Nonlinear Stationary Stuctures in Nonthermal Plasmas

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Bernstein, Greene and Kruskal [1] showed the existence of an unlimited class of solutions to the Vlasov equation (BGK solutions) containing stationary potential structures. They proved that essentially arbitrary one-dimensional potential distributions can be derived, if a suitable number of particles trapped in potential troughs are added.

In the specific case of double layers (DL), which may be regarded as a BGK equilibrium with a potential drop ϕ_0 through the layer, some specific conditions must be fulfilled. One condition is the presence of free and trapped (or reflected) electrons and ions; a DL with only one type of trapped particle is exceptional, so usually all four classes of particles are required. Double layers are found in a wide variety of plasma environments, from discharge tubes to space plasmas, and are especially common in current-carrying plasmas.

In the present work we investigate the existence of stationary potential structures in nonthermal plasmas. Our model is based on nonthermal distribution functions, since the plasma distributions near a DL, for example, are usually strongly non-Maxwellian. Here the free and trapped electron populations are modeled by the family of κ distributions, which has been proven to be appropriate for modeling non-Maxwellian plasmas. Some preliminary results are shown and discussed.

Theory

Let us consider a Vlasov plasma of electrons and ions. As the intensity of the linear waves in such a plasma is increased, nonlinear effects become important. Here we discuss the BGK modes, named after Bernstein, Greene and Kruskal [1]. There are many practical applications of BGK modes, such as the nonlinear stage of a Landau damped Langmuir wave and the theory of DLs.

We already know that any distribution that is a function only of the constants of motion of the individual particles is a solution of the Vlasov equation. Here we are interested only in equilibrium solutions that do not depend explicitly on time. A BGK mode involves an equilibrium distribution function in the presence of a spatially varying electrostatic potential, which is produced self-consistently by the distribution function through Poisson's equation. Then for each particle species the Vlasov equation is (1D)

$$\left[v\frac{\partial}{\partial z} - \frac{q_j}{m_j}\frac{d\varphi}{dz}\frac{\partial}{\partial v}\right] f_j(z,v) = 0, \tag{1}$$

where $v \equiv v_z$ and $j = e, i$. The potential $\varphi(z)$ must be determined self-consistently through Poisson's equation

$$\frac{d^2\varphi}{dz^2} = 4\pi e\left[\int_{-\infty}^{\infty} f_e(z,v)dv - \int_{-\infty}^{\infty} f_i(z,v)dv\right]. \tag{2}$$

If the ion and electron velocities are large enough, none of them are trapped in the electrostatic potential wells. We can also consider the case where some of the electrons and/or ions are trapped. It turns out that almost any potential $\varphi(z)$ can be constructed by choosing appropriate distributions of trapped/untrapped electrons and trapped/untrapped ions.

Here we assume fixed ions and consider the case of trapped and untrapped electrons modeled by two different κ distributions. Working in the wave frame, the Vlasov equation (1) is solved by

- Free electrons - $E = u^2 - \phi \geq 0$

$$f_{\kappa f}(u) = A_\kappa \left\{ 1 + \frac{\left[\sigma\sqrt{(u^2-\phi)} + u_D\right]^2}{(\kappa - 3/2)} \right\}^{-\kappa} \tag{3}$$

- Trapped electrons - $E = u^2 - \phi < 0$

$$f_{\kappa t}(u) = A_\kappa \left\{ 1 + \frac{\beta\left(u^2-\phi\right) + u_D^2}{(\kappa - 3/2)} \right\}^{-\kappa} \tag{4}$$

where $\sigma = sgn(u)$, $u = v/v_T$, $\phi = e\varphi/k_B T$ and $A_\kappa = \frac{N_0\Gamma(\kappa)}{[\pi(\kappa-3/2)]^{1/2} v_T \Gamma(\kappa-1/2)}$. u_D is the phase velocity of the structure (electron hole) in the laboratory frame and β is the trapping parameter, which is introduced to allow the description of different states of trapped particles: $\beta = 0$ represents an electron distribution which is flat in the trapped region, and $\beta < 0$ describes a situation in which there is a deficit of trapped particles [2].

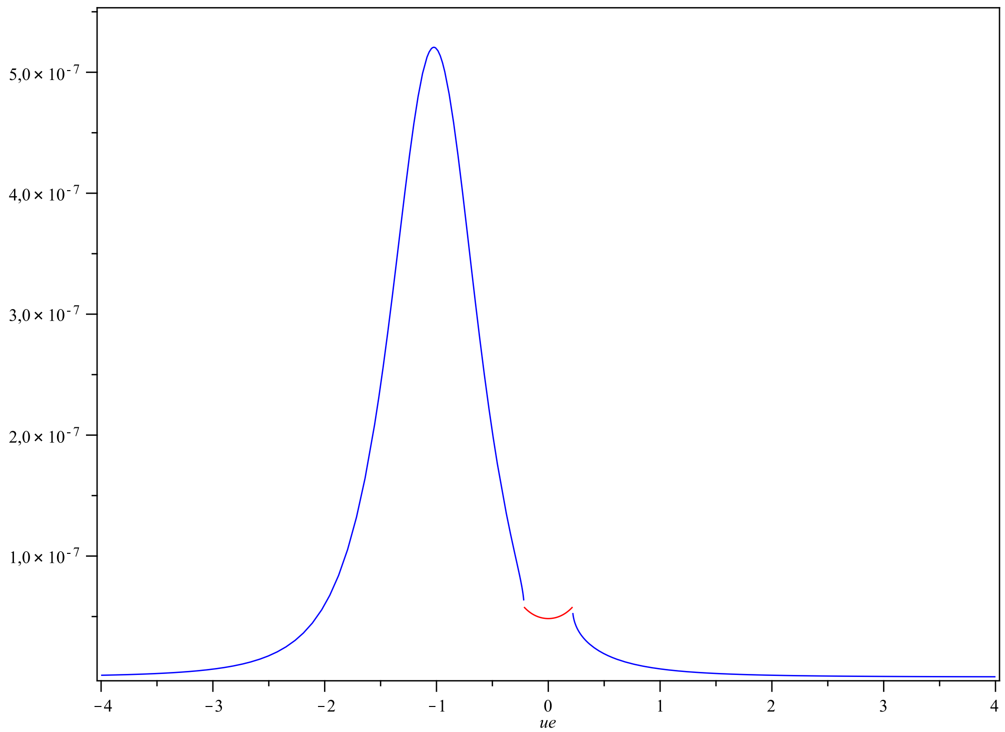

Figure 1: Electron distribution function for $\beta = -3$ and $\kappa = 2$ (electron hole in red)

Preliminary Results

First, we determine the normalized density $n_e(\phi)$ given by

$$n_e(\phi) = \frac{A_\kappa \left(\kappa - 3/2\right)^\kappa}{N_0} \left[I_1\left(\phi\right) + I_2\left(\phi\right) + I_3\left(\phi\right)\right], \tag{5}$$

where

$$I_1\left(\phi\right) = \int_{-\infty}^{-\sqrt{\phi}} \left\{\left(\kappa - 3/2\right) + \left[-\sqrt{\left(u^2 - \phi\right)} + u_D\right]^2\right\}^{-\kappa} du, \tag{6}$$

$$I_2\left(\phi\right) = \int_{-\sqrt{\phi}}^{\sqrt{\phi}} \left[\left(\kappa - 3/2\right) + \beta\left(u^2 - \phi\right) + u_D^2\right]^{-\kappa} du, \tag{7}$$

$$I_3\left(\phi\right) = \int_{\sqrt{\phi}}^{\infty} \left\{\left(\kappa - 3/2\right) + \left[\sqrt{\left(u^2 - \phi\right)} + u_D\right]^2\right\}^{-\kappa} du. \tag{8}$$

In Figs. (2) and (3) we show the influence of the parameters κ and β in the normalized electron density.

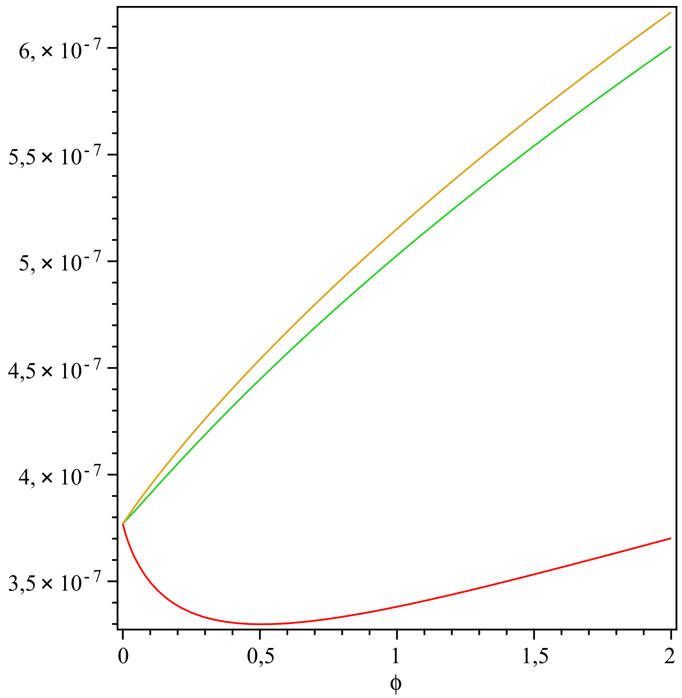

Figure 2: $n_e(\phi)$ for $\beta = 0$ and $\kappa = 1.6$(red), 2.5(green) and 4.8(yellow)

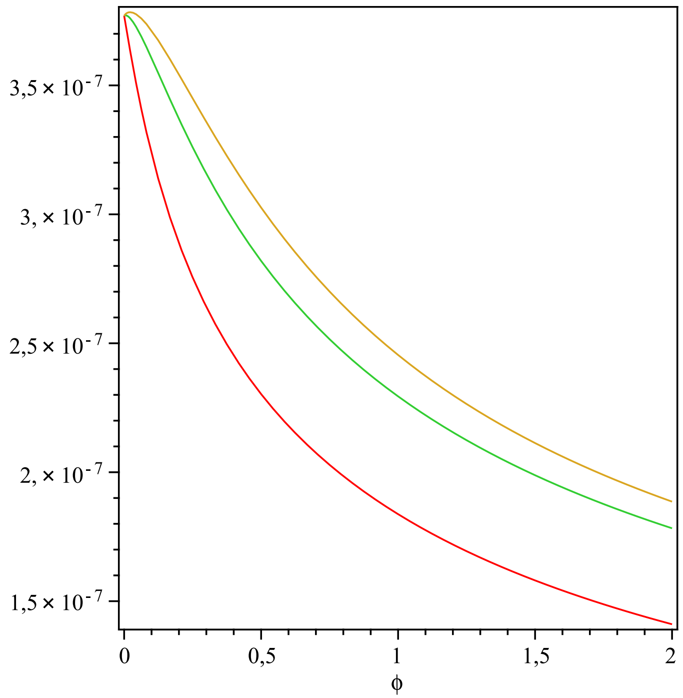

Figure 3: $n_e(\phi)$ for $\beta = -3$ and $\kappa = 1.6$(red), 2.5(green) and 4.8(yellow)

Next Step

We now will proceed to determine the normalized electrostatic potential ϕ given by

$$\frac{d^2\phi}{d\zeta^2} = 2[n_e\,(\phi) - 1], \tag{9}$$

where $\zeta = \omega_{pe}z/v_t$. We intend to analyze the influence of various parameters on the behaviour of ϕ. We also will consider the case of trapped and untrapped ions and the possible electrostatic structures associated to them.

References

1. I. B. Bernstein, J. M. Greene, and M. D. Kruskal, *Phys. Rev.* 108, 546 (1957).

2. H. Schamel, *Physica Scripta* T2/1, 228 (1982).