Dynamic Ising Model: Reconstruction of Evolutionary Trees

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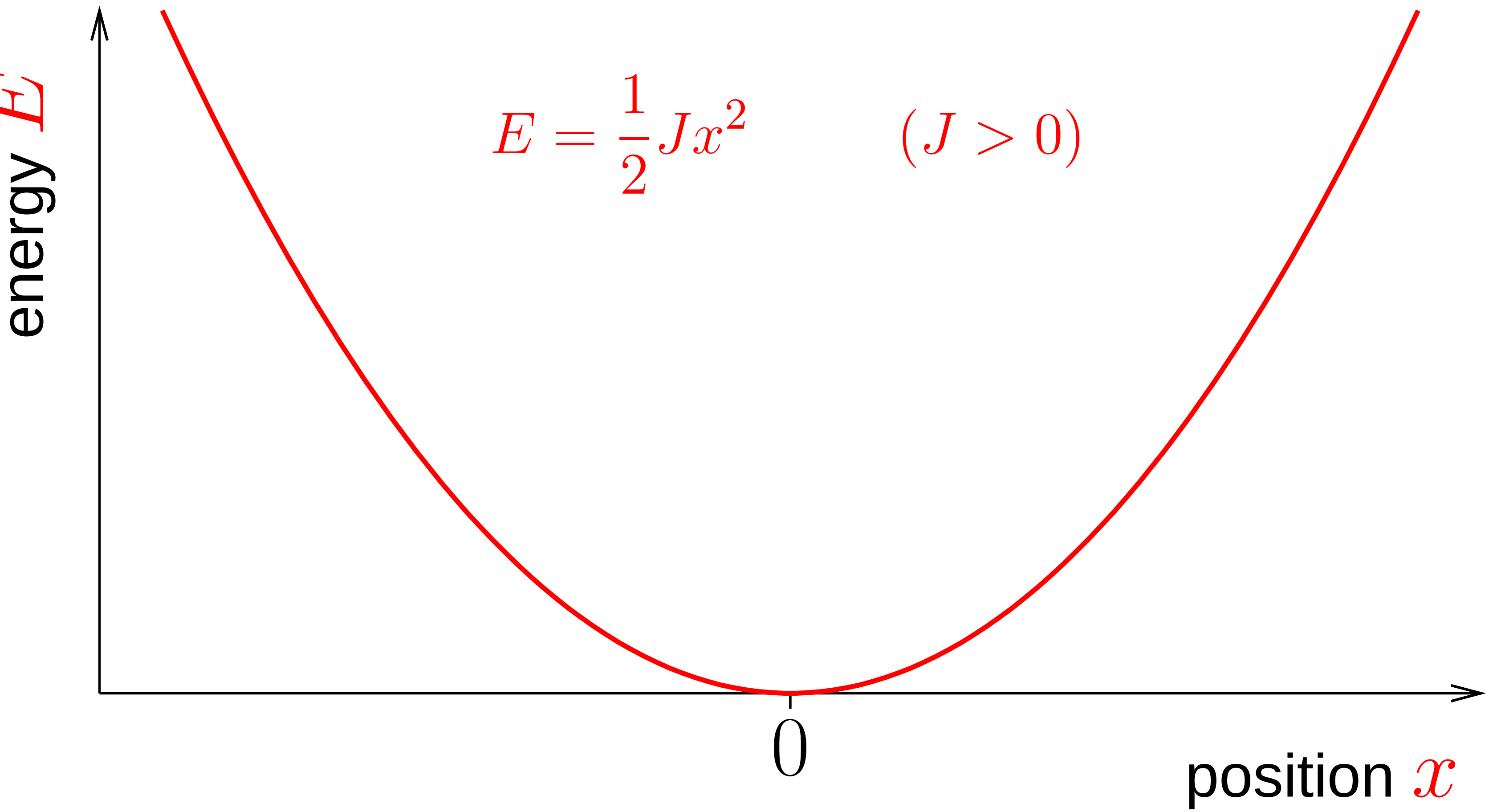
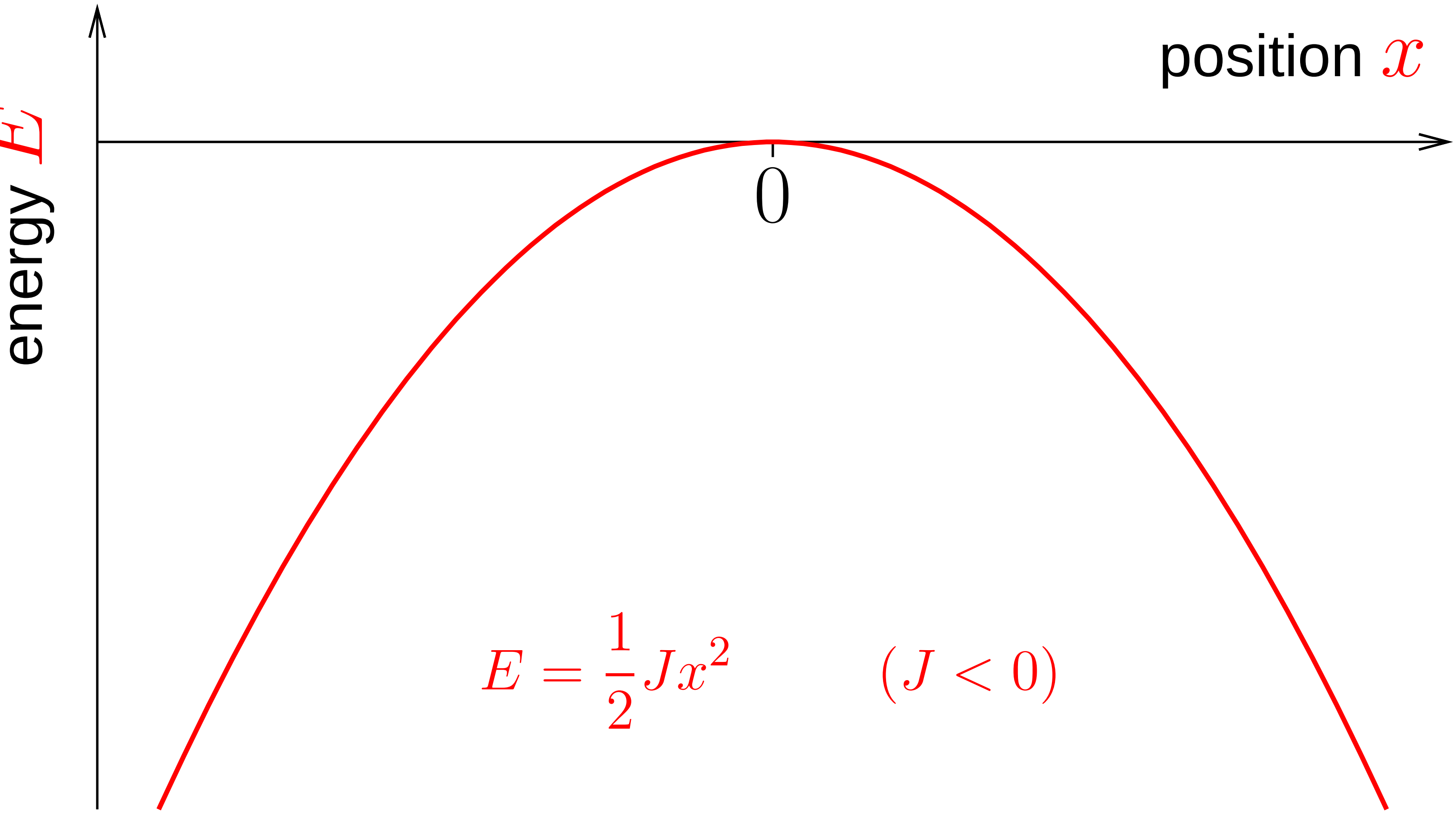
One particle (confinement)



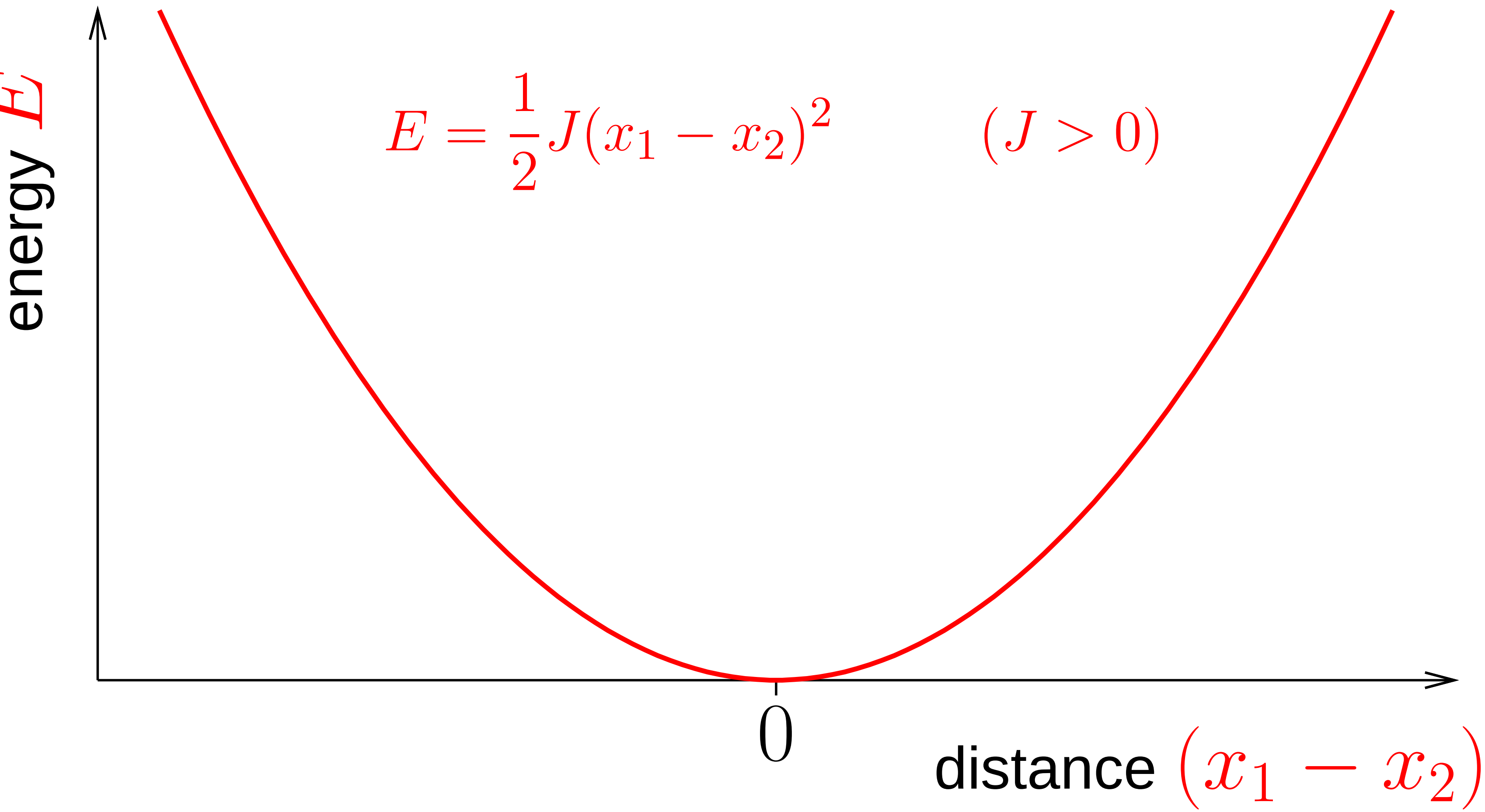
One particle tied to the origin by a spring.

One particle (runaway)



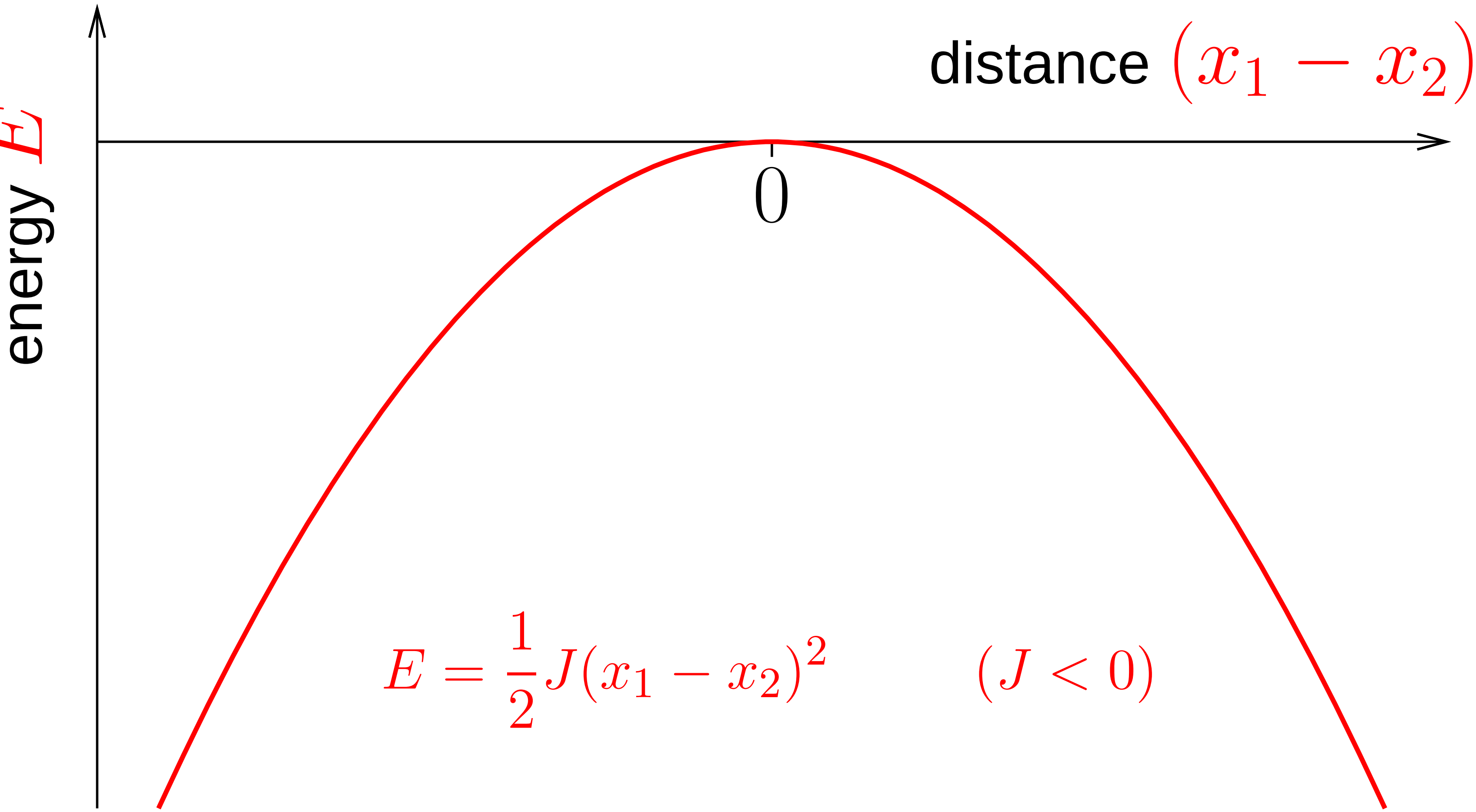
One particle repelled away from the origin by a anti-spring.

Two attracting particles



Two particles tied to each other by a spring.

Two repelling particles



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$$\text{update}: \quad \begin{cases} x_i(t+\Delta t) = x_i(t) + v_i(t)\Delta t + \frac{1}{2}\frac{F(x_i)+F(\bar{x}_i)}{2}\Delta t^2 \\ v_i(t+\Delta t) = v_i(t) + \frac{F(x_i)+F(\bar{x}_i)}{2}\Delta t \end{cases}$$

Verlet

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Onsager’s exact solution (1944) for the thermodynamic behaviour of the uniform ferromagnet in two dimensions is a paradigmatic scientific achievement.

Back to the current model

$$\frac{1}{2}J_{ij}(x_i - y_j)^2 = \frac{1}{2}J_{ij}x_i^2 + \frac{1}{2}J_{ij}x_j^2 - J_{ij}\,x_i x_j$$

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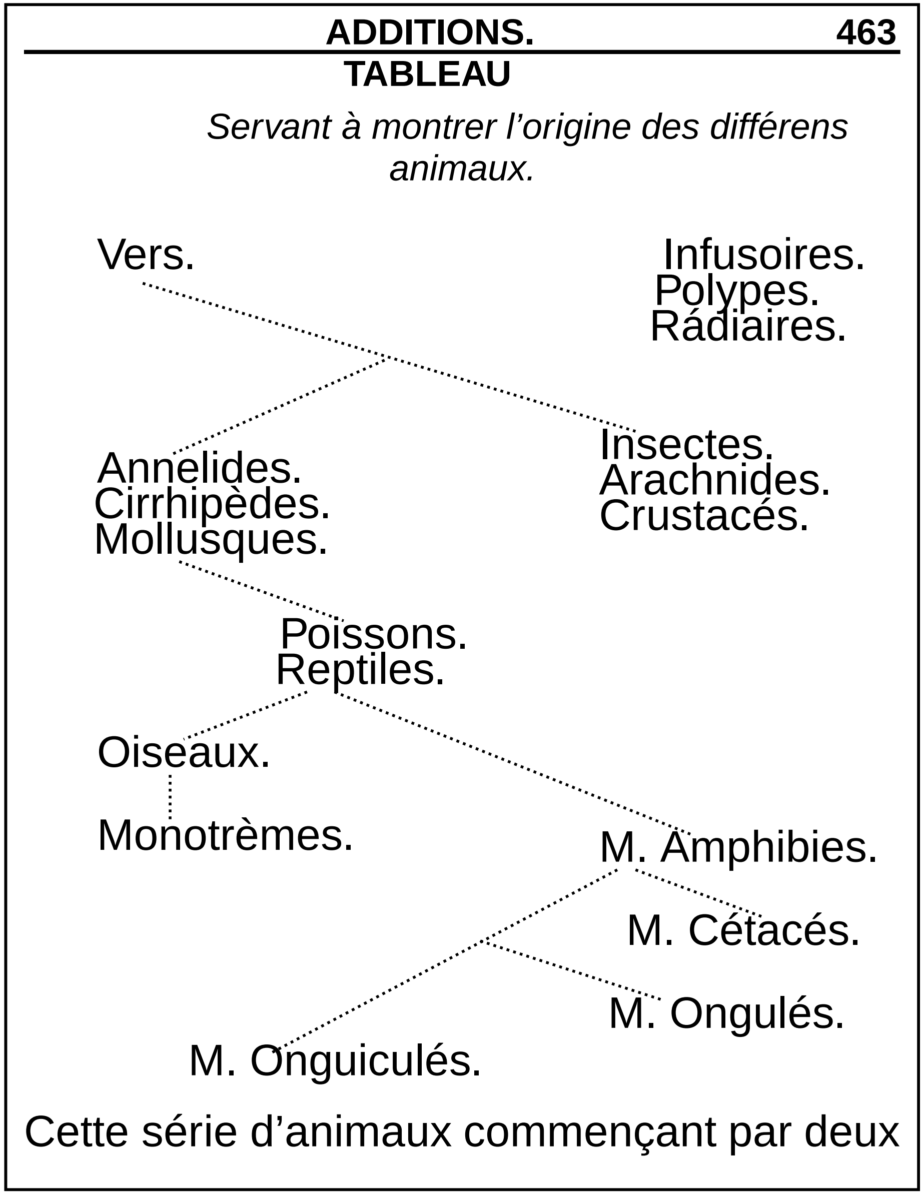
That is why I called it Dynamic Ising Model.

The first evolutionary tree



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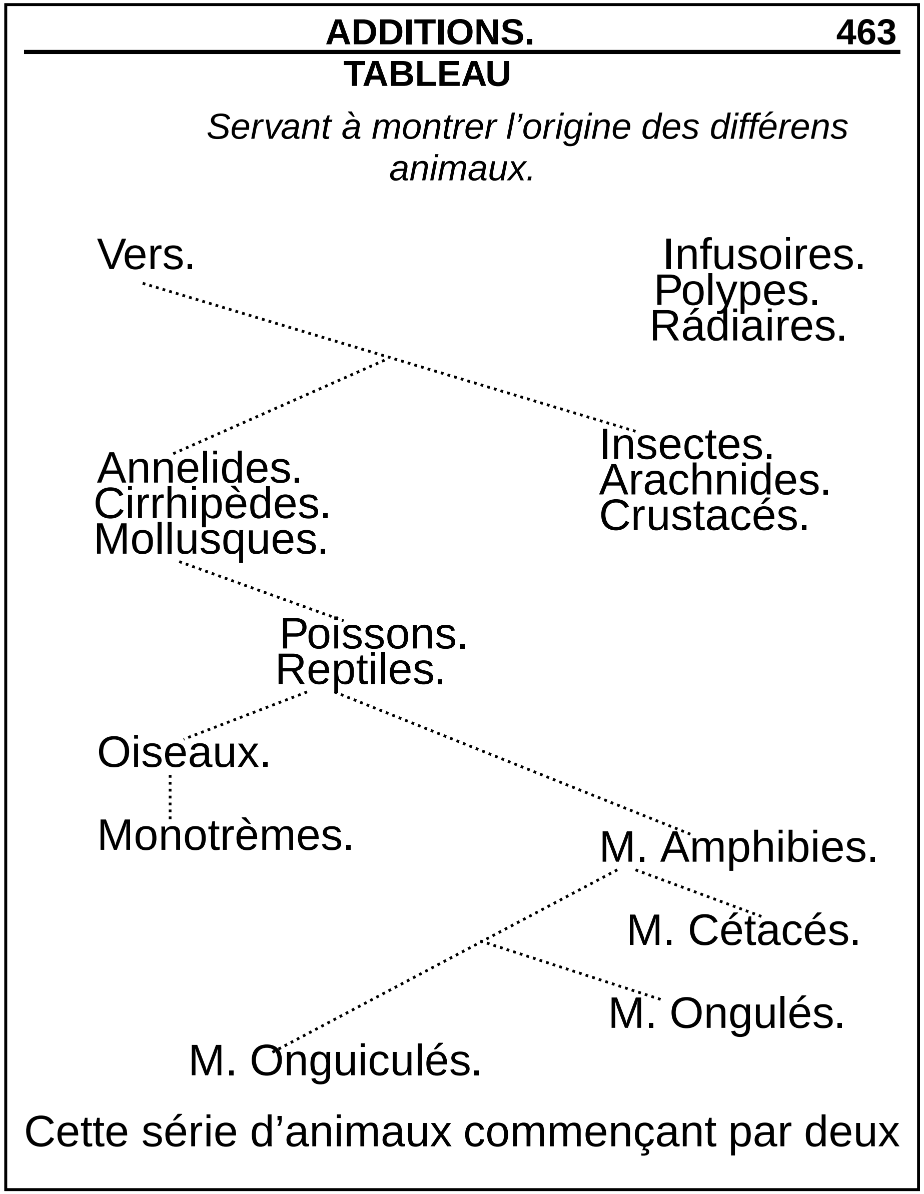
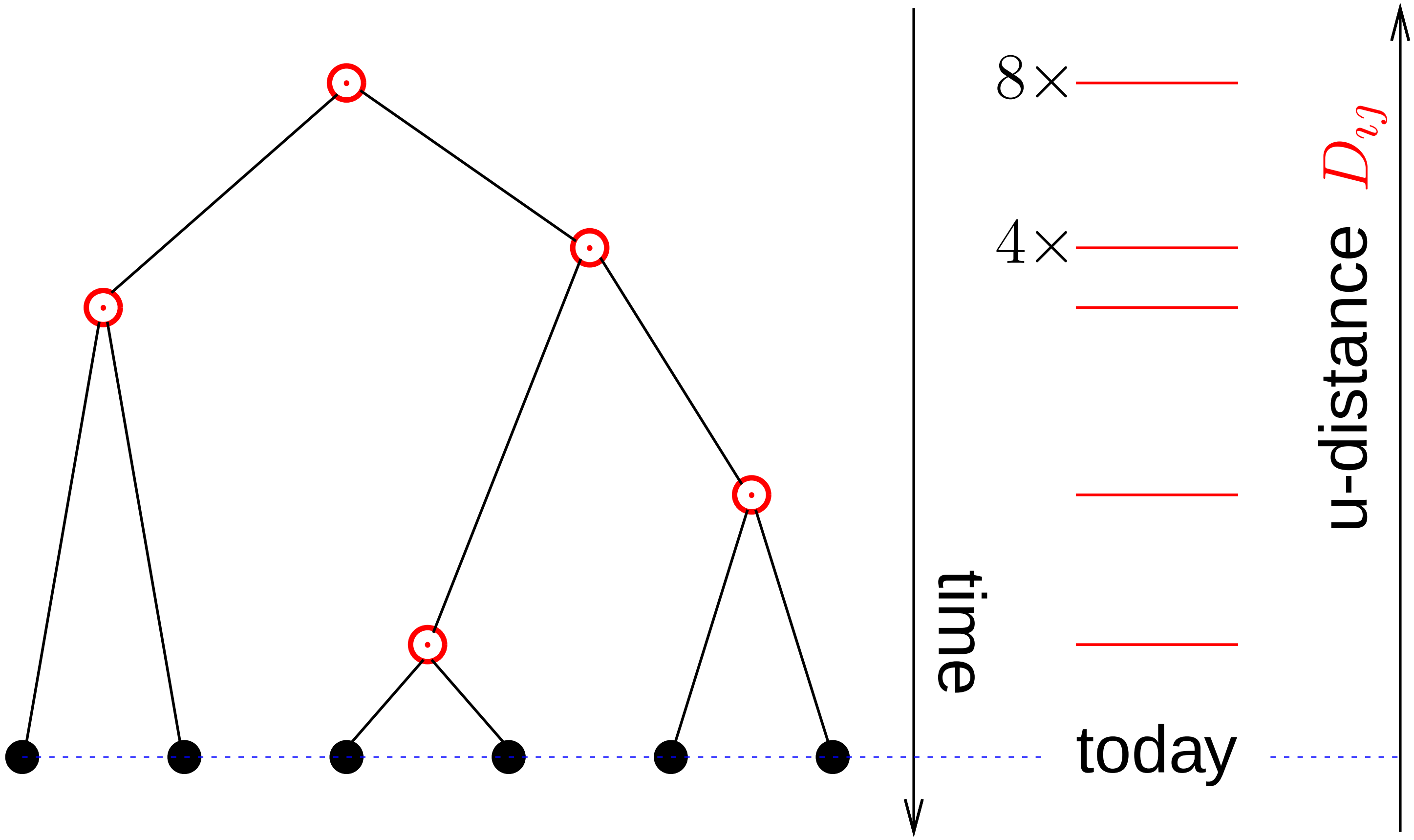


La Phylosophie Zoologique

Lamarck (1809)

Darwin's birth year

Another evolutionary tree



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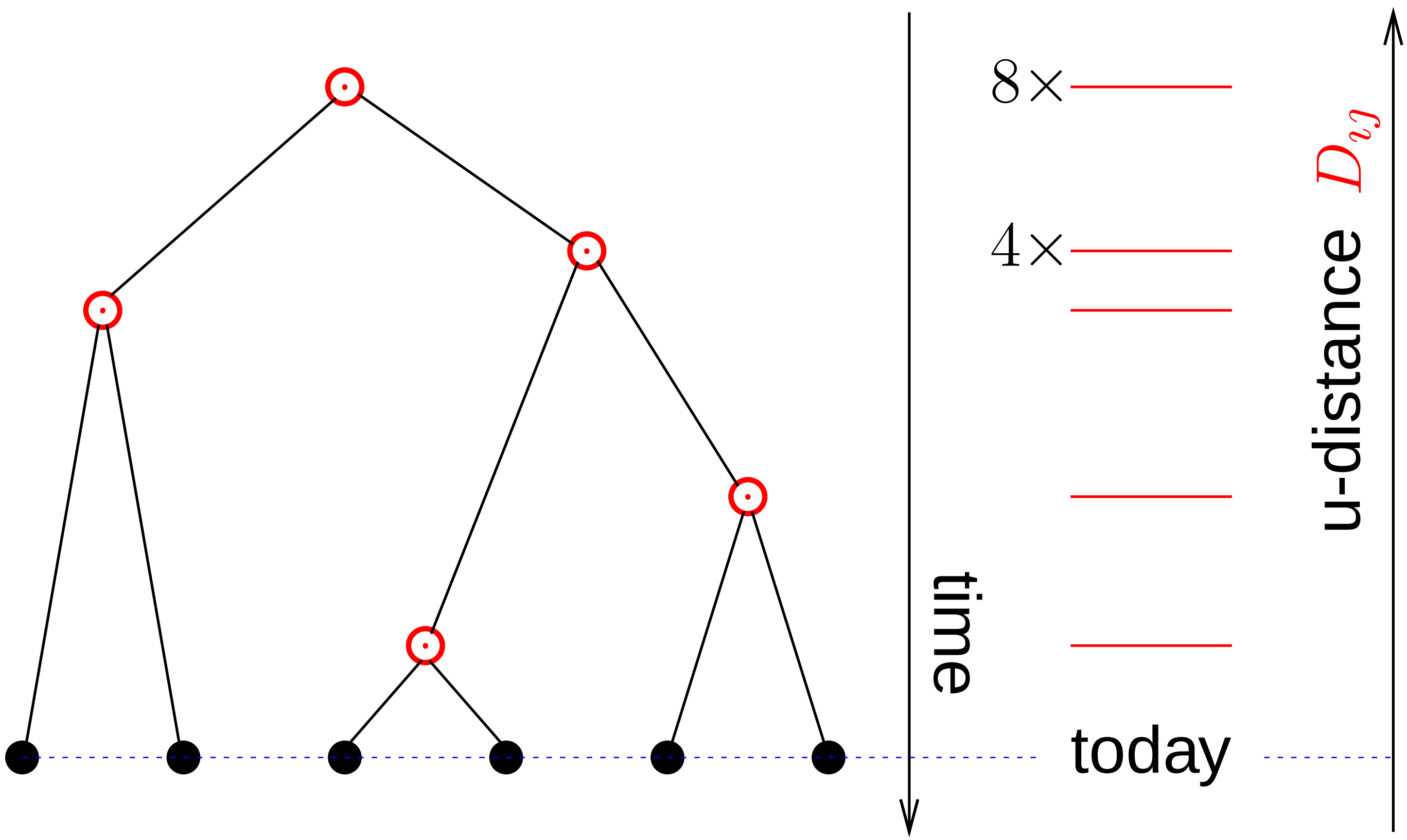
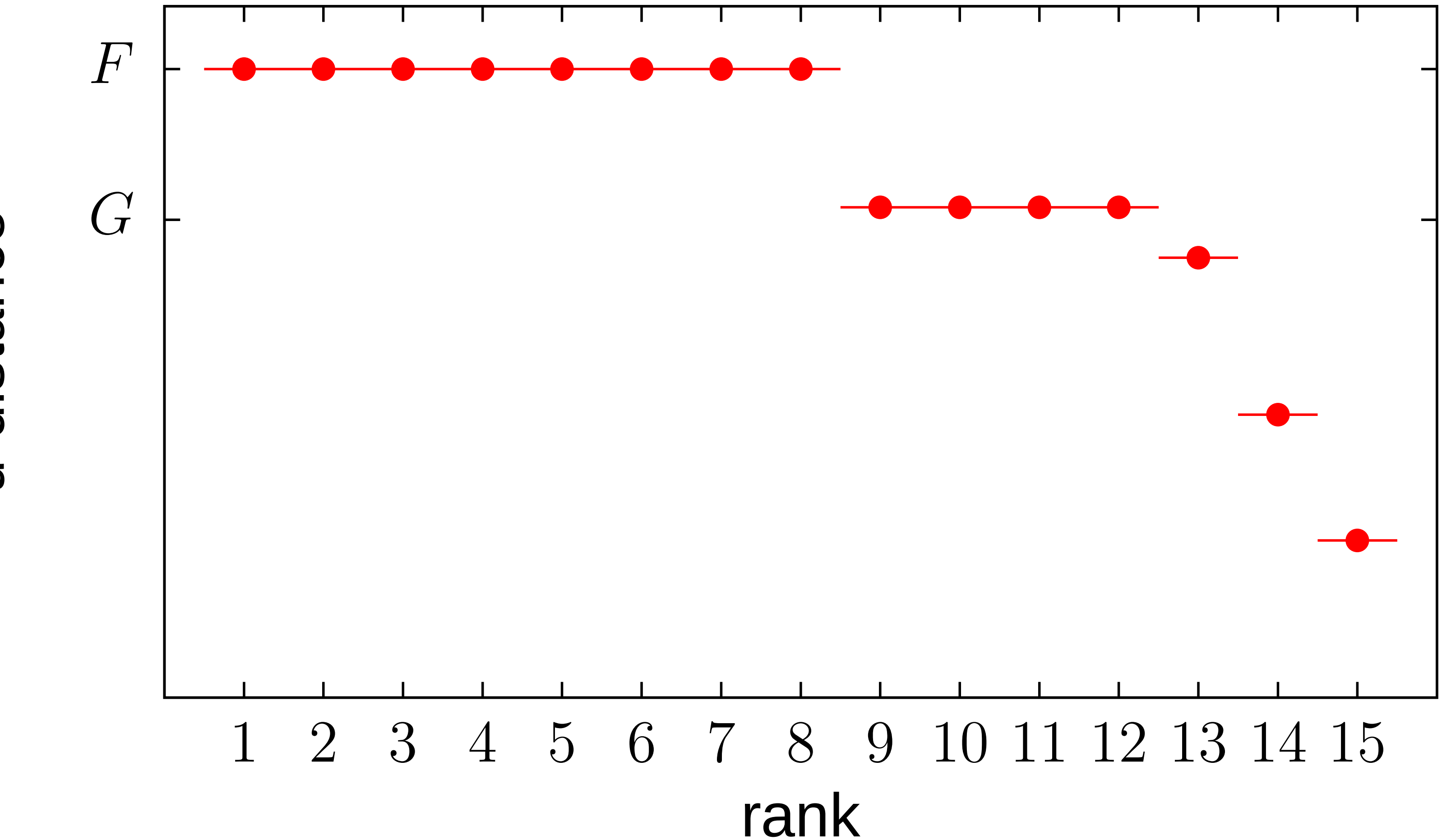


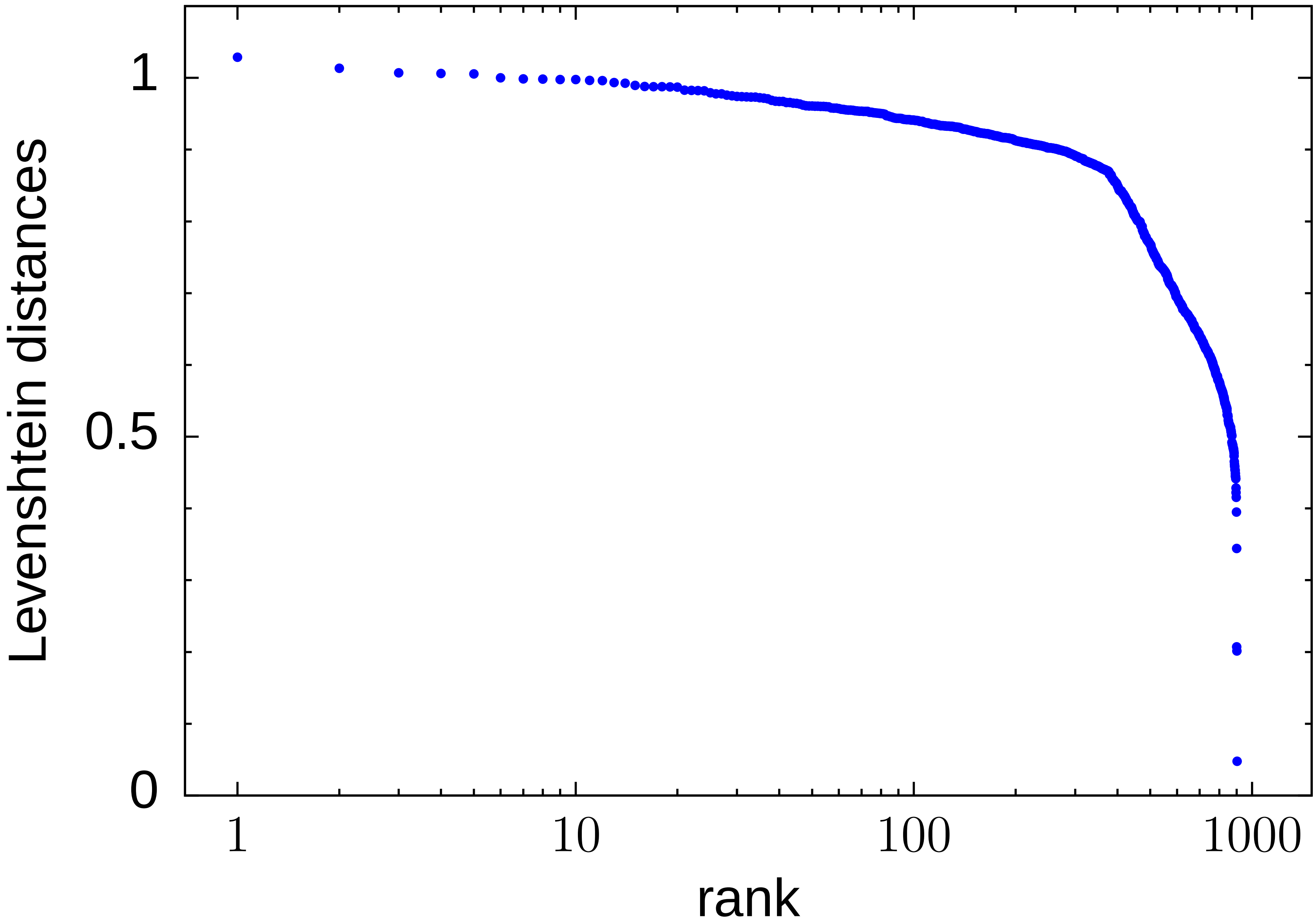
$$J_{ij} = D - D_{ij}$$

Parameter D is adjusted in between the two higher levels.

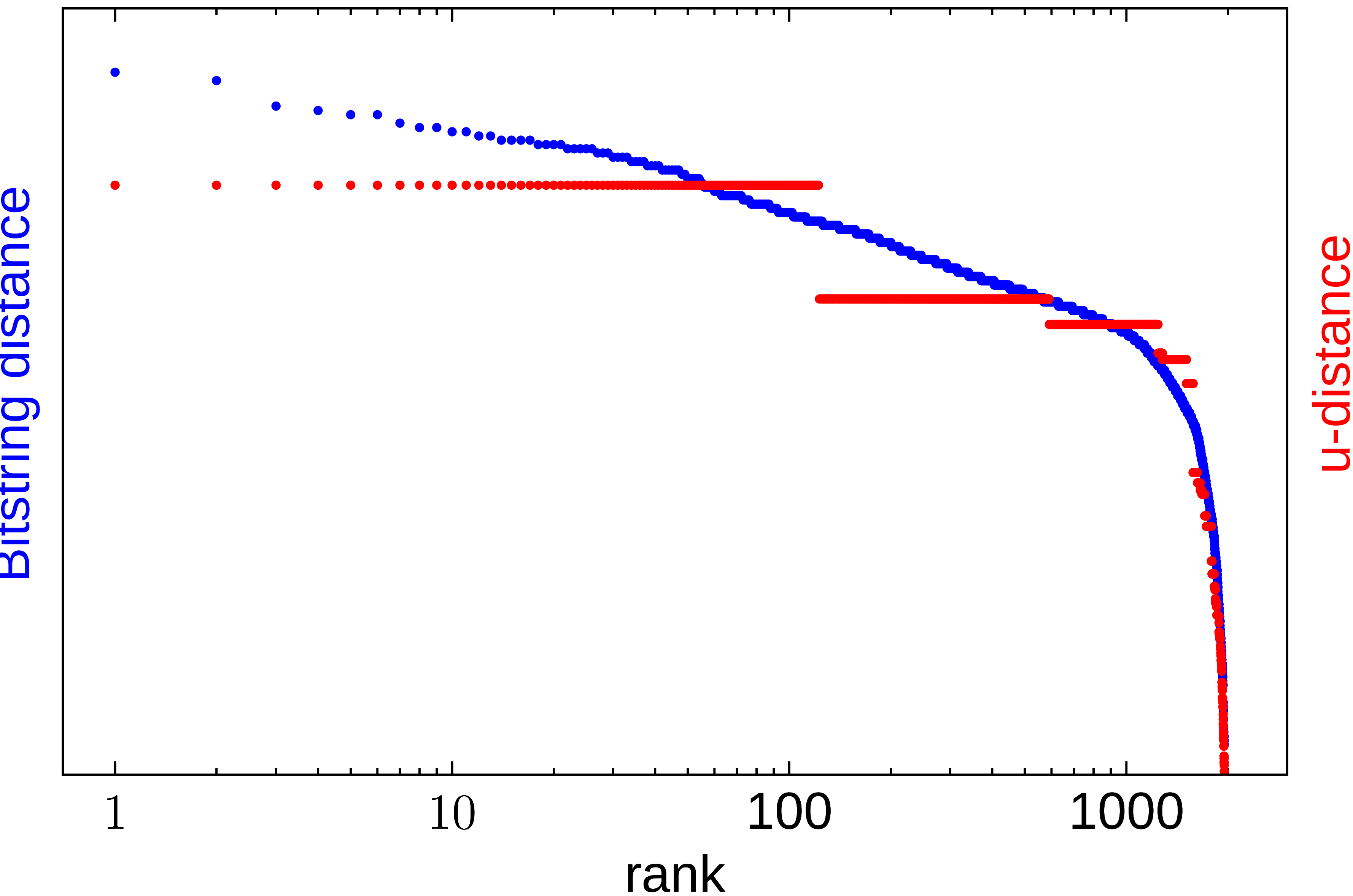
Rank plot, same tree



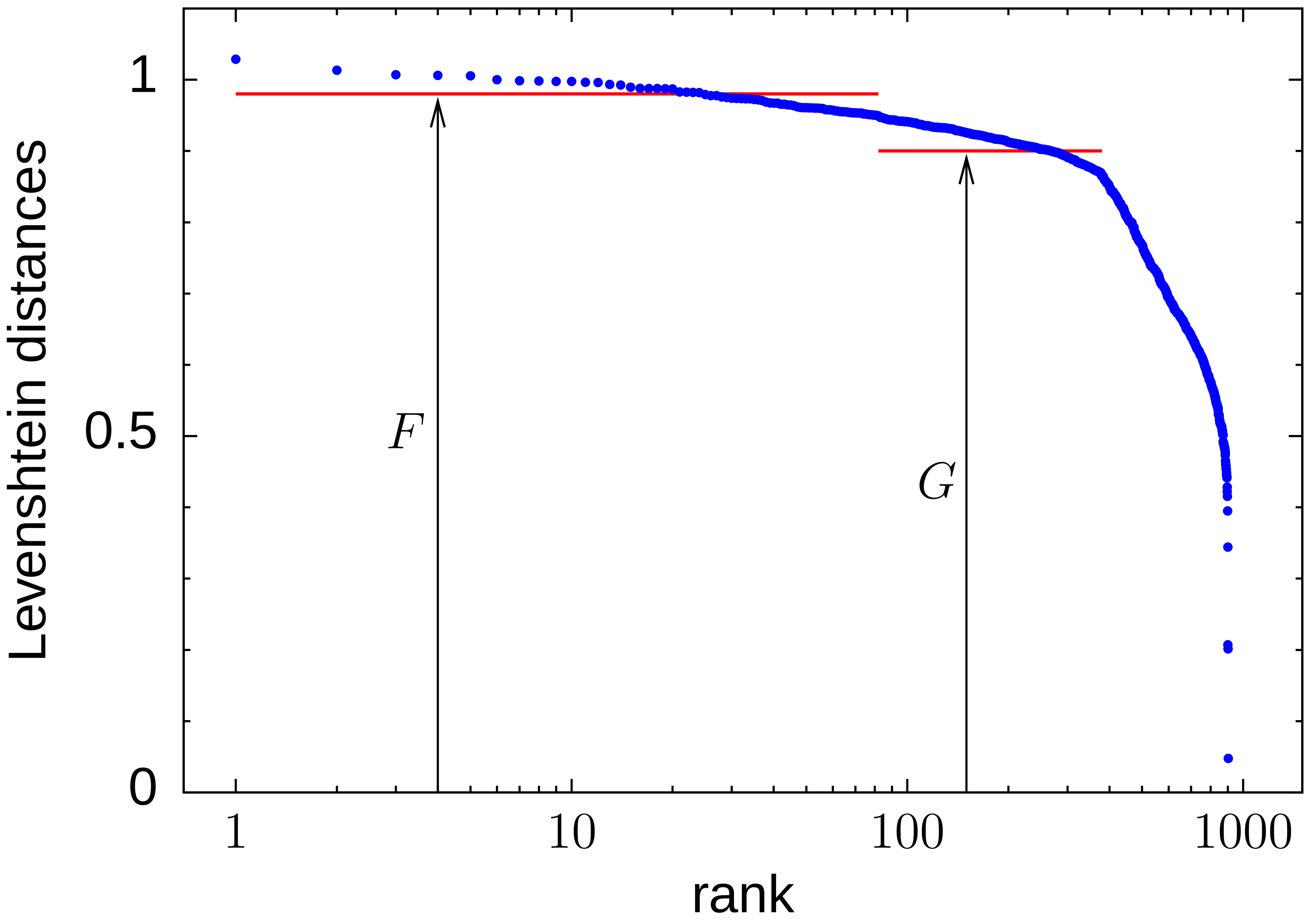
Tupian family, 43 languages



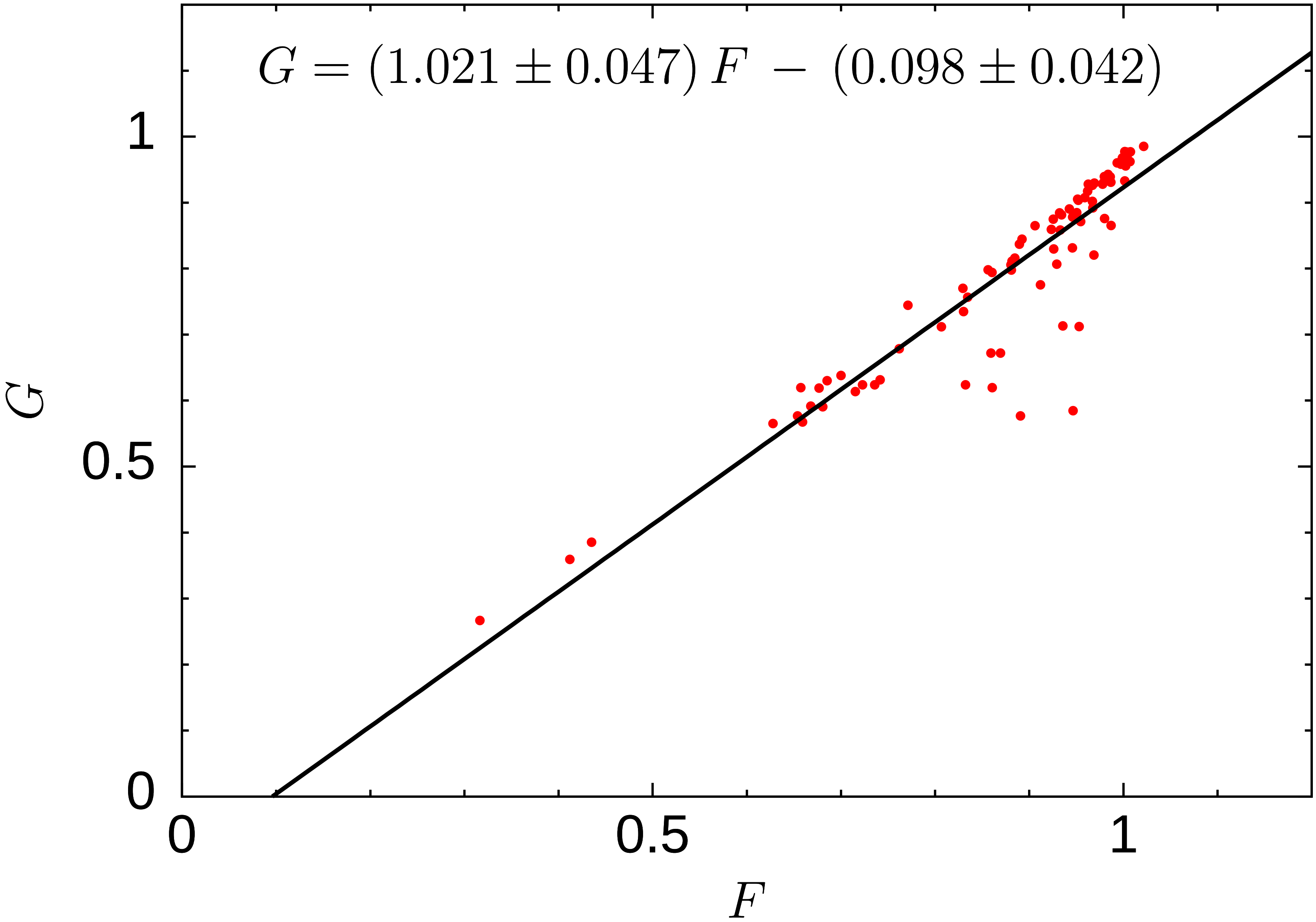
Simulated tree, 63 languages



Measures for each real family



89 real families



Our group at UFF

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Armando Ticona (La Paz)
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Klauko Mota
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Marlon Ramos