Lecture 4
GWs from Binary Systems

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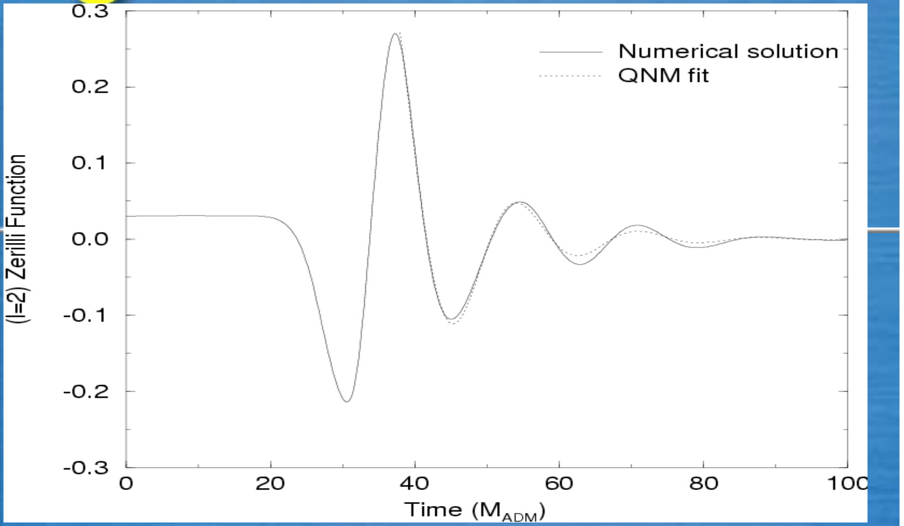
Black-Hole "Ringing"

- The newly formed BH is ringing till settles down to the stationary Kerr state (QNMs).
- The **amplitude** of the ringdown waves and their energy depends on the distortion of the BH.
- Typical frequencies: 10^{-4}-10^{4} Hz
- Binary system merging
- Supermassive BHs absorbing smaller BHs or stars
- The ringing due to the excitation by the fallback material (after the collapse) might last for secs
- The **energy** emitted in GWs by the falling material is: $\Delta E \gtrsim 0.01\mu c^2(\mu/M)$.



$$\omega \approx \frac{1}{M}(0.37+0.19a) \approx 12kHz\left(\frac{M_\odot}{M}\right)$$

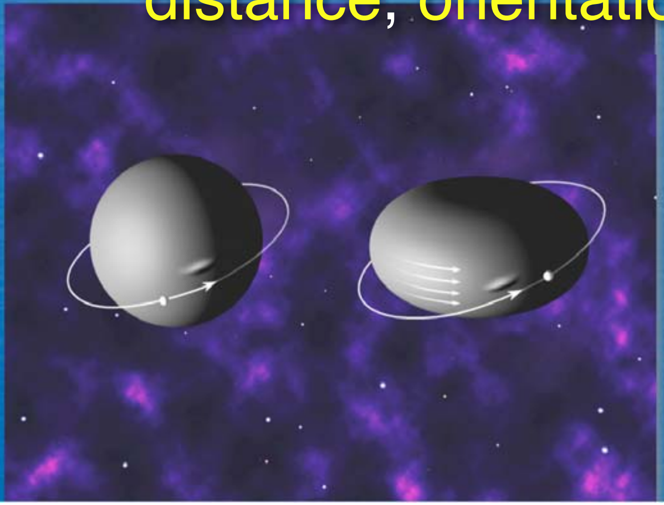
$$\tau \approx M(1.48+2.09a) \approx 0.05ms\left(\frac{M}{M_\odot}\right)$$

$$h_{\text{eff}} \approx 2\times10^{-21}\left(\frac{\varepsilon}{0.01}\right)\left(\frac{d}{10\text{Mpc}}\right)^{-1}\left(\frac{\mu}{M_\odot}\right)$$

•**GWs from BHs:** a unique probe of their existence

Compact Binary Inspirals

- Late-time dynamics of compact binaries is highly relativistic, dictated by non-linear GR effects
- Post-Newtonian theory, which is used to model the evolution, is now known to $O(v^7)$
- The shape and strength of the emitted radiation depend on many parameters of binary system: masses, spins, distance, orientation, sky location, …



Three archetypal systems

1. Double Neutron Stars (NS-NS)
2. Neutron Star-Black Hole (NS-BH)
3. Double Black Holes (BH-BH)

Coalescence of Compact Binaries

- During the frequency change from 100-200Hz GWs carry away $5\times10^{-3}M_{\odot}c^2$.
- In LIGOs band
  - **NS/NS** (~16000 cycles)
  - **NS/BH**(~3500 cycles)
  - **BH/BH**(~600 cycles)
- The GW amplitude is:
- Larger total mass improves detection probability.

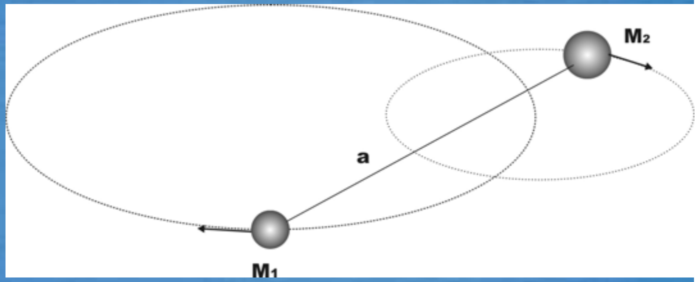
events/y	LIGO-I	LIGO-II
NS/NS	~0.05	~60-500
BH/NS	~0.02	~80
BH/BH	~0.8	~2000
Total	0.8	$\gtrsim$2000

- ✓ **Phase effects are important**, if the signal and the template get out of phase their cross correlation will be reduced.
- ✓ **High accuracy templates** are needed for accurate detection.

$$h \approx 7.5\times10^{-23}\left(\frac{M}{2.8M_{\odot}}\right)^{2/3}\left(\frac{\mu}{0.7M_{\odot}}\right)\left(\frac{f}{100Hz}\right)^{2/3}\left(\frac{100Mpc}{r}\right)$$

Rotating Quadrupole (a binary system)

THE BEST SOURCE FOR GWs



- **Radiated power**

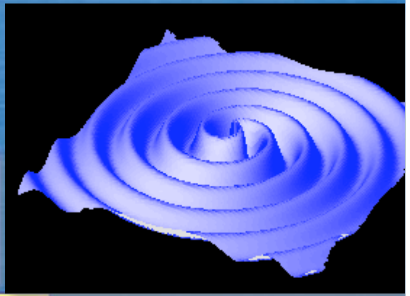
$$-\frac{dE}{dt}=\frac{32}{5}\frac{G}{c^5}\mu^2a^4\omega^6=\frac{32}{5}\frac{G}{c^5}\frac{M^3\mu^2}{a^5}$$

- Energy loss leads to **shrinking of their orbital separation**

$$\frac{da}{dt}=-\frac{64}{5}\frac{G^3}{c^5}\frac{\mu M^2}{a^3}$$

- **Period changes** with rate

$$\frac{\dot{P}}{P}=-\frac{96}{5}\frac{G^3}{c^5}\frac{\mu M^2}{a^4}$$



- ...and the system **will coalesce** after

$$T_{inspiral}=\frac{5}{256}\frac{c^5}{G^3}\frac{a_0^4}{\mu M^2}$$

- The **total energy loss** is

$$\Delta E_{rad}=\frac{G}{2}\mu M\left(\frac{1}{a_0}-\frac{1}{a}\right)$$

- Typical **amplitude** of GWs

$$h\approx 5\times10^{-22}\left(\frac{M}{2.8M_{\odot}}\right)^{2/3}\left(\frac{\mu}{0.7M_{\odot}}\right)\left(\frac{f}{100Hz}\right)^{2/3}\left(\frac{15Mpc}{r}\right)$$

An interesting observation

The **observed frequency change** will be:

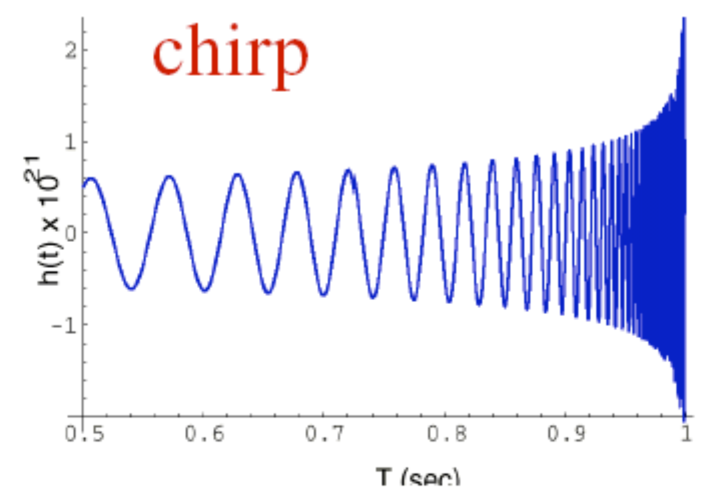
$$\dot{f} \sim f^{11/3} \mathcal{M}_{chirp}^{5/3}$$

$$\mathcal{M}_{chirp}^{5/3} = \mu M^{2/3}$$

The **corresponding amplitude** will be :

$$h \sim \frac{\mathcal{M}_{chirp}^{5/3} f^{2/3}}{r} = \frac{\dot{f}}{f^3 r}$$

- ✓ Since both **frequency** and its **rate of change** are **measurable quantities,** we can immediately **compute the chirp mass.**
- ✓ The **third relation** provides us with a **direct estimate of the distance of the source**
- ✓ **Post-Newtonian** relations can provide **the individual masses**



Binary systems (examples)

PSR 1913+16

$M_1=M_2\sim1.4M_\odot$, P=7h 45m 7s, r=5kpc,

$h_{earth}\sim10^{-20}$, $f\sim10^{-4}$Hz, $T_{insp}\sim3\text{x}10^8$yr

$dP_{theo}/dt=-7.2\times10^{-12}$s/yr $dP_{obs}/dt=-(6.9\pm06)\text{x}10^{-12}$s/yr

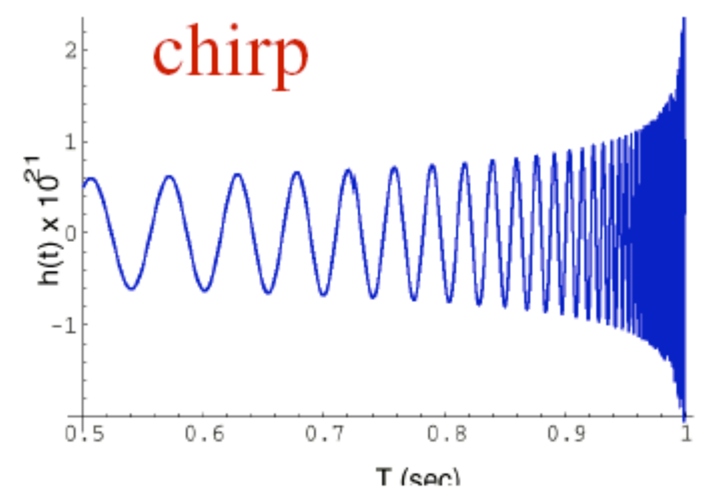
The LIGO/VIRGO binary (10-1000Hz)

$M_1=M_2\sim1.4M_\odot$, $f_0=10$Hz, $f_{final}=1000$Hz,

$T_{insp}\sim15$min, after ~15000 cycles (inspiral/merging 300Mpc)

$M_1=50M_\odot$, $M_2\sim50M_\odot$, $f_0=10$Hz,

$f_{final}=100$Hz, (inspiral/merging 400Mpc)



$$f_{BH}\sim12kHz\left(\frac{1M_\odot}{M}\right)$$

The LISA binary (10^{-5}-10^{-2}Hz)

$M_1=M_2\sim10^6M_\odot$, $f_0=10^{-4}$Hz, $f_{final}=0.01$Hz, (inspiral/merging at r~3Gpc)

$M_1=M_2\sim10^5M_\odot$, $f_0=10^{-4}$Hz, $f_{final}=0.1$Hz, (inspiral/merging at r~3Gpc)

$M_1=M_2\sim10^4M_\odot$, $f_0=10^{-3}$Hz, $f_{final}=1$Hz, (inspiral at r~3Gpc)

Smaller Stars/BHs plunging into super-massive ones

Coalescing binary signal

- The waveform 2nd PN approximation:

$$\bar{h}(f)=\mathcal{N}\cdot f^{-7/6}\cdot\exp\left[i\sum_{k=1}^{6}\psi_k(f)\lambda_k-i\frac{\pi}{4}\right]$$

$\mathcal{N}$: normalization constant

$\psi_k=\psi_k(f,f_a)$: functions of the frequency only

$\lambda_k=\lambda_k(t_c,\Phi_c,\tau_0,\tau_1,\tau_{1,5},\tau_2)$: signal parameters

$t_c,\ \Phi_c$: time and phase of coalescence

- "Newtonian case"

$$\bar{h}(f)=\mathcal{N}\cdot f^{-7/6}\cdot\exp\left[2\pi i\lambda_1(t_c,\Phi_c,\tau_0)-i\pi/4\right]$$

$$\mathcal{N}\sim\frac{\mathcal{M}^{5/6}}{R}\left[F_+^2(1+\cos^2 i)^2+4F_\times^2\cos^2 i\right]^{1/2}\quad,$$

$$\tau_0=\frac{5}{256}\frac{\mathcal{M}^{-3/5}}{(\pi f_a)^{5/3}}$$

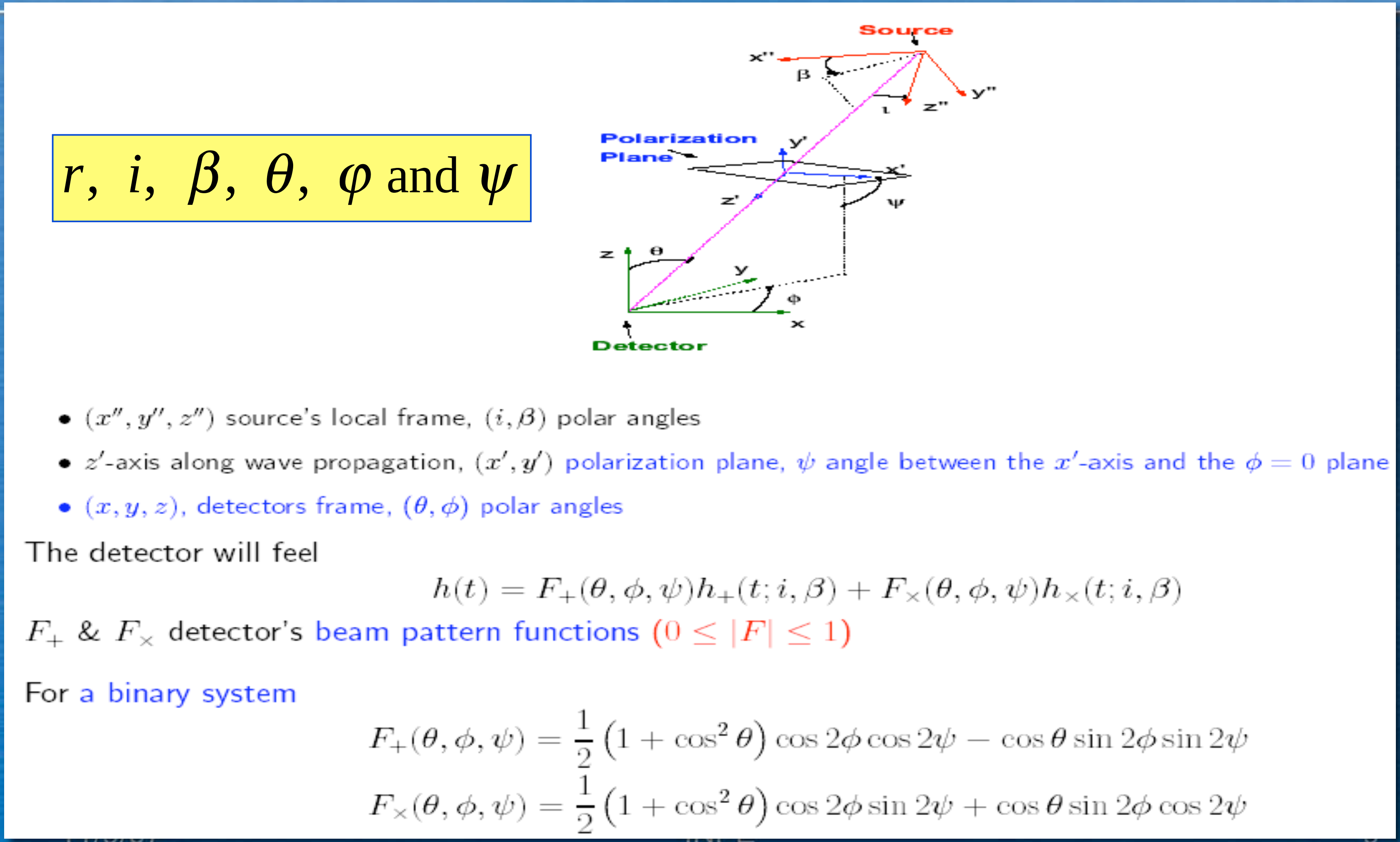
- The signal depends on 4 parameters
  - The amplitude $\mathcal{N}$
  - The phase Φ_C
  - The time parameter τ_0
  - The chirp mass $\mathcal{M}=\mu^{3/5}M^{2/5}$

$$\Phi_I=\arctan\left[\frac{\langle h_c|s\rangle}{\langle h_s|s\rangle}\right]$$

$$\mathcal{N}=\frac{\langle h_c|s\rangle\sin\Phi_I+\langle h_s|s\rangle\cos\Phi_I}{\langle h_s|h_s\rangle}$$

Emission, Propagation and Detection

6 parameters determine the position of the binary on the sky



$r,\ i,\ \beta,\ \theta,\ \varphi$ and ψ

- (x'', y'', z'') source's local frame, (i, β) polar angles
- z'-axis along wave propagation, (x', y') polarization plane, ψ angle between the x'-axis and the $\phi = 0$ plane
- (x, y, z), detectors frame, (θ, ϕ) polar angles

The detector will feel

$$h(t) = F_+(\theta,\phi,\psi)h_+(t;i,\beta) + F_\times(\theta,\phi,\psi)h_\times(t;i,\beta)$$

F_+ & $F_\times$ detector's beam pattern functions $(0 \leq |F| \leq 1)$

For a binary system

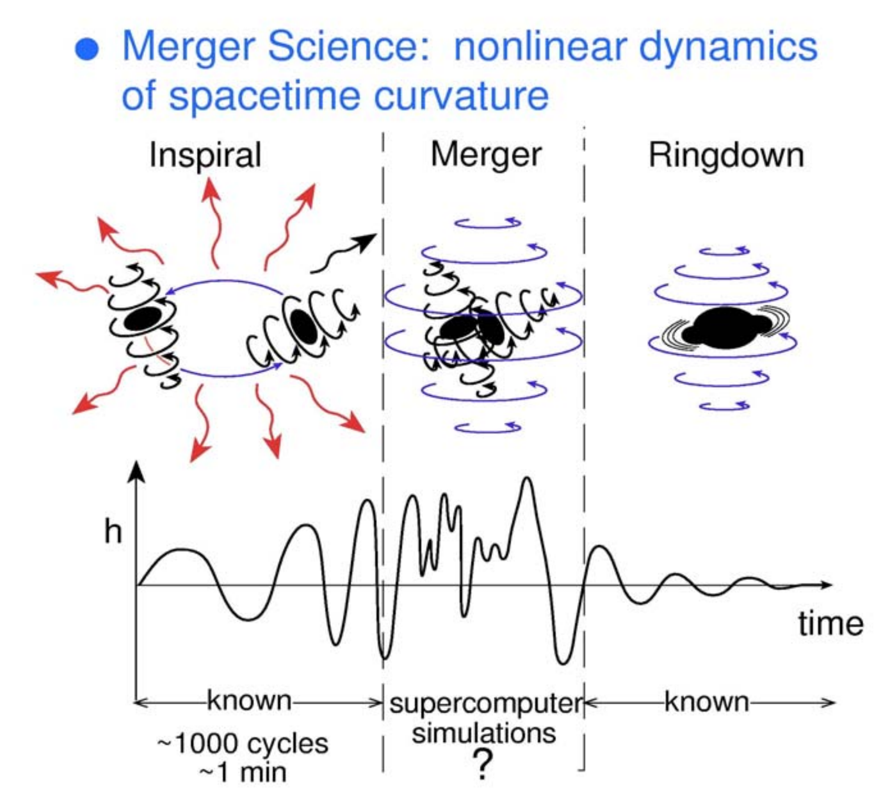
$$F_+(\theta,\phi,\psi) = \frac{1}{2}\left(1+\cos^2\theta\right)\cos 2\phi\cos 2\psi - \cos\theta\sin 2\phi\sin 2\psi$$

$$F_\times(\theta,\phi,\psi) = \frac{1}{2}\left(1+\cos^2\theta\right)\cos 2\phi\sin 2\psi + \cos\theta\sin 2\phi\cos 2\psi$$

Gravitational Waves from Binaries

Generically, there are 3 regimes in which binaries radiate:

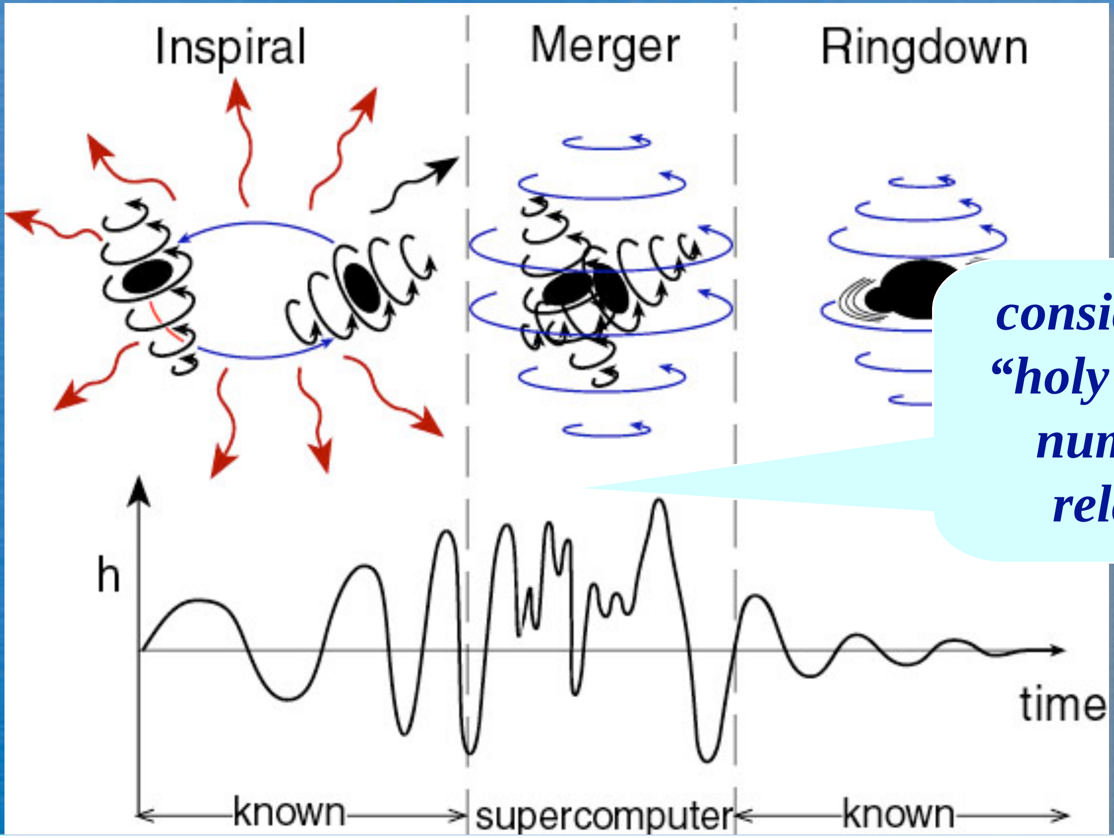
- **Orbital in-spiral**: PN-approximations or point-particle orbits.
- **Plunge/merger** after the last stable orbit: numerical simulations or point-particle orbits.
- **Ring-down** of the disturbed black hole as it settles down to a Kerr hole: perturbation theory of black holes.



- Merger Science: nonlinear dynamics of spacetime curvature

Final merger of BH binary

- Strong-field merger is brightest GW source, luminosity $\sim 10^{23} L_{SUN}$
- **Requires *numerical relativity* to calculate dynamics & waveforms**
- Waveforms scale with **masses**, **spins** → apply to ground-based & LISA



considered the "holy grail" of numerical relativity

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A Brief History of binary black hole simulations….

- 1964: Hahn & Lindquist: try to evolve collision of 2 "wormholes"
- 1970s: Smarr and Eppley: head-on collision of 2 BHs, extract GWs
  - Pioneering efforts on supercomputers at Livermore Natl Lab
- 1990s: LIGO moves ahead & work on BBH problem starts again..
  - Work on 2-D head-on collisions at NCSA
  - NSF Grand Challenge: multi-institution, multi-year effort in 3-D

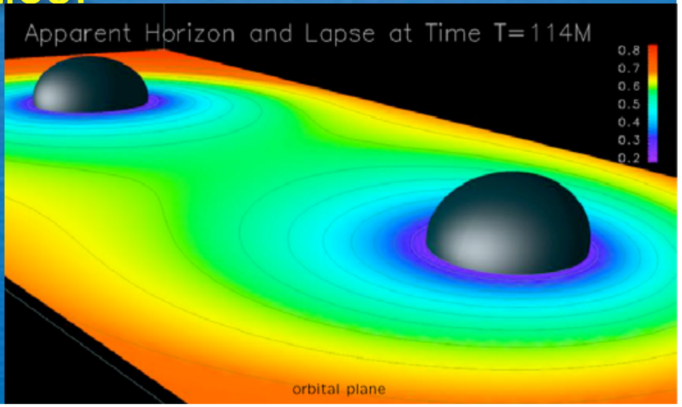
→ ***This is <u>really</u> difficult! Instabilities, issues in formalisms, etc…***

  - Diaspora: multiple efforts (AEI, UT-Austin, PSU, Cornell…)
  - Difficulties proliferate, instabilities arise, codes crash….
  - ***"Numerical relativity is impossible…"***
- 2000 & beyond: LIGO/GEO/VIRGO and LISA spur more work
  - New groups: Caltech, UT-Brownsville, LSU, Jena, GSFC…
  - ***Since 2004, breakthroughs & rapid progress***

    → orbits, at last!

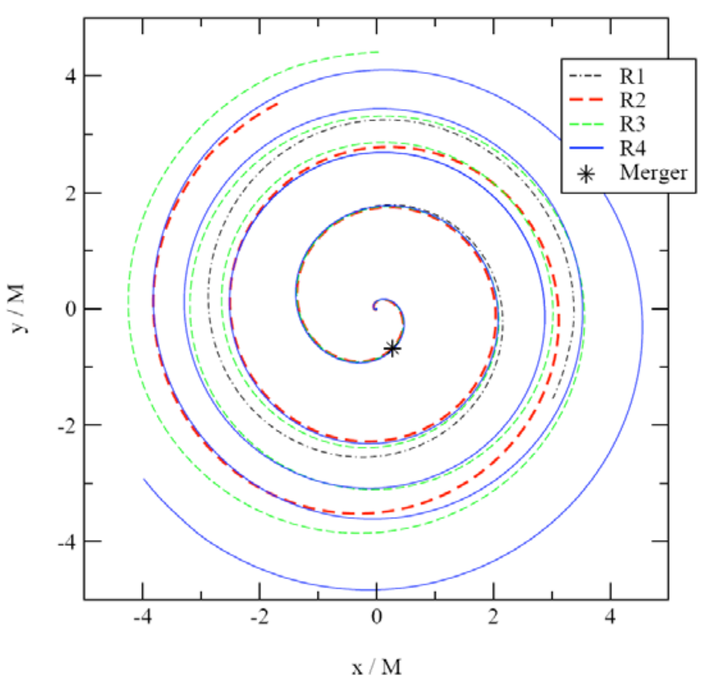
Recent progress…on a broad front

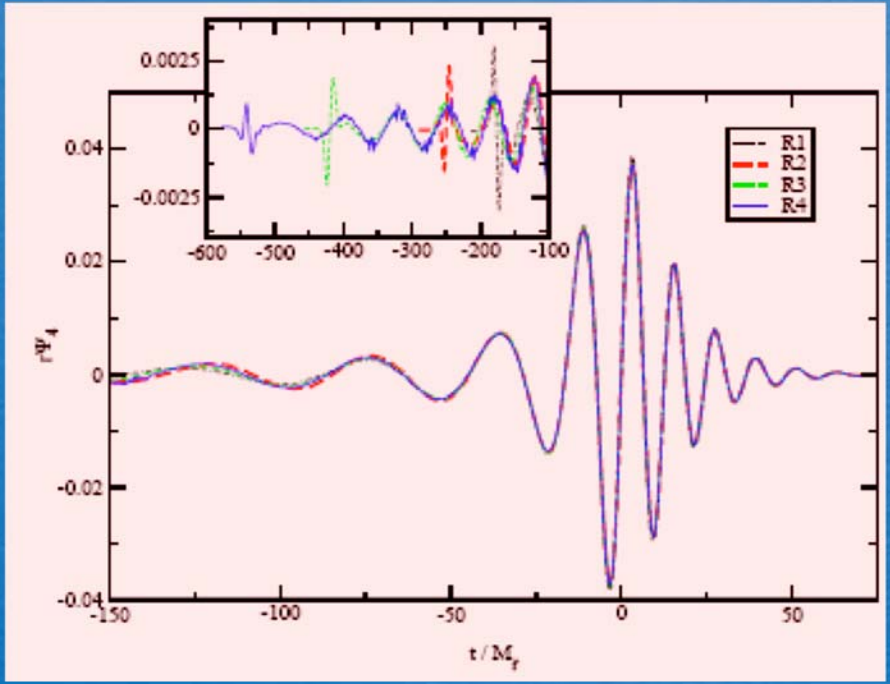
- Evolutions of BH binary with unequal mass, spinning BHs
  - start on approx quasi-circular orbits near last stable orbit
  - stable evolution over multiple orbits, plunge, merger, ringdown
- Independently written codes and different software
  - Finite differences; spectral methods
- Different formulations of the Einstein equations
  - 1^{ST} & 2^{nd} order PDEs; which variables to use; role of constraints
- How to handle the BHs: excision; “punctures”
- Gauge or coordinate conditions: co-moving coords; moving BHs
- Variable grid resolution to handle multiple scales:
  - $\lambda_{GW} \sim (10 – 100)M$
  - Mesh refinement; spectral decomposition



Universal waveform...

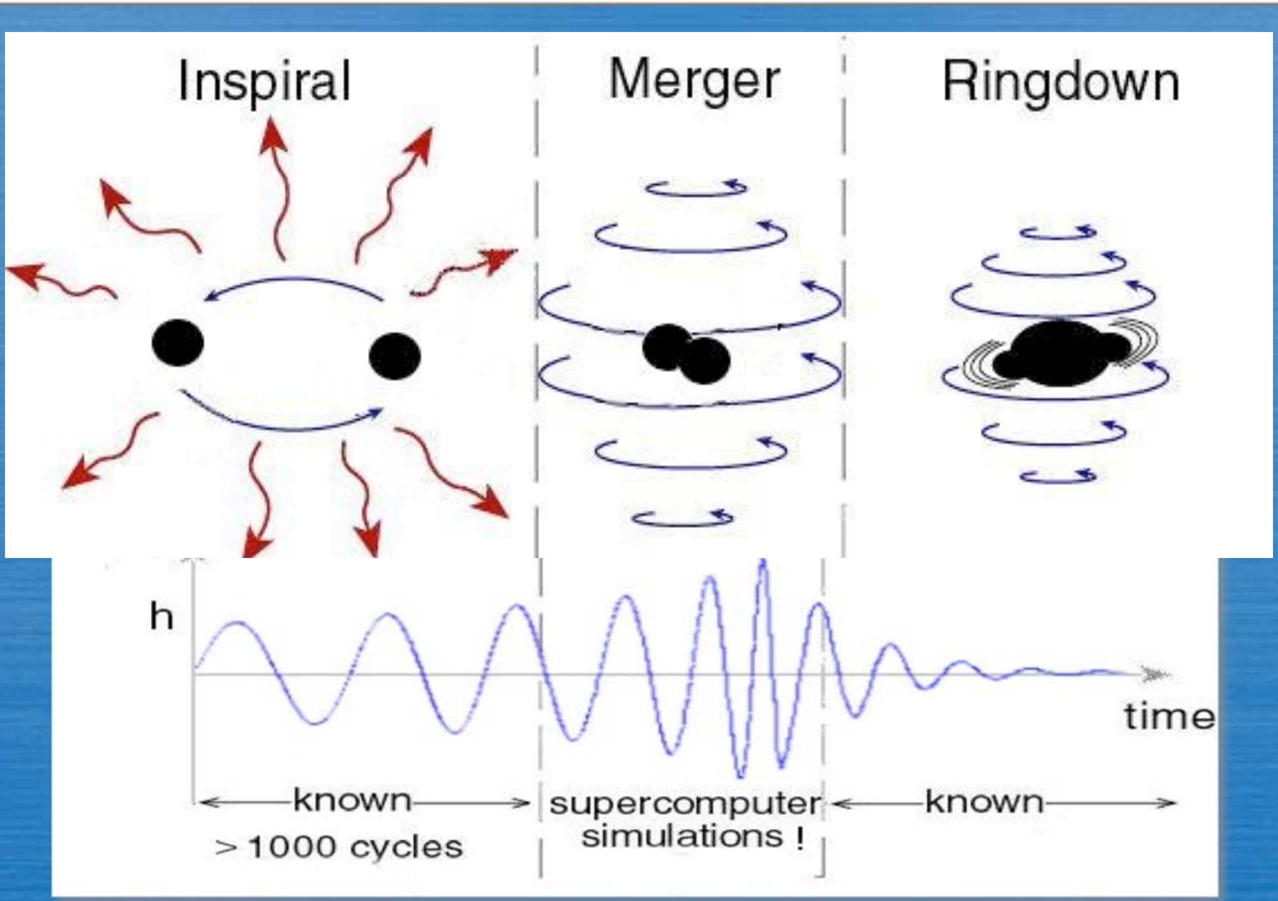
All runs agree to within < 1% for final orbit, merger & ringdown





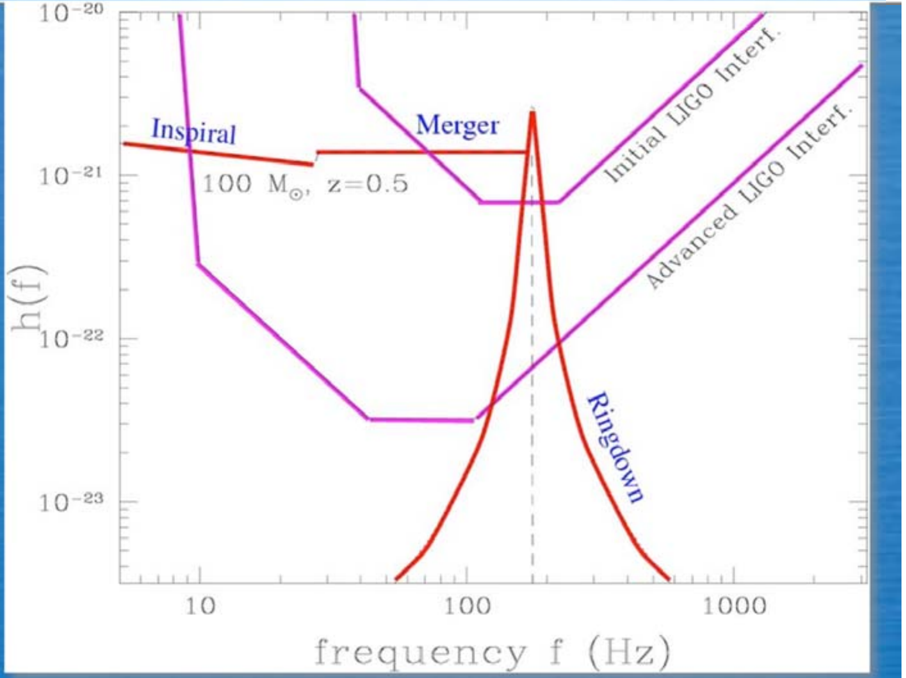
- Baker et al 2006, Pretorious 2006, Brugman et.al 2004 (Jena), LSU-AEI 2004, Penn-State 2005, Brownsville 2005

The emerging picture...



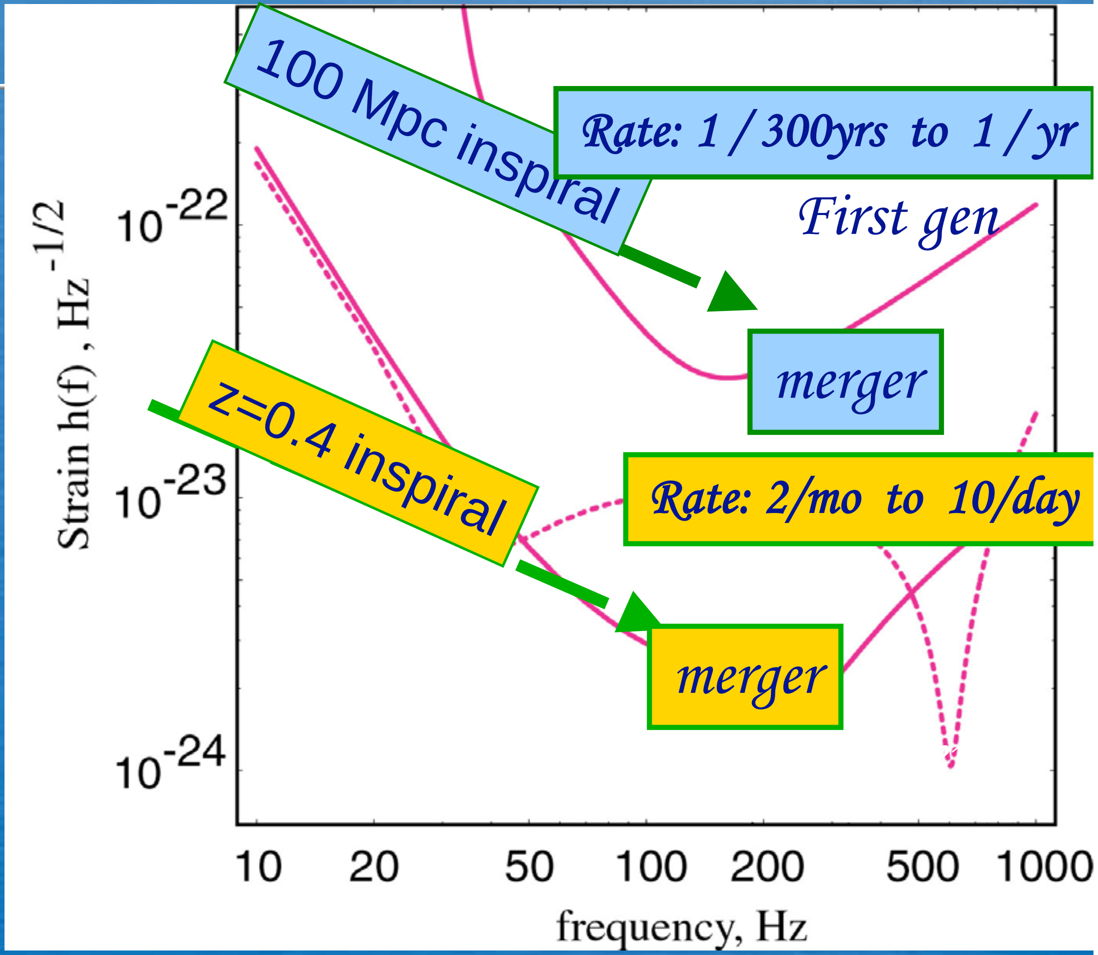
BH/BH coalescence

- The inspiral, merger, and ringdown waves from **$50M_\odot$ BH binaries** as observed by initial and advanced LIGO.
- The energy spectra are coming from crude estimates (10% of the total mass energy is radiated in merger waves and 3% in ring-down waves).
- We observe that the inspiral phase is not visible with initial LIGO, for this case Numerical Relativity is important.



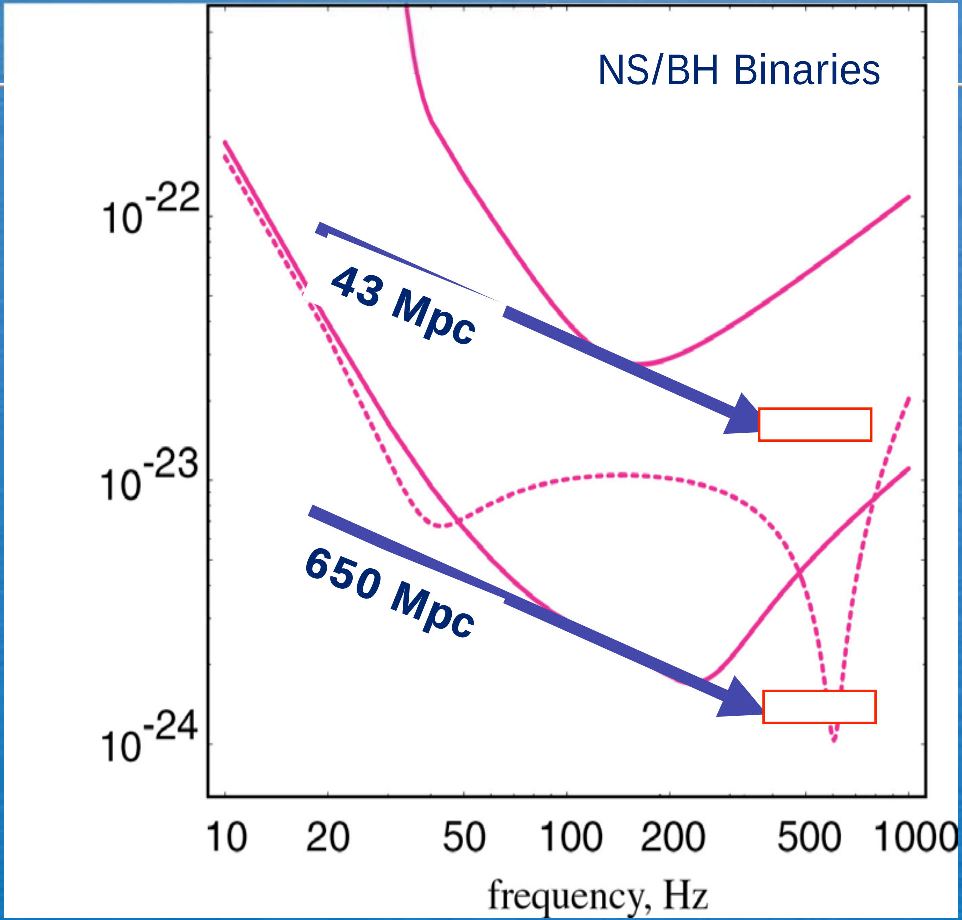
Possible First Source: Binary Black Hole Coalescence

- $10 M_\odot + 10 M_\odot$ BH/BH binary
- Event rates based on population synthesis,
- mostly globular cluster binaries.
- Totally quiet!!



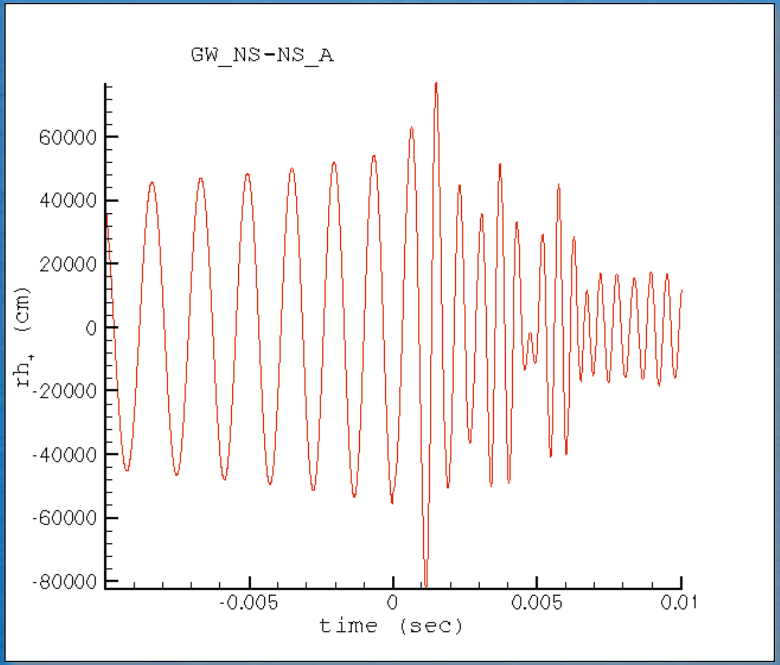
NS-BH inspiral and NS Tidal Disruption

- **NS-BH Event rates**
  - Based on *Population Synthesis*
- Initial interferometers
  - Range: 43 Mpc
  - 1/1000 yrs to 1per yr
- Advanced interferometers
  - Range: 650 Mpc
  - 2 per yr to several per day



Merging phase: NS/NS & BH/NS

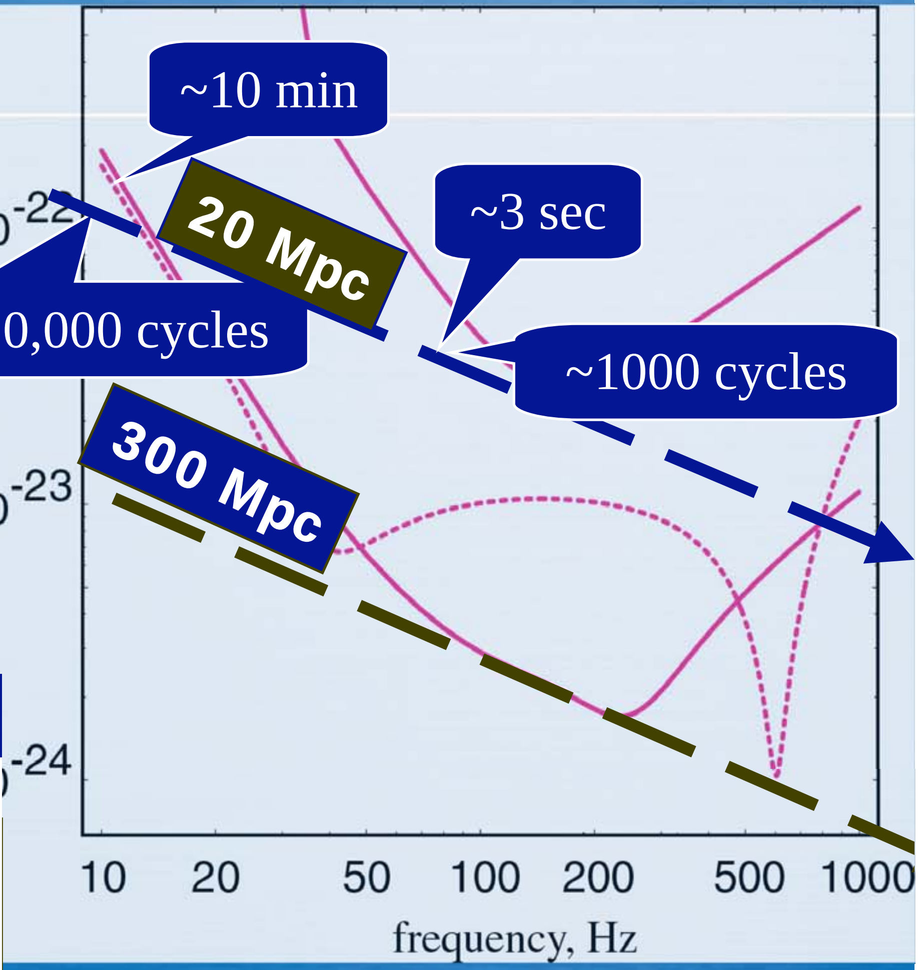
- **Tidal disruption of a NS by a BH**
  - GWs could carry information about the EOS of NS eg. estimation of NS radius (15% error).
  - The disruption waves lie in the band 300-1000Hz
  - A few events per year at 140Mpc (LIGO-II)
- **Merging of NS-NS**
  - Imprint of the NS radii just before merging (**$f \lesssim 1$kHz**)
  - During the merging we could get important information about the EOS (**$f \gtrsim 1$kHz**)



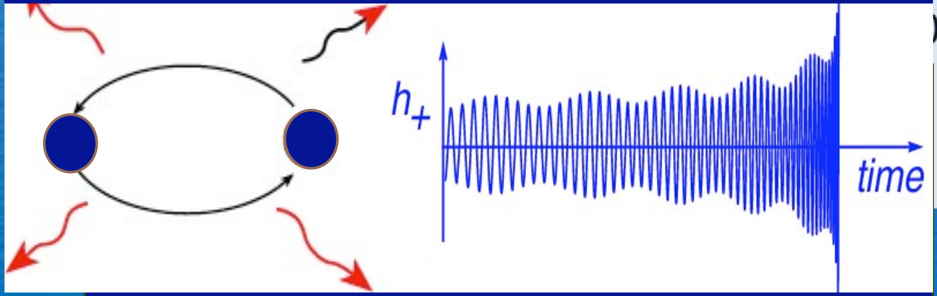
Neutron Star Binary Inspiral

NS-NS coalescence event rates

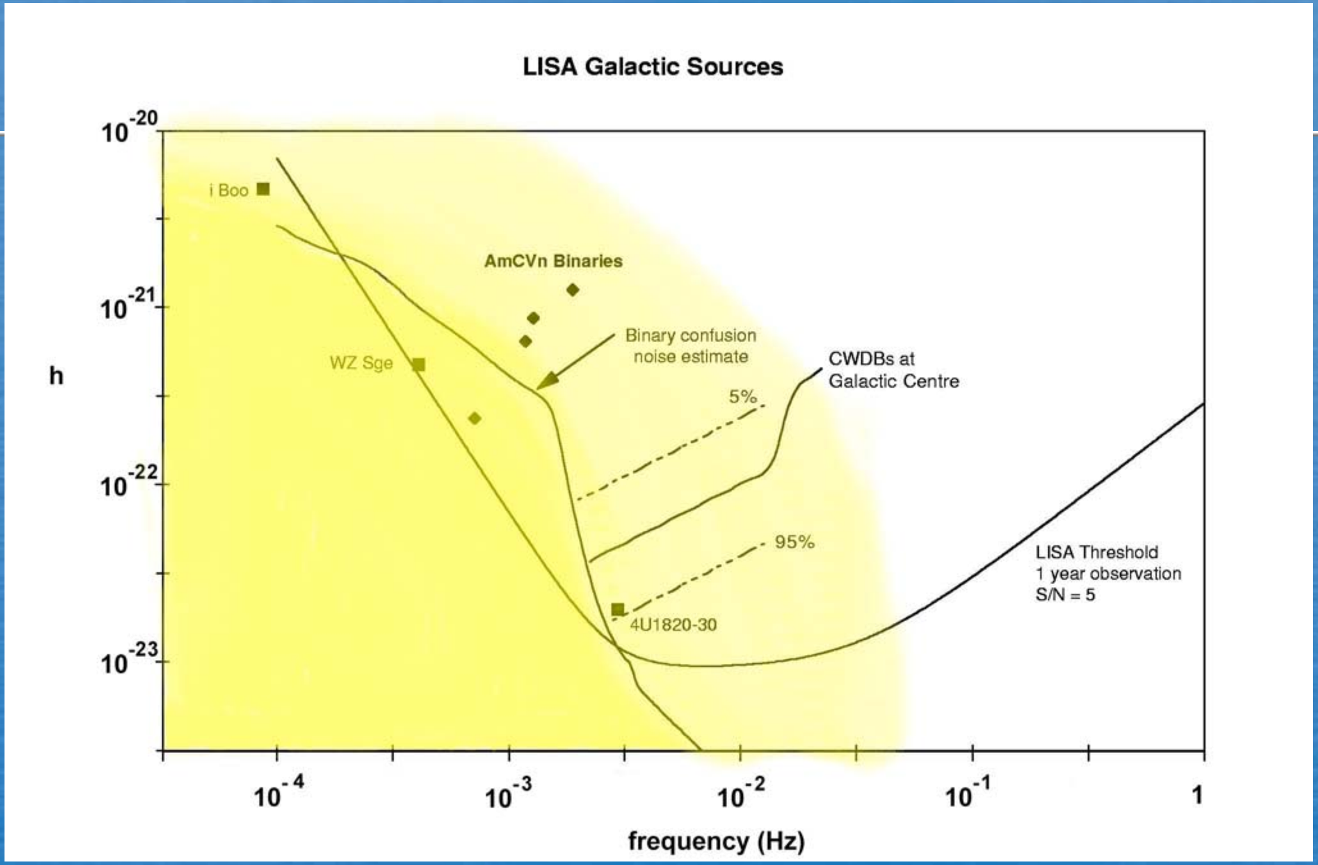
- Initial interferometers
  - Range: 20 Mpc
  - 1 per 40 yrs to 1 per 2 yrs
- Advanced interferometers
  - Range: 300Mpc
  - few per yr to several per day
- The discovery of a new binary pulsar have increased the rate upwards by an order of magnitude



Signal shape very well known



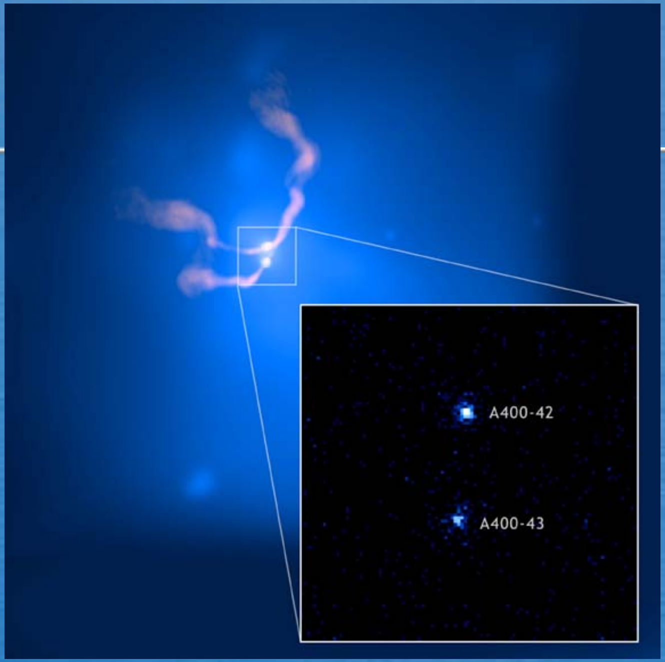
LISA and Compact Galactic Binaries



LISA will see all the compact white-dwarf and neutron-star binaries in the Galaxy.

Massive Black Holes in Galaxies

- Detected masses from 10^6 to 10^9 $M_\odot$. Smaller masses possible.
- Galaxy mergers should produce BH mergers.
    - Rate uncertain: 1/yr for 10^6 $M_\odot$ at $z=1$?
- Protogalaxy mergers may be richer. Phinney: possibly 10^3/yr for 10^5 $M_\odot$ at $z = 7$.
- Stellar BHs fall into massive BHs more often, but weaker radiation.



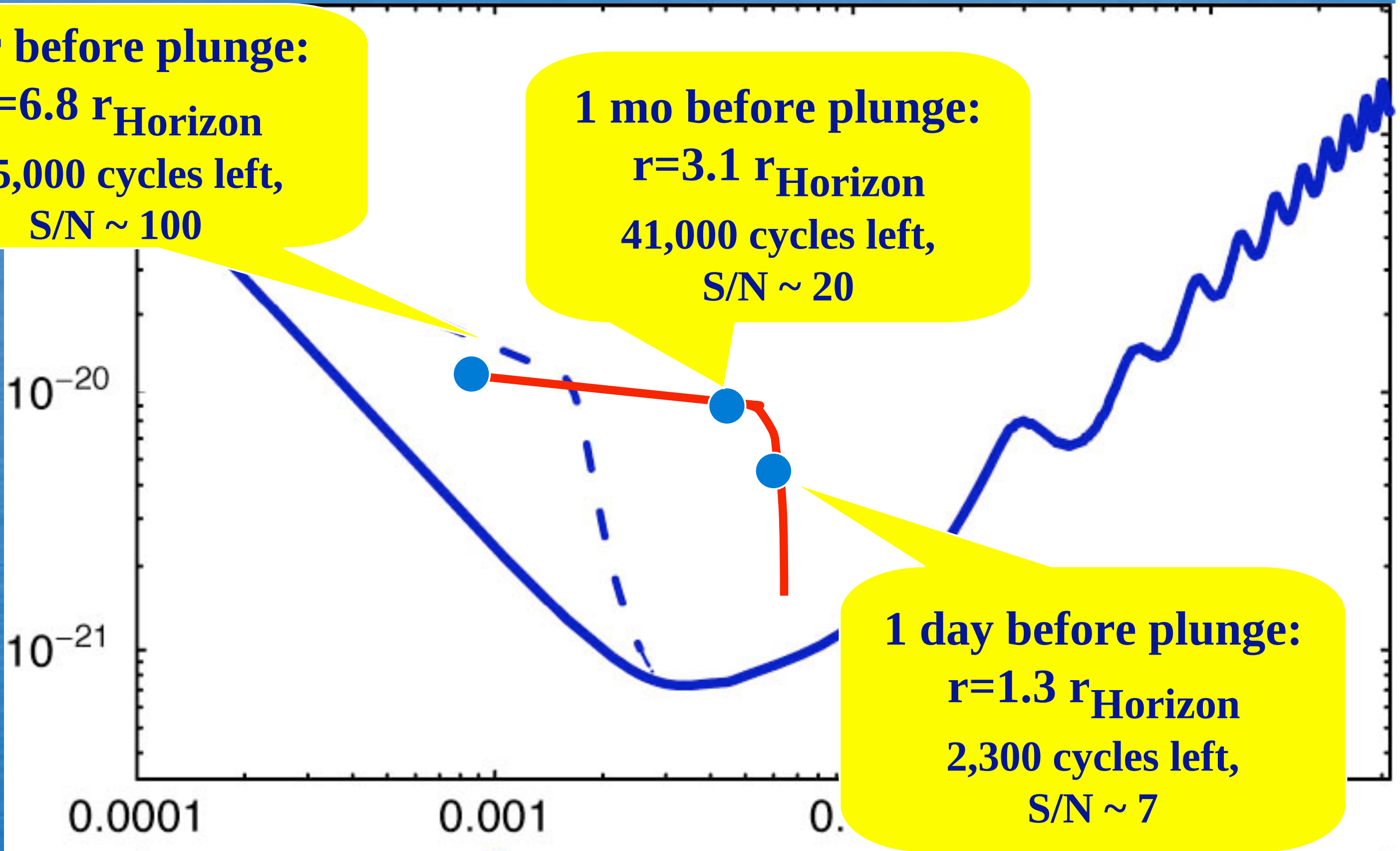
- ***LISA can test GR in the dynamical,***
  strong field regime…if we know the merger waveforms
- When $m_1 \neq m_2$, GW emission is asymmetric → recoil kick

EXAMPLE: Circular, Equatorial orbit; $10\ M_{\odot} + 10^6\ M_{\odot}$; fast spin

Problems:

1. Seeing them under SMBHs

2. Separating them from each other

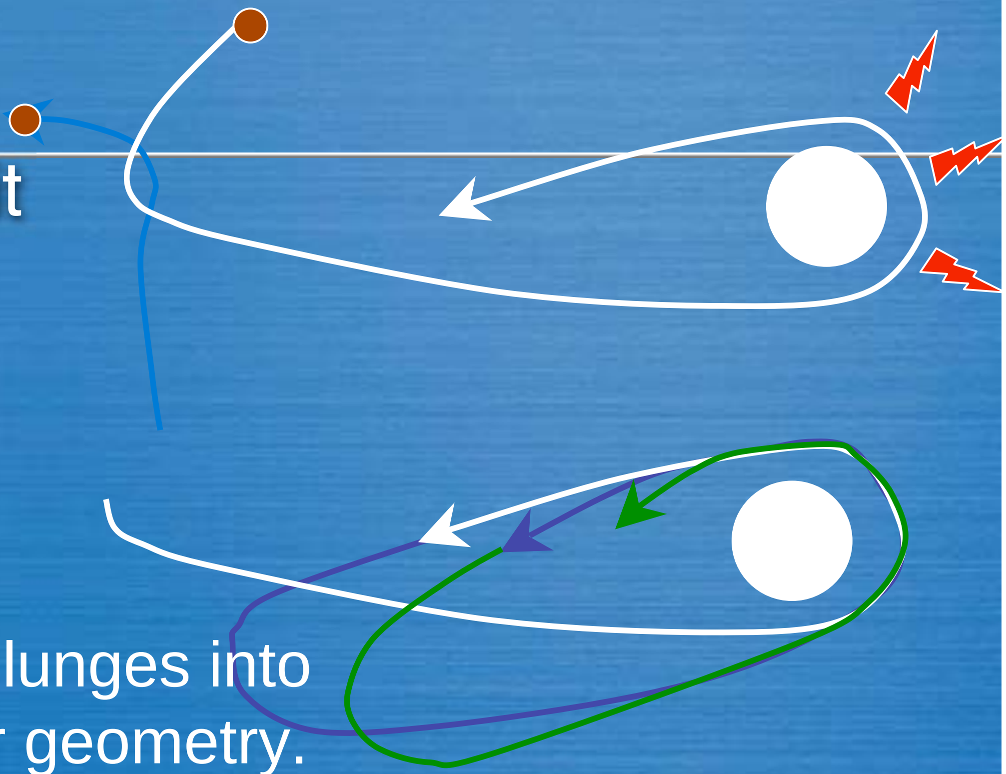


1 yr before plunge:
$r=6.8\ r_{Horizon}$
185,000 cycles left,
S/N ~ 100

1 mo before plunge:
$r=3.1\ r_{Horizon}$
41,000 cycles left,
S/N ~ 20

1 day before plunge:
$r=1.3\ r_{Horizon}$
2,300 cycles left,
S/N ~ 7

Small Objects Spiral into Massive Hole



- Scatter into tight orbit

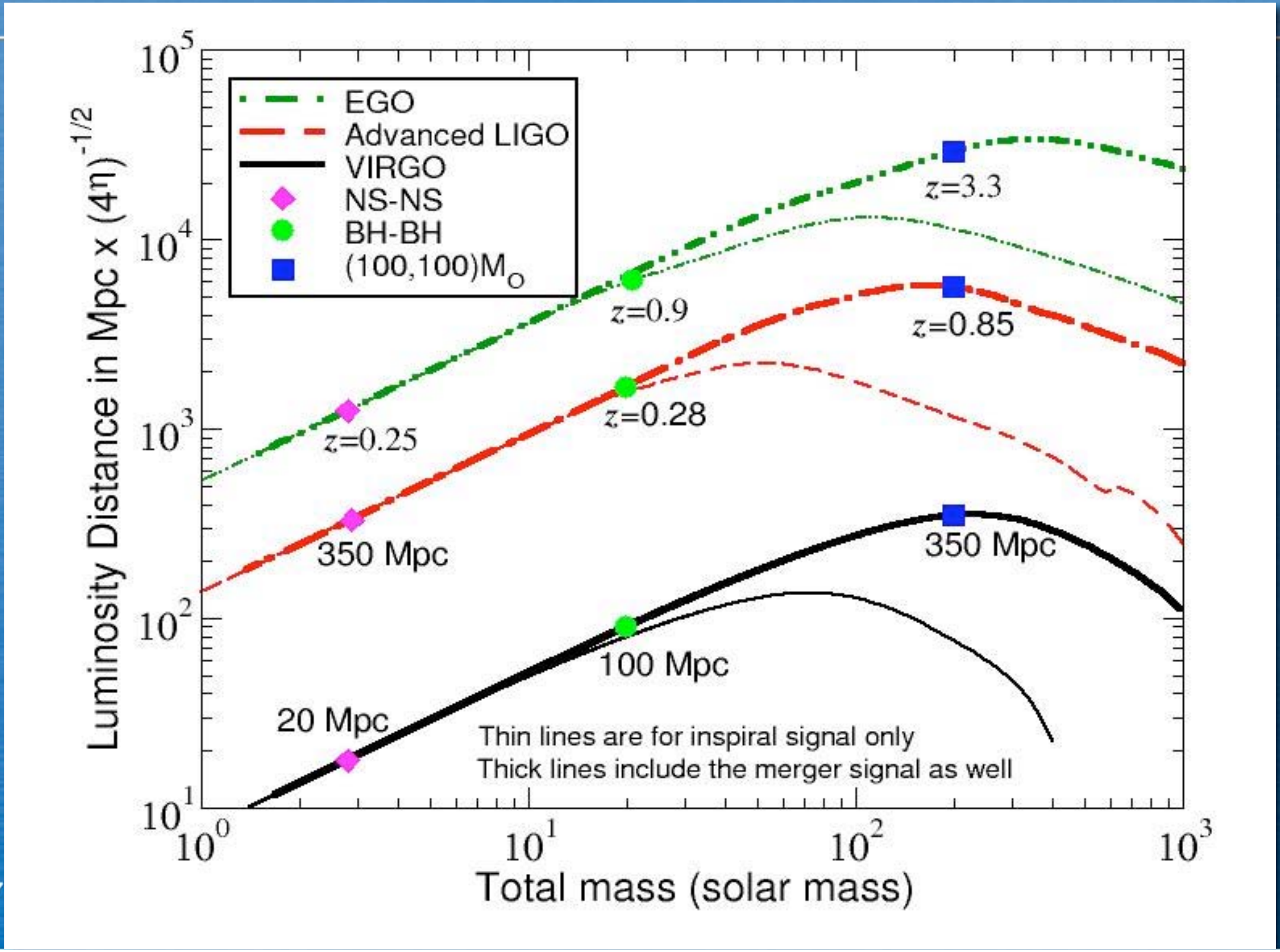
Subsequent evolution:

- Gradual decrease of eccentricity
- Little change of periholian
- Relativistic precession

Still quite eccentric when plunges into hole. Waveform maps Kerr geometry. *Test No-Hair Theorems.*

A few per year at ~ 1 Gpc [Sigurdsson & Rees]
May be somewhat larger for ~10 $M_\odot$ holes [Phinney]

Inspirals can be seen to cosmological distances



Thin lines are for inspiral signal only
Thick lines include the merger signal as well