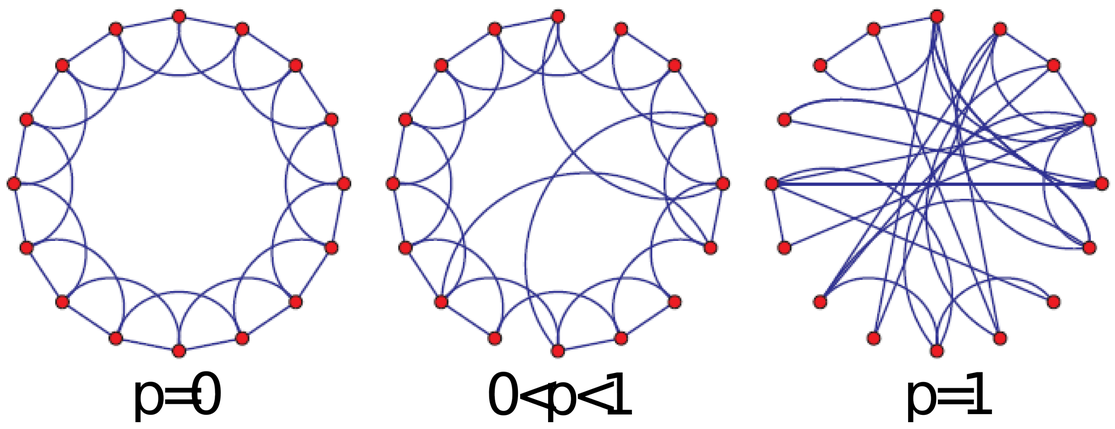
Bifurcation Analysis of an Opinion Formation Model in a Small-World Network

Orahcio Felício de Sousa, Marcio Argollo de Menezes e Thadeu J. P. Penna

Instituto de Física (Niterói) e Instituto de Ciências Exatas ICEx (Volta Redonda) - Universidade Federal Fluminense

The Model

In this opinion formation model (Bagnoli, 2009), an individual can change its opinion by influence of their neighborhood. The opinions are represented by spins $\sigma_i \in 0, 1$. The interactions between the individuals follow a Watts-Strogatz small-world (SW) model, with a fraction p of random edges. The opinion formation model is of Ising type, with antiferromagnetic pair interactions modeling anticonformism, and ferromagnetic plaquette terms modeling the social norm constraints.



Dynamics

The influence of the neighbors is given by

$$h_i = \frac{\sum_{ij} a_{ij}\sigma_j}{k_i}$$

where a_{ij} comes from the adjancency matrix and k_i is the number of neighbors of the i^{th} individual. The individuals change its opinion according the following rules:

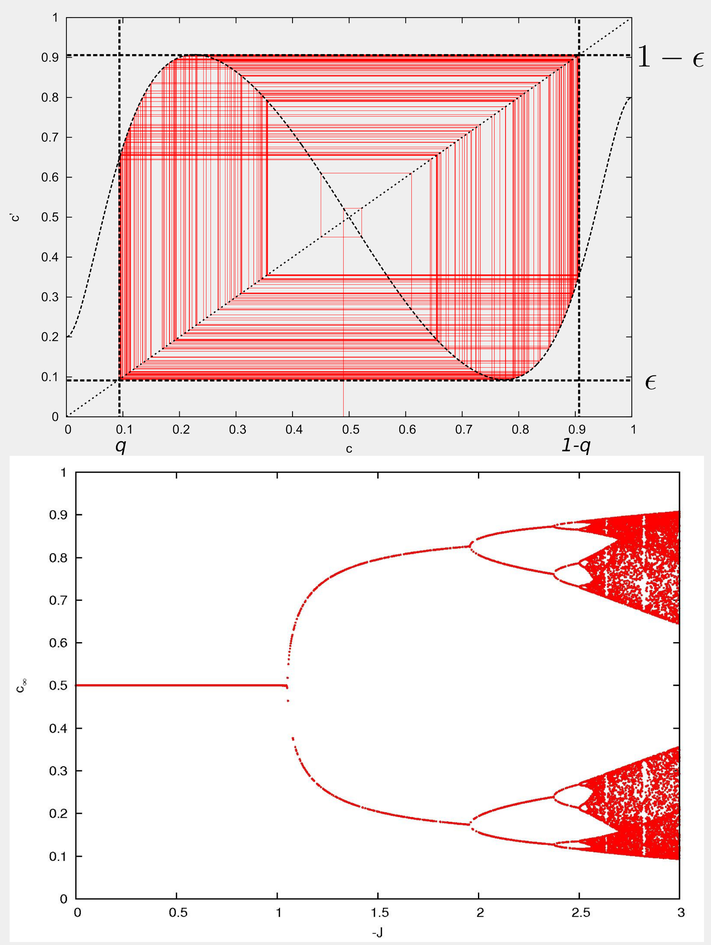
$$\tau(1|h) = \begin{cases} \epsilon & \text{if } h < q \\ (1 + \exp(-2J(2h-1)))^{-1} & \text{if } q \leq h \leq 1-q \\ 1-\epsilon & \text{if } h > 1-q \end{cases}$$

The quantity of interest is $c = \sum \sigma_i$, the density of active sites.

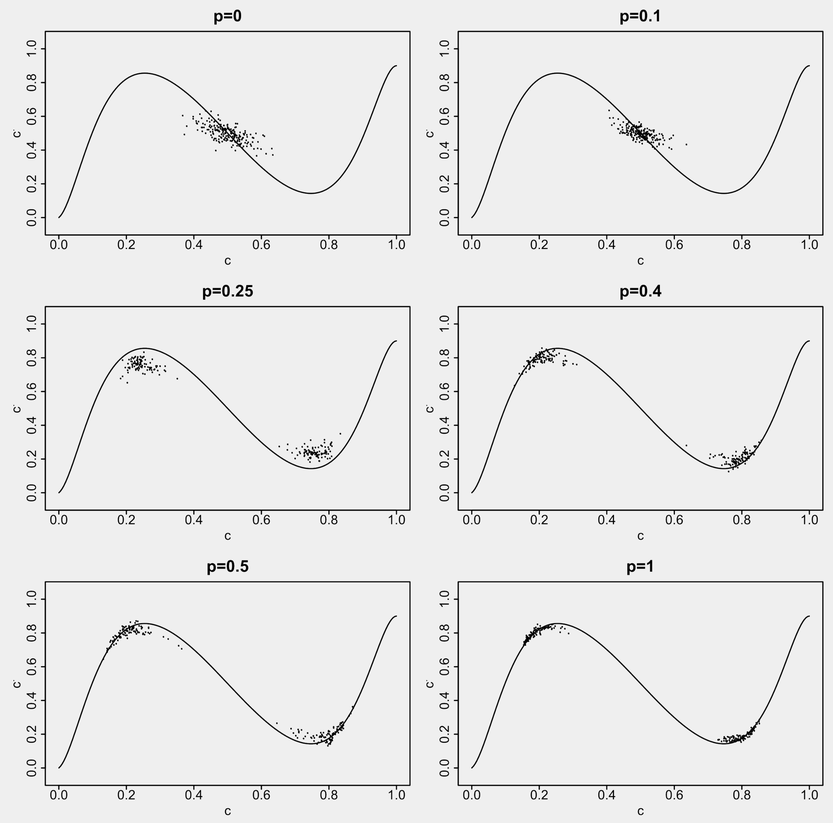
Special Limits

- For $\epsilon = 0$ (infinite plaquette terms) we have the absorbing states $s = 0$ and $s = 1$ if the social pressure is above (below) the threshold $1 - q$ (q), respectively.
- $J > 0$ or ferromagnetic interactions model a conformistic society while for $J < 0$ (anti-ferro) model an anti-conformistic society. Whereas in the ferromagnetic case leads to two absorbing states, the anti-ferro leads to an active phase where the former absorbing states are unstable.
- $q = 0$ and $q = 1$ represents the kinetic Ising model.

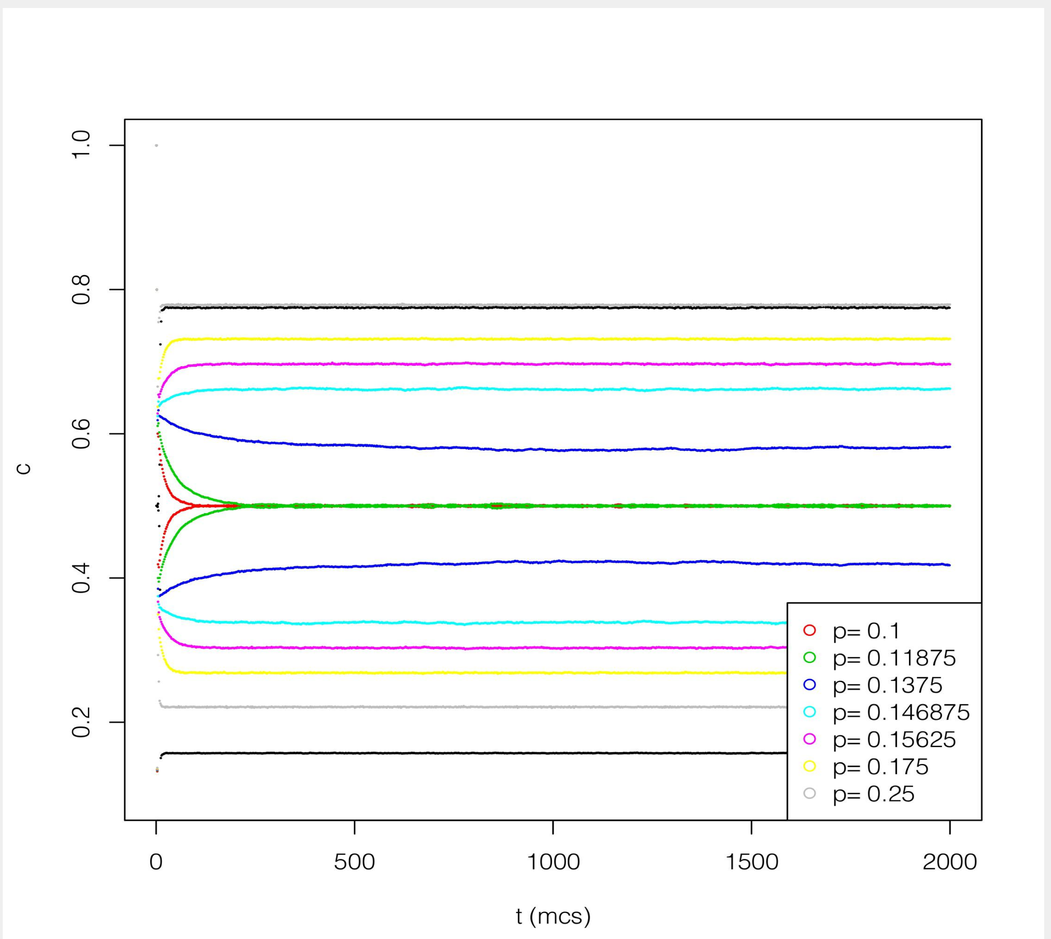
Mean Field



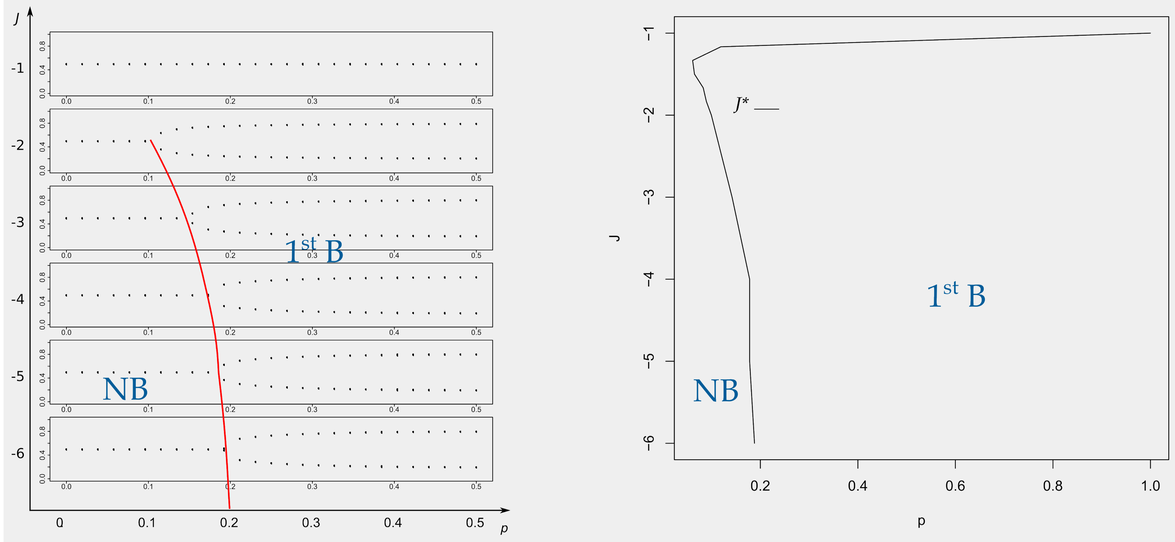
Results in a SW network



Time Evolution



First Bifurcation



$N = 5 \cdot 10^5$, $K = 20$, $\epsilon = 0.1$, $q = 0.1$

Conclusions

- $p = 1$ approaches the mean field predictions;
- Bifurcation points takes a long simulation to equilibrium;
- Phase diagram suggests a small p value for the $J^\star$ value predicted in the mean field approach.
- The geometrical transition observed by Argollo et. al at $p = 0$ does not appears to be modified by opinion dynamics.

References

- Watts, D.J.; Strogatz, S.H. (1998). Collective dynamics of 'small-world' networks. Nature 393 (6684): 409–10
- F. Bagnoli, F. Franci and R. Rechtman, Phys. Rev. E 71, 046108 (2005)
- Bagnoli, F., Barnabei, G., Rechtman, R. 2009, arXiv:0909.0117
- M. Argollo de Menezes, C. F. Moukarzel and T. J. P. Penna. Europhys. Lett. 50 574, (2000).