Core Collapse

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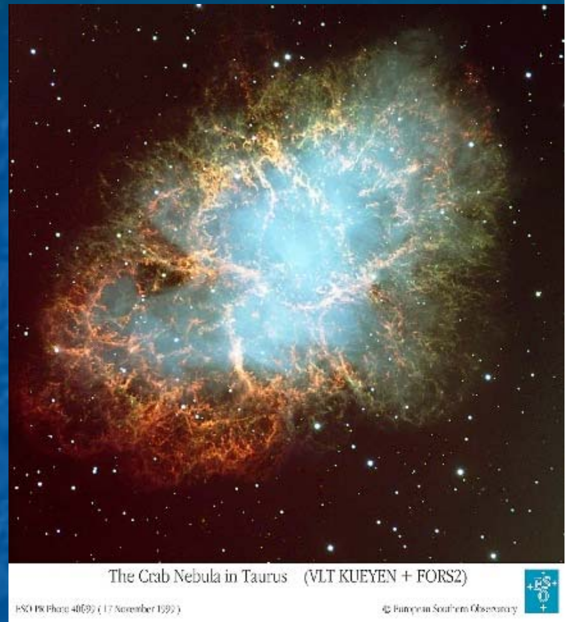
Based partially on material provided by H. Dimmelmeier

Suggested Reading

- Chris L. Fryer and Kimberly C. B.New, ``Gravitational Waves from Gravitational Collapse'', Living Rev. Relativ., 6, 2, (2003)
- Ewald Mueller, ``Simulation of Astrophysical Fluid Flow", in Computational Methods for Astrophysical Fluid Flow, Saas-Fee Advanced Course 27, Springer, Berlin, 343, (1998)
- Nikolaos Stergioulas, ``Rotating Stars in Relativity'', Living Rev. Relativity, 6, 3, (2003),
- Jose A.Font, ``Numerical Hydrodynamics in General Relativity", Living Rev. Relativity, 6, 4, (2003).

Observations of Core Collapse SN

- Core Collapse Supernovae is part of the stellar evolution.
  - Crab nebula is the remnant of SN observed in 1054ad and left behind a 30 Hz pulsar
- Galactic SN events can be observe with naked eye.
  - Last 2 observed in 1572 ad and 1604 ad
  - Closest core collapse SN in our lifetime SN 1987A in LMC (23/2/1987). Progenitor star $18M_{\odot}$



Supernova rates in the Local Group

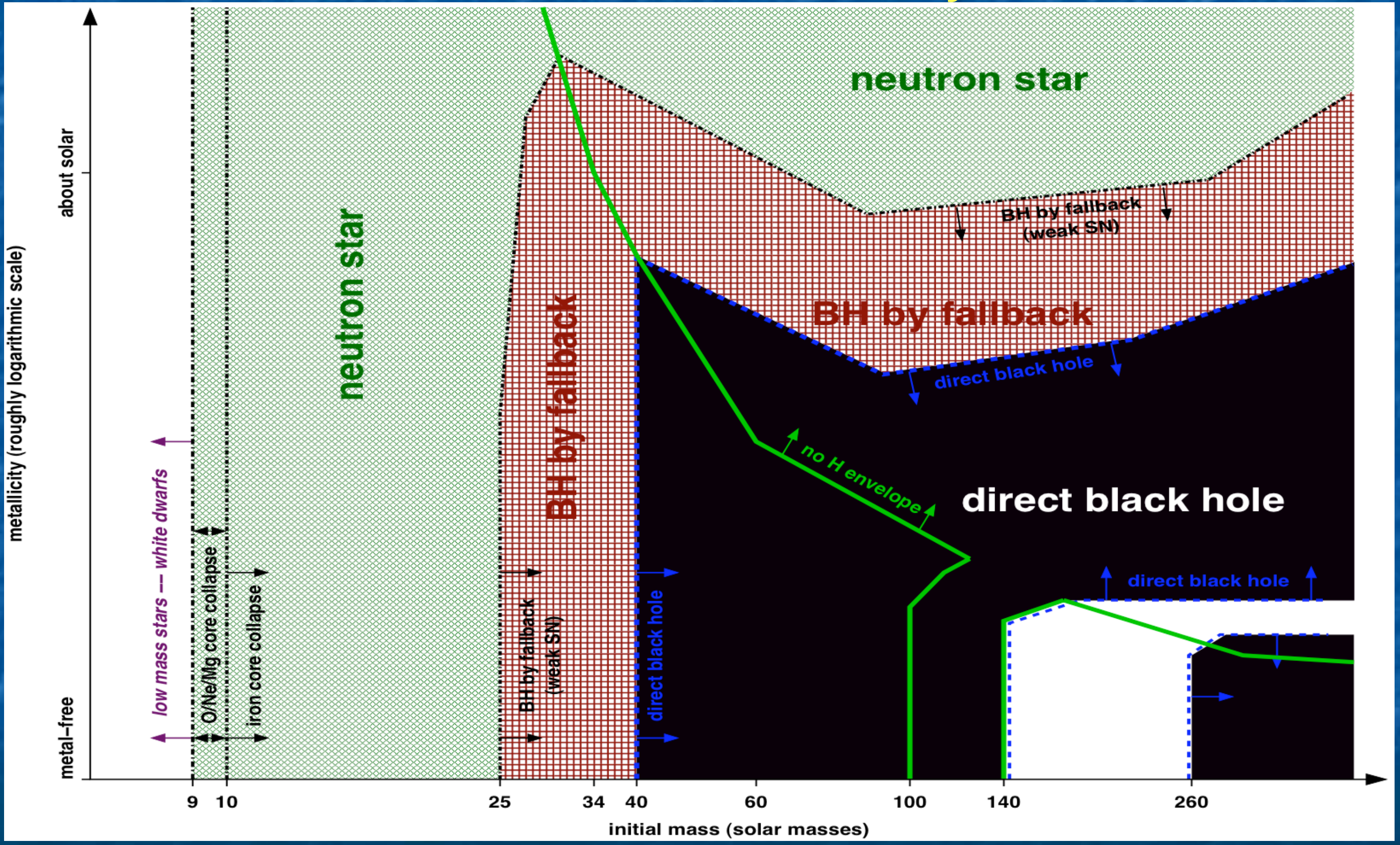
- Core-Collapse Supernovae (SN Type II, Ib/c):
  - Progenitor stars of $\geq 8M_{\odot}$ and lifetimes <50Myr
  - Associated with star forming regions; not in elliptical galaxies
- SN rate estimates based on star formation rate & initial-mass function, and/or galaxy morphology and SN statistics
- Local Group of Galaxies: D~3 Mpc
  - Milky Way, Andromeda (M31, D~800kpc), M33 (D~850kpc) and ~30 small satellite galaxies (LMC & SMC,…)
- Galactic rate: ~ 1.5/cy ($\pm$1.0/cy) [Cappellaro et al. 1989]
  - SMC: ~0.065/cy
  - LMC: ~0.23/cy
  - M31: ~0.21/cy
  - M33: ~0.16/cy

Types of Gravitational Collapse

- Collapse of stellar core:
  - Regular SN core collapse to NS (9-25$M_\odot$) [90%]
  - Core collapse of a very massive star to BH
    - Delayed ~20-50$M_\odot$ (Collapsar type II)
    - Instantaneous ≥ 40-50$M_\odot$ (Collapsar type I)
- Collapse of a Population III star to BH (~$10^2 M_\odot$)
- Collapse of a SMS to a SMBH (10^4-$10^8 M_\odot$)
- Accretion induced collapse of compact object:
  - Collapse of a WD to NS : $M_{WD} > 1.4\ M_\odot$
  - Collapse of a massive NS to BH : $M_{NS} > 1.4\text{-}3\ M_\odot$
- Collapse of merging NS to BH : $M_{remnant} > 3\text{-}6\ M_\odot$

Gravitational collapse of Stellar Cores

The fate of stellar core depends on the progenitor initial mass and metallicity



Heger et al., ApJ 2003

Collapse & GWs

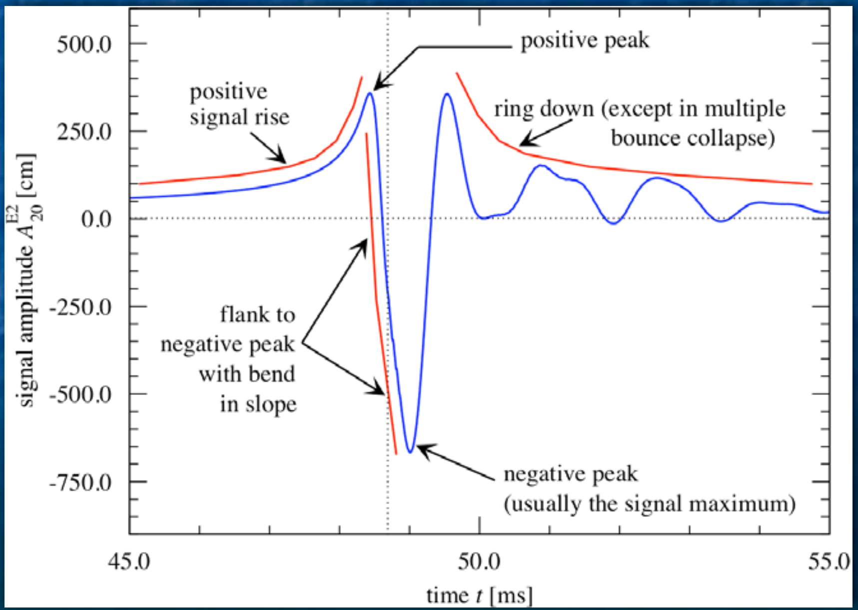
- EM waves
  - Observe SN explosion as shock breaks through stellar surface
  - Hours after core collapse, light curve is only "echo" of driving engine
- Neutrinos
  - A few tenths of msecs after collapse
  - Flux decays with square of distance!
- Gravitational Waves
  - Decouple from matter directly after generation
  - Amplitude decays linearly with distance!

Information from GW Detection

- Rotational state of progenitor core
- Equation of State (EoS) at high densities
- Role and strength of magnetic field
- Composition of compact object
- Emission mechanism for GWs

Strong interactions between,

- **observation,**
- **theory**
- **numerical simulations**



Collapse is extremely complicated

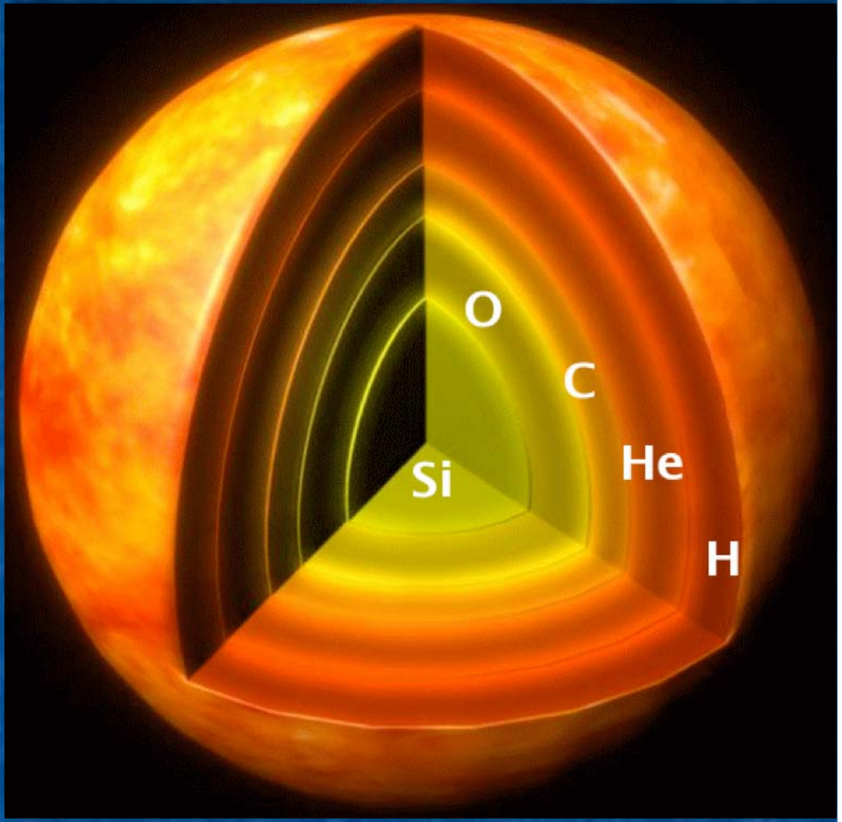
- Gravitational Physics : GR, as typically 2M/R~0.3 -1
- Stellar initial models: Stellar evolution
- Equation of State : Particle and nuclear physics
- Fluid flow and correct treatment of possible shock front : Modern hydrodynamics
- Possible influence of neutrinos : Boltzmann transport
- Rotation and other anisotropies : Multidimensional treatment
- Magnetic fields : MHD
- Very different grids and time scales: Special grids, adaptive mesh refinement

✓Difficult, very diverse, and partially uncertain physics
✓Challenging numerical and computational problems

Simulations still strongly rely on approximations

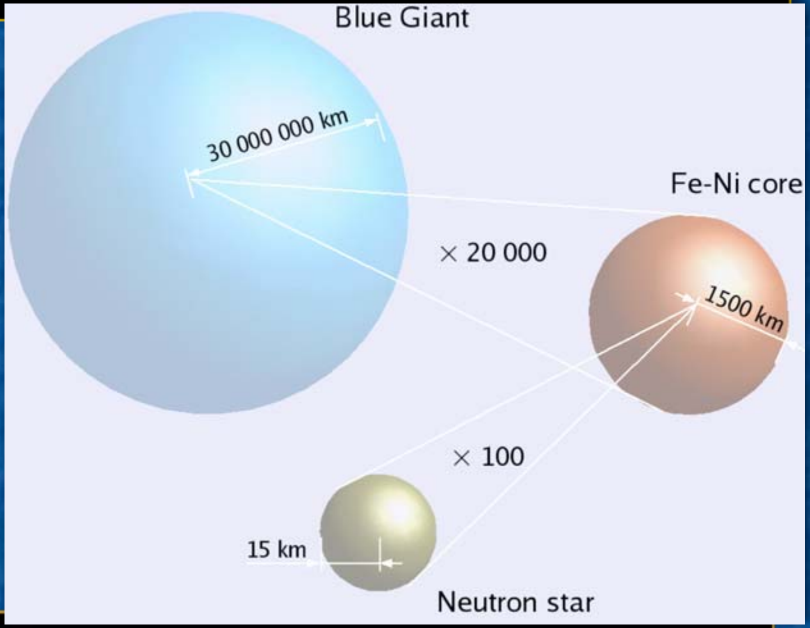
Structure of the Progenitor

- Consider blue giant star with $R \sim 50 R_{\odot}$ and $M \sim 9\text{-}15 M_{\odot}$
- Due to successive nuclear burning: (H → He, He → C, C → O, O → Si) Star develops onion-like structure.
- Interfaces of active thermonuclear burning separate shells of different composition.
- Stars with this mass experience all possible burning stages.
- Several million years after its birth: Star develops central core of iron-group nuclei (so-called "iron core"; Si → Fe, Ni).



The Iron Core

- Iron core consists of ashes from Si burning.
- These cannot be fused to heavier elements (Fe has highest nuclear binding energy).



Properties of final iron core:

- ✓ $\rho_{center} \sim 10^{10}$ g/cm^3
- ✓ $T_{center} \sim 10^{10}$ K (relativistic degenerate Fermi gas)
- ✓ $R_{core} \sim 1500$ km
- ✓ Mainly supported by electron degeneracy pressure ($P=K\rho^{\gamma}$, $\gamma=4/3$)

At this stage:

- Iron core becomes unstable due to electron captures and photo-disintegration of nuclei
- This reduces pressure and effectively pushes adiabatic index γ the critical value below 4/3

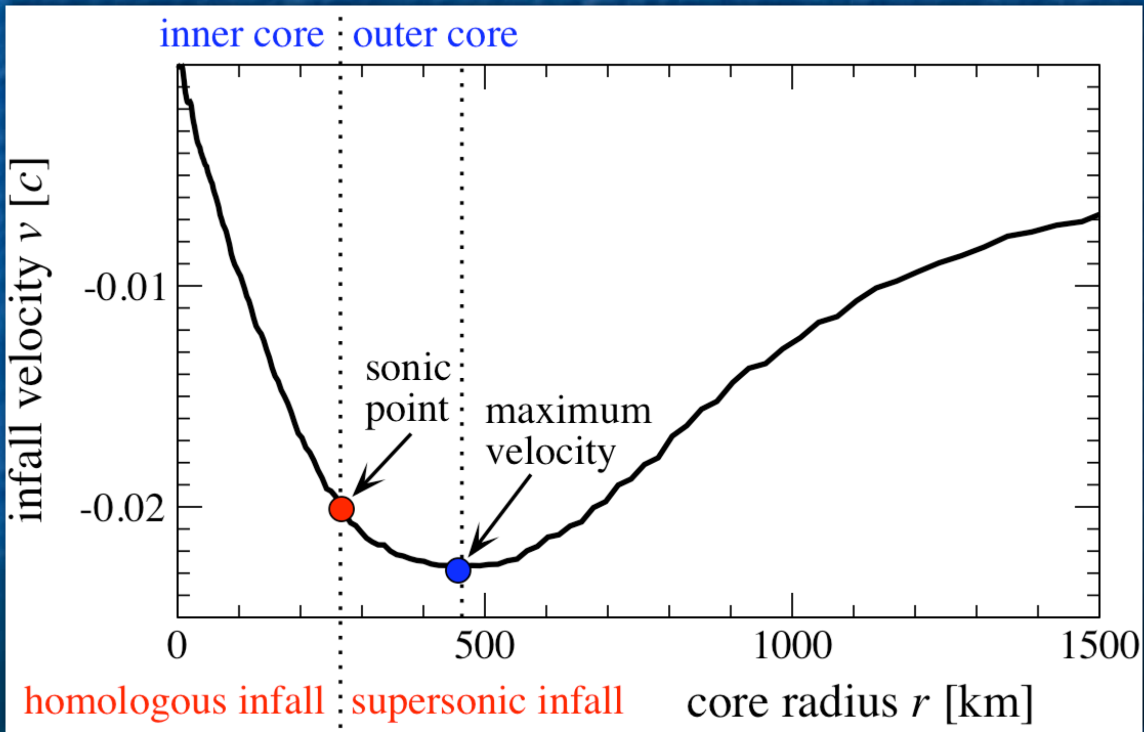
Iron core starts to collapse rapidly!

Contraction Phase

Iron core splits into two parts:

INNER CORE : Up to sonic point $M_{inner\ core} \sim 0.7M_{\odot}$ (depending on the EoS (subsonic)

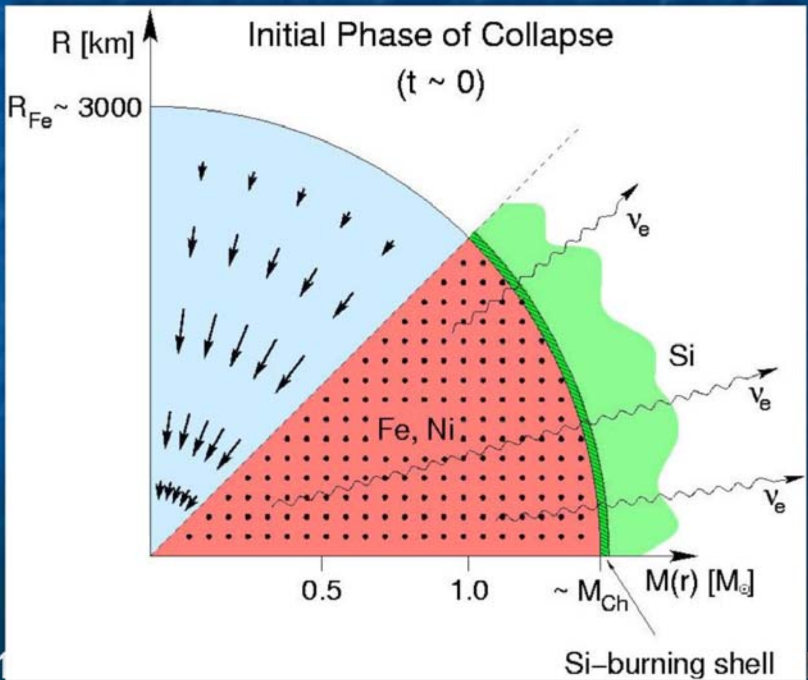
OUTER CORE : Beyond sonic point. Supersonic collapse, decoupled from inner core (supersonic).

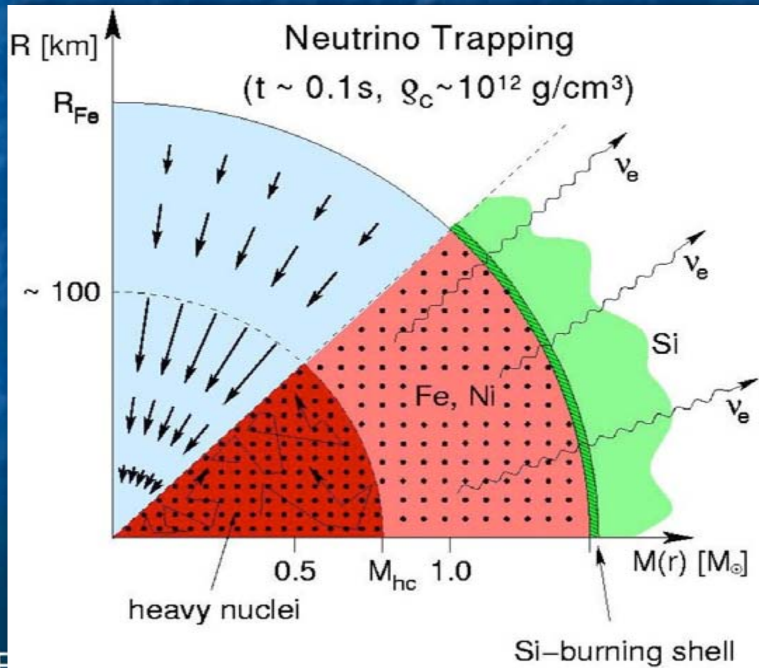


Neutrino Trapping

During contraction: Neutrinos are generated by electron capture, electron fraction Y_e decrease

- **$\rho \leq 10^{12}$ g/cm^3** : ν escape freely.
  - They carry away a bit of energy. ➔ Collapse is almost adiabatic.
- **$\rho \geq 10^{12}$ g/cm^3** : ν are trapped inside matter due ν-e^- scattering
  - Diffusion time scale > collapse time scale. ➔ Collapse is practically adiabatic.





Stiffening of the EoS and Core Bounce

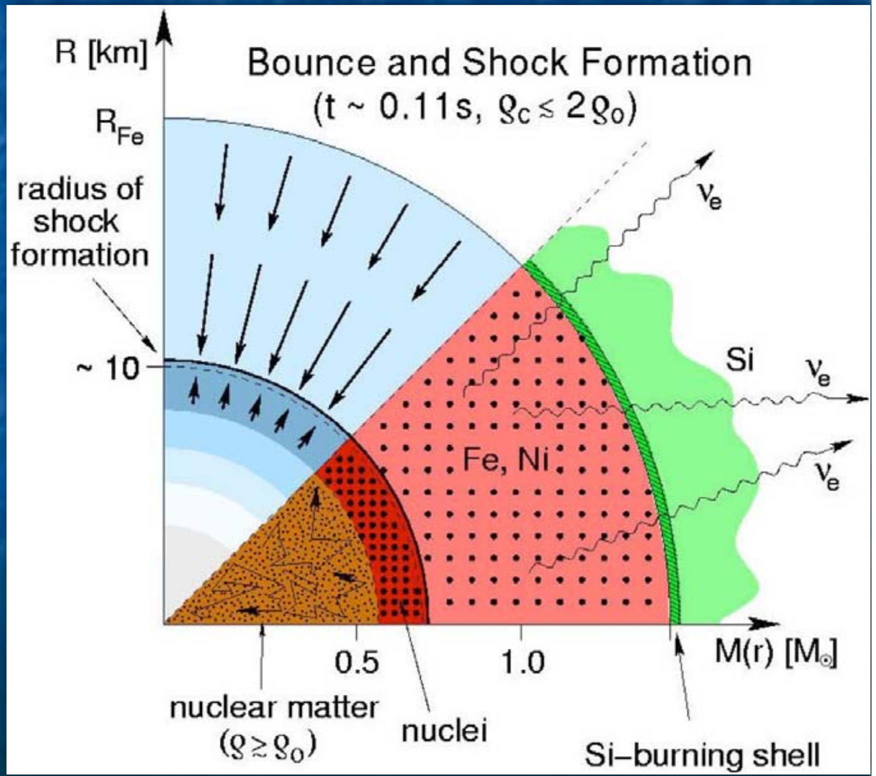
After $t_{collapse}$~100 ms:

- ✓ Collapse velocity reaches $v_{infall\ max} \geq 0.1$ c
- ✓ Central density is at $\rho_{center} \geq \rho_{nucl} \sim 2x10^{14}$ g/cm^3

- Nuclei transform into coherent nuclear matter
- Repulsive nuclear forces act:
  - ✓ EoS stiffens ($\gamma \geq 2$)
  - ✓ Pressure increase strongly

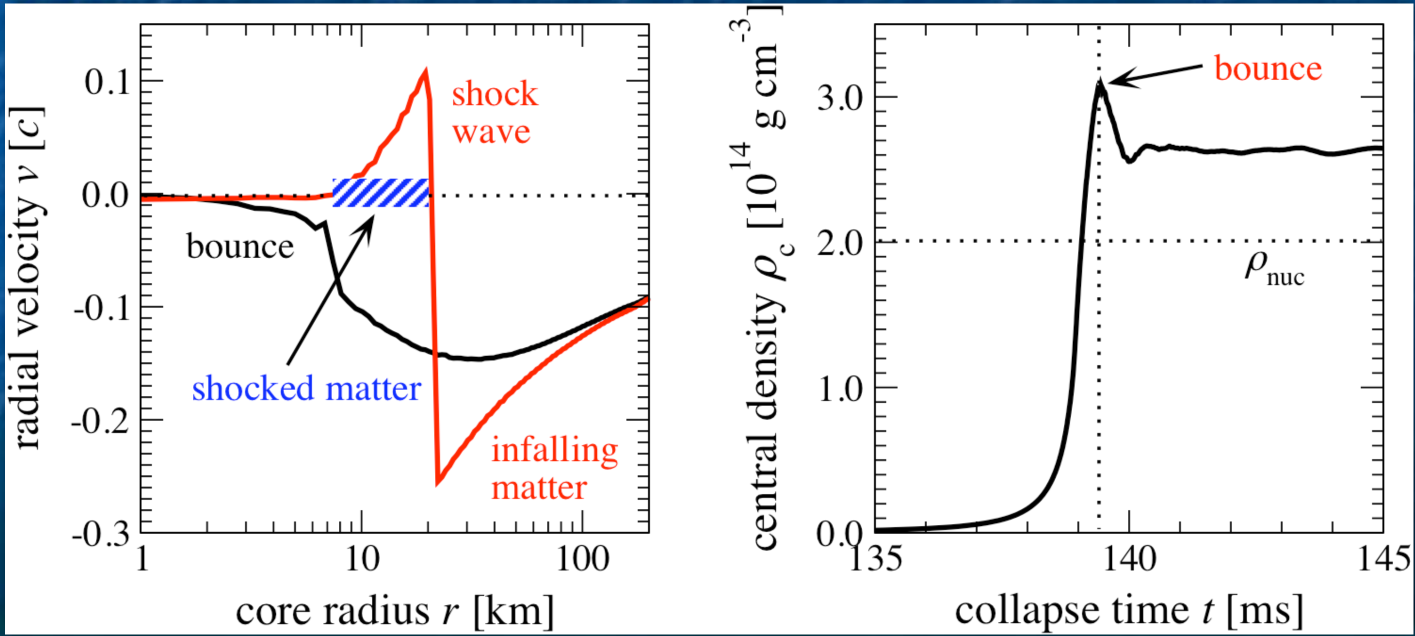
Due to inertia:

- ✓ Inner core overshoots equilibrium position and rebounds
- ✓ After some ring-down oscillations: Inner core acquires new equilibrium and forms hot proto-neutron star (PNS)



Formation of the Prompt Shock

- Near the center the infalling mass shells are stopped
- Pressure waves accumulate at sonic point (R_{sonic} ~ 30-40km) within ~ 1ms
- Prompt shock forms and propagates through outer core (initial energy : **E~ 5-8x10^{51} erg**)
- As shock travels out, matter from outer core continues to fall in.



Stalling of the Prompt Shock

- As the prompt shock propagates out : it dissociates Fe nuclei into free nucleons
- Severe energy losses ~8MeV/nucleon or **$1.6x10^{51}$erg/0.1M$_\odot$**

Thus:

✓Shock consumes entire kinetic energy still within iron core

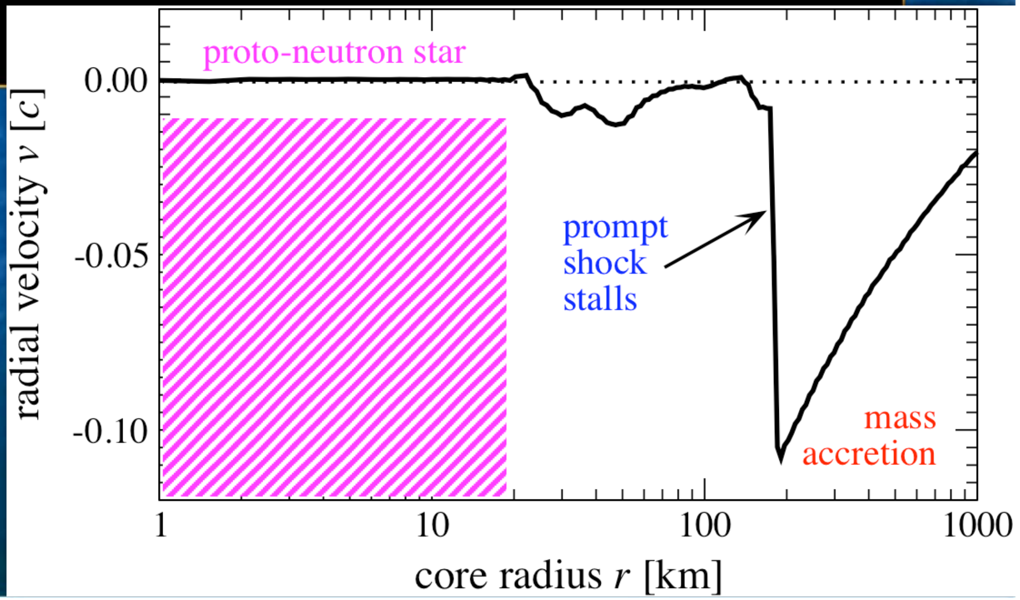
✓Very soon after bounce (at ~3ms) : shock turns into standing accretion shock.

✓Further shock expansion till ~100ms due to heating from accreted infalling matter

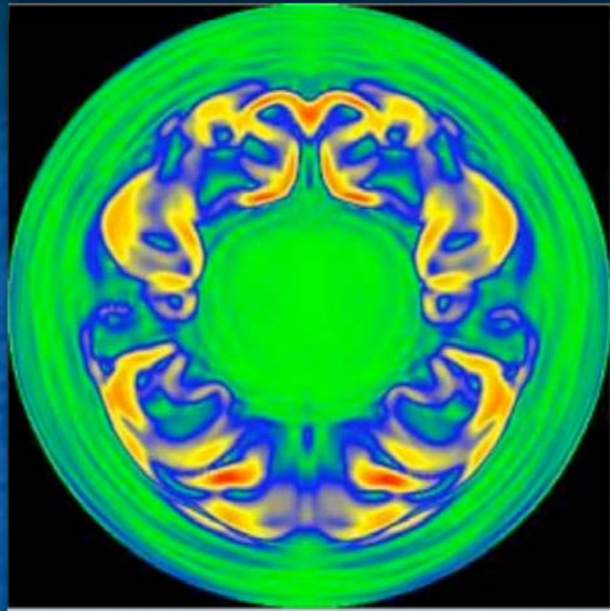
✓PNS grows in mass and size

This means:

- Prompt explosion mechanism fails!



Cooling of the Proto-Neutron Star



Shortly after core bounce:

- Hot PNS further deleptonizes (Y_e~0.3 >0 during bounce).

At $t_{post\ bounce}$ ~10 ms:

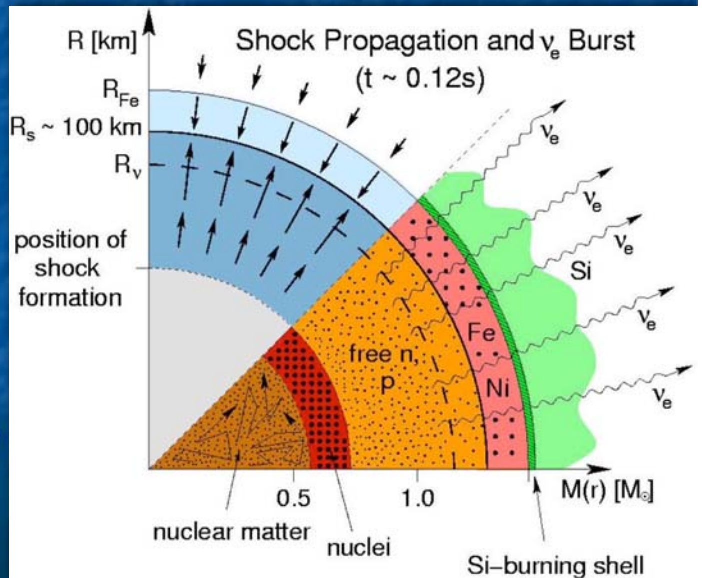
- Neutrino sphere recedes into PNS. ➔ ν's stream out.

Neutrino sphere is transition region between:

- Optically thick for (diffusion limit).
- Optically thin for (free streaming limit).

Additionally:

- PNS becomes convectively unstable In other words: Hot PNS star begins to boil!
- This strongly enhances neutrino luminosity.
- **Finally:** PNS cools, shrinks, and evolves into neutron star.

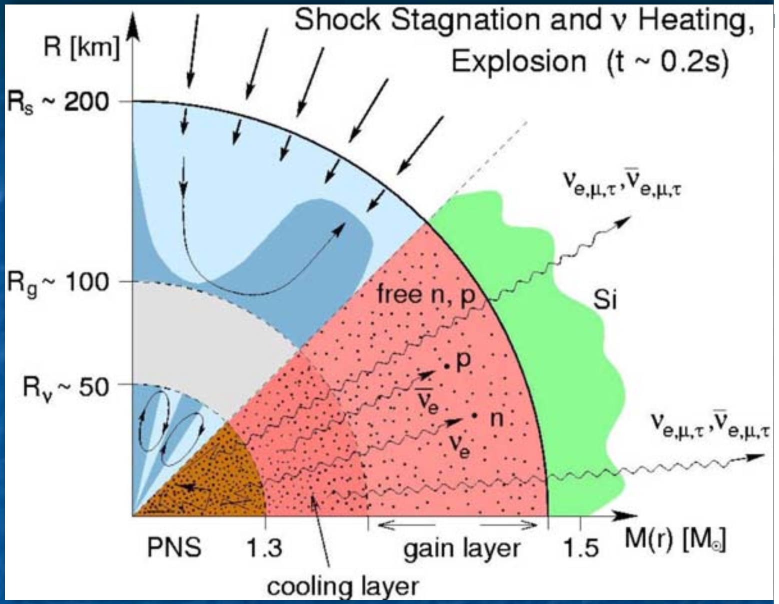


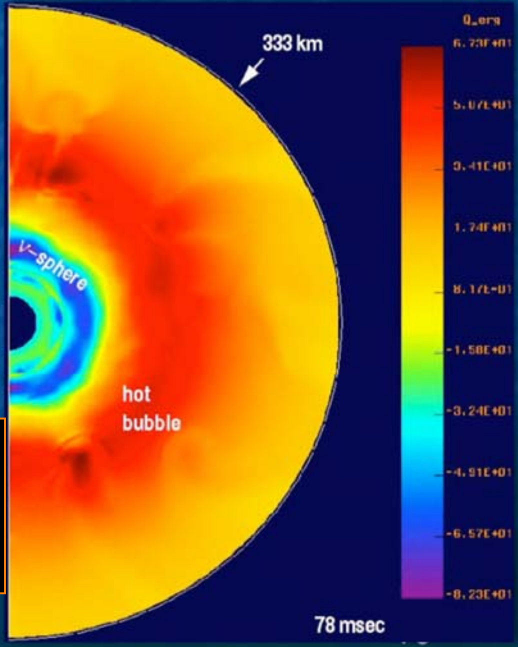
Revival of the Shock Front

- Wilson (1982) discovered that the stalled shock was revived several 100ms after core bounce

  ➔ **Successful explosion**

- Neutrinos from PNS scatter in hot bubble region behind stalled shock.
- They transfer energy & momentum to this "gain region" ($E_{transfer} \sim 10^{51}$ erg < $0.01 E_\nu$ with $E_\nu \sim 3 \times 10^{53}$ erg)
- Post-shock region is heated, shock is driven outwards.
- Convective instabilities may be crucial for reviving stalled shock!





Still : So far numerical simulations do not obtain robust explosions…

The Role of Magnetic Fields

From pulsars we know:
Neutron stars have magnetic fields (up to 10^{15} G for magnetars).

Conclusions from recent numerical studies:

Weak initial fields ($B \leq 10^{11}$ G)

-- Change neither collapse dynamics

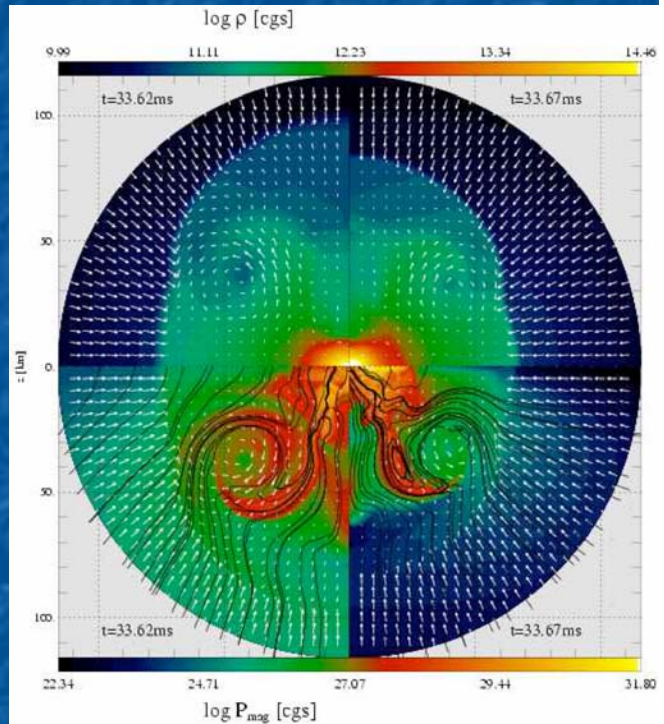
-- nor resulting gravitational wave signal

Strong initial fields ($B \geq 10^{11}$ G)

-- Slow down core efficiently even retrograde rotation occurs.

-- They cause qualitatively different dynamical evolution and GW signal.

-- Highly bipolar, jet-like outflows occur.



Investigations of secular evolution and magnetic instabilities (e.g. MRI) must be performed.

Obergaulinger, Aloy, Mueller, A&A, 2006; Shibata et al., PRD, 2006
Cerda-Duran, Font, Dimmelmeier, A&A, 2007

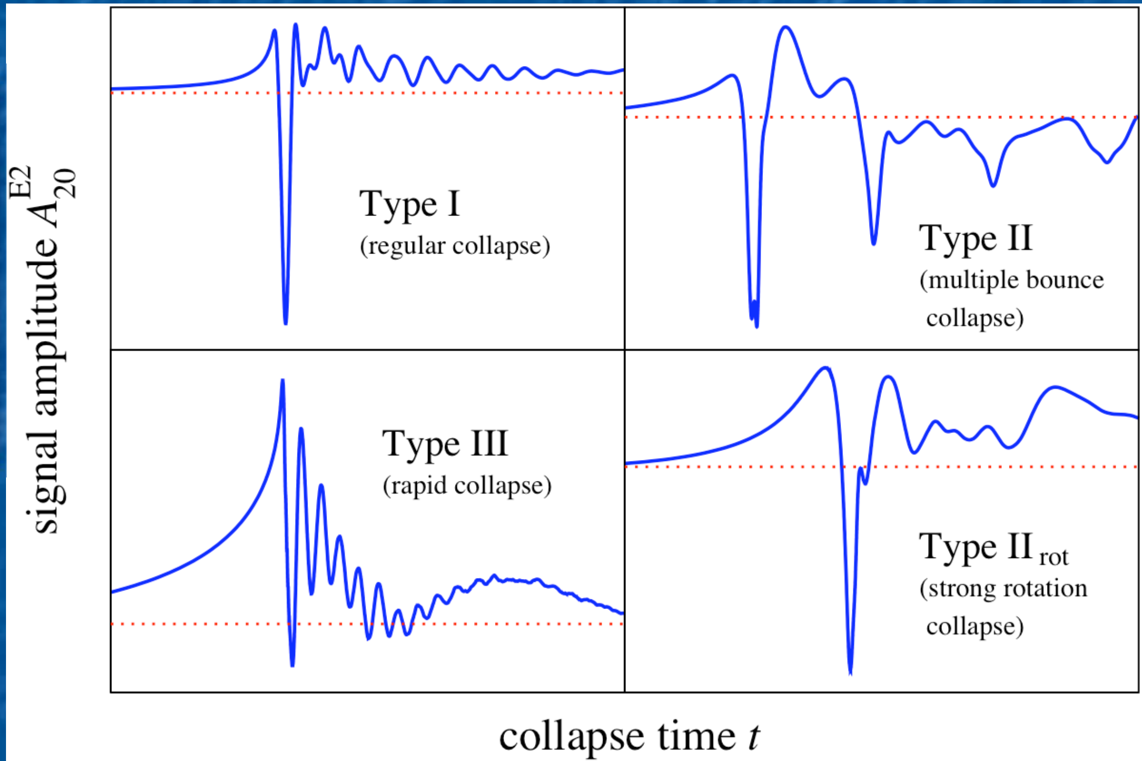
Collapse Dynamics & Burst Waveform Types

From early numerical studies of SN collapse became clear that the waveform type reflects collapse dynamics of core bounce!

Typical values :

$h \sim 10^{-20}$ at 10 kpc

f~500-1000 Hz.



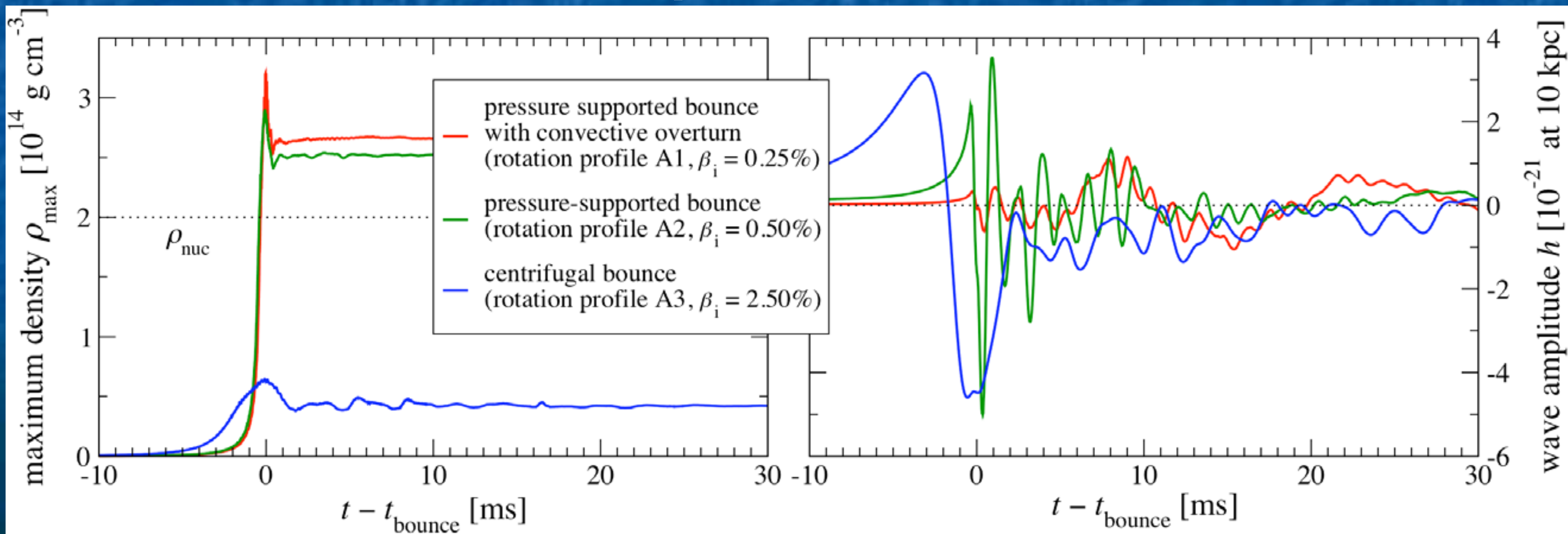
Hillebrandt and Wolff, in ``Nucleosynthesis'', 1985, Moenchmeyer et al., A\&A, 1991; Janka, Zwerger, Moenchmeyer, A&A, 1993; Zwerger and Mueller, A&A, 1997; Dimmelmeier, Font, Mueller, A&A, 2002; Dimmelmeier et al., PRD, 2005; Cerda-Duran et al., A&A, 2005; Shibata and Sekiguchi, PRD, 2005, 2006

GR Simulations with Microphysical EoS

New sophisticated studies of rotational core collapse in GR:

[Ott et al., PRL, 2007; Dimmelmeier et al., PRL, 2007]

- ✓ Coupled relativistic gravity (CFC or BSSN) and hydrodynamics.
- ✓ Initial models from stellar evolution code. [Woosley, Heger, Weaver, RMPh, 2002]
- ✓ Tabulated microphysical EOS with approximate neutrino treatment (deleptonization and neutrino pressure). [Shen et al., PTPh, 1998; Liebendoerfer, APJ, 2005]



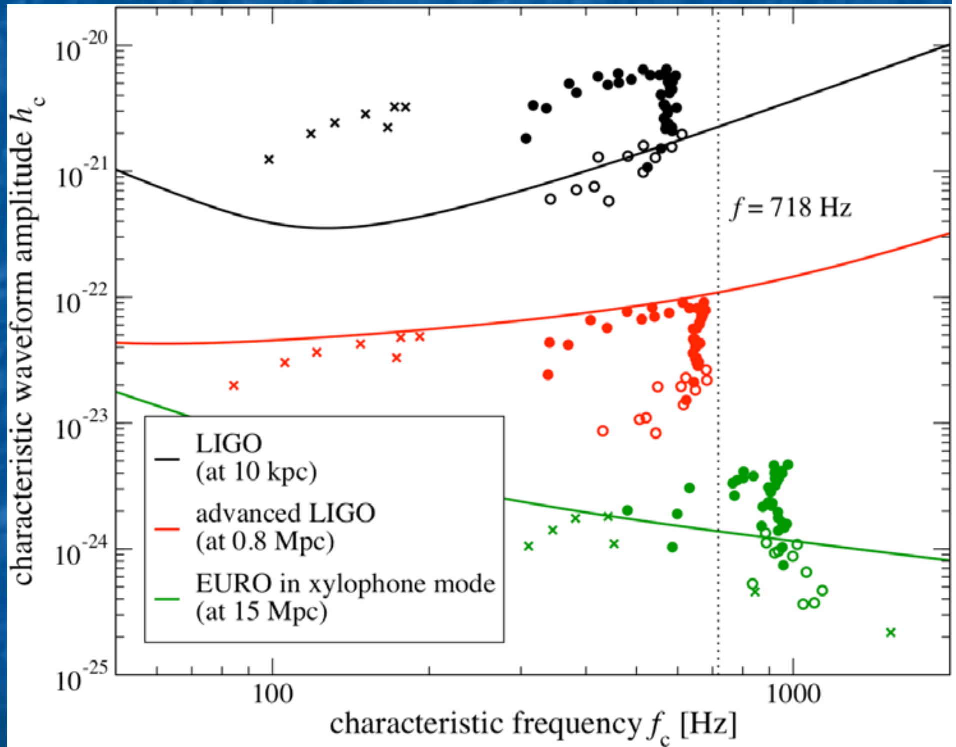
Detection Prospects for GW Detectors

Signals with moderate or slow rotation: (54 models)

- Almost identical peak in frequency spectrum
- Signal frequency is practically independent of rotation rate

Predictions for detectability:

1. Signal from within Milky Way detectable by current detectors.
2. Extragalactic signal maybe detectable by 2nd generation detectors (possibly with narrow-banding).
3. Signal from Virgo cluster probably detectable by 3rd generation detectors.



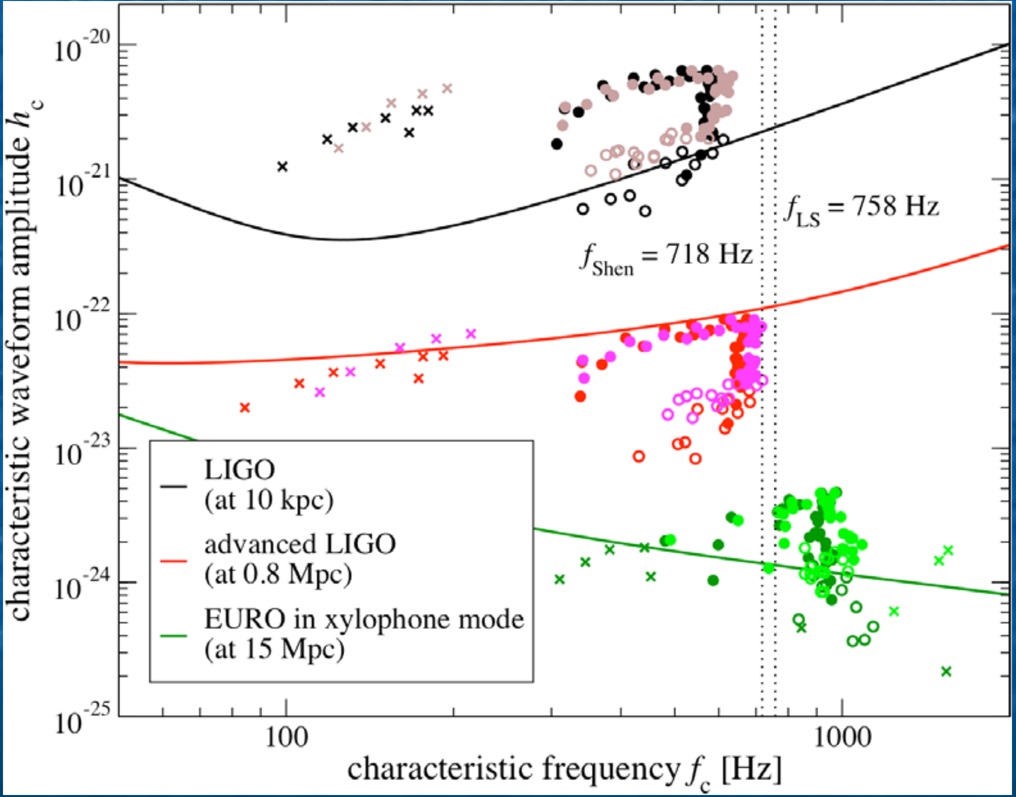
Only in Virgo cluster we may have sufficient event rate of ≥1 per year!

Dimmelmeier et al., PRL, 2007]

The Influence of the EoS on the Core Bounce Signal

Recent comparison of two alternative EOSs for SN core collapse shows:

Signal spread due to different rotation states is more pronounced than influence of EOS.



Weak influence of EOS on :

- PNS structure,
- GW amplitude,
- GW frequency.

It will be very hard to determine EOS details from core bounce signal!

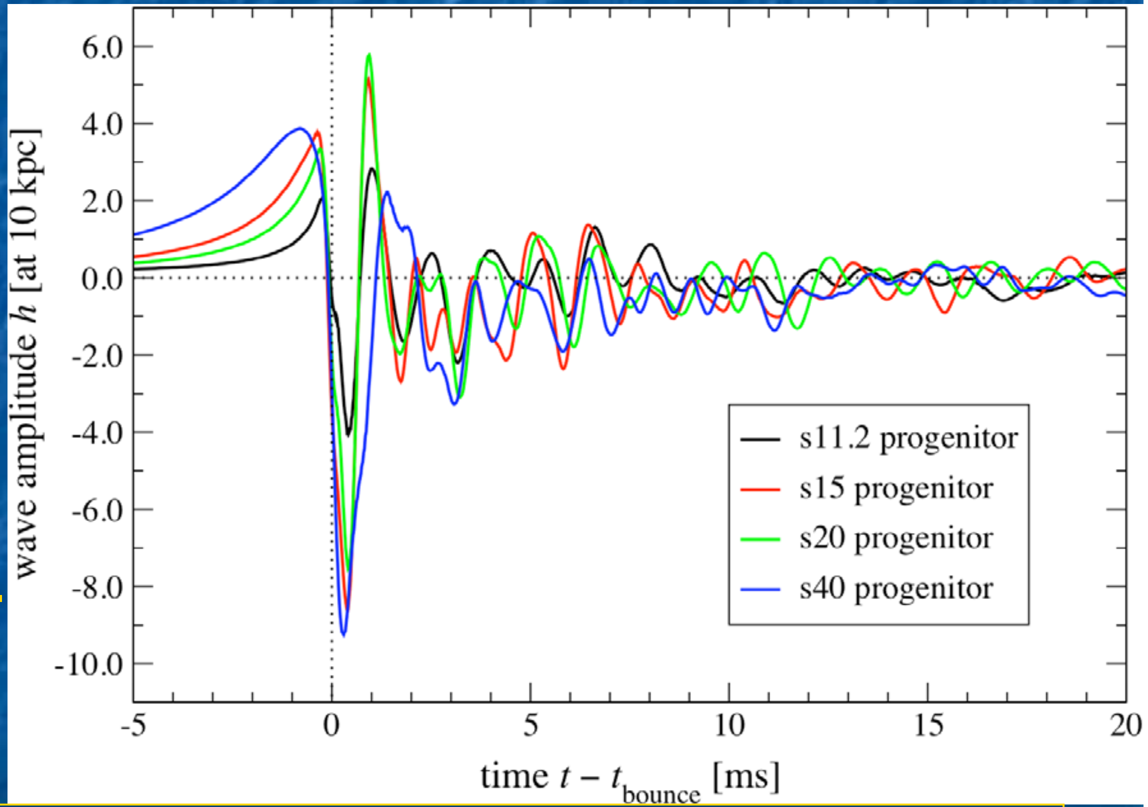
The Influence of the Progenitor Mass

Variation of progenitor mass (M =11M$_\odot$, 15M$_\odot$, 20M$_\odot$, 40 M$_\odot$)shows (with fixed initial rotation profile): Signal shape and frequency remain essentially unaltered [Woosley, Heger, Weaver, RMPh, 2002]

Tough challenge to infer

- initial rotation rate,
- initial rotation profile,
- nuclear EOS, and
- progenitor mass

from core bounce signal.



It is unlikely to reveal much information about pre-collapse core from bounce signal alone!

The Influence of Magnetic Fields

New study of magnetic field effects on signal from core bounce shows:

Contribution to gravitational wave signal from magnetic field is negligible

[Cerda-Duran, Font, Dimmelmeier, A\&A, 2007, Sotani, Yoshida, KK PRD, 2007]

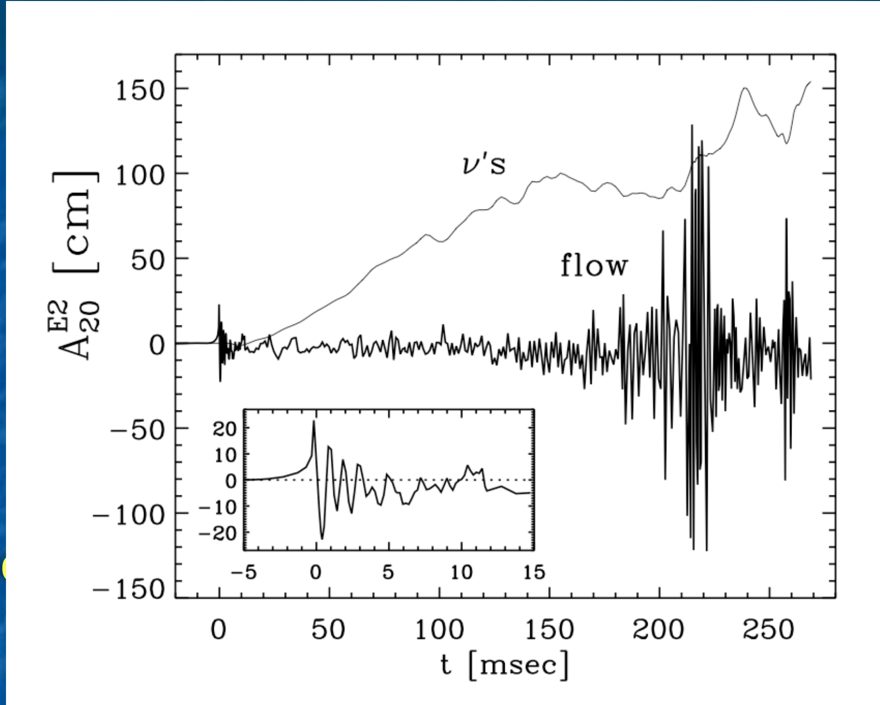
Reason:

- Initial field is weak (astrophysical motivation).
- **Field only winds up** during collapse → Weak amplification.
- Possible instability mechanisms act only after core bounce.

But if magnetic field is sufficiently amplified after core bounce: It can influence structure of PNS and thus gravitational wave emission!

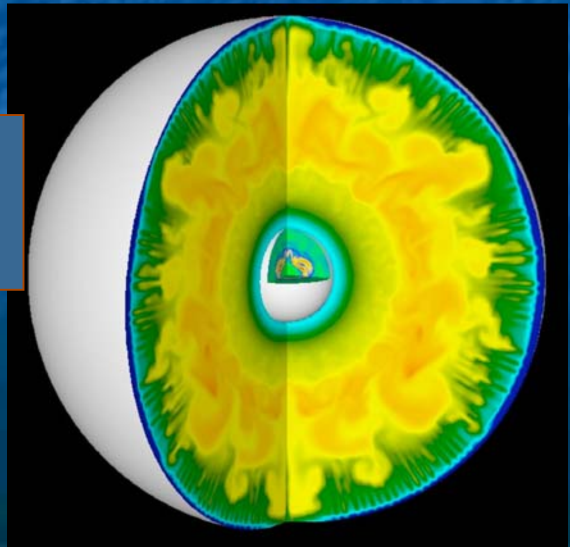
Gravitational Waves from Convection

- Important for PNS
- Can develop even for non-rotating models
- Additional to burst signal
- Two source mechanisms:
  - Convective boiling of NS
  - Anisotropic neutrino emission/absorpti
- Various signals from core bounce, convection, and neutrinos can have comparable amplitudes!



Long emission time scale can yield relatively high energy for continuous signal!

Signal strength : $h \sim 10^{-21}$ at 10 kpc
Frequency : f ~ 200 - 800 Hz

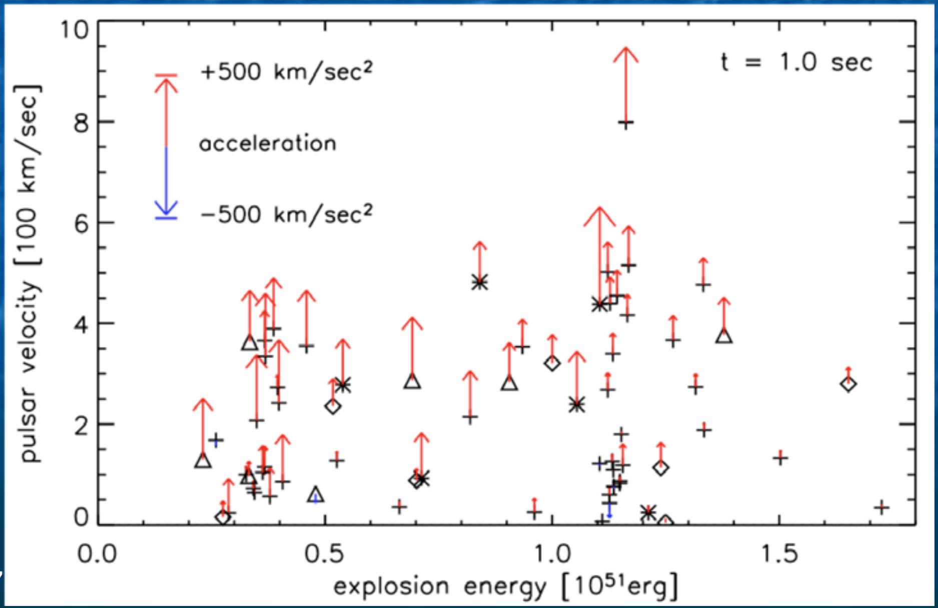


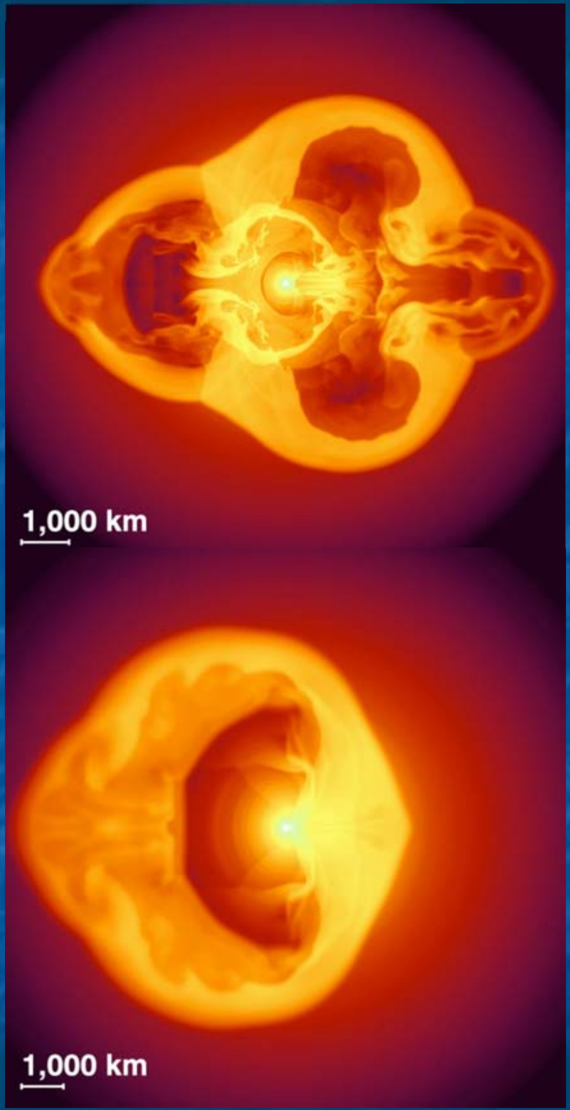
The *l=1* Instability - NS Kicks

Analog to convection even in spherical models:

Large scale anisotropic instabilities can develop!

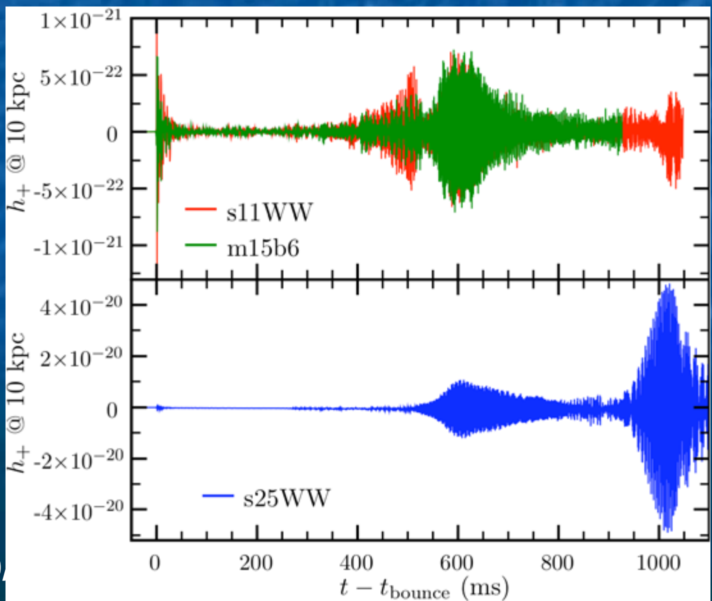
- Example: Hydrodynamic *l=1* instability.
- Simulations in Newtonian gravity, with sophisticated microphysics and initial models. [Scheck et al., PRL, 2004]
- Recoil of neutron star due to this instability could explain ``neutron star kicks"!

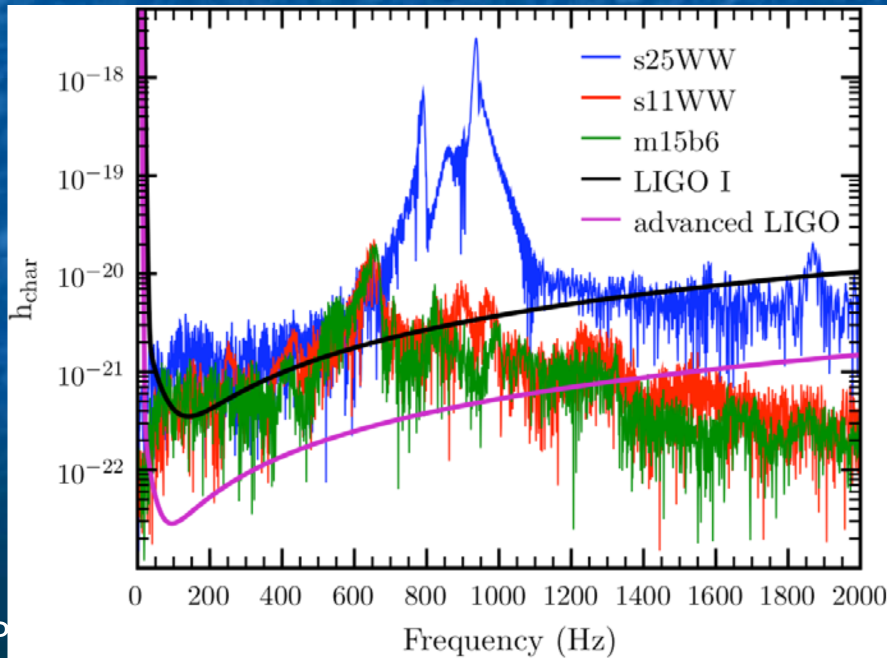




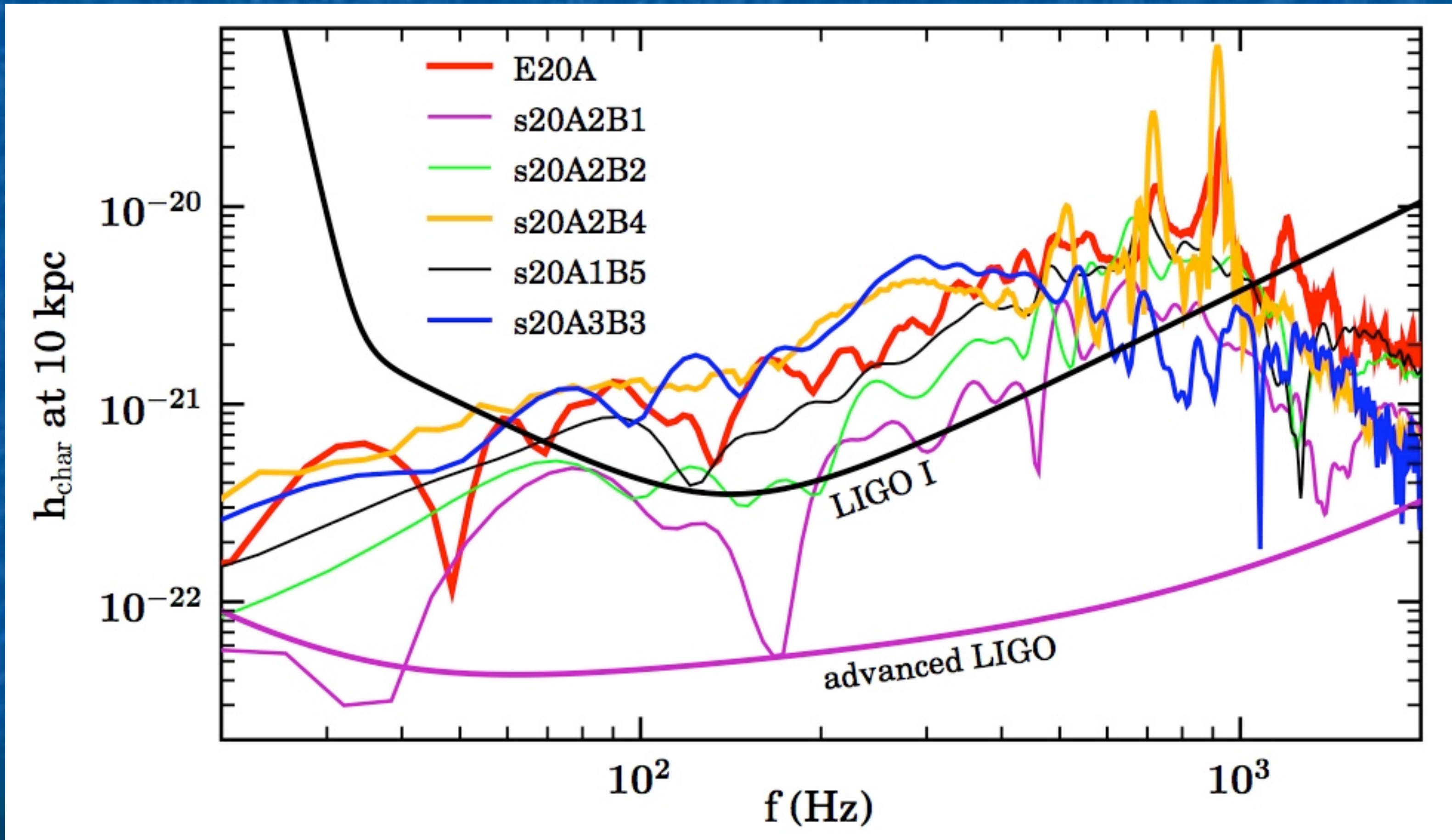
GW from the *l=1* g-mode Instability

- Such an instability could even act as direct engine for supernova explosion [Burrows et al., ApJ, 2006]
- Transfer of energy from pulsations in PNS to shock by acoustic waves [Ott et al., PRL, 2006]
- Unstable *l=1* modes excited by turbulence and accretion, couple to harmonic at ${}_2f_{mode}$
- Simulations done in Newtonian gravity, influence of general relativity not exactly known.
- Signal strength : h ~ 10^{-21} - $4\mathrm{x}10^{-20}$ at 10 kpc,
- Frequency : f ~ 800-1000 Hz





An integrated picture of GW emission during collapse



Ott etal 2006

Accretion Induced Collapse

Two types of accretion induced collapse:

``Regular'' AIC: White dwarf in binary system accretes mass from companion.

- Material burns degenerately. → Nova explosion.
- Material burns non-degenerately. → Collapse to NS (similar to collapse of stellar iron core).
- From estimated formation rate: Need volume out to 100 Mpc.
- GW emission from: Core bounce (rotation!), convection, and dynamical/secular instabilities.

AIC of (proto)-neutron star to BH:

- NS in binary system accretes mass from companion, or
- PNS accretes mass from fall-back in supernova core collapse, or
- massive NS from binary merger collapses after angular momentum redistribution.

Formation rates are rather unclear.

- GW from contraction and ring down of BH (QNM).
- If short-lived matter torus around black hole is accreted: Standard scenario for short GRB.

Maximum Amplitudes

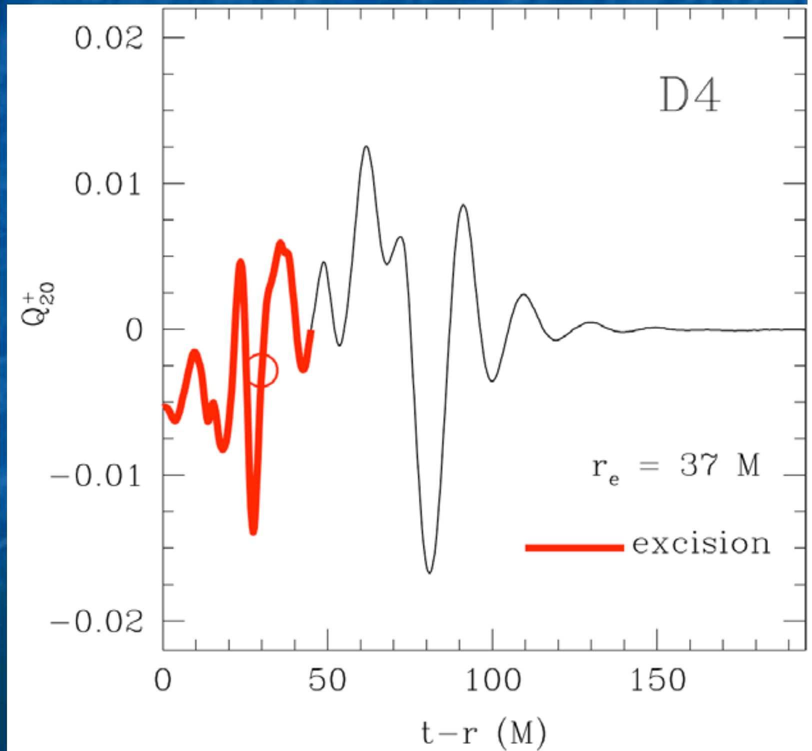
- **Rotating Collapse & Core Bounce (promising):**
  - $E_{GW} \sim 2x10^{-8} M_{\odot}c^2$
  - δf~200-800Hz
  - h_{max}~10^{-21} at 10 kpc
  - duration 1-10 ms
- **Neutrino Driven Convection & Shock Instability**
  - Generic to all core-collapse SN (duration : 100ms -1s)
  - Broad spectrum:1Hz-1kHz
  - $E_{GW} \sim 10^{-9} - 10^{-10} M_{\odot}c^2$
  - h_{max}~10^{-22} at 10 kpc
- **Non-axisymmetric Rotational Instabilities**
  - Great potential, but details unknown & very model dependent
  - Up to and above h_{max}~10^{-20} at 10 kpc depending on progenitor
  - Narrow-band emission
  - Problem: need for ms initial periods conflicting with pulsar birth spin rates

Stellar Core Collapse to a Black Hole

- Almost all simulations of collapsars assume Newtonian gravity [Aloy et al., ApJL, 2000]
- Only recently: 3d simulations of black hole formation become available! [e.g. Shibata, PThP, 2000; APJ, 2003; Baiotti et al., PRD, 2005; PRL, 2005; PRL, 2006]

Several challenges:

- No approximation of full GR possible.
- Wave extraction via quadrupole formula not feasible.
- Singularity appears in center of BH.
- Avoidance of singularity (e.g. by slicing conditions). ➔ Grid stretching.
- Excision of singularity. ➔ Exploit that physical information cannot pass outside through horizon.
- Regularization of singularity. ➔ Original puncture approach.
- “Ignore" singularity. ➔ Moving puncture approach.



NUMBERS for Core Collapse

- GW burst from core collapse

$$h \simeq 10^{-20} \frac{10kpc}{d}, \; f \approx 600 Hz$$

- GW from convective boiling…

$$h \simeq 10^{-21} \frac{10kpc}{d}, \; f \approx 700 Hz$$

- GW from the coupling of an unstable l=1 g-mode in PNS to higher harmonics…

$$h \simeq 4 \times 10^{-20} - 10^{-21} \frac{10kpc}{d},$$
$$f \approx 800 - 1000 Hz$$

- GWs from QNM oscillations

$$h \simeq 10^{-20} - 3 \times 10^{-19} \frac{10kpc}{d},$$
$$f \approx 1 - 5kHz$$

Garching group+collaborators 2004-06

Summary

- We may expect ~1-2 SNe/cy in our galaxy, ~2 SNe/cy out to 60kpc, and ~ 3 SNe /cy in the entire Local Group
- Dominant GW emission processes in SNe relevant for current LIGOs : Post-bounce convection and possibly core g-modes
- Significant GW emission from rotating collapse/bounce and the occurrence of rotational instabilities are unlikely (perhaps in 1 out of 10^3-10^4 SNe)
- Most conservative estimates: Only galactic (D~10kpc) SNe likely to be detected by current LIGOs.
- Advanced instruments: factor of 10 in sensitivity will only slightly increase the expected detectable event rate, but will facilitate the extraction of physical data.

Why do we need rapid rotation?

Several GW emission mechanisms during NS formation rely on rapid rotation:

1. *Core-bounce signal in axisymmetric collapse*

   For slow rotation detectable only within Galaxy, but rapid rotation allows larger distances. Nonlinear couplings may enhance GW emission.

2. *Dynamical bar-mode instability*

   Need $T/|W|>0.24$. If bar persists for many periods, signal detectable out to the Virgo cluster.

3. *Low T/|W| m=2 instability*

   Ne

   h_e

4. *Lo*

   Ne m=2 mode excitation, only detectable in our Galaxy.

But, are NS born rapidly rotating?

5. *CFS f-mode instability*

   Needs $T/|W|>0.08$ to operate. If $T/|W|>0.25$ and $\alpha\sim1$, detectable to *100*Mpc!

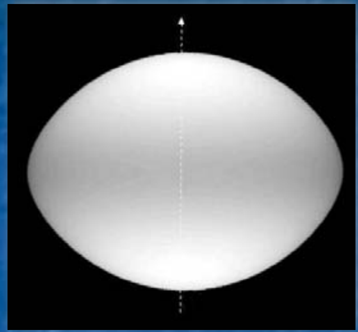
6. *r-mode instability in young strange stars*

   Needs millisecond initial periods. For $\alpha\sim10^{-3}$ there may be several sources in our Galaxy at any time – detectable with a few weeks integration.

Typical Progenitors

A large fraction of progenitor stars are initially rapidly rotating:

The average rotation of OB type stars on the main sequence is 25% of break up speed.



About 0.3% of B stars have $\Omega > 67\%$ of breakup, e.g.of Regulus in Leo: 86% of breakup.

But: Magnetic Torques can Spin Down the Core!

$$S = \frac{B_r B_\phi}{4\pi}$$

Spruit & Phinney 1998, Spruit 2002, Heger, Woosley & Spruit 2004

When the progenitor passes through the Red Supergiant (RSG) phase it has a huge envelope of several hundred times the initial radius.

The core's differential rotation produces a magnetic field by dynamo action that couples the core to the outer layers, transferring away angular momentum. This leads to slowly rotating neutron stars at birth (~10-15ms).

Is there a way out of this?

By-Passing the RSG Phase

Massive Stars ($M>25M_\odot$) evolve very rapidly. Two advantages:

a) There is not sufficient time to slow down the core effectively!

b) A strong wind (WR phase) will expel the envelope, preventing slow down of core by magnetic torques.

A strong wind (high mass-loss rate) allows NS to be formed instead of a BH, but could also carry away a lot of angular momentum.

Mass-loss rate is lower if the star has low metallicity.

In addition, rapidly rotating WR stars may lose mass mainly at the poles (temperature is higher there) => angular momentum loss is lower.

Rapidly rotating cores produced by right mixture of high mass and low metallicity

Massive rapidly rotating cores => millisecond NS => magnetars e.g. Wheeler et al.2000

Observational evidence:

1) magnetar produced by 30-40$M_\odot$ progenitor Gaensler et al.2005

2) magnetar with > 40$M_\odot$ progenitor in star cluster Muno et al.2005

Additional Paths to Rapid Rotation

1) Rotational mixing in OB stars: Woosley & Heger 2005

Rapid rotation in massive OB stars can induce deep rotational mixing, preventing the RSG phase (stars stay on main sequence).

Woosley & Heger (2005) estimate that 1% of all stars with mass $>10M_\odot$ will produce rapidly rotating cores.

2) Loss of envelope in binary evolution:

If a binary companion strips the outer envelope of a massive star before core collapse, the RSG phase is avoided.

(see Fryer & Kalogera 2001, Pfahl et al. 2002, Podsiadlowski et al. 2003, Ivanova & Podsiadlowski 2003)ß

3) Fall-back accretion (see e.g. Watts & Andersson, 2002)

4) Binary WD mergers

Suggested as ms pulsar formation mechanism in globular clusters. (Middleditch 2003)

Also, suggested as alternative magnetar formation mechanism, with event rate 0.3/year at ~ 40Mpc. (Levan et al. 2006)

Alternative Ways to Rapid Rotation

1) Higher progenitor mass:

Table 4: Pulsar Rotation Rate With Variable Remnant Mass[a]

Mass	Baryon[b] ($M_\odot$)	Gravitational[c] ($M_\odot$)	$J(M_{\rm bary})$ (10^{47} erg s)	BE (10^{53} erg)	Period[d] (ms)
12 $M_\odot$	1.38	1.26	5.2	2.3	15
15 $M_\odot$	1.47	1.33	7.5	2.5	11
20 $M_\odot$	1.71	1.52	14	3.4	7.0
25 $M_\odot$	1.88	1.66	17	4.1	6.3
35 $M_\odot$ [e]	2.30	1.97	41	6.0	3.0

(Heger, Woosley & Spruit 2004, Heger & Woosley 2005)

Stellar evolution proceeds much faster for higher masses - there is less time to slow down the core. If a binary companion strips the outer envelope of a massive star before core collapse, a rapidly rotating NS forms.

(see Fryer & Kalogera 2001, Pfahl et al. 2002, Podsiadlowski et al. 2003, Ivanova & Podsiadlowski 2003)

Rapid initial rotation will wind-up magnetic field lines, leading to the formation of **magnetars.**

(Gaensler et al. 2005)

Conclusions

1. Typical core collapse will lead to slowly rotating NSs - most GW mechanisms not operating/not detectable at good event rates.
2. But, there are several ways to produce rapidly rotating NSs at birth, but only in ~1% of SN events.
3. Still, the strongest GW mechanisms (those detectable beyond the Virgo cluster) may have good event rates for advanced LIGO/VIRGO type of detectors.
4. Need to focus more on strongest GW mechanisms both theoretically and by narrow-banding/improving detectors in 1 - 3kHz range.