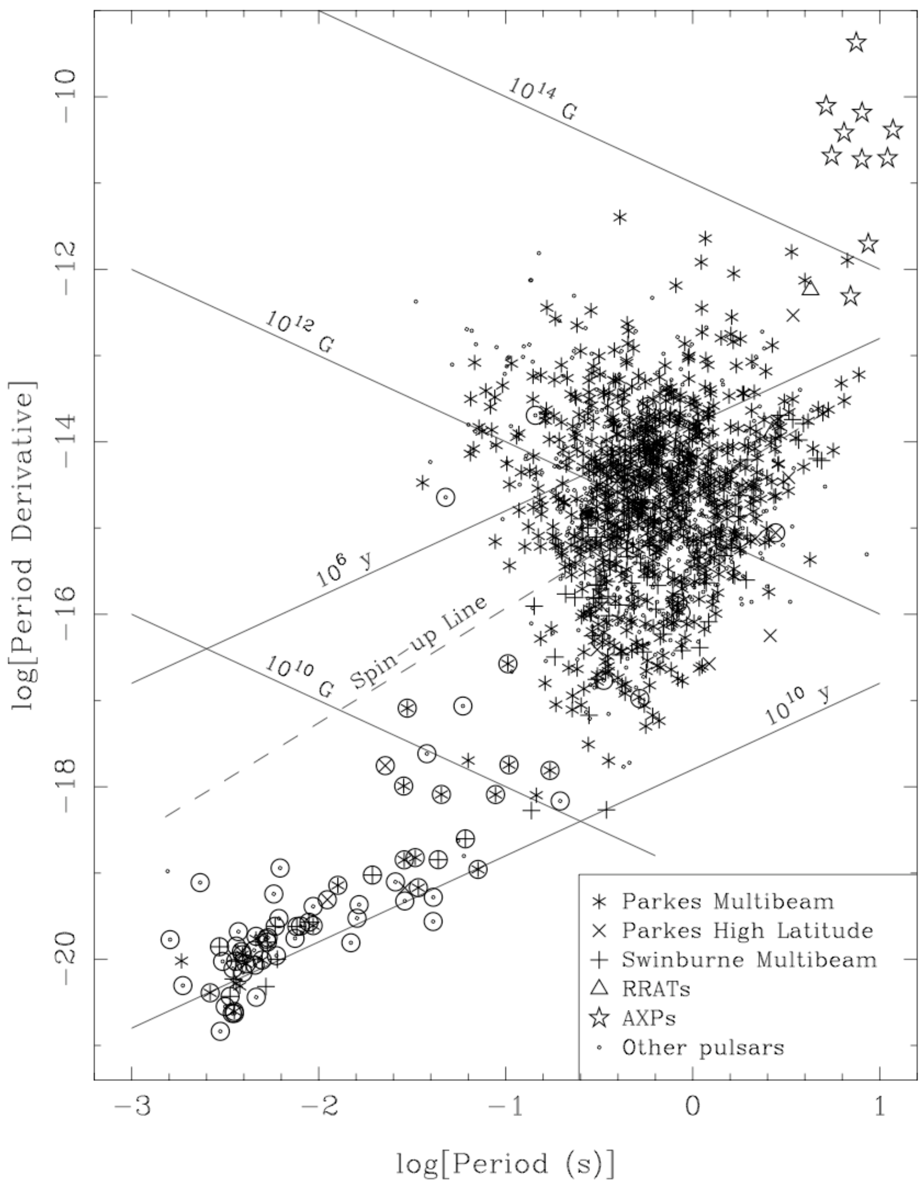
Chandra Hubble

An Accretion Powered Pulsar

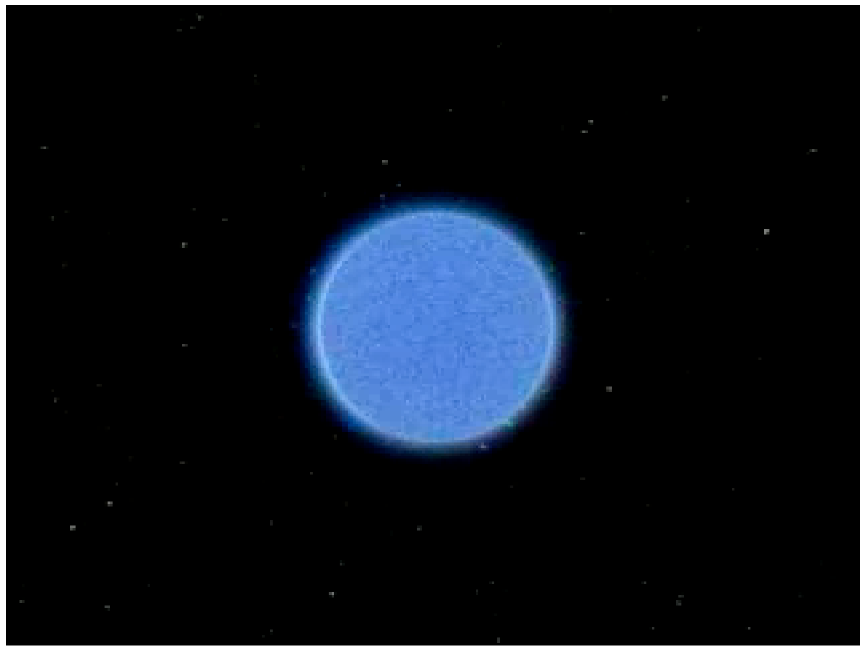


- magnetic field of the primary (the neutron star) channels the flow to the polar caps: an X-ray pulsar
- the angular momentum transported to the neutron star causes it to spin up

P=Pdot diagram of pulsars



A Galactic Black Hole



Our Supermassive Black Hole

- this is the longest Chandra exposure image of our Galactic Center Sagittarius A*
- supermassive black holes feed off of nearby stars and ISM gas
- timescale of flaring events suggests they are occurring near the event horizon