Universidade Federal do Ceará

Transport on Exploding Clusters

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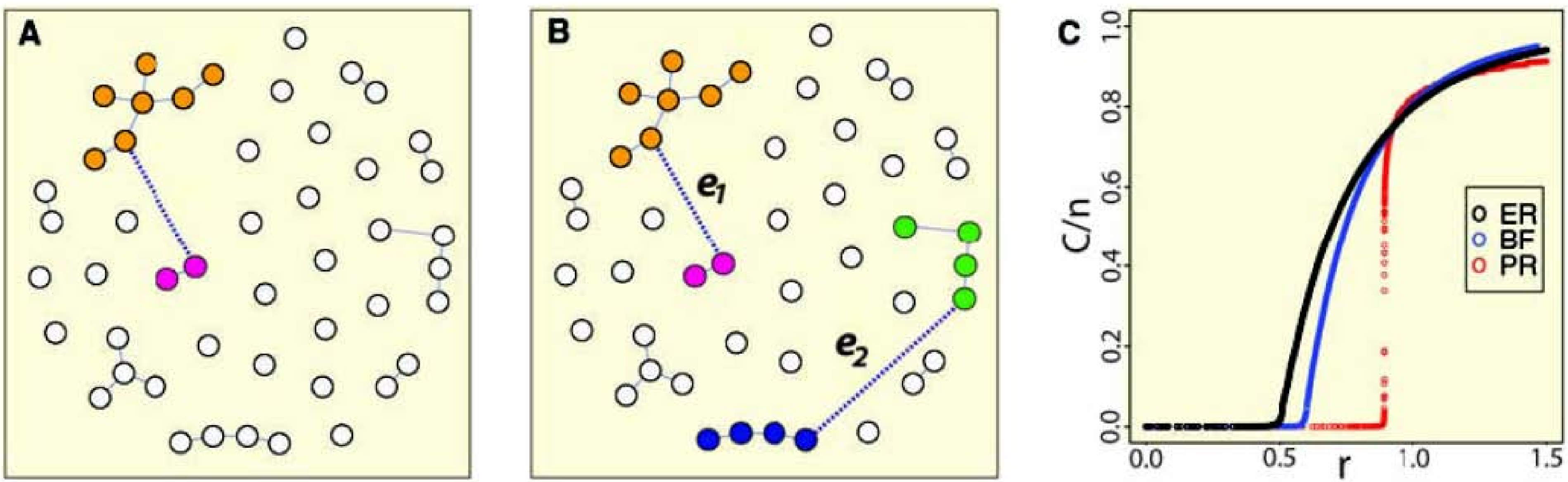
Product Rule (PR)

1) Consider a fully connected graph.

2) Select randomly two bonds and occupy the one which creates the smaller cluster.

Dimitris Achlioptas

Classical Percolation **Product Rule (PR)**



D. Achlioptas, R. M. D'Souza, and J. Spencer, Science 323, 1453 (2009)

REPORTS

Explosive Percolation in Random Networks

Dimitris Achlioptas,[1] Raissa M. D'Souza,[2,3*] Joel Spencer[4]

Networks in which the formation of connections is governed by a random process often undergo a percolation transition, wherein around a critical point, the addition of a small number of

One of the most studied phenomena in probability theory is the percolation transition of ER random networks, also known as the emergence of a giant component. When m edges have been added, if $r < ½$, the largest component remains miniscule, its number of vertices C scaling as $\log n$; in contrast, if $r > ½$, there is a component of size linear in n. Specifically, $C \approx (4r - 2)n$ for r slightly greater than ½ and, thus, the fraction of vertices

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Explosive Percolation Transition is Actually Continuous

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Recently a discontinuous percolation transition was reported in a new "explosive percolation" problem for irreversible systems [D. Achlioptas, R. M. D'Souza, and J. Spencer, Science 323, 1453 (2009)] in

Explosive Percolation is Continuous, but with Unusual Finite Size Behavior

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(Dated: March 24, 2011)

We study four Achlioptas type processes with "explosive" percolation transitions. All transitions are clearly continuous, but their finite size scaling functions are not entire holomorphic. The

Continuity of the Explosive Percolation Transition

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(Dated: March 25, 2011)

The explosive percolation problem on the complete graph is investigated via extensive numerical simulations. We obtain the cluster-size distribution at the moment when the cluster size heterogeneity becomes maximum. The distribution is found to be well described by the power-law form with the decay exponent $\tau = 2.06(2)$, followed by a hump. We then use the finite-size scaling method to make all the distributions at various system sizes up to $N = 2^{37}$ collapse perfectly onto a scaling curve characterized solely by the single exponent τ. We also observe that the instant of that collapse converges to a well-defined percolation threshold from below as $N \to \infty$. Based on these observations, we show that the explosive percolation transition in the model should be continuous, contrary to the widely-spread belief of its discontinuity.

PACS numbers: 64.60.ah, 64.60.aq, 36.40.Ei

Best-of-m Model: Geometry and Transport Properties

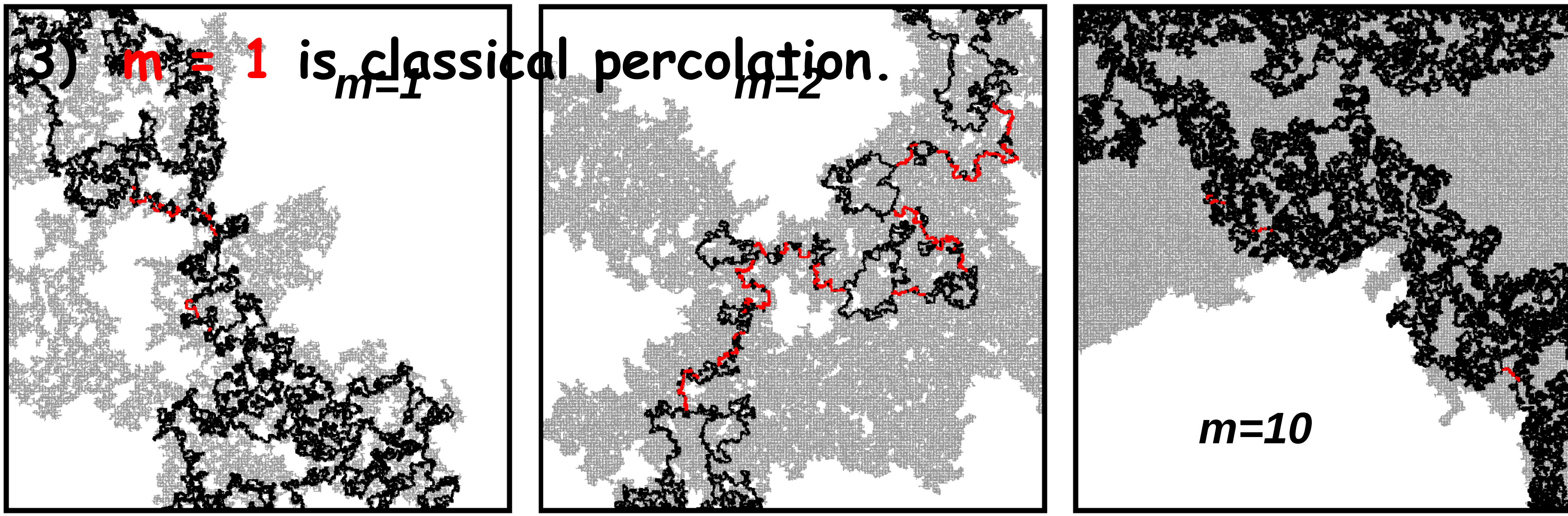
1) Select randomly **m** bonds and occupy the one which creates the smaller cluster.

2) This is a straightforward generalization of the product rule (PR) which corresponds to **m = 2**.

3) **m = 1** is classical percolation.

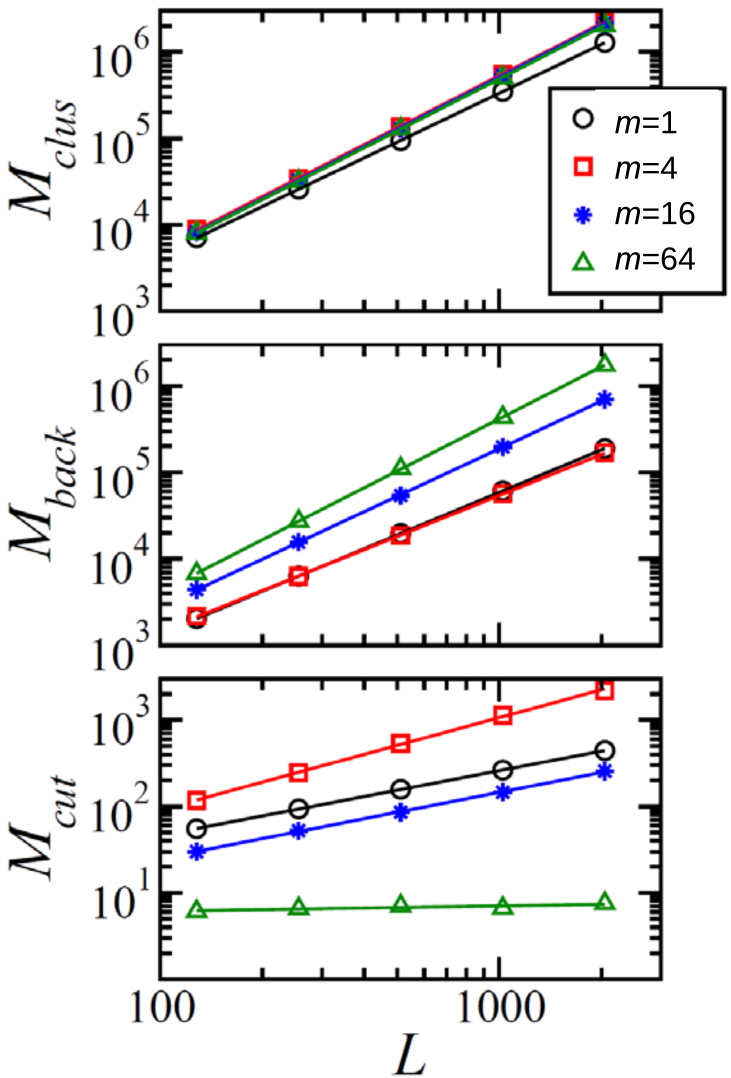
at the critical point

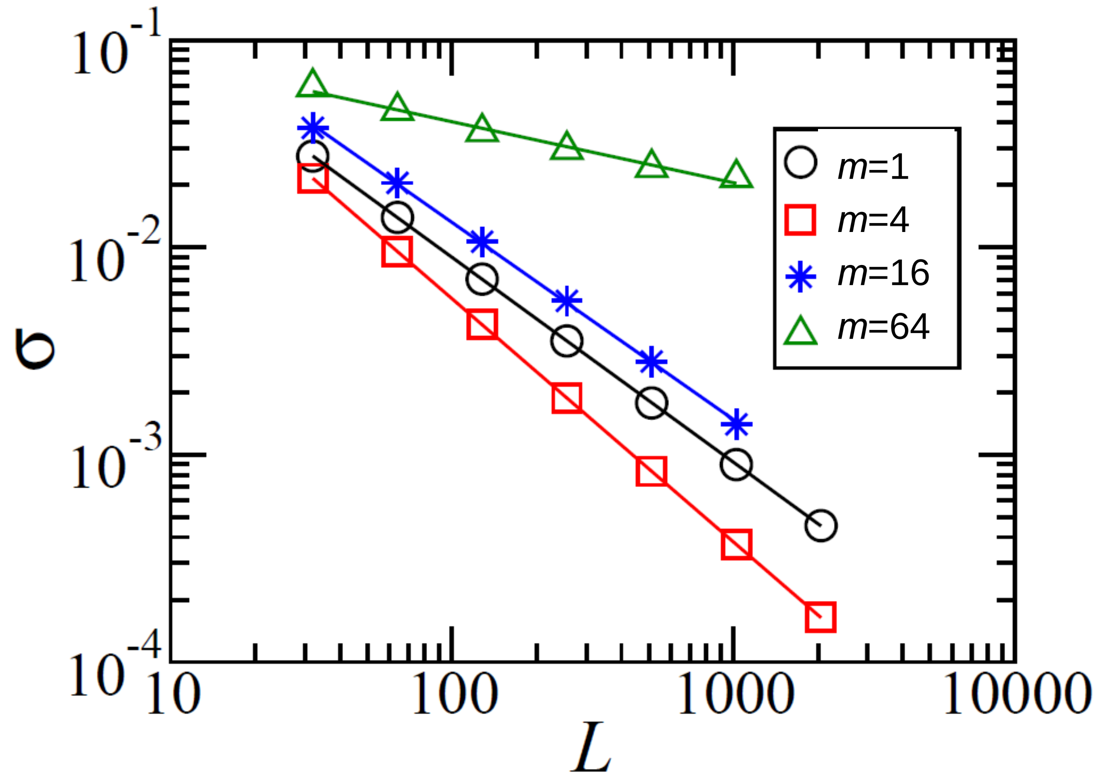
grey: largest cluster
black: conducting backbone
red: cutting bonds



JSA, Herrmann, Moreira, and Oliveira, *Phys. Rev. E* 83, 031133 (2011)

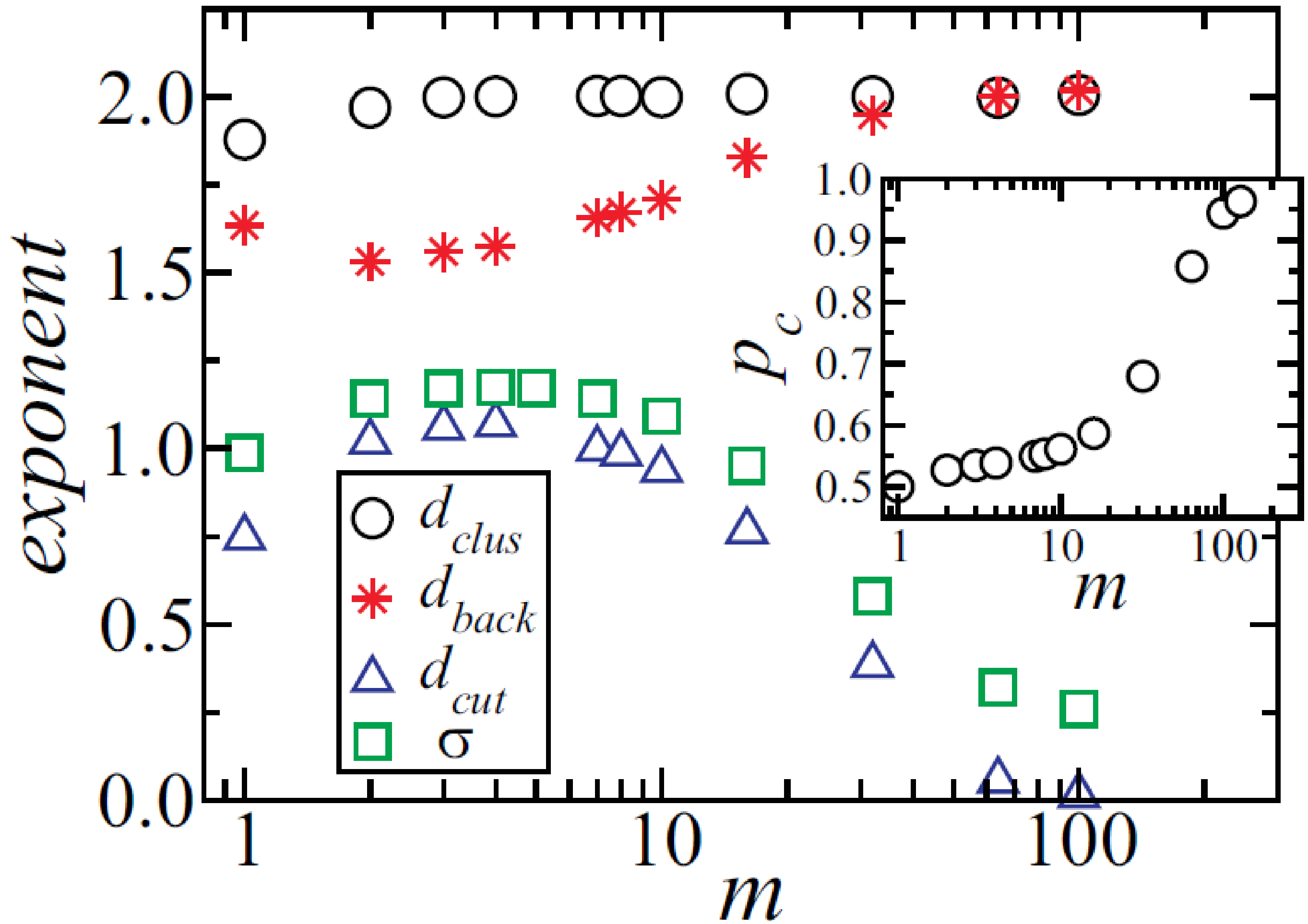
Best-of-m Model: Geometry and Transport Properties





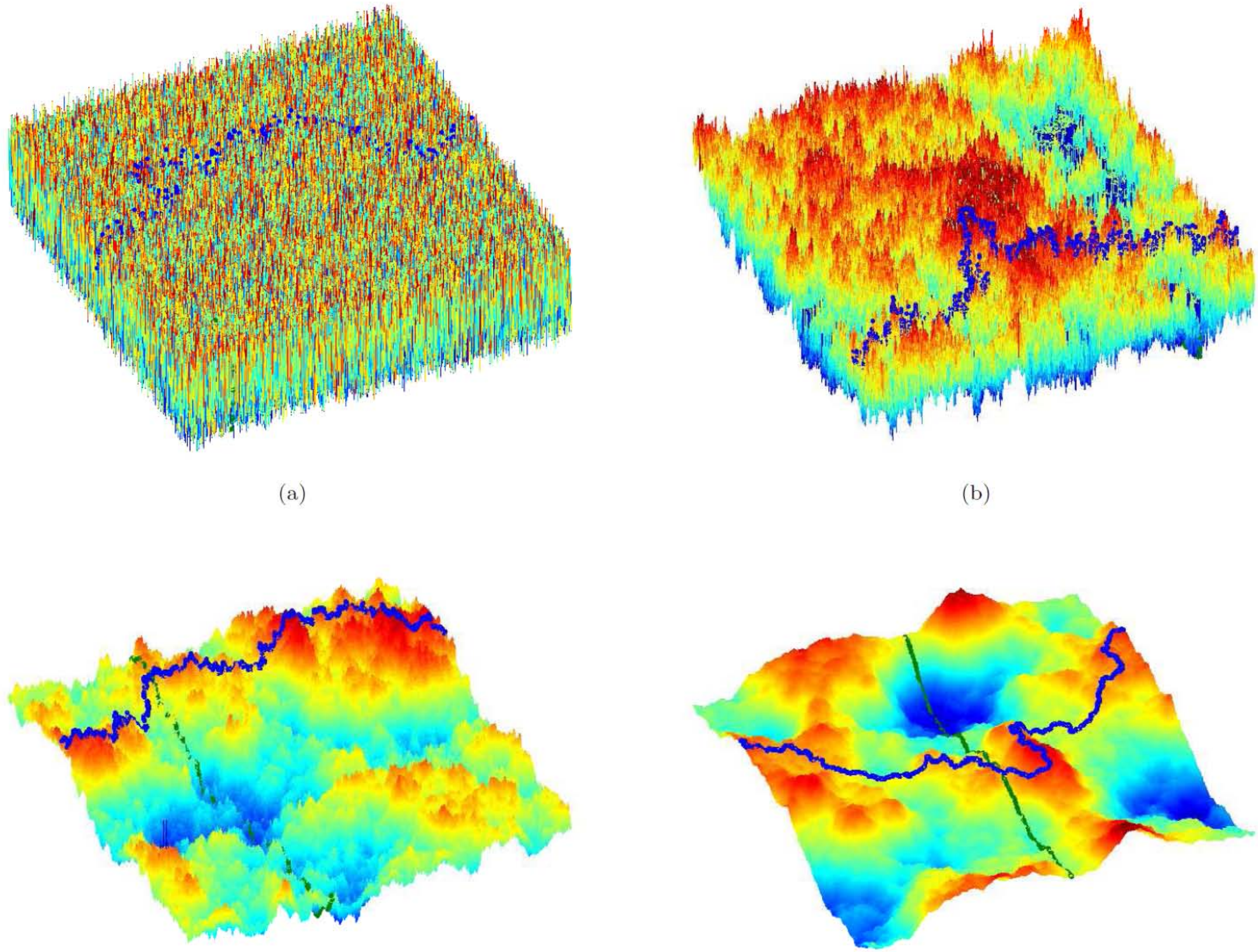
electrical conductivity

Best-of-m Model: Geometry and Transport Properties



Exponents depend non-monotonically on m!

Correlated Landscapes



Fractional Brownian Motion $\Rightarrow \left\langle h(\vec{r})h(\vec{r}+\vec{R}) \right\rangle \propto \left|\vec{R}\right|^{-\gamma}$

Optimal Paths under strong disorder

1) On a square lattice of size L with fixed BC's at the top and bottom and periodic BC's in the transversal direction, we assign to each site **i** a given "energy" value ε_i given by,

$$\varepsilon_i = \exp[\beta(p_i - 1)]$$

where p_i is a random variable uniformly distributed in [0,1]. This is equivalent to choose ε_i from a power-law distribution,

$$P(\varepsilon_i) \sim 1/\varepsilon_i$$

(now normalizable) with maximum cutoff $\varepsilon_{\max} = e^{\beta}$.

2) The energy of a path is the sum of all energies of its sites. The optimal path (OP) is defined here as the one among all paths connecting the bottom to the top of the lattice that has the smallest energy.

3) In the strong disorder limit, optimal paths are self-similar with fractal dimensions given by $D_f \approx 1.22$ [Cieplak, Maritan & Banavar, *Phys. Rev. Lett.* (1994)]. This path has the same fractal dimension as the watershed.

Optimal Path Crack

JSA, Oliveira, Moreira & Herrmann, PRL 103, 225503 (2009)

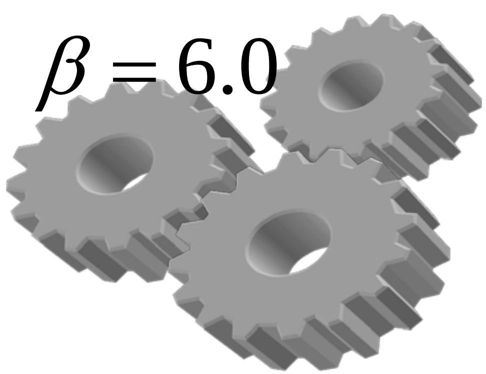
1) The Dijkstra algorithm [Dijkstra, *Num. Math.* (1959)] is used to calculate the first OP connecting the bottom to the top of the network;

2) The site in the OP having the highest energy is permanently blocked (i.e., an irreversible "micro-crack" is formed);

3) The next OP is calculated, from which the highest energy site is again removed and so on, and so forth;

4) The process continues iteratively until the system is disrupted, i.e., we can no longer find any path connecting bottom to top. The disrupting path also has the same fractal dimension as the watershed.

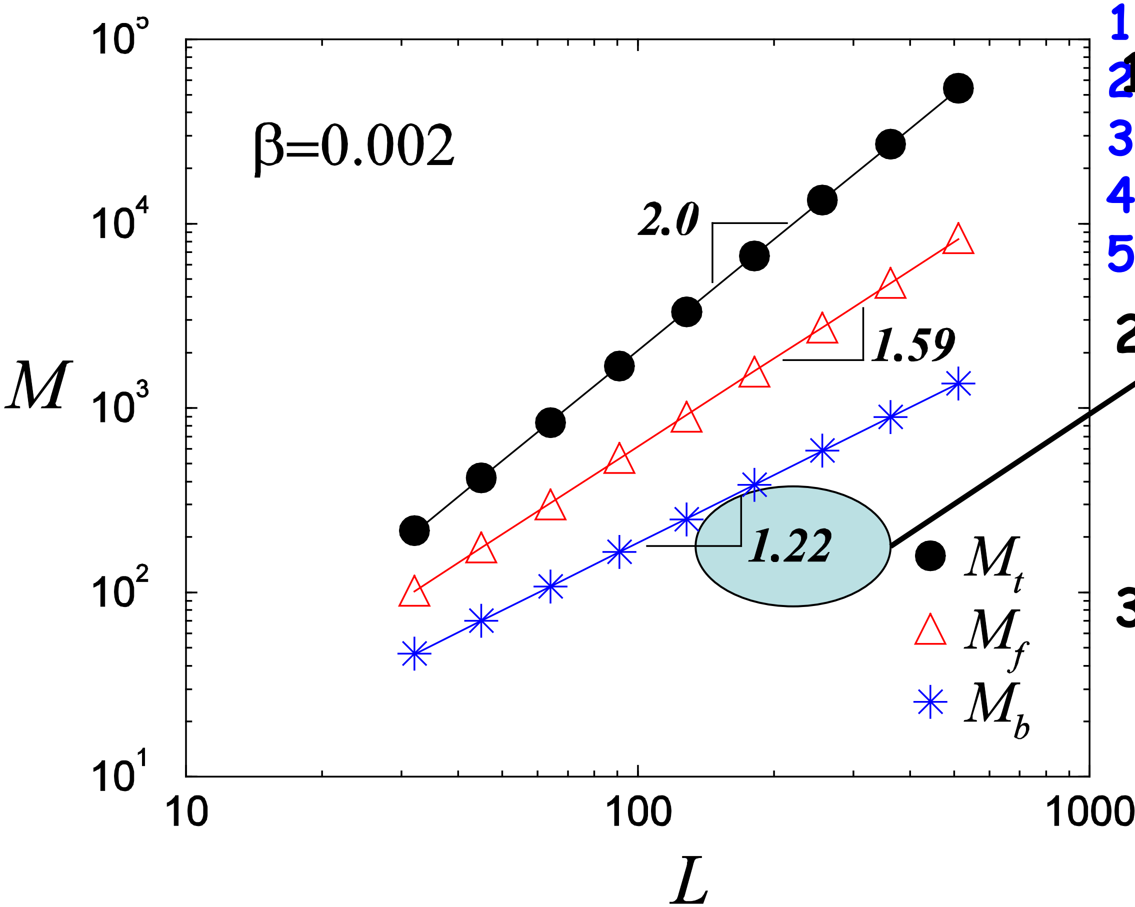
$\beta = 6.0$



Quantitative Results

JSA, Oliveira, Moreira & Herrman, PRL (2009)

- Simulations with 1000 realizations of lattices for each different size 32≤L≤512 and distinct values of the disorder parameter β.
- Weak disorder $\Longrightarrow$ clear scaling laws.



1) OP's under strong disorder
2) Disordered Polymers
3) Strands in IP
4) Paths on MST's
5) Hulls of Explosive Percolation

1) $M_b \sim L^{D_b}$ (backbone)

with $D_b = 1.22 \pm 0.02$

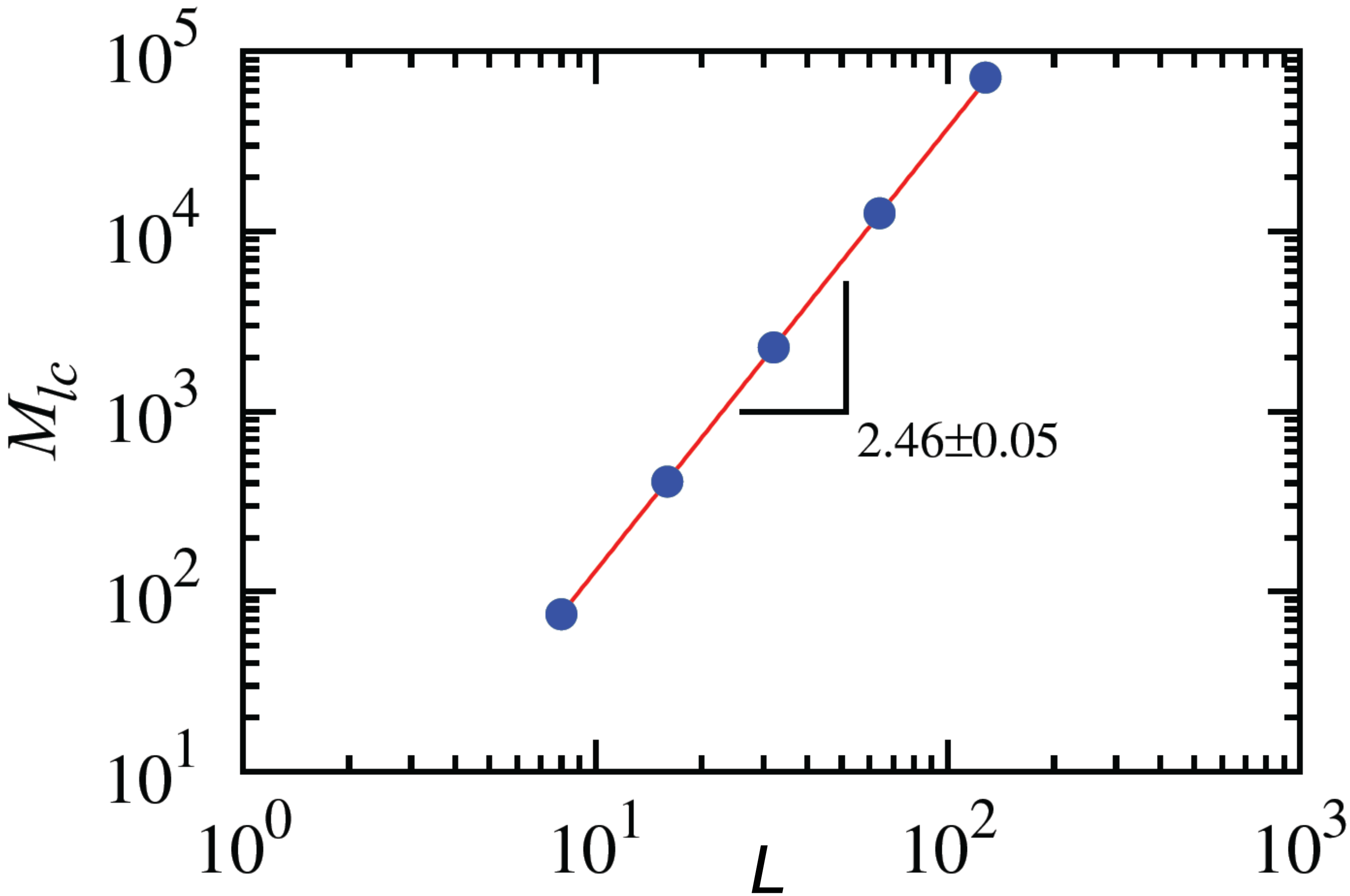
2) $M_f \sim L^{D_f}$ (OPC fracture)

with $D_f = 1.59 \pm 0.02$

3) $M_t \sim L^{D_t}$ (all cracks)

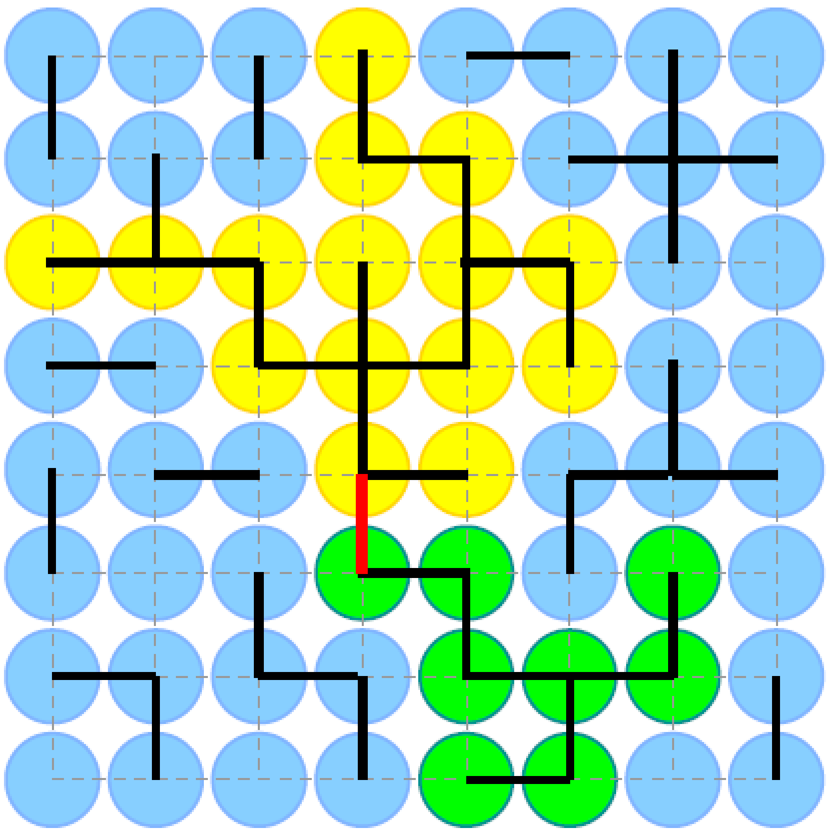
with $D_t = 2.00 \pm 0.01$

Optimal Path Crack in 3D



In 3d, watersheds, OPC's and the surface of clusters generated with the Gaussian model of explosive percolation, all have the same fractal dimension!

Bridge Percolation



A bridge (or anti-red bond) is a bond which, if occupied, would create a first spanning cluster.

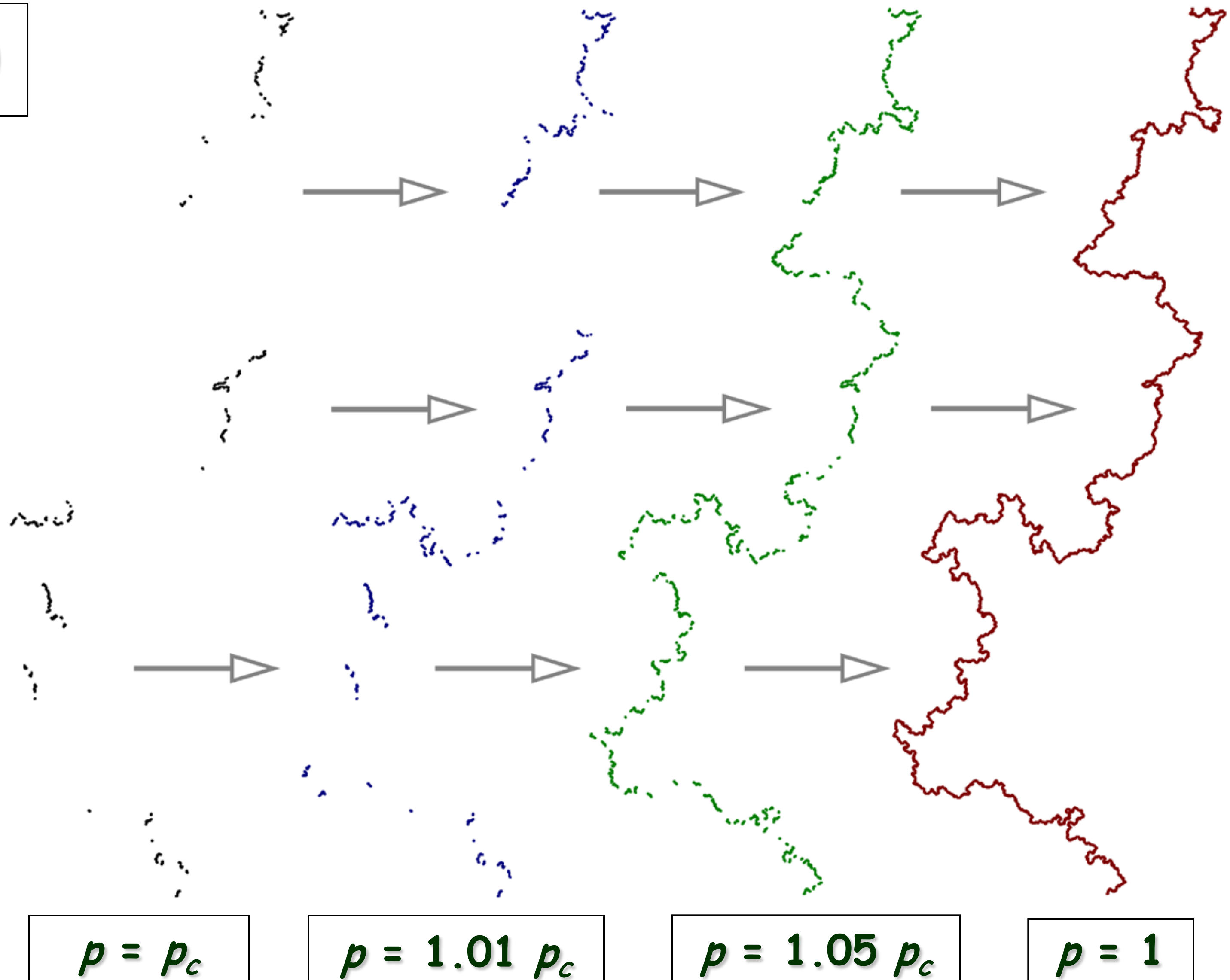
Be p_1 the probability to occupy a bridge. The weight of a configuration is then given by,

$$p^O (1-p)^E (1-p_1)^B$$

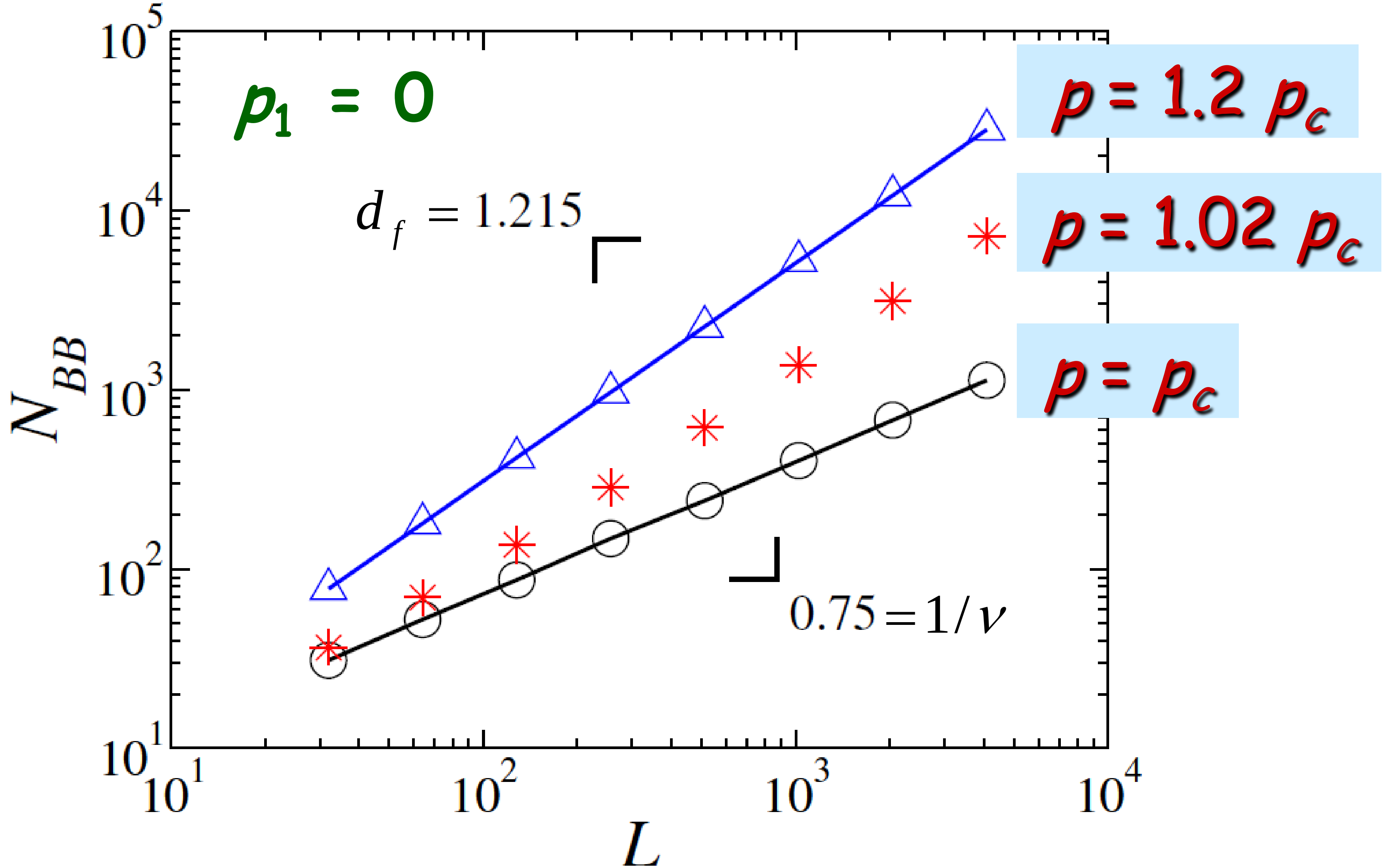
where O, E and B are the number of occupied bonds, empty bonds and bridges, respectively.

Bridge Percolation

$p_1 = 0$

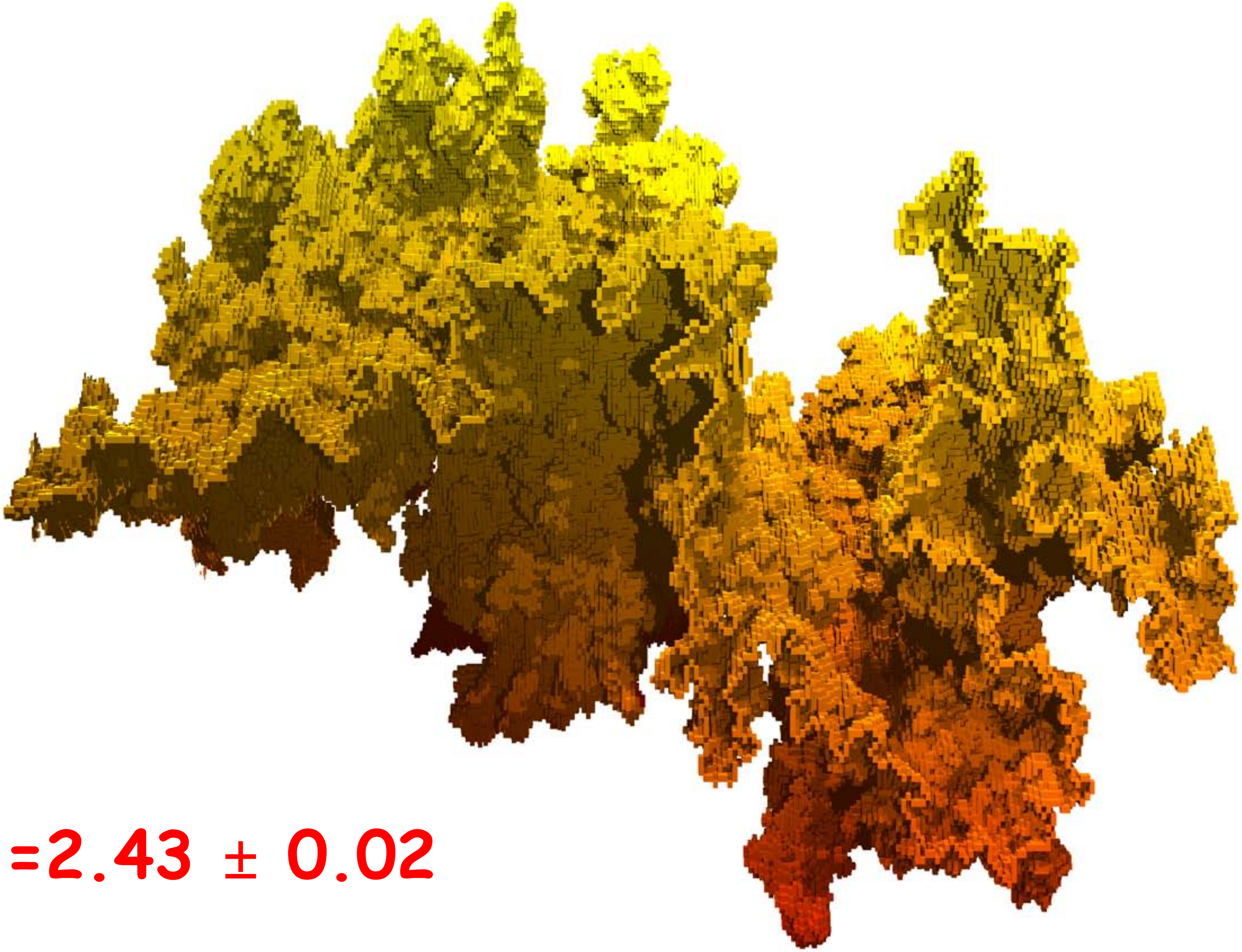


Bridge Percolation



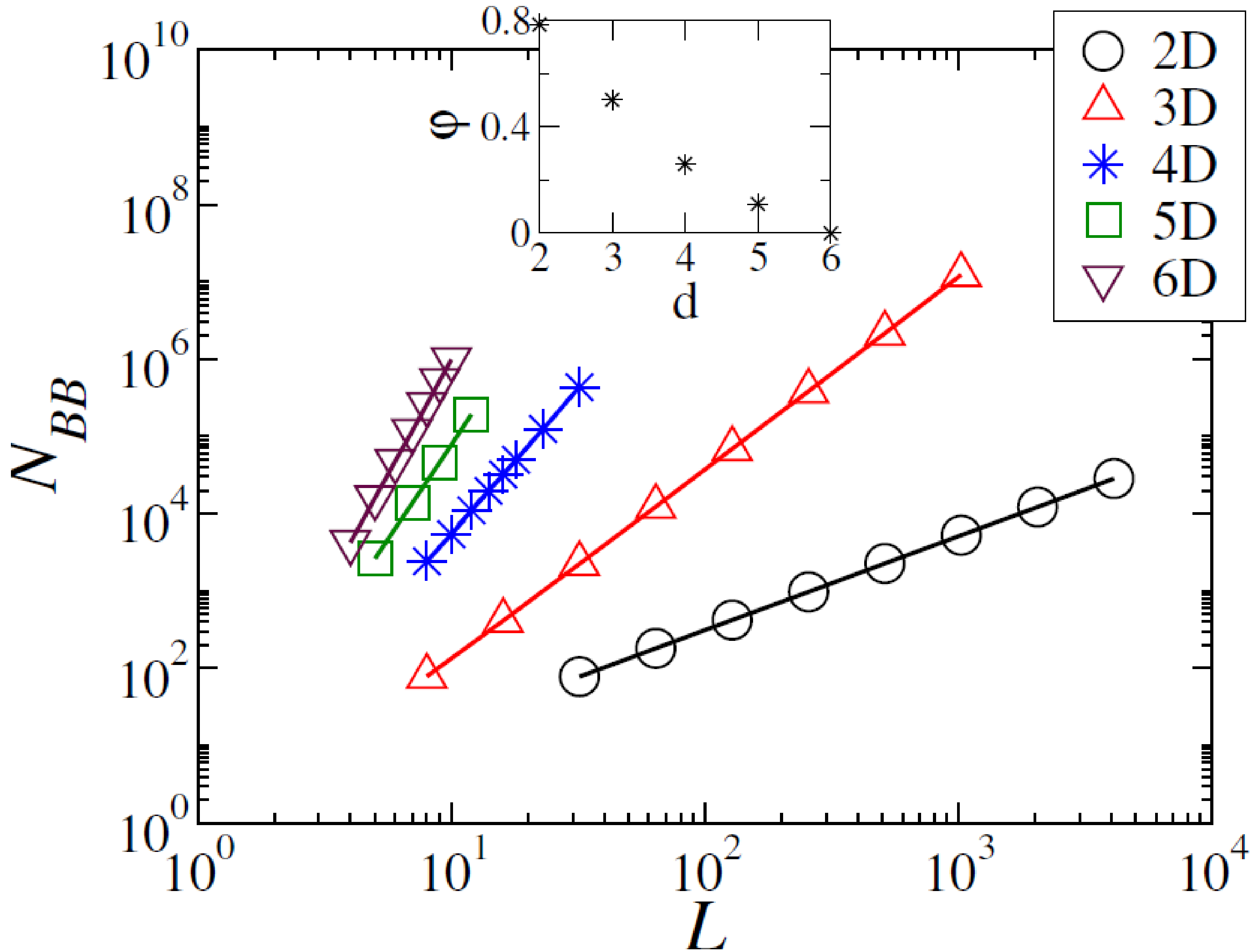
Bridge Percolation in 3D

p_1



$d_f = 2.43 \pm 0.02$

Bridge Percolation in higher dimensions



$p_1 = 0$

$$\varphi = d - d_f$$

Above the upper critical dimension, the set of bridges is dense!

Conclusions

- The backbone of the fracture constituted of OPC's is apparently (not proved) disorder independent. It is also a self-similar object with fractal dimension $D_b \approx 1.22$.
- This dimension is (statistically) similar to the ones obtained for OP's under strong disorder [Schwartz et al., *PRE* (1998)], Disordered Polymers [Cieplak et al., *PRL* (1994)], strands in Invasion Percolation [Cieplak et al., *PRL* (1996)], paths on Minimum Spanning Trees [Dobrin et al., *PRL* (2001)], and the hulls of Explosive Percolation clusters [Araújo & Herrmann, PRL (2010)].
- Watersheds on uncorrelated landscapes and bridge percolation also exhibit the same fractal dimension, namely, $D_f \approx 1.21$.
- Exploding percolation clusters generated with the best-of-m rule can have anomalous transport properties.

Outlook

- Is the PR rule a first or second order transition?
- Can one formulate a rigorous equivalence between the models?