The Multilateral Negotiation Regime: Motivation, Context, Setup, Experimental Design and Results *

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Introduction

This project introduces the general concept, tests and a final version for prototyping of the-called Multilateral Negotiation Regime (MNR), aiming at the procurement of school textbooks by the Brazilian National Fund for Education Development (FNDE), a Central Procurement Agency (CPA), on behalf of most Brazilian public schools (at Federal, State and Municipal levels) in the sample consuming those books (schools). The MNR was inspired on a multi-round sealed bid combinatorial auction, and in the present format it resembles more a discriminatory auction for separate goods where the bidders enjoy economies of scope and quote a fixed fee related to common fixed costs across the production of the different goods. Another important feature of the MNR is that the prices are somewhat hedonic, in the sense that they reflect different quality perception by the schools. This quality perception is constructed by the schools on a shortlist of items, and it may be influenced (distorted) by sales agents of the bidders. The quality scale is then revealed to the CPA by the schools before the MNR is run. The current setting is a negotiated procedure, but anecdotal evidence suggests that the CPA is wasting money, as they negotiate separately the purchase of the first choice of each schools, and only once, more than ten years ago (no record thereof available), they resorted to buy the second choice to increase their bargaining position. We shall also add side constraints in order to prevent suppliers from being awarded an excessive share of the lots.

1 Background of the National Textbook Program[1]

The National Textbook Program (PNLD after its Brazilian acronym) is the oldest of the programs aimed at the distribution of courseware to the students of the Brazilian public schools, and it began under another denomination in 1937. Throughout these 80 years, the program was perfected and had different names and forms of execution. Currently, the PNLD is focused on Brazilian basic education, except pre-school. See the program's history below:

1937 - Decree-Law no. 93, of December 21, 1937, creates the National Book Institute.

1938 - Decree-Law no. 1.006, dated 12/30/38, establishes the National Textbook Commission (CNLD), establishing its first policy of legislation and control of production and circulation of textbooks in the country.

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[1]This section is a merged summary of the program descriptions: (i) furnished by the CPA directly to the team and (ii) scraped from the Program's website (https://www.fnde.gov.br/programas/programas-do-livro/livro-didatico/historico).

1945 - By Decree-Law no. 8.460, of 12/26/45, the legislation on the conditions of production, importation and use of the textbook is consolidated, restricting to the teacher the privilege of choice of the book to be used by the students.

1966 - An agreement between the Brazilian Ministry of Education (MEC) and the United States Agency for International Development (USAID) allows the creation of the Technical Book and Textbook Committee (Colted), with the objective of coordinating actions related to elaboration, publishing and distribution of the textbooks. The agreement assured to MEC sufficient resources for the free distribution of 51 million books over a period of three years. By ensuring government funding from public funds, the program acquired continuity.

1970 - Ordinance No. 35 of the Ministry of Education, implements the system of joint publishing with national publishers, with resources from the National Book Institute (INL).

1971 - The National Book Institute (INL) begins to carry on the Textbook Program for Elementary Schools (Plidef), taking on the administrative and management responsibilities of the financial resources, thus far in charge of Colted. The counterpart of the States to the program funding becomes necessary and is initiated, due to the termination of the MEC/USAID agreement.

1976 - By the Decree 77,107, the government becomes in charge of buying a great portion of the books to distribute to a part of the schools and of the States. With the extinction of INL, the National School Material Foundation (Fename) becomes responsible for the implementation of the textbook program. The funds come from the National Fund for Education Development (FNDE) and from minimal counterparts established for the participation of the States. Due to insufficient resources to serve all public elementary school students, the vast majority of municipal schools are excluded from the program.

1983 - Fename is replaced by the Student Assistance Foundation (FAE), which incorporates the Plidef. On that occasion, the working group in charge of scrutinizing textbook problems proposes that teachers participate in the choice of books, as well as the extension of the program to include further years of elementary education.

1985 - With the issuance of Decree 91,542 in September, Plidef is replaced by the National Textbook Program (PNLD), which entails several changes, such as:

- Appointment of the textbook shortlist by the teachers;
- Reutilization of the book, implying the abolition of the disposable book and the improvement of the technical specifications for its production, aiming at a greater durability and enabling the implantation of textbook banks;
- Extension of the supply to 1st and 2nd grade students in public and community schools;
- End of the financial participation of the states, by transferring the control of the decision making process to FAE while preserving the shortlisting by teachers at their own discretion.

1992 - The distribution of books is under threat by budget constraints and there is a decline in the scope of distribution, by restricting supply to as far as the 4th grade of elementary education.

1993 - Resolution CD FNDE No. 6 earmarks, in July 1993, funds for the acquisition of textbooks for students in public education networks, thus establishing a regular flow of funds for the acquisition and distribution of textbooks.

1993/1994 - Criteria for evaluation of textbooks are defined, with the publication of the "Definition of Criteria for Evaluation of Textbooks" by MEC, FAE and UNESCO.

1995 - Gradually, universalization of the distribution of the textbook in elementary school is resumed: in 1995, Mathematics and Portuguese; in 1996, Science; and in 1997, Geography and History.

1996 - The shortlisting of the textbooks by a committee of experts is started, and the first Textbook Guide from the 1st to 4th grades is published. The books were evaluated by the committee according to criteria previously discussed, which improved the quality of the works selected. The publication of the Textbook Guide was started to enable teachers of public schools to choose titles for pupils in the early years of elementary school, as a result of the technical assessment carried out by the Ministry of Education under the PNLD, following criteria disclosed in a public call. This procedure has been perfected, and has been applied to date. Books that present conceptual errors, induction to errors, outdatedness, prejudice or discrimination of any kind are excluded from the Guide.

1997 - With the extinction of the Student Assistance Foundation (FAE) in February, the responsibility for the implementation of the PNLD is transferred to the National Fund for the Development of Education (FNDE). The program is expanded and the Ministry of Education continues to acquire textbooks for literacy learning, Portuguese language, Mathematics, Science, Social Studies, History and Geography for all students in grades 1 to 8 of public primary education (current grades 2 to 9).

2000 - The distribution of Portuguese language dictionaries for the use of 1st to 4th grade students in 2001 is included in the PNLD and, for the first time in the history of the program, the textbooks are delivered in the year before the academic year of their use. The books for 2001 were delivered by December 31, 2000.

The PNLD is the result of coordinated action between the Ministry of Education and the FNDE. MEC works with the definition of the pedagogical policies and strategies of the Program. In turn, the FNDE operates the acquisition and distribution of the books.

The program seeks to provide the constitutional right that students of basic education have to receive books and learning materials. Currently, the Program serves students of elementary and secondary education, with several subjects.

The PNLD has two characteristics that increase its efficiency and effectiveness: decentralized choice and centralized purchasing. It provides for meeting the specific demands of schools, since the choice is decentralized. However, by making a centralized purchase, its bargaining power enables the CPA to negotiate more advantageous prices by appropriating a share of the scale economies enjoyed by the publishers with the centralized transactions.

It is worth noting that the PNLD does not serve all schools in the public network, but only those whose education departments have joined the Program. Currently, 96.16% of the country's school systems (federal, state, municipal and district) receive the books they have purchased. When the school system is no longer interested in receiving the books of the Program, they may request the suspension of remittances or the exclusion of the Program.

The PNLD is executed every year, however, the coverage of each stage of education (1st to 5th year or 6th to 9th grade or high school) occurs alternately and in triennial cycles. Thus, every three years, the books of a teaching stage are completely replaced. Teachers make a new choice of book collections of all subjects, and are then forwarded in November the books, some of them consumable and some of them reusable. The latter are used by students and at the end of the year, returned to the schools so that in the following year the incoming students also use them. In the former case the student can complete activities in the book itself, which thus needs not to be returned at the end of the year.

Books are also supplied to students of the years that are not object of the acquisition process. Reusable books are subject to damage and loss, and some of them are never returned, so a replacement lot is ordered every year. If the number of students increase, the corresponding number of books is also added to the replacement order. In addition, yearly consumable books are sent to all students at all stages of teaching.

Thus, for each educational stage, once every three years the selection and acquisition of all the books takes place. Then, for two years, some copies of books of replacement and complementation of the consumables are sent to the school. Titles that are consumable are shipped every year in sufficient quantity to all students. Books purchased in one year are distributed the following year.

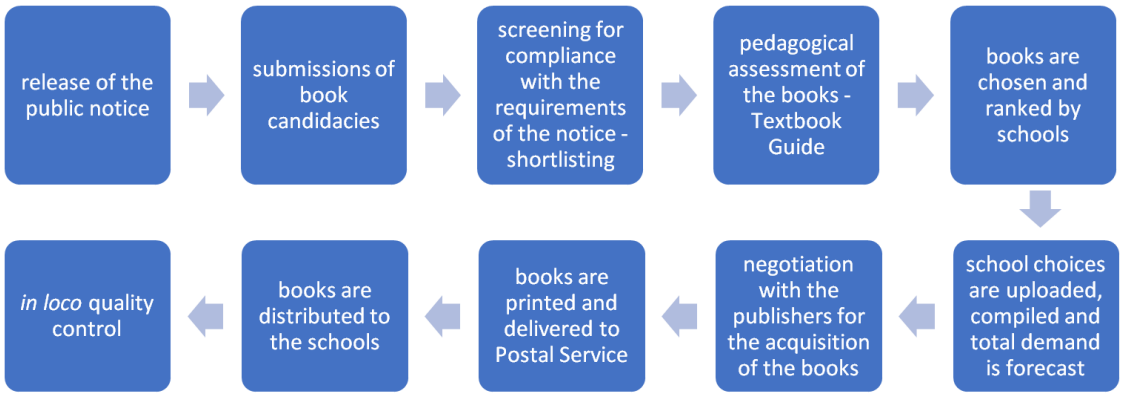
To acquire and distribute the books, the PNLD is executed according to the following steps: release of the public call, applications, shortlisting (by committee), rating (by committee), selection (by schools), aggregation of choices, manufacturing, distribution and field inspections (see Figure 1)

Each edition of the PNLD begins two years before, with the release of the pubic call. It describes the subjects, the characteristics of the books and information about the application and shortlisting process. Publishers then submit their books in application for participating in the PNLD. The books go through the screening, which verifies if they meet the physical characteristics required in the public notice.

The shortlisted books are then sent for assessment by the Ministry of Education. To carry out the task, experts from public universities are commissioned, who then write the reviews that will compose the Textbook Guide.

The document is supposed to be consulted by the teachers, at the time of the selection, that is, the year prior to the students' use of the books. This step is performed by the schools in democratic meetings of teachers of each discipline (curriculum component). Then the chosen collections are

Figure 1: Stages of the National Textbook Program



ranked, and each ranking is entered into PNLD's system by the school's principal, using a personal password.

In order to know how many books will be sent to each school, the School Census[2] numbers are used. However, books are purchased one year prior to use and at that time the most up-to-date data available in the census are from the previous year. Thus, the data is lagged for two years. To correct this information a projection is made by FNDE, using Inep estimates of approvals, transfers and new enrollments. The aggregation of the products of the number of students per grade and corresponding subjects generates the numbers of books to be delivered. The same multiplication provides the estimated demands for each first-choice title. These numbers are put on the table for the price bargaining. The books are acquired through a simplified negotiated procedure on the grounds that the copyright of each book chosen by the schools belongs to only one publisher, which characterizes the singularity of the object, and therefore renders a tendering procedure unfeasible. The negotiation is done by the publisher, in person or by e-mail, and by a commission appointed for this process.

The book prints are manufactured by the publishers within a specific term determined by the FNDE. After the production of the books and before their distribution, some copies are collected for the accomplishment of the quality control. These samples are submitted to several tests at the Technological Research Institute in São Paulo (IPT), which evaluates the quality of printing, gluing, and paper. If non-conformities are found in this process, fines are charged to the publishers.

The books produced are dispatched to the Brazilian Postal Service, which is responsible for the distribution. They are delivered directly to schools or, in the case of rural schools, to the headquarters or warehouses of the Education Departments that they belong to, and which are responsible for the final shipment. This step requires heavy logistical operation, given the size of the country and the difficulty of access to the most remote points. Still, most books reach destinations before the start of the school year.

Schools are also important players. They should promote the choice of books with the democratic participation of teachers, use the books according to the pedagogical proposal during the three years of service, carry out continuous control of delivery to students and return of books for them, among other activities.

According to the stage of teaching attended, the PNLD has served between 29 and 39 million students, and its annual spending on books has been increasing from 1 to 1.3 billion reais (source: https://www.fnde.gov.br/programs/book-programs/textbooks/statistical data).

1.1 Communication with education networks

The good performance of the PNLD depends to a large extent on the manager who works at each end, in the school system headquarters and in each school. This is because there are steps that are taken by them, such as the choice of books, the relocation of surplus books between schools and the undoing of the book at the end of the Program (they have to be sent for recycling). Therefore, the FNDE has worked to increase communication with this public.

[2]The School Census is undertaken by a Research Institute of the Ministry of Education, the Institute for Educational Reserach and Studies — Inep.

Given the size of Brazil and the number of books distributed by PNLD, distribution logistics is one of the major challenges and highlights of the program. In PNLD 2016, for example, in a few months, 128.5 million books were distributed to around 121 thousand addresses in all 5.5 thousand Brazilian municipalities, including large metropolises and the most distant localities.

The challenge is even greater because the parcels forwarded are not uniform. Each school receives a different number of books, of each series, from various collections chosen. So the orders placed differ from one another and range from 50 to 5,000 books, for example. In order to send the correct quantity of each title chosen, there are computerized systems, which also assist in tracking the orders forwarded. This enables the control of the entire process.

The books usually leave the Southeastern Region (São Paulo, Rio de Janeiro...), where most publishers are concentrated, and reach their destinations on truckloads, by airmail, in boats, or in bicycles. In some delivery locations, the order takes 15 days to arrive from the state capital due to difficult access. In Amazonas, for example, this time frame may be higher due to weather conditions, because if it is raining heavily, it is not possible to continue traveling because wood and other forest materials descend through the river. On the other hand, if there is a drought, the boats are not able to navigate the river.

2 Description of the awarded contracts

2.1 Description of the current model

Currently each school lists a first and a second choice for each subject, and submits this list to the CPA. The CPA aggregates the first-choice titles of each school and opens a bargaining process with a same publisher of a subset of these books. The CPA has an estimate of costs per size (weight and volume) of the book and economies of scope, and uses these estimates (annually updated by sector price indices) to formulate its price proposal for each of the producers. This negotiation occurs because each book's author has an exclusivity clause with one single publisher, which would, in theory, impede the use of tendering (in Brazilian Procurement Law 8666/93 this is called "ineligibility for bidding"). There are rounds of counteroffers, the details of which we do not know quite well, but which, for the purposes of the present research, can be modeled as a set of alternate offers à la Rubinstein (Rubinstein, 1982). The CPA always has the option to substitute the second choice book $j\prime_m$ for the book that is object of the bargain j_m, but notice that the ordered pairs of books vary according to the school, thus this substitution has to be done for each subset of schools with the same ordered pair, and hence the aggregated demand of each subset has to be shifted to the second-choice title of that particular subset. The heterogeneity of preferences, therefore, renders the negotiation process particularly complex and exhausting for the CPA.

The current steps are therefore three:

1. Vendors submit their products to the Ministry of Education, who shortlist them according to a list of criteria.

2. Once the shortlisting outcome is released to the entire population, schools make an ordered list of preferences - currently two. Buying units may be influenced by sales promoters.

3. The first options are grouped under contract, one per supplier. The same agreement can and should contain quantities of each and every book that was selected first for each of the subjects. Schools that did not exercise their right of choice receive the most chosen book within their region.

4. A bargaining process of the purchasing center begins with each of the suppliers. If no agreement is reached, the CPA redistributes that supplier's quantities to their respective second options.

The acquired books are delivered to a same location appointed by the Postal Service. No change is expected in this, so the distances of suppliers to the point of delivery do not vary with the spatial distribution of the group of consumer units.

2.2 Mathematical specification [Technical section: may be skipped]

This subsection describes the same setting above with a mathematical notation.

The CPA has to procure books for S schools. Each school $s \in \mathcal{S} = \{1, 2, ...S\}$ procures a bundle of book collections (a vector) $\mathcal{D}(s)$ whose element $\mathcal{D}(s, d)$ is the quantity of book collections $b \in \mathcal{B}(d)$ to be consumed by the students of that school when learning the subject $d \in \mathcal{M}$. Each school rates by themselves (i.e., using their own scoring criteria) each of the goods available before the auction begins. The set $b \in \mathcal{B}(d)$, on its turn, is a subset of a broader set $\mathcal{J}(d)$, and the selection of this subset is exogenous to the school s – a simplification, because the school can always opt out from the present Centralized Procuring System, and buy books with other funding sources[3]. Therefore we have a fixed number of "markets" $M = \#(\mathcal{M})$ in the textbook industry, with production functions quite alike, and with a number of common costs (see below). In each market $d = 1, ...M$ there exists an exogenous number $N(d)$ of products marketed by the main firms of that industry. The number of manufacturers is $J(d)$, but $N(d) \geq J(d), \forall d$, as some corporations may have a variety of options available under different brands (i.e., different book collections developed by different authors).

Before the actual auction, a qualification phase takes place, whereby a committee of experts of the Ministry of Education (whereto the CPA is subordinated), and independent of the CPA, shortlists the goods following a pre-specified scoring function. The goods are submitted to a Request for Candidacies and are rated by this committee strictly under technical criteria, but these ratings are currently not public – the shortlisting works as a pass-or-fail test only.

After the shortlisting phase, there survive $n(d)$ out of the $N(d)$ initial candidates, therefore $n(d) \leq N(d)$. A catalogue is subsequently posted on line containing the shortlisted goods in each market d, each one with a brief description. Again $n(d) \geq \jmath(d), \forall d$, where $\jmath(d)$ is the number of surviving firms in each market d.

As mentioned before, there exist S schools. They benefit from the services of the CPA, who act on behalf of them in a similar manner to the old-fashioned "centralized warehouse model" (see, for example, OECD (2011), p.16), as they conduct the whole logistic and contractual aspects of the procurement process[4]. More precisely:

1. The CPA procures the goods: they conduct the negotiations and they award the frame contract

2. The CPA funds the payments for the goods consumed by all the S schools

3. The CPA provides the delivery of the goods: a delivery firm receives the goods shipped by the suppliers to their warehouses, sort the packages and ship them forward to the schools over the entire national territory.

Thus, ultimately, the schools' only duties within the current Centralized Procuring System are (i) opting in or out of the system; (ii) selecting their most preferred options of goods. Nothing else.

As mentioned before, each school s consumes a quantity $\mathcal{D}(s, d)$ of goods in each market[5]. The school s accesses the catalogue, and from the brief description available, they may select as many as k options of stock-keeping units (b) in each market, in decreasing order of preference. Currently $k = 2$, but the CPA has agreed to increase it to $k = 3$. Some schools may also opt not to make a choice[6]. In this case they are aware that they will receive the book corresponding to a fraction (with equal probability for each bidder) of the remaining demand assigned to the brand with the greatest demand in their region (let us call it "the majority rule"). We shall assume henceforth that their choice will be exercised in all the markets or in none.

Another constraint of the problem is that the k options should belong to the portfolios of different suppliers — in other words, in case the first choice is supplied by a firm $e \in (E)$, the second one should be supplied by $\tilde{e} \neq e$. This latter constraint will add to the political sensitiveness of the project and should not be understated. In fact, at present there are many schools who list two goods of the same supplier. In fact, if we restrict one good for each supplier, they will have

[3] Many of the schools who have opted out of the system resort to the so-called "teaching kits", which comprise sets of lecture notes and possibly other learning resources (such as digital media) prepared by school franchise conglomerates.

[4] The difference from the "Monopoly" model is that the schools are free to opt out of the system and receive the funds for their own procurement procedures, without having to prove to the CPA that they can get better prices.

[5] We could simplify the analysis further by assuming that $\mathcal{D}(s, d) = \gamma(s), \forall d$.

[6] Notice that this is very different of opting out of the system.

an even greater incentive to proliferate brands, and that is why we have to impose a ceiling on the total quantities awarded to the same economic group (and the definition of economic group must follow the Brazilian Antitrust case law). But the restriction is intended to preserve competition. If the same supplier is bidding for more than one good, competition is falsified.

2.2.1 The current procurement system

The CPA aggregates the first choice of each and every school. Let:

- $\xi_1(s,b,d) = 1$ if s chooses b as first choice (or is assigned b by the majority rule) in market d and zero otherwise;
- $G(b) = \sum_s \xi_1(s,b,d) \cdot \mathcal{D}(s,d)$, where $b \in \mathcal{B}(d)$;
- $e \in \mathcal{E}$ be a company or economic group (in our case, a publisher or publishing conglomerate) supplying various brands (titles) b in each market d (subject)
- $e(b)$ the function that identifies the publishing group owning the brand b
- $D(e) = \sum_{bs.t.e(b)=e} G(b)$

A bargaining procedure is then open by the CPA with the supplier whose bundle of goods has been included in any first choice.

In fact, the bargaining procedure comprises price curves for each of the goods[7] and some discounts are applied by aggregating the number of goods to be produced by a same firm or economic group, as the CPA has a cost estimate normalized by size (weight and volume) of a title and of the economies of scope (in case the firm supplies goods in different markets), and they apply these estimates (which are regularly updated by industrial price indices) so as to formulate their price proposal to each one of the suppliers[8].

In theory, one should expect the bargain to proceed as a Rubinstein-style alternate-offers game. Should the game indeed proceed after the first offer, though, the CPA would have a quite complex problem to solve, as they would have to reallocate lots of schools to their second choices (and there might be $\jmath_d - 1$ distinct second choices), and then resume bargains with each of them.

Therefore, in our view, if the CPA wants to save money in the procurement process, lowering the first offer, and they do not want a long and tiresome sequence of alternate offers, they should resort to an iterative procedure that emulates an auction, receiving bid submissions in rounds, and responding with quantity allocations to each bidder. That is what we describe in the upcoming sections.

The current stages are therefore, the following:

1. Suppliers submit their products to a Ministry, who organizes (by themselves or by subcontracting through a selection process of their own) a committee of experts to shortlist the products into a Web catalogue, following a list of criteria[9].

2. Once the catalogue is available on line, anyone can consult it, although access to the full digital content of the books is only available for the agents of the schools in charge of the selection[10]. In particular, the schools elect an ordered list of preferences — currently two options are selected. Preferences are merely ordinal; the schools do not rate the products with numerical scores. It is very important to note that the schools may be heavily influenced by sales representatives, who distribute samples and small gifts to the public officers that ultimately vote in the local decision procedure[11].

[7] In the case of textbooks, for each of the **titles**.

[8] As mentioned below, this negotiation takes place because each title is owned exclusively by a same firm, an in Brazil this prevents a from being conducted. This is the so-called "Ineligibility of bidding", whereby the government negotiates directly with the monopolist of that supply.

[9] In the last shortlisting process, there was one committee for each d, who defined their own criteria; it seems the new rules indicate a predefined set of criteria imposed to the committee, who is guided by the Ministry, and is not restricted to experts.

[10] In Chile, for example, the entire catalogue is restricted to the schools.

[11] The CPA staff reckons that these influence activities may be to blame for reinforcing the market power of the largest sellers.

3. Each and every school's first choice of a same product is aggregated into a single contract to be negotiated with that product's supplier. If the same supplier has more than one product in a same market d or across different markets, they are aggregated into the same contract as well. In other words, a same contract may and must comprise the sum of quantities of all books belonging to a same supplier's portfolio and selected as first choice in each of the M markets, plus the quantities demanded by the schools that have been allocated to these suppliers under the majority rule (because the school did not exercise their right to name and order their first and second choices. In our notation, the contract with the supplier e covers all the $G(b)$ such that $e(b) = e$, plus the quantities allocated to them by the majority rule.

4. A bargaining process is initiated by the CPA with each one of the suppliers $e \in \mathcal{E}$ of products nominated by the schools. If no agreement is reached, the CPA should redistribute the quantities of those first choices to the suppliers of the respective second choices.

The selected products are delivered to a same warehouse of the Brazilian Postal Service, who has been hired by the CPA for the entire logistics of the social program. Recently we were told that the CPA was considering a requirement to decentralize logistics, so that the procurement model would not favour firms located closer to the current warehouse. But we were told not to take this extension into consideration.

3 Premises for the solution proposed

After a dialogue with the CPA, we identified an opportunity to introduce some price competition into the procurement model, while preserving the privilege of ordering preferences assigned to the schools. For maintaining the plurality of choices by the schools, the new model should still place some weight on the quality level of the good as perceived by the schools. In fact, the good in question has some features of a credence good[12]. The school officers decide what book is best suited for the population served by that school. We could proceed in suggesting some reputation system that could empower the population served to rate the good and generate a reputation score to be used in future procurement procedures. This suggestion, however, has not been incorporated into the current project, and may raise politically sensitive concerns, therefore we have to deal with this idea very cautiously in the future.

Quality perception, however, is a construct influenced by the current suppliers, and the data generating process for these rates should be guarded against manipulation and bribery. In his article, for example, Burguet (2017) describes a situation where an inspector may misrepresent the quality of a service performed by a contractor in exchange for a bribe from the latter. Another article (Burguet and Che (2004)) studies competitive procurement administered by a corrupt agent who is willing to manipulate his evaluation of contract proposals in exchange for bribes. If the agent is corrupt and has large manipulation power, bribery makes it costly for the efficient firm to secure a sure win, so in equilibrium the efficient firm loses the contract with positive probability. The optimal scoring rule for the buyer de-emphasizes quality relative to price and does not fully handicap, and may even favor, the efficient firm.

The optimal scoring rule is left for section 6 below. Here we shall focus, first, on the cost function used by the CPA to calculate their reservation price curve. Next we focus on the stages of the selection process and on the options for the submission of bids (i.e., the auction format).

3.1 The cost function used for the reservation price curve [Technical section: may be skipped]

In a general setting, the good b may be seen as a bundle of units standardized by the material utilized, size and weight[13].

Now let:

- The titles supplied by a publisher e be indexed by $i(e) = 1, ...n_e$

[12] A good whose quality can only be ascertained by an expert on behalf of the consumer(Darby and Karni, 1973).

[13] In our textbook case, the CPA pays the publisher for a book according to the number of signatures — a standardized unit of measurement for publishing contained in that book.

- The respective numbers of signatures manufactured and delivered of each $b\,(e)$ are $q_1, ..., q_{n(e)}$

From Fiuza (2017), we have learned that the CPA currently calculates their reservation price based on a total cost curve that may be described by a function such as:

$$C\left(q_1, ..., q_{n(e)}\right) = \sum_i \left(c_i \cdot x_i + v_i \cdot q_i + \phi_r \cdot q_i - \omega_r \cdot q_i^2\right) + \Phi_{\tilde{r}} \cdot \sum_j q_j - \Omega_{\tilde{r}} \cdot \left(\sum_j q_j\right)^2 \quad (1)$$

such that the average cost curve for a title i may be written as:

$$C_i\left(q_1, ..., q_{n(e)}\right) = c_i + f\left(q_i, r\right) + F\left(\sum_j q_j, \tilde{r}\right) \quad (2)$$

where:

- $x_i = 1$ if book i is produced, zero otherwise
- c_i accounts for the project (fixed) cost, or at least the part that is sunk by the publisher to comply with the CPA's requirements
- v_i accounts for the average logistic cost
- ϕ_r and ω_r are coefficients for the average printing cost, both of which vary according to the signature quantity tranche r — such that $f\left(q_i, r\right)$ is a monotonically (and convex) decreasing function of the quantity of signatures
- $\Phi_{\tilde{r}}$ and $\Omega_{\tilde{r}}$ are the scope economy coefficients applied to the average printing cost, both of which vary according to the aggregate signature quantity tranche $\tilde{r}$— such that $F\left(\sum_j q_j, \tilde{r}\right)$ is a non-decreasing concave function of the aggregate quantity of signatures across all titles

Note that the CPA has never provided any engineering explanation for the last component, that acts as a discount demanded from the publishers and their economic groups, simply based on the accumulated number of units (in our present setting, the signatures) produced. We have concluded that these discounts could only be explained by common overhead costs. That is why we adapt the total cost curve to be approximated by a simplified affine function:

$$C(q_1, ..., q_n) = K + \sum_i \tilde{c}_i \cdot x_i + \tilde{v}_i\left(q_i\right) \cdot q_i$$

where:

- K is a common avoidable fixed cost that, divided by the total number of signatures, will produce the convex decreasing component of the average cost due to economies of scope.
- $\tilde{c}_i$ comprises the avoidable (fixed) project cost for content and typesetting
- $\tilde{v}_i\left(q_i\right)$ embodies all printing costs, such as paper, labor and capital, which, according to the engineer estimate of the CPA, is not a constant variable unit cost, but rather entails a slightly decreasing average cost

Following this specification, we could say that the average cost for a particular good i is:

$$C_i(q_1, ..., q_n) = \frac{K_i}{q_i} + \frac{\tilde{c}_i}{q_i} + \tilde{v}_i\left(q_i\right)$$

where K_i is the share of K allocated to collection i by some rule, such as Fully Distributed Cost or so. This specification underlies the preference of the CPA for a price curve bid segmented in "tranches", i.e., quantity ranges, for each title[14], plus a K lump-sum component. According to the

[14]Thus, a bid consists of a number of tranches (quantity range) and a unit ask-price for each of them.

spreadsheets provided by the CPA, the number of relevant tranches are 19 (see Fiuza (2017))[15]. More on that in the next subsection.

Relative to other examples of procurement of divisible projects, and mechanisms to assign sales by sellers to schools, in the present case the cost function described above, characterized by increasing returns to scale (non-increasing average costs), implies a tendency to natural monopoly, and makes it challenging to construct a supply function as an aggregation of bids (this is in contrast, for instance, to spot markets for electric power, where no generator has economies of scale at the market level of output. However, lessons from these markets may still be learnt.)

On the other hand, this cost specification signifies that our setting is different from many cases of provision of public services to different constituencies (such as school lunch, transportation, garbage collection).

Here, the costs for publishers are, to all effect, only a function of the total number of copies supplied, independently of how these are distributed across schools. That signifies that there is no such supply heterogeneity as in the abovementioned services, and that the definition of lots is of no consequence in this case.

Last but not least, under the assumption that the main source of decreasing average cost is the existence of fixed cost (because the marginal cost is not so significantly decreasing), we adopt a pay-as-you-bid approach, i.e, a discriminatory auction. That means that a publisher who is selected to, say, supply n tranches with sizes and unit prices respectively (q_i, p_i), $i = 1, 2, ..., n$, obtains a payment of $\sum_{i=1}^{n} q_i p_i$, and not a payment of $p_i \sum_{i=1}^{n} q_i$[16].

The argument in favour of this design is that the marginal cost curve (which will be behind the bid under this proposal) is flatter, i.e., less decreasing, than the average cost curve, so that the issue of avoiding monopolization is less pressing.[17]

3.2 Remarks on the preferences of the schools:

As mentioned in subsection 2, the CPA bears the whole cost of procuring the titles, paying for their manufacturing and shipping to the central warehouse, and for the logistics to the schools. The funds used for that purpose could alternatively be transferred by the Federal Government to subnational governments and be used for minor purchases not restricted to the Ministry's shortlisting. In fact, when the school (School) states that they do not want to receive the program's own titles, they are opting out of the system, and therefore are excluded from the total demand of the CPA for the books.

But when an individual school opts in, they are, in a sense, "free-riding" on a purchase made on their behalf, and they do neither bear any burden for paying too much for a book nor enjoy any profit for saving money on that purchase. This is a typical agency problem, whereby the Agent in charge of ranking the books for consumption by students have utility functions disalligned from the objective function of the Principal (the CPA). But, in fact, what is the CPA's welfare function to be maximized? This is the heart of the matter.

The seminal literature on procurement by Laffont and Tirole (1993), for example, starts from a welfare function calculated as the consumer surplus, i.e, it is based on the demand curve for the product. When the authors allow for an unverifiable quality (see p.215), they pose a gross consumer surplus $V^q(q, u, \theta)$, where q stands for quantity, u for quality, and θ for remaining demand shifters. The underlying inverse demand curve assumed by the authors, however, assume that the consumers pay for the good, and the authors elaborate on strategies of identification

[15] Thus, the size of the tranches is set by the buyer, and only the prices are the decision of the publisher. We could well propose to let each publisher choose their own tranches, if some or all of their own thresholds differ from the official ones. For the algorithm, we believe, this would not impact much. But note that in order to test it *and* to implement it, we would need to have *each* publisher inform, not only the unit price for each tranche, but also the first lower bound and all the upper bounds of each one (as the upper bound of tranche n equals the lower bound of tranche $n-1$). Furthermore, one should wonder whether it is worth requiring from the publishers so much information. They should be very comfortable in not having to decide their own tranches: as printing is mostly outsourced, part of the costs should be quite alike. Of course, they may be comfortable simply because it reduces uncertainty and may (perhaps) facilitate collusion. So this is a decision to be made by the CPA, in our view.

[16] Here we have not included a common overhead fixed fee component.

[17] The "natural monopoly" trap comes from the fact that, faced with downward sloping "supply functions" (bids), the larger the amount assigned to a seller the lower the total cost of satisfying the demand. Unless monopolization is not a concern, the mechanism will then have to arbitrate measures to limit the quantity assigned to each supplier. The less related to costs these limits are, the less efficient the outcome will be.

of that demand curve[18]. Understandably we are not able to perform such inference, therefore we have to resort to alternative forms of eliciting from the Agents (the schools) some assessment of the difference of valuation between two titles of the same subject. In other words, we are looking for a **truth-telling mechanism**. We need such assessment because otherwise we are not able to value the trade-off between saving cost of acquisition and delivering high quality goods to the schools (and ultimately to the students themselves). This trade-off is usually accounted for by scoring functions[19] in a form $\nu(p,u)$, where $\frac{\partial \nu}{\partial p} < 0$ and $\frac{\partial \nu}{\partial u} > 0$. Here are some approaches available for such function:

1. schools could state their cardinal preferences for each book, using some function $\mathcal{R}(s,b) \in [0,10]$. One could impose some restrictions on $\mathcal{R}(s,\cdot)$, such as imposing that the best ranked is necessarily equal to 10, or rescaling the marks accordingly. The problem with this approach is obvious: schools influenced (in a legal or illegal, ethical or unethical manner) by sales representatives will tend to magnify their Willingness to Pay (WTP) for the titles marketed by these sales reps. Even if we impose a floor rating for the second and third options, the tendency is that schools become more and more targeted by sales reps as the publishers realize that boasting their marks will increase the own CPA's WTP. Even budget ceilings and/or market share ceilings in the Winner Determination Problem (see below) are not able to discourage this targeting; they are only able to cap these efforts.

2. We could aggregate the same "marks" or ratings of the schools across all schools stating the same ordinal preferences, and to that end we could use statistics that would smooth out the influence of schools targeted by the sales reps. This could be done in a variety of forms. For example, the schools would be demanded to provide multicriteria assessments and the formula would discard or penalize extreme marks throughout the criteria. Even if we do not use multicriteria, we could discard extreme ratings or simply use medians or other quartiles, rendering the aggregate rating less vulnerable to manipulation.

3. We could well adopt a value function that incorporate some information from the schools that would not affect their own probability of getting the titles they most prefer. For that purpose we should ask the schools their differences in WTP for the titles, but we shall not use for the valuation any cardinal information from the schools for titles belonging to their most preferred triple of titles.

4. The CPA could assign a value to the bonus related to the shadow cost of the funds available in their budget for book acquisition. As we mention below, this was the selected approach.

3.3 Stages of the Multilateral Negotiation Regime:

1. Preliminary steps:

   (a) Suppliers submit their products to a Ministry, who organizes (by themselves or by subcontracting through a selection process of their own) a committee of experts to shortlist the products into a Web catalogue, following a list of criteria.

   (b) The schools elect an ordered list of preferences with k_b options— ideally three options, that is, $k_b = 3$, to strike a balance between increasing competition and keeping the number of package bids manageable. The maximum *potential* number of such ordered triples is therefore:

$$\sum_d A_{\jmath_d}^{k_b} = \sum_d \frac{\jmath_d!}{(\jmath_d - k_b)!} \tag{3}$$

2. The schools with the same orders of preference are aggregated to a same lot. The actual potential number, however, depends on the numbers of titles supplied by same economic groups, because *no school shall be allowed to choose more than one title from the same group.*

   The numbers of students corresponding to each triple shall vary widely, because the number of schools with the same order of preferences varies, and so do their respective sizes.

[18] "*Using the observability of p [price] and q, it is possible to eliminate the unobservable service quality level [u] in the consumers' gross valuation of the commodity.*"

[19] See Dimitri et al. (2006) for a comprehensive survey of this subject.

3. In case we decided to use the schools' own valuing to calibrate our bid bonus, we would require that, besides informing their ordered triple, each school provided their WTP regarding at least three titles not included in that triple[20]. This could be done in two different ways:

   (a) We might randomly present three extra titles besides the ones included in the triple, and which necessarily are not from an economic group in common with titles of that triple (this should be performed after the schools have already selected their titles).

   (b) schools who are not so influenced by sales reps may be willing to search more than three titles before shortlisting the triple. We might thus impose a constraint that they browsed at least six titles of different groups and ask them to rate the six of them for completing the task.

4. Schools who do not exercise their right to choose shall receive titles selected by a low-pay auction, as they are manifestly indifferent to the titles [21]. Note that the questions about WTP should be multiple choice ones, not open questions. For each ordered pair we would present only three values for the differences of WTP. These values mihgt come, for instance, from price differences (not normalized by number of signatures) collected by webscraping in the private market.

5. For each lot we should impute to each ordered pair (first versus second, and second versus third) the median differences[22] of WTP across all schools who did *not* choose any of the titles in their triples and informed these differences in WTP for the same ordered pair. Or we would simply assign amounts decided by the CPA, which could be equal for a same ordered pairs across subjects or could vary according to the subject. Let us call them a *bonus* and denote it by $dW_l\,(1,2)$ and $dW_l\,(2,3)$, where l identifies the lot.

6. Now the MNR shall proceed in the following way:

   (a) The buyer informs each supplier present in the Web catalogue the total demand for each title of their portfolios. It is important to notice that *the bidders are not informed by the Buyer about the distribution of the preferences: neither who ranked them first, second, or third, nor where, nor how many students are in each school of their sets*[23]. They are to be informed, as well, that the demands for titles of the same subject do not intersect.

   (b) Each bidder submits a price curve for each b of its portfolio, such that the unit cost is $p_1(b)$ for the first q_1 signatures bought, $p_2(b) < p_1(b)$ for the next q_2 books bought, and so on, *plus* a fixed common overhead fee K. Therefore to buy q signatures of title b in subject d, such that $\sum_{r=1}^{i-1} q_r(b) \leq q \leq \sum_{r=1}^{i} q_r(b)$, the CPA pays the price

   $$P(q,d,b) = \sum_{r=1}^{i-1} p_r(b)q_r(b) + p_i(b)\Big(q - \sum_{r=1}^{i-1} q_r(b)\Big).$$

   and the total payments from the CPA to the bidder e are

   $$\sum_{bs.t.e(b)=e} P(q,d,b) + K_e$$

   (c) The CPA runs the WDP algorithm (see section 5) and informs to each bidder the numbers of signatures awarded for each title they have submitted, and the corresponding payment to be earned upon delivery (end of first round).

[20] In order not to raise suspicions, we might well require ratings for the six titles, and then simply discard the information about the triple. We might also test for differences in distributions.

[21] More details on that in the sections 5 and 6. For the time being, let us only note that low pay auction means that I do not differentiate WTP across titles, so they compete in equal footing.

[22] As we are restricting the values to three, we will get the intermediate value of the triple most of the times, except when one of the extremes reaches 50% of the responses or more.

[23] Of course, they may observe signals of the choices, as the schools are required to publicize their choices. The CPA claims, however, that they do not observe it immediately: the schools would take time to publicize it and this delay would exceed the elapsed time until the negotiation took place, so the publishers would be able to observe only part of them and infer the remainder. Still this publicization may be getting faster. Besides that, the observation may be asymmetric across publishers, and that is why we tested that in our experiment.

(d) Second round: the bidders are able to revise down all or some of the $p_r(b)$ and or K, and submit them again.

(e) The CPA informs the new allocation. If the MNR has two rounds, the session is closed, a standstill period starts for appeals, and after that period, if no revision of the WDP is called, the contracts are awarded[24]. If the MNR has three rounds, the bidders have a new chance to revise down their prices and fixed fee, the WDP algorithm is run, the final allocation is announced, and everything else is performed in the same chronological order (standstill, possible revision, awards).

3.4 Further considerations based on previous literature and project experience

In this section we will elaborate on further considerations for the design of the solution. We will incorporate insights from the procurement and auctions literature as well as the experiences of our consultants from other projects of their own. We will focus on three issues in particular. First, how to aggregate the preferences of the schools and what can be learned from the purchasing of framework agreements. Second, what are the benefits of employing a multi-stage mechanism. Third, how to manage long-run competition in the market for school books.

Aggregating Preferences. Even though the problem of the CPA resembles the purchasing of framework agreements (OECD, 2011), it is unique in its own way. Framework agreements, sometimes also referred to as catalogue buying, are usually negotiated by a central purchasing unit. The entities belonging to this unit can then choose from a catalogue with the negotiated prices and choose the products that they need. At this the entities operate on a budget or use their own funds to pay for the products. Thus, they are sensitive to the price trade-offs between products that are substitutes. Even though each organization could in principle be in charge of their own purchases, the rationale is that, by managing the procurement process centrally, the procurement agency may be able to exploit the purchasing power of a large buyer while providing variety to satisfy consumers' heterogeneous needs. Therefore, it is not surprising that Framework Agreements have been widely adopted worldwide: in 2010, the European Union awarded EUR 80 billion using Framework Agreements in 2010, equal to 17% of the total value of public procurement in the EU (EU, 2012). As the organizations are price sensitive and the central purchasing authority is not able to manage demand, their only power stems from excluding suppliers from the market. Thus, the central purchasing authority induces price competition by excluding certain suppliers from the catalogue and then letting the users choose themselves among the remaining products.

The main trade-off faced by a procurement agency when buying differentiated products is the following. On one hand, consumers have heterogeneous preferences and, therefore, increasing product variety may increase consumer satisfaction. On the other hand, reducing the number of products bought may increase suppliers' incentives to bid aggressively, so that their products have a better chance to be part of the small selection of items. Saban et al. (2015) studies the optimal mechanism on how to resolve this trade-off. They implement their solution in collaboration with the Chilean government procurement agency. Gur et al. (2017) study the performance of standard auctions for framework agreements. Apart from those contributions the literature is silent on the design of framework agreements. This is surprising given the huge economical impact of those agreements.

What makes the problem of CPA unique and even more challenging than the problems considered in the literature, is that schools are not sensitive to the book prices. The books are delivered to the schools free of charge. Thus, schools, in theory, are rational in choosing their most preferred option and labeling all other options as not acceptable. Therefore, all the competitive pressure has to come from the procurement mechanism of the CPA.

Schools cannot be encouraged to reveal the cardinal trade-offs between different books, because the purchasing mechanism is sensitive to manipulations of this assessment, e.g., by means of lobbying of sales representatives. We can conclude that the discussion in Subsection **??** captures nicely the options available to the CPA to ask for cardinal preferences of the schools given the literature and the constraints faced.

Multi-stage procedures. One of the foremost questions to answer when designing a negotiation procedure is whether to procure with a call for a single (sealed-bid) round of offers or whether

[24]The name *standstill* is mostly used in Europe after the award, but in Brazil administrative appeals are usually taken before award as well.

to opt for a dynamic multi-stage implementation. The advantage of a single round procedure is that with many bidders and complex bid functions preparing and running even a singe round of offers can be a time consuming and challenging task. Thus, a single round of offers reduces those efforts. Moreover, single-stage procedures minimize the scope of tacit bidder collusion. To see this, consider the difference between a descending auction and a first-price sealed bid auction. In a descending auction, if the bidders agreed to stop bidding at the reserve price, this agreement is stable through the threat that if one bidder deviates and bids lower the other bidder observe this and engage in competitive bidding. This is not possible in a sealed-bid auction. In a sealed bid auction, whenever bidders agree to bid the reserve price, each bidder has a strong incentive to undercut this agreement (Robinson, 1985).

The advantage of of multi-stage mechanisms is that they allow for information revelation and price discovery along the way. There are two lines of reasoning for this. First, multi-round auctions in an environment with common values allow bidders to draw inference from the bids of others and update their beliefs about the common value. This leads to a more efficient price discovery and in expectation to lower prices. The seminal model to analyze this was developed by Milgrom and Weber (1982). A recent overview of the vast body of literature can be found in Kagel and Roth (2016). The use of dynamic auctions in Spectrum allocation is mostly motivated by this price discovery effect (See Milgrom (2000) for the Simultaneous Multiple Round Auction — SMRA — and Ausubel and Baranov (2014) for the Combinatorial Clock Auction — CCA) Second, during a multi-stage procedure bidders update their beliefs about their winning probability and engage in costly information-acquisition which was not worthwhile ex-ante. The comparison between English and second-price auction has been carried out by Compte and Jehiel (2007) and Rezende (2018). Both papers find that the dynamic auction yields more information acquisition, efficiency and lower prices. The comparison between Dutch and first-price auctions has been carried out by Gretschko and Wambach (2014) and Miettinen (2013). Both papers find that the dynamic auction yields more information acquisition and higher efficiency. However, the first-price auction can result in lower prices.

One way of addressing the trade-off between the advantages of single- and multi-stage auctions is to conduct sealed-bid auctions over several rounds (**?**). It is important that bidding in the first rounds has consequences for the following rounds. In particular a bidder who bids too high or remains inactive during the first rounds should be excluded from the subsequent round. Otherwise, bidders can behave like a "snake-in-the-grass", i.e., not revealing any information and only waiting on information revealed by other bidders. In this case, multi-stage auctions do not produce the desired effect. A prominent auction that was conducted as a 2-round first-price auction is the auction for the transmission rights of the German soccer league (Bundesliga). The rule was that a bidder who bids 20% above the second-highest bids would receive the rights already in round 1. Thus, serious bidding already in the first round was encouraged. For a description of the rules see the decision of the Bundeskartellamt (Bundeskartellamt, 2015).

Managing long-run competition. Given the economies of scale that are associated with the high fixed costs in the market at hand, the short-run efficient solution is to allocate as much demand to one publisher as possible. As auctions tend to find efficient solutions, the allocation may only include just a small subset of publishers. In the long run this would hurt the CPA, because some publishers may eventually exit the market and the supply structure consequently shrink. Thus, some management of long-run competition is necessary. In general, there are three ways of managing the long-run competition in an auction: subsidies, set-asides, and caps.

The idea of subsidies is problematic. Apart from having to guess the overhead cost, subsidies create sometimes absurd incentives for participation in the auction. That is, bidders may want to participate just for the subsidy without the interest of winning any of the volume. Moreover, if fixed costs are not considered in the winner determination problem, subsidies may lead to many more publishers winning than efficient even in the long run. A good example for such a situation are constant marginal costs. The auction would allocate volume to too many suppliers and result in large subsidy payments. A smarter way of subsidizing a group of bidders is to grant them a bonus. A bonus of $x\%$ guarantees that a bidder in the favoured group would win even if his price is $x\%$ higher. The problem with this approach is to chose the correct group of bidders to subsidize.

A better way to restrict market power is to use set-aside or impose caps. Set-aside are usually used to promote entry. A part of the volume is literally set aside and only a subset of the bidders is allowed to bid on this volume, e.g., new entrants. A problem with this, as above, is to choose the right group of bidders. Caps are a tool to prevent concentration in an already existing market.

A cap rule enforces that none of the bidder can win more then $x\%$ of the market. A through discussion of this instruments in the context of spectrum auctions can be found in Cramton et al. (2011).

In the situation at hand it seems to be that entry is limited and that the objective of the CPA is to safeguard a certain market structure. Moreover, caps cannot be imposed on the lots that are from schools that provided a ranking of the books. This is to ensure that the preferences of schools are honored. Thus, caps seem to be the reasonable choice. For example, the CPA could impose the following cap: no firm is allowed to have more then $x\%$ of the market, unless this share is reached with ranked lots only.

There is a trade-off between safeguarding market structure and achieving attractive prices in the auction. Imposing caps decreases competition in the auction and may lead to auction failure in case of insufficient supply, given that larger bidders are limited. A through discussion of those risks can be found in Marx (2013).

4 The initial chosen procedure

We started with a setting with multi-stage sealed bids:

1. Sellers are informed of the (unidentified) lots in which they are competing, and or of the maximum quantity they can cast.
2. Each seller submit a price curve or a price proposal with "steps" (quantity discounts for each production range)
3. The buyer maximizes the value in relation to the allocation of the lots and returns to the sellers their initially purchased quantities
4. Sellers have a chance to revise their price curves down (even if only part of it) and resubmit them to the Buyer
5. The buyer maximizes value with respect to batch allocation and returns vendors their final allotted quantities (if there are 2 rounds)
6. If there are 3 rounds, repeat steps 4 and 5.

4.1 Constraints and Scenarios

We also tested for the following side constraints:

1. Maximum number of books sold by a single publisher or company;
2. Separate aggregated lots from schools that did not fully rate their preferences. In the scoring auction, all selected publishers can bid, and their rankings are a statistic of all school assessments. Out of these lots, the dispute is for a lower price (since the schools did not express any preference, or did it incompletely).

4.2 Steps of value formation

To circumvent the problem of sales representatives' influence, the following truth-telling mechanism was envisaged: At the time of choice, the school would report:

- 1st option
- 2nd option
- 3rd option

After the school informed the options, the system would draw two more books for the school to evaluate, rank and value the quality difference. These books would necessarily be from other publishing groups not chosen by the school. The reason for that is to not have the information of value contaminated by the influence activities of the publishers. They should focus on having your titles included in the list. We would only use this information of value in the lots of other schools (that had selected those pairs of titles), not in those that informed it.

To raise the response rate to this questionnaire, it was also proposed that the selection process be validated only after the school reports the relative value between the proposed pair of titles, perhaps giving a longer period for it to access the content and respond, or by preventing them from recording this value without accessing the content, or by asking some objective and short questions that would force them to justify the relative note. This policy could also be heralded as a "doublecheck" of which they have dedicated themselves to the choice.

After the closure of the choices, the FNDE would compute some statistics of the relative scores of each ordered pair. For example, you can use the median of the relative notes. This would indicate how much each school would be willing to pay more for a title A in relation to the title B. The conversion of the note into a value in reais (R$) would be agreed with the FNDE in light of its budget availability. Note that in pair (A, B) the difference in WTP (willingness to pay) may be different from that estimated in pair (B, A) because they come from different schools. These differences of WTP we will call "bonuses". Now FNDE's objective was to minimize the cost of its purchases, and to make some trade-off of this lower cost for more quality.

We borrow from the literature of score auctions the concept of Quality Score Monetary Value. We chose a very simple, linear, functional specification. Each price charged for a book b would therefore receive a bonus according to its classification by the school. The third-placed would not receive any bonus (ie the third-place bonus would be normalized to zero). The first-place bonus would be the sum of the WTP differences between the first and second, and between the second and third. The second-place bonus would be only the WTP difference between itself and the third-place. We would then add the schools that chose the same ordered triples in lots.

Lot I chose:
1. A
2. B
3. C

It is different from Lot II, which chose:
1. A
2. C
3. B

And both are different from Lot III, which chose:
1. B
2. A
3. C

And also different from Lot IV, which chose:
1. B
2. C
3. A

Assume that the WTP differences of the pairs are as follows: (A,B) = 2; (B,C) = 1; (A,C) = 1,5; (B,A) = 2,5; (C,A) = 1,5; (C,B) = 2

- Then the bonuses in lot I would be: 3 for A (first-place); and 1 for B (second-place).
- In lot II it would be: 3.5 for A; and 2 for C.
- In lot III it would be: 4 for B; and 1.5 for A.
- In lot IV it would be: 2.5 for B; and 1.5 for C.

We would use the same bonus (A, C) on lots II and III, but it would not work for lot IV. We would also use the same bonus (B, C) on lots I and IV, but it would not work for lot II.

After discussions on the pros and cons of this approach, between the FNDE and the project team, however, it has been assessed that this approach is too costly, since it requires large incremental changes in the system that collects school choices.

It is worth noting that all systems were being redone to internalize all projections of the pupil and the intersection of information of the choices with these projections. FNDE was unable to provide technical personnel for the development of the prototype, as had been agreed at the

beginning of the project. Note also that schools will already have an extra burden in selecting a third title to inform the FNDE. In addition, it is already planned to forbid two or more titles of the same economic group, so as not to distort competition in lots. At present it is forbidden to list titles from the same publisher, but titles from different publishers belonging to the same economic group are still allowed.

For all this, and after consulting all those involved in the preparation of the MNR experiment, it was decided to postpone sine die the implementation of this consultation. This will avoid a burnout in the relationship with schools. Introducing the changes more gradually will allow for an increase in mutual trust, and an intermediate evaluation of the new model with the simplification described below.

In the solution to be adopted in the first year, schools will continue being called to rank the titles. The only changes remain, therefore, that they choose 3 titles from different economic groups. For disciplines that do not have a sufficient number of titles that meet this criterion, the number of titles chosen should be reduced to two.

FNDE continues to collect the ordered triples and discreetly assigns the difference between WTP between the 1st and 2nd titles, and the difference between WTP between the 2nd and 3rd titles. In other words, the bonus to be assigned to the titles only varies with the placement of the title in the lot, but it is restricted to be the same for all pairs of 1st and 2nd place, and between 2nd and 3rd place.

In the example given above, title A would receive the same bonus in lots I and II, because it is the first one; let's call it dW_1. In lot III, their bonus would be different: dW_2. Similarly, title B would earn the same bonus b_1 in lots III and IV but would win the bonus dW_2 in lot I. Title C would win the bonus b_2 in lots II and IV. That makes a total of eight bonuses, that is, two per lot, as it should be. And we would only have two bonus values for all ordered triples.

How do we transpose the information about the choices into the winners' determination module?

Answer: We created the variables ξ_1, ξ_2 and ξ_3 as functions of the lot s, of the discipline of collection b, where $\xi_j = 1$ if and only if the schools in the batch have listed collection b in the j-th position of the collection list of discipline d, $j = 1, 2$ or 3. Then, as a general rule, from the choices made by the schools of the lot s (they are grouped in the same lot because they made exactly the same choices), we can have the sum of ξ_j in the subject d being equal to 3. Note that in particular we can have choices such that the lot has: two $\xi_1 = 1$ and one $\xi_3 = 1$ (first place tie), or one $\xi_1 = 1$ and two $\xi_2 = 1$ (tie in second place), or three $\xi_1 = 1$ (triple tie in first place)

Note also that if the school has not made its choices, it will be assigned $\xi_1 = 1$ for all collections (the school is indifferent to all, so all are the first option).

The determination of the winner is to assign $\nu(s, d, b) = 1$ to collection b when this is the collection to be delivered to the schools of the lot s in the subject d, and $\nu(s, d, b) = 0$ for all the other collections of that discipline in that lot.

Note that the bonuses described above do not always represent the same percentage in the price.

4.3 The score and the Winner Determination Problem

The CPA requires a downward sliding price as the quantity purchased increases. In a bilateral negotiation with inelastic demand, as is the current picture, this is the best way to extract part of the economies of scale of the publishers. The CPA insists on keeping prices down, and therefore the proposal of each publisher is a price curve.

But in a multilateral negotiation this gives an additional advantage to the big publishers, because they compete with the small ones in different scales. We tempered this advantage in three ways:

1. Each publisher quotes separately in its proposal the fixed (avoidable) cost of the collection, K_b, which is paid to it for any positive quantity in the collection b;

2. In addition to the K_b cost, it quotes K_e, which is paid once for the entire contract, provided that one or more collections are sold, no matter which;

3. The price curve defines the marginal price, more precisely the unit price of the tranche. So if in the first tranche (say, from 1 to 10 thousand notebooks) the unit price is 5 and in the second tranche (10,001 to 30,000) it is 4.5, the FNDE pays $K_e + K_e + 5 \times 10,000 + 4.5 \times (q - 10,000)$ if you buy any quantity q within the second tranche.

There are two more features that meet the CPA's desire to remedy the high concentration of the supply structure and its high entry barriers

4. Impose an award constraint, that is, a ceiling for the participation of any economic group, either in quantity of signatures sold or in terms of revenue, both as a market share of the CPA's purchases.

5. For schools that do not exercise their full choices (or make no choice, at all), the choice will be for the lowest price, no bonus. The aforementioned award limit will be cumulative with these lots, and we can also create a floor for awarding to smaller publishers in this lot or cumulatively with the other lots (set asides).

5 The Winner Determination Problem

5.1 The mathematical formulation [Technical]

The WDP looks for a set of books that maximizes a score function. We shall discuss alternatives for the score function in section 6, but here it is important to emphasize that we have to take into account preferences, prices, side constraints (such as market share caps or minimum number of awarded suppliers), and the maximal budget B the CPA is ready to spend to buy these books. Let us call $\tilde{w}(B)$ the optimal value of this WDP.

The WDP will be used in the auction process as follows. At each round, each editor $e \in E$ provides the bid $((p_1(d,b), q_1(d,b)), \ldots, (p_T(d,b), q_T(d,b)))$ for each discipline d and each book b of this discipline from editor e, where T is the maximum number of signature tranches (for the time being, let us assume that all the tranches' sizes are preset by the CPA). For this set of bids, we can determine, by dichotomy, the largest value $\bar{B}$ of the budget above which no increase in the total mark of the books can be obtained. Similarly, we can determine the lowest budget $\underline{B}$ below which no solution to the problem exists.

For a budget between $\underline{B}$ and $\bar{B}$ the maximal satisfaction of the schools is obtained solving the WDP corresponding to this budget. Therefore, all we need to know to determine which books would be bought on the basis of these bids is to determine parameter B in the WDP. A first possibility would be to obtain an approximation of the function $\tilde{w}(\cdot)$ on the interval $[\underline{B}, \bar{B}]$ computing its values for a set of points in this interval. This curve can be used to choose B offering good trade-off between the cost and the total score of the books bought. In a more systematic manner, we can estimate a budget $B(\alpha)$ such that we are at most $100\alpha\%$ below the maximal possible total mark of the books, i.e., to estimate the minimal budget $B(\alpha)$ such that $\tilde{w}(B(\alpha)) \geq (1-\alpha)\tilde{w}(\bar{B})$ for some $0 < \alpha < 1$. Of course if this value $B(\alpha)$ is above the maximal amount of money the CPA can spend to buy the books then we will take $B = B_0$, meaning that in every case we will choose the budget $B = \min(B_0, B(\alpha))$.

Following the format of a discriminatory auction, the unit cost is $p_1(d,b)$ for the first q_1 books bought, $p_2(d,b) < p_1(d,b)$ for the next q_2 books bought, and so on. Therefore to buy q book signatures of title b in subject d, such that $\sum_{j=1}^{i-1} q_j(d,b) \leq q \leq \sum_{j=1}^{i} q_j(d,b)$, the CPA pays the average price

$$p(d,b,q) = \frac{P(d,b,q)}{q} = \frac{\sum_{r=1}^{i-1} p_r(b)q_r(b) + p_i(b)\Big(q - \sum_{r=1}^{i-1} q_r(b)\Big) + K_b}{q}.$$

We define the following decision variables:

- $x_e, e \in \mathcal{E}$ where $\mathcal{E}$ is the set of editors: $x_e = 1$ if the total amount of sales for editor e is greater than zero and $x_e = 0$ otherwise;
- $x_b = 1$ if the amount of sales of collection b is greater than zero, and $x_b = 0$ otherwise;
- $y(d,b,\ell) \in \{0,1\}, d = 1, \ldots, D, b \in \mathcal{B}(d), \ell = 1, \ldots, m$: $y(d,b,\ell) = 1$ if and only if some books of type b for subject d are bought with price $p_\ell(d,b)$ per signature;
- $z(s,d,b,\ell) \in \{0,1\}, s \in \mathcal{S}, d = 1, \ldots, D, b \in \mathcal{B}(d), \ell = 1, \ldots, m-1$: $z(s,d,b,\ell) = 1$ if and only if some books of type b for subject d are bought with price $p_\ell(d,b)$ for school s;

- $n(s, d, b, \ell)$: number of signatures of title b of subject d bought for school s with price $p_\ell(d, b)$;
- $\nu(s, d, b) = 1$ if the school s receives book b for subject d

The list of exogenous variables (i.e., they are provided by the CPA as "parameters" of the problem) is the following:

- $\xi_i(s, d, b) = 1$ if school s ranks b as i-th choice, $i = 1, 2, 3$
- $w(s, d, b)$: the net utility (consumer surplus) enjoyed by the school s when consuming title b of subject d. Note that this utility may depend on the total budget available B.

It is important to notice that we are using the school as the unit of consumption for allocation purposes. But actually the schools are grouped into lots. These lots will receive the same books, and the sum of their demands is what matters both for accumulating the quantities along the tranches and for grouping books with the same (pairwise) WTP differences.

Thus, henceforth one should read "lot" wherever we refer to "school".

We obtain the WDP:

$$
\begin{cases}
\max \sum_{s\in\mathcal{S}} \sum_{d=1}^{D} \sum_{b\in\mathcal{B}(d)} \sum_{\ell=1}^{m} w(B, s, d, b) n(s, d, b, \ell) \\
\sum_{\ell=1}^{m} n(s, d, b, \ell) = \mathcal{D}(s, d) t_b \nu(s, d, b), \ \forall s, d, \ell, \\
\sum_{b\in\mathcal{B}(d)} \sum_{\ell=1}^{m} \frac{n(s, d, b, \ell)}{t_b} = \mathcal{D}(s, d), \ \forall s, d, \ell, \\
y(d, b, \ell) \geq z(s, d, b, \ell) \ \forall s, d, b, \ell, \\
0 \leq \sum_{s\in\mathcal{S}} n(s, d, b, \ell) \leq y(d, b, \ell) q_\ell(d, b) \ \forall d, b, \ell, \\
\sum_{s\in\mathcal{S}} \sum_{d=1}^{D} \sum_{b\in\mathcal{B}(d)} \sum_{\ell=1}^{m} p_\ell(d, b) n(s, d, b, \ell) \leq B, \\
\sum_{s\in\mathcal{S}} \sum_{d=1}^{D} \sum_{b\in\mathcal{B}(d):\mathcal{E}(d,b)=e} \sum_{\ell=1}^{m} p_\ell(d, b) n(s, d, b, \ell) \\
\leq P_e\Big(\sum_{s\in\mathcal{S}} \sum_{d=1}^{D} \sum_{b\in\mathcal{B}(d)} \sum_{\ell=1}^{m} p_\ell(d, b) n(s, d, b, \ell) \Big), \forall \text{ editor } e, \\
y(d, b, \ell) \in \{0, 1\}, z(s, d, b, \ell), \ \forall s, d, b, \ell, e.
\end{cases}
$$

5.2 Interpretation of the WDP

Description:

1. We minimize the sum of the fixed costs plus the amounts paid in each tranche according to the allocations of the books, with their respective numbers of signatures, among the various lots of schools, deducting from each book its respective bonus, which depends on its position in the triple list and which are the other books in this list, also in each batch;
2. We cannot allocate more than one collection in the same discipline to schools of the same lot;
3. The tranche in which the signatures of the last copy are located must correspond to the accumulated number of signatures of all the copies allocated to the lots awarded for that collection;
4. The number of signatures of the copies allocated to the schools in each subject must meet the estimated demand for the student's projection (be the total per collection / school, or for all collections);
5. The signatures of a collection for a lot of schools may span more than one tranche;
6. The same tranche may accumulate more than one lot of schools in one discipline;

7. An awarding constraint applies;

8. There is still missing a constraint for the set asides;

9. The minimization is with respect to the indicators $\nu(s, d, b)$, which, as already mentioned, only assume values 0 and 1 (and it is 1 only when collection b of discipline d is allocated to schools in lot s). Prices are inputs of the problem, and are submitted by the publishers throughout the negotiation.

As seen above, the demands of schools are yielded by the forecast of the number of pupils. The indicators ξ and dW are imputed by a decision rule from the CPA (which, in turn, should be based on budgetary criteria and on what is learned from experiments conducted with students). The number of signatures in each book comes from the Textbook Guide. Note that dW is only assigned to pairs in which an order exists. When two or more collections tie, not only dW does not exist, but also necessarily: EITHER $\xi_2 = 0$ (tie in 1st place) OR $\xi_3 = 0$ (tie in second place) OR all titles tie.

In summary, the Winner Determination Problem (WDP) consists of a program of optimization (more precisely, minimization) of acquisition costs, allowing for bonuses for better evaluated books (these bonuses soften price competition between books, as if they were applying "preference margins" to collections preferred by schools). The program receives as inputs from Simec (the CPA's platform):

1. Selected book lists (coded in variables ξ_1, ξ_2, and ξ_3) by lot;

2. Differences in willingness to pay for ordered pairs of books (defined by the CPA);

3. Requests for copies of each discipline in each batch of schools (student's projection);

4. Number of signatures of each collection.

   The program receives as inputs from the publishers:

5. Price curves in 19 tranches, fixed costs per collection (K_b) and by publisher (K_e).

The program produces as outputs:

- For publishers: the total number of copies awarded in each collection (even with the publications of the schools' choices, the publishers will not be able to know in which batches they were selected as first, second or third choice, or even at a lower price in case of indifference, at most, consulting SIMAD or its successor, they will find out which schools your books will have been sent to);

- For the CPA:

  – the quantities of copies of each collection in each discipline in each lot (which are always equal to zero or equal to the demand of that lot), or, equivalently, the $\nu(s, d, b)$, multiplied by the demands of the produce one of these two results;

  – the amounts paid per signature of each collection in each tranche (which allows to calculate the average price and total expenses); and

  – the bonuses assigned in each lot to the respective collections.

The CPA receives from Simec which schools make up each lot in each subject (note that lots may be completely different for each subject). Knowing which collections were assigned to each lot, and knowing which schools make up the lot, then by consequence (thanks to WDP restrictions), by sending a number of copies of that collection equal to the demand of each school, the sum of those numbers in the lot necessarily equals the quantities calculated in output 2(a) above.

6 The Value Function

We start from a simple demand function for a book based on price and perceived quality u. WLOG, let a school s rank their ordered triple $(1, 2, 3)$ (book 1 is the most preferred title). Regardless of the method used for imputing to this triple their WTP differences, or bonuses, let $dW(d, 1, 2)$

and $dW(d,2,3)$ be the respective differences between books 1 and 2, and between books 2 and 3, within the catalogue of discipline d, as informed by **all** the schools.

Now let us assume that we can calculate a maximum WTP $\bar{w}(B,s,d)$ for each school, that is, the maximum welfare enjoyed when using the most preferred title (in our example, title 1) of subject d. We do not have any proposal to have such a huge number of estimates[25], so let us start with a simple rule of thumb. Let $\bar{w}(B,s,d) = \bar{T}(d) \times \bar{V}(\bar{T},B) = \bar{w}(B,d) \forall s$, where $\bar{T}(d)$ is the average number of signatures for a book of the subject d (or the number that the CPA deems as optimal according to some rule of thumb of their own)[26] and $\bar{V}(\bar{T},B)$ is the maximum average price per signature that the CPA deems fair, given a budget B. The latter function establishes thus the most direct connection between the available budget B (or $B(\alpha)$) and the WTP of the CPA on behalf of the schools. Thus, any utility function that is decreasing in price and increasing in the WTP will do. For example it could be a linear function such as:

$$\begin{aligned} w(B,s,d,b) = \bar{w}(B,d) - dW(d,1,2) \cdot \xi_2(s,d,b) \\ -(dW(d,2,3) + dW(d,1,2)) \cdot \xi_3(s,d,b) - \beta(d) \cdot p(d,b,q^b) \cdot t_b = \\ = a_b - \beta(d) \cdot p(d,b,q^b) \cdot t_b \end{aligned} \tag{4}$$

where t_b is the number of signatures of one unit of book b, q^b is the total quantity of signatures bought by the CPA, and $\beta(d)$ is a parameter also to be calibrated from external sources, and which may vary with the subject d. Now, notice that we can see $w(B,s,d,b)$ as a unit demand by a student. In this sense, we can normalize $\beta(d)$ to 1, so that the student enjoys the maximum WTP $w(s,b)$ when consuming book b at price zero, necessarily enjoys a positive surplus when the price is between 0 and a_b, and can maximize their surplus by choosing the b that gives them the greatest surplus. Also notice that the demand is for books, not signatures. That is why we use a utility function based on prices scaled up by the number of signatures. Finally, notice that for emulating a low-pay auction for those schools that did not exercise their right to select an ordered triple, the element a_b is equal to $\bar{w}(B,d)$ for all books b competing in subject d, and all books are eligible for those schools (or lots).

We may come up with other specifications that preserve the same features:

1. It is increasing in the WTP (proxy for quality)
2. It is decreasing in price.

This includes simple specifications such as:

1. Quadratic: $w(B,s,d,b) = \left(a_b - \beta(d) \cdot p(d,b,q^b) \cdot t_b\right)^2$
2. Log-linear: $w(B,s,d,b) = ln(a_b) - ln(\beta(d) \cdot p(d,b,q^b) \cdot t_b)$ (where a_b is the multiplication of $\bar{w}(B,d)$ by multiplicative discount factors, i.e., percentage discounts)

Notice that the schools may be aggregated into lots (or strata, if you prefer). This diminishes the dimension of the summations to a more manageable order. Also notice that as long as the value (net utility) function is separately additive in $p(d,b,q^b)$, i.e., it can be described by some function:

$$w(B,s,d,b) = \kappa(\bar{w}(B,d), dW(d,1,2), dW(d,2,3)) - \chi(\beta(d), p(d,b,q^b) \cdot t_b),$$

we can transform the maximization of $w(B,s,d,b)$ into a minimization of acquisition costs:

$$\lambda(B,s,d,b) = \chi(\beta(d), p(d,b,q^b) \cdot t_b) + \tau(\bar{w}(B,d), dW(d,1,2), dW(d,2,3) - \bar{w}(B,d),$$

where $\tau(\cdot)$ is a function decreasing in dW.

Therefore, we would convert the maximization of the WDP in section 5 into a minimization of $\lambda(B,s,d,b)$[27]. For example, in the linear specification, the CPA would have a WDP like the

[25] In a universe of 40,000 schools and 10 subjects, we would need 400,000 estimates. Of course, we have to aggregate the schools into lots, and for 13 books, there are $\frac{13!}{10!} = 1,716$ different lots for each subject. Again, in a universe of 10 subjects, we get $1,716 \times 10 = 17,160$ estimates.

[26] An advantage of this rule of thumb is that the CPA is able to induce the industry to converge to a reasonable book size; the current rule privileges books with more pages and therefore more signatures.

[27] By the way, notice that $\bar{w}(B,d)$ is a constant, so we can safely drop it from the minimand without affecting the solution, therefore we do not have to care about a good estimation of the maximum WTP.

following[28]:

$$
\begin{cases}
\min \sum_{s\in\mathcal{S}}\sum_{d=1}^{D}\sum_{b\in\mathcal{B}(d)}\sum_{\ell=1}^{m}(p_\ell(s,d,b)n(s,d,b,\ell)+\sum_e K_e x_e+\sum_b K_b x_b \\
-\sum_b\sum_{b'\neq b}\xi_1(s,d,b)\xi_2(s,d,b')\mathcal{D}(s,d)t_b\nu(s,d,b) \\
\times\Big(dW(s,d,b,b')+\sum_{b"\neq b'}dW(s,d,b',b")\xi_3(s,d,b")\Big) \\
-\sum_b\sum_{b'\neq b}dW(s,d,b,b')\xi_2(s,d,b)\xi_3(s,d,b')\mathcal{D}(s,d)t_b\nu(s,d,b) \\
\sum_{\ell=1}^{m}n(s,d,b,\ell)=\mathcal{D}(s,d)t_b\nu(s,d,b),\ \forall s,d,\ell, \\
\sum_{b\in\mathcal{B}(d)}\sum_{\ell=1}^{m}\frac{n(s,d,b,\ell)}{t_b}=\mathcal{D}(s,d),\ \forall s,d,\ell, \\
y(d,b,\ell)\geq z(s,d,b,\ell)\ \forall s,d,b,\ell, \\
0\leq\sum_{s\in\mathcal{S}}n(s,d,b,\ell)\leq y(d,b,\ell)q_\ell(d,b)\ \forall d,b,\ell, \\
\sum_{s\in\mathcal{S}}\sum_{d=1}^{D}\sum_{b\in\mathcal{B}(d)}\sum_{\ell=1}^{m}p_\ell(d,b)n(s,d,b,\ell)\leq B, \\
\sum_{s\in\mathcal{S}}\sum_{d=1}^{D}\sum_{b\in\mathcal{B}(d):\mathcal{E}(d,b)=e}\sum_{\ell=1}^{m}p_\ell(d,b)n(s,d,b,\ell) \\
\leq P_e\Big(\sum_{s\in\mathcal{S}}\sum_{d=1}^{D}\sum_{b\in\mathcal{B}(d)}\sum_{\ell=1}^{m}p_\ell(d,b)n(s,d,b,\ell)\Big),\forall\text{ editor }e,) \\
y(d,b,\ell)\in\{0,1\},z(s,d,b,\ell),\ \forall s,d,b,\ell,e.
\end{cases}
$$

where

- $dW(s,d,b,b')$ is the difference in WTP between books b and b', when these books are ranked exactly in this order, i.e., $b\succeq b'$ (b is weakly preferred to b'); it is calculated and provided by the CPA (and works as a parameter); notice that it may be zero, and this only happens when $b\sim b'$ (i.e., when the school is indifferent between b and b'), and also note that if there is a tie, there are two $\xi_1=1$ or two $\xi_2=1$ or three $\xi_1=1$. For such lots, either one of the bonus (in case of a single tie) or both of them (in case all options are deemed equal) are zeroed. Also notice that for the lots of schools which did not state any preference for some d, we shall also set $\xi_1=1,\forall b$, given this d. For this particular lot, there is no bonus at all, because both $\xi_2=0$ and $\xi_3=0$.
- P_e is the award constraint valid for any firm e

Notice that we rebate the bonus dW_1+dW_2 for each lot where the book is bought and is a first choice, and we rebate only dW_2 when the book is bought and is a second choice. Also notice that this notation also applies wherever the school is indifferent between the first and the second choice, because then $dW_1=0$, and likewise $dW_2=0$ when it is indifferent between the second and the third choice. This reinforces the effect of having one of the ξs equal to zero.

7 Experimental design

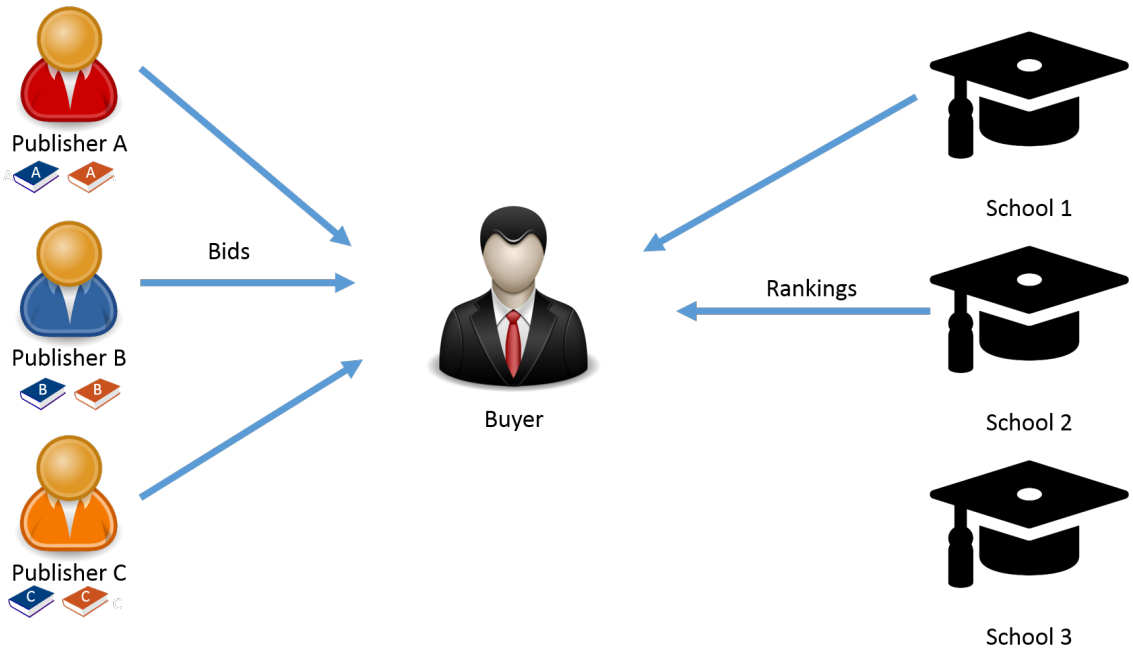
7.1 Basic setup

In the experimental setting, three[29] ex-ante symmetric publishers (Publisher A, Publisher B, and Publisher C) compete to provide 12 million books for *History* and 12 million books for *Math* to the

[28]Note that the indicator functions for ranks do not cover all lots, because for the so-called residual no WTP difference has been reported. This signifies that the whole residual lot will be awarded to the lowest price. We can also run a low-pay non-uniform auction separately for this lot. If we do so, publishers already operating in a high scale tranche will not start from the lower price related to that tranche. On the other hand, if we keep the residual lot in the same WDP, and since we use K_e, smaller publishers may reap greater scale economies by being awarded this residual lot, which is approx. 11% of the market for each subject.

[29]Even though the number of bidders is substantially larger in reality, we should keep the number of bidders as small as possible in the experiment. First, having more bidders implies that we need a larger total number of

Figure 2: Procurement situation



buyer. The buyer needs to purchase 12 million books for each subject. This quantity is allocated across 3 schools (School 1, School 2, and School 3). Each school needs books for *History* and *Math*. In what follows we describe the baseline treatment and comment on the other treatments below. All monetary values are expressed in experimental currency units (ECU). An illustration of the situation is provided in Figure 2.

7.1.1 Schools

In the experiment the schools are automatized. School 1 and School 2 attribute different values to different books in each subject and submit rankings representing their preferences. School 3 attributes the same value to the books of each publisher in both subjects. Hence, school C does not submit a ranking to the buyer.

If, for example, school 1 prefers the book of Publisher B over the book of Publisher A and the book of Publisher A over the book of Publisher C for *Math*, School 1 submits the following ranking R_1^{Math}=(Publisher B, Publisher A, Publisher C). This ranking implies that the buyer assigns an advantage a^{Math} to Publisher B and that Publisher C is not allowed to bid for the demand of School 1.

We assume that schools' preferences are independent both across schools and across subjects. For each publisher each position in each ranking has a probability of 1/3.

Furthermore, let d_i denote the demand of school i for books. We assume that each school has a demand of 4 million books for each subject.[30]

participants which can be problematic given the rather small subject pool. Second, increasing the number of bidders and thereby the level of competition will probably reduce effect sizes making the empirical analysis more complex.

Having more bidders implies observing more bids, but we rather care about the performance of different mechanisms, here the number of observations is independent of the number of bidders. Then having more bidders is just more expensive, as it requires more participation.

The best way to examine the influence of competition in an experiment is to vary the level of competition.

[30] Assuming this symmetry allows us to keep bid vectors reasonably small. Publishers need to submit 6 bids per period.

If, for example, we assume that there is a total demand of 10 million books and School 1 and School 2 both have a demand of 4 million books, publishers must submit a bid vector containing 10 bids in order to cover all possible quantities.

If we introduce randomness regarding the school size the number of bids becomes much greater. Even if we have a setting in which the size of School C is known and School A and School B have either both a demand of 4 million (symmetric case) or have a demand of 2 and 6 million (asymmetric case) the decision becomes much to complex. Furthermore, it is not clear what effect the uncertainty should have. Our consultants' experience suggests that such a design would result in pure noise. Furthermore, introducing heterogeneity here reduces the comparability of

7.1.2 Buyer

In the experiment, the buyer is automatized and is not played by a participant. The buyer aggregates the rankings of the schools and creates up to three lots for each subject depending on the ranking. A publisher ranked first for a given subject receives a bonus a^{Math} or $a^{History}$ respectively, which will be considered in the awarding decision. A publisher ranked second is allowed to place a bid and a publisher ranked third is not allowed to bid for the specific lot.

If, for example, Publisher B is ranked first and Publisher is C ranked second for *Math* the buyer will select Publisher C if the price per million books Publisher C requests is less than a^{Math} larger than the price per million books that Publisher B requests.

7.1.3 Size of the lots

Independent of the ranking of schools there is always a lot of size 4 million books for each subject representing the demand of school 3. From the point of view of the publishers, the lot sizes of school 1 and school 2 will depend on whether they were ranked first, second or third by this school. Thus, even though there is no uncertainty about the size of the schools, there is uncertainty about the demand from the point of view of the publishers. The demand is either 4, 8 or 12 million books.

7.1.4 Publishers

Each of the three publishers is played by one participant in the experiment. The cost of publisher $i \in \{A, B, C\}$ are given by a fixed cost K^j per subject with $j \in \{History,\ Math\}$[31] and variable costs per million books c_i^j, i.e.,

$$C(q_i^{History}, q_i^{Math}) = K^{History} \cdot \chi(q_i^{History}) + c_i^{History} \cdot q_i^{History} + K^{Math} \cdot \chi(q_i^{Math}) + c_i^{Math} \cdot q_i^{Math}.$$

Here $q_i^j \in \{0, 4, 8, 12\}$ denotes how many million books j are provided by publisher i and $\chi(q_i^j)$ is an indicator function such that $\chi(q_i^j) = 1$ whenever $q_i^j > 0$ and zero otherwise. The costs c_i^j are independently and identically distributed for all publishers and all books. The fixed costs K^j are assumed to be the same for all publishers.

- Variable costs c_i^j are uniformly distributed between 0 and 100 ECU.
- $K^j = 10$ ECU for all publishers and both subjects.

7.2 Information

Prior to the procurement mechanism, the buyer observes his value for each of the books for each of the publishers. The publishers observe their own variable cost of production c_i^j for each of the books j but not the cost of production of their competitors. Moreover, the publishers observe the total quantity of books they can bid on, but not their ranks for the lots. Depending on the treatment, the publishers also observe their ranks.

7.3 Procurement mechanism

7.3.1 Bids

Depending on the rankings a publisher submits a bid vector containing between 2 and up to 10 entries $(b_4^{History}, b_8^{History}, b_{12}^{History}, b_4^{Math}, b_8^{Math}, b_{12}^{Math})$. To rule out losses, bids are required to be not smaller than costs, i.e. $b_{i,q}^j \geq K^j + q \cdot c_i^j$. The bids are capped by a reservation price equal to the highest possible cost realization of 100 ECU plus fixed costs.

bidding behavior in different periods.

[31] If publishers do not automatically demand K in addition to their bid, we should have a subject specific fixed cost. Otherwise it is not clear where to add the fixed costs or we need to allow for package bids which are very cumbersome to place.

7.3.2 Winner determination

For the determination of the winning publisher for the lots of School 1 and 2 we consider the bonuses $a^{History}$ and a^{Math} assigned to publishers ranked first. The value adjusted bid of the publisher $i \in \{A, B, C\}$ ranked first for subject $j \in \{History,\ Math\}$ is given by $b_i^j - a^j \cdot q_i^j$. The value adjusted bid of the publisher k ranked second for book j is given by b_k^j. The procurement mechanism selects the bids that minimize the sum of the value adjusted procurement costs for the lots of School 1 and School 2 and the non-adjusted prices for the lots of School 3.

7.3.3 Timing – Stopping Rule

The procurement mechanisms consist of several rounds in which publishers submit bids. Let k denote the number of rounds and let t denote the current round.

1. In rounds $t < k$ publishers submit their bids and observe their (preliminary) share. They do not observe the bids or shares of their competitors.
2. In round $t = k$ publishers submit their final bids and observe their share.

A publisher's bid for a certain book must be (weakly) decreasing over time, i.e. publishers are not allowed to increase their bids in later rounds.

8 Treatments

8.1 Within-subject variation

In the first half of periods each subject participates in a series of two-round mechanisms. In the second half of periods each subject participates in a series of three-round mechanisms. In order to control for order effects, we employ a counterbalanced design. This implies that half of the participants start with a series of three-round mechanisms.

- Stopping rule:
  (a) 2 rounds
  (b) 3 rounds

8.2 Between-subject variation

- Information:
  (a) Publishers observe if they are allowed to bid
  (b) publishers observe their ranks
- Set asides/Quotas:
  (a) Each publisher can provide up to 24 million books
  (b) Each publisher can provide at most 8 million books per subject

9 Organization of the experiment

9.1 Recruitment

Before the experiments start all potential participants must register in the subject pool. Once the subject pool is set up, members of the subject pool can be invited to experimental sessions. If possible, invitations should be session specific to avoid selection effects. The assignment to session should be random and the number of subjects invited to a specific experimental session should be as small as possible to further reduce selection effects. Participants should not be able to sign up for an experimental session as groups. Each member of the subject pool must only be allowed to participate in a single experimental session.

9.2 Experimental procedure

The experimental sessions must be conducted as anonymous as possible. At least, participants must not be able to communicate or to identify each other during the experimental sessions. Participants must know that there will not be deception, unlike in psychological experiments, and how actions translate into payments. At the beginning of the experiment each participant receives written instructions and reads them on his or her own. Afterwards these instructions is read aloud by an experimenter in front of the participants to ensure public knowledge. Participants then have the opportunity to ask the experimenter questions regarding the instructions in private. When all questions are answered, participants need to answer a quiz in private. The quiz tests if each participants understands the instructions. Only when all participants answered all quiz questions correctly the experiment starts.

9.3 Cohorts

Let a *cohort* denote the group of participants that interact in one experimental session. Each cohort provides one independent observations. At the beginning of each period the members of one cohort are randomly assigned to different markets. Given that three publishers interact in each market, each cohort should consist of at least 12 participants to rule out repeated game effects. In order to have enough statistical power to conduct non-parametric tests we need at least 6 cohorts per treatment.[32]

9.4 Duration

One experimental session should not last longer than two hours including the payment of subjects. Previous experience shows that the preparation (registration of participants, seating, reading the instructions, answering questions, and quiz) takes between 15 and 30 minutes depending on the complexity of the setting.

In most related experiments subjects participate in a sequence of 25 to 50 periods. Having a large number of periods allows us to observe subjects behavior for several different cost realizations. Furthermore, participants get accustomed to the game and can learn. In many experiments subjects behavior gets closer to equilibrium predictions over time.

- Quiz to ensure understanding
- 50 periods[33]
- Payment equal to the sum of profits of each period

Most related experiments consist of 30 to 50 periods. While it was the norm to pay each period, in many new experiments only a few rounds are paid to avoid supergame effects. The payoff relevant periods are then randomly determined at the end of the experiment.

10 Good experimental practice

The design is based on former studies that conduct lab experiments to test real world market design. See, for example, Burtraw et al. (2009); Goeree and Holt (2010); Ledyard et al. (1997); Plott (1994).

[32] Given that we want to test the influence of the stopping rule with a within-subject design and the influence of information and set asides with a between subject design, we need at least $2 \times 2 \times 12 \times 6 = 288$ participants. Brunner et al. (2010); Holt et al. (2007) report that subjects participated in more than one treatment.

[33] The duration of the experiment is mainly determined by the time participants need to understand the instructions and answering the test questions correctly. Reducing the number periods would only help little to reduce the duration of sessions. The reason to run 25 or 50 periods is not necessarily to increase statistical power, it rather allows for learning and to observe time trends. Having many rounds also ensures that we can observe all relevant situations for each subject. The more random draws we have, the more observations we need. If we really want to randomly draw school sizes we need much more observations compared to the case where school sizes are fixed.

In order to reduce duration and noise we should keep the design as simple as possible.

If necessary we can have subjects who participate in more than one session. However, we should try to limit it to 2 participations if necessary 3 participations. It is preferable to that no subject participates in one treatment more than once.

11 Experiment outcomes

Please refer to the *Report of the Experiments* by Moacyr Silva and Otto Nascimento.

12 Interpretation of outcomes

Please refer to the file *Textbook Procurement Experiment.pdf*

13 Final model

In light of the experiments reported, we hereby propose the following *Multilateral Negotiation Regime*:

1. The Ministry of Education publishes the Request for Candidacies and proceeds the shortlisting stage with the assistance of a committee of experts and practitioners; the outcome is the Textbook Guide.
2. FNDE defines the amounts for each bonus and the award constraint, such that:
   (a) No publisher is allowed to be awarded a share greater than a cap P except if the whole share corresponds to lots where the books have been ranked first or second option.
   (b) The bonus is only enjoyed by the publisher in lots where their book is ranked first or second without any tie.
3. FNDE publishes the rules of the MNR and schedules the deadlines for the submission of the sealed bid price curves for each set of collections owned by each publisher and included in the Textbook Guide. The rules include a simple description of the WDP formula, the award constraints, the description of the rounds, instructions for login, and the publication of a table with the amounts of bonuses.
4. On the first date of the published schedule the sealed bids are opened. FNDE runs the system described by the system of equations **(6)** (section **??**) and returns to each publisher the quantity (greater or equal to zero) of each book collection, the set of which will produce the amount awarded to the corresponding publisher.
5. Publishers are allowed to revise down any segment of their price curves and resubmit them (Second sealed bid round).
6. Again, quantities along each publisher's price curve are announced. Contracts for these quantities are finally awarded.

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