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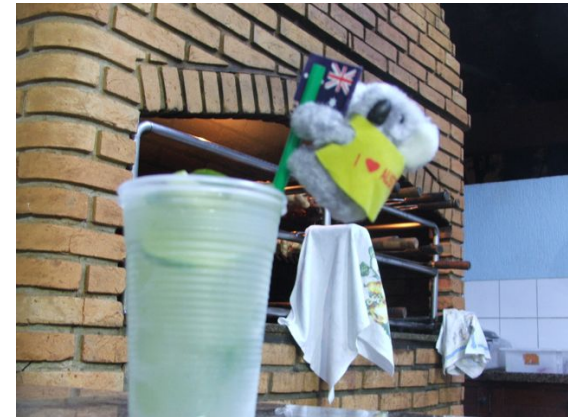
# Pulsars and gravitational waves: 4 Detecting the waves

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# Purpose of this lecture series

- Provide an overview of pulsars
- Provide an overview of gravitational waves
- **Show how, in theory, pulsar observations can be used to detect gravitational waves**
- Describe issues with the current data sets
- Describe unsolved problems
- Provide enough information that you can process pulsar observations and develop tools to search for gravitational waves



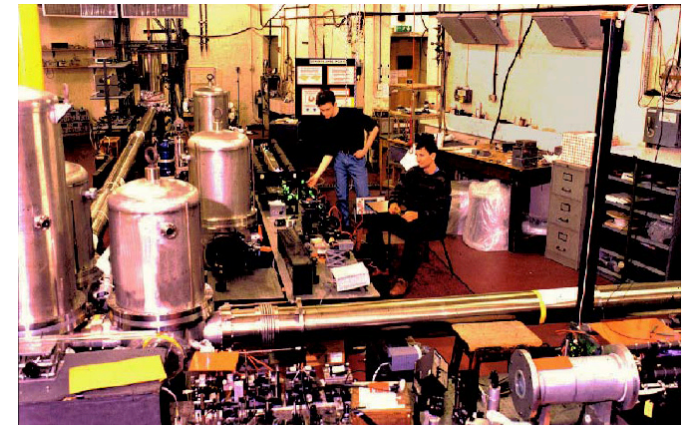
# Attempts to detect gravitational waves



Resonant bars  
1960s



Doppler tracking  
1970s onwards



Interferometers  
~1977 onwards

# LIGO and VIRGO

- *“To achieve its goal, LIGO must detect movements as small as one thousandth the diameter of a proton, which is the nucleus of a hydrogen atom.”*
- *“Achieving this degree of sensitivity requires a remarkable combination of technological innovations in vacuum technology, precision lasers, and advanced optical and mechanical systems”*
- Sensitive to  $10^2$  Hz gravitational waves.



Hanford, Washington



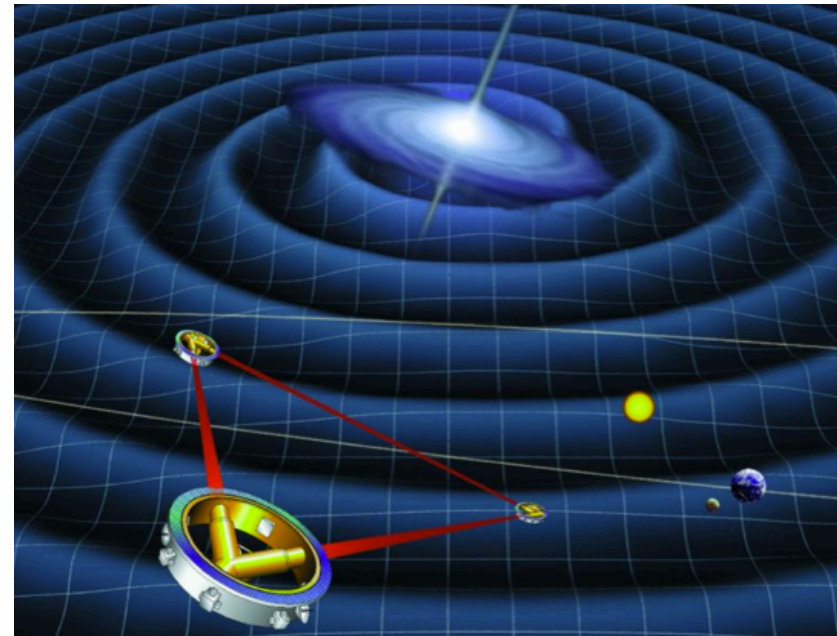
Livingston, Louisiana



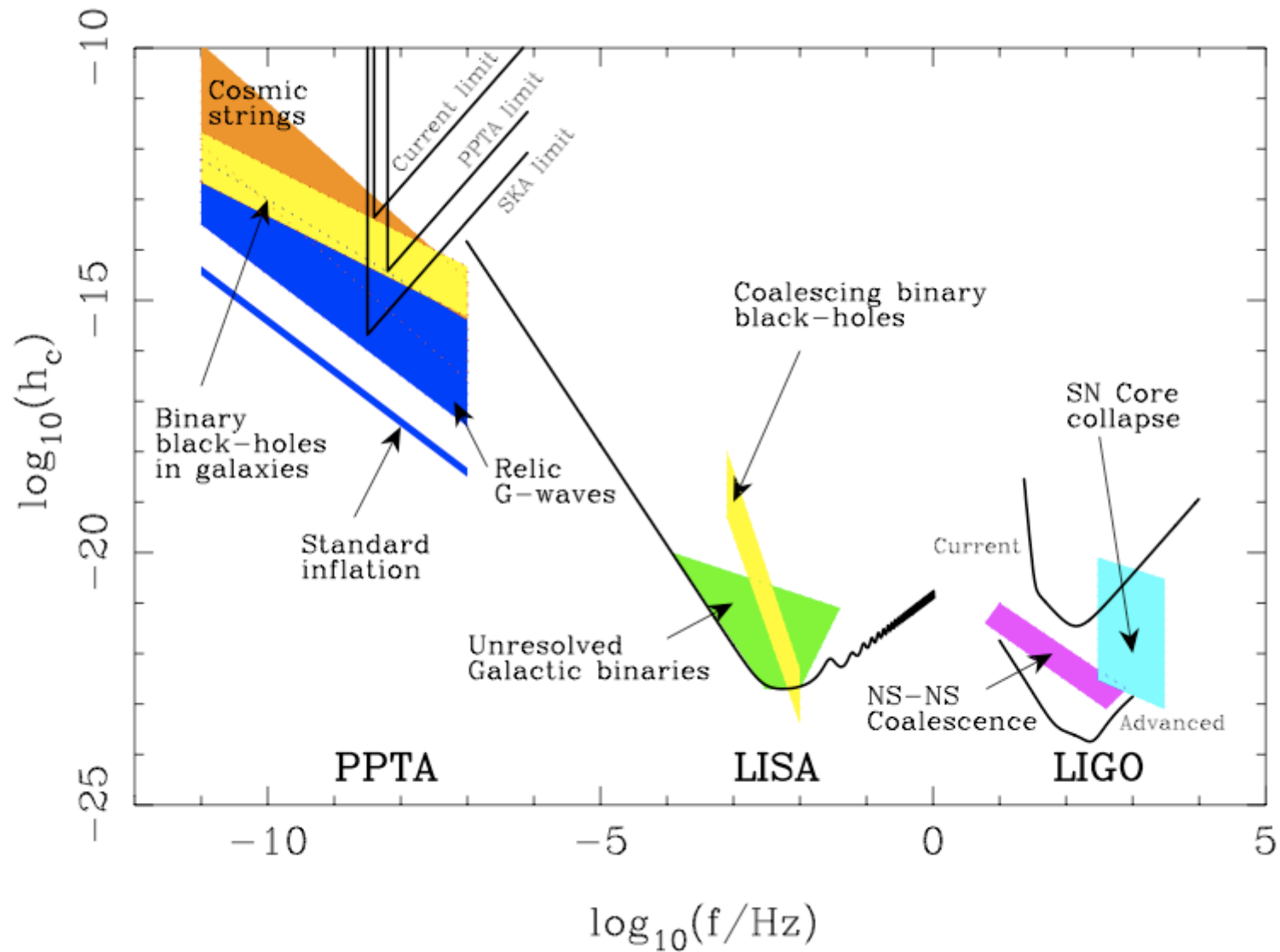
Eurelios 2000

# LISA

- LISA: 5-million-km arms
- Sensitive to  $10^{-3}$  Hz gravitational waves
- USA funding cut
- ESA continuing
- “A future minor role for NASA in the ESA-led mission has not been ruled out.” – LISA website
- “After studying several configurations, a new baseline for transfer, orbit and layout has been identified that will be refined in the coming month with the help of European industry. The new baseline employs less costly orbits, and simplifies the design of LISA by reducing the distance between the satellites and employing four rather than six laser links.”

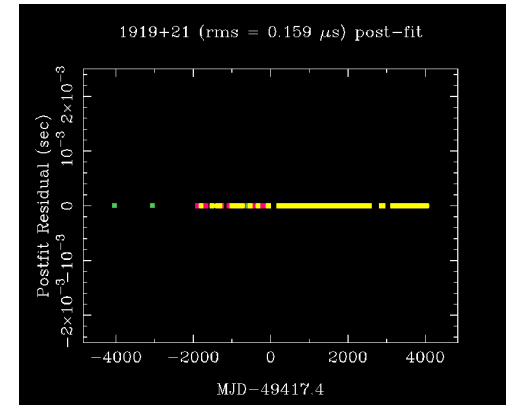


# The spectrum of the gravitational wave experiments



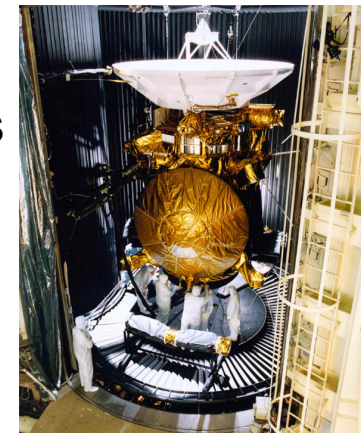
# Estabrook & Wahlquist, Detweiler, Sazhin (~1979)

- Pulsar timing experiments may allow detection of gravitational wave signals - stochastic background or single sources
- Sensitive to gravitational wave wavelengths comparable with the observing data length (i.e GW frequency  $\sim$ nHz)
- However, the expected induced GW signal is small!



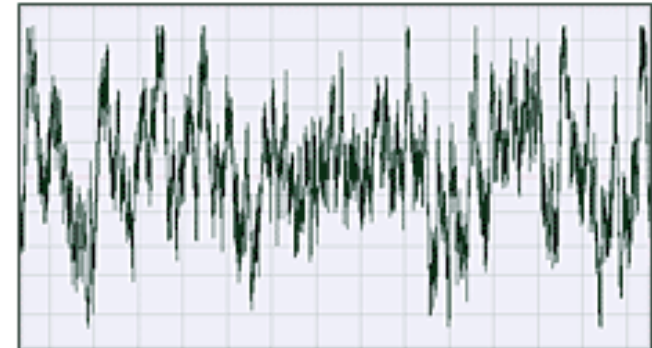
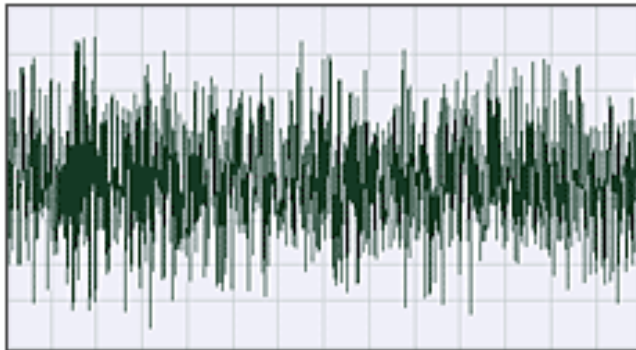
Simulated GW background signal ...  
it's very weak!

Much analysis  
came from doppler  
tracking of  
satellites



# The problem

- We wish to detect a signal that is:
  - 1) correlated between different pulsar data sets
  - 2) is very weak compared with the measurement error
  - 3) has a “red noise” spectrum (for a background) or is sinusoidal (for a single source)



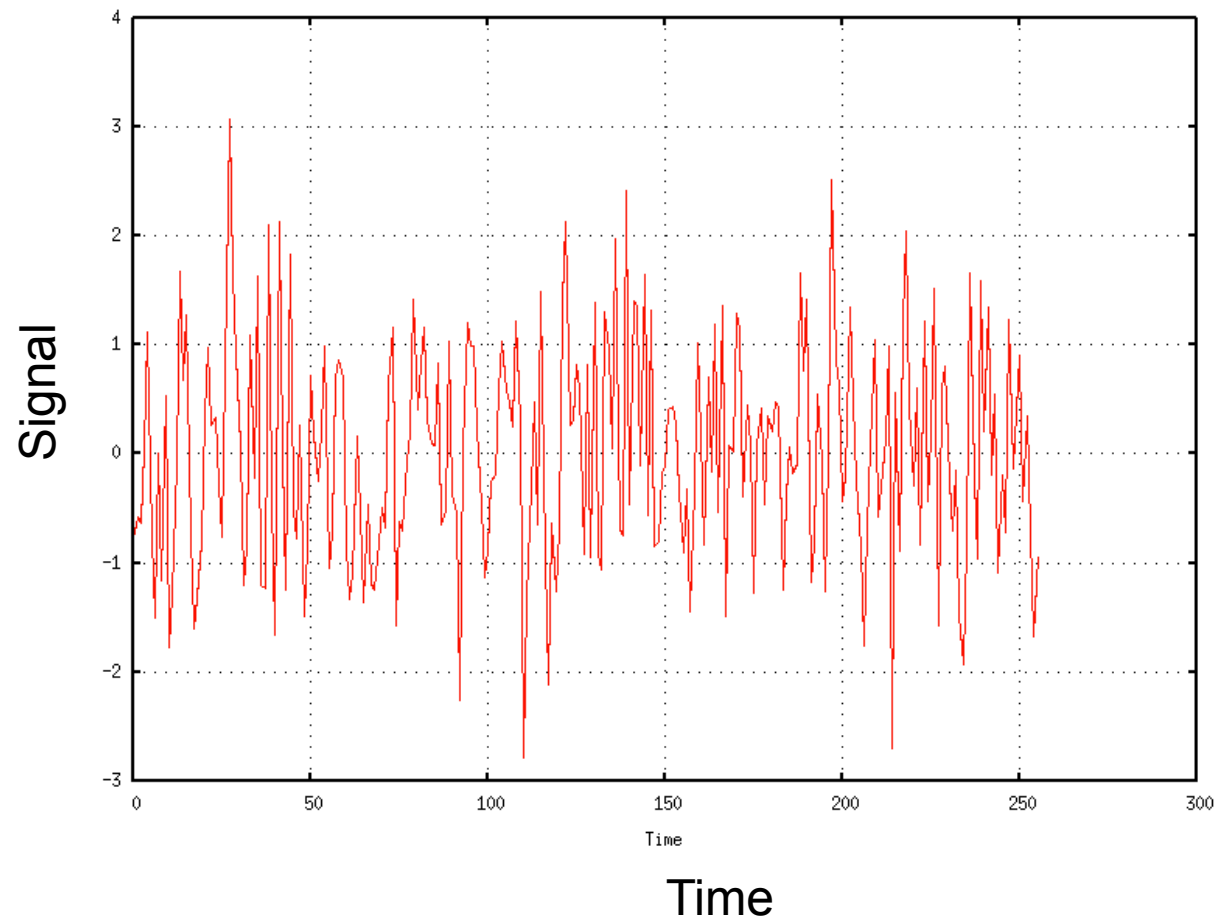
Our datasets are like this!

# Detecting gravitational waves

- How do you look for a signal within a dataset?

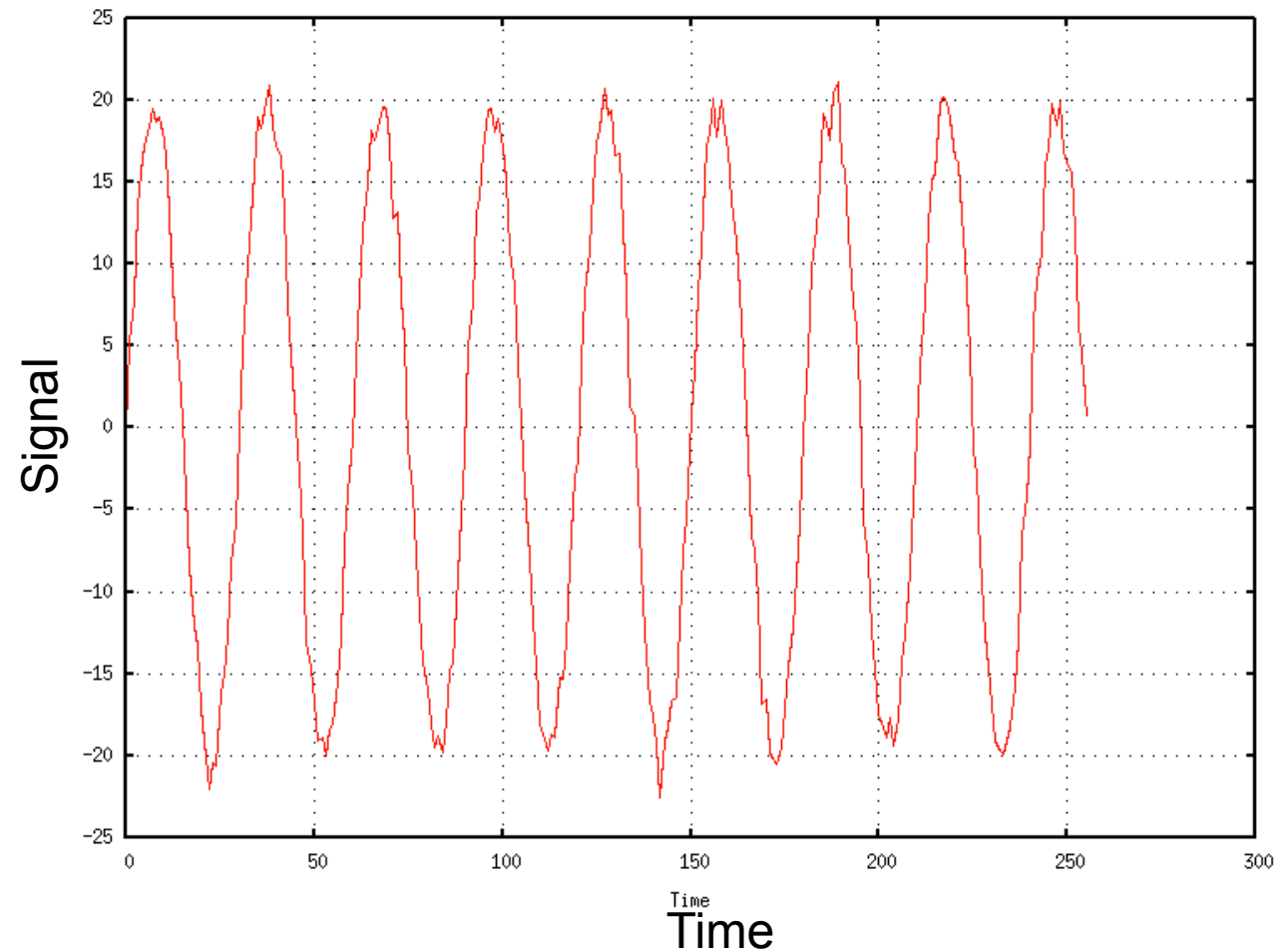
# Looking for a sinusoid within a time series

- Signal without a sinusoid



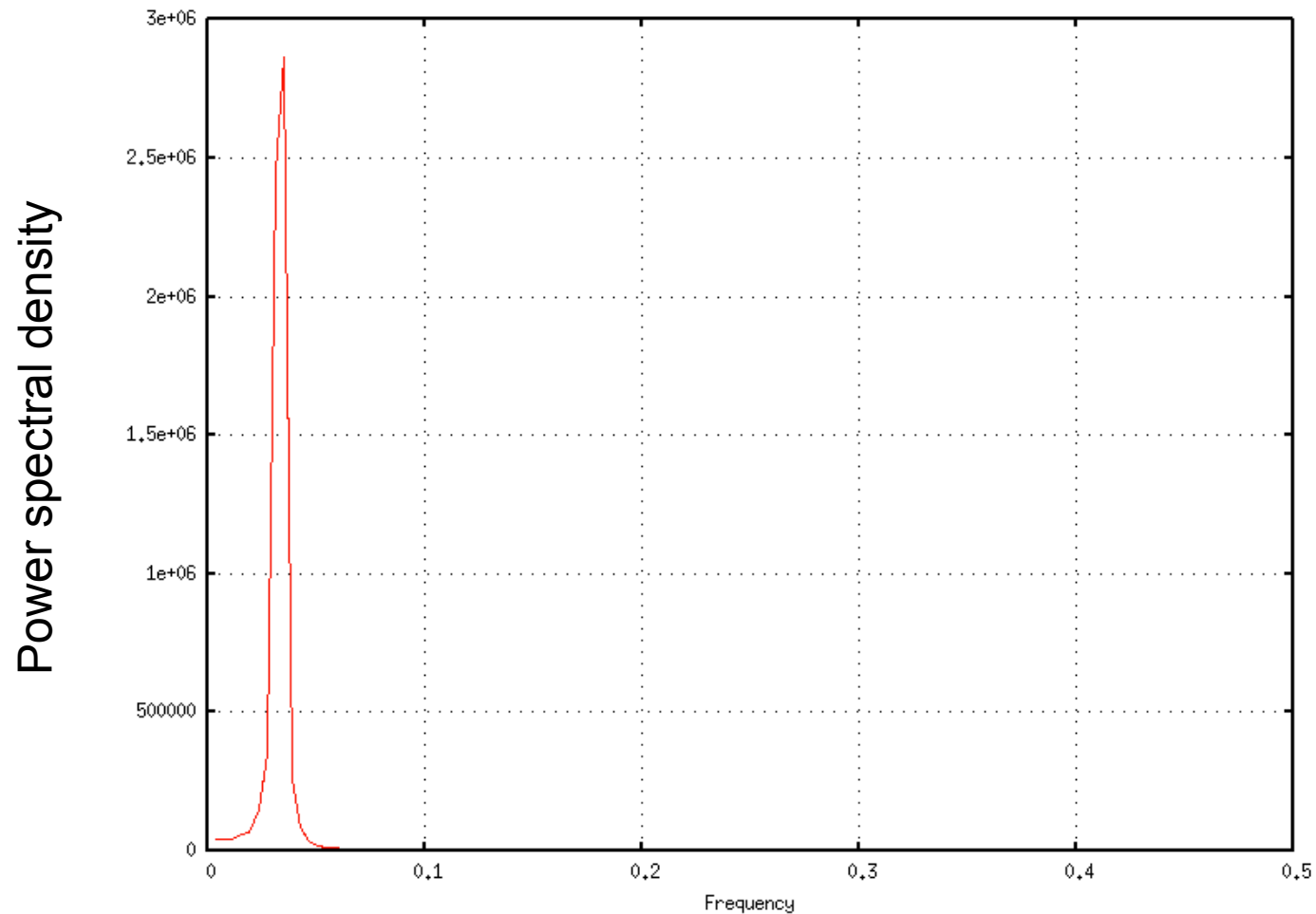
# A bright signal is easy to find

- Signal + strong sinusoid



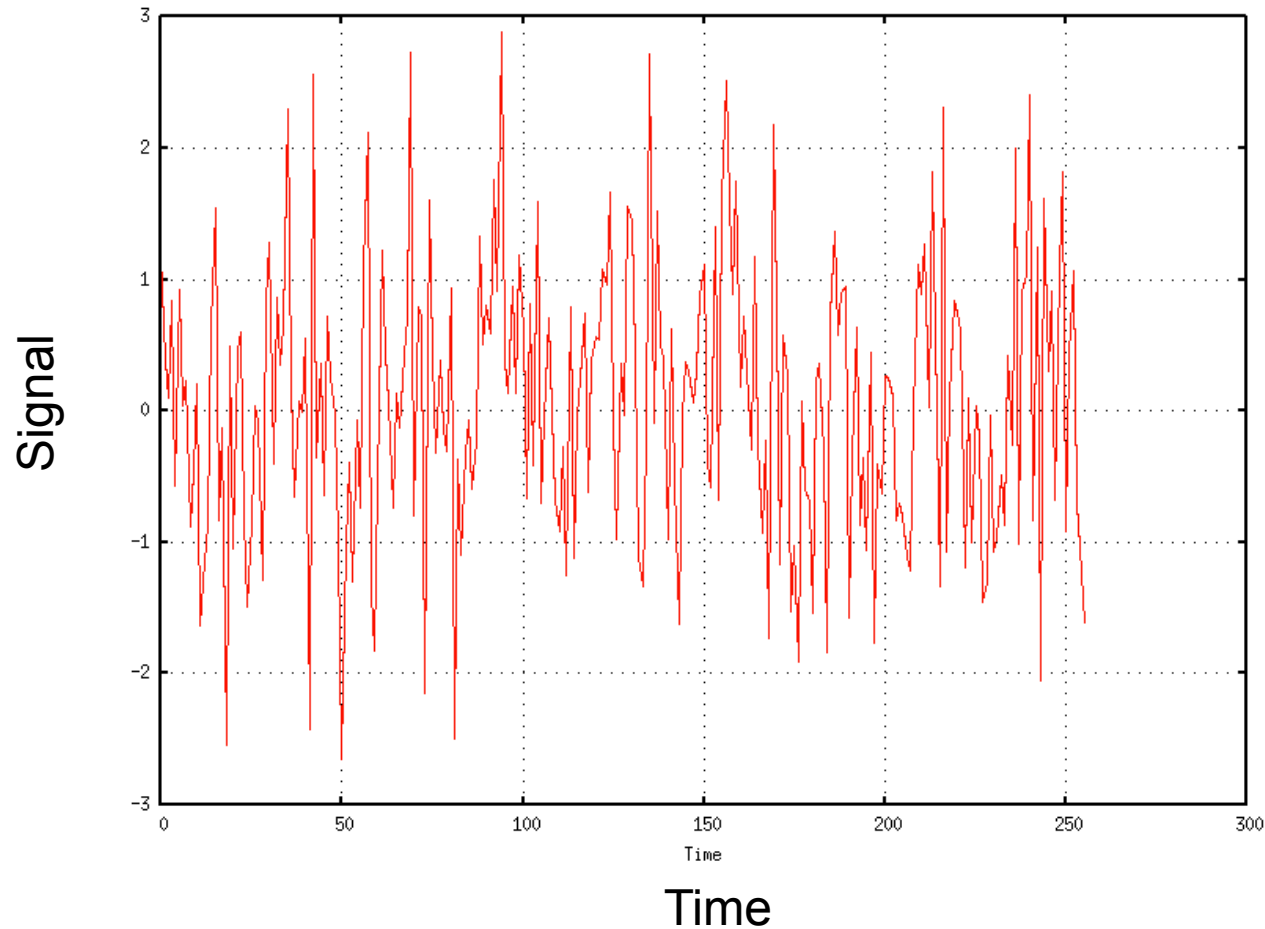
# Power spectrum

- Power spectrum

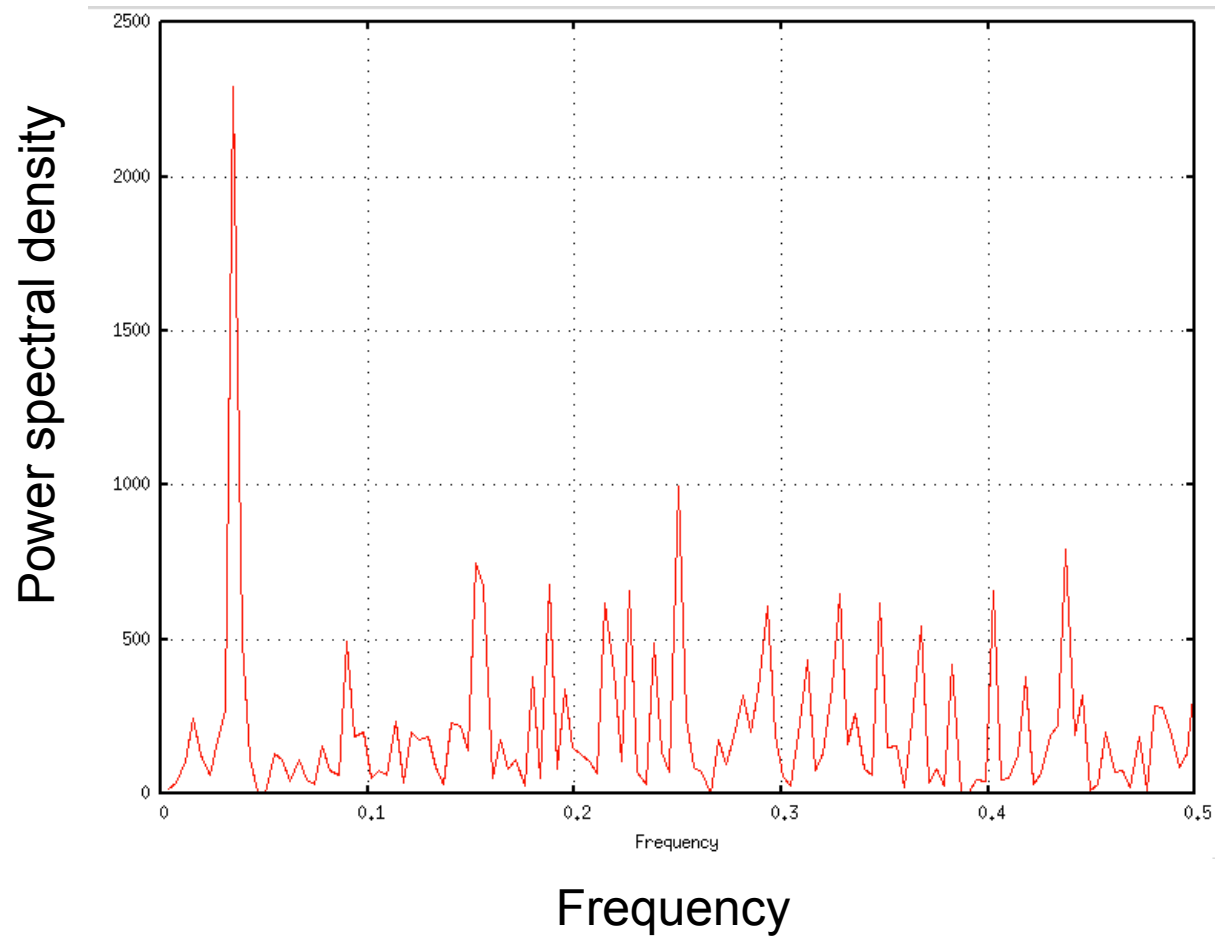


# A small signal is not so easy to find

- Low amplitude Sinusoid (amplitude = 0.5)



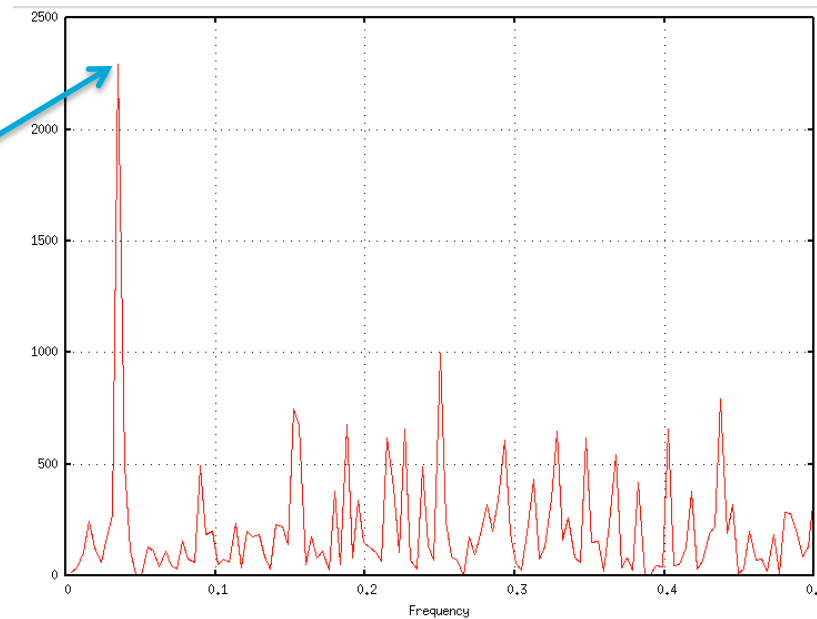
- Low amplitude sinusoid (amplitude = 0.5)



# Monte-carlo approach to detection

- Develop a “statistical parameter” that represents what you’re looking for.
- $S$  = maximum value in the power spectrum
- Record this value for the real data (amplitude of sinusoid in data = 0.5,  $S = 2327$ )

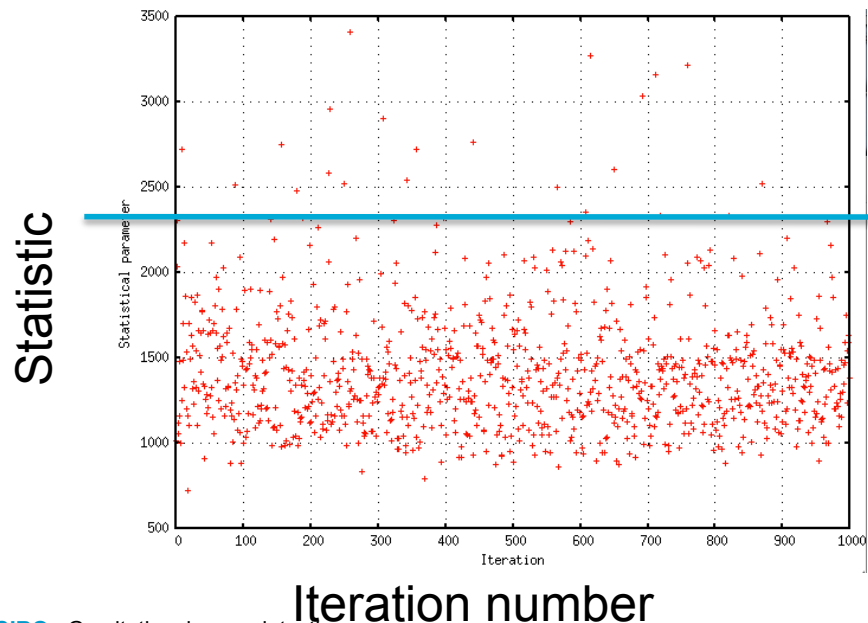
$$S_{\text{act}} = 2327$$



Frequency

# Monte-Carlo approach to detection

- Create a simulated data set that has the same statistical property as the actual data, but does not include the signal
- Measure the statistical parameter for the simulated data. Repeat many times
- Determine the probability that the measured value could have occurred by chance!
- If this probability is high then do not claim a detection!

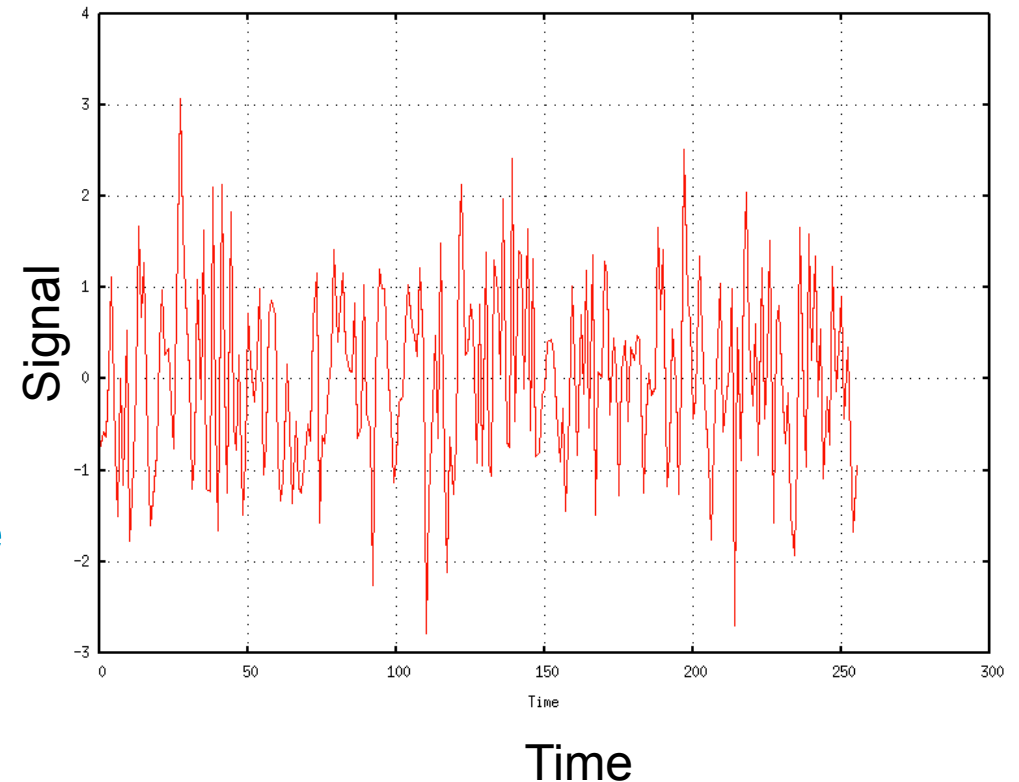


22 times out of 1000  
gives a false detection  
of this signal (2%)

Would you believe it?

# Limits versus detection

- Question 1: “Is there a sinusoidal signal in my data – which is definitely a sinusoid?”
- Question 2: “What is the maximum amplitude of a sinusoidal signal that could be hiding in my data?”
- Example: What limit can be placed on the amplitude of a sinusoid at a given frequency?



# A limit

- Obtain a reasonable statistical parameter from the real data (e.g., power at a given frequency)
- Simulate a data set with a sinusoid at given frequency of amplitude  $A$
- Repeat a large number of times and determine the percentage of simulations that produce a statistic larger than the real data
- If more than 95% of the simulations produce  $S > S_{\text{act}}$  then decrease  $A$  and repeat
- If fewer than 95% of the simulations produce  $S > S_{\text{act}}$  then increase  $A$  and repeat
- Record  $A$  that gives  $S > S_{\text{act}}$  for 95% of the realisation
  
- **This  $A$  will be smaller than the amplitude of a sinusoid that could be “detected” with high confidence**

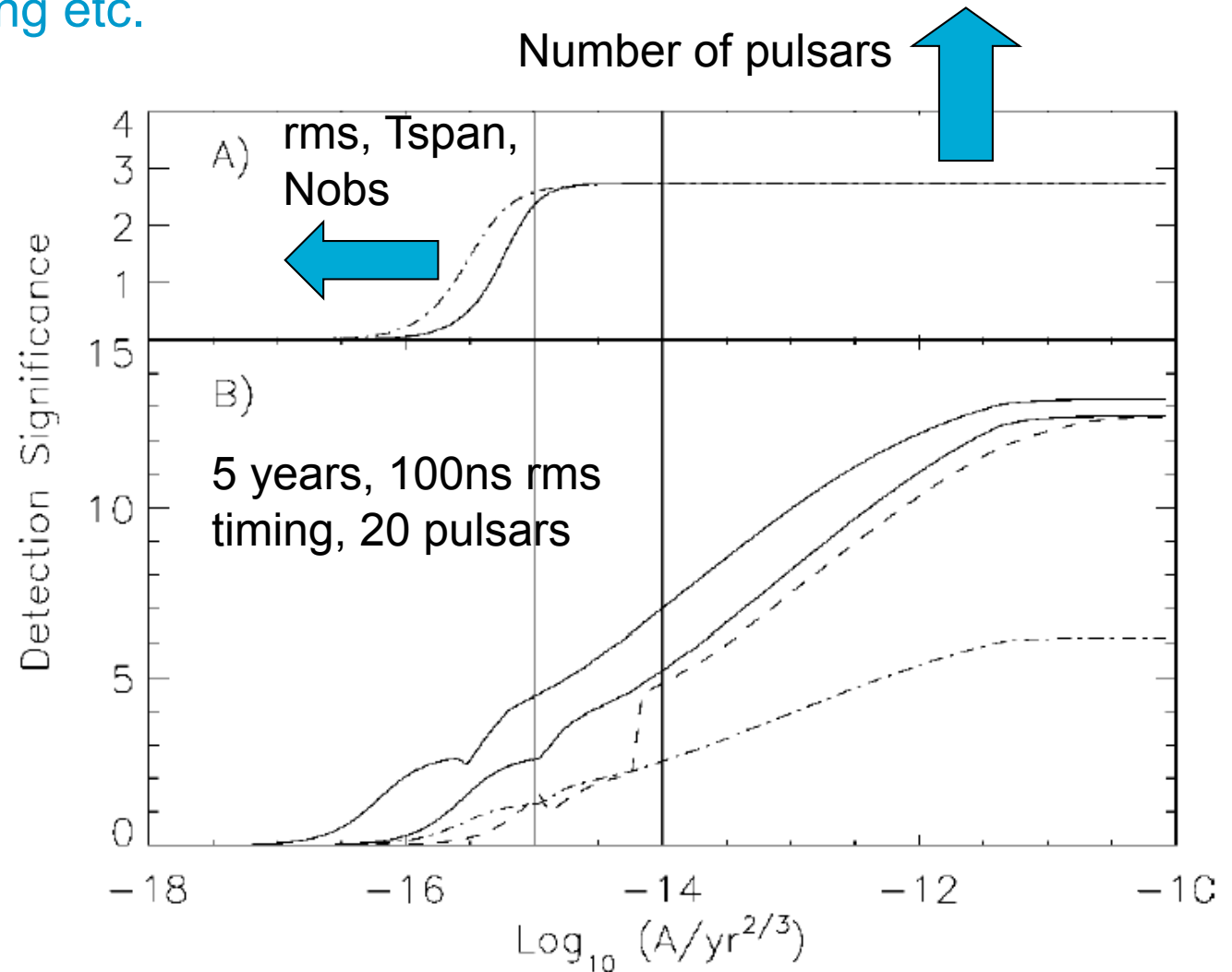
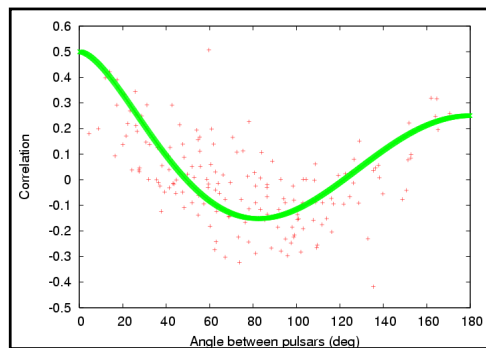
# Defining the question

- What is the probability that the signal in my data is caused by a gravitational wave?
- What is the largest gravitational wave that could be in my data (without me knowing about it)?
- What is the largest gravitational wave with a frequency of  $1 \times 10^{-8}$  Hz?
- What is the largest gravitational wave of any frequency?
- *Some papers in the literature have not got this completely correct!*
- Note: can use “frequentist” or “Bayesian” methods to answer such questions.

# The basic ideas – Jenet et al. (2005)

- Use simulated data. Assume all data sets are “white” and have same sampling etc.

Calculate how well can you measure the expected angular correlation for a gravitational wave background?

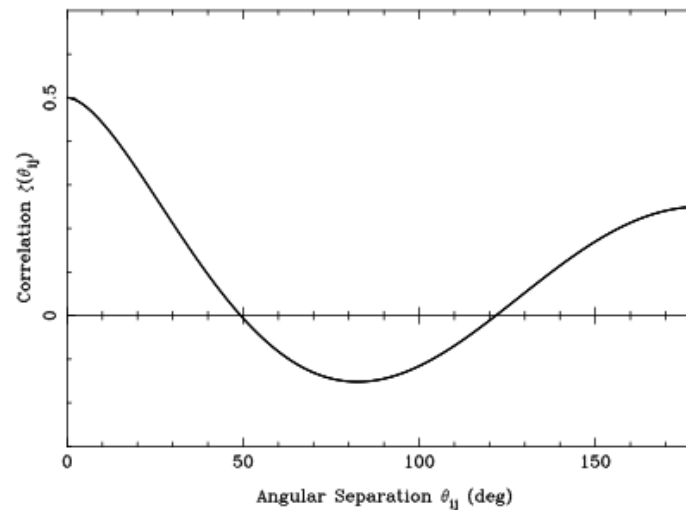


# Summary of Jenet et al. (2005)

- For detection:
- Need at least 20 pulsars
- Need to have data spanning  $> 5$  years (with approximately 1 observation every 2 weeks)
- Need rms timing residuals  $< 100$ ns
- Must deal the expected “red” noise
- Don't yet have the data sets that achieve the required sensitivity

# Yardley et al. (2011)

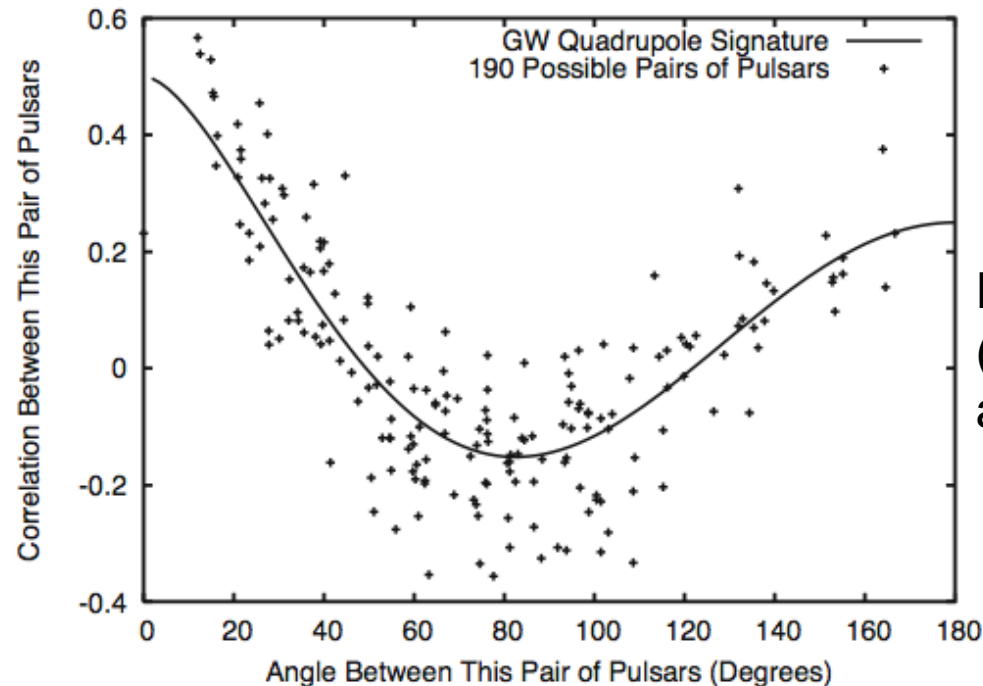
- Use 20 pulsars from the early Parkes data
- For all pulsars, the GWB will induce timing residuals with a steep red power spectrum
- The induced residuals are correlated between different pulsar pairs.
- Used a **frequentist** approach



# Detecting the GWB signal

- The GWB signal induces correlated residuals between pulsars.
- For an array of 20 pulsars, there are 190 different pulsar pairs.

!!!Simulated  
GWB Signal!!!



Hellings & Downs  
(1983), Jenet et  
al. (2005)

- Hence we can detect the GWB if we can detect this variation of the correlation between the residuals of each pulsar pair.

# Yardley et al. (2011) method

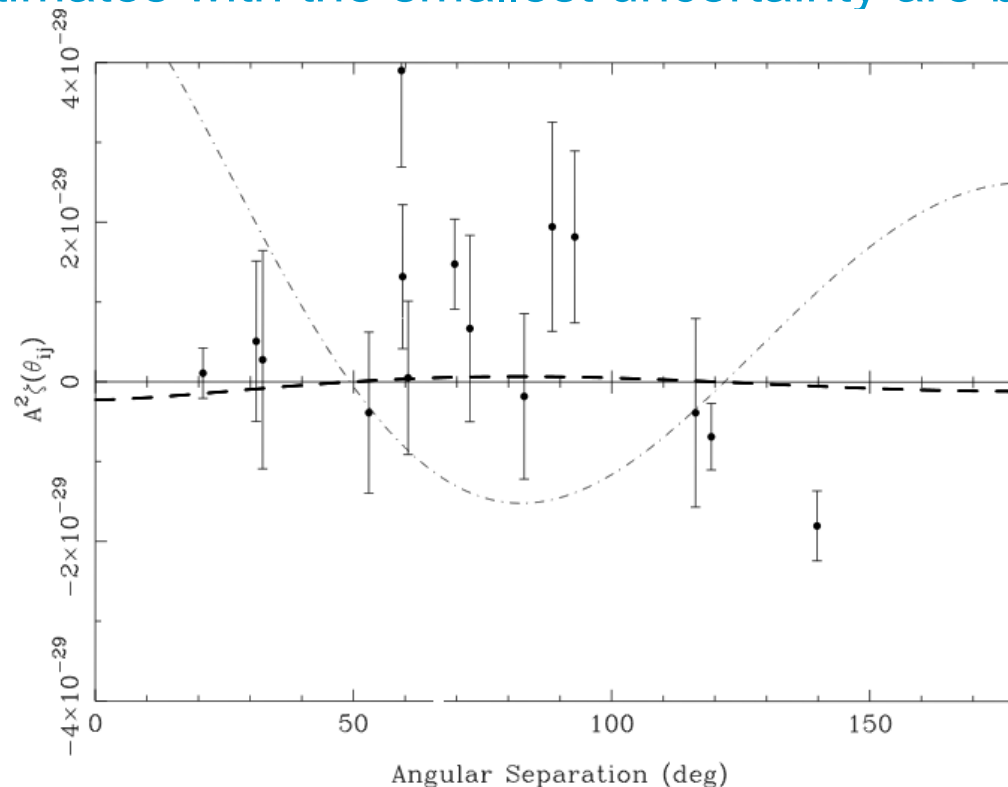
- Difficult to determine cross-correlation as the data sets are irregularly sampled and each observation has a different uncertainty.
- Yardley method:
  - Obtain a pair of pulsars (i and j) and determine the overlapping data span
  - Remove a quadratic polynomial fit to the region of the overlapping data span for each pulsar
  - Form the power spectra for each pulsar  $P_i(f)$  and  $P_j(f)$
  - Determine the cross power spectrum:  $X_{ij}(f) = P_i(f)P_j(f)^*$
  - Sum the cross power spectrum to obtain the **zero lag covariance**
  - Determine the amplitude of the GWB that would give that zero lag covariance. Calculate a weighted mean over all pairs to obtain the amplitude
  - Remove biases by Monte Carlo simulation – simulate (using tempo2) a GWB with given amplitude and compare with the resulting amplitude from this method

# Technique for detecting the GWB signal in a pulsar timing array

- For a subset of the Parkes Pulsar Timing Array observations, the 15 covariance estimates with the smallest uncertainty are below:

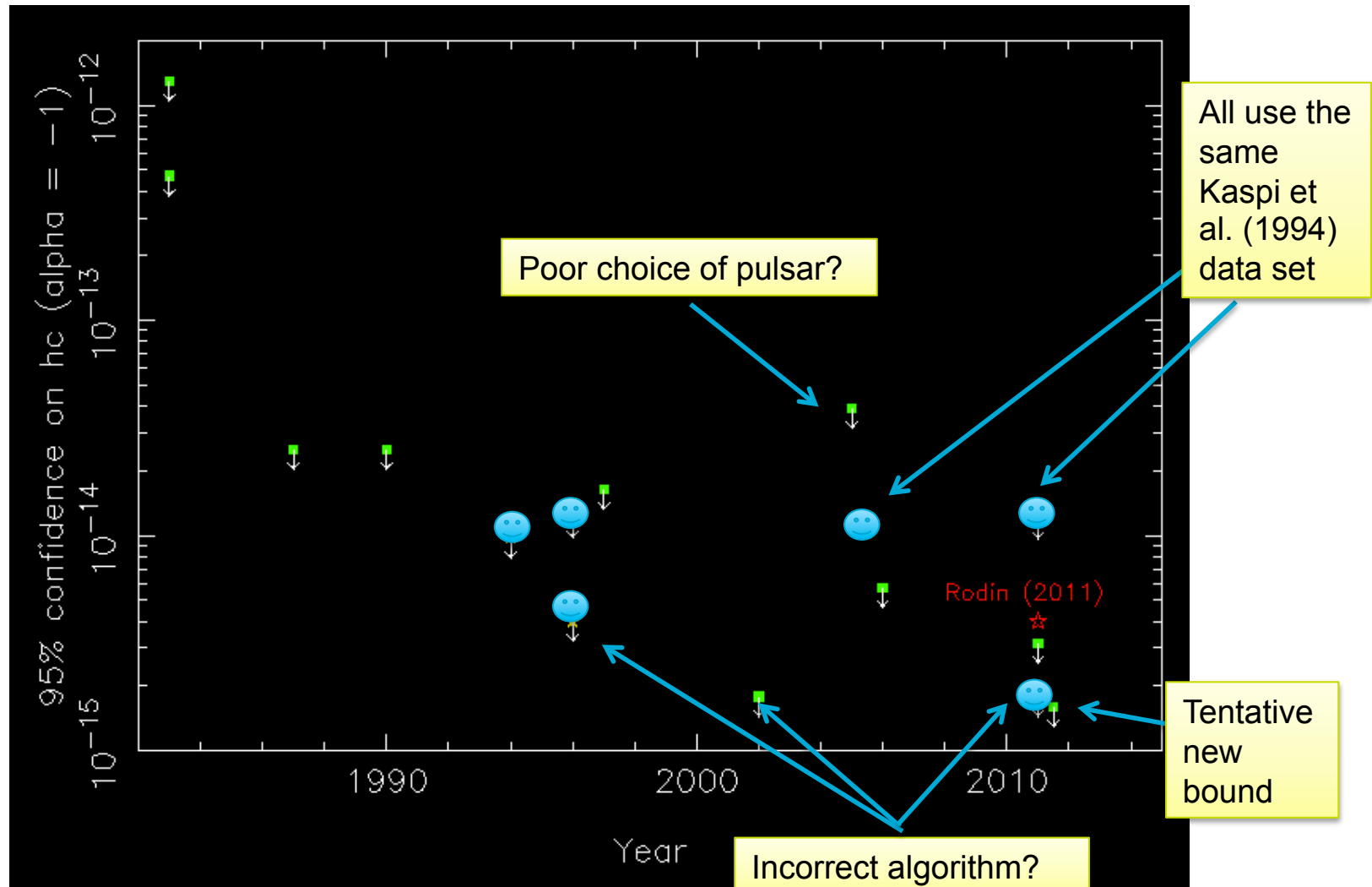
NB! The y-axis is “correlation” times “amplitude squared”.

Yardley et al. (2011)



- We have not made a detection of the GWB signal.

# Published limits on gravitational wave background (95% confidence)

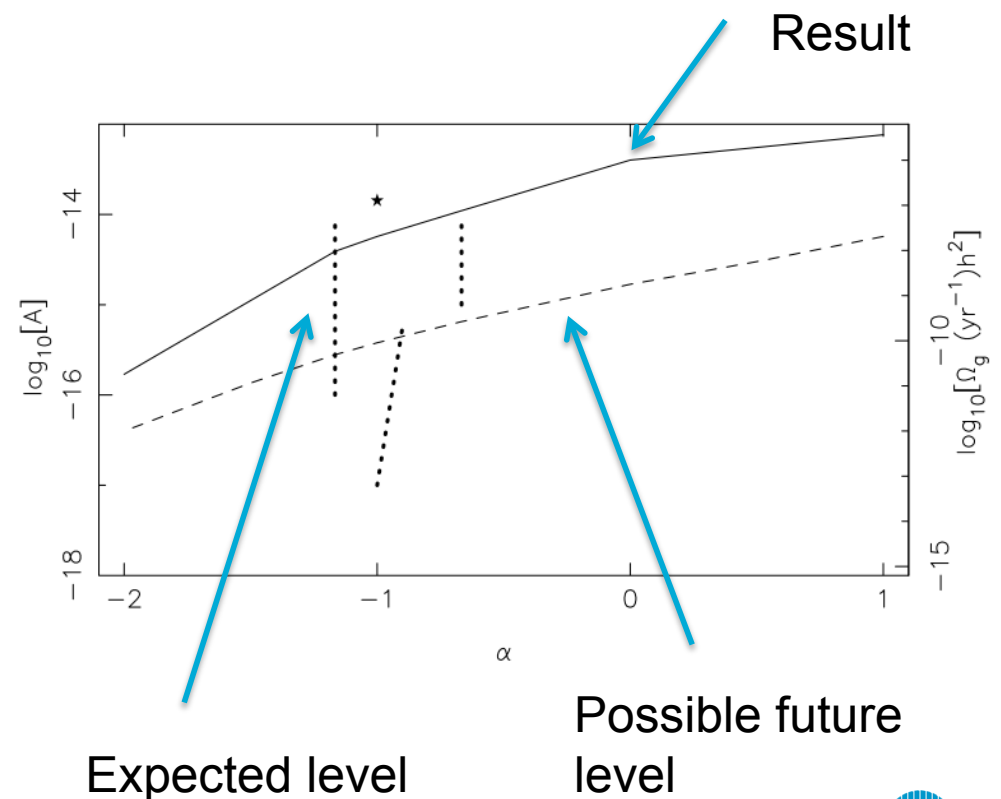


☹ Use same data set

# The Jenet et al. (2006) method

- Use pulsars from the Parkes pulsar timing array project that have “white” timing residuals
- Inject a simulated gravitational wave background
- Increase/decrease the gravitational wave amplitude until it is detectable 95% of the time
- Problem: only applicable to white data sets (almost no data sets are white)

$$h_c(f) = A \left( \frac{f}{\text{yr}^{-1}} \right)^\alpha.$$

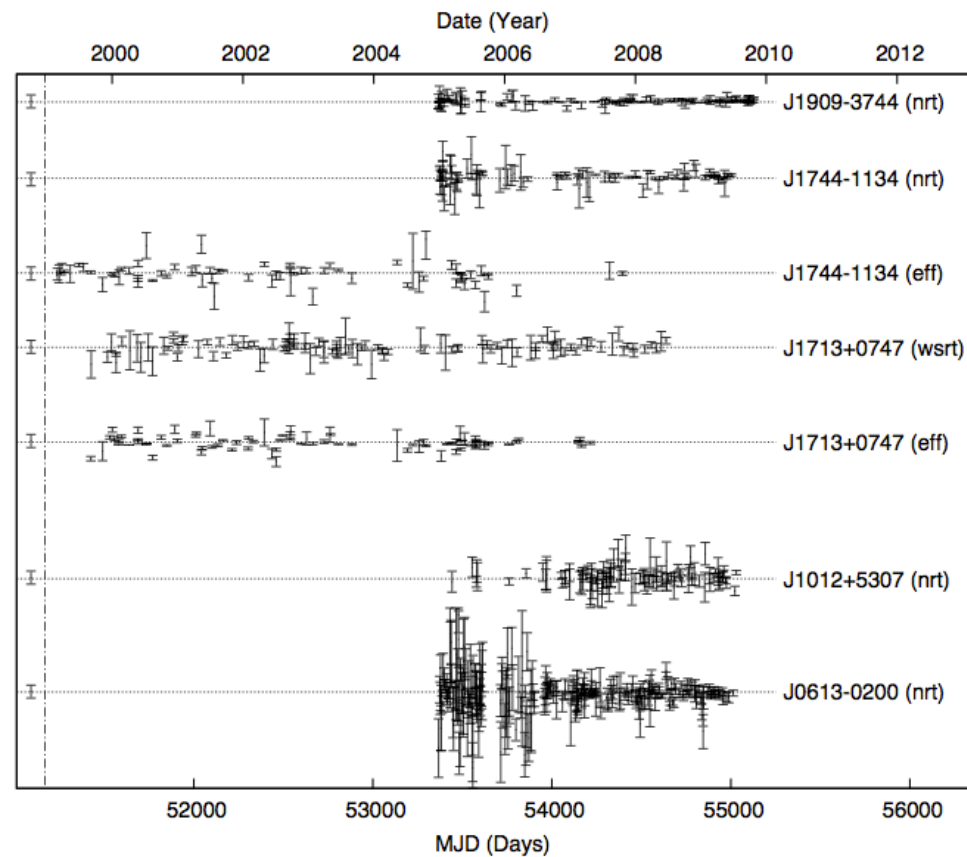


# Implications from the Jenet et al. (2006) result

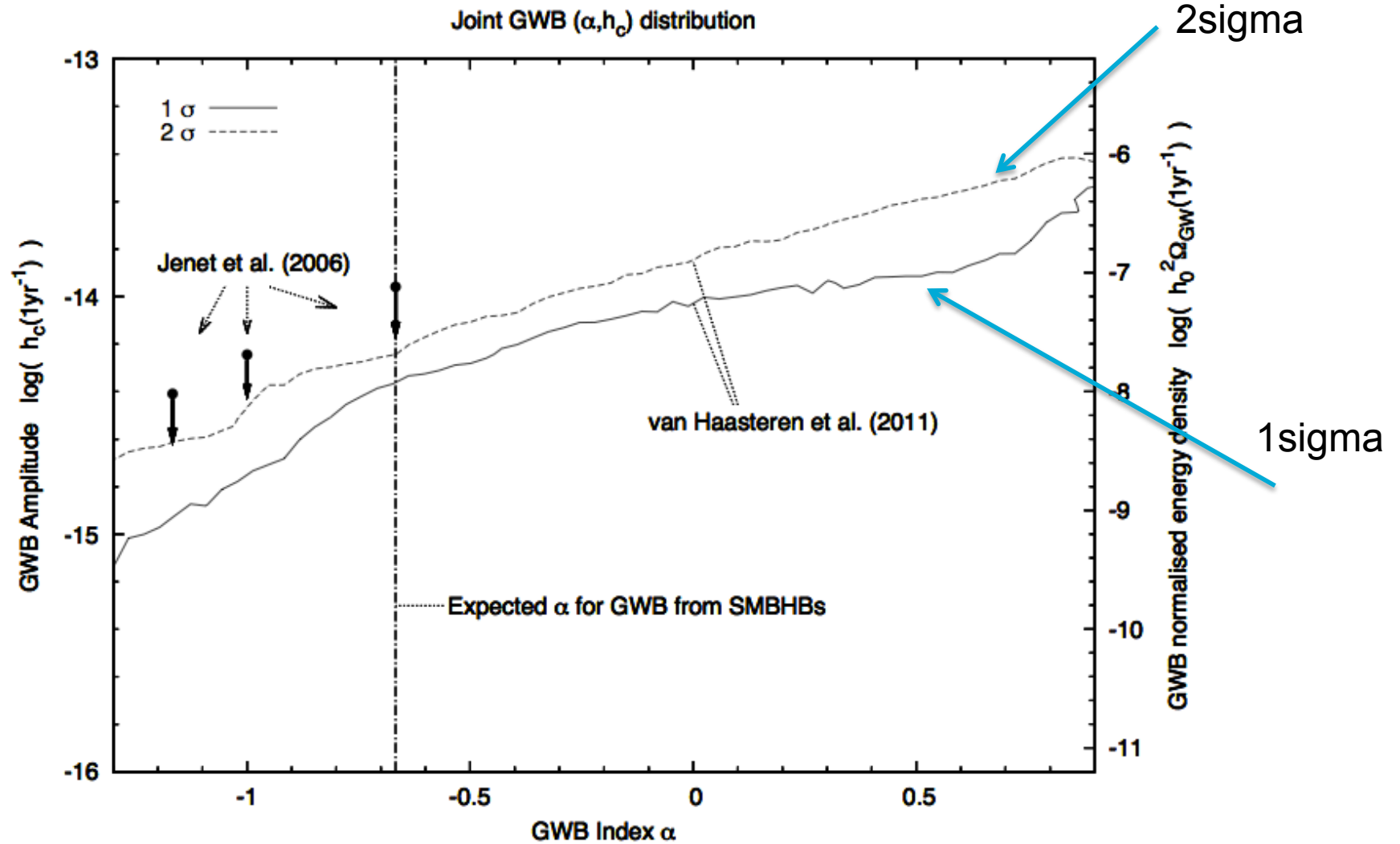
- 135 citations to paper (so far) on topics such as:
- “Prospects for detecting dark matter halo substructure with pulsar timing”
- “Search for cosmic strings in the COSMOS survey”
- “Signals of Inflationary Models with Cosmic Strings “
- “Constraint on the early Universe by relic gravitational waves: From pulsar timing observations”
- “Constraining the Coalescence Rate of Supermassive Black-hole Binaries Using Pulsar Timing”
- “Gravitational-Wave Constraints on the Abundance of Primordial Black Holes”
- ...

# Van Haasteren et al. (2011)

- Developed Bayesian detection/limit method
- Apply to European Pulsar Timing Array data sets



# Van Haasteren et al. (2011)



# Other gravitational wave background methods

- Funke (1978) “Possible detection of gravitational waves using correlation techniques”
- Bertotti, Carr & Rees (1983) “Limits from the timing of pulsars on the cosmic gravitational wave background”
- Hellings & Downs (1983) “Upper limits on the isotropic gravitational wave background from pulsar timing analysis”
- Romani & Taylor (1983) “An upper limit on the stochastic background of ultralow-frequency gravitational waves”
- Stinebring, Ryba, Taylor & Romani (1990) “Cosmic gravitational-wave background – Limits from millisecond pulsar timing”
- Mashhoon & Scitz (1991) “Pulsar timing and upper limits on a cosmic background of gravitational waves”
- Kaspi, Taylor & Ryba (1994) “High-precision timing of millisecond pulsars”
- Thorsett & Dewey (1996) “Pulsar timing limits on very low frequency stochastic gravitational radiation”
- McHugh, Zalamansky, Verotte & Lantz (1996) “Pulsar timing and the upper limits on a gravitational wave background”
- Kopeikin (1997) “Binary pulsars as detectors of ultralow-frequency gravitational waves”
- ... Zalamansky, Robert, Verotte & Taris (1997); Lommen & Backer (2001); Jenet, Hobbs, Lee & Manchester (2005); Jenet et al. (2006); Anholm et al. (2008); van Haasteren et al. (2008); Finn & Lommen (2010); van Haasteren et al. (2011); Yardley et al. (2011); Rodin (2011) ...

# Our current method (Shannon et al., in preparation)

- Must deal with red noise in the pulsar data
- Must account for the different fitting, data spans, measurement error for different pulsars
  
- Step 1: Obtain power spectra for each pulsar data set
- Step 2: Determine the low-frequency power spectral density (weighted average for different pulsars)
- Step 3: Simulate data with same white noise + a gravitational wave background
- Step 4: Increase amplitude of the background until the low-frequency power spectral density is greater than the measured value

# Our current method

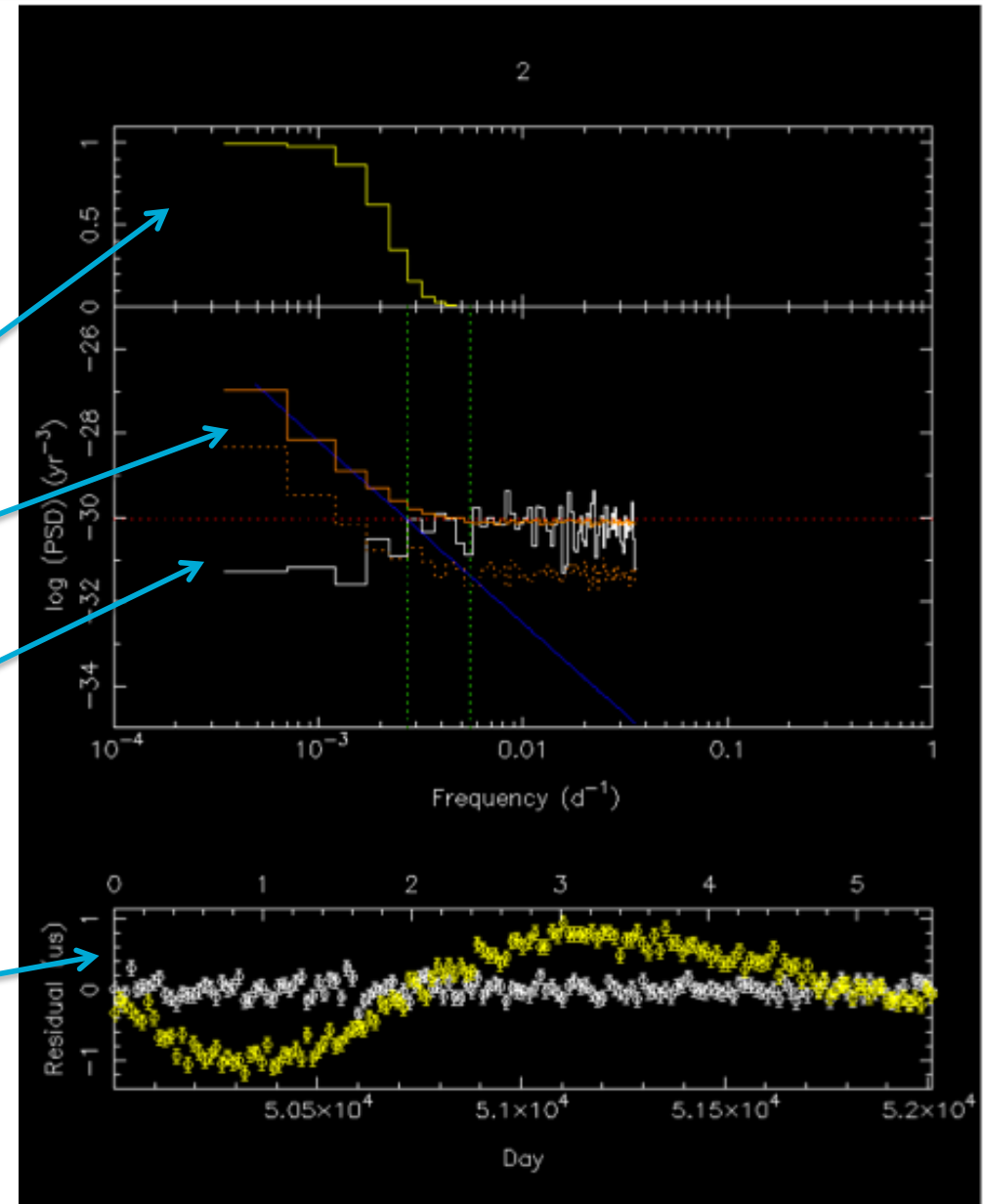
- Simulated bright gravitational wave background signal

Weighting function

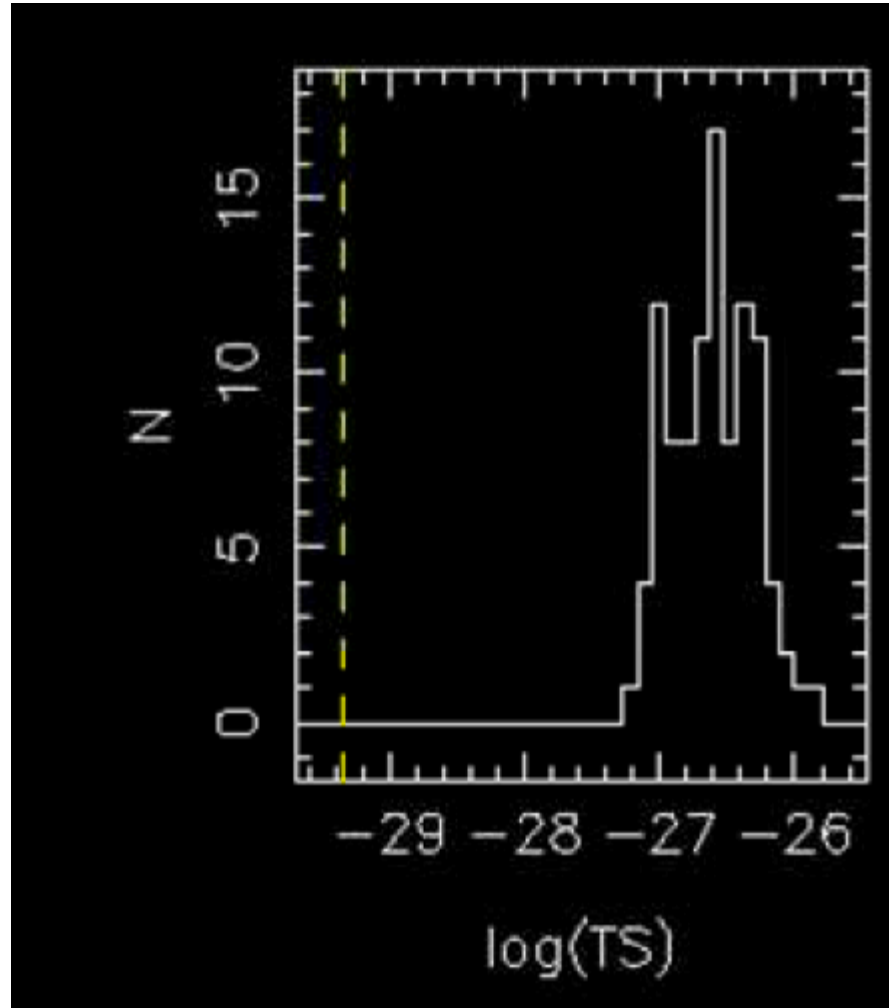
Average simulated power spectrum

Observed power spectrum

White = real data time series  
Yellow = simulated

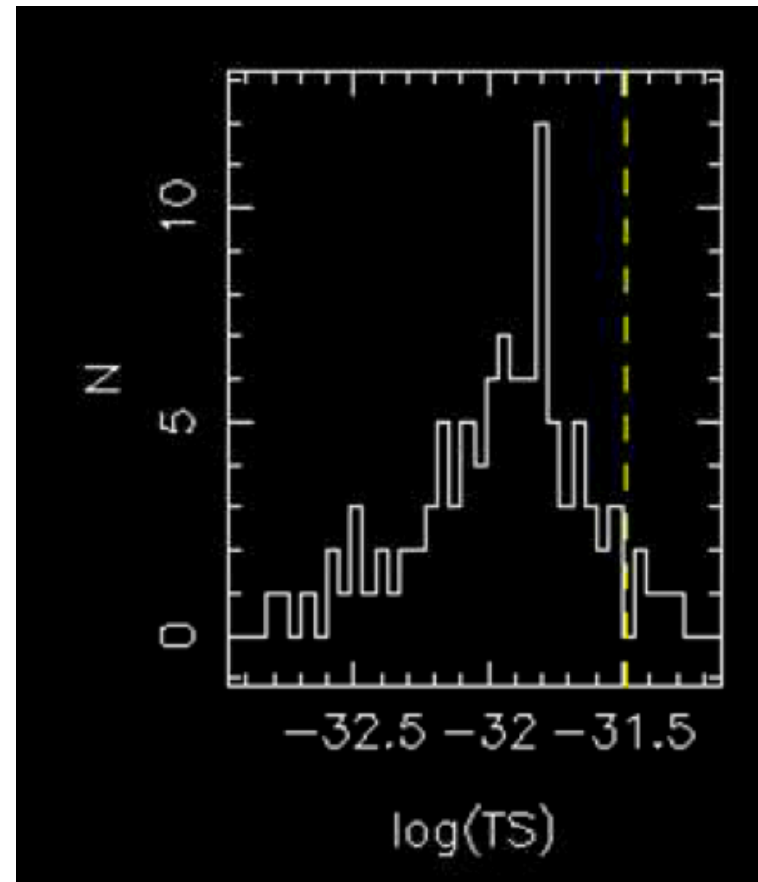
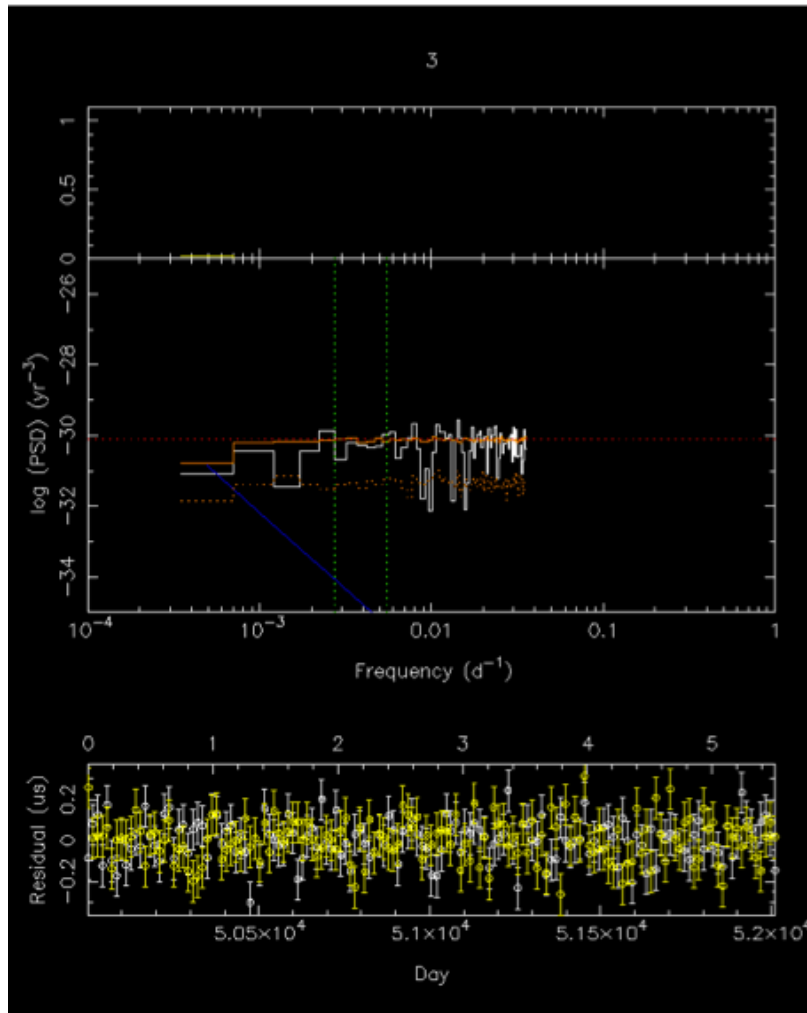


# Our current method



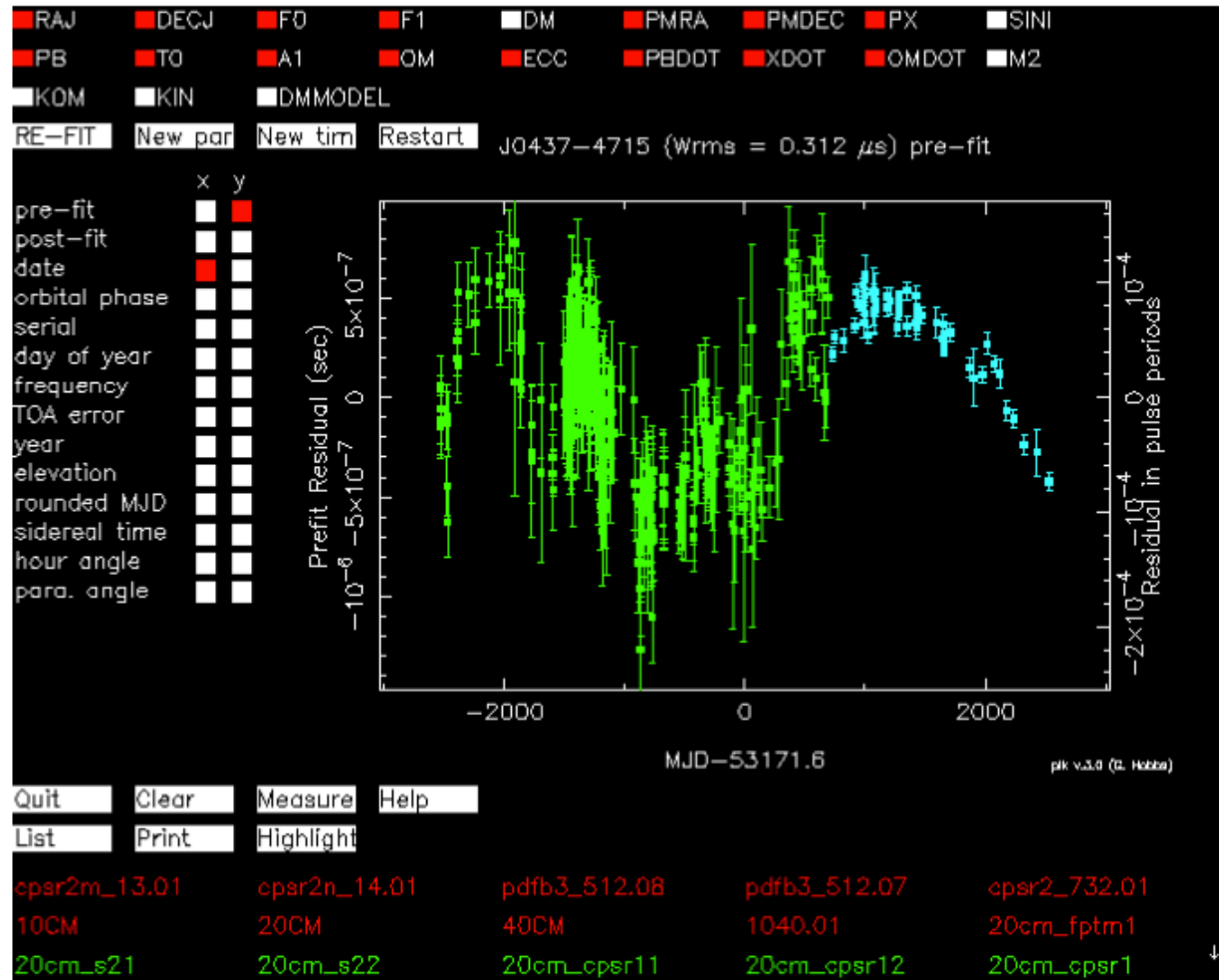
# Our current method

- No gravitational wave signal simulated



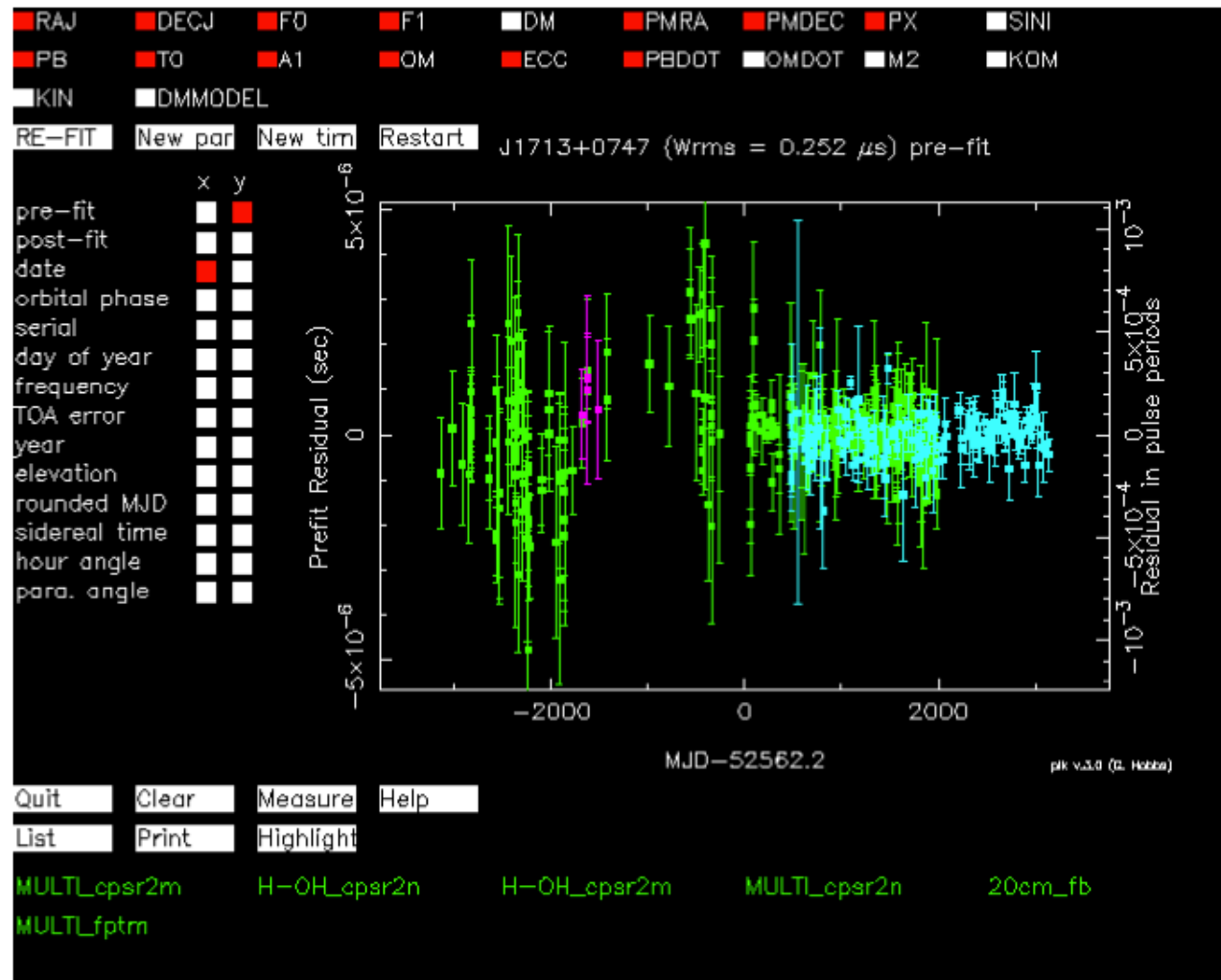
# Initial analysis of 3-best PPTA pulsars

**14 years  
(excluded  
fptm data)**



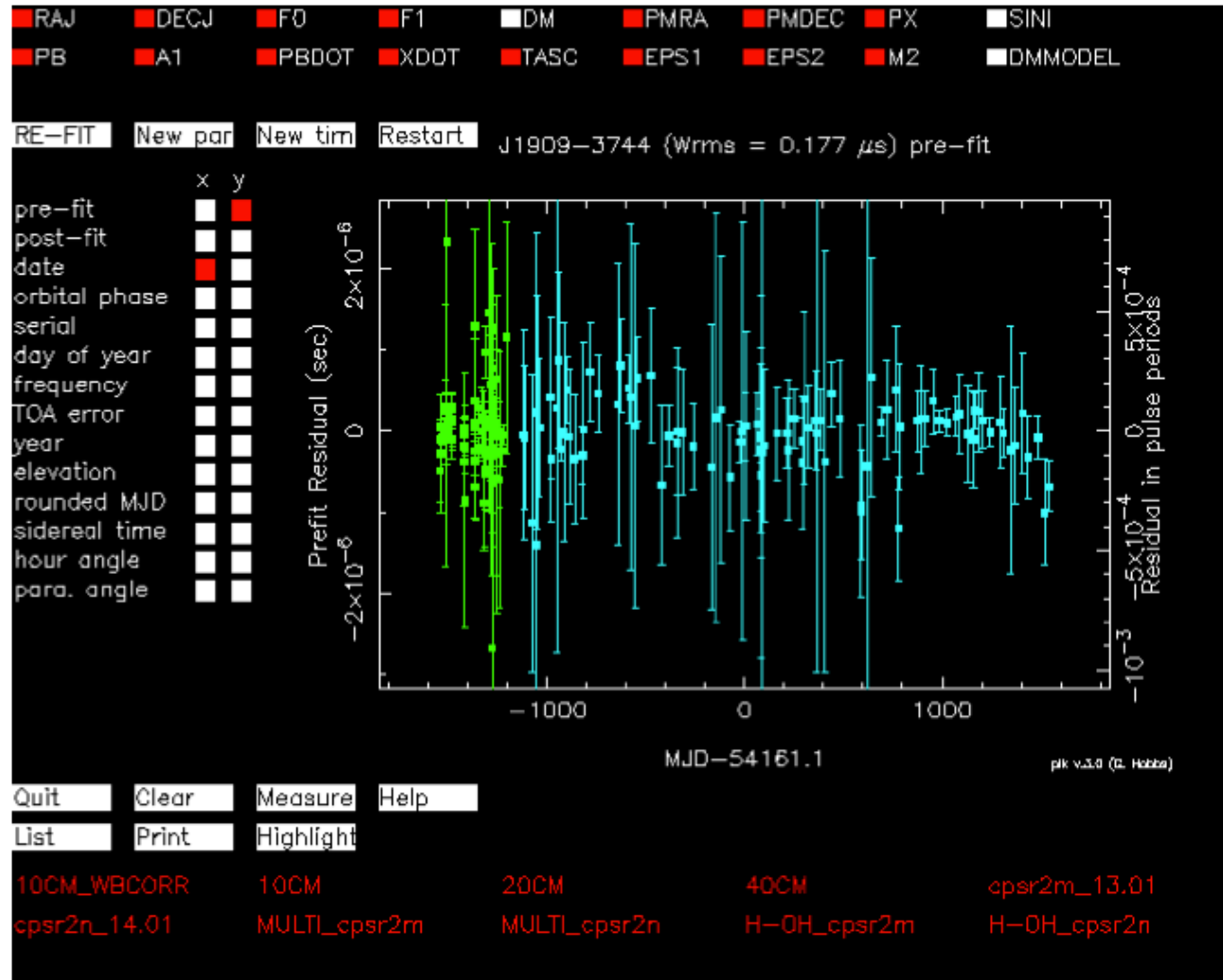
# Initial analysis of our 3-best pulsars

18 years



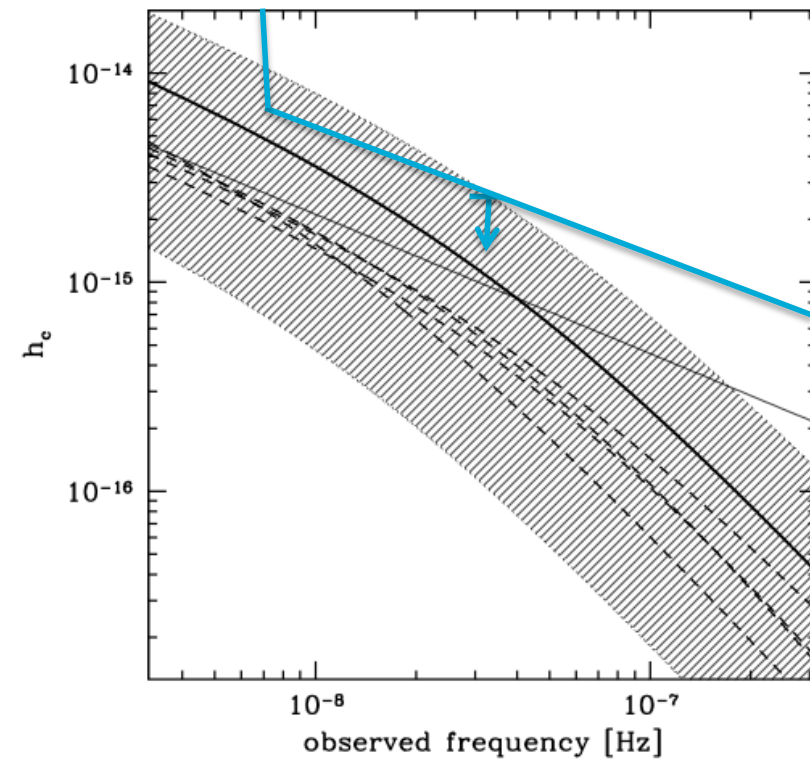
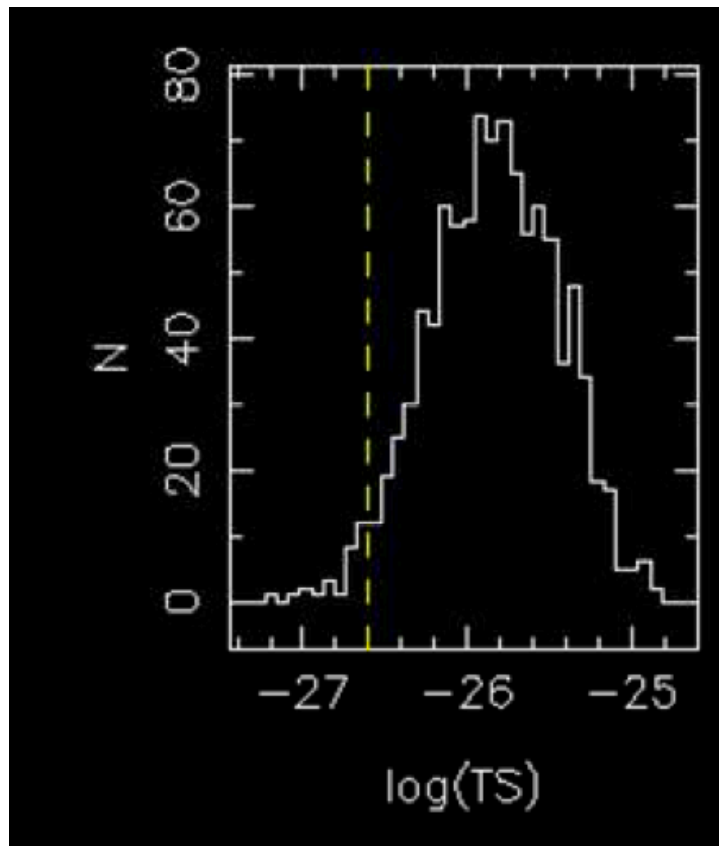
# Initial analysis of our 3-best pulsars

8 years



# Initial analysis of our 3-best pulsars

- Can rule out gravitational waves with  $A = 4 \times 10^{-15}$ , 97% of the time!



Sesana et al. (2008)

# Individual sources: Yardley et al. (2010)

- Try to place a limit on the existence of individual gravitational wave sources
- Use observations of 18 pulsars from the Parkes pulsar timing array project

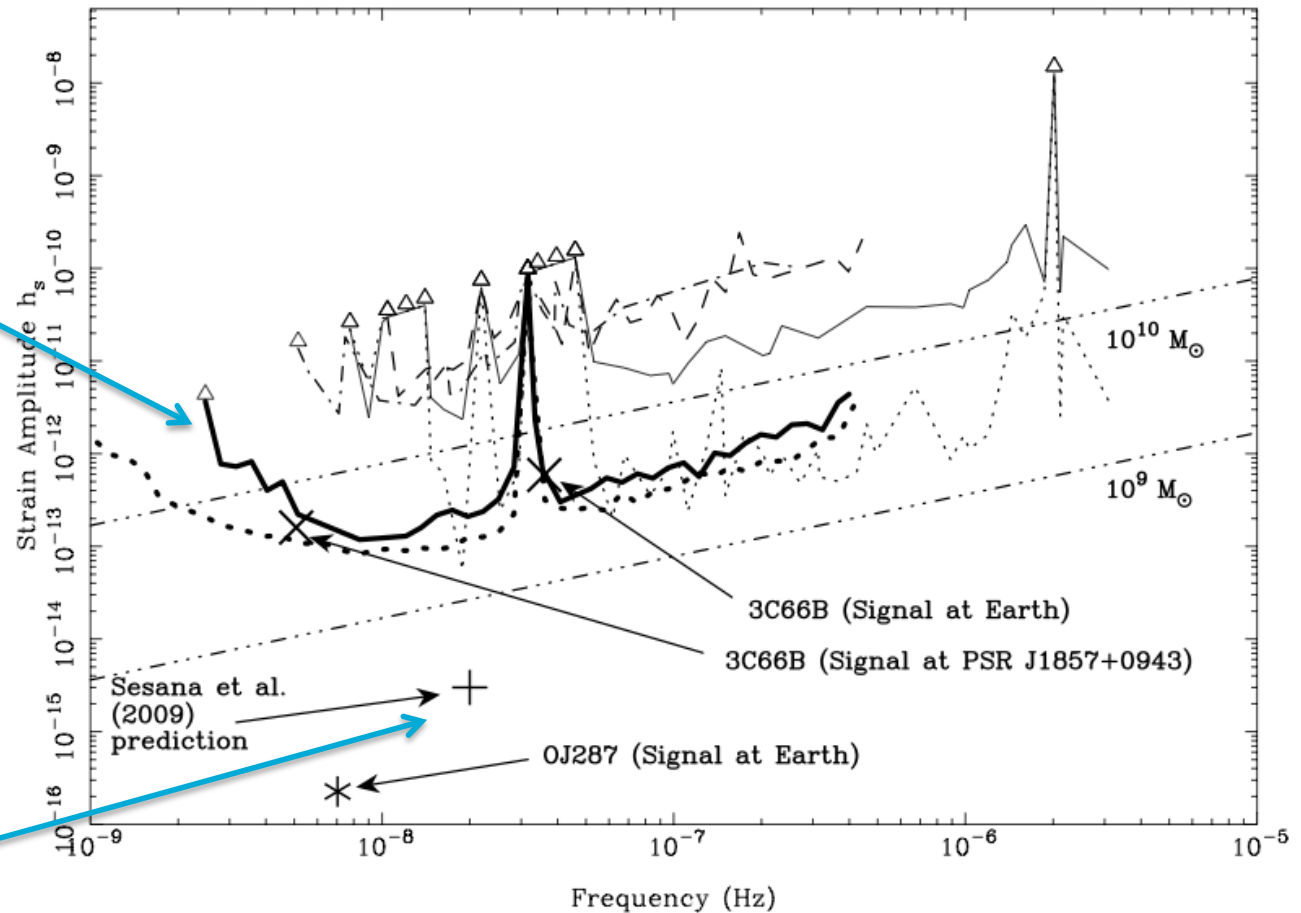
## **The sensitivity of the Parkes Pulsar Timing Array to individual sources of gravitational waves**

D. R. B. Yardley,<sup>1,2\*</sup> G. B. Hobbs,<sup>1</sup> F. A. Jenet,<sup>3</sup> J. P. W. Verbiest,<sup>4</sup> Z. L. Wen,<sup>5</sup>  
R. N. Manchester,<sup>1</sup> W. A. Coles,<sup>6</sup> W. van Straten,<sup>7</sup> M. Bailes,<sup>7</sup> N. D. R. Bhat,<sup>7</sup>  
S. Burke-Spolaor,<sup>1,7</sup> D. J. Champion,<sup>1,8</sup> A. W. Hotan<sup>9</sup> and J. M. Sarkissian<sup>1</sup>

# Individual sources: Yardley et al. (2010)

Our sensitivity curve

Likely source



More sensitive if we know where the source is!

# Individual sources

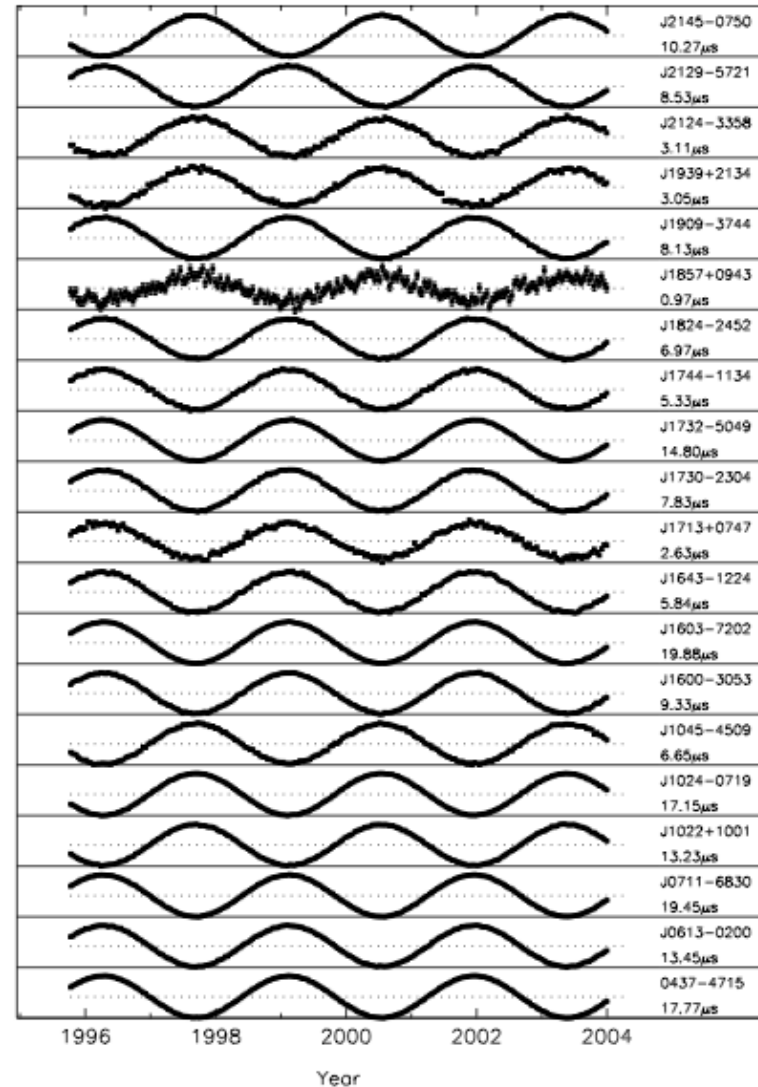
- Let's try and detect a sinusoidal signal corresponding to a bright single gravitational wave source (ignore the pulsar term)
- Assume that I happen to know the frequency of the gravitational wave emission

Fit a sinusoidal signal with the correct angular functional form (this assumes that I know where the source is)

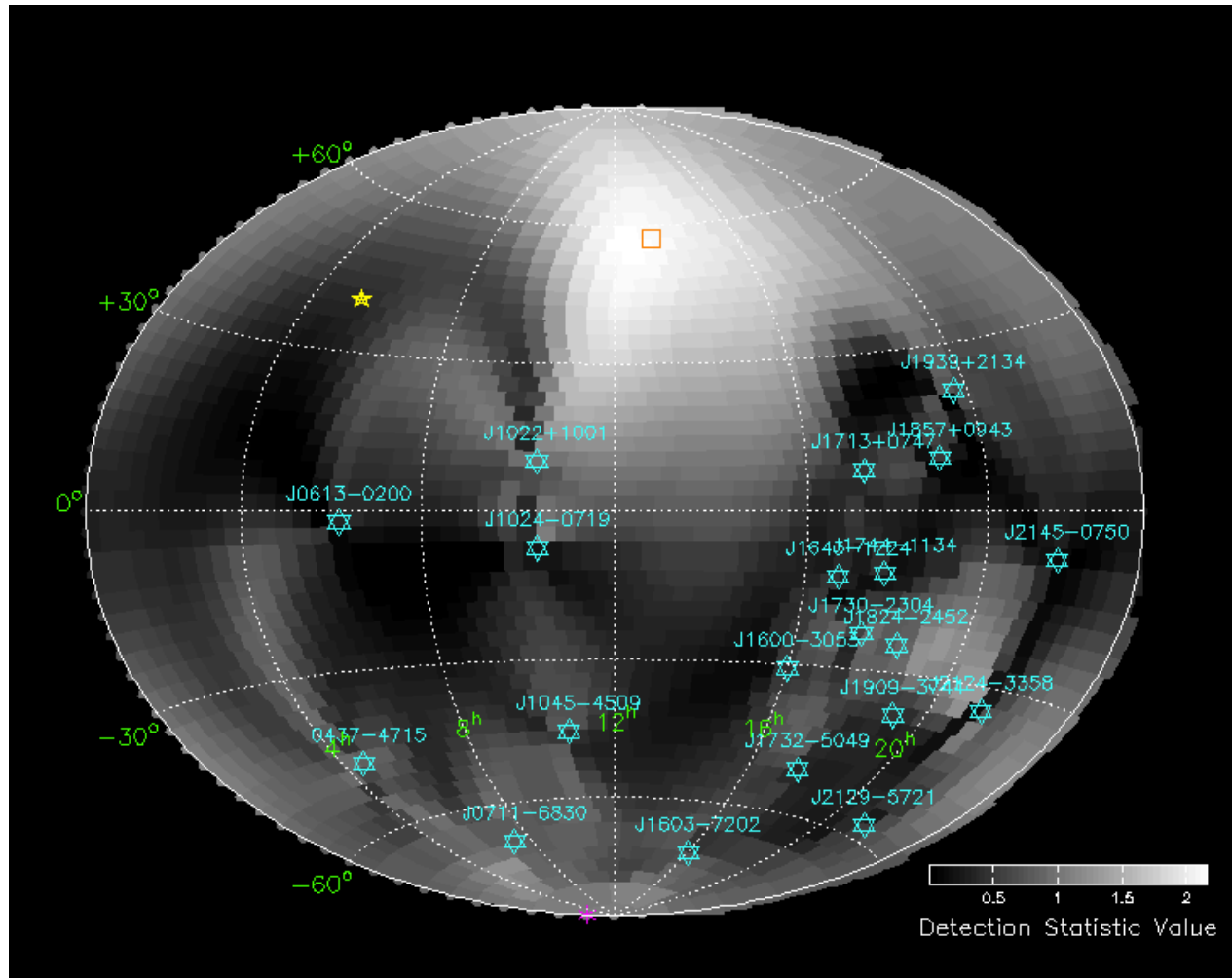
I don't know where the source is => try every possible position!

# Simulated data

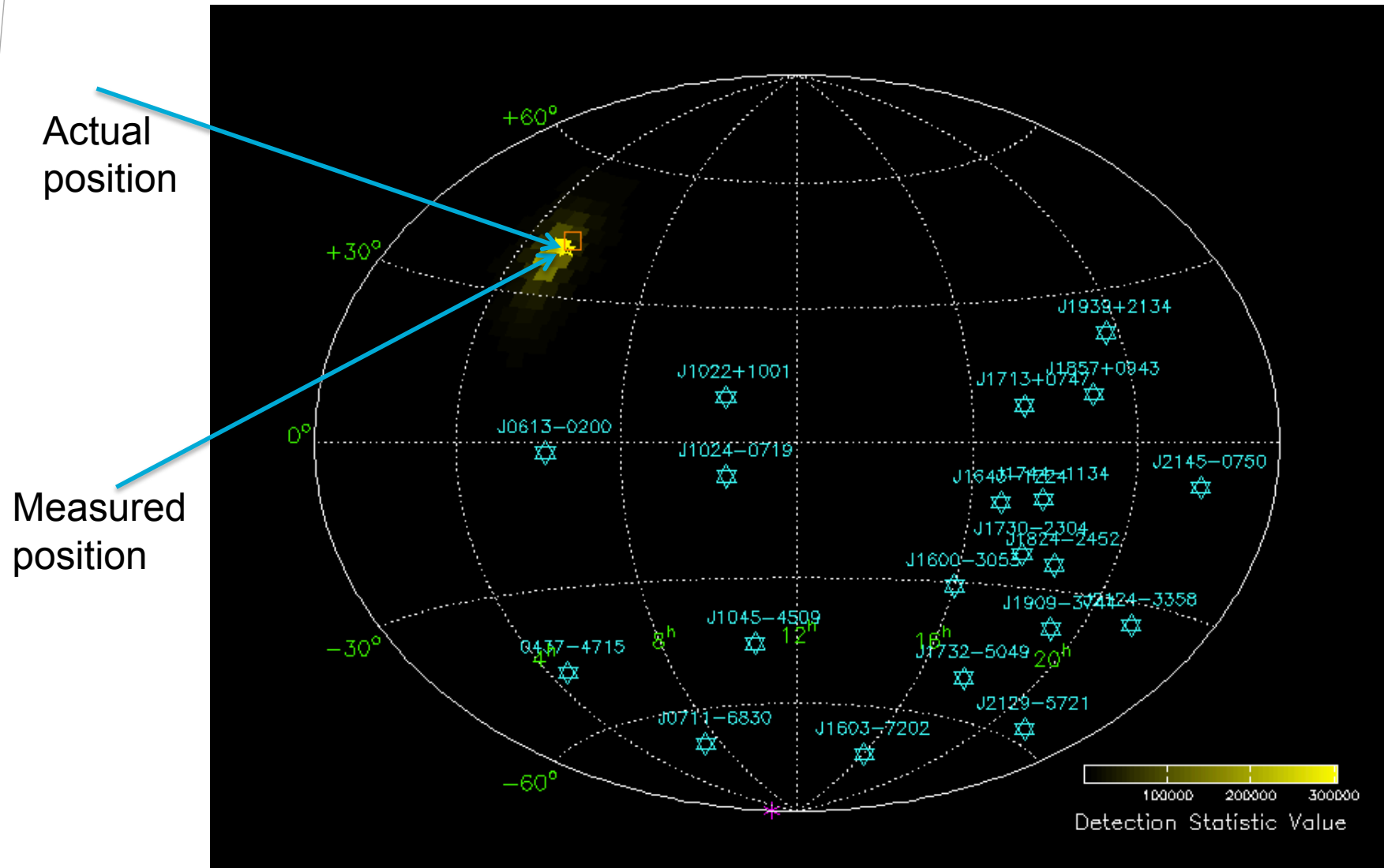
- Very strong gravitational wave source:  $A_+ = 10^{-12}$ ,  $A_x = 0$



# No signal simulated (just white noise)

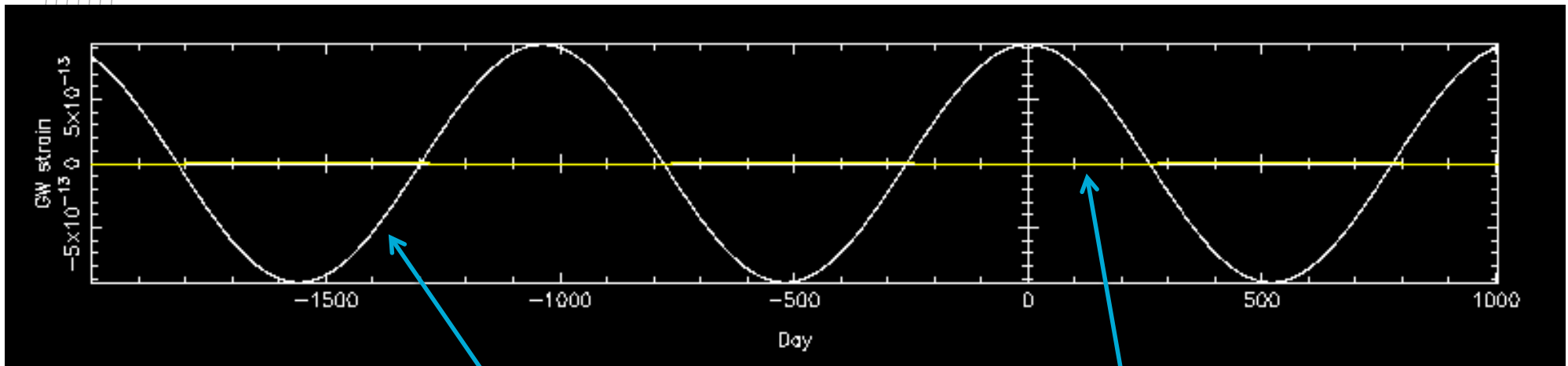


# Very strong source: $A_+ = 10^{-12}$ , $A_x = 0$ (no pulsar term)



# The measured GW strain signal

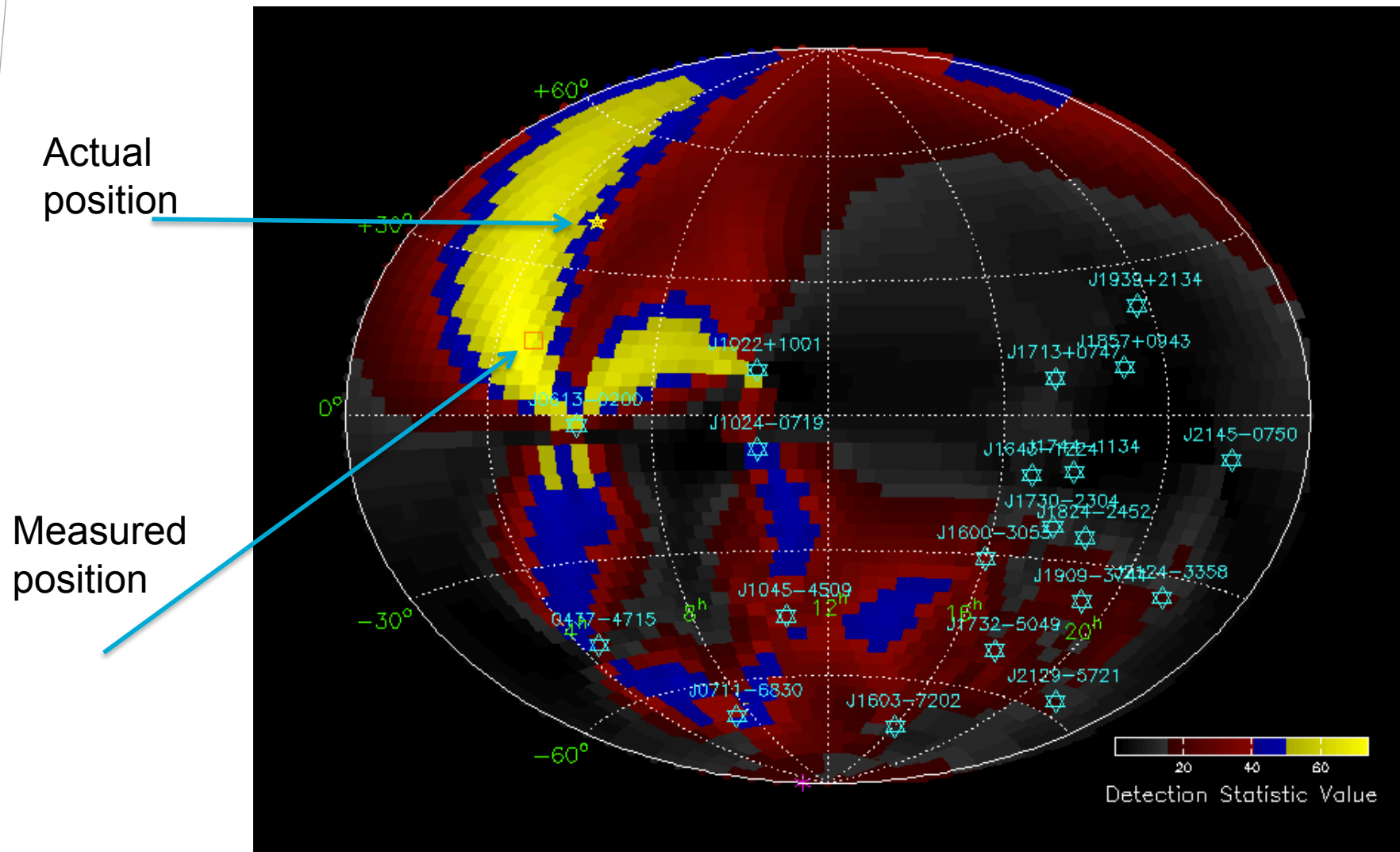
- Resulting signal



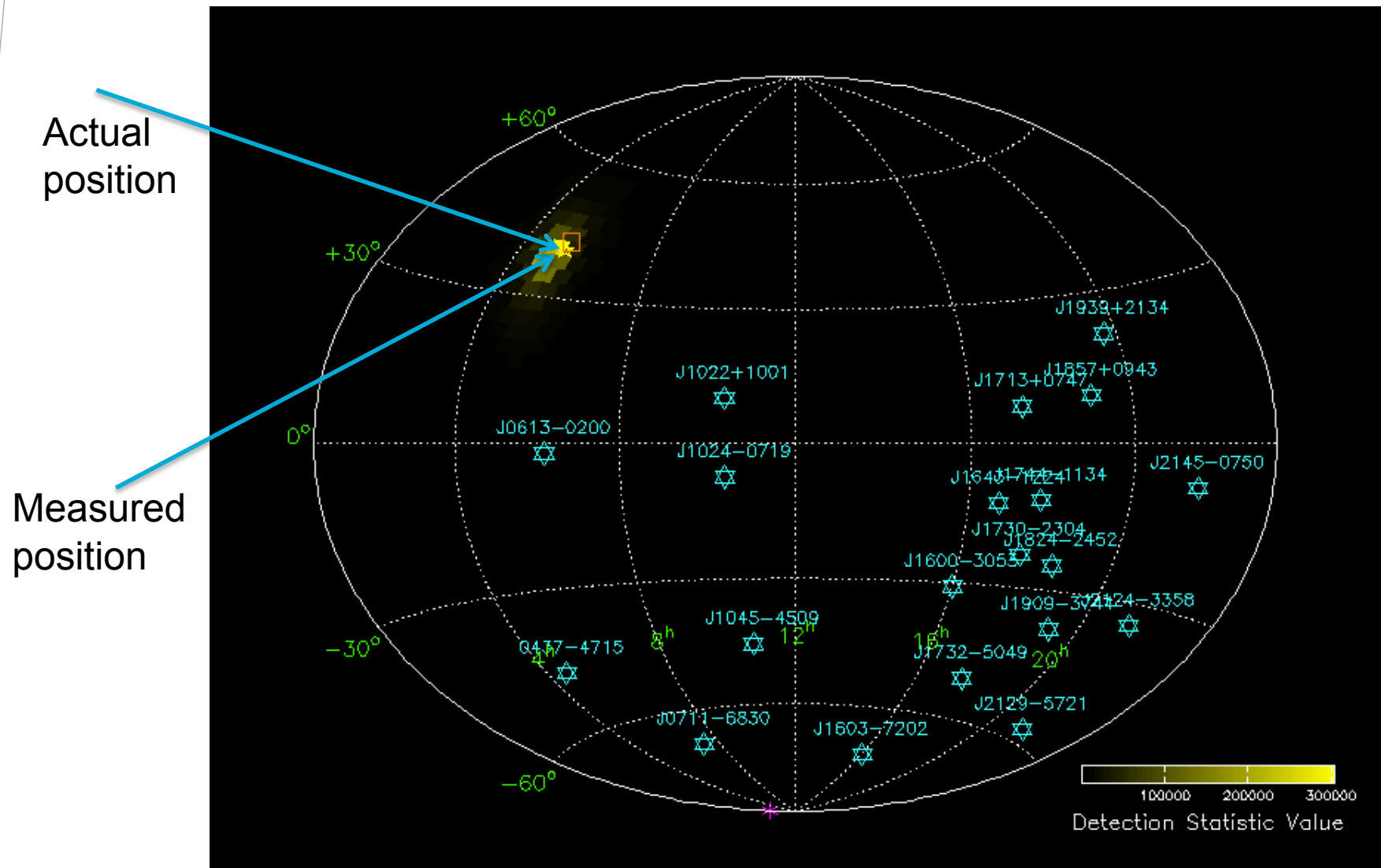
A+

Ax

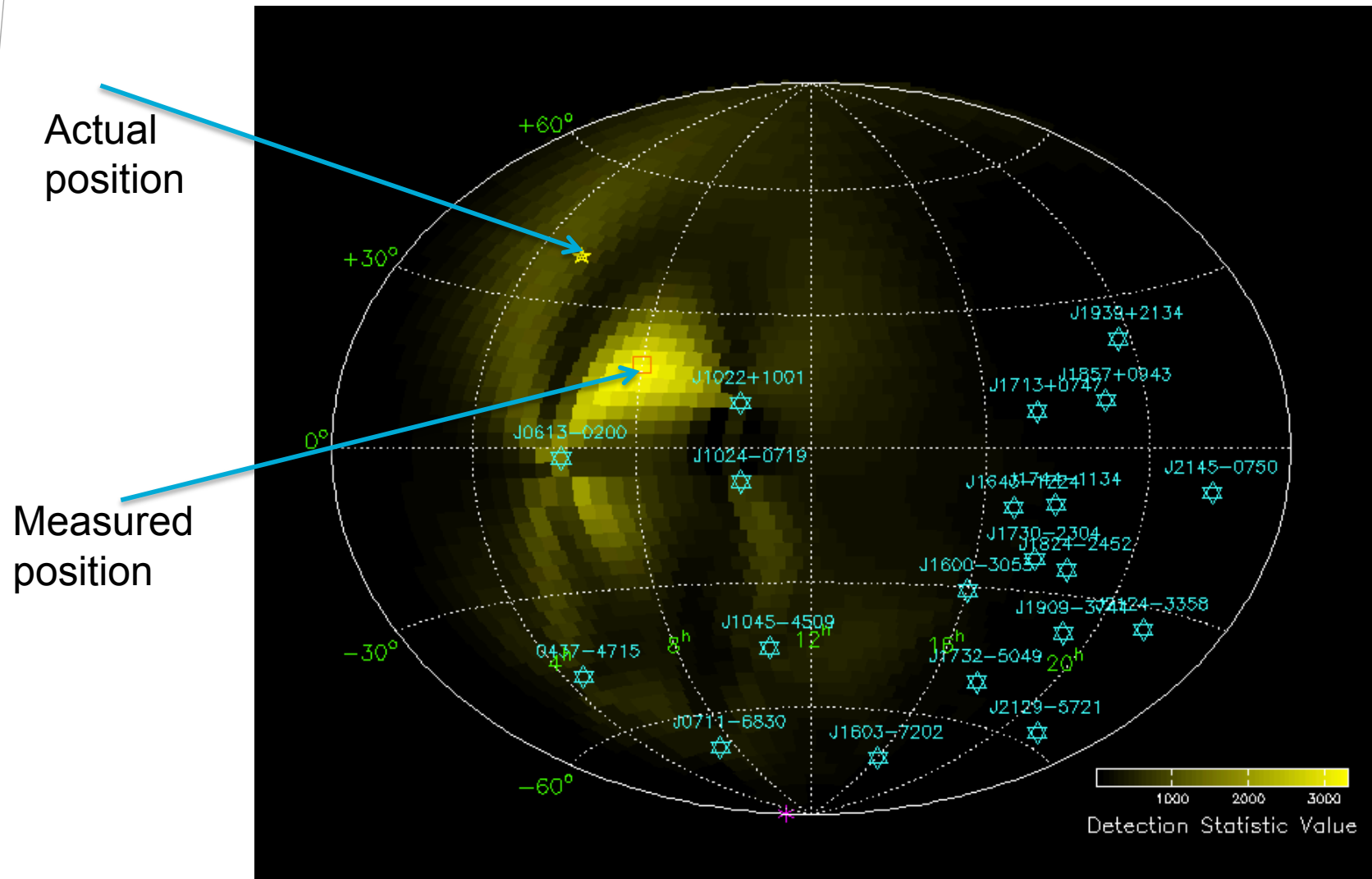
# A weaker source: $A_+ = 10^{-14}$



Recall: very strong source:  $A_+ = 10^{-12}$ ,  $A_x = 0$   
(no pulsar term)



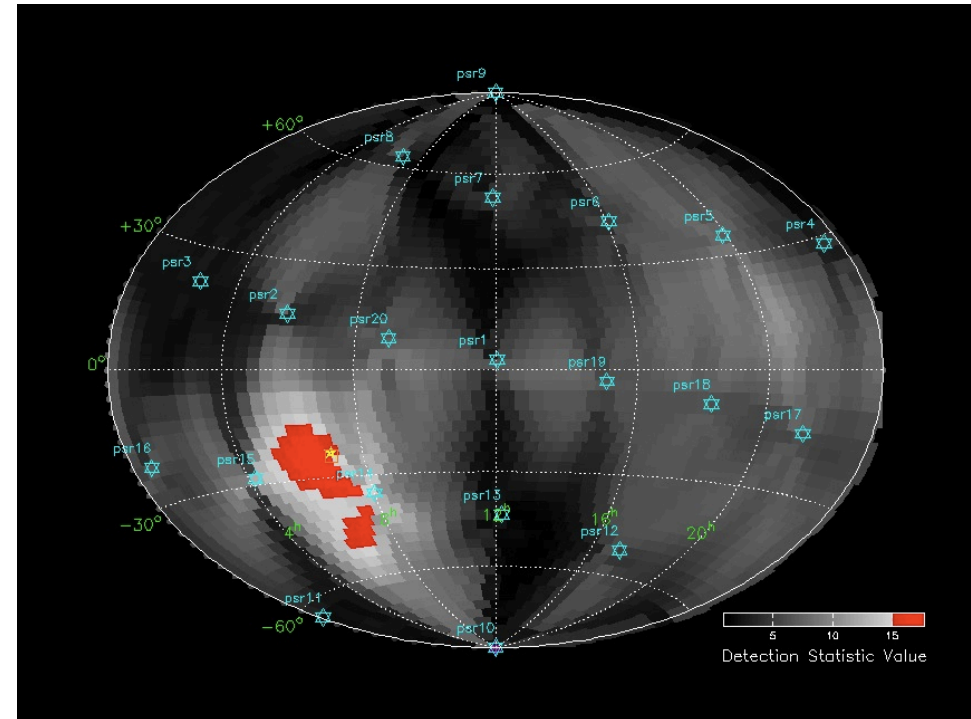
# Very strong source: $A_+ = 10^{-12}$ , $A_x = 0$ (with pulsar term)





# Burst gravitational wave sources

- Work currently being carried out by E. Petroff.
- Do not fit for a single sinusoidal signal
- Instead fit for arbitrary functional form,  $f(t)$ , defined using a harmonic series, or by linear interpolation
- Have to change statistic used for “detecting” the signal



# Conclusions

- We require ~20 pulsars, observed for ~5 years, with rms timing residuals ~100ns to detect the gravitational wave background
- Have developed techniques for searching for the waves
- Number of gravitational wave sources so far detected:

0

Tomorrow: what can we do after we have detected gravitational waves?  
What other fun stuff can we do with our existing data sets?

# Spot the koala ... any questions?

