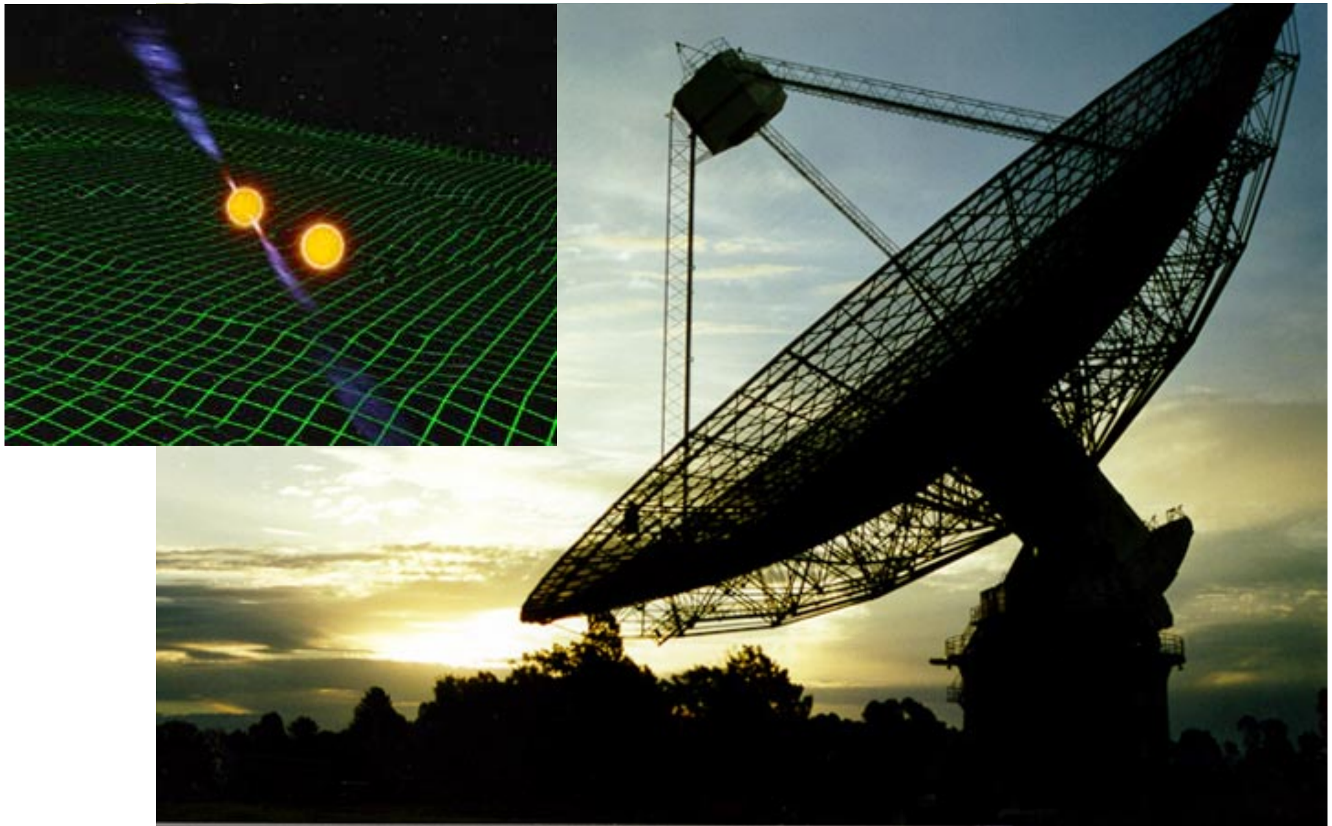

Pulsars and gravitational waves: 2

The pulsar timing method and properties of gravitational waves

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Purpose of this lecture series

- Provide an overview of pulsars
- Provide an overview of gravitational waves
- Show how, in theory, pulsar observations can be used to detect gravitational waves
- Describe issues with the current data sets
- Describe unsolved problems
- Provide enough information that you can process pulsar observations and develop tools to search for gravitational waves

Lecture series

- Lecture 1: overview (a bit of everything)
- Lecture 2: the pulsar timing method. How gravitational waves influence pulsar observations
- Lecture 3: Expected gravitational wave sources. Current data sets
- Lecture 4: Techniques to search for gravitational waves
- Lecture 5: a bit of fun and the future!

Review so far

- Pulsars are rapidly rotating neutron stars
- They emit a beam of radiation that produces the observed radio pulses
- They are incredibly stable rotators
- The pulses can be used like the tick of a clock
- Different phenomena cause variations in the measured pulse arrival times. The different phenomena can be distinguished by searching for correlations between different pulsars.
- The phrase “Pulsar timing residuals” represents the difference between the predicted and observed pulse arrival times

Purpose of lecture 2

- Describe the pulsar timing method in lots of detail

- Aim 1: you should be able to download the software used for "pulsar timing", obtain some data files and process the data
- Aim 2: you should understand the basic terminology used when talking about "pulsar timing"

- Aim 3: understand how gravitational waves affect pulsar timing residuals
- Aim 4: be able to simulate the effects of gravitational waves on pulsar data.

Part 1: Getting pulsar data files

- Part 1: Getting pulsar data files
- Part 2: Pulsar timing
- Part 3: Gravitational waves

Getting some data sets:

The Parkes data archive: http://datanet.csiro.au/dap/

An advert … pulsar data from the Parkes radio telescope is available online (Virtual Observatory compatible)

The Parkes data archive: http://datanet.csiro.au/dap/

- Browse Datasets
- Pulsar Search
- Geospatial Search
- Acknowledgements

ATNF Pulsar Observation Search

Query pulsar observations taken at the Parkes radio telescope. Publications that make use of the data from this archive should include an acknowledgement statement. Please refer to the statements provided here.

Source Name / Position	
Source Name:	J1022+1001
Cone Search:	
Right Ascension:	hh:mm:ss.ss (J2000)
Declination:	dd:mm:ss.ss (J2000)
Search Window:	arcmin

Observation	
Project ID:	
Observation Date: (dd/mm/yyyy)	to
Observation Date: (MJD)	to

Search

Frequency / Band	
Frequency: (MHz)	to
Band Name:	Any 70cm (~0.44 GHz) 50cm (~0.66 GHz) 20/18cm (1.2-1.8 GHz)
Observation Mode:	All
Backend:	All
Frontend:	All

Configure Results Columns

Can type in a pulsar name

The Parkes data archive: http://datanet.csiro.au/dap/

Project ID
- ☑ P456 (3216)
- ☑ P140 (99)
- ☑ P361 (5)

Backend
- ☑ CPSR2 (1812)
- ☑ PDFB3 (361)
- ☑ WBCORR (317)
- ☑ PDFB1 (302)
- ☑ PDFB4 (279)
- ☑ PDFB2 (249)

Observation Type
- ☑ raw (1732)
- ☑ preprocessed (1588)

Frontend
- ☑ 1050CM (1083)
- ☑ MULTI (953)
- ☑ MULT_1 (630)
- ☑ H-OH (391)
- ☑ (125)
- ☑ Unknown (125)

Search Window: [ ] arcmin

Observation Mode: All
Backend: All
Frontend: All

Observation
Project ID:
Observation Date: (dd/mm/yyyy) to
Observation Date: (MJD) to

Search

Configure Results Columns

Search Results:

Only the first 1000 records have been returned. Max 50 files can be downloaded at a time.

◉ tar ○ zip Download file(s)

0 MB to be downloaded

☐	Project ID	Filename	File Size (MB)	Preview	Last Modified	Source	RA	Dec	Backend
☐	P456	a050628_063734.rf	184.90		2011-03-14 12:55:30	J1022+1001	10:22:58	10:01:53	WBCORR
☐	P456	a050715_041726.rf	257.30		2011-03-14 12:55:40	J1022+1001	10:22:58	10:01:53	WBCORR
☐	P456	a050716_071833.rf	108.50		2011-03-14 12:55:45	J1022+1001	10:22:58	10:01:53	WBCORR
☐	P456	a050824_010837.rf	245.20		2011-03-14 12:55:51	J1022+1001	10:22:58	10:01:53	WBCORR

The Parkes data archive: http://datanet.csiro.au/dap/

Search Results:

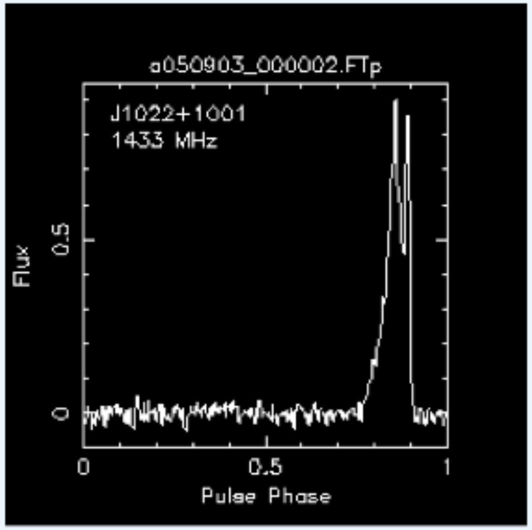
Only the first 1000 records have been returned. Max 50 files can be downloaded at a time.

◉ tar ○ zip [Download file(s)]

0 MB to be downloaded

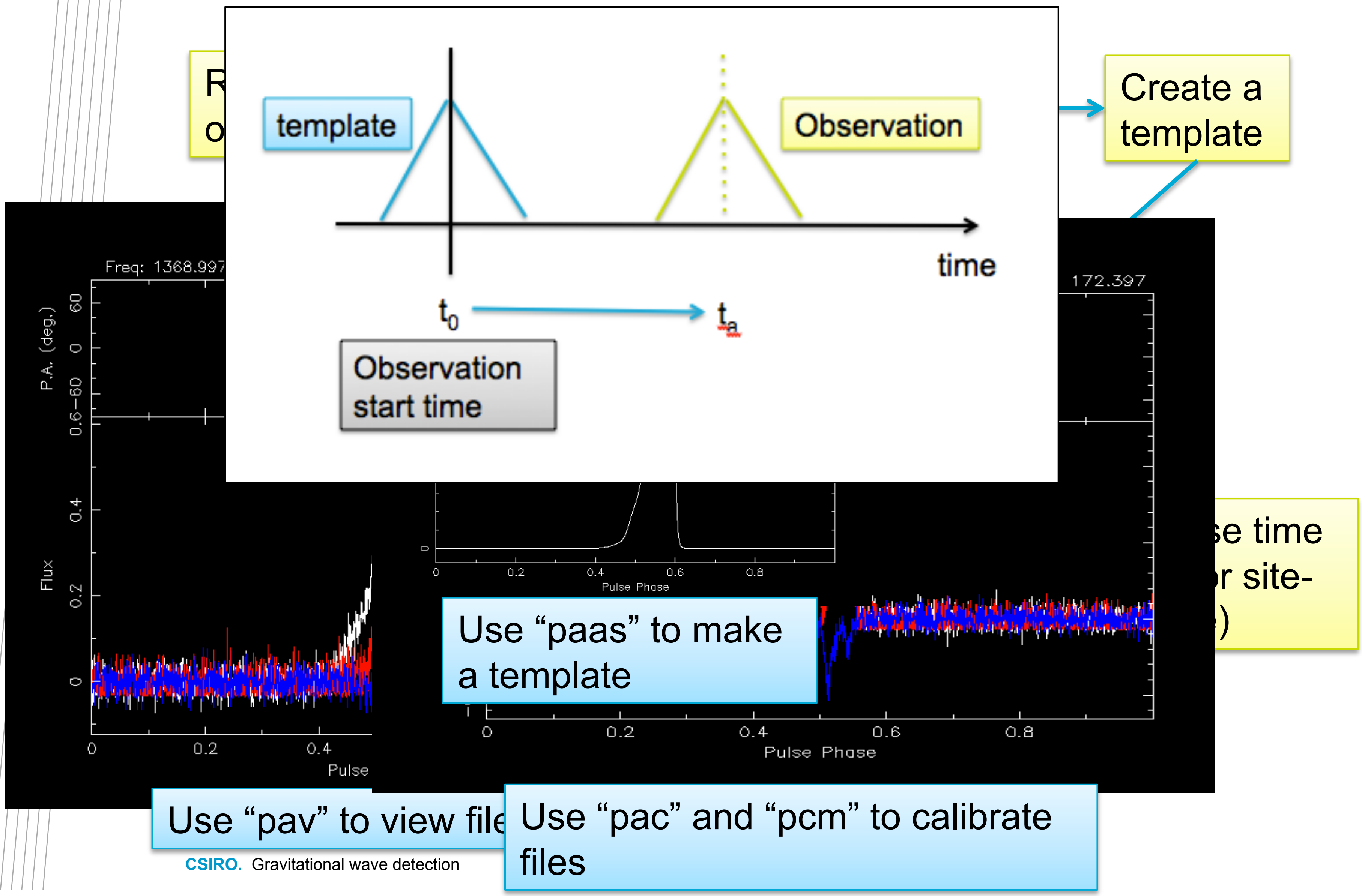
☐	Project ID	Filename	File Size (MB)	Preview	Last Modified	Source	RA	Dec	Backend
☐	P456	a050628_063734.rf	184.90		2011-03-14 [illegible]	J1022+1001	10:22:58	10:01:53	WBCORR
☐	P456	a050715_041726.rf	257.30		[illegible]	[illegible]	[illegible]	53	WBCORR
☐	P456	a050716_071833.rf	108.50		[illegible]	[illegible]	[illegible]	53	WBCORR
☐	P456	a050824_010837.rf	245.20		[illegible]	[illegible]	[illegible]	53	WBCORR
☐	P456	a050825_015826.rf	124.60		[illegible]	[illegible]	[illegible]	53	WBCORR
☐	P456	a050825_023324.rf	124.60		[illegible]	[illegible]	[illegible]	53	WBCORR
☐	P456	a050903_000002.rf	65.30		[illegible]	[illegible]	[illegible]	53	WBCORR
☐	P456	a051002_215233.rf	124.60		[illegible]	[illegible]	[illegible]	53	WBCORR
☐	P456	a051002_222516.rf	124.60		[illegible]	[illegible]	[illegible]	53	WBCORR
☐	P456	a051003_234653.rf	64.30		2011-03-14 12:56:15	J1022+1001	10:22:58	10:01:53	WBCORR

Preview of a050903_000002.rf



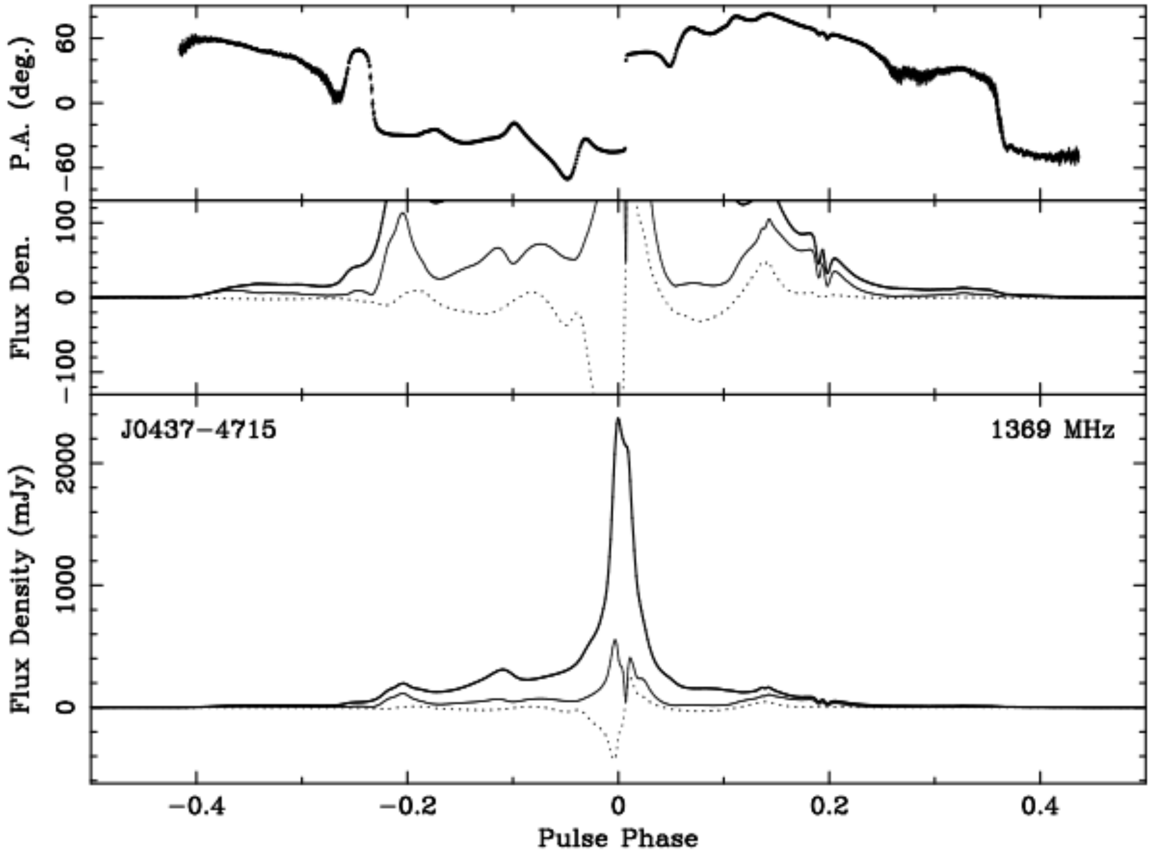
CSIRO

Processing the data: from raw data to site-arrival-time



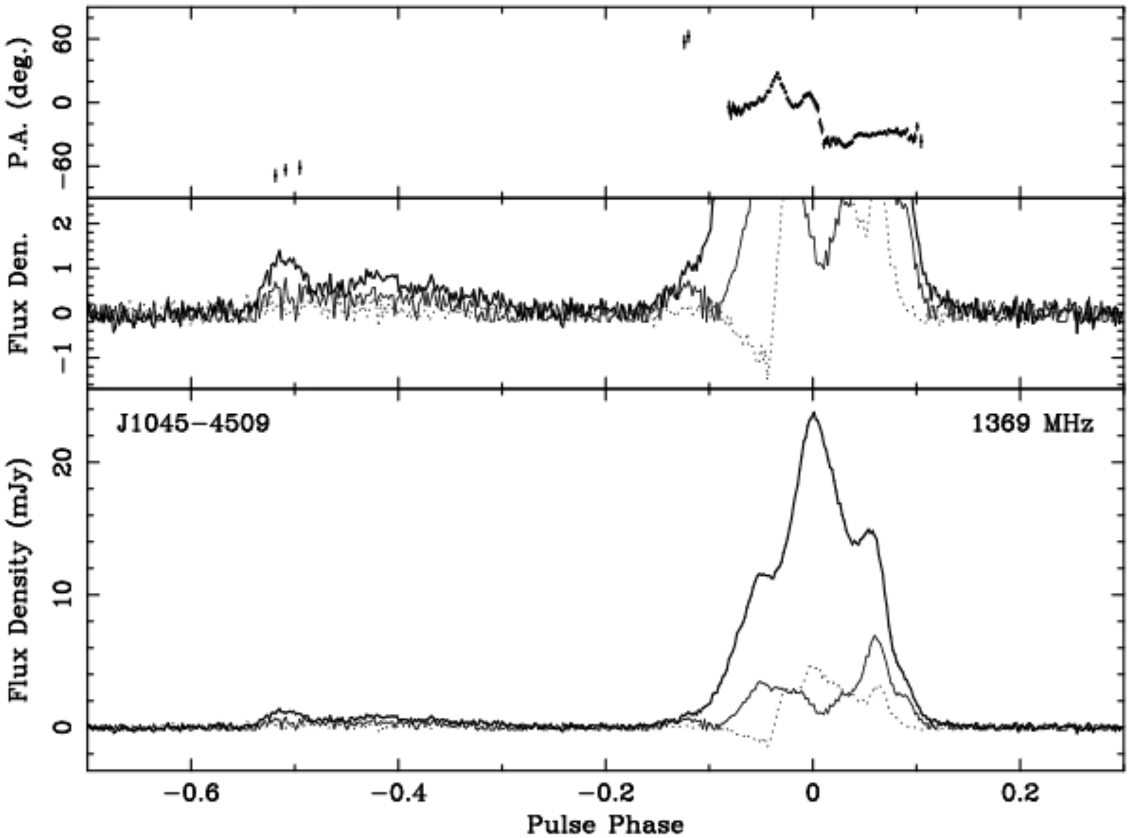
An aside: pulse profile shapes

- Yan et al. (2011): 1.4GHz profiles from the Parkes Pulsar Timing Array project



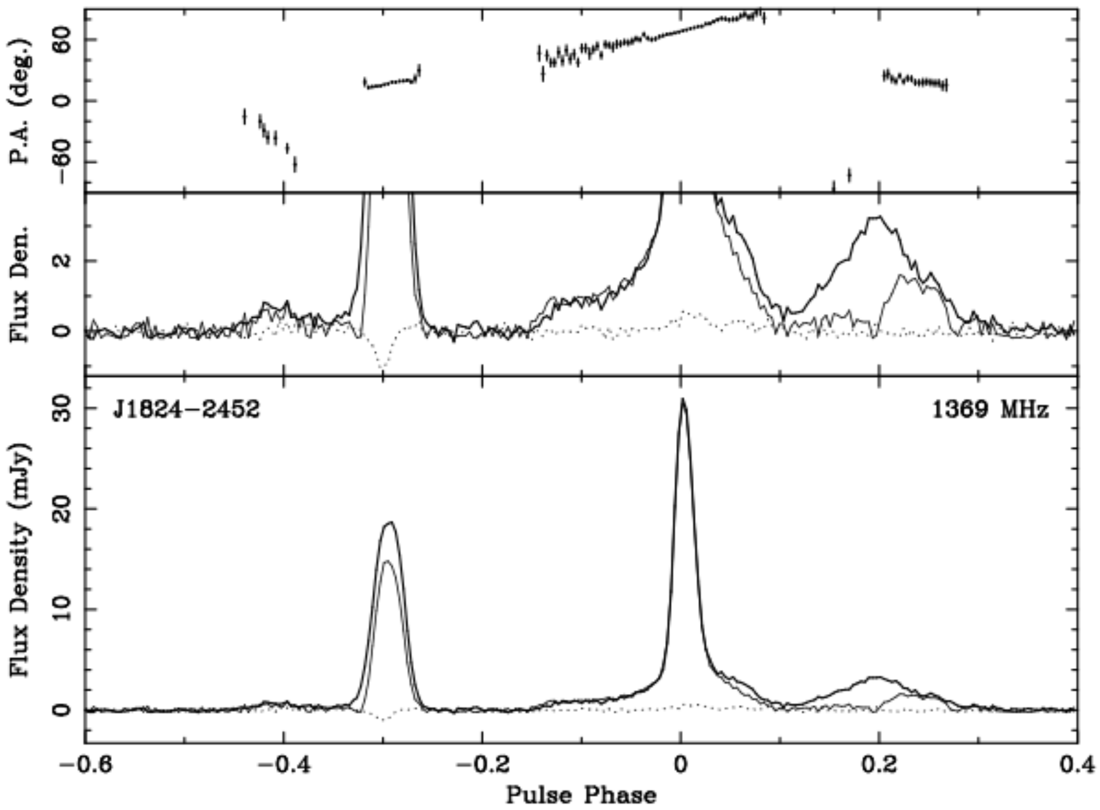
An aside: pulse profile shapes

- Yan et al. (2011): 1.4GHz profiles from the Parkes Pulsar Timing Array project



An aside: pulse profile shapes

- Yan et al. (2011): 1.4GHz profiles from the Parkes Pulsar Timing Array project



Part 2: Pulsar timing

- Part 1: Getting pulsar data files
- Part 2: Pulsar timing
- Part 3: Gravitational waves

Pulsar timing

- Start with measurements of pulse arrival times and a basic model for the pulsar's position, pulse period and orbital parameters
- Get: pulsar timing residuals, improved model of pulsar's parameters

- TEMPO2 is a software package. It implements "pulsar timing" algorithms.
- Throughout I'll discuss TEMPO2, but the ideas are valid for TEMPO1/PSRTIME/TIMAPR …

- Major TEMPO2 developers (software and algorithms): G. Hobbs, R. Edwards, R. Manchester, W. Coles, X. You, M. Keith, F. Jenet, D. Yardley, J. Verbiest, …

What can you do with pulsar timing? Examples from 2010/2011

- Searching for gravitational wave signals (e.g., Yardley et al. 2011, Yardley et al. 2010, Abbott et al. 2010, van Haasteren et al. 2010)
- Using pulsars as navigational aids (Ruggiero 2011)
- Statistical analysis of timing residuals (e.g., Na, X. S. et al., 2011, Hobbs et al. 2010)
- Studying emission geometry (e.g., Noutsos et al., 2011)
- Searching for gamma-ray pulsars (Ray et al., 2010)
- Determining pulsar masses (Demorest et al. 2010)
- Studying pulsations from main-sequence stars (Ravi et al. 2010)
- Tests of relativity (Weisberg et al. 2010)
- Measuring Jupiter's mass (Champion et al. 2010)
- Observations of glitches (Chukwude et al. 2010)
- Analysis of accreting millisecond X-ray pulsar (Patruno et al. 2010)
- Relativistic spin precession (Manchester et al. 2010)

A parameter file (.par)

Pulsar name, astrometric, rotational, dispersion measure and orbital parameters

```
PSRJ     J0613-0200
RAJ      06:13:43.9750585                   1
DECJ     -02:00:47.17238                    1
F0       326.60056219017465751              1
F1       -1.0228669646983851338e-15         1
PEPOCH   53114
POSEPOCH        53114
DMEPOCH  53114
DM       38.77910000000000001
PMRA     1.7864748030585594928    1
PMDEC    -10.270236825031680 75   1
PX       0.91177639168508902443   1
BINARY   DD
PB       1.1985125752330855253    1
T0       53113.947671230393077    1
A1       1.0914444592811573305    1
OM       45.451457136345105293    1
ECC      5.2933858304387 94e-06   1
START    53431.313924448706732
FINISH   55619.299962597727951

TZRMJD   54526.290614015242458
TZRFRQ   1373.0139999999998963
TZRSITE 7

EPHVER   5
CLK      TT(TAI)
EPHEM    DE414
MODE     1
```

Fit for this parameter?

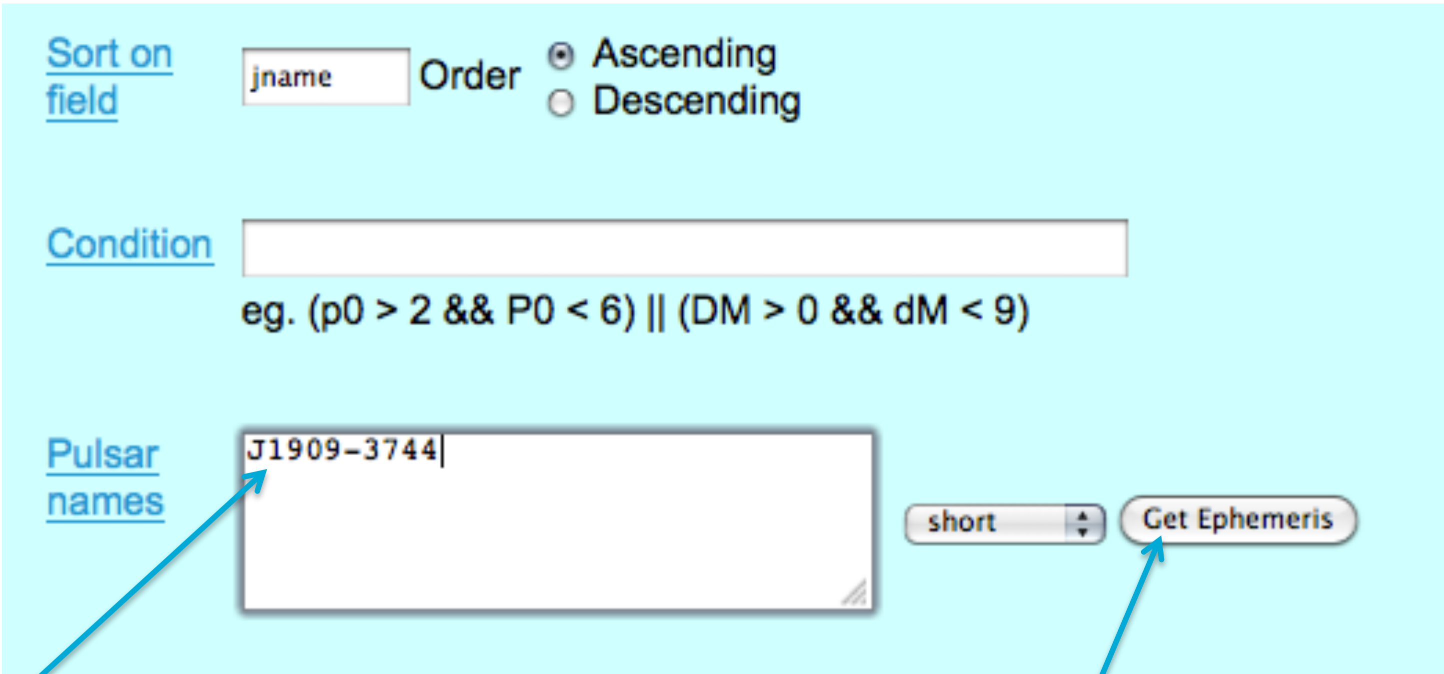
Realisation of terrestrial time

Solar system ephemeris

Weighted fit?

Getting an initial parameter file

- Can use the ATNF pulsar catalogue
- http://www.atnf.csiro.au/research/pulsar/psrcat
- Includes all published pulsars



Type pulsar name here

Click on "Get Ephemeris"

An arrival time file (.tim)

States that this is a tempo2 format file

Site arrival time (MJD)

Telescope code

```
FORMAT 1
/pulsar/20cm/m2002-12-10-13:00:00.czFTp  1404.734 52618.5903043780479 1.909 7 -f MULTI_CPSR2n -cal 1
/pulsar/20cm/m2002-12-10-13:39:58.czFTp  1404.734 52618.5903043780479 1.909 7 -f MULTI_CPSR2n -cal 1
/pulsar/20cm/n2002-12-10-13:00:00.czFTp  1341.331 52618.5902648849556 1.494 7 -f MULTI_CPSR2m -cal 1
/pulsar/20cm/n2002-12-10-13:39:58.czFTp  1341.331 52618.5902648849556 1.494 7 -f MULTI_CPSR2m -cal 1
/pulsar/20cm/m2002-12-14-14:41:47.czFTp  1404.736 52622.6211812124650 2.459 7 -f MULTI_CPSR2n -cal 1
/pulsar/20cm/n2002-12-14-14:41:47.czFTp  1341.427 52622.6211782913951 3.076 7 -f MULTI_CPSR2m -cal 1
/pulsar/20cm/m2002-12-15-14:25:37.czFTp  1405.235 52623.6455892166404 1.496 7 -f MULTI_CPSR2n -cal 1
/pulsar/20cm/n2002-12-15-14:25:37.czFTp  1341.500 52623.6455900526348 1.145 7 -f MULTI_CPSR2m -cal 1
/pulsar/20cm/m2003-02-04-09:24:09.czFTp  1404.736 52674.4137128970834 1.846 7 -f MULTI_CPSR2n -cal 1
/pulsar/20cm/n2003-02-04-09:24:09.czFTp  1341.267 52674.4137041290017 1.883 7 -f MULTI_CPSR2m -cal 1
/pulsar/50cm/m2003-02-07-08:21:20.czFTp  658.954 52677.3610004327656 0.766 7 -f 50CM_CPSR2 -cal 1
/pulsar/50cm/m2003-02-07-10:01:43.czFTp  659.220 52677.4386400805335 0.745 7 -f 50CM_CPSR2 -cal 1
/pulsar/50cm/m2003-04-13-10:03:28.czFTp  660.190 52742.4260621791924 1.468 7 -f 50CM_CPSR2 -cal 1
/pulsar/20cm/m2003-06-05-00:56:26.czFTp  1404.493 52795.0485182670624 1.596 7 -f MULTI_CPSR2n -cal 1
/pulsar/20cm/n2003-06-05-00:56:26.czFTp  1340.976 52795.0485170472312 1.130 7 -f MULTI_CPSR2m -cal 1
/pulsar/20cm/m2003-06-05-01:32:02.czFTp  1404.480 52795.0848331525075 1.408 7 -f MULTI_CPSR2n -cal 1
/pulsar/20cm/n2003-06-05-01:32:02.czFTp  1340.987 52795.0847944005109 1.060 7 -f MULTI_CPSR2m -cal 1
/pulsar/50cm/m2003-06-06-06:12:48.czFTp  660.002 52796.2746158076829 0.572 7 -f 50CM_CPSR2 -cal 1
/pulsar/20cm/m2003-07-25-23:30:54.czFTp  1404.445 52846.0215328027366 1.034 7 -f MULTI_CPSR2n -cal 1
```

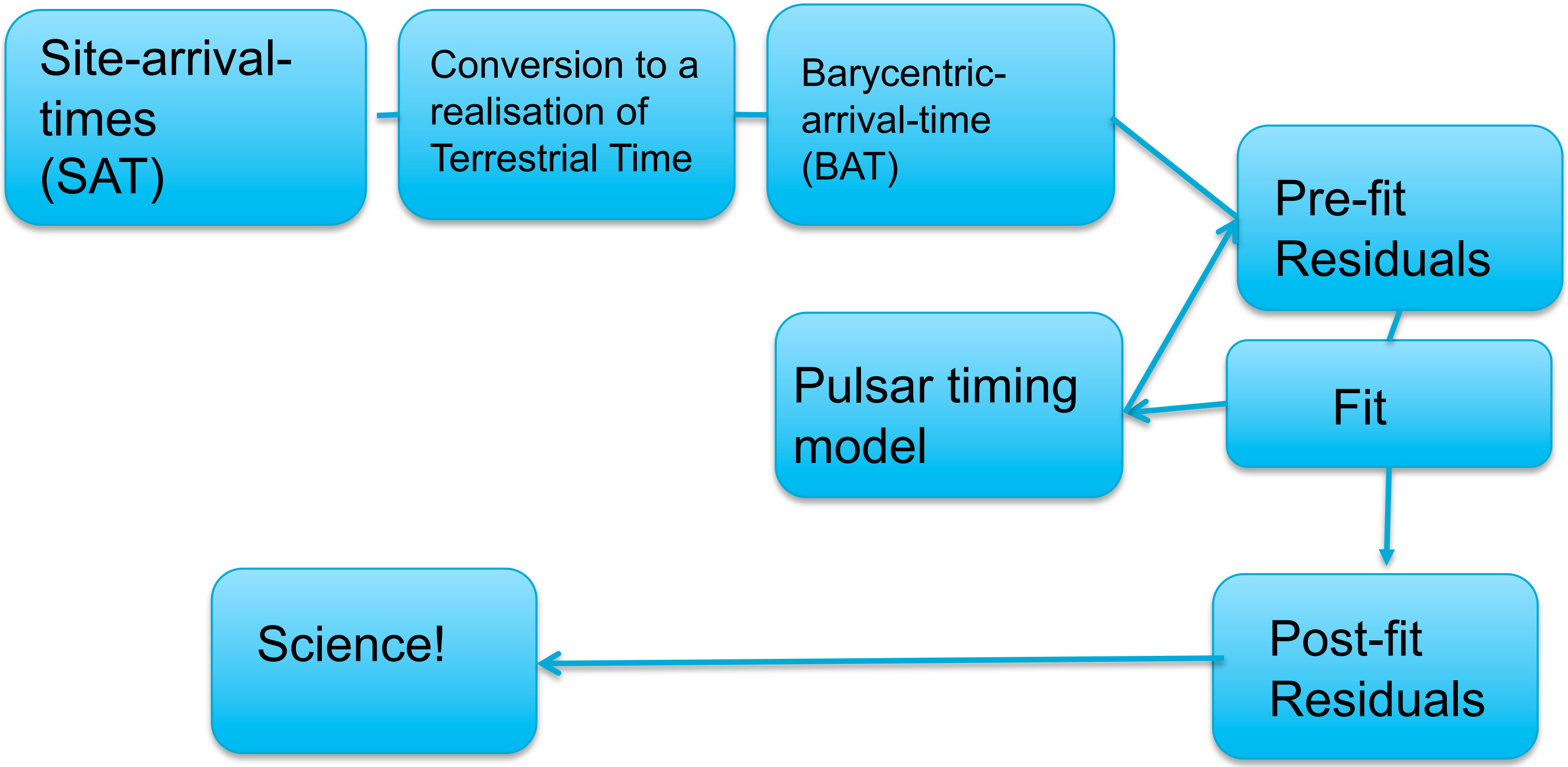
Filename or identifier

Observing frequency (MHz)

Uncertainty on arrival time (us)

User defined flags

Basic idea



Site-arrival-times (SAT)
Conversion to a realisation of Terrestrial Time
Barycentric-arrival-time (BAT)
Pre-fit Residuals
Pulsar timing model
Fit
Post-fit Residuals
Science!

How does tempo2 work?

- Details in Hobbs, Edwards & Manchester (2006) and Edwards, Hobbs & Manchester (2006)

Clock corrections

Einstein delay

Shapiro delay

Secular motion (e.g., radial velocity)

$$\Delta t = \Delta_C + \Delta_A + \Delta_{E_\odot} + \Delta_{R_\odot} + \Delta_{S_\odot} - D/f^2 + \Delta_{VP} + \Delta_B$$

Conversion of site-arrival-time to pulse emission time

Atmospheric delays

Roemer delay

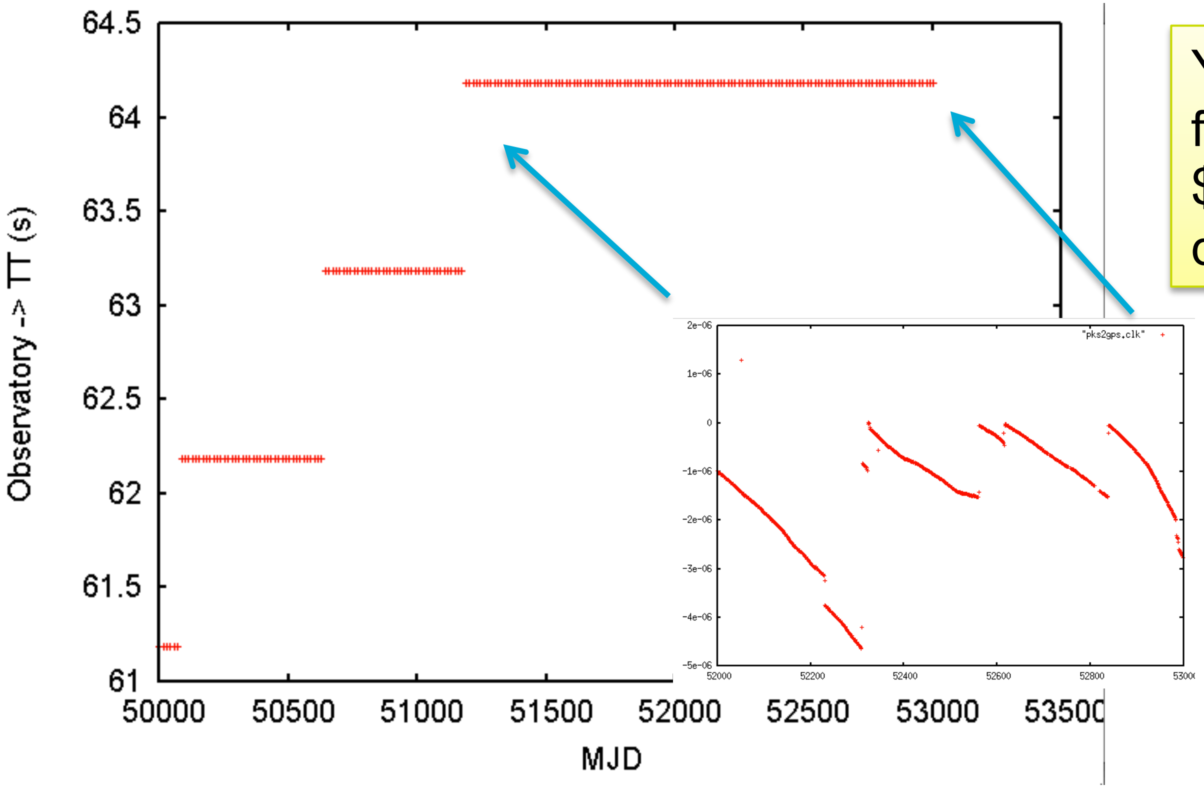
Dispersive delay

Orbital motion

Details: clock correction

$$\Delta t = \Delta_{C} + \Delta_{A} + \Delta_{E_\odot} + \Delta_{R_\odot} + \Delta_{S_\odot} - D/f^2 + \Delta_{VP} + \Delta_{B}$$

- Must convert from the observatory time standard to a realisation of terrestrial time TT. Use set of text files containing the difference between two time standards: pks2gps.clk, gps2utc.clk, utc2tai.clk, tai2tt_bipm2011.

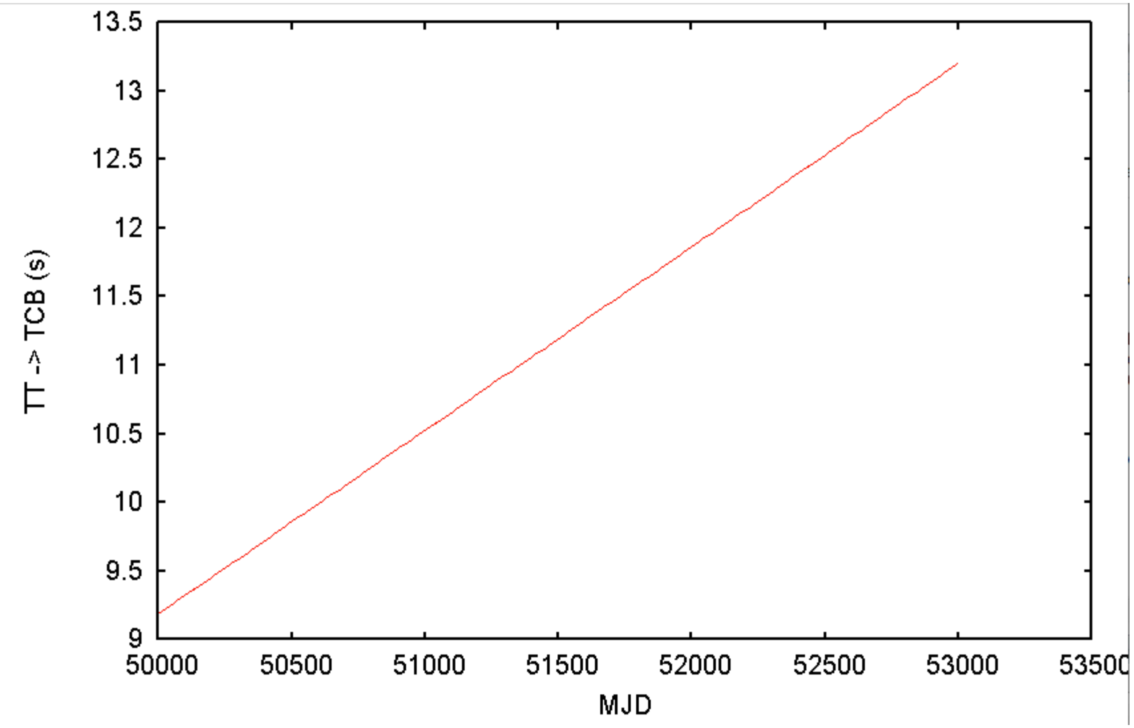


You'll find these files in $TEMPO2/clock directory

Details: Einstein delay

$$\Delta t = \Delta_{\rm C} + \Delta_{\rm A} + \Delta_{{\rm E}_\odot} + \Delta_{{\rm R}_\odot} + \Delta_{{\rm S}_\odot} - D/f^2 + \Delta_{\rm VP} + \Delta_{\rm B}$$

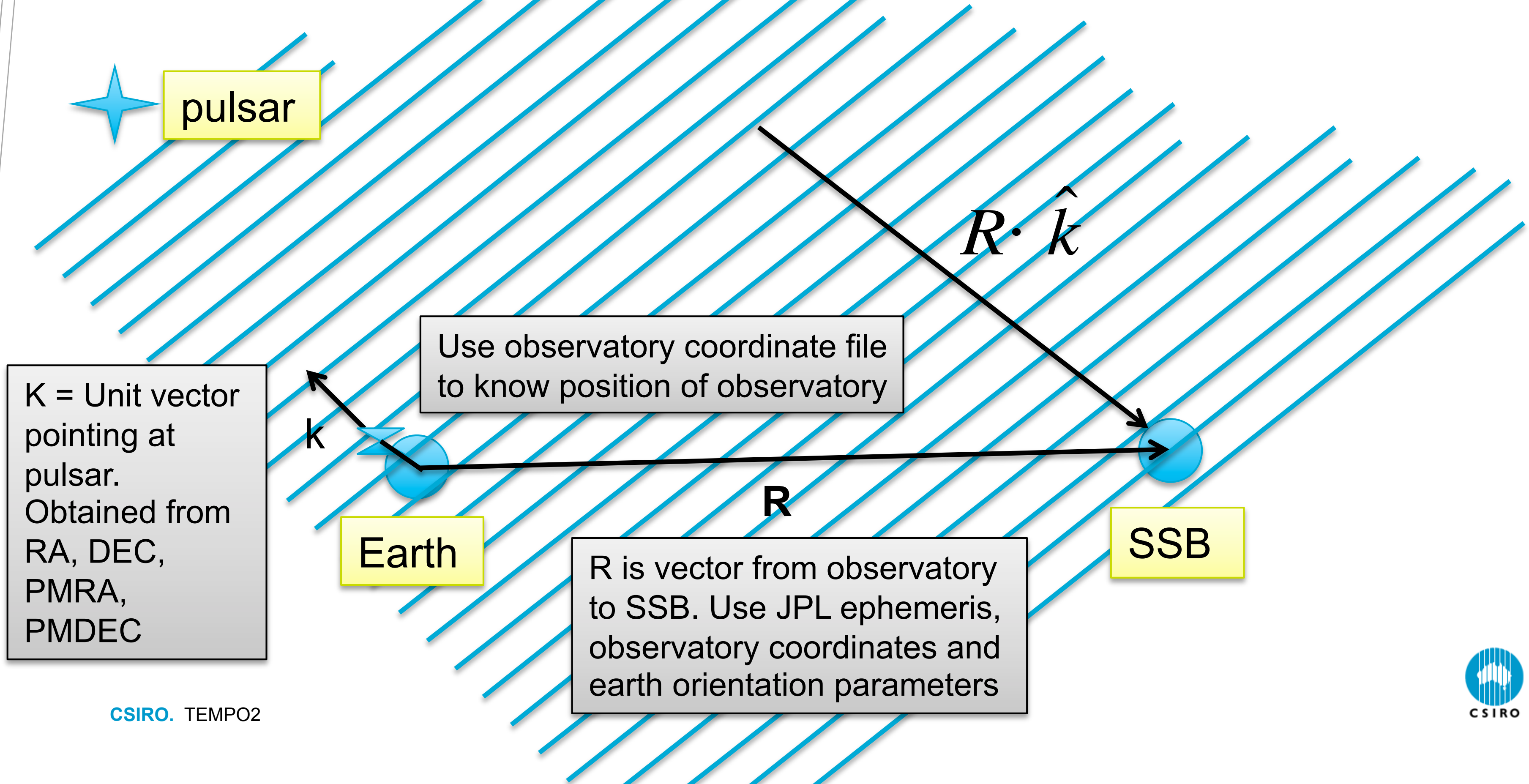
- Pulse arrival times (in terrestrial time) must be converted to the time frame of the Solar System Barycentre.
- TEMPO2 uses the tabulated results of Irwin & Fukushima (1999)



Details: Roemer delay

$$\Delta t = \Delta_{\mathrm{C}} + \Delta_{\mathrm{A}} + \Delta_{\mathrm{E}_\odot} + \Delta_{\mathrm{R}_\odot} + \Delta_{\mathrm{S}_\odot} - D/f^2 + \Delta_{\mathrm{VP}} + \Delta_{\mathrm{B}}$$

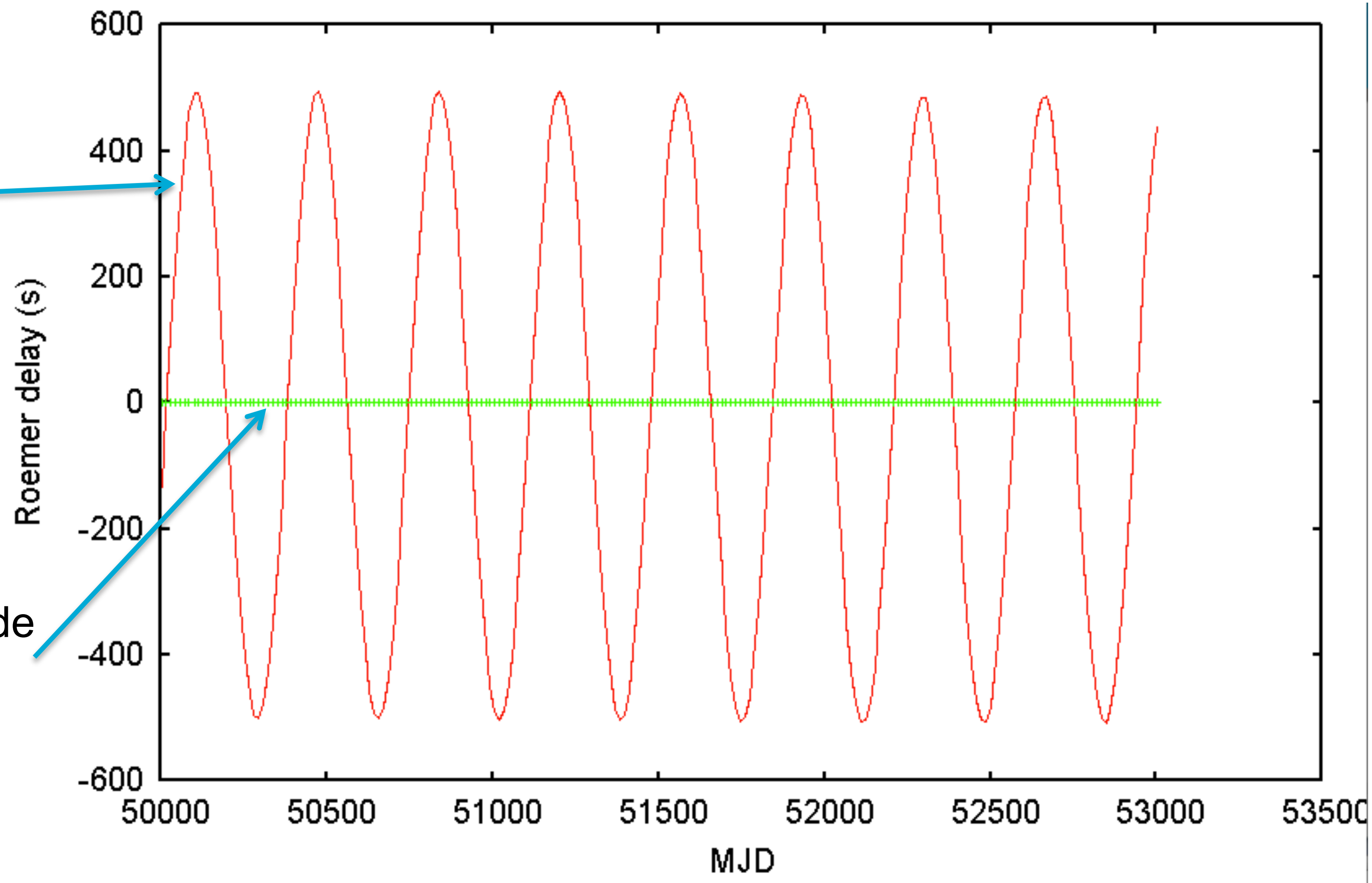
- The Roemer delay is the vacuum light travel time between the pulse arriving at the observatory and the equivalent arrival time at the SSB.



Roemer delay

Pulsar in ecliptic

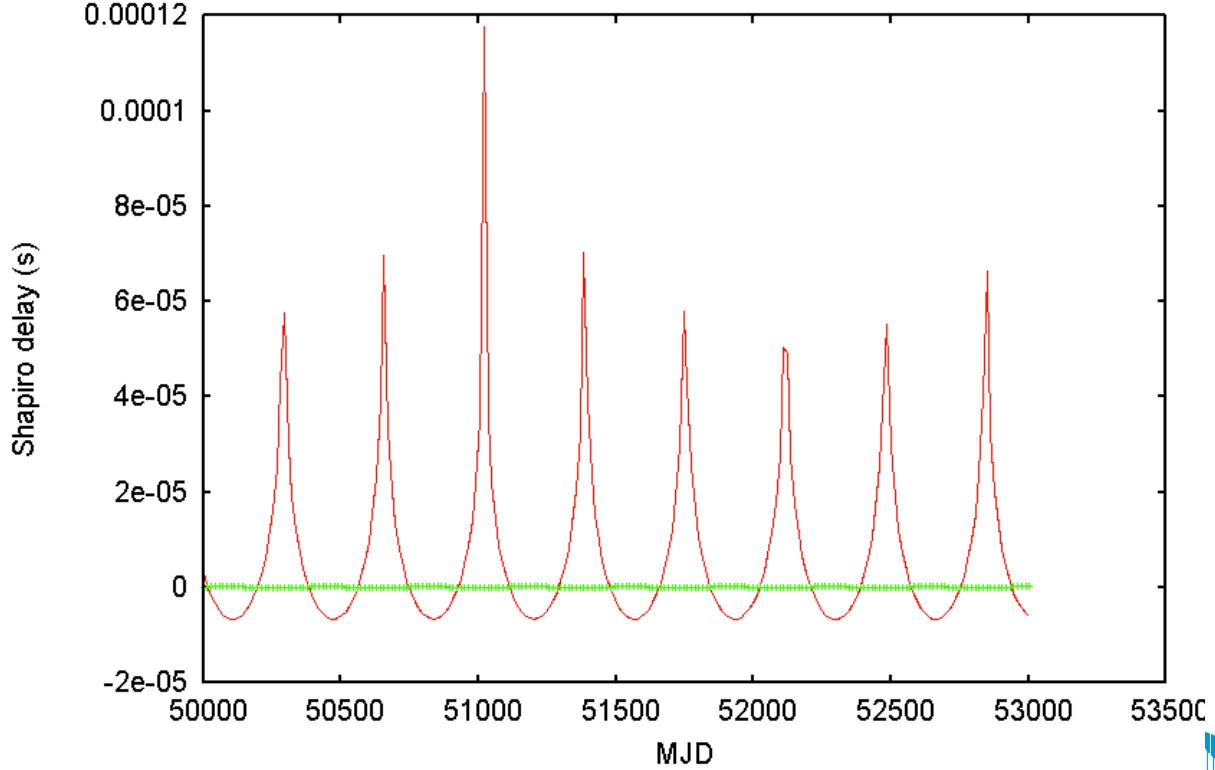
Ecliptic latitude = 90 deg



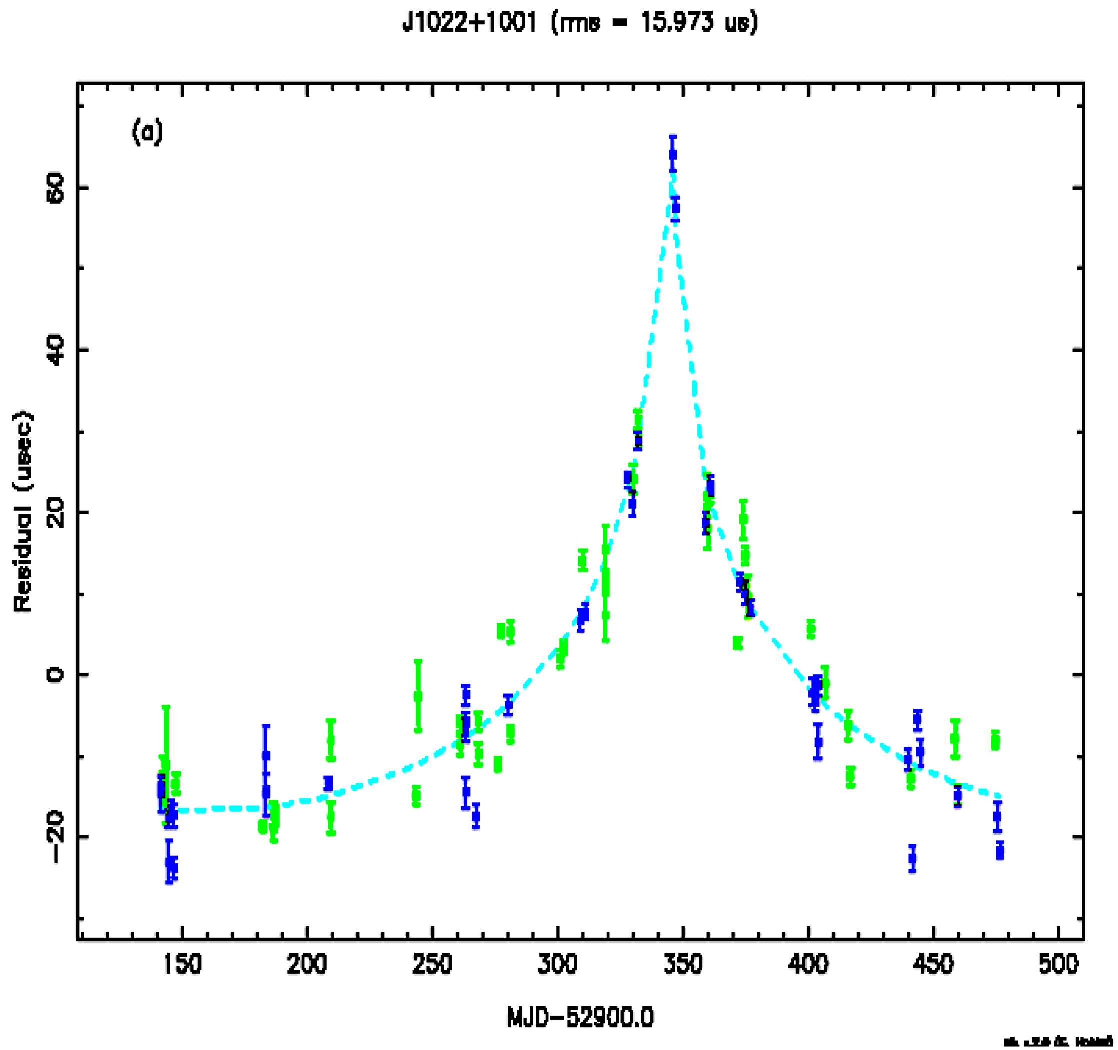
Details: Shapiro delay

$$\Delta t = \Delta_C + \Delta_A + \Delta_{E_\odot} + \Delta_{R_\odot} + \Delta_{S_\odot} - D/f^2 + \Delta_{VP} + \Delta_B$$

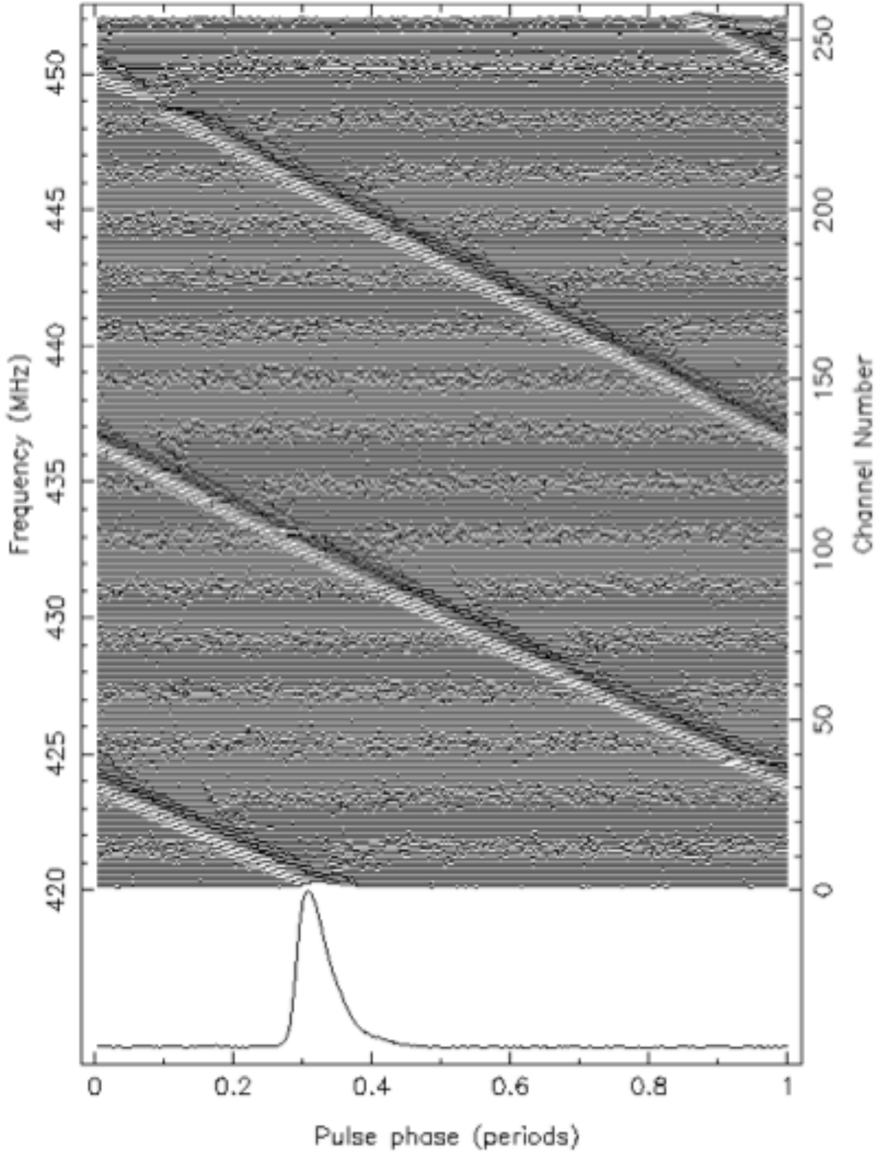
- Time delay caused by the passage of the pulse through large gravitational fields. Tempo2 includes delay caused by Sun (< 110us), Jupiter (<180ns), Saturn (<58ns), Neptune (<12ns) and Uranus (<10ns)



An aside: the Solar Shapiro Delay



Pulse dispersion

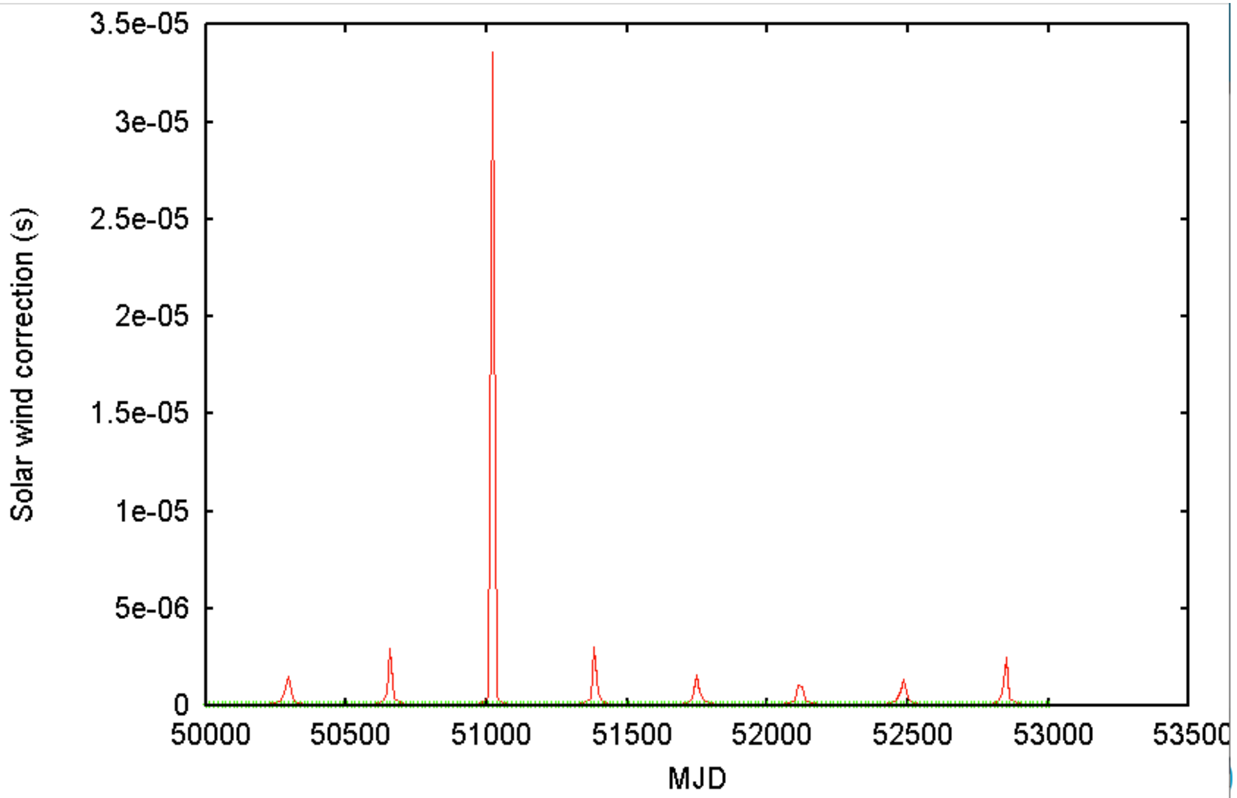


Details: dispersive delay

$$\Delta t = \Delta_C + \Delta_A + \Delta_{E_\odot} + \Delta_{R_\odot} + \Delta_{S_\odot} - D/f^2 + \Delta_{VP} + \Delta_B$$

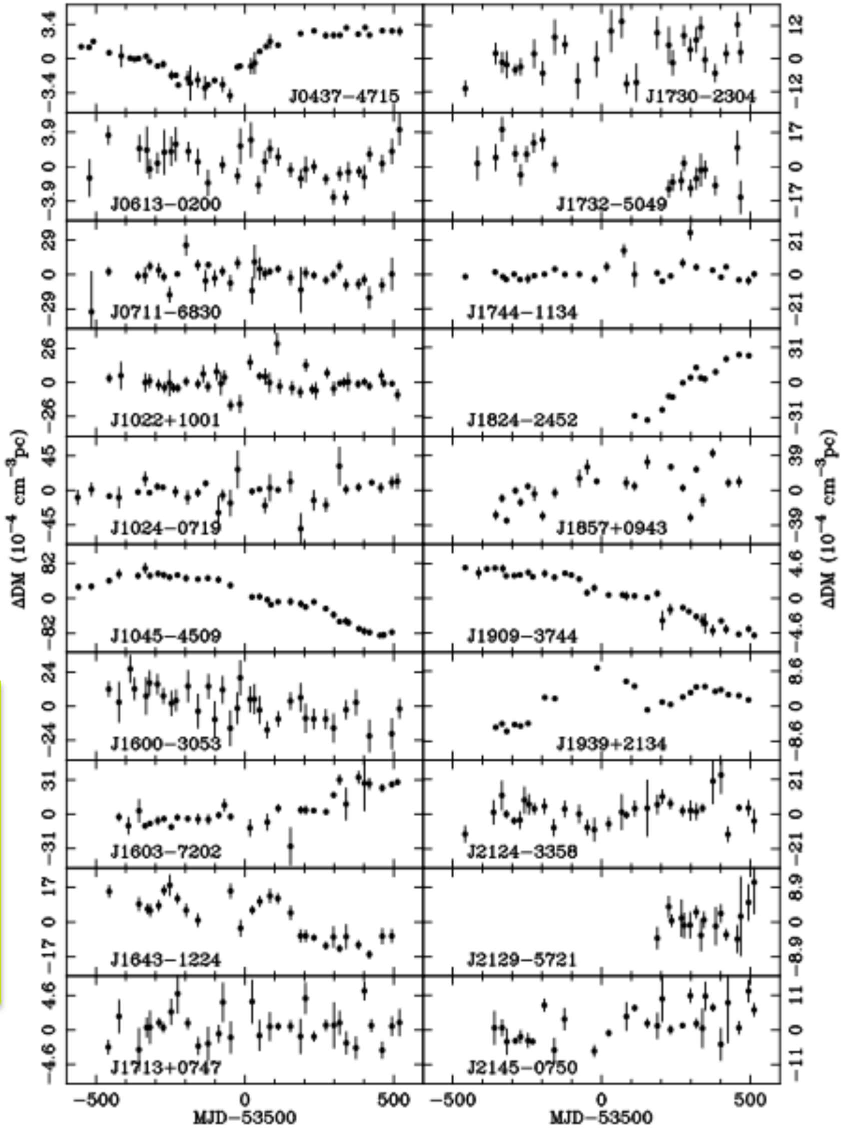
- D is the dispersion constant. DM (cm^{-3}pc) = 2.410 x 10^{-16} D
- f is the frequency of the radiation at the Solar System barycentre

$$DM = \int n_e dl$$



Dispersion measure

- You et al. (2007)

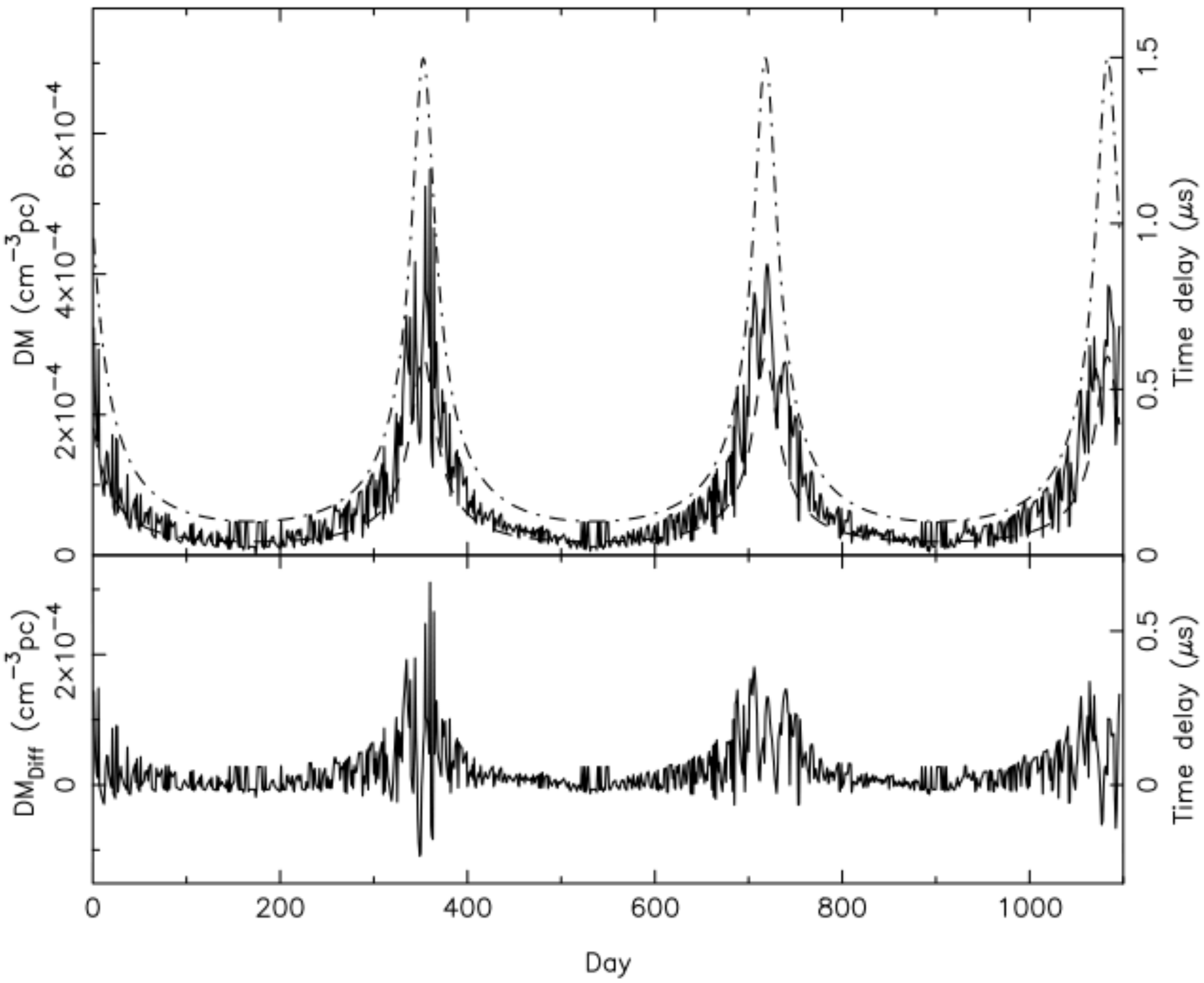


Bored?
Have a computer?

Try:
http://pulseatparkes.atnf.csiro.au/distance/

Solar wind

- You et al. (2007), MNRAS



Details: binary system

$$\Delta t = \Delta_C + \Delta_A + \Delta_{E_\odot} + \Delta_{R_\odot} + \Delta_{S_\odot} - D/f^2 + \Delta_{VP} + \Delta_B$$

Parameter	Description	Symbol
PB	Binary period of pulsars (d)	P_b
ECC	Eccentricity of orbit	e
A1	Projected semimajor axis of orbit (lt-s)	x
T0	Epoch of periastron (MJD)	T_0
OM	Longitude of periastron (°)	ω
TASC	Epoch of ascending node (MJD)	T_{asc}
EPS1	$e \sin \omega$	η
EPS2	$e \cos \omega$	κ
KOM	longitude of the ascending node	Ω
KIN	inclination angle	i
SHAPMAX	$-\ln(1 - \sin i)$	s_x
OMDOT	Periastron advance (deg yr^{-1})	$\dot{\omega}$
PBDOT	The first time derivative of binary period	$\dot{P}_b$
ECCDOT	Rate of change of eccentricity (s^{-1})	$\dot{e}$
A1DOT	Rate of change of semimajor axis (lt-s s^{-1})	$\dot{x}$
GAMMA	Post-Keplerian 'gamma' term (s)	γ
XPBDOT	Rate of change of orbital period minus GR prediction	
EPS1DOT	Rate of change of EPS1	$\dot{\eta}$
EPS2DOT	Rate of change of EPS2	$\dot{\kappa}$
MTOT	Total system mass ($M_\odot$)	M
M2	Companion mass ($M_\odot$)	m_2
DTHETA	Relativistic deformation of the orbit	d_θ
XOMDOT	Rate of periastron advance minus GR prediction (deg yr^{-1})	
SINI	Sine of inclination angle	s
DR	Relativistic deformation of the orbit	d_r
A0	The first aberration parameter	A
B0	The second aberration parameter	B
BP	Tensor multiscalar parameter	β'
BPP	Tensor multiscalar parameter	β''
AFAC	Aberration geometric factor	

- For pulsars in binary systems, tempo2 includes parameters describing the orbital motion.
- Various binary models exist. Suggest using "T2" model that combines most earlier binary models

Using the pulsar timing model

- Have pulse emission time in the pulsar frame.
- Predict using the pulsar timing model

Pulse frequency (and time derivatives)

Reference phase

$$\phi(t) = \sum_{n \geqslant 1} \frac{\nu^{(n-1)}}{n!} \left(t_{\mathrm{e}}^{\mathrm{psr}} - t_{\mathrm{P}}\right)^n + \phi_0.$$

Phase of pulse sequence

Pulse emission time

Time at which $d\phi/dt = \nu$

- Can also include simple model of glitch events

CSIRO

Timing residuals

Pulse phase

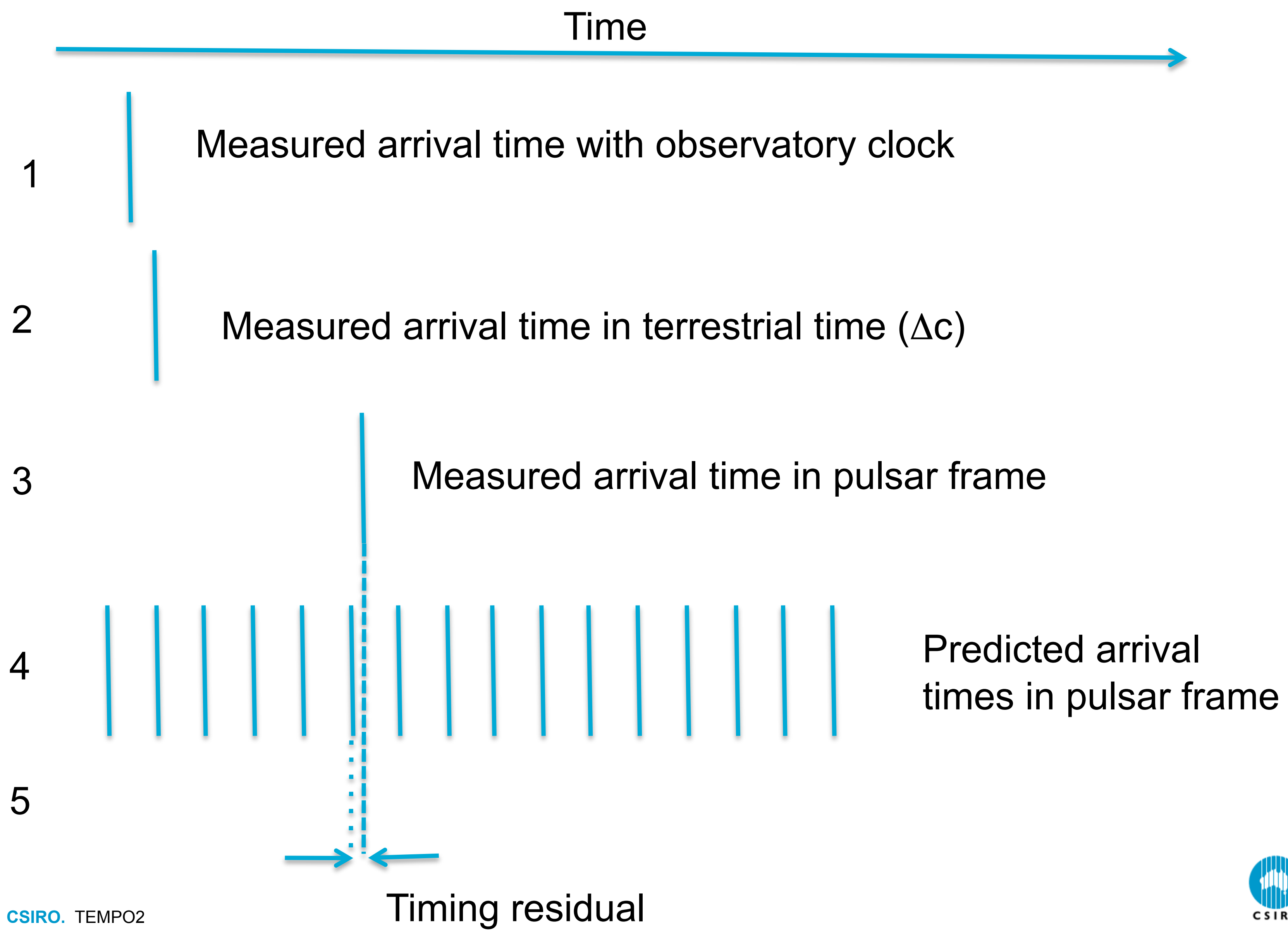
Nearest integer to ϕ_i

$$R_i = \frac{\phi_i - N_i}{\nu}$$

Timing residual for i'th observation

Pulse frequency

Example

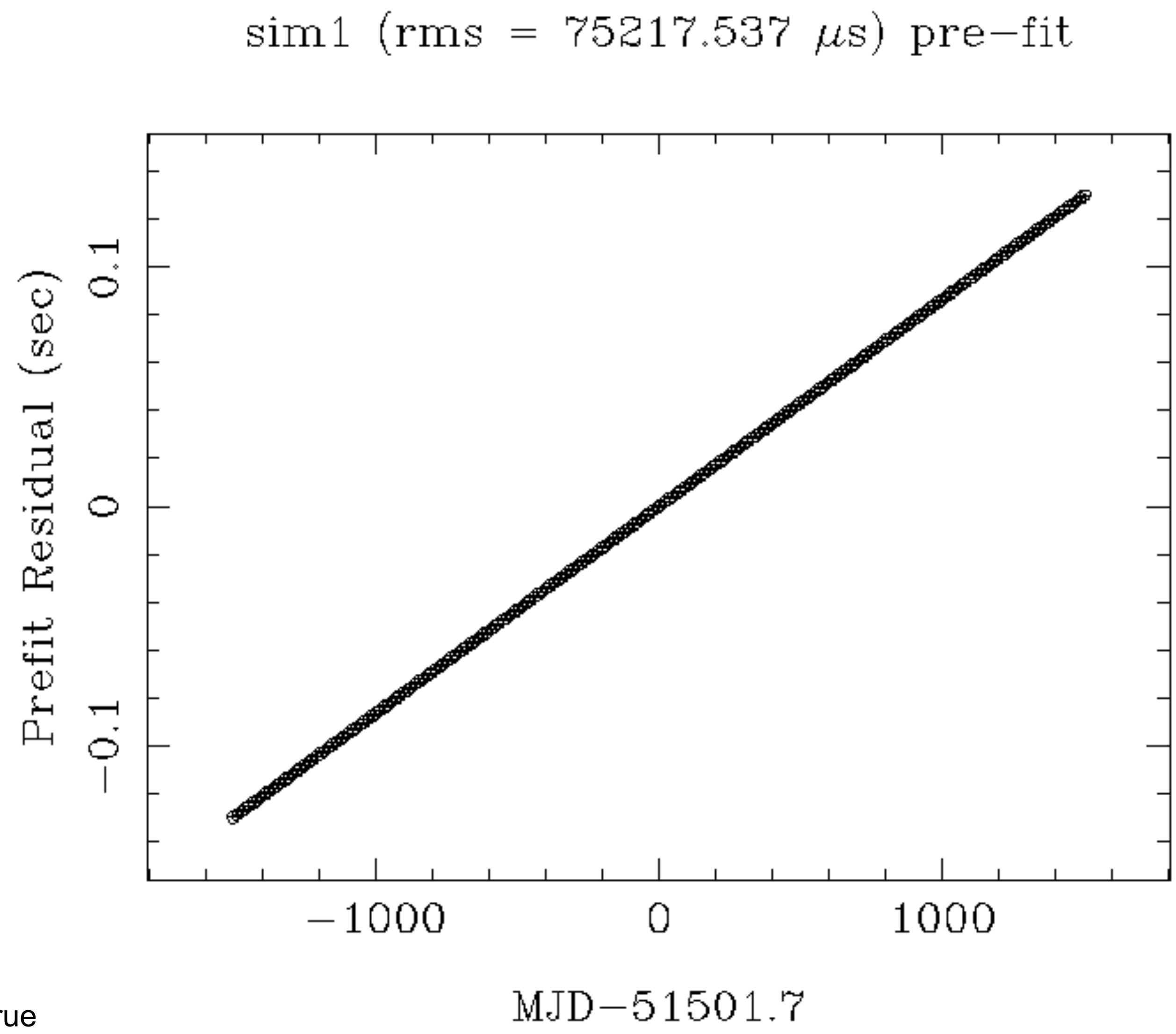

Time
1
Measured arrival time with observatory clock
2
Measured arrival time in terrestrial time (Δc)
3
Measured arrival time in pulsar frame
4
Predicted arrival times in pulsar frame
5
Timing residual

Pulsar timing residuals: let's say it again!

- If pulsar model predicts the observations perfectly (and the conversion from the observatory to pulsar frame is perfect) then R = 0 (within measurement uncertainty).

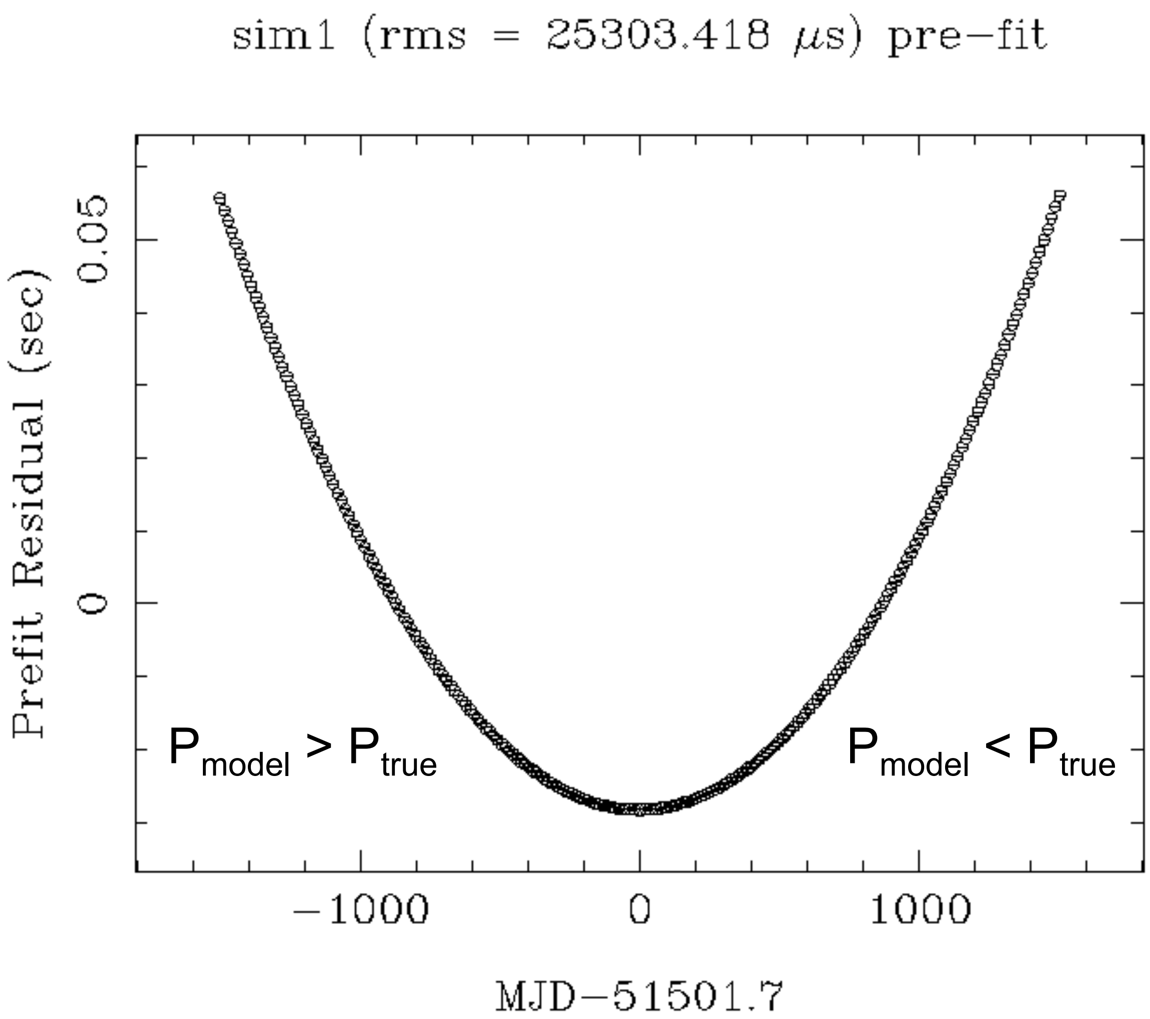
- If R != 0 then the pulsar model is (1) not accurate or (2) does not include a physical process that affects the measured arrival times or (3) the correction from the observatory to pulsar frame is not correct.

Pulsar timing residuals: incorrect F0

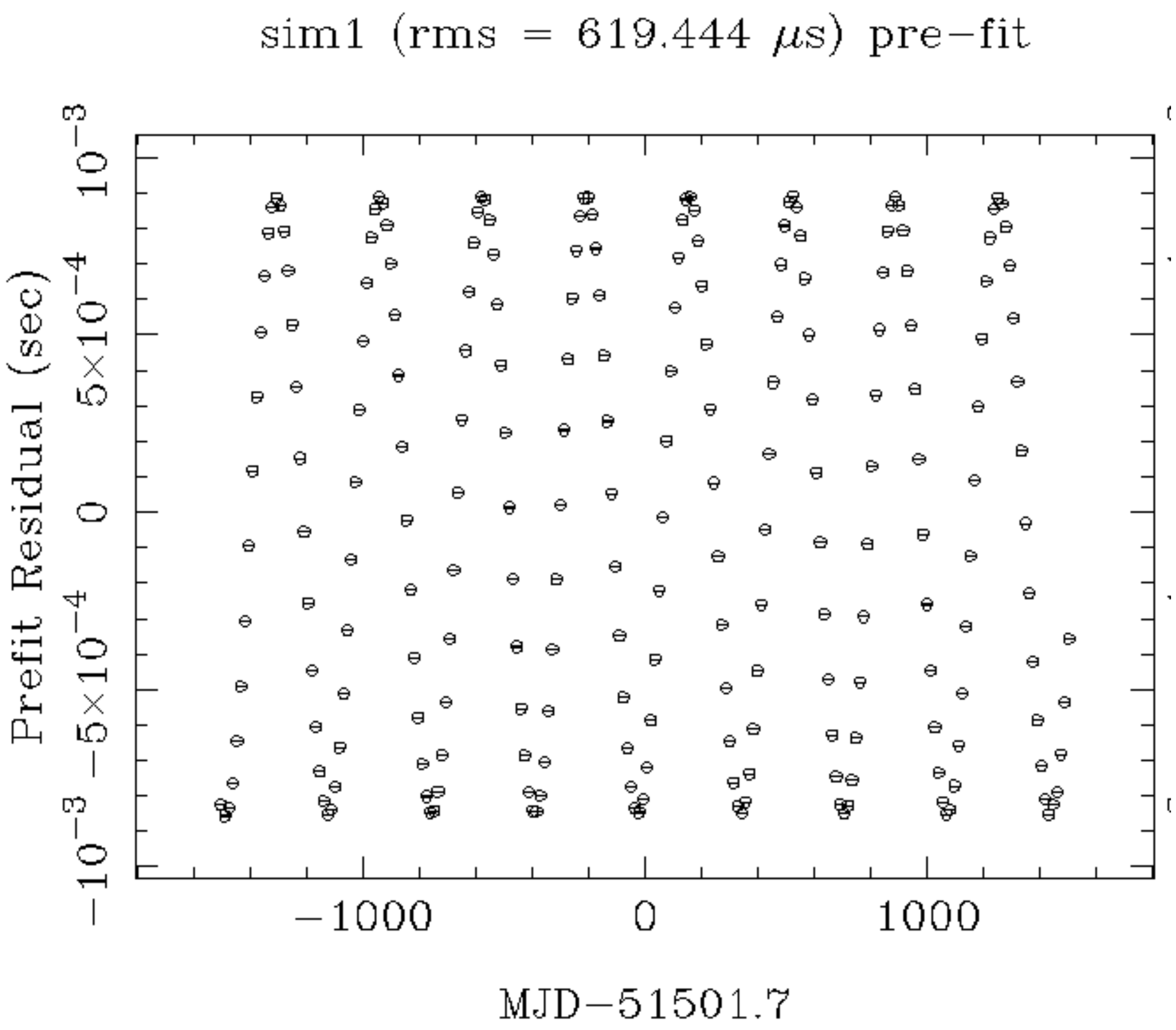


$P_{model} < P_{true}$

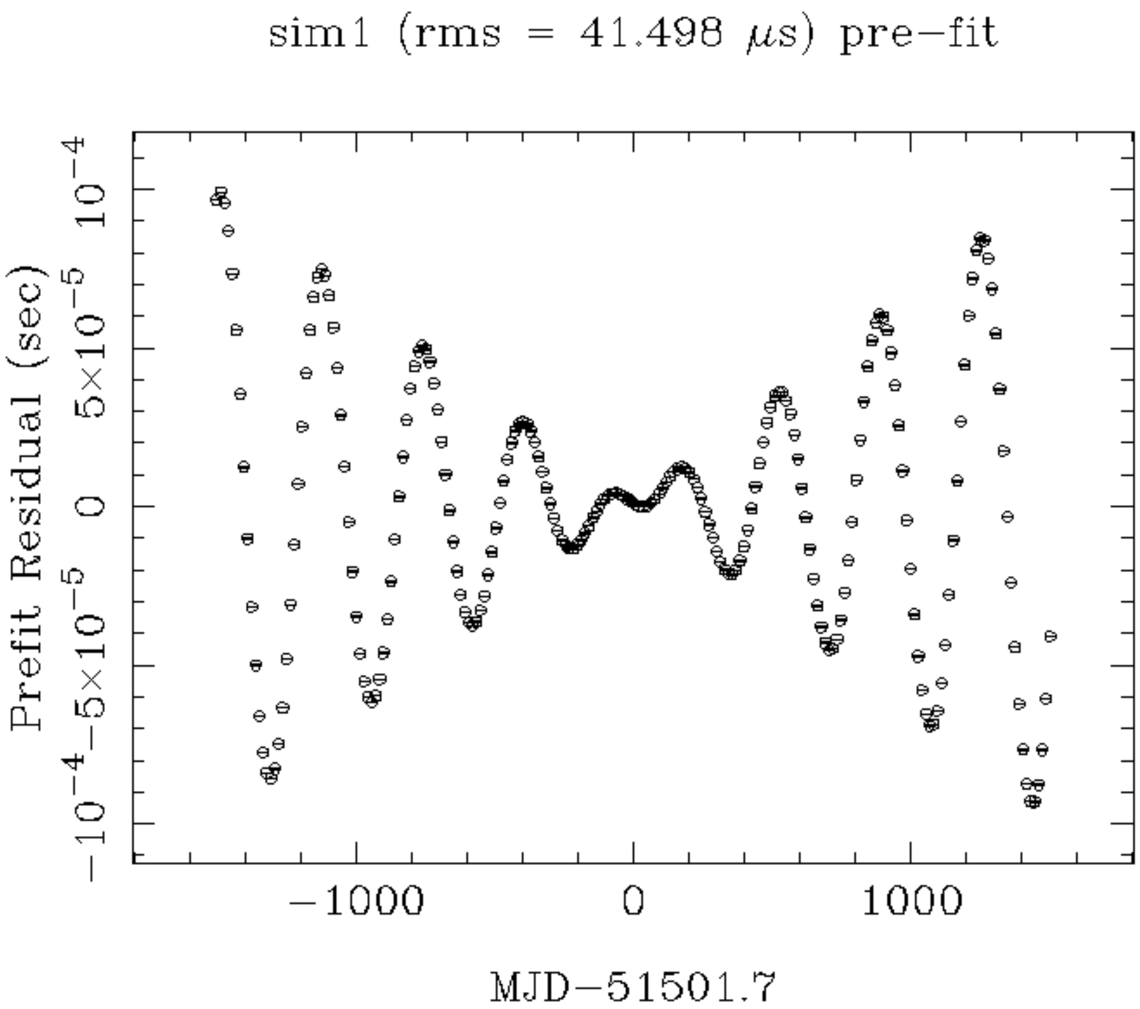
Pulsar timing residuals: incorrect F1



Pulsar timing residuals: incorrect position

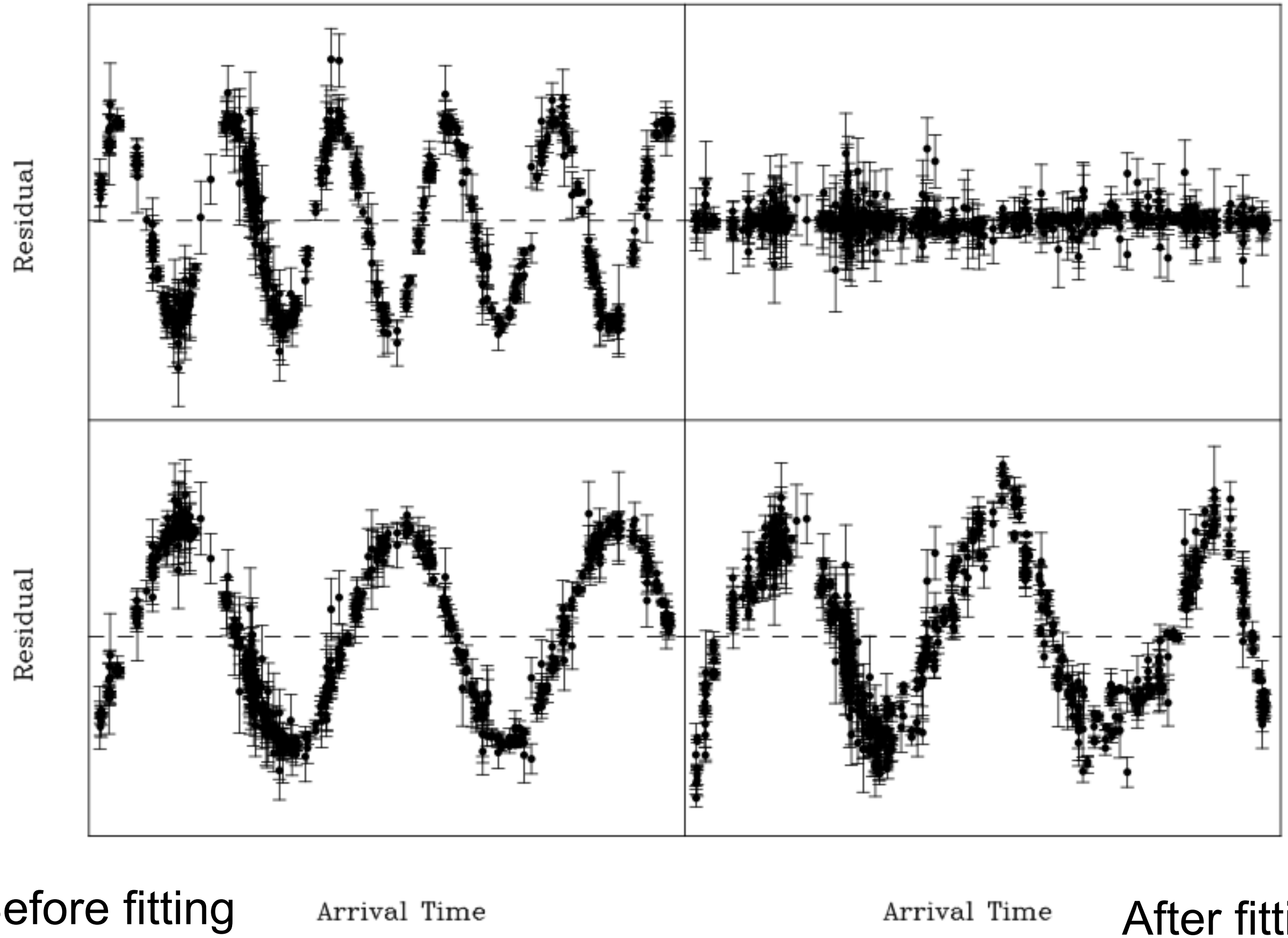


Pulsar timing residuals: incorrect proper motion



Absorbing a gravitational wave signal

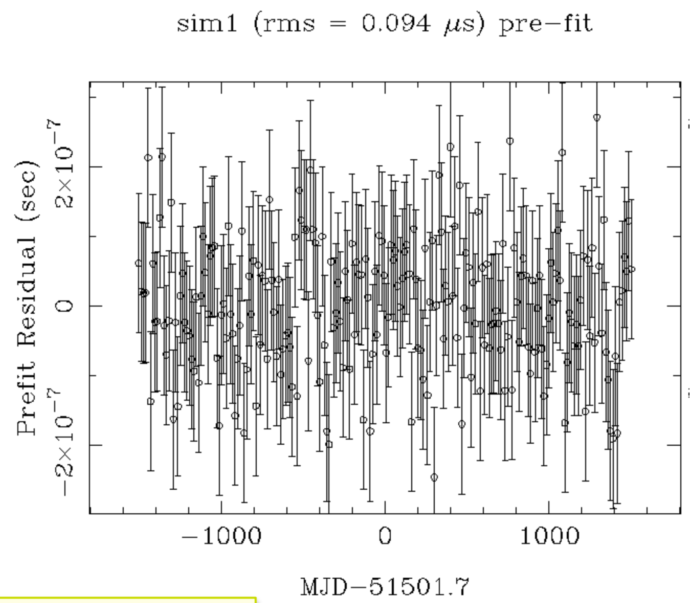
- Yardley (2010) MNRAS

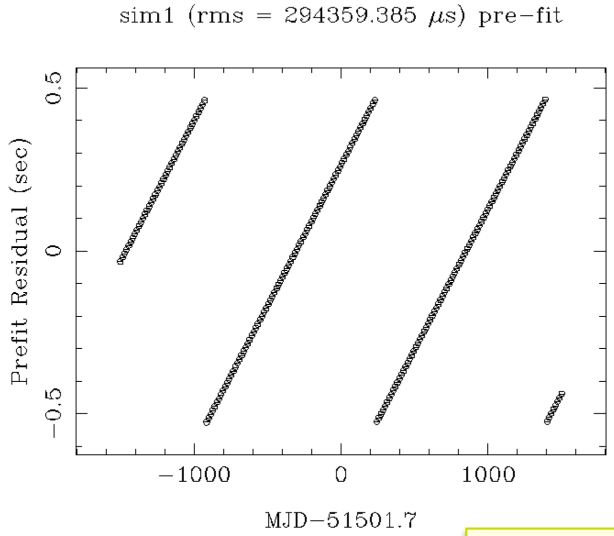


Before fitting

After fitting

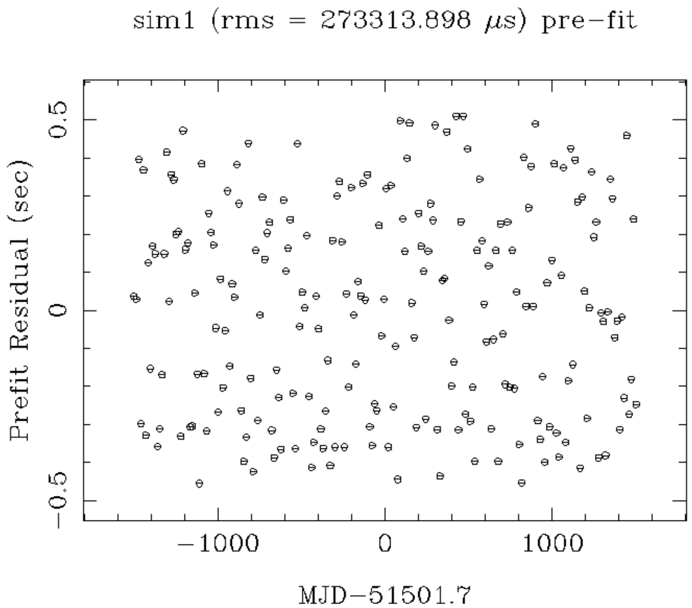
Pulsar timing residuals





Phase wraps

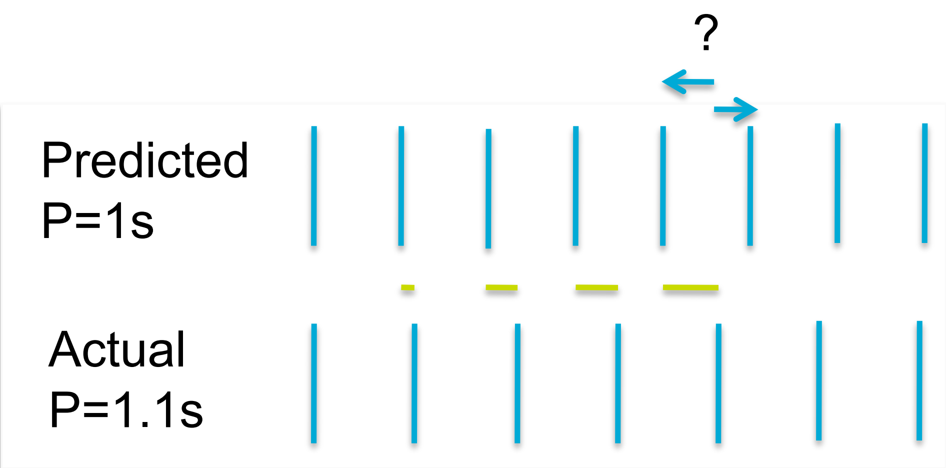
Good parameters

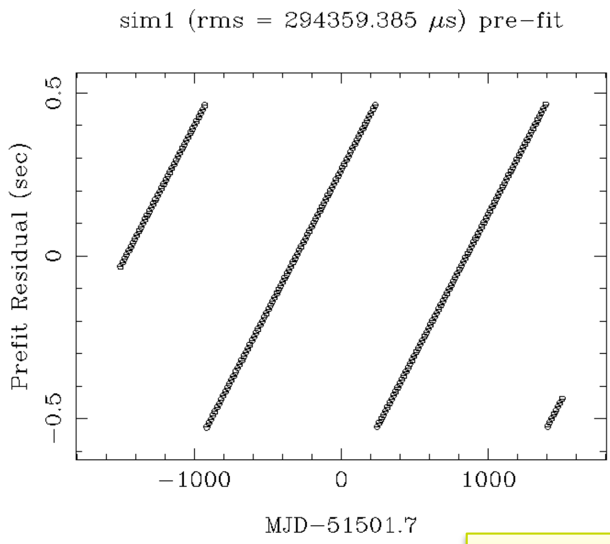


Do not have a phase connected solution

Phase wraps

Always chooses the closest pulse!

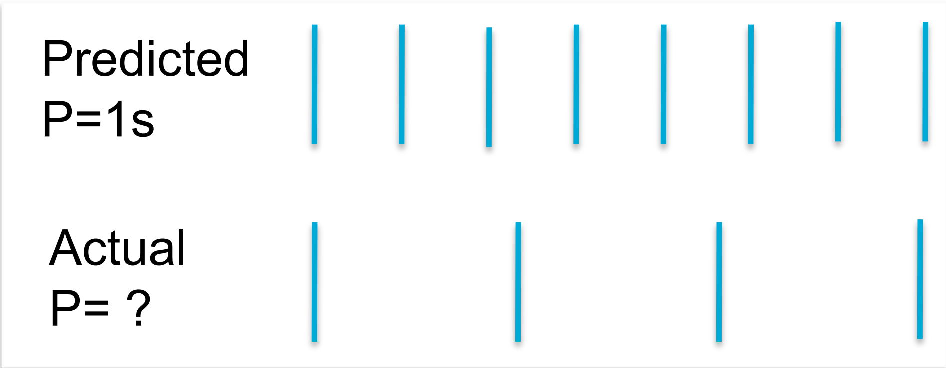


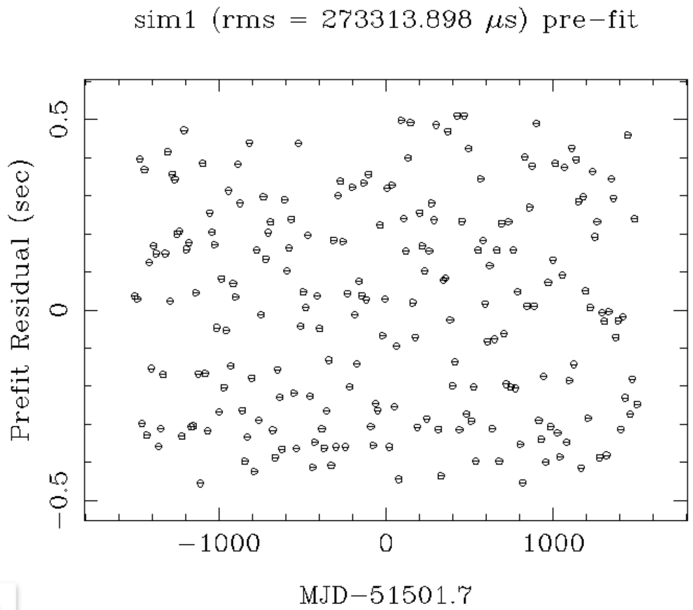


Phase wraps

No phase connection

Do not have a phase connected solution





A few notes

TEMPO2 is written in C/C++ (with a little Fortran) and runs on linux and MacOS.

- Main tempo2 website: http://www.atnf.csiro.au/research/pulsar/tempo2 (see "tutorials" and "Documentation")

- Main tempo2 download site:
- http://sourceforge.net/projects/tempo2

- Tempo2 email distribution list:
- Sign up at http://pulsarastronomy.net (click on "Mailing Lists")
- Contact: george.hobbs@csiro.au, use feedback form

Part 3: Gravitational waves

- Part 1: Getting pulsar data files
- Part 2: Pulsar timing
- Part 3: Gravitational waves

Part 3: Gravitational waves and pulsars

Gravitational waves

- I am not an expert in general relativity

In general relativity, the metric determines the spacetime geometry

$$ds^2 = g_{\mu\nu} dx^\mu \, dx^\nu$$

Einstein's equation determines the dynamics of the metric:

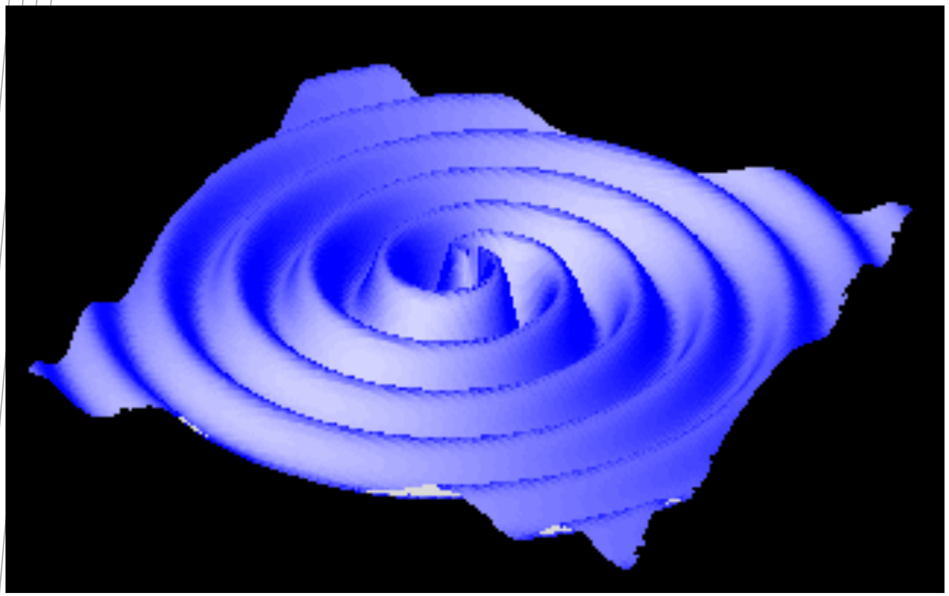
$$G_{\mu\nu}(g) = 8 \pi T_{\mu\nu}$$

Taking $g_{\mu\nu}$ to be of the following form:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

We get a wave equation for h:

$$- \partial^2 h_{\mu\nu} / \partial^2 t + r^2 h_{\mu\nu} = 4\pi T_{\mu\nu}$$



"A gravitational wave is a fluctuation in the curvature of space-time which propagates as a wave."

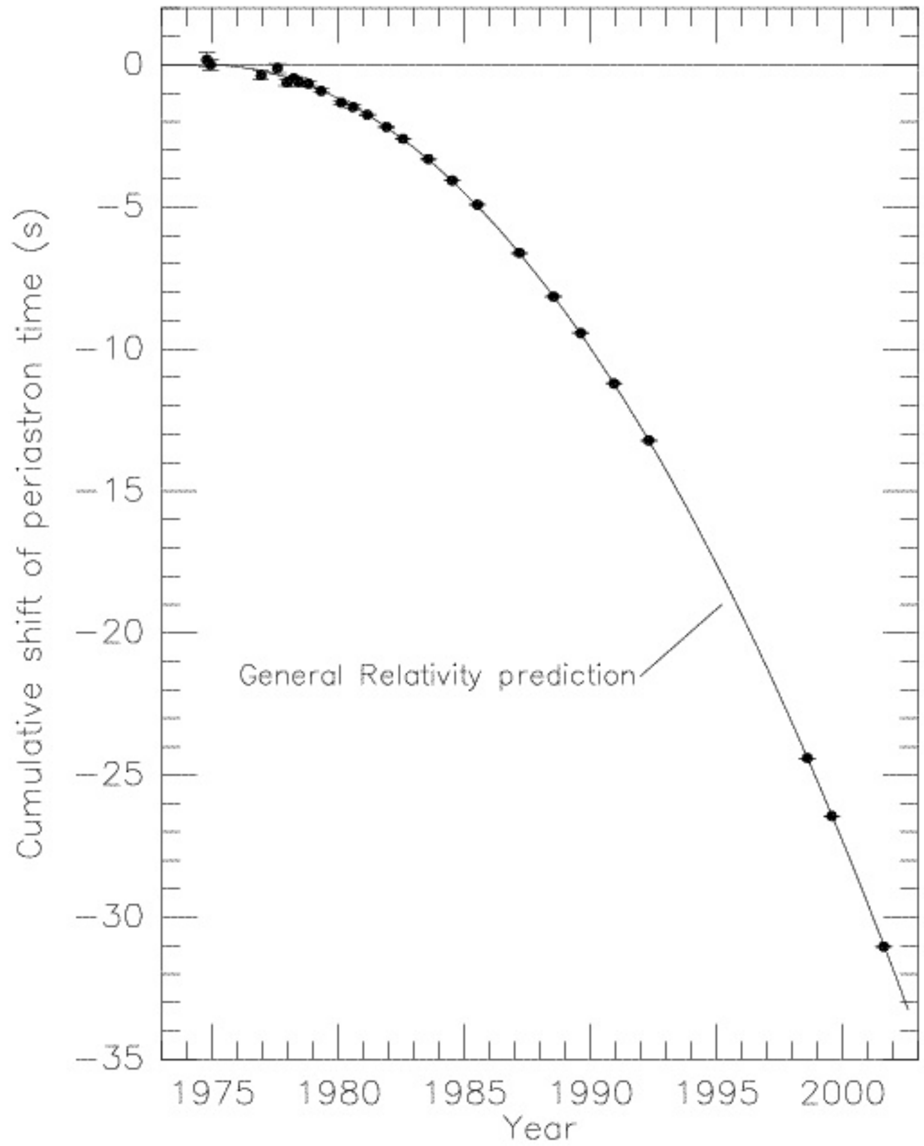
In general relativity, gravitational waves travel at the speed of light

Gravitational waves are generated by the motion of masses

How do we know that gravitational waves exist?

• Prediction based on measured Keplerian parameters and Einstein's general relativity due to emission of *gravitational waves* (1.5cm per orbit)

•After ~250 MYr the two neutron stars will collide!

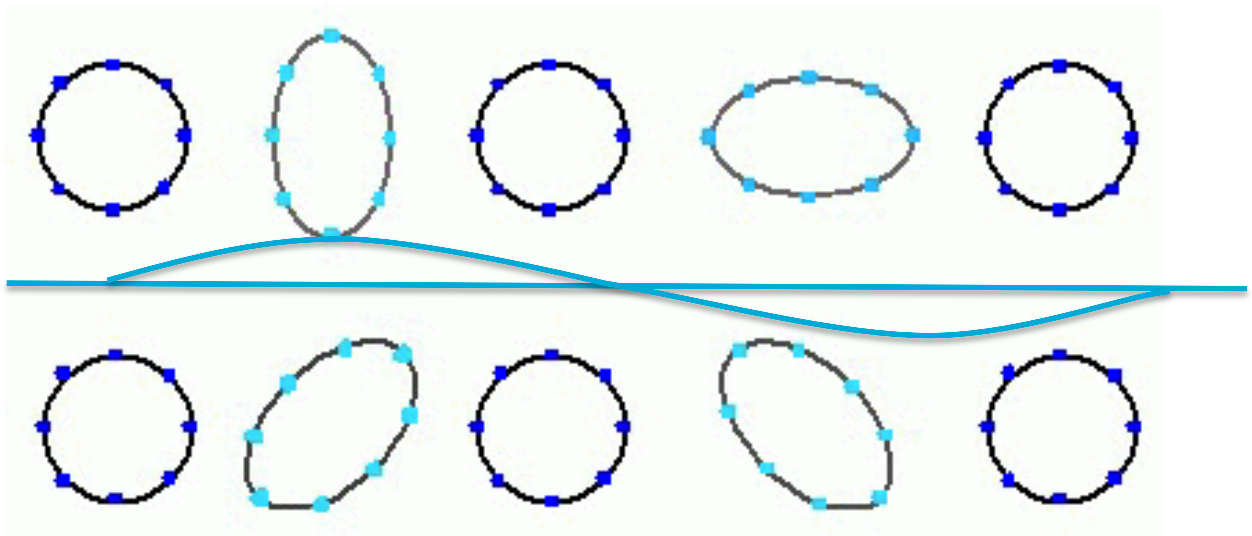


(Weisberg & Taylor 2003)

Describing gravitational waves

Use standard terminology: wave-length (or frequency), amplitude and polarisation properties

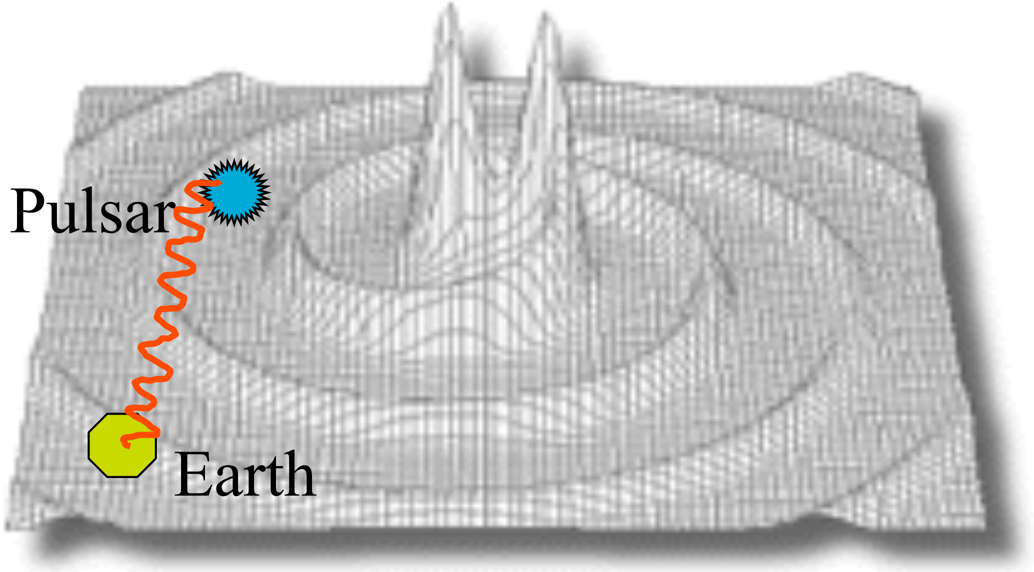
A+



Ax

Can have linearly, circularly or elliptically polarised waves

Gravitational waves



Note: pulsar distances are not known precisely enough to predict the pulsar term.

GW strain at Earth

The "Earth" term

$$\frac{\delta\nu}{\nu} = -\mathcal{H}^{ij}(h_{ij}(t_e, x_e^i) - h_{ij}(t_e - d, x_p^i))$$

$$R(t) = -\int_0^t \frac{\delta\nu(t)}{\nu} dt$$

GW strain at pulsar (assumed distance d)

The "pulsar" term

The geometrical factors

- Pulsar coordinates are described in “right ascension” and “declination”
- Must describe the gravitational wave source, the wave propagation and polarisation angle using consistent coordinate system
- See Hobbs et al. 2010 (MNRAS) or Lee et al. (2011)

$$
\begin{aligned}
P_+ &= (k_p \cdot \lambda)^2 - (k_p \cdot \beta)^2 \\
P_\mathrm{x} &= 2(k_p \cdot \lambda)(k_p \cdot \beta) \\
\gamma &= k_p \cdot k_g
\end{aligned}
$$

Angular factors

$$
\begin{aligned}
k_p \cdot \beta &= -\sin\beta_p \cos\beta_g + \cos(\lambda_g - \lambda_p)\cos\beta_p \sin\beta_g \\
k_p \cdot \lambda &= \cos\beta_p \sin(\lambda_g - \lambda_p) \\
k_p \cdot k_g &= \cos\beta_g \cos\beta_p \cos(\lambda_g - \lambda_p) + \sin\beta_g \sin\beta_p
\end{aligned}
$$

Putting it all together

$$R_e(t) = \text{Real}\left(\int_0^t \frac{P_+ A_+(t) + P_\times A_\times(t)}{2(1-\gamma)} dt\right)$$

For a non-evolving source, the terms $A_{+,\times}$ are given by

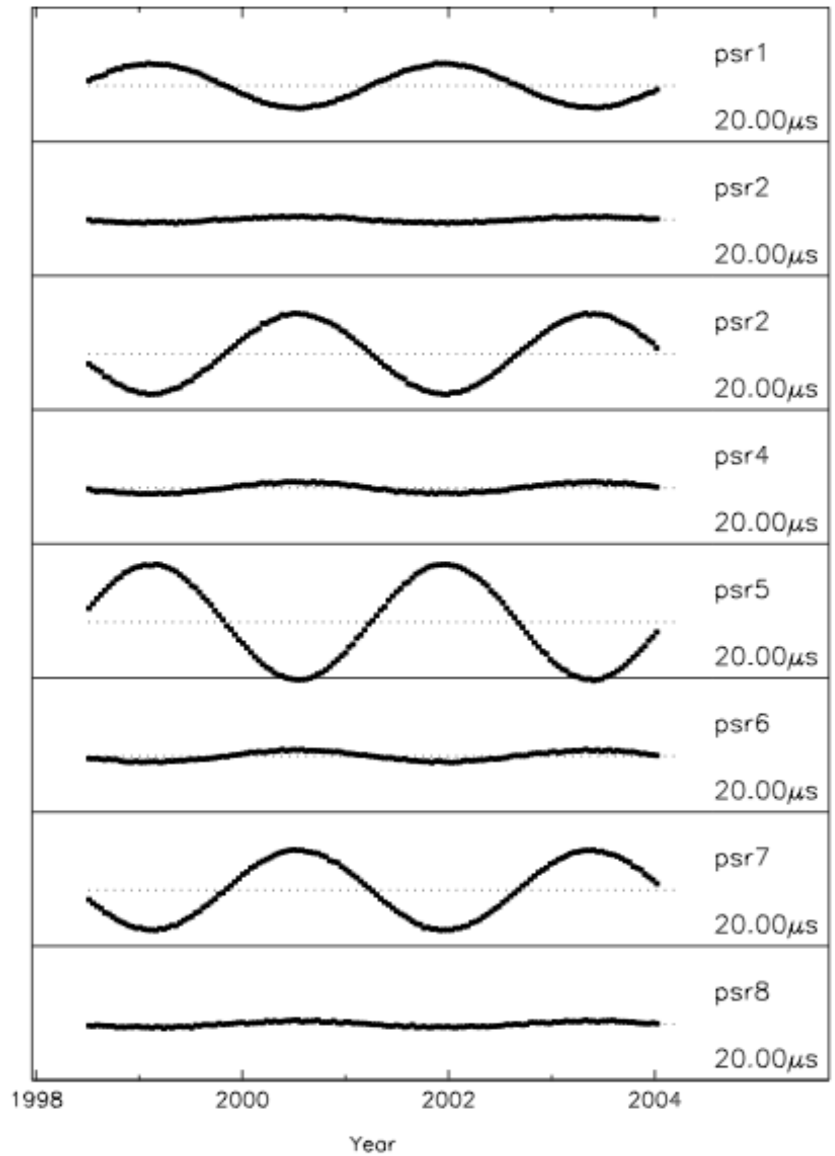
$$A_{+,\times} = A_{+,\times} e^{i\omega_g t}$$

Pulsar term is the same except for extra phase:

$$\Delta_\phi = (1 + k_p \cdot k_g)\frac{D\omega_g}{c}$$

Single GW sources: non-evolving: only Earth term

- Can simulate "perfect" pulse arrival times (given a pulsar timing model).
- Can add the induced residuals from a non-evolving (sinusoidal) GW source.
- Can use tempo2 to form timing residuals
- (Here simulate a GW signal only in A+ polarisation, pulsars in ring separated by 45 degrees)

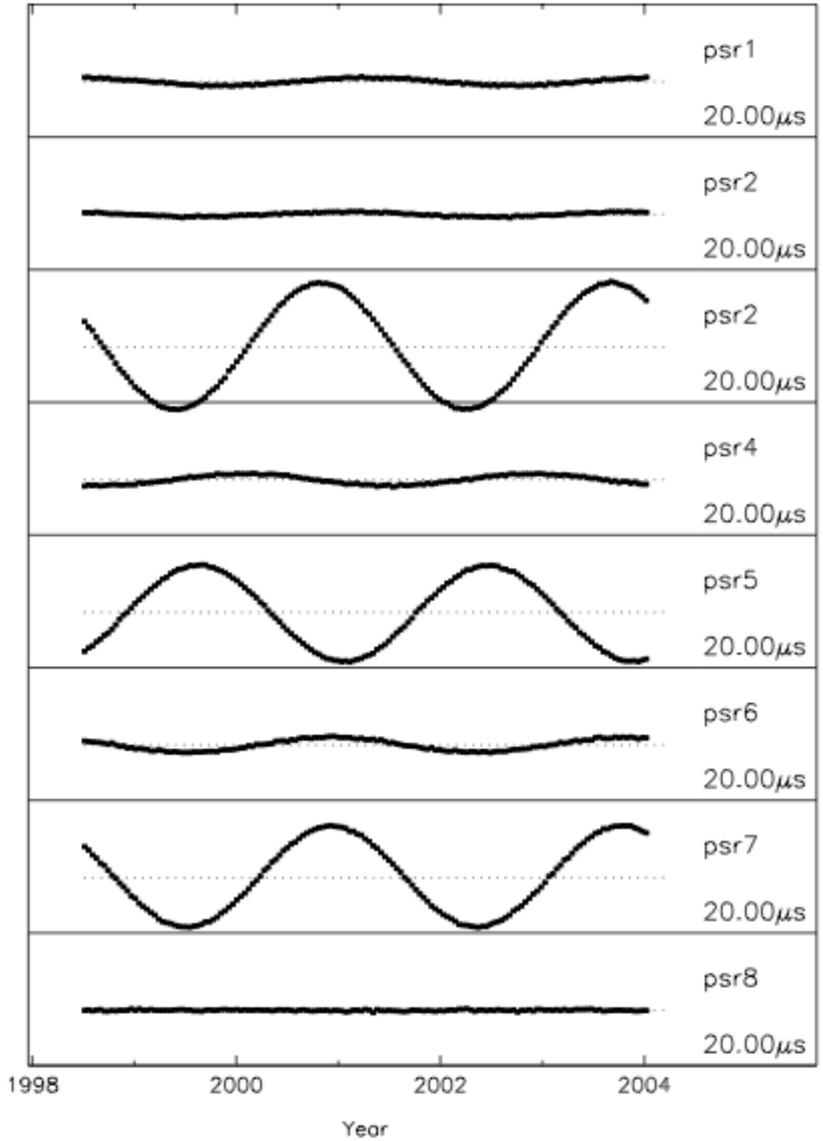


Single GW sources: non-evolving: Earth term and pulsar term

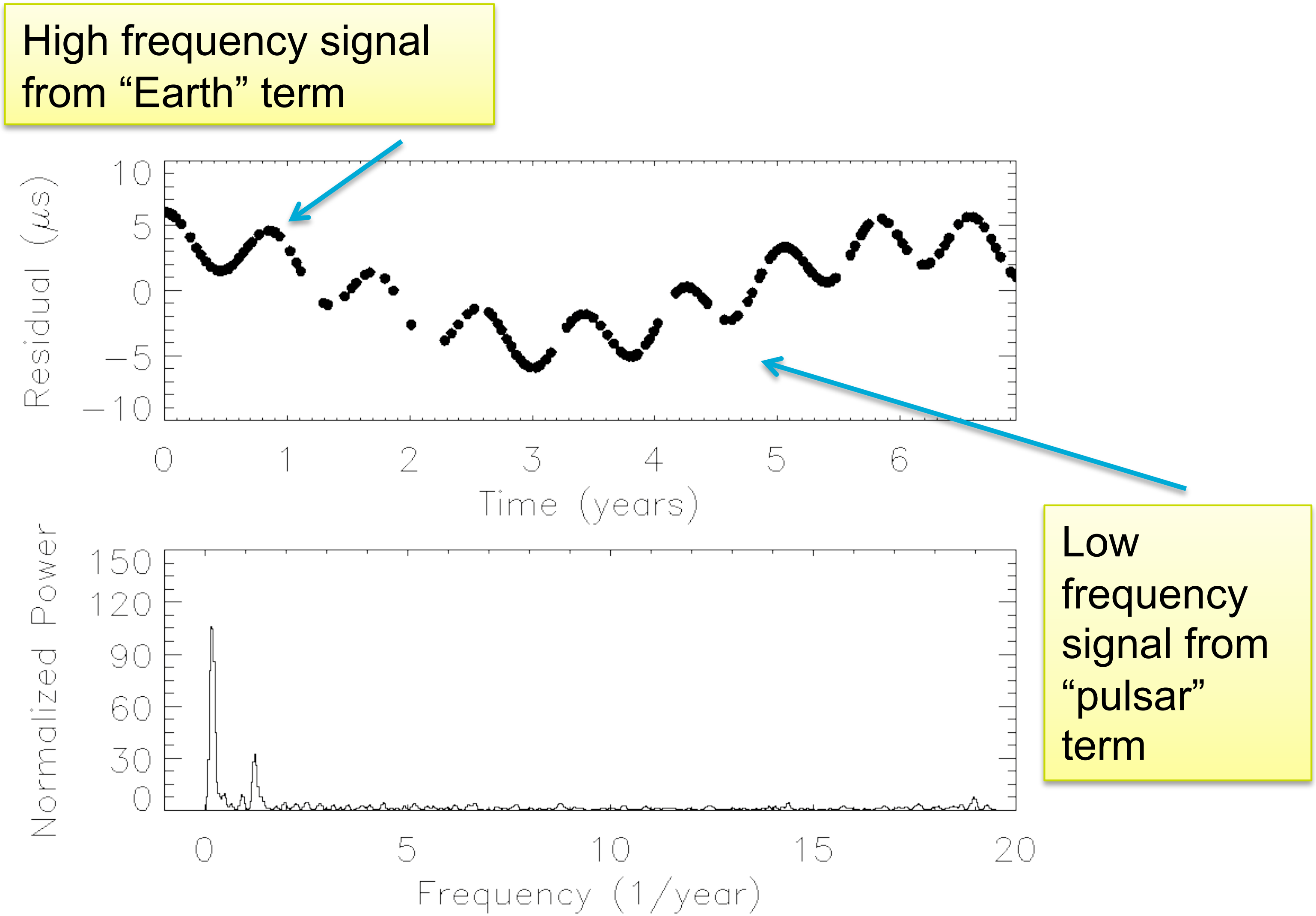
- Can simulate “perfect” pulse arrival times (given a pulsar timing model).
- Can add the induced residuals from a non-evolving (sinusoidal) GW source.
- Can use tempo2 to form timing residuals

- (Here simulate a GW signal only in A+ polarisation)

The pulsar term adds in an unknown phase shift



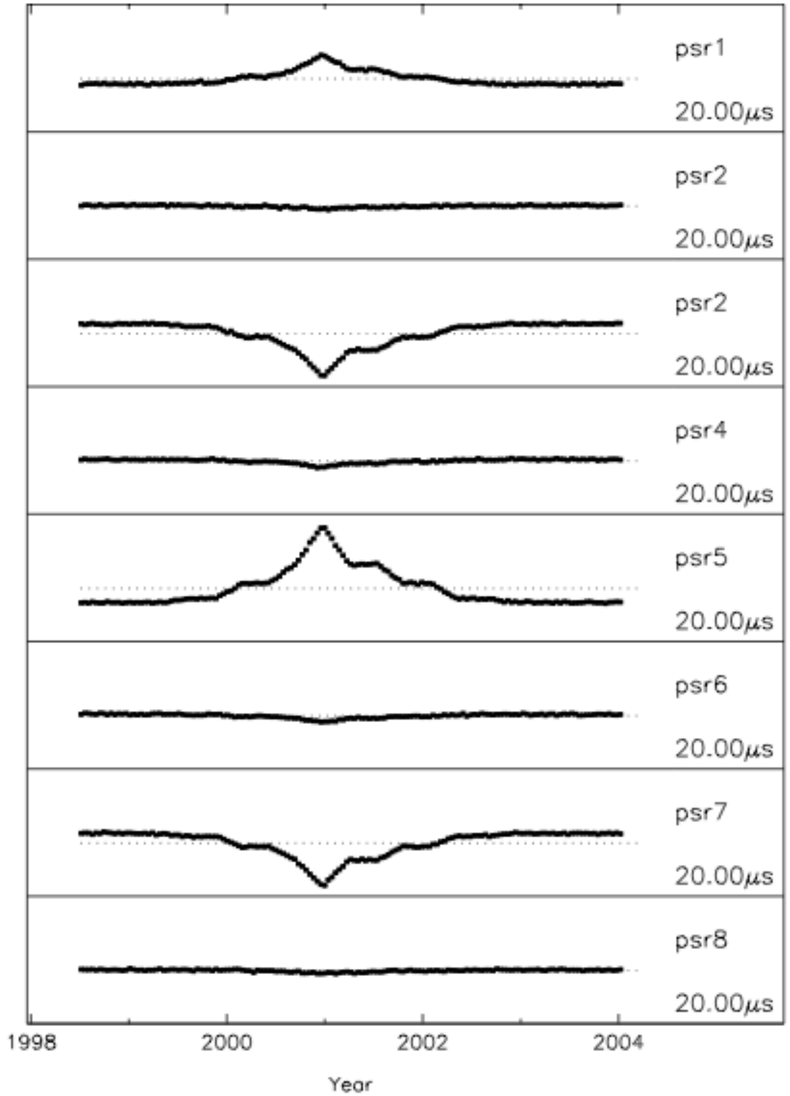
Single GW sources: evolving source: Earth term and pulsar term



Single GW sources: burst source

- Can simulate "perfect" pulse arrival times (given a pulsar timing model).
- Can use tempo2 to form timing residuals for an arbitrary GW waveform

- (Here simulate a GW signal only in A+ polarisation)



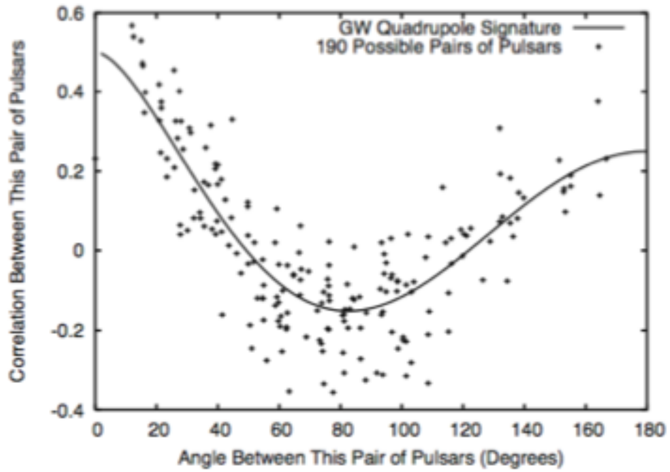
A gravitational wave background

$$h_{\mu\nu} = \text{Re}\left[\sum_j A_{\mu\nu_j} e^{i\vec{k}_j\cdot\vec{x} - i\omega_j t}\right]$$

$$R(t,\hat{k}) \;=\; -\int_0^t \sum_{s=0}^{N-1} \mathcal{H}(\hat{k},\hat{\eta_s})^{ij}\left(h_{ij}(t_e, x_e, \hat{\eta_s}) - h_{ij}(t_e - d, x_p, \hat{\eta_s})\right) dt_e$$

This is the same for all pulsars. This depends on the pulsar.

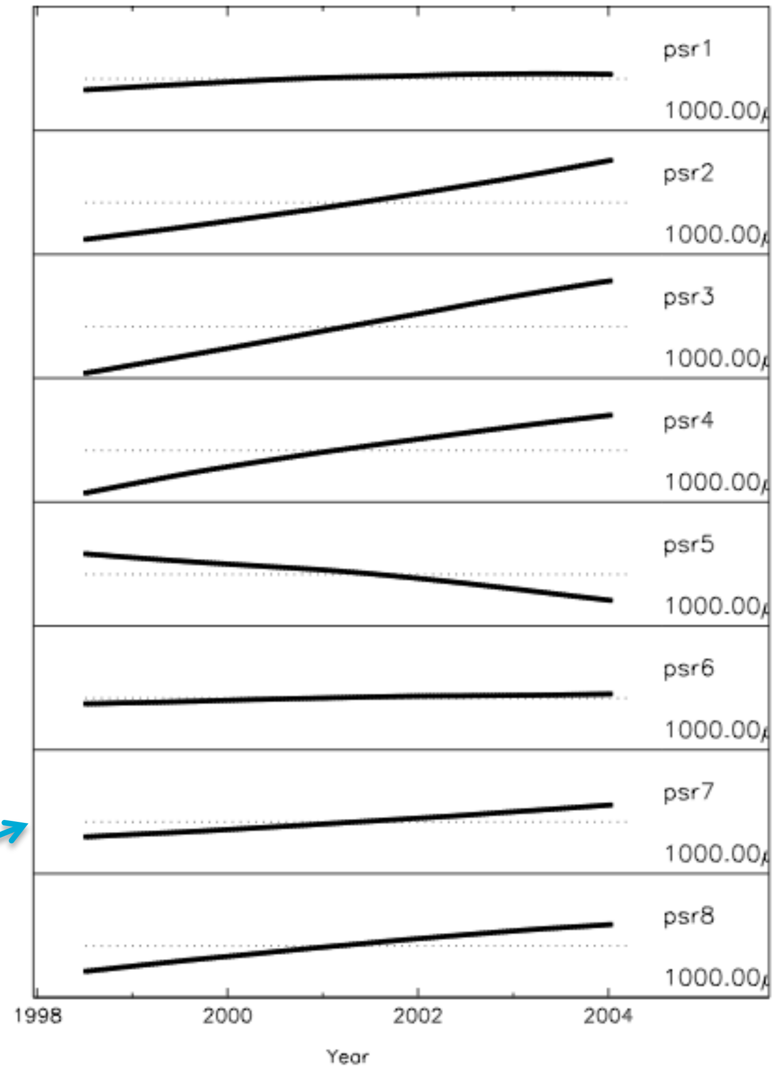
Hellings & Downs (1983):



Simulating a GW background

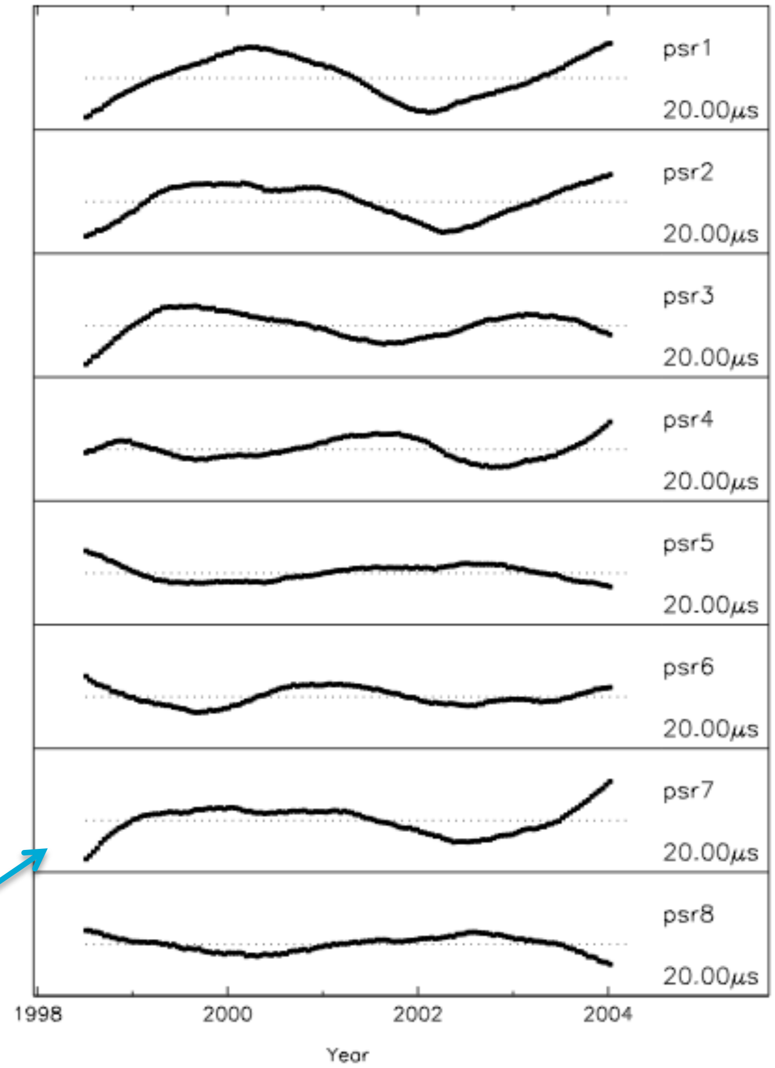
- 1) can simulate a large number of single sources with random positions, frequencies and polarisation properties (described later in this lecture series)
- 2) can simulate time series following a power law $P(f) = Af^{\alpha}$ that are correlated according to Hellings & Downs (1983)

1ms timing residuals



Simulating a GW background: issue with fitting

- 1) can simulate a large number of single sources with random positions, frequencies and polarisation properties (described later in this lecture series)
- 2) can simulate time series following a power law $P(f) = Af^{\alpha}$ that are correlated according to Hellings & Downs (1983)



20us timing residuals for simulated background (unrealistically large background)

Expected sizes

- Single source: peak amplitude ~10ns (work by Sesana et al.)
- Burst source: ?
- Gravitational wave background: induced residuals ~50ns (work by Sesana et al.)

- In the next talk we'll discuss where these numbers came from
- In talk 4 we'll discuss methods used to search for these gravitational waves

Conclusion

- Described how raw pulsar data files can be obtained from the CSIRO data access portal (http://datanet.csiro.au/)
- Described the pulsar timing method
- Showed how to download and use tempo2 (http://www.atnf.csiro.au/research/pulsar/tempo2)
- Discussed gravitational waves
- Determined the induced timing residuals caused by gravitational waves

- Next lecture:
- Actual pulsar data sets
- Sources of gravitational waves