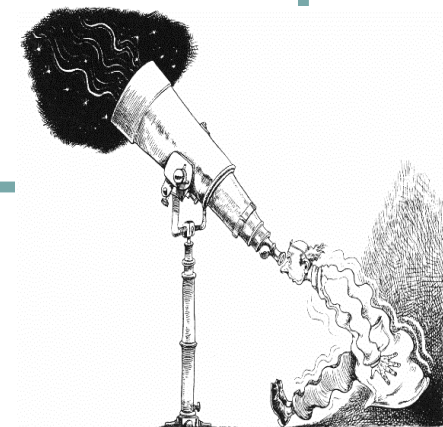




Gravitational Waves

Kostas Kokkotas

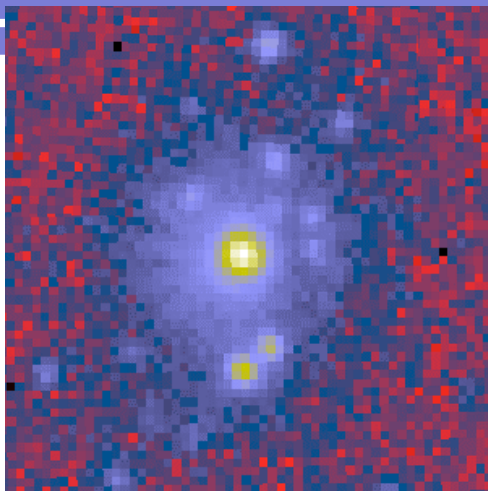
Eberhard Karls University of Tübingen
&
Aristotle University of Thessaloniki



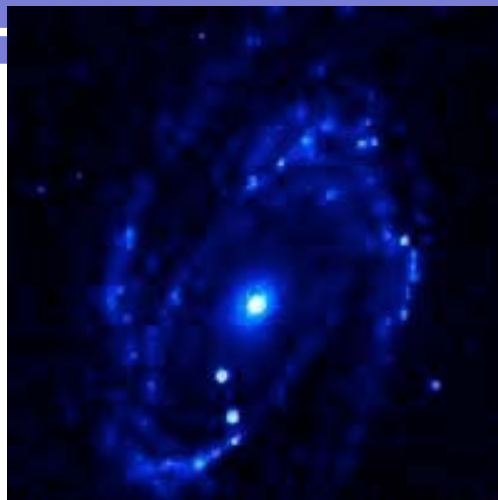
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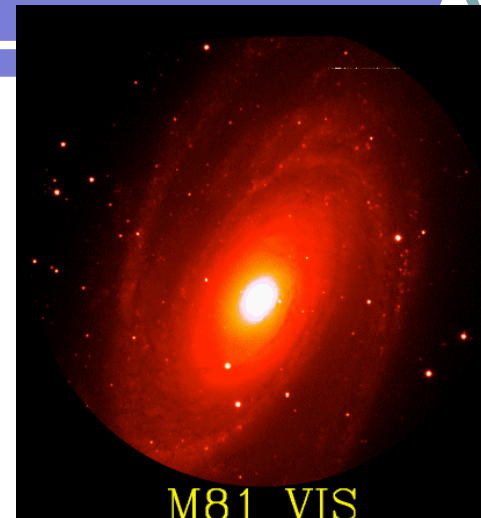
M81 galaxy



X-ray: 10 nm

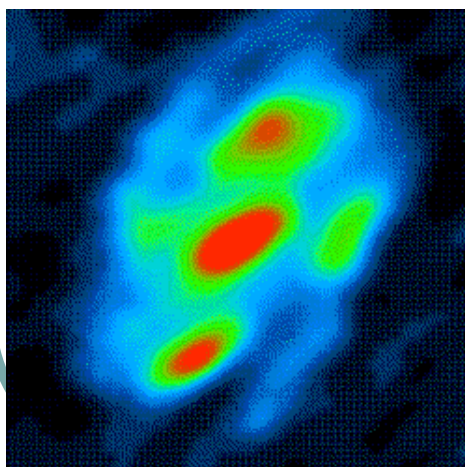


UV: 200 nm



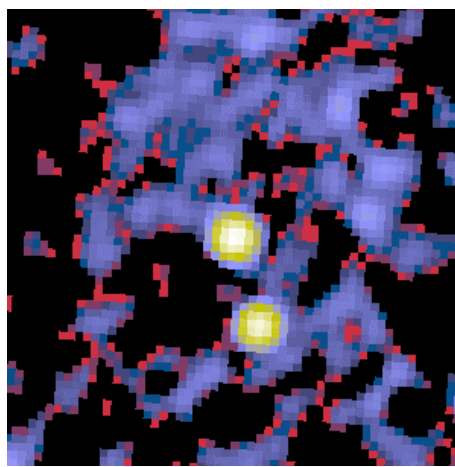
M81 VIS

Visible: 600 nm



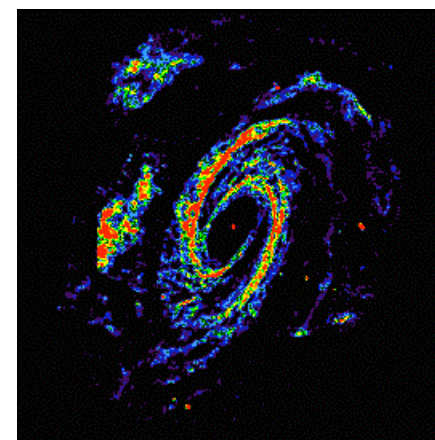
Infrared: 100 mm

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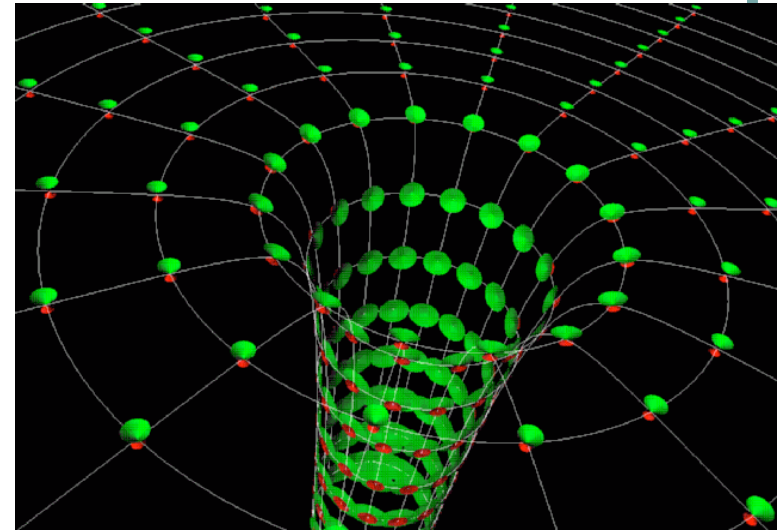
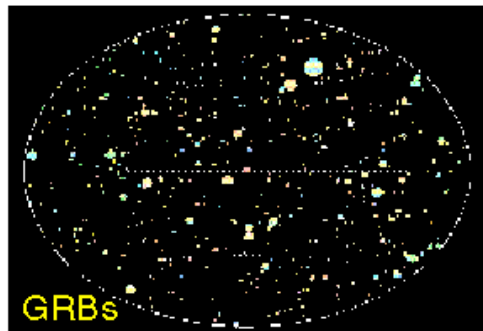
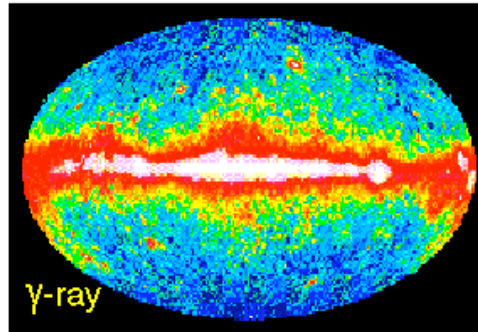
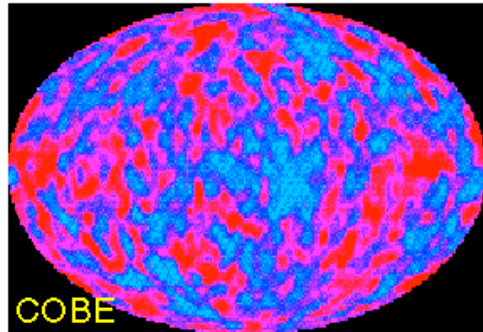
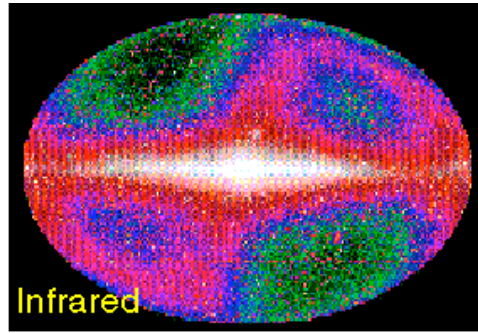
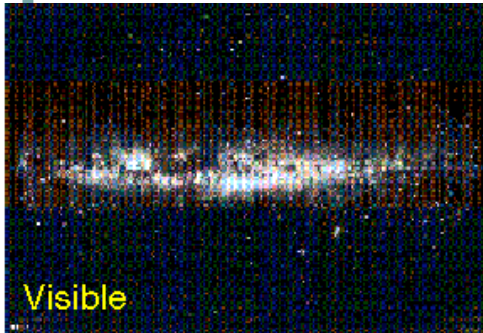
Radio: 21cm

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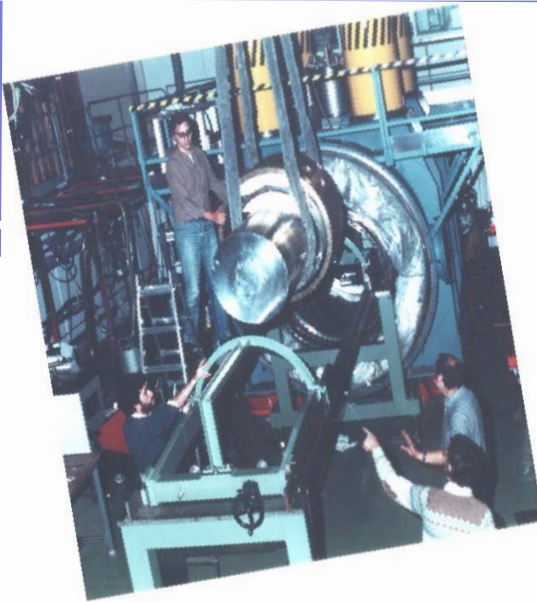


Radio – HI filter

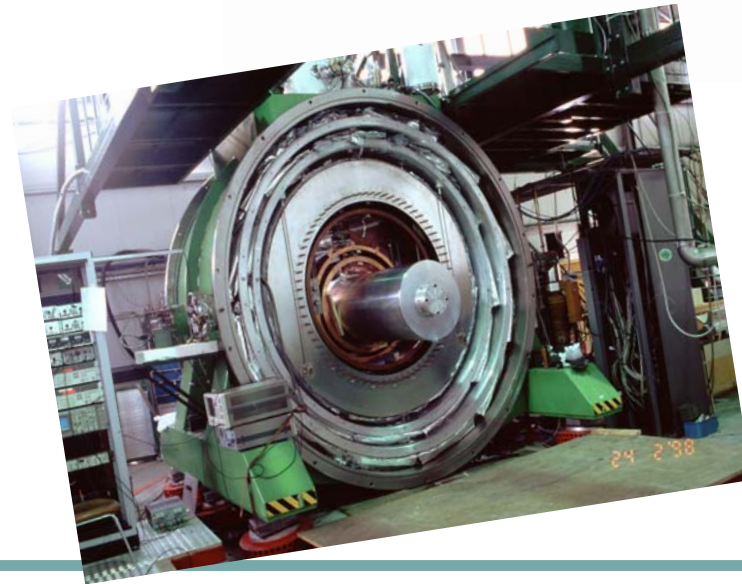
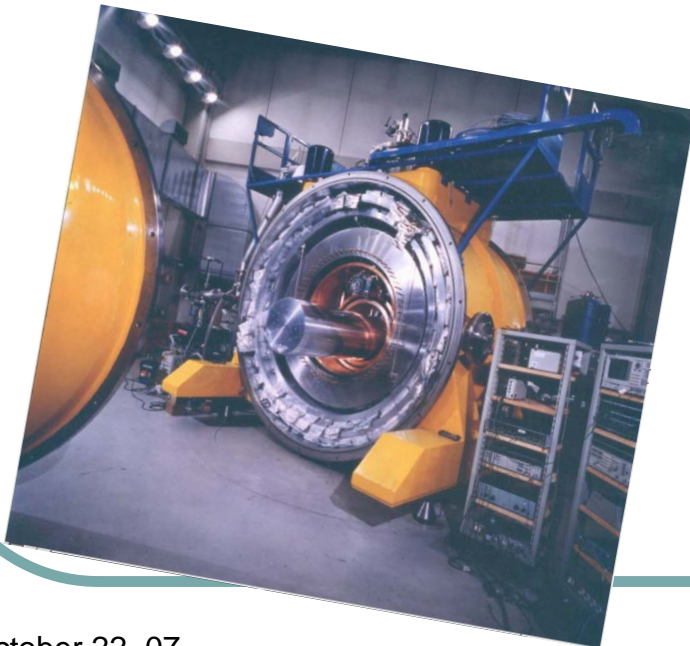
A New Window on the Universe



Gravitational Waves will provide a new way to view the dynamics of the Universe



ALLEGRO AURIGA EXPLORER NAUTILUS NIOBE



October 22, 07

INPE

4

Gravitational Waves: an international dream

GEO600 (British-German)
Hannover, Germany



TAMA (Japan)
Mitaka



October 22, 07



LIGO (USA)
Hanford, WA & Livingston, LA



AIGO (Australia),
Wallingup Plain, Perth



VIRGO (French-Italian)
Cascina, Italy

About the lectures...

- Gravitational Waves: Introduction
- Sources of Gravitational Waves
 - Collapse
 - Neutron Stars - Black Holes
 - Binary Systems

Gravitational Waves

- **Why Gravitational Waves?**
 - Fundamental aspect of General Relativity
 - Originate in the **most violent events** in the Universe
 - Major **challenge to present technology**
- **Why we have not seen them yet?**
 - They **carry enormous amount of energy** **but**
 - They **couple very weakly** to detectors.
- **How we will detect them?**
 - **Resonant Detectors** (Bars & Spheres)
 - **Interferometric Detectors** on Earth
 - **Interferometers** in Space

Gravitational vs E-M Waves

- **EM waves are radiated by individual particles**, **GWs are due to non-spherical bulk motion of matter**. I.e. the information carried by EM waves is stochastic in nature, while the GWs provide insights into coherent mass currents.
- **The EM will have been scattered many times**. **In contrast, GWs interact weakly with matter and arrive at the Earth in pristine condition**. Therefore, GWs can be used to probe regions of space that are opaque to EM waves. Still, the weak interaction with matter also makes the GWs fiendishly hard to detect.
- Standard astronomy is based on **deep imaging of small fields of view**, while **gravitational-wave detectors cover virtually the entire sky**.
- **EM radiation has a wavelength smaller than the size of the emitter**, while **the wavelength of a GW is usually larger than the size of the**

Neutrinos: are more like EM waves than GW in most respects, except...

Propagate through most things like GW, so you can see dense centers

But neutron stars don't generate so many ν after first few minutes

Morale: **GWs carry information which would be difficult to get by other means.**

Uncertainties and Benefits

- **Uncertainties**
 - The **strength** of the sources (may be orders of magnitude)
 - The **rate of occurrence** of the various events
 - The **existence of the sources**
- **Benefits**
 - **Information about the Universe that we are unlikely ever to obtain in any other way**
 - Experimental **tests of fundamental laws of physics** which cannot be tested in any other way
 - **The first detection of GWs will directly verify their existence**
 - By comparing the arrival times of EM and GW bursts we can **measure their speed** with a fractional accuracy $\sim 10^{-11}$
 - From their polarization properties of the GWs we can verify GR prediction that the waves are **transverse** and **traceless**
 - From the waveforms we can directly identify the **existence of black-holes.**

Information carried by GWs

- **Frequency**

$$f \sim 10^4 \text{ Hz} \Rightarrow \rho \sim 10^{16} \text{ gr/cm}^3$$

$$f \sim 10^{-4} \text{ Hz} \Rightarrow \rho \sim 1 \text{ gr/cm}^3$$

$$f_{dyn} \sim \left(\frac{GM}{R^3} \right)^{1/2} \sim (G\rho)^{1/2}$$

- **Rate of frequency change**

$$\dot{f}/f \sim (M_1, M_2)$$

- **Damping**

$$\tau \sim M^3/R^4$$

- **Polarization**

- Inclination of the symmetry plane of the source
- Test of general relativity

- **Amplitude**

- Information about the strength and the distance of the source ($h \sim 1/r$).

- **Phase**

- Especially useful for detection of binary systems.

GW Frequency Bands

- High-Frequency: 1 Hz – 10 kHz
 - (Earth Detectors)
- Low-Frequency: 10^{-4} – 1 Hz
 - (Space Detectors)
- Very-Low-Frequency: 10^{-7} – 10^{-9} Hz
 - (Pulsar Timing)
- Extremely-Low-Frequency: 10^{-15} – 10^{-18} Hz
 - (COBE, WMAP, Planck)

Gravitation & Spacetime Curvature

Newton

$$\nabla^2 U = 4\pi G \rho$$

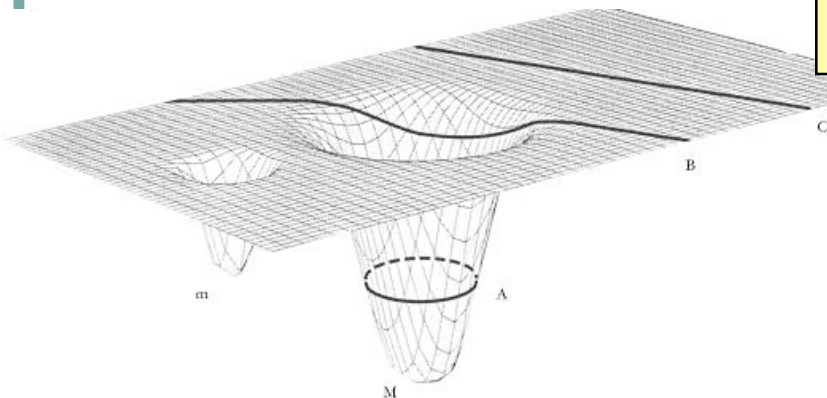
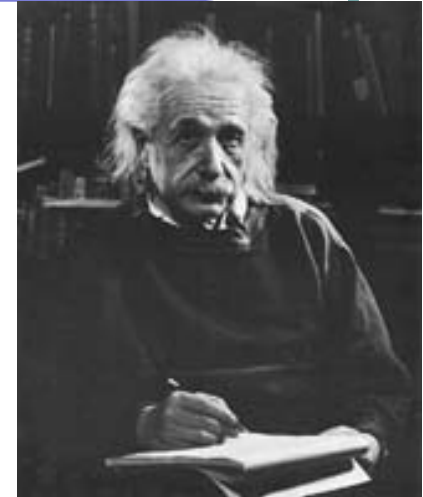
$$\frac{d^2 \vec{x}}{dt^2} = \nabla U$$

Einstein

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$$\frac{d^2 x^\mu}{ds^2} \sim f(g^{\mu\nu})$$



- **Matter** dictates the degree of **spacetime** deformation.
- **Spacetime** curvature dictates the motion of **matter**.

GWs fundamental part of Einstein's theory

Linearized Gravity

- Assume a **small perturbation** on the background metric:
- The **perturbed Einstein's equations** are:

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1$$

$$h_{\alpha\beta;\mu}{}^{;\mu} + g_{\alpha\beta} h^{\mu\nu}{}_{;\nu\mu} - 2h_{\mu(\alpha}{}^{;\mu}{}_{;\beta)} + 2R_{\mu\alpha\nu\beta} h^{\mu\nu} - 2R_{\mu(\alpha} h_{\beta)}{}^{\mu} = kT_{\alpha\beta}$$

- Far from the source (**weak field limit**)...
- And by **choosing a gauge**:
- Simple wave equation:**

$$\tilde{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h_{\alpha}{}^{\alpha}$$

$$h^{\mu\nu}{}_{;\nu} = 0$$

$$\left(-\frac{\partial^2}{\partial t^2} + \nabla^2\right) h^{\mu\nu} = \partial_{\lambda} \partial^{\lambda} h^{\mu\nu} = kT^{\mu\nu}$$

Lorentz or De Donder gauge

Transverse-Traceless (TT)-gauge

- Plane wave solution

$$\tilde{h}^{\mu\nu} = A^{\mu\nu} e^{ik_a x^a}$$

$$A^{\mu\nu} k_\mu = 0$$

$$k^\mu k_\mu = 0$$

- TT-gauge (wave propagating in the z-direction)

$$A^{\mu\nu} = h_+ \varepsilon_+^{\mu\nu} + h_\times \varepsilon_\times^{\mu\nu}$$

$$\varepsilon_+^{\mu\nu} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\varepsilon_\times^{\mu\nu} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- Riemann tensor
- Geodesic deviation
- ...and the **tidal force**

$$R_{j0k0}^{TT} = -\frac{1}{2} \frac{\partial^2}{\partial t^2} h_{jk}^{TT}$$

$$\frac{d^2 \xi_k}{dt^2} \approx -R_{k0j0}^{TT} \xi^j = \frac{1}{2} \frac{\partial^2 h_{jk}^{TT}}{\partial t^2} \xi^j$$

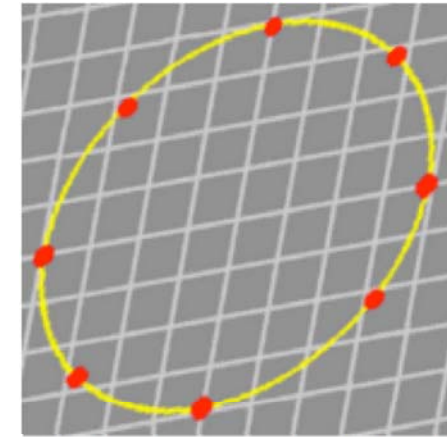
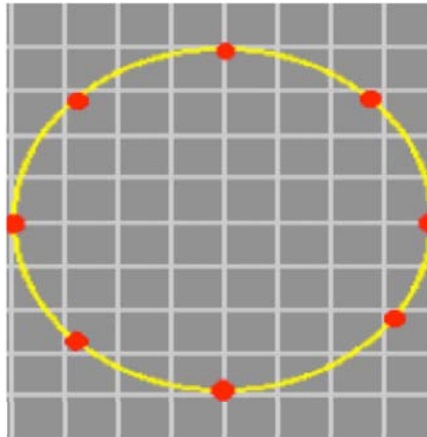
$$f^k \sim m \cdot R_{0j0}^k \cdot \xi^j$$

Gravitational Waves II

$$h^{\mu\nu} = h_+ \varepsilon_+^{\mu\nu} \cos[\omega(t - z)]$$

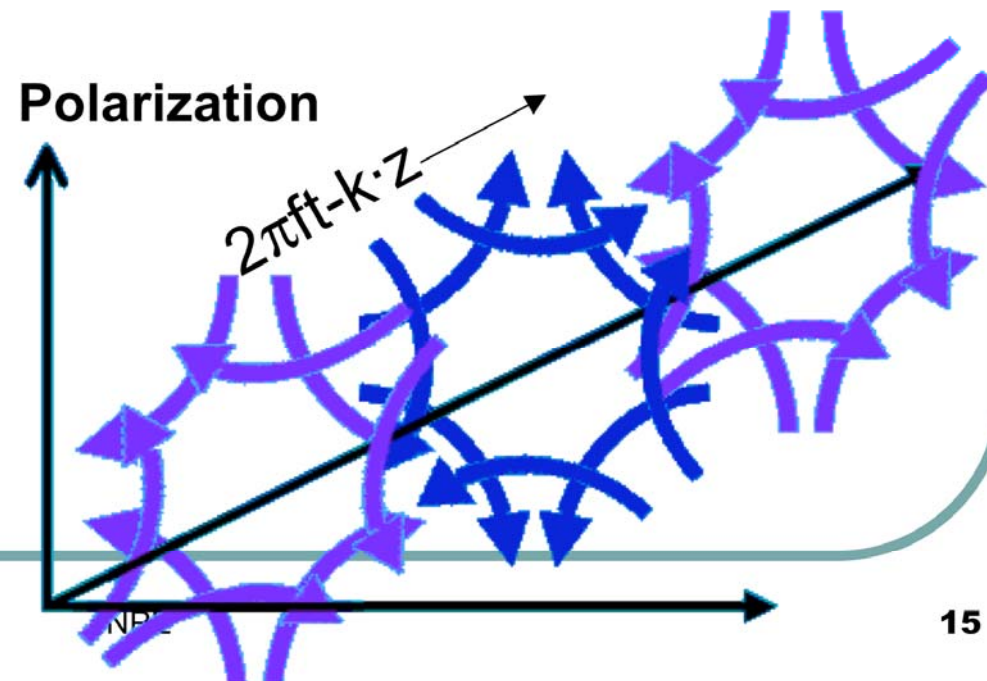
$$\Delta x = -\frac{1}{2} h_+ \cos[\omega(t - z)] x$$

$$\Delta y = \frac{1}{2} h_+ \cos[\omega(t - z)] y$$

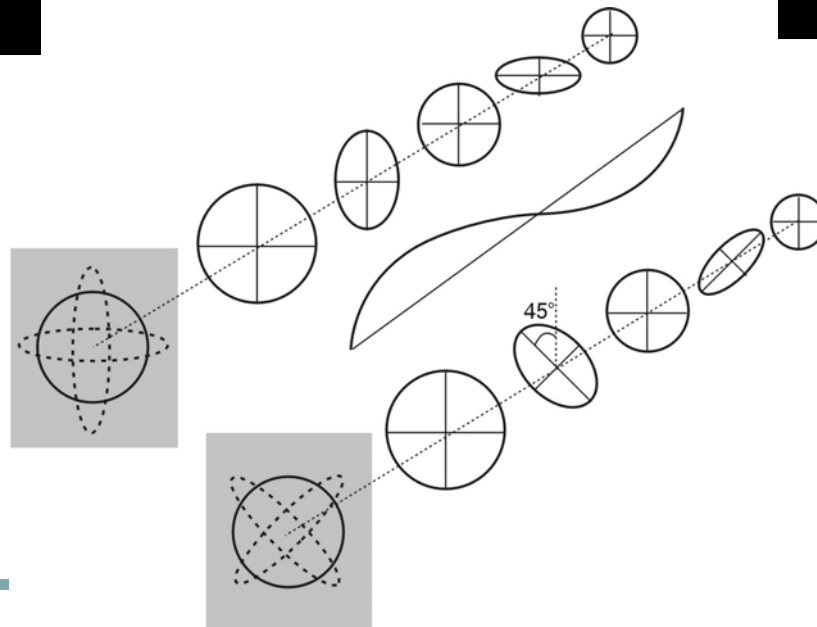
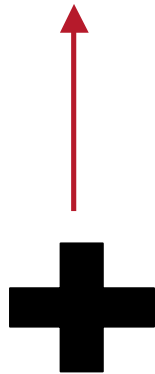
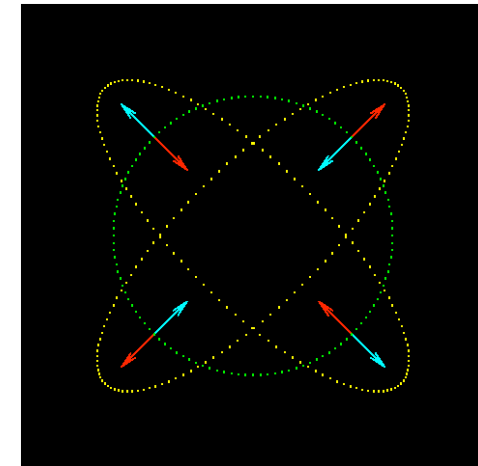
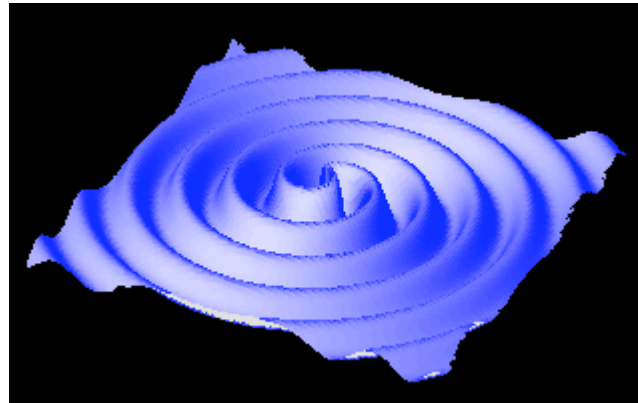
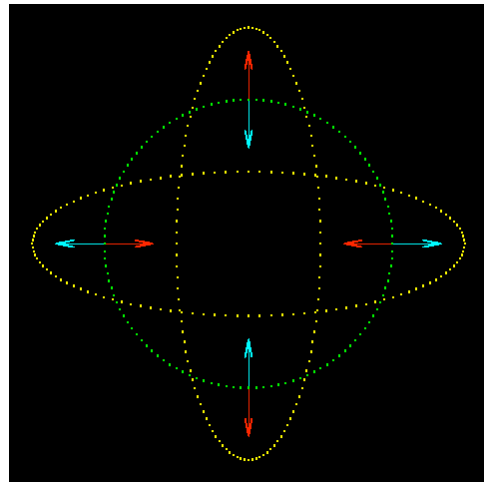


...in other words

$$\frac{\Delta l}{l} = h$$



GW Polarizations



Stress-Energy carried by GWs

GWs exert forces and do work, they must carry energy and momentum

- The energy-momentum tensor in an arbitrary gauge

$$t_{\mu\nu}^{(GW)} = \frac{1}{32\pi} \left\langle \tilde{h}_{\alpha\beta;\mu} \tilde{h}^{\alpha\beta}_{;\nu} - \frac{1}{2} \tilde{h}_{;\mu} \tilde{h}_{;\nu} - \tilde{h}^{\alpha\beta}_{;\beta} \tilde{h}_{\alpha\mu;\nu} - \tilde{h}^{\alpha\beta}_{;\beta} \tilde{h}_{\alpha\nu;\mu} \right\rangle$$

- ...in the TT-gauge:

$$t_{\mu\nu}^{(GW)} = \frac{1}{32\pi} \left\langle \tilde{h}^{jk}_{;\mu}{}^{TT} \cdot \tilde{h}_{jk;\nu}{}^{TT} \right\rangle$$

- ...it is divergence free

$$t_{\mu;\nu}^{\nu(GW)} = 0$$

- For waves propagating in the z-direction

$$t_{00}^{(GW)} = -\frac{1}{c} t_{0z}^{(GW)} = \frac{1}{c^2} t_{zz}^{(GW)} = \frac{1}{16\pi G} \frac{c^2}{c^2} \left\langle \dot{h}_+^2 + \dot{h}_\times^2 \right\rangle$$

- for a SN exploding in Virgo cluster the energy flux on Earth

$$t_{0z}^{(GW)} \approx \frac{\pi c^3}{4 G} f^2 \left\langle h_+^2 + h_\times^2 \right\rangle = 320 \times \left(\frac{f}{1\text{kHz}} \right)^2 \left(\frac{h}{10^{-21}} \right)^2 \frac{\text{ergs}}{\text{cm}^2 \text{sec}}$$

- The corresponding EM energy flux is:

$$\sim 10^{-9} \text{ erg} \cdot \text{cm}^{-2} \cdot \text{sec}^{-1}$$

Wave-Propagation Effects

GWs affected by the large scale structure of the spacetime exactly as the EM waves

- The magnitude of h_{jk}^{TT} falls off as $1/r$
- The polarization, like that of light in vacuum, is parallel transported radially from source to earth
- The time dependence of the waveform is unchanged by propagation except for a frequency-independent **redshift**

We expect

- **Absorption, scattering and dispersion**
- Scattering by the background curvature and tails
- Gravitational focusing
- **Diffraction**
- **Parametric amplification**
- Non-linear coupling of the GWs (frequency doubling)
- Generation of background curvature by the waves

$$\frac{f_{\text{received}}}{f_{\text{emitted}}} = \frac{1}{1+z}$$

The emission of grav. radiation

If the energy-momentum tensor is varying with time, GWs will be emitted

- The retarded solution for the linear field equation

$$\left(-\frac{\partial^2}{\partial t^2} + \nabla^2 \right) \tilde{h}^{\mu\nu} = k T_{(\text{matter})}^{\mu\nu}$$

- For a point in the radiation zone in the slow-motion approximation

$$h^{\mu\nu} = 2 \int \frac{T^{\mu\nu}(t - |\vec{x} - \vec{x}'|, \vec{x}')}{|\vec{x} - \vec{x}'|} d^3x'$$

$$h^{\mu\nu} \approx \frac{2}{r} \int T^{\mu\nu}(t - r, \vec{x}') d^3x' \sim \frac{2}{r} \frac{\partial^2}{\partial t^2} [Q^{jk}(t - r)]^{TT}$$

- Where Q_{kl} is the quadrupole moment tensor

$$Q^{kl} \equiv \int \rho(t, x^k) \left(x^k x^l - \frac{1}{3} r^2 \delta^{kl} \right) d^3x$$

- **Power emitted in GWs**

$$L_{GW} = -\frac{dE}{dt} = \frac{1}{5} \frac{G}{c^5} \sum_{ij} \langle \ddot{Q}_{ij} \cdot \ddot{Q}_{ij} \rangle$$

Angular and Linear momentum emission

- **Angular momentum emission**

$$\frac{dJ_i^{GW}}{dt} = \frac{2}{5} \sum_{jkl} \epsilon_{ijk} \langle \ddot{Q}_{jl} \cdot \ddot{Q}_{lk} \rangle$$

- **Linear momentum emission**

$$\frac{dP_i^{GW}}{dt} = \frac{2}{63} \sum_{jk} \langle \ddot{Q}_{jk} \cdot \ddot{Q}_{jki} \rangle + \frac{16}{45} \sum_{jkl} \epsilon_{ijk} \langle \ddot{Q}_{jl} \cdot \ddot{P}_{lk} \rangle$$

Q_{ijk} : mass octupole moment

P_{ij} : current quadrupole moment

Back of the envelope calculations!

- **Characteristic time-scale** for a mass element to move from one side of the system to another is:

$$T \sim \frac{R}{v} \sim \frac{R}{(M/R)^{1/2}} = \left(\frac{R^3}{M} \right)^{1/2}$$

- The **quadrupole moment** is approximately:

$$\ddot{Q}_{ij} \sim \frac{MR^2}{T^3} \sim \frac{Mv^2}{T} \sim \frac{E_{ns}}{T} \sim \left(\frac{M}{R} \right)^{5/2}$$

- **Luminosity**

$$L_{GW} \sim \frac{G}{c^5} \left(\frac{M}{R} \right)^5 \sim \frac{G}{c^5} \left(\frac{M}{R} \right)^2 v^6 \sim \frac{c^5}{G} \left(\frac{R_{Sch}}{R} \right)^2 \left(\frac{v}{c} \right)^6$$

$$\frac{c^5}{G} = 3.63 \times 10^{59} \text{ erg} / \text{s} = 2.03 \times 10^5 M_{\odot} c^2 / \text{s}$$

- The **amplitude** of GWs at a distance r ($R \sim R_{Schw} \sim 10 \text{ Km}$ and $r \sim 10 \text{ Mpc} \sim 3 \times 10^{19} \text{ km}$):

$$h \sim \frac{\ddot{Q}}{r} \sim \frac{1}{r} \left(\frac{MR^2}{T^2} \right) \sim \left(\frac{M}{r} \right) \left(\frac{M}{R} \right) \sim 10^{-19}$$

- **Radiation damping**

$$\tau_{react} = \frac{E_{kin}}{L_{GW}} \sim \left(\frac{R}{M} \right)^{5/2} T \sim \left(\frac{v}{c} \right) \left(\frac{R}{R_{Sch}} \right)^3 T$$

What we should remember...

- **Length variation**

$$\frac{\Delta l}{l} = h$$

- **Amplitude**

$$h^{jk} \approx \frac{2}{r} \ddot{Q}^{jk}$$

- **Power emitted**

$$L_{GW} = -\frac{dE}{dt} = \frac{1}{5} \frac{G}{c^5} \sum_{ij} \langle \ddot{Q}_{ij} \ddot{Q}_{ij} \rangle$$

Vibrating Quadrupole

- The position of the two masses
- The quadrupole moment of the system is



- The radiated gravitational field is

- The emitted power

- And the damping rate of the oscillator is

$$x = \pm[x_0 + \xi \sin(\omega t)] \quad , \quad x_0 \ll \xi$$

$$Q^{kl}(t-r) \approx \left[1 + \frac{2\xi}{x_0} \sin \omega(t-r) \right] Q_0^{kl}$$

$$Q_0^{kl} = \begin{pmatrix} -2mx_0^2 & 0 & 0 \\ 0 & -2mx_0^2 & 0 \\ 0 & 0 & 4mx_0^2 \end{pmatrix}$$

$$h^{kl} = \frac{2}{3} \left(\frac{\xi}{x_0} \right) \frac{\omega^2}{r} \sin[\omega(t-r)] Q_0^{kl}$$

$$-\frac{dE}{dt} = \frac{G}{45c^5} \langle \ddot{Q}_{kl} \ddot{Q}_{kl} \rangle = \frac{16}{15} \frac{G}{c^5} (mx_0 \xi)^2 \omega^6$$

$$\gamma_{rad} = -\frac{1}{E} \left\langle \frac{dE}{dt} \right\rangle = \frac{16}{15} \frac{G}{c^5} mx_0^2 \omega^4$$

Two-body collision

- The radiated power
- The energy radiated during the plunge from $z=\infty$ to $z=-R$
- If $R=R_{Sch}$ ($M=10M_{\odot}$ & $m=1M_{\odot}$)

$$-\frac{dE}{dt} = \frac{8}{15} \frac{G}{c^5} m^2 (3\dot{z}\ddot{z} + z\ddot{\dot{z}})^2$$

$$-\Delta E = \frac{4}{105} \frac{1}{R^{7/2}} \frac{G}{c^5} m^2 (2GM)^{5/2}$$

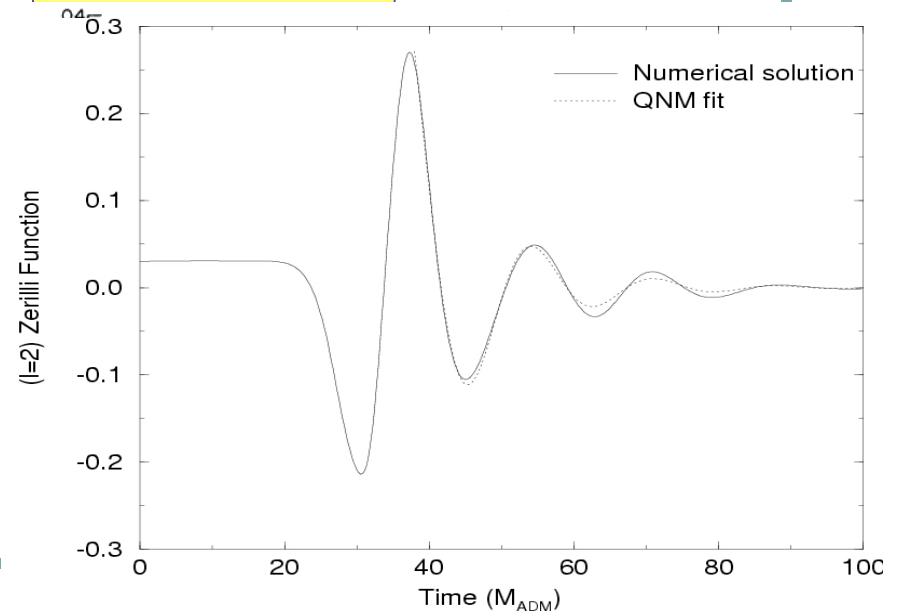
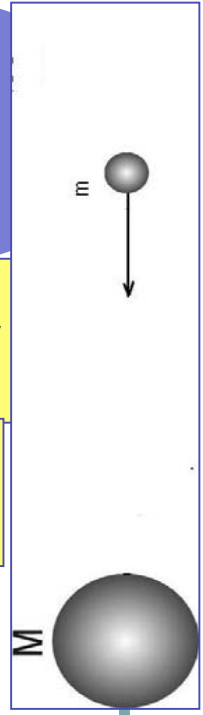
$$\frac{1}{2} m \dot{z}^2 = \frac{GmM}{|z|}$$

$$-\Delta E = 0.0190 mc^2 \frac{m}{M}$$

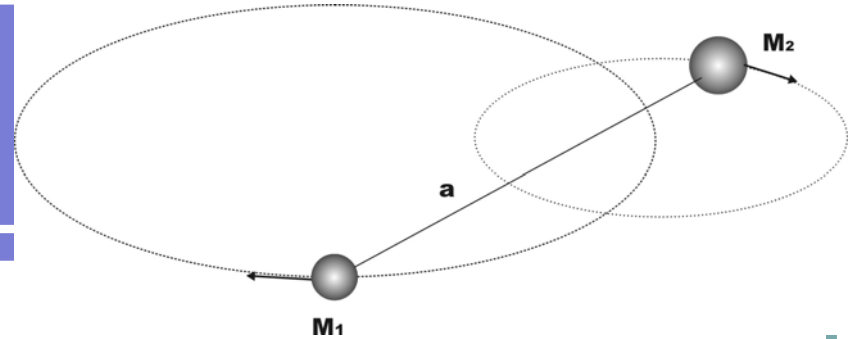
$$-\Delta E_{true} = 0.0104 mc^2 \frac{m}{M} \rightarrow 2 \times 10^{51} \text{ erg}$$

$$\Delta t \approx R/v \approx R/c \approx 30 \text{ km} / c \approx 10^{-4} \text{ s}$$

$$f \approx 10^4 \text{ Hz}$$



Rotating Quadrupole (a binary system)



THE BEST SOURCE FOR GWs

- Radiated power
- Energy loss leads to **shrinking of their orbital separation**
- **Period changes** with rate
- ...and the system **will coalesce** after
- The **total energy loss** is
- Typical **amplitude** of GWs

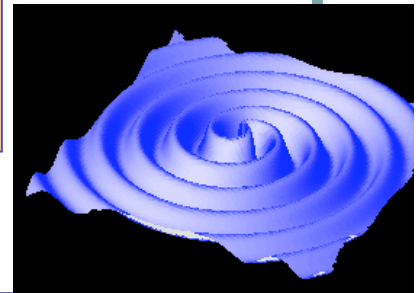
$$-\frac{dE}{dt} = \frac{32 G}{5 c^5} \mu^2 a^4 \omega^6 = \frac{32 G M^3 \mu^2}{5 c^5 a^5}$$

$$\frac{da}{dt} = -\frac{64 G^3 \mu M^2}{5 c^5 a^3}$$

$$\frac{\dot{P}}{P} = -\frac{96 G^3 \mu M^2}{5 c^5 a^4}$$

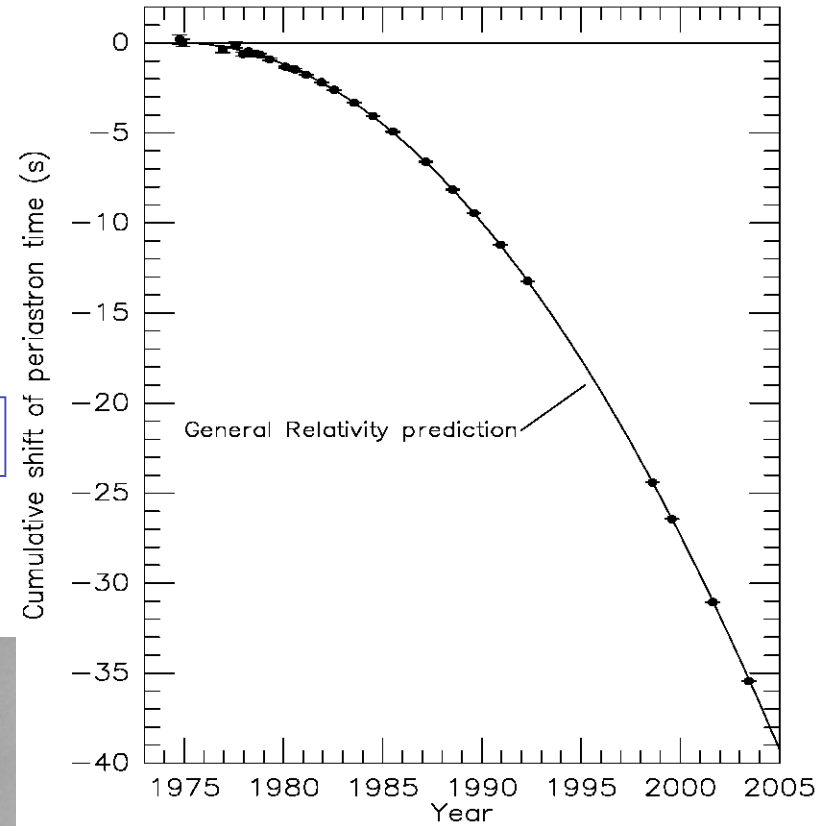
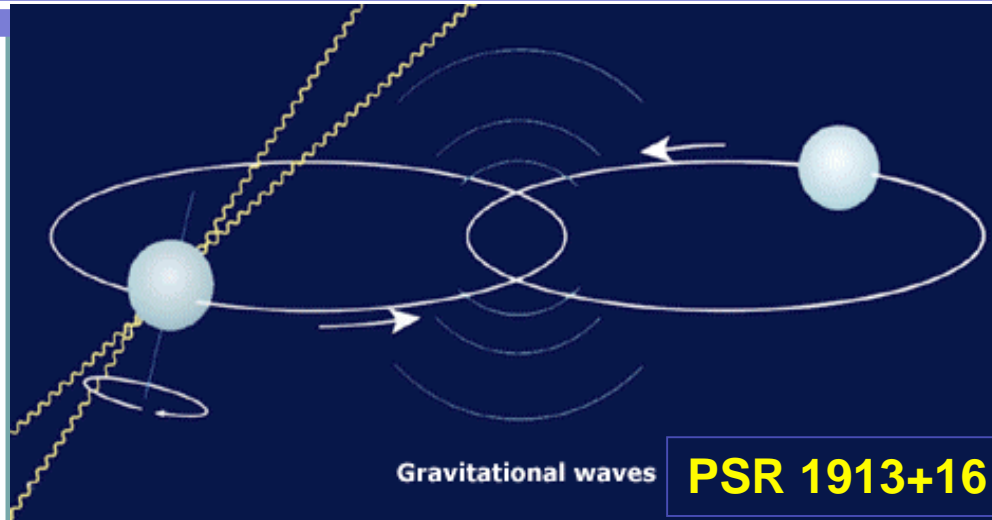
$$T_{\text{inspiral}} = \frac{5 c^5 a_0^4}{256 G^3 \mu M^2}$$

$$\Delta E_{\text{rad}} = \frac{G}{2} \mu M \left(\frac{1}{a_0} - \frac{1}{a} \right)$$



$$h \approx 5 \times 10^{-22} \left(\frac{M}{2.8 M_{\odot}} \right)^{2/3} \left(\frac{\mu}{0.7 M_{\odot}} \right) \left(\frac{f}{100 \text{ Hz}} \right)^{2/3} \left(\frac{15 \text{ Mpc}}{r} \right)$$

First verification of GWs



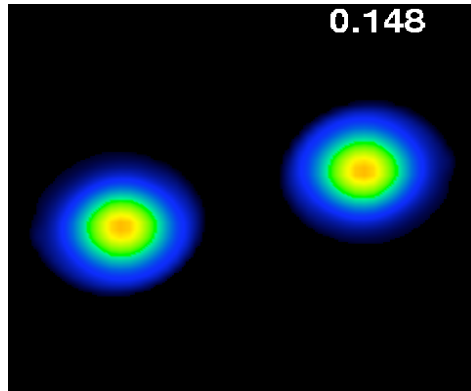
Nobel 1993

Hulse & Taylor

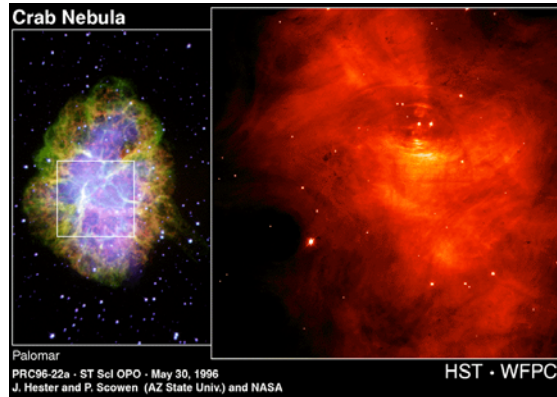


$$\frac{\dot{P}_{b,corrected}}{\dot{P}_{b,GR}} = 1.0013 \pm 0.0021$$

GW sources for Interferometric and Acoustic Detectors



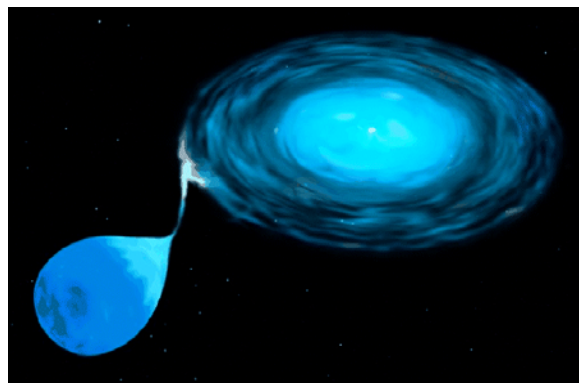
BH and NS Binaries



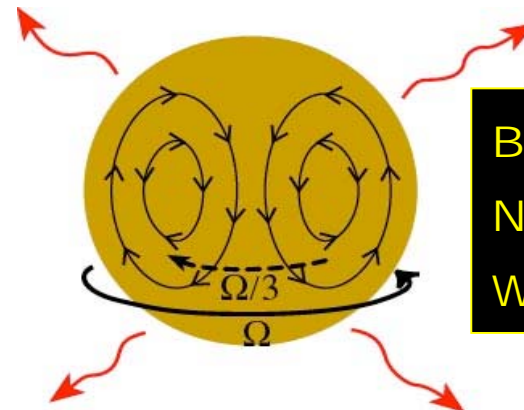
Supernovae, BH/NS formation

$$L_{GW} \sim \left(\frac{M}{R} \right)^5$$

$$h \sim \epsilon \cdot \left(\frac{M}{r} \right) \cdot \left(\frac{M}{R} \right)$$



Spinning neutron stars in X-ray binaries



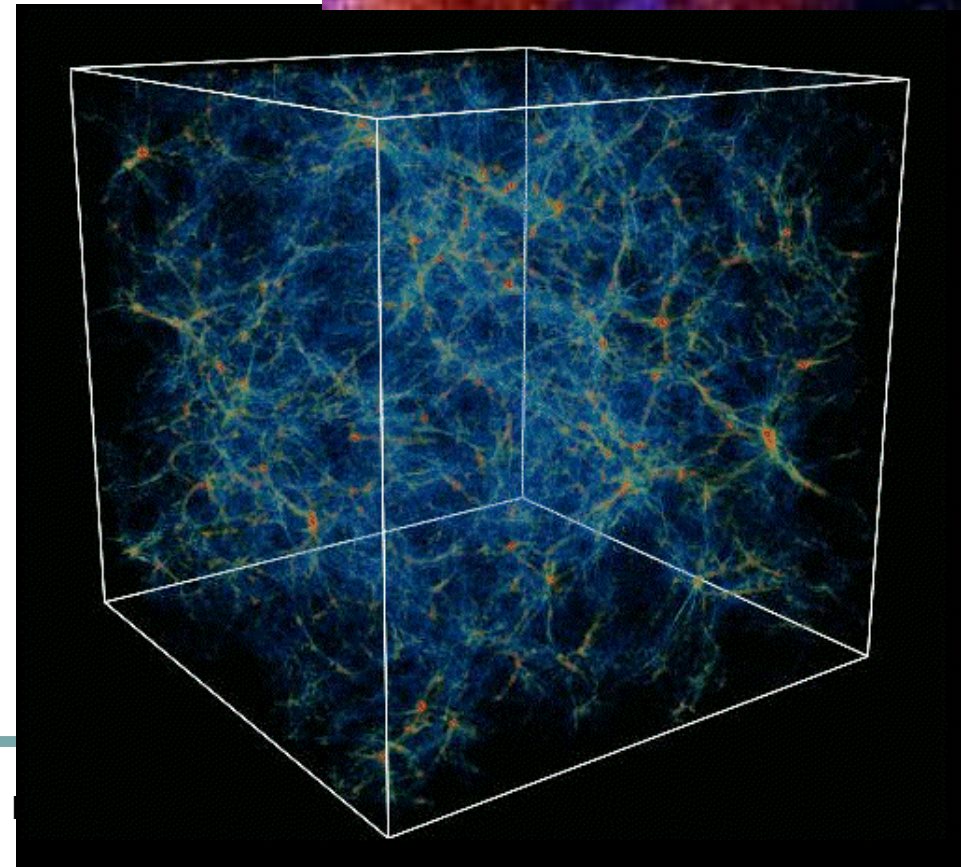
Young Neutron Stars

- Black Holes : $M/R=0.5$
- Neutron Stars : $M/R \sim 0.2$
- White Dwarfs : $M/R \sim 10^{-4}$

+ stochastic sources

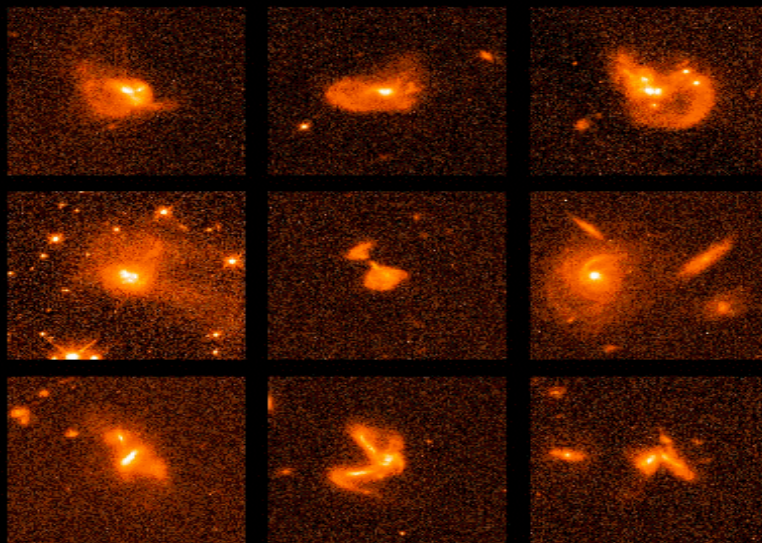
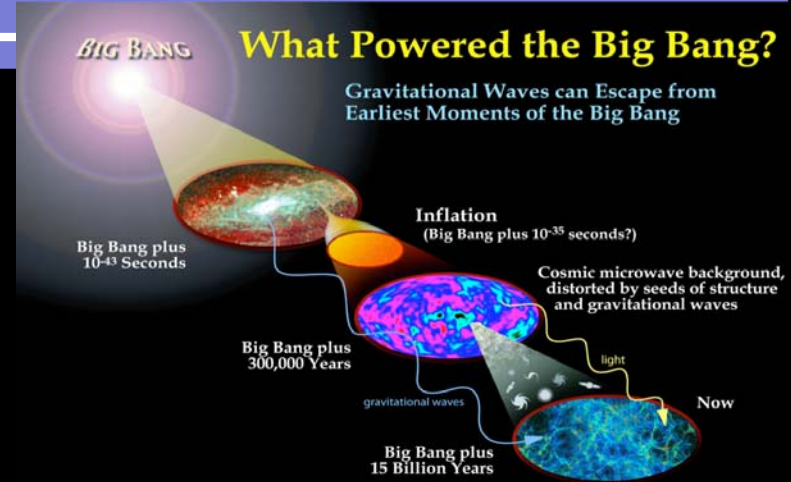
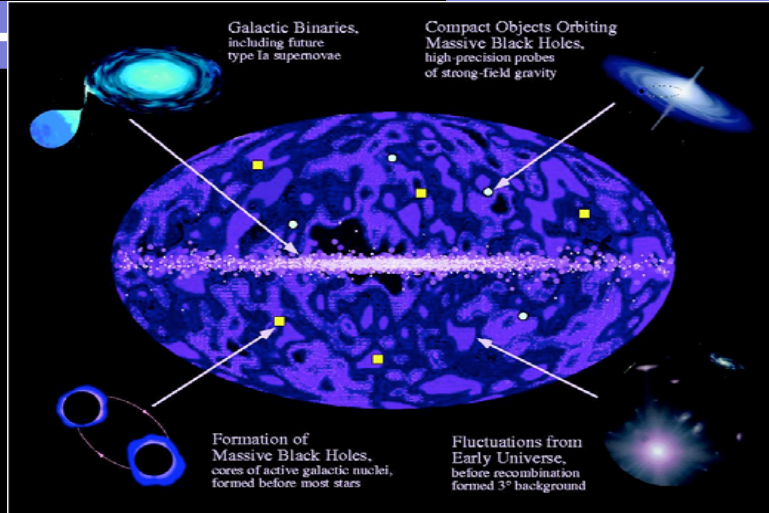
(contribute to a noisy background)

- ✓ Big Bang,
- ✓ early expansion of the Universe,
- ✓ cosmic strings,
- ✓ unresolved sources...

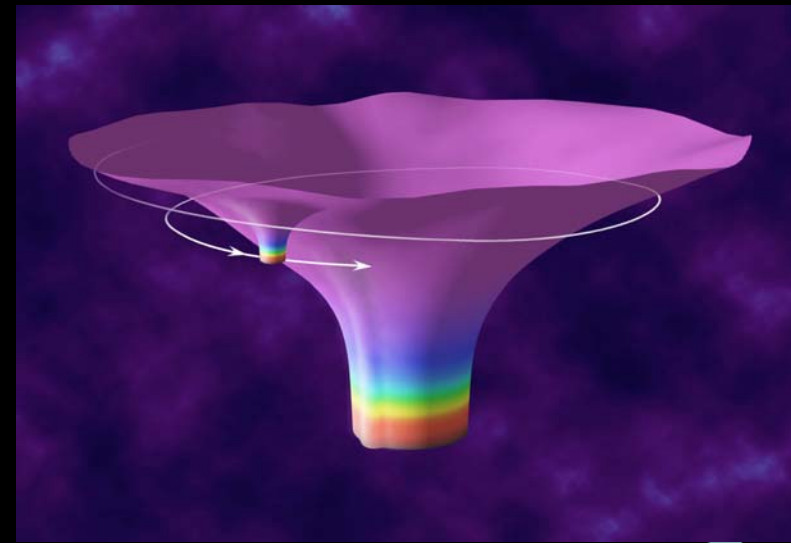


Sources in LISA

Galactic Binaries



Galaxy mergers



Capture orbits

A Quadrupole Detector



Tidal force is the driving force of the oscillator

- Plane wave
- Displacement & Tidal force
- Equation of motion
- Solution

$$h^{\mu\nu} = h_+ \varepsilon^{\mu\nu} e^{i(\omega t - kz)}$$

$$\xi_x \approx L h_+ e^{i\omega t} \quad f_x \approx m L h_+ \omega^2 e^{i\omega t}$$

$$\ddot{\xi} + \dot{\xi} / \tau + \omega_0^2 \xi = -\frac{1}{2} \omega^2 L h_+ e^{i\omega t}$$

$$\xi = \frac{\omega^2 L h_+ e^{i\omega t} / 2}{\omega_0^2 - \omega^2 + i\omega / \tau}$$

$$\xi_{\max} = \omega_0 \tau \cdot L \cdot h_+ / 2 = Q \cdot L \cdot h / 2$$

- **Cross section**

$$\sigma = \frac{32\pi}{15} \frac{G}{c^3} \omega_0 \cdot Q \cdot M \cdot L^2$$

- **Weber's detector:**
 $M=1410\text{kg}$, $L=1.5\text{m}$,
 $d=66\text{cm}$, $\omega_0=1660\text{Hz}$,
 $Q=2 \times 10^5$.

$$\sigma_{\text{Weber}} \approx 3 \times 10^{-19} \text{ cm}^2$$

Quadrupole Detector Limitations

Problems

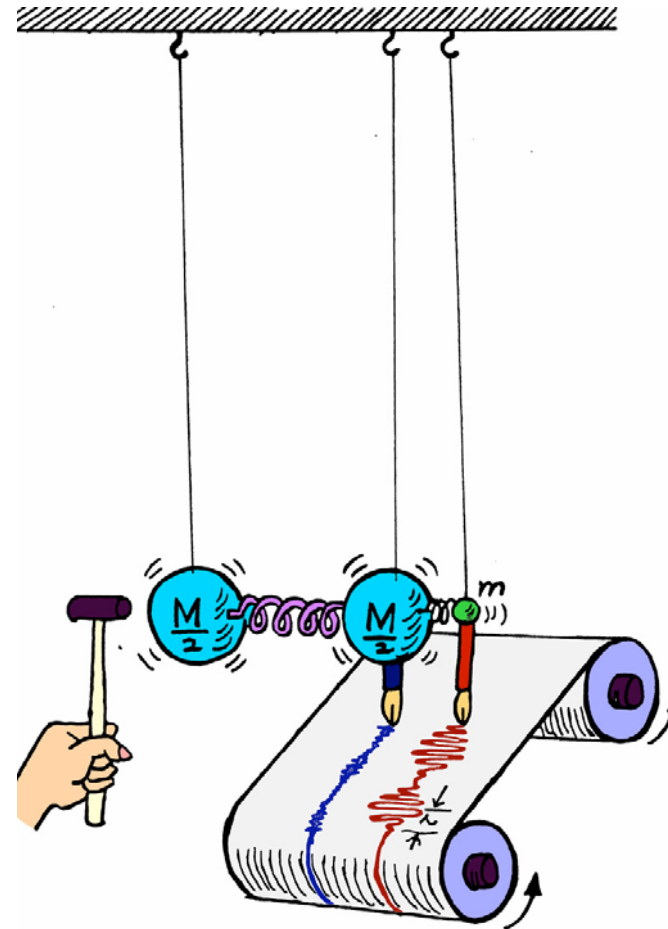
- Very small cross section
 $\sim 3 \times 10^{-19} \text{ cm}^2$.
- Sensitive to periodic GWs **tuned** in the right frequency of the detector
- Sensitive to bursts **only** if the pulse has a substantial component at the resonant frequency
- The **width of the resonance** is:
- Thermal noise limits our ability to detect the energy of GWs.
- The excitation energy has to be greater than the thermal fluctuations $E \gtrsim kT$

$$\Delta\nu \sim \gamma / 2\pi \sim 10^{-2} \text{ Hz}$$

$$h_{\min} \geq \frac{1}{\omega_0 L Q} \sqrt{\frac{15kT}{M}} \sim 10^{-20}$$

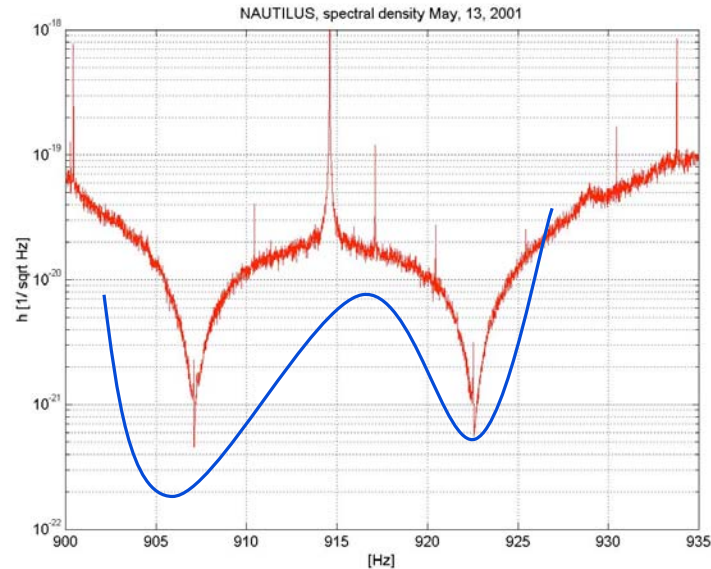
Improvements to resonant detectors

- Have higher Q
- Operate in extremely low temperatures (mK)
- Larger masses
- Different geometry
- Better electronic sensors



Modern Bar Detectors

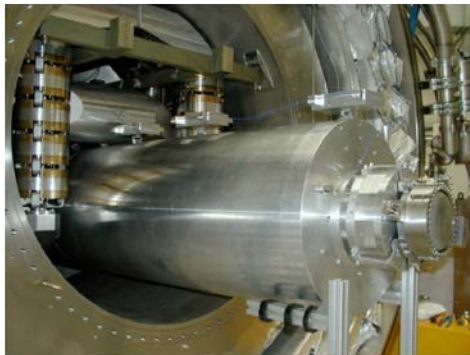
	WEBER	NAUTILUS
mass(kg)	1410	2270
Length(m)	1.53	2.97
ω_0 (Hz)	1660	910
$Q = \omega/\gamma$	2×10^5	2.3×10^6
$\sigma (\omega_0)_{\text{abs}}$ (cm ²)	2×10^{-19}	70×10^{-19}
Typical pulse sensitivity h	10^{-16}	9×10^{-19}



International Gravitational Events Collaboration

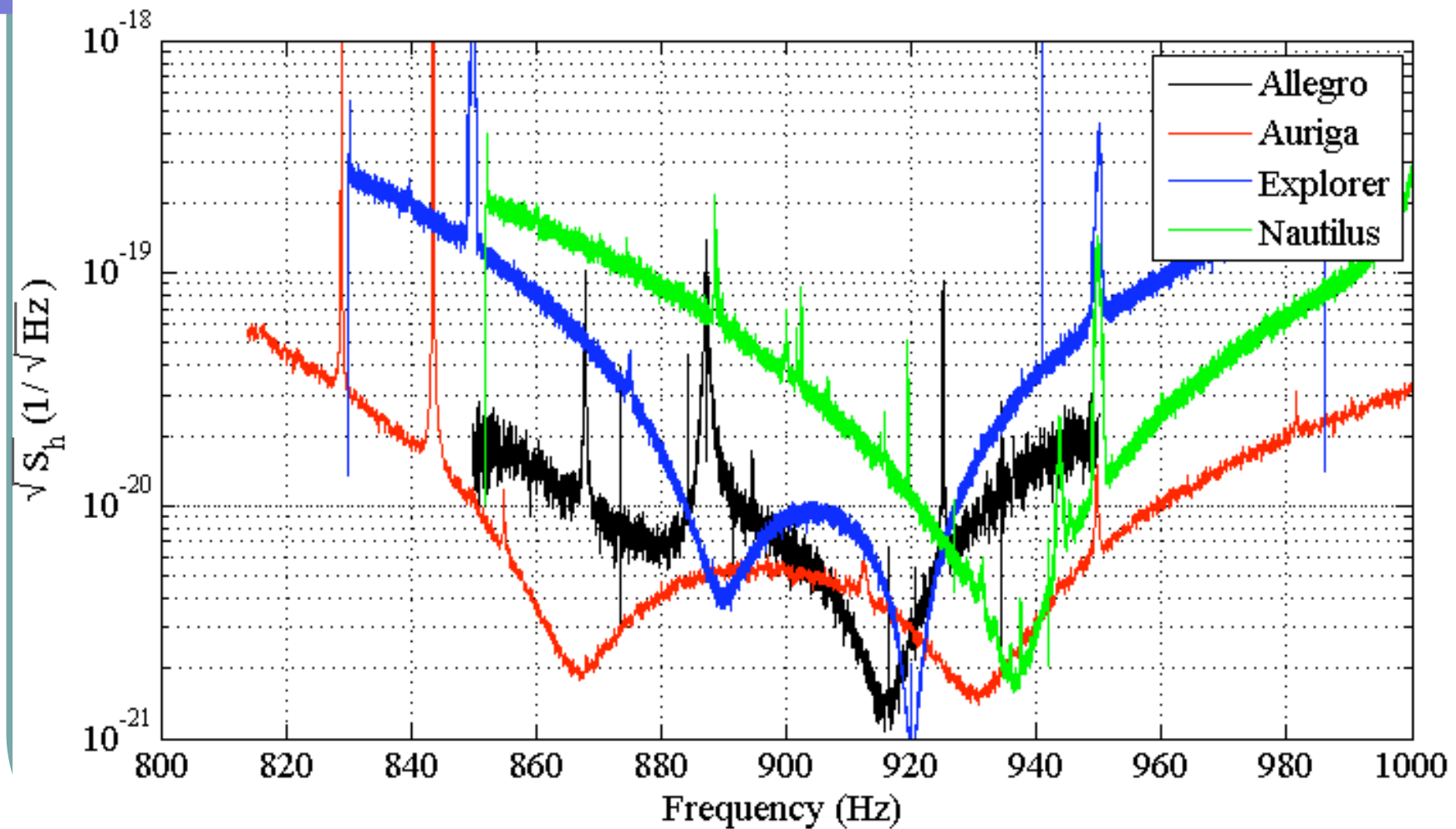
ALLEGRO- AURIGA – ROG (EXPLORER-NAUTILUS)

- The “oldest” resonant detector EXPLORER started operations about 16 years ago.
- This kind of detector has reached a high level of reliability.
- The duty factor is greater than 90% .



New Advanced Acoustic Detectors are in RD phase

Sensitivity of Resonant Detectors



Exploiting the resonant-mass detector technique: the spherical detector

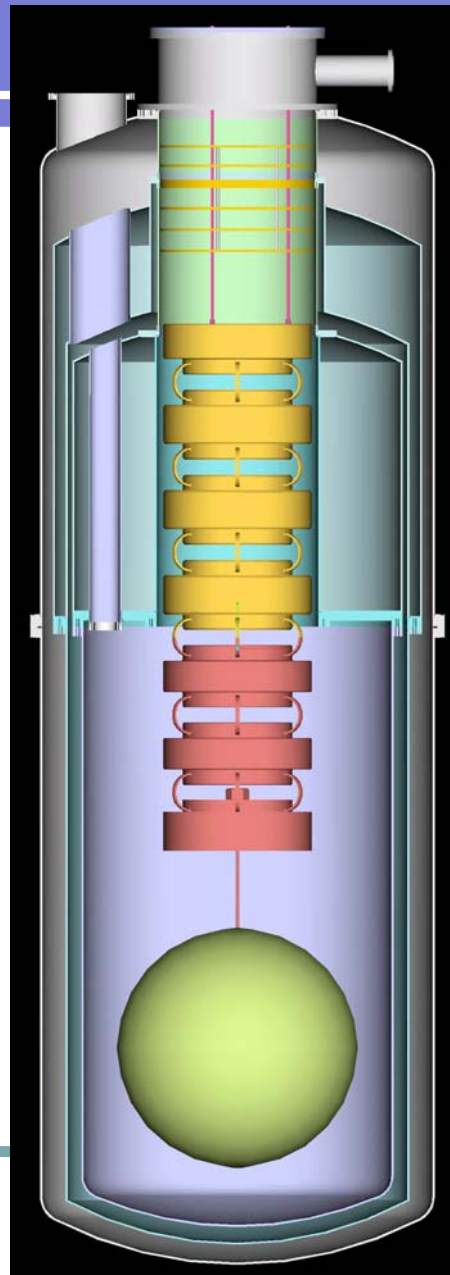
$M = 1-100$ tons

Sensitivity:

$10^{-23} - 10^{-24} \text{ Hz}^{-1/2};$

$h \sim 10^{-21} - 10^{-22}$

- **Omnidirectional**
- Capable of **detecting source position**
- Capable of **measuring polarization**



MINIGRAIL
Leiden (Netherlands)

MARIO SHENBERG
Sao Paulo (Brasil)

SFERA
Frascati (Italy)

CuAl(6%) sphere
Diameter = 65 cm
Frequency = 3 kHz
Mass = 1 ton

Exploiting the resonant-mass detector technique: the spherical detector



Laser Interferometers

- The output of the detector is

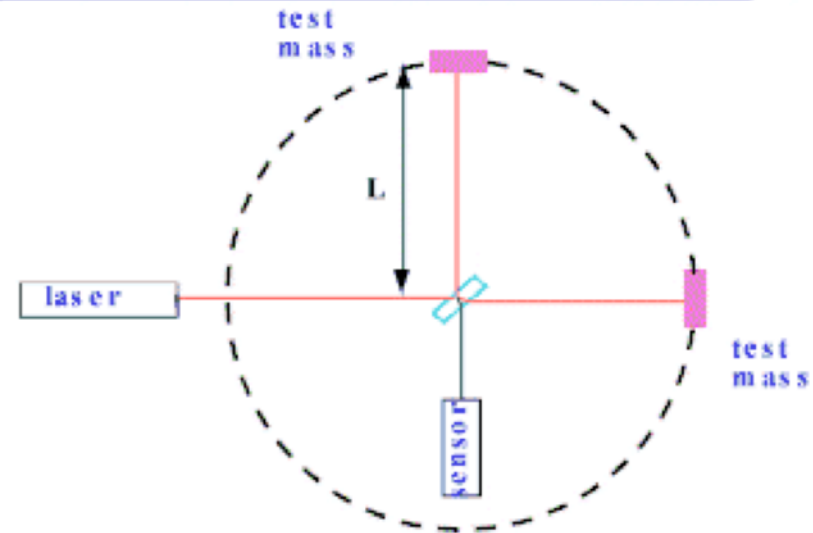
$$\frac{\Delta L}{L} = F_+ h_+(t) + F_\times h_\times(t) = h(t)$$

- Technology allows measurements $\Delta L \sim 10^{-16} \text{ cm}$.
- For signals with $h \sim 10^{-21} - 10^{-22}$ we need arm lengths $L \sim 1 - 10 \text{ km}$.
- Change in the arm length by ΔL corresponds to a phase change

$$\Delta\phi = \frac{4\pi b \Delta L}{\lambda} \sim 10^{-9} \text{ rad}$$

- The number of photons reaching the photo-detector is proportional to laser-beam's intensity $[\sim \sin^2(\Delta\phi/2)]$

$$N_{\text{out}} = N_{\text{input}} \sin^2(\Delta\phi/2)$$



OPTIMAL CONFIGURATION

- ✓ Long arm length L
- ✓ Large number of reflections b
- ✓ Large number of photons (but be aware of radiation pressure)
- ✓ Operate at interface minimum $\cos(2\pi b \Delta L / \lambda) = 1$.

GW detector network



LIGO Hanford WA
(4km & 2km)

LIGO Livingston LA
(4km)



VIRGO (3km)



TAMA (300m)

AIGO

GEO (600m)



Noise Sources I

- **Photon Shot Noise**

- The number of emerging photons is subject to statistical fluctuations
- Implies an uncertainty in the measurement of ΔL

$$\delta N_{out} \sim \sqrt{N_{out}}$$

- **Radiation Pressure Noise**

- Lasers produce radiation pressure on the mirrors
- Uncertainty in the measurement of the deposited momentum leads to an uncertainty in the position of the mirrors

$$h_{min} = \frac{\delta(\Delta L)}{L} = \frac{\Delta L}{L} \sim \frac{1}{bL} \sqrt{\frac{\hbar c \lambda}{2\pi\tau I_0}}$$

$$h_{min} = \frac{\tau}{m} \frac{b}{L} \sqrt{\frac{\tau \hbar I_0}{c \lambda}}$$

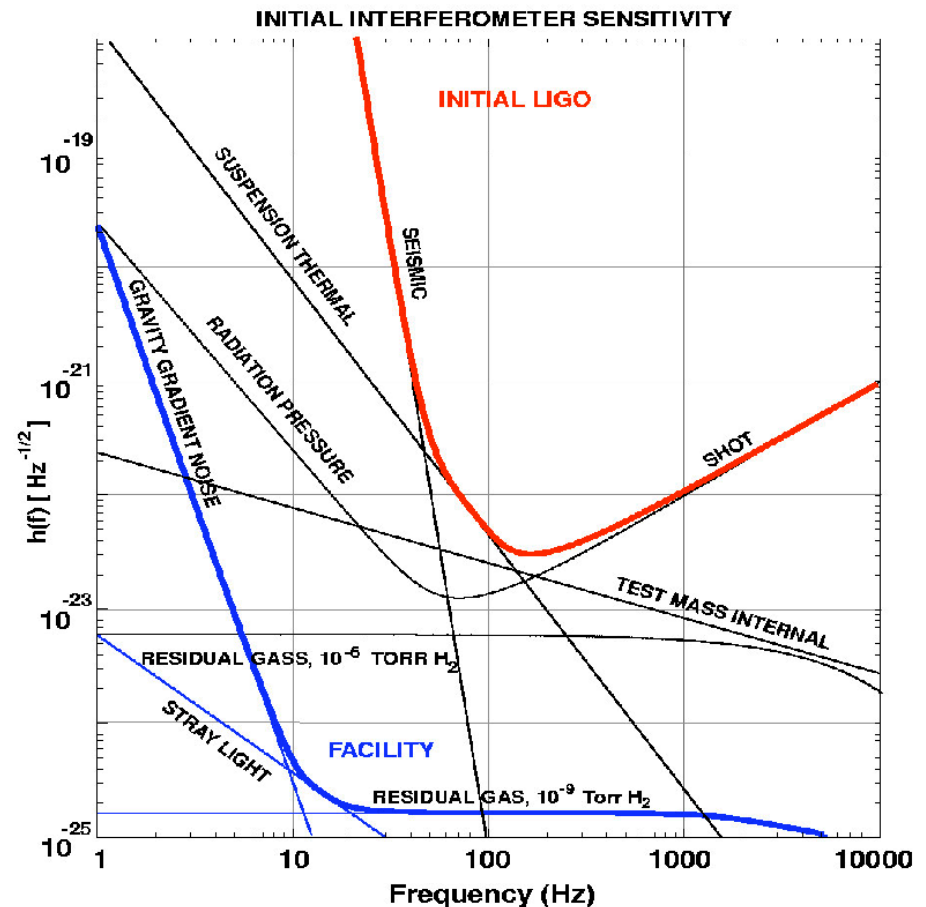
- **Quantum Limit**

- If we try to minimize PSN and RPN with respect to laser power we get a **minimum detectable strain**
- Heisenberg's principle sets an additional uncertainty in the measurement of ΔL
- ($\Delta L \Delta p \gtrsim \hbar$, if $\Delta p \sim m \Delta L / \tau \dots$) and the **minimum detectable strain** is

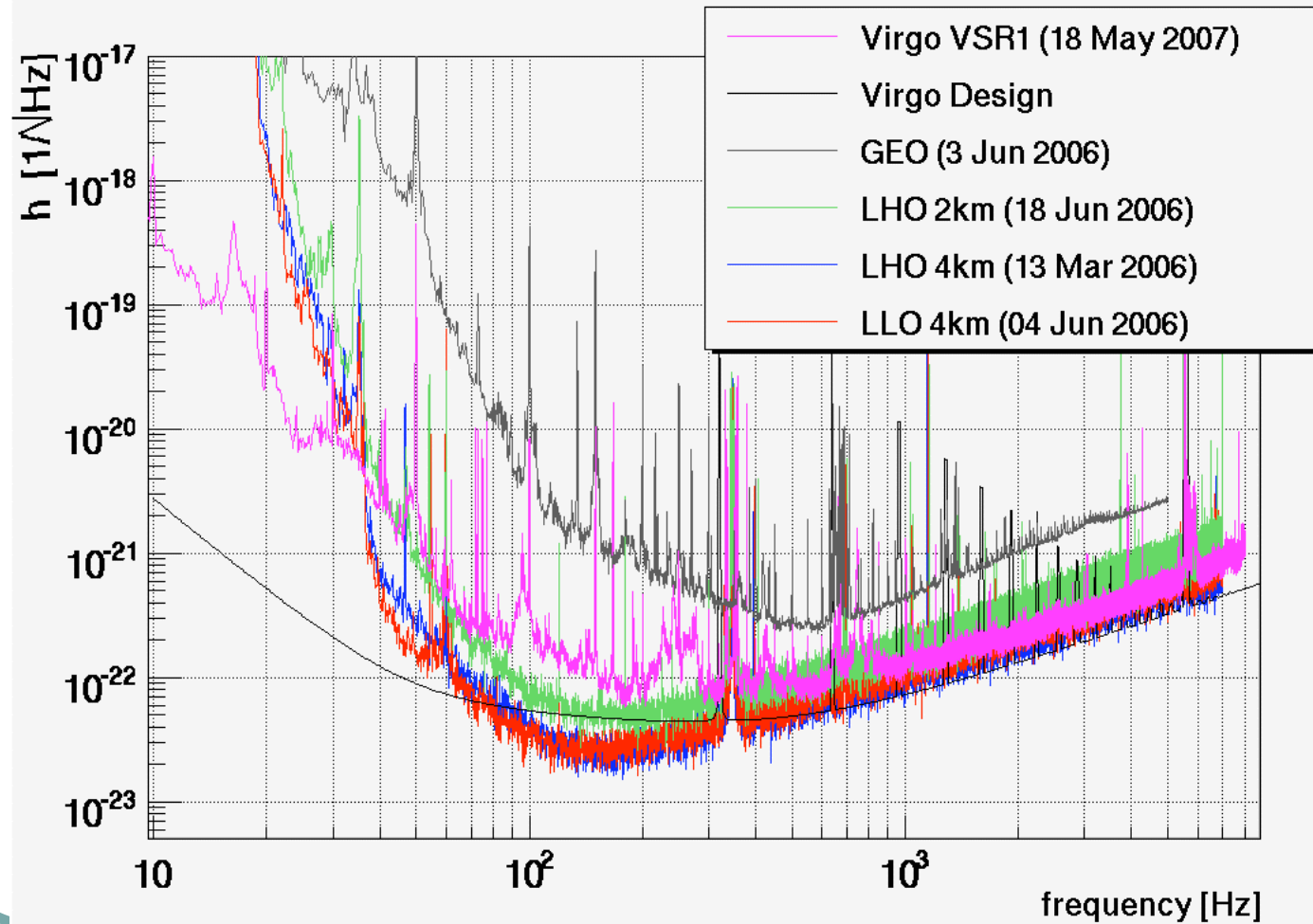
$$h_{min} = \frac{1}{L} \sqrt{\frac{\tau \hbar}{m}}$$

LIGO/Virgo Sensitivity

- **Seismic Noise**
 - Important **below 60Hz**.
Dominates over all other types of noise.
- **Residual gas-phase noise**
 - Statistical fluctuations in the residual gas density induce fluctuations of the refraction index and as a result on the monitored phase shift.
 - Vacuum pipes (**$\sim 10^{-9}$ torr**)



GW network sensitivity



Interferometers and Sources

×100 in 10-20 years is possible

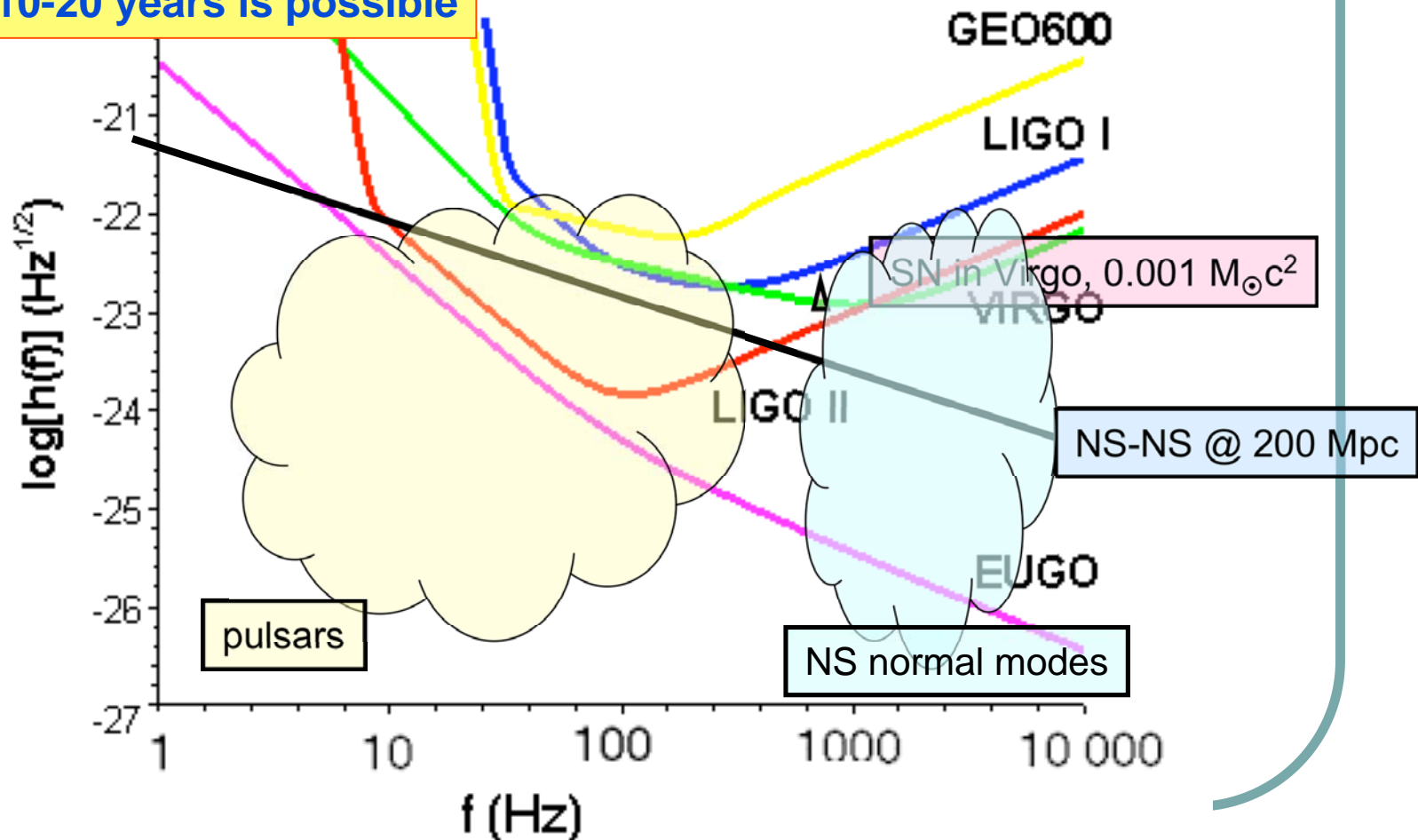
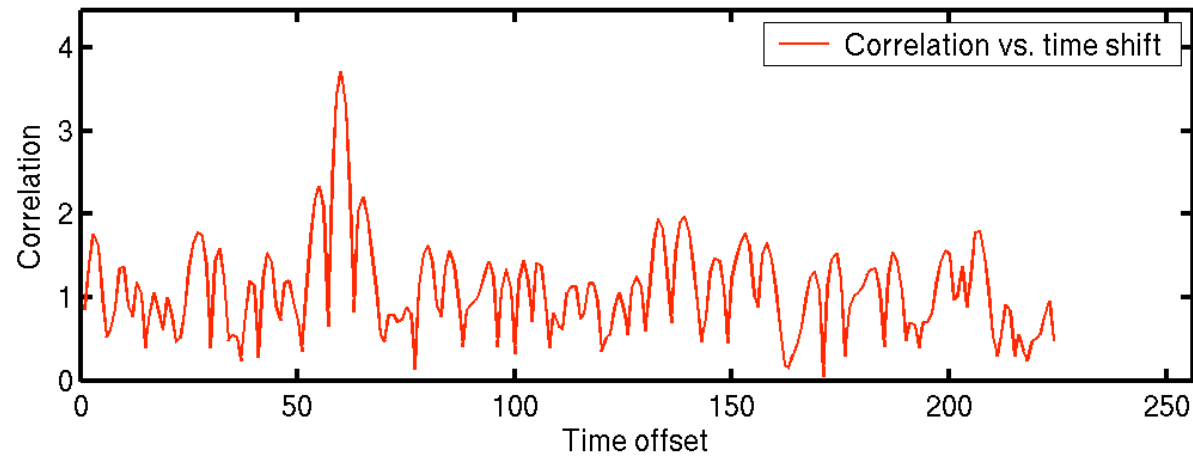
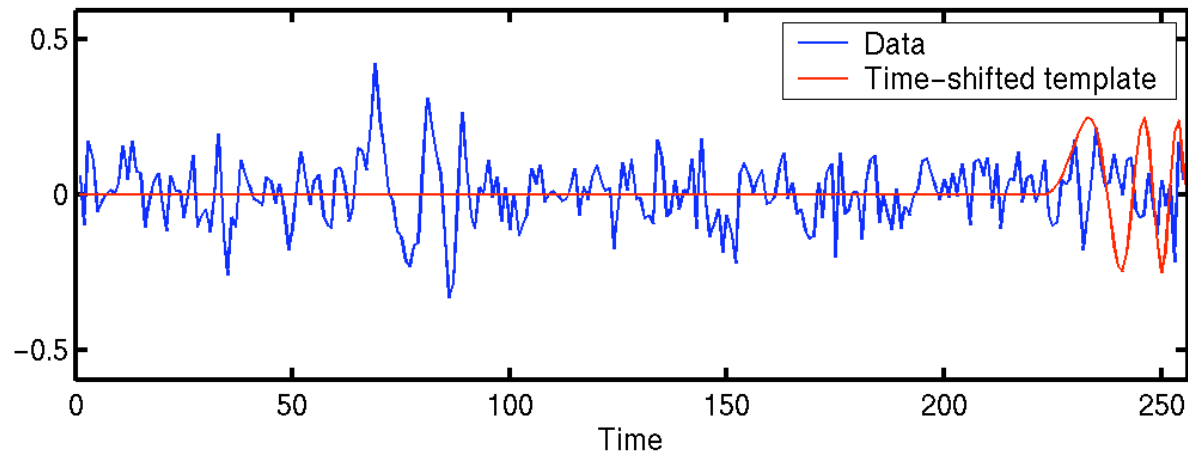


Illustration of Matched Filtering



Moral from previous slide

- ✓ The signals are very weak buried into the noise of the detectors
- ✓ We will be able to extract them ONLY if we have a quite good knowledge of the incoming signal.
- ✓ Even small errors will not allow detection

Status of Detectors

- Resonant bar detectors are operating in a stable mode but at low sensitivity compared with...
- **LIGO is currently carrying out a science run at design sensitivity.**
- Searches for all major categories of sources are underway and will at least set upper limits.
- **Detections are possible!**
- Enhancements in **~3 years** will increase the reach by a factor of **3**
- **An upgrade (Advanced LIGO/Virgo) is planned early next decade**
- Detections are 'guaranteed'

Emphasis on high frequency window

GWs produced from highly relativistic objects (high M/R or M/R^3) which corresponds to frequencies **700Hz -4kHz**

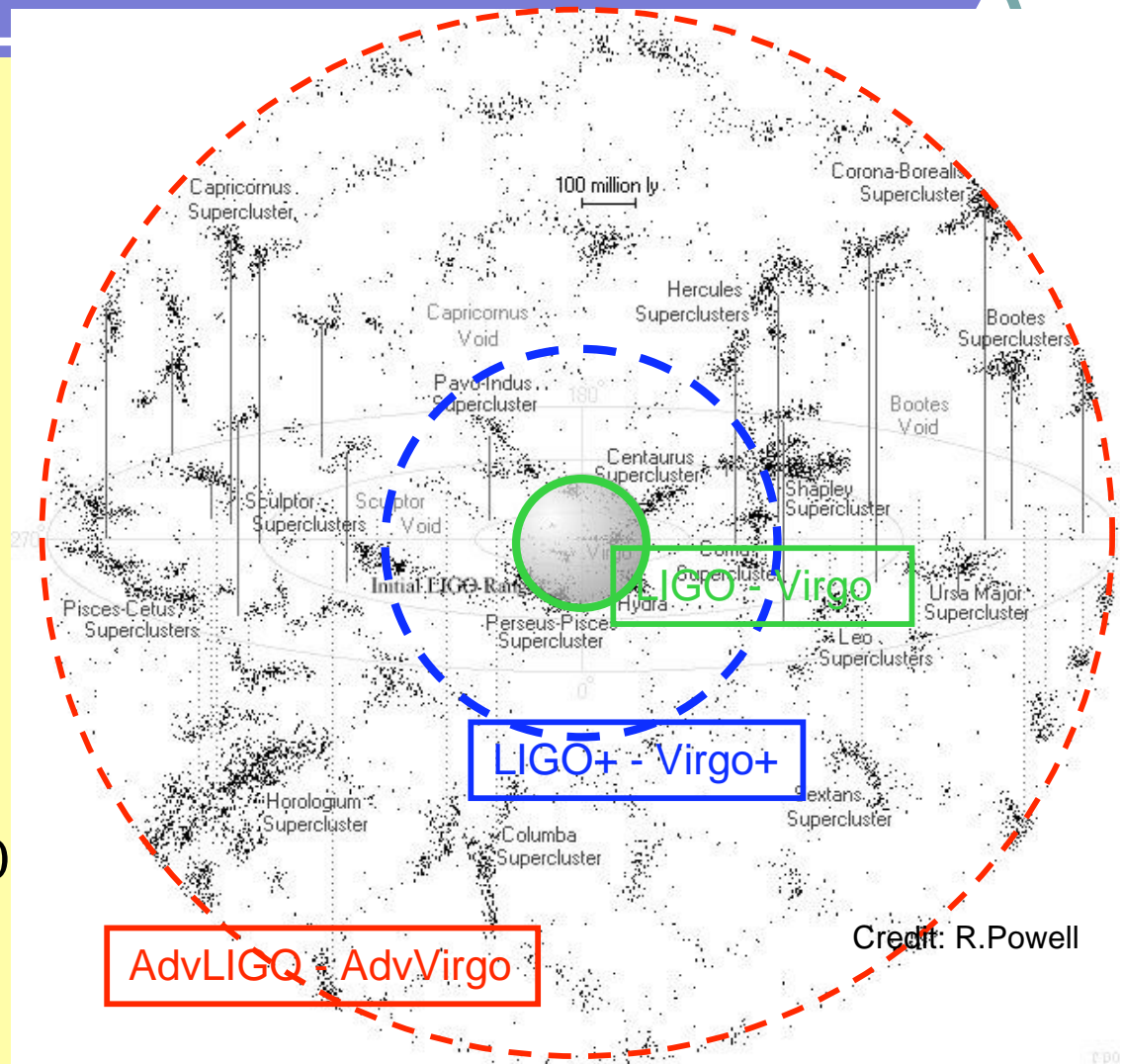
Science data runs to date

- Since Autumn 2001 GEO and LIGO have completed **5 science** runs
 - Analysis completed for S1/2 and (most) papers published;
 - For S3/4 analysis – 2 papers published and many more in preparation
 - Some runs done in coincidence with TAMA and bars (Allegro)
 - [LIGO now at design sensitivity](#)
- ‘Upper Limits’ have been set for a range of signals
 - Coalescing binaries
 - Pulsars
 - Bursts (including GRBs)
 - Stochastic background
- **>20 major papers published or in press since 2004** (work from a collaboration (LSC) of more than 400 scientists)

**S5: started on 4th Nov. 2005 at Hanford (LLO a few weeks later) -
GEO joined initially for overnight data taking, then 24/7
18 months data taking in coincidence**

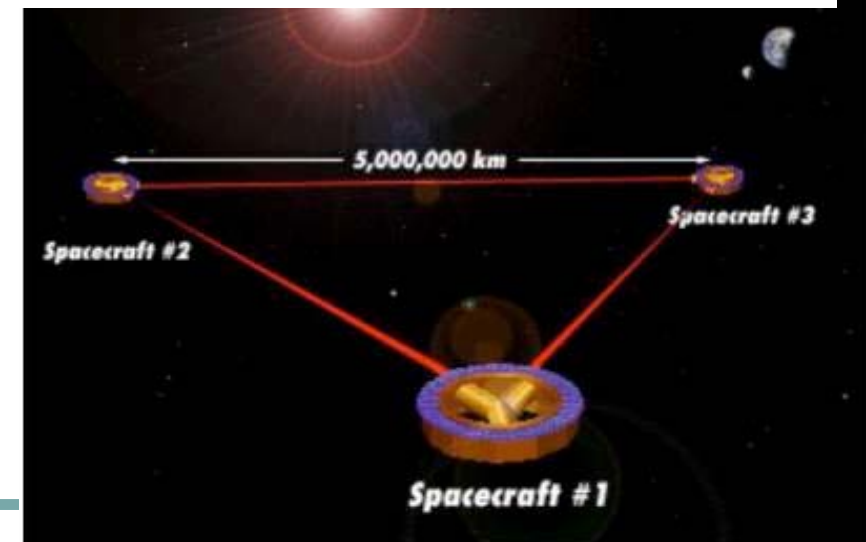
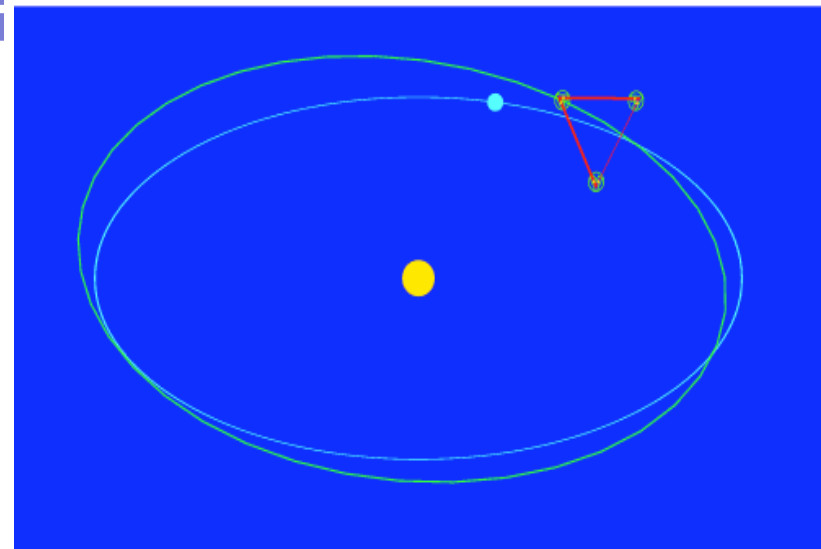
Towards GW astronomy

- ✓ Present detectors will test upper limits
- ✓ Even in the optimistic case rate too low to start GW astronomy
- ✓ Need to improve the sensitivity
- ✓ Increase the sensitivity by 10 increase the probed volume by 1000
- ✓ Plans to improve the present detectors

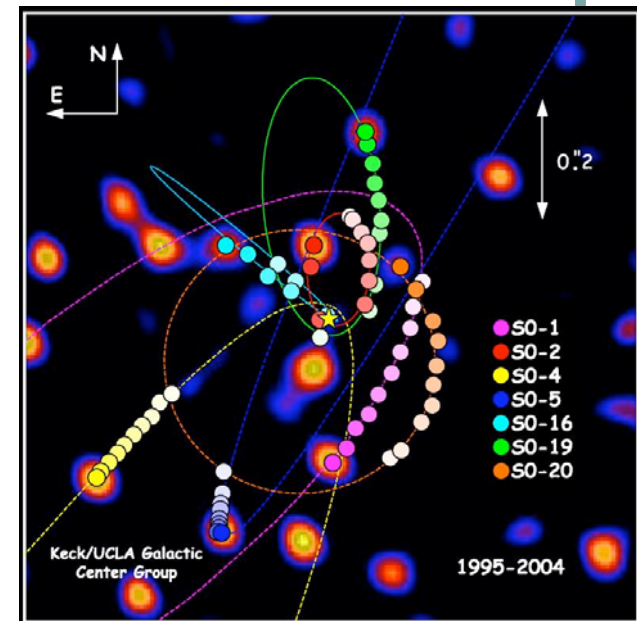
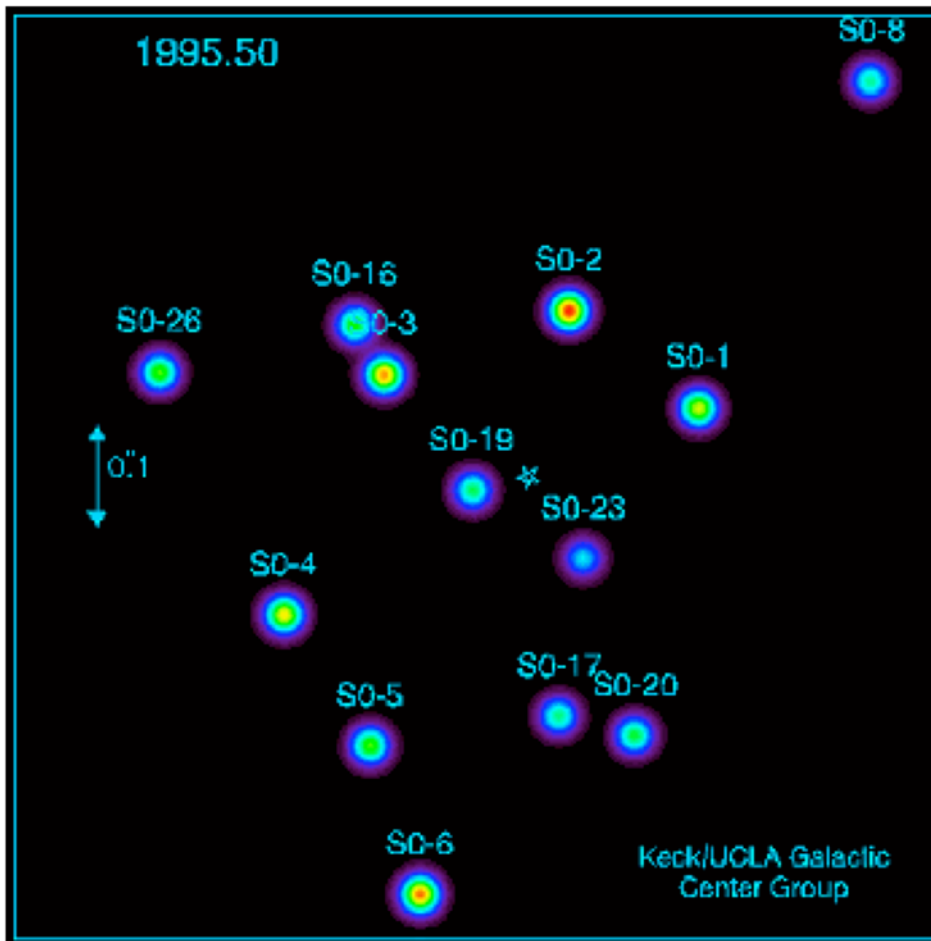


LISA the space interferometer

- LISA is **low frequency detector**.
- With arm lengths **5,000,000 km** targets at **0.1mHz – 0.1Hz**.
- Some sources are very well known (**close binary systems in our galaxy**).
- Some other sources are extremely strong (**SM–BH binaries**)
- LISA's sensitivity is roughly the same as that of LIGO, but at **10^5 times lower frequency**.
- Since the gravitational waves energy flux scales as $F \sim f^2 \cdot h^2$, it has **10 orders** better energy sensitivity than LIGO.

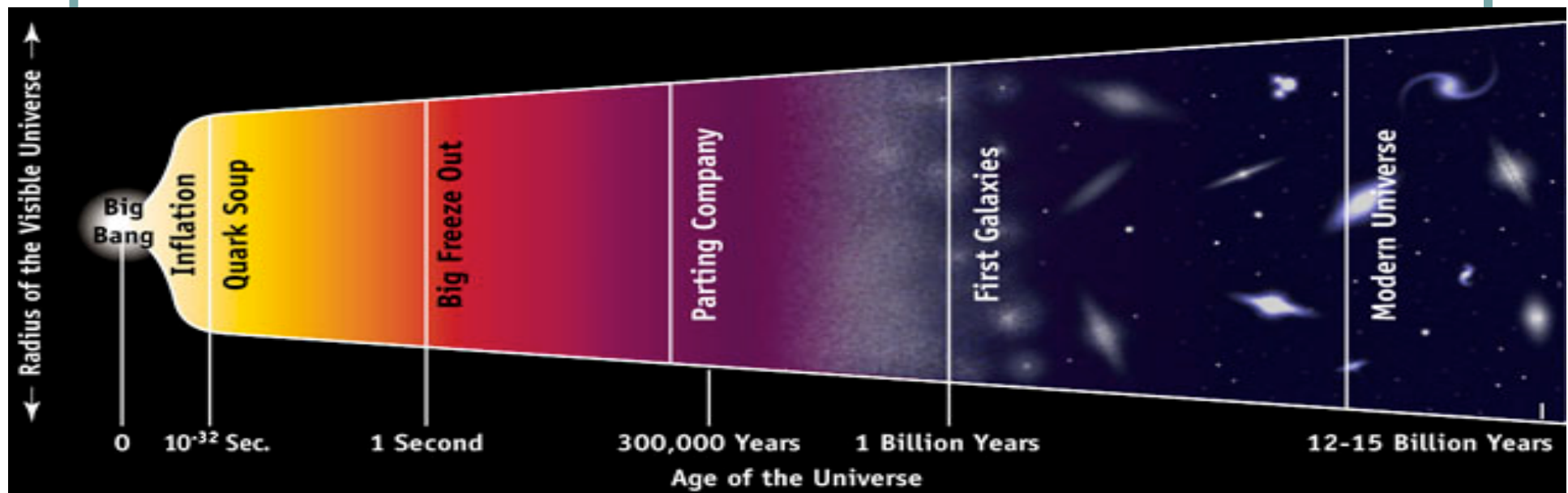


The BH in our Galactic Center

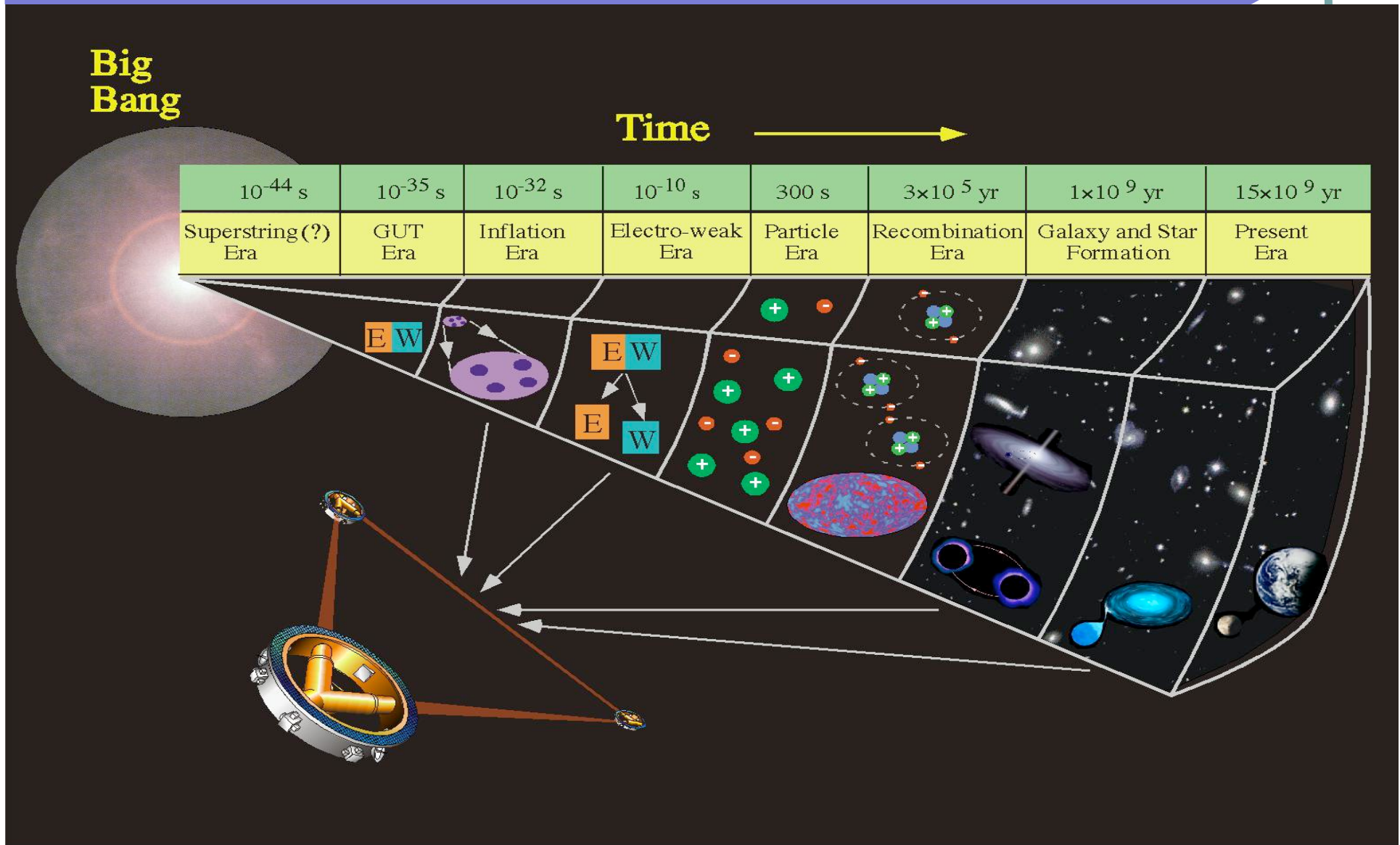


Stochastic Sources

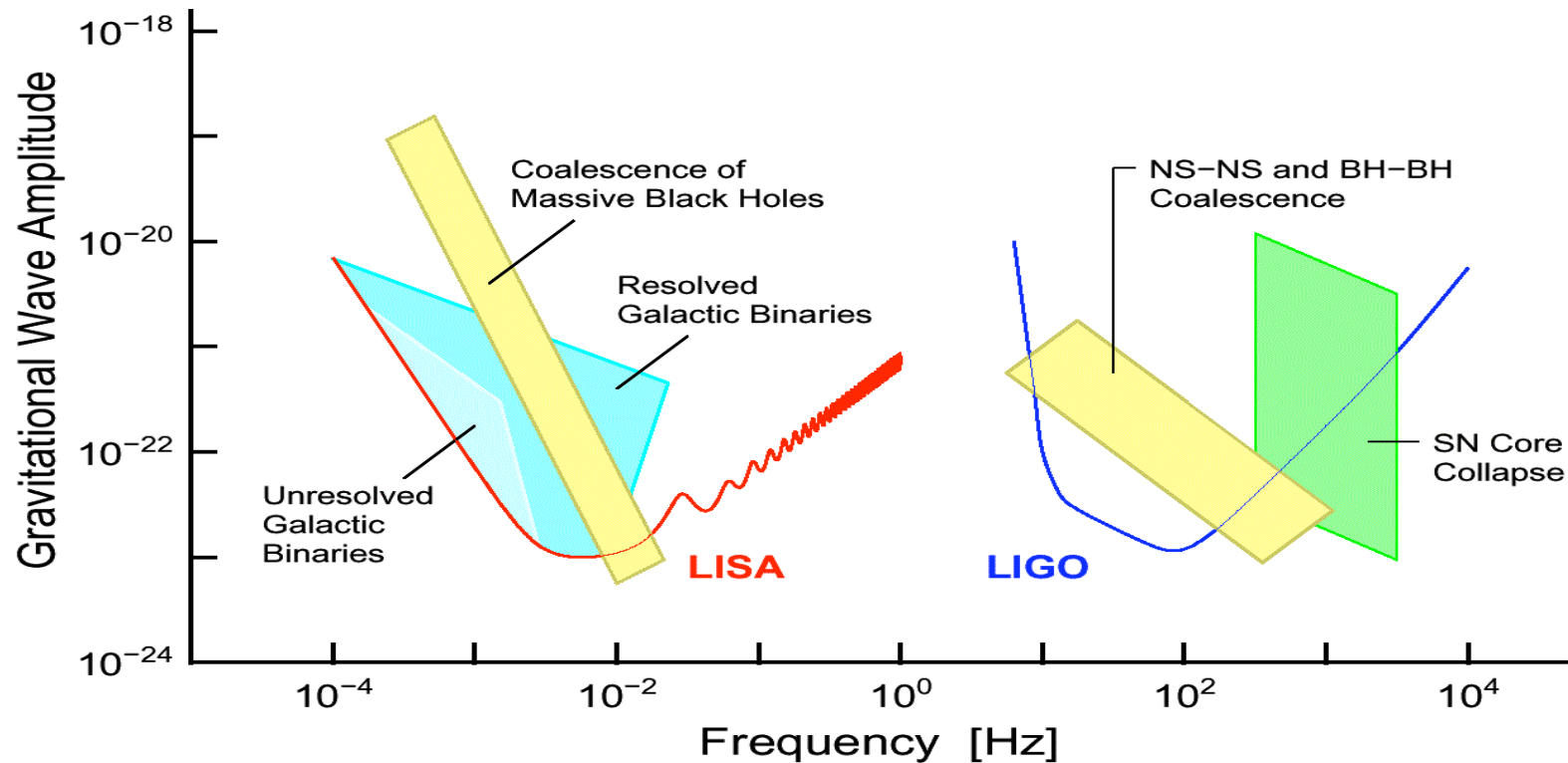
- Stochastic backgrounds
 - Astrophysically generated and from the Big Bang
 - Strength and spectrum of astrophysical backgrounds, production of early-universe radiation, relation to fundamental physics (string theory, branes, ...)



LISA: Fundamental Physics, Astrophysics and Cosmology

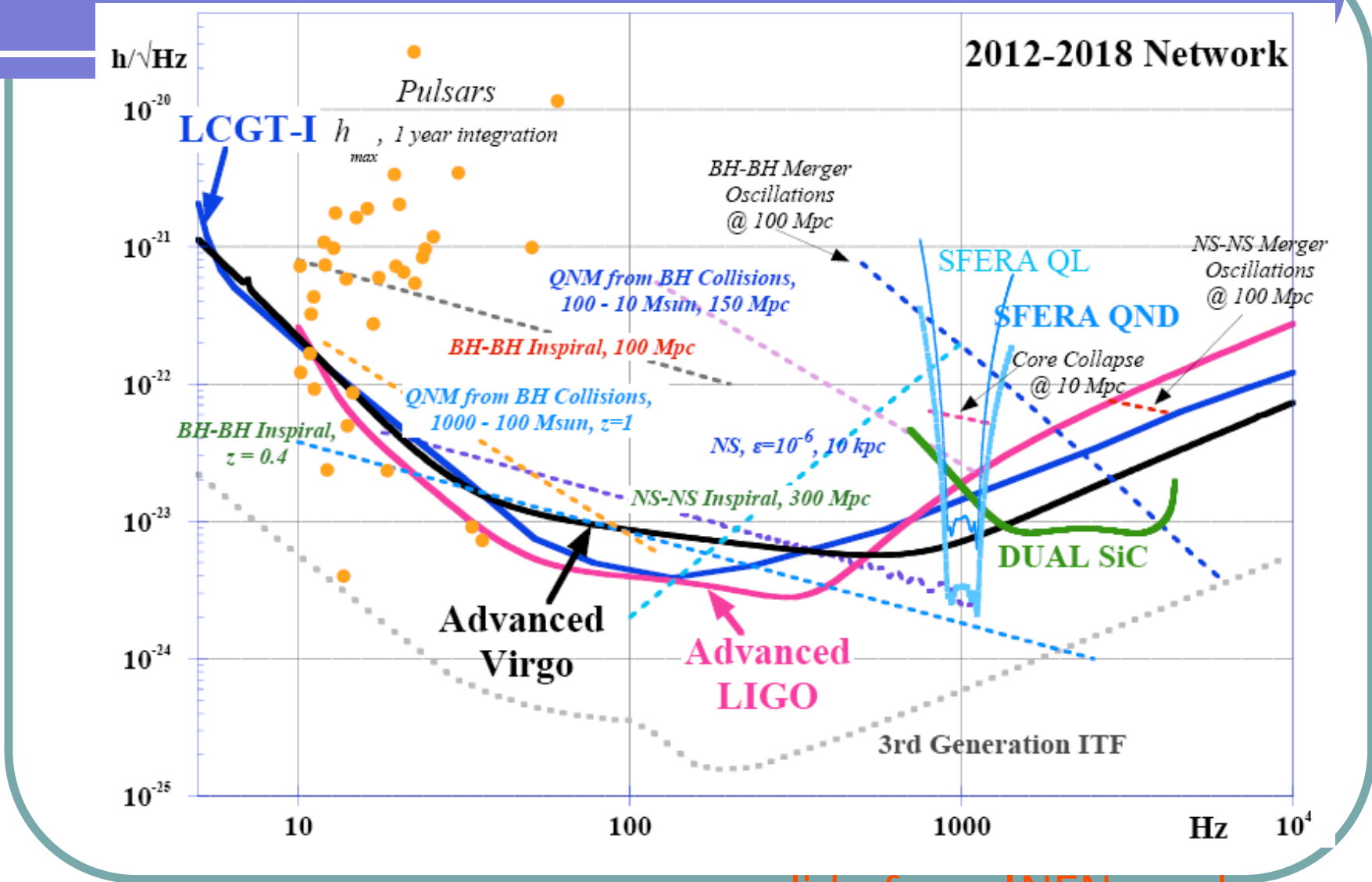


Gravitational Wave Spectrum...



Complementary observations, different frequency bands, and different astrophysical sources...

2012 - 2018



- slide from INFN roadmap