Decision making in complex systems

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Big picture

Complex (social) systems are particularly challenging:

- The number of variables is very large
- The relevant variables are often unknown and hard to measure
- The time scales which the variables evolve are not well separated from each other
- There is just one realization
- The observer participates in the system and modifies the environment (for instance, the Lucas's critique in macroeconomic and public policy)
- Nonlinear network dependence (representative agent assumption does not work)

Big picture

- The existence of feedback
- The rules of the game and the agents are evolving over time
- The dynamics of information flow is fundamental
- The existence of large degree of randomness and heterogeneity, memory and anticipation
- The systems are highly non-stationary
- The existence of incentive problems

☺ Helbing, D. and Balietti, S. From Social Simulation to Integrative System Design. Forthcoming in European Physical Journal Special Topics, 2011.

☺ Johnson, N. F., Jefferies, P. and Hui, P. M. Financial Market Complexity: What Physics Can Tell Us About Market Behaviour. Oxford University Press, 2003.

Big picture

Arguments that will guide this presentation are based on:

- Optimization
- Control

Big picture

Why does optimization play a role here?

- Although there are several principles behind agents decisions, in some sense we believe that social agents try to maximize some measure of satisfaction.

Big picture

How about control?

- Human beings always have tried to interfere in nature or social systems in order to improve their adaptation.
- Intervention in complex systems is possible:
  - Seminal example: Chaos control methodology (Ott, Grebogi and Yorke, 1990)
  - New examples: Controlling by deletion (Motter, 2004) and controlling self-organized criticality (Cajueiro and Andrade, 2010)

Big picture

How about control?

☺ E. Ott, C. Grebogi and J. A. Yorke. Controlling chaos. Physical Review Letters 64, p. 1196-1199, 1990. [ISI 3078]

☺ A. E. Motter. Cascade control and defence in complex networks. Physical Review Letters 93, p. 098701, 2004. [ISI 132]

☺ D. O. Cajueiro and R. F. S. Andrade. Controlling self-organized criticality in Abelian Sandpiles. Physical Review E 81, p. 015102, 2010. [ISI 3]

Talk

- Human dynamics of tasks execution
- Social complex networks and the influence on the decision making process
  - Are complex networks likely to arise in social interactions?
  - Flow of information in complex networks
  - Enforcing social behavior in complex networks
  - Navigation in complex networks
- Controlling self-organized criticality

Queueing theory

Motivation

- Queueing theory involves the mathematical study of queues or waiting lines
- The formation of waiting lines is occurs whenever the current demand for a service exceeds the current capacity to provide that service
- These decisions are often difficult since one cannot accurately predict when units will arrive to seek service or how much time will be required to provide that service
- Providing too much service would involve excessive costs
- Not providing enough service capacity would cause the waiting line to become excessively long at times
- Excessive waiting also is costly in some sense, whether it be a social cost, the cost of lost customers...

Queueing theory

Standard modeling

The basic process assumed by most queuing models are the following:

- Customers requiring service are generated over time by an *input source*
- The customers enter the queuing system and join a queue. At certain times a member of the queue is selected for service by some rule known as the *service discipline* (Ex. FIFO)
- The required service is then performed for the customer by the *service mechanism*, after which the customer leaves the queueing system
- The time required to receive service is called *waiting time* and the time between consecutive arrivals is referred to as the *interarrival time*

Queueing theory

Examples

- Commercial service systems: barber shop, cafeteria line
- Transportation service systems: cars waiting at a traffic light, airplanes waiting to land, cars waiting to park
- Business-industrial services systems: maintenance systems, banking lines
- Social service systems: Judicial system, mails being waited to be answered
- Health care systems: hospitals, ambulances

Queueing theory

Role of exponential distribution

Characteristics of queueing systems are determined by two statistical properties:

- Interarrival times
- waiting times

The standard distribution used to model these variables is the so-called exponential distribution:

$$P(T \leq t) = 1 - e^{-\alpha t}$$

$$P(T > t) = e^{-\alpha t}$$

Empirical findings

Stylized facts

- Empirical evidence has shown that the dynamics of inter-event times driven by human actions may not be random and not well approximated by exponential distributions
- These processes are characterized by bursts of rapidly occurring events separated by long periods of inactivity

Empirical findings

Examples

- Internet activity: Paxson and Floyd (1996)
- financial asset activity: Masoliver, Monteiro and Weiss (2003)
- E-mail activity: Barabasi (2005)

Barabasi's framework

- Model of human activity where the distribution of the inter-event time is a consequence of a decision queue process
- The most relevant protocols for driving human dynamics is a protocol based on the execution of the high priority item (the others are FIFO and random)
- In the protocol based on the execution of the high priority item, while high priority tasks are executed as soon as they are added to the list, low priority tasks wait for a long time until all high priority tasks are executed
- He shows numerically that the distribution of inter-event times follows a power law

Literature review

- Vazquez (2005) and Anteneodo (2009) provide exact results for Barabasi's model
- Grinstein and Linsker (2006) map the variable length priority model considered above onto a model of biased diffusion deriving asymptotic distributions for the inter-event times
- Kentsis (2005) and Barabasi and Oliveira (2006) argue that other mechanisms contribute for the distributions of waiting times such as deadlines, time dependence of priorities and the social context of the problem etc
- Blanchard and Hongler (2007) relax the assumption that the priorities of tasks do not change over time and studies queueing systems where deadlines are assigned to the incoming tasks

Our work

☺ D. O. Cajueiro and W. L. Maldonado Role of optimization in the human dynamics of task execution. Physical Review E 77, p. 035101, 2008.

- We investigate the assumption that people execute tasks on a protocol that execute firstly the high priority item.
- We suppose that people assign priorities to the tasks on their lists in order to minimize some cost index, i.e., a cost associated to the fact of not processing a given collection of tasks in a given time step
- We built a discounted stochastic dynamic programming model with two types of tasks (low and high priority tasks) and a cost per stage for keeping a number of low and high priority tasks without processing

Setup of the problem

- There are two queues waiting for a service on a single server
- Let $g(x_L, x_H)$ be the current cost of having state (x_L, x_H) which is the state of the system, x_L (x_H) is the number of tasks in the low (high) priority queue

The dynamics of these queues are modeled as follows. At each discrete time step with probability

- $\lambda\rho$ a new task arrives in the queue formed by high priority tasks
- $\lambda(1-\rho)$ a new task arrives in the queue formed by low priority tasks

Setup of the problem

- Within each of the queues the tasks are executed on a FIFO basis.
- With probability $\mu u(x_L, x_H)$ the first task of the high priority queue is executed and with probability $\mu(1 - u(x_L, x_H))$ the first task of the low priority queue is executed.
- We assume here that $u(x_L, x_H)$ is a state dependent control variable that the agent will choose in order to minimize the total cost function $J_u(x_L, x_H) = E^u_{x_L, x_H}[\sum_{t=1}^{\infty} \alpha^t g((x_L(t), x_H(t)))]$, where $E^u_{x_L, x_H}[\cdot]$ is the expected value conditioned to the current state (x_L, x_H) and the state control variable u and α is the discount factor.

Solution of the problem

Bellman equation

Due to the principle of optimality and the Banach fixed point theorem, if the minimum cost function $J(x_L, x_H) = \min_{u(x_L,x_H)\in[0,1]} J_u(x_L, x_H)$ exists, it must be given by the unique solution of the Bellman equation, that may be written as

$$J(x_L, x_H) = F(x_L, x_H) + \min_{u(x_L,x_H)\in[0,1]} u(x_L, x_H)G(x_L, x_H)$$

where

$$\begin{aligned} F(x_L, x_H) &= g(x_L, x_H) + \lambda\rho(1-\mu)[\alpha J(x_L, x_H + 1)] \\ &+ \lambda(1-\rho)(1-\mu)[\alpha J(x_L + 1, x_H)] \\ &+ (1-\lambda)\mu[\alpha J(x_L - 1, x_H)] \\ &+ \rho\lambda\mu[\alpha J(x_L - 1, x_H + 1)] \end{aligned}$$

Solution of the problem

Bellman equation

$$
\begin{aligned}
& G(x_L, x_H) = \\
& (1-\lambda)\mu[\alpha(J(x_L, x_H - 1) - J(x_L - 1, x_H))] \\
+ \quad & \rho\lambda\mu[\alpha(J(x_L, x_H) - J(x_L - 1, x_H + 1))] \\
+ \quad & (1-\rho)\lambda\mu[\alpha(J(x_L + 1, x_H - 1) - J(x_L, x_H))]
\end{aligned}
$$

Results

Linear costs

We assume that $g(x_L, x_H) = h_L x_L + h_H x_H$, for $0 < h_L < h_H$, i.e., the current cost of having one additional high priority task in the queue is larger than having one additional low priority task in the queue.

It is easy to show that the cost function is given by

$$J(x_L, x_H) = c + c_L x_L + c_H x_H$$

Furthermore,

$$G(x_L, x_H) = \mu \frac{\alpha}{1-\alpha}(h_L - h_H)$$

is always negative implying that $u(x_L, x_H) = u = 1$ for every state (x_L, x_H).

- This result is consistent with Barabasi's results.

Results

Quadratic costs

We assume that $g(x_L, x_H) = h_L x_L^2 + h_H x_H^2$, for $0 < h_L < h_H$.

- The solution of the Bellman equation depends explicitly on the signal of the function $G(x_L, x_H)$
- Three different regions will arise
  - Region A the domain of (x_L, x_H) where $G(x_L, x_H) > 0$
  - Region B the domain of (x_L, x_H) where $G(x_L, x_H) = 0$
  - Region C the domain of (x_L, x_H) where $G(x_L, x_H) < 0$.

Results

Quadratic costs

We have found that the minimum cost function is given by

$$J(x_L, x_H) = \begin{cases} J^A(x_L, x_H) \text{ if } (x_L, x_H) \in A \\ J^B(x_L, x_H) \text{ if } (x_L, x_H) \in B \\ J^C(x_L, x_H) \text{ if } (x_L, x_H) \in C \end{cases}$$

and the optimal control is given by

$$u(x_L, x_H) = \begin{cases} 0 \text{ if } (x_L, x_H) \in A \\ u \in [0, 1] \text{ if } (x_L, x_H) \in B \\ 1 \text{ if } (x_L, x_H) \in C \end{cases}$$

Results

Quadratic costs

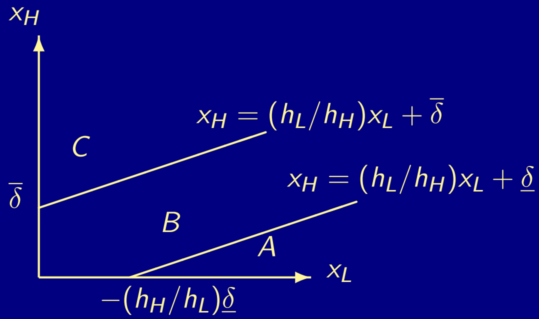


Figura: The regions A, B and C in the plane $x_L - x_H$.

Results

Quadratic costs

- Differently from the linear costs case, several types of protocol are possible.
- Region C considers a protocol based on the execution of the high priority task.
- Region A considers a protocol based on the execution of the low priority task. It occurs in order to avoid that the size of the queue of the low priority tasks do not increase too much. "Too much" here is measured by the ratio h_L/h_H.
- Region B does not determine a protocol. It can be a random protocol (mixed strategy) or simply a protocol such the one considered in region C or region A. Figure 1 shows the geometry of these regions in the plane $x_L - x_H$.

Results

Quadratic costs

It is not difficult to show that the expected value of the state obeys the following dynamics

$$E_t[x(t+1)] = E_t\begin{bmatrix} x_L(t+1) \\ x_H(t+1) \end{bmatrix} = \begin{bmatrix} x_L(t) \\ x_H(t) \end{bmatrix} + \begin{bmatrix} \lambda(1-\rho) - \mu(1-u(x_L(t), x_H(t))) \\ \lambda\rho - \mu u(x_L(t), x_H(t)) \end{bmatrix}$$

which has infinite fixed points if and only if $\lambda = \mu$ and $u(x_L, x_H) = u = \rho$.

Results

Quadratic costs

Consider only the most interesting situation which is the fixed-length-queue, i.e., $\lambda = \mu$.
Assuming that $\lambda = \mu$, $u^B = \rho + \epsilon$ and $\epsilon > 0$ and let $e = (1, -1)'$, then the expected value of the system is governed by

- $E_t[x(t+1)] = x(t) + \lambda \epsilon e$ if it is in region B
- $E_t[x(t+1)] = x(t) - \lambda \rho e$ if it is in region A
- if the state is in region C, the expected state will certainly come to region B and not come back to this region

Therefore:

- The dynamics takes place in the line passing by $x(0)$ and following the direction e.
- If the expected state is in region B it goes into the direction of region A and viceversa.
- This dynamics is equivalent to the one dimensional system

$$y(t+1) = \begin{cases} y(t) + t^{+} \text{ if } y(t) \leq 0 \\ y(t) - t^{-} \text{ if } y(t) > 0 \end{cases}$$

defined on the interval $(-t^{-}, t^{+}]$, where $t^{+} = \lambda\epsilon$ and $t^{-} = \lambda\rho$

Results

Quadratic costs

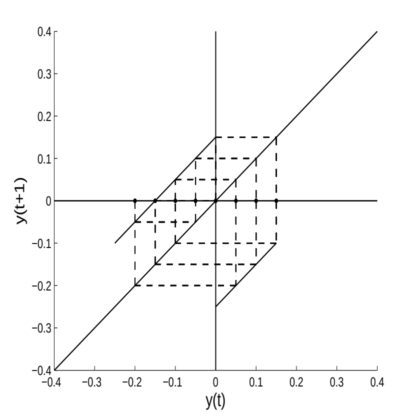


Figura: The evolution of $y(t)$ for $y_0 = -0.2$, $\lambda = 0.5$, $\rho = 0.5$ and

Results

Quadratic costs

- We can conclude that the stochastic process that defines the length of each queue is not stationary.
- The dynamics of the expected value of the length of the queue exhibits a complex behavior: infinitely many cycles or a ω-limit set being a dense subset in the interval. The intuition behind that complex dynamics is quite reasonable.
- In the region close to the frontier $x_H = (h_L/h_H)x_L + \underline{\delta}$ that separate A and B, we can observe the following: if the expected state is in A, its dynamics moves toward region B, since the priority is of L .
- Once the expected state is in B, the dynamics takes it back to the region A, since in this case in average the priority is of H (due to the condition $u^B = \rho + \epsilon$ and $\epsilon > 0$).

Results

Quadratic costs

- Because the frequency of tasks arriving is equal to that of attending them, a cyclical or complex dynamics emerges close to the referred frontier.
- Figure above shows the case where this system is a limit cycle.
- A similar situation involving regions B and C arises in the case of $\epsilon > 0$ and $\rho = u^B + \epsilon$. In these situations, the protocol is ruled by the protocols considered in regions A and B in the former case and by the protocols considered in regions B and C in the later case.
- For $\lambda \neq \mu$, either the expected value goes to infinite, converges to 0, to axis $x_L = 0$ or to axis $x_H = 0$, following different routes. Furthermore, different kinds of protocols are possible.

Final Remarks

- In the human dynamics of the tasks execution decisions the priority of one task is not always defined as being the most important current task.
- The dynamics of the work executions depends on the cumulated tasks of short run priorities, the importance of each kind of task and the intertemporal discount factor.
- We have found that the dynamics of the expected state of the system may be complex, exhibiting cycles of any order or with limit set being a dense subset of the interval depending on the parameter values of the model.

Final Remarks

- It is worth noting that complex dynamics in the solution of dynamic programming problems are usually obtained for low discount factors [Montrucchio and Sorger (1996)]. However, in our quadratic case, complex dynamics arises for discount factors of any size.

Are complex networks likely to arise in social interactions?

☺ Cajueiro, D. O. Agent preferences and the topology of networks. Physical Review E, v. 72, p. 047104, 2005.

- Agents choose their neighbors in order to maximize their satisfactions (or needs)
- We can define utility functions as functions that present a tradeoff between benefits and costs of making connections

Are complex networks likely to arise in social interactions?

- Small world networks: Two types of connections arise (a) connections between close (low cost) agents that bring some benefit; (b) connections between distant (high cost) agents that bring huge benefits.
- Scale-free networks: Either more connected nodes present smaller costs or higher benefits.
- Due to the homogeneity, regular (determinist homogeneity) and random (random homogeneity) are not expected.

How the flow of information influence the decision making process?

A solution based on the MG framework

☺ D. O. Cajueiro and R. S. De Camargo. Minority game with local interactions due to the presence of herding behavior. Physics Letters A v. 355, p. 280-284, 2006.

☺ B. A. Mello, V. M. C. S. Souza, D. O. Cajueiro and R. F. S. Andrade. Network evolution based on minority game with herding behavior European Physical Journal B v. 76, p. 147-156, 2010.

- Main principle: Agents herd more informed agents.
- Main result: The volatility of the system (usually) increases in the presence of herding behavior.

Enforcing behavior in complex networks

☺ D. O. Cajueiro. Enforcing social behavior in an Ising model with complex neighborhoods. Physica A 390, p. 1695-1703, 2011.

- How should one proceed if this one wants to influence individual behavior in a networked population?

Enforcing behavior in complex networks

Is this useful?

- This may be useful for instance to marketing companies interested in political marketing, diffusion of new products or changing habits of consumers in favor of a given company.
- This can also be useful to interested governments that intend to fight against habits such as smoking and drug consumption or to reduce criminality in a neighborhood of a city.

Enforcing behavior in complex networks

Related literature

- There are some works [Baik and Kim Int. Rev. Law Econ. 21 (2001) 271-285; Ferrer Eur. Econ. Rev. 54 (2010) 163-180] that study punishment for criminal behavior of a small group of individuals that interact in very simple neighborhoods. ☺ We work in a population explicitly modeled by a variation of the Ising model and we work with complex neighborhoods of many agents.
- There have been some attempts to characterize how consumer networks are formed [Kiss and Bichler Decis. Support Syst. 46 (2008) 233-253] ☺ We provide a nice justification for the importance of these works.

Enforcing behavior in complex networks

Framework

☺ S. N. Durlauf. PNAS 96, 10582-10584, 1999.

- Our system is formed of N agents (or N groups of agents with the same relevant characteristics).
- Each of them shares one of the two opposite behaviors, denoted by $\sigma_i = \pm 1$ (for instance, smoke or not smoke), for $i = 1, \cdots, N$.

Enforcing behavior in complex networks

Framework

- The decision of each agent towards one of these opinions depends on three different ingredients:
  - Agent's ability to support his own opinion.
  - A given external influence towards one of the opinions (such as current media).
  - The effect of the other agents' decision in this agent's decision. This setup is standard in the literature of social interactions and opinion formation.

☺ We include the possibility that an external interested party (such as government or a marketing company) is able to control part of the external influence that an agent is submitted.

Enforcing behavior in complex networks

Framework

We assume here (without lack of generality) that the opinion in favor of this desired behavior is $+1$. We also assume that the opinion of agent i changes from σ_i to $\tilde{\sigma}_i$, from one period to the next one, according to

$$\tilde{\sigma}_i = \begin{cases} \sigma_i \text{ with probability } \; p_i^u \\ -\sigma_i \text{ with probability } \; 1 - p_i^u \end{cases}$$

Enforcing behavior in complex networks

Framework

where

$$p_i^u = \frac{\exp(-\beta I_i^u)}{\exp(-\beta I_i^u) + \exp(\beta I_i^u)},$$

$\beta = 1/T$ is the inverse of the *social temperature*, a measure of the degree of randomness of the agents, and

$$I_i^u = -x_i - \sigma_i(h_i + u_i) - \sum_{\substack{j=1 \\ j \neq i}}^{N} r_{ji}\sigma_j\sigma_i = I_i - \sigma_i u_i$$

is the so-called *social impact* exerted on every individual. If $I_i > 0$ then the individual is inclined to change his opinion. On the other hand, if $I_i < 0$, the opposite happens.

Enforcing behavior in complex networks

Optimal policy

- We are looking for the optimal level of *enforcement* u_i that the interested party should introduce in the system in order to influence the behavior of an individual i of the population.
- We assume that the interested party will choose the aggressive campaign in order to maximize the probability of the agents in the next period performing the desired behavior.
- We assume that the cost with the enforcement law is constrained to K, i.e., $\sum_{i=1}^{N} u_i = K$.

Enforcing behavior in complex networks

Optimal policy

We show that the optimal policy $u^\star$ and $\mu^\star$ that solve this problem is given by

$$u_i^\star = \frac{I_i}{\sigma_i} + \frac{K}{N} - \overline{I/\sigma}, \text{ for } i = 1, \cdots, N$$

where $\overline{I/\sigma} = \frac{1}{N}\sum_{k=1}^{N} \frac{I_k}{\sigma_k}$.

Enforcing behavior in complex networks

Optimal policy

For the optimal policy, one gets

$$\tilde{p}_i^{u^\star} = \exp\left[\beta(K/N - \overline{I/\sigma})\right]/2\cosh\left[\beta(K/N - \overline{I/\sigma})\right].$$

Therefore, the optimal policy equalizes the probabilities $\tilde{p}_i^u$, for $i = 1, \cdots, N$, for both types of agents.

- The intuition behind this policy is simple:
  - If an individual is strongly inclined to the desired decision, the interested party does not need to worry about him.
  - The interested party may even reduce the resources directed to him.
  - This happens because he has a high self-support parameter in favor of the desired action, the field has a strong positive effect on him or his peers strongly affect his behavior towards the desired decision.

Enforcing behavior in complex networks

Complex neighborhoods

How do complex neighborhoods affect the optimal enforcement law $u^{\star}$?

- We show that distribution of the enforcement laws depends strongly on the topology of the complex neighborhood.

Enforcing behavior in complex networks

Complex neighborhoods

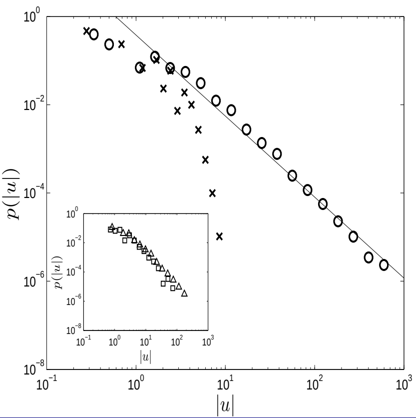


Figura: Probability distribution of the absolute values of the optimal

Enforcing behavior in complex networks

How may one identify the role of each individual in the network?

- We show that the Lagrange multipliers associated to an associated optimization problem can be used to identify the role of each individual in the network.

Enforcing behavior in complex networks

How may one identify the role of each individual in the network?

An example: Solve the problem $max_{u_1,u_2,u_3} u_1 + 2u_2 + 3u_3$ subject to $u_1 + u_2 + u_3 = 1$ and $u_1 \geq 0, \quad u_2 \geq 0, \quad u_3 \geq 0$

- It is easy to show that the solution of this problem is given by $(u_1, u_2, u_3, \lambda_1, \lambda_2, \lambda_3, \mu) = (0, 0, 1, 2, 1, 0, 3)$.
- Note that if we could have $u_1 = \delta_1 < 0$ and $u_2 = \delta_2 < 0$, we would be able to increase the maximum of the function $u_1 + 2u_2 + 3u_3$ by $-2\delta_1 - \delta_2$ units.
- The effect of reducing the value of u_1 by $-\delta_1$ units is larger than the effect of reducing the value of u_2 by $-\delta_2$ units.
- Therefore, individual 1 is a better resource provider than individual 2. Thus, $\lambda_1 > \lambda_2$.

Navigation in complex networks

What is that?

A walk consists of stepping from node to node of the complex network via the links between them.
Examples:

- Displacement of a walker in a city
- Transmission of information between computers
- Internet searching
- Accessing a member of an organization

Navigation in complex networks

Why are people interested in that?

- They may identify the ability of communication of two individuals in the network.
- They may investigate how the topology of the network constraints the communication (displacement) among (of) agents in the network.
- They may create optimal topologies for searching.
- They may design techniques to identify the most important agents in the network.

Navigation in complex networks

Random walkers X directed walkers

- "Random walkers": a random walker located at a specific node chooses one of the neighbors of this node based on some transition matrix in order to continue the walk [Noh and Rieger. PRL 92, 118701 (2004), Yang. PRE 71,016102 (2005), Costa e Travieso. PRE 75, 016102 (2007)]
- "Directed walkers": each step the walker takes the shortest path to the target [Sneppen *et al.*. EPL 69, 853 (2005). Rosvall *et al.* PRL 94, 028701 (2005). Rosvall *et al.* PRE 72, 046117 (2005)]

Navigation in complex networks

Optimal Navigation in complex networks

☺D. O. Cajueiro. Optimal navigation in complex networks. Physical Review E, 79, 046103, 2009.
☺ What drives the behavior of the walker?

Navigation in complex networks

Optimal Navigation in complex networks

- Assume that the sites of a city are represented by a network G with n nodes $V(G) = \{1, 2, \cdots, t, \cdots, n\}$ where t is a special node called target.
- In each node, a traveler has to choose between making the next step of his walk randomly at a cost C_N or using the link that will certainly approximate him to the target t with cost $C_N + C_I$, where C_N is the constant cost of one step navigation and C_I is the constant cost of asking people for the correct direction.

Navigation in complex networks

Optimal Navigation in complex networks

- We assume that the traveler makes the decision in order to minimize the cost of the trajectory given by

$$J(i) = \min_{\pi \in \Pi} E_{\pi} \left[\sum_{k \in \mathcal{P}(i,t)} g(k, u(k))/i \right]$$

where $\mathcal{P}(i,t)$ is the path from node i to the target t, the expectance $E^{\pi}[\cdot/i]$ is conditional to the policy π and to the node i, $\pi = \{u(1), u(2), \cdots, u(t), \cdots, u(n)\}$ is an admissible policy that belongs to the set of admissible policies Π and $u(k)$ is the admissible control that belongs to the set of admissible controls $U(k) = \{0, 1\}$, $\forall k$.

Navigation in complex networks

Optimal Navigation in complex networks

- This problem can be solved numerically using dynamic programming. The convergence is ensured by the Banach Fixed Point Theorem.

Navigation in complex networks

Optimal Navigation in complex networks

- We show that two extreme regimes arise (depending on the cost of information) – one dominated by directed walkers and the other by random walkers.
- We show that the critical point of the transition from one regime to the other is a function of the connectivity and the size of the network.
- We show that this approach can be used to generalize several concepts presented in the literature concerning random navigation and direct navigation.
- We show that the investigation of the extreme regimes dominated by random walkers and directed walkers is not sufficient to correctly assess the characteristics of navigation in complex networks.

Navigation in complex networks

Optimal navigation and the centrality of the nodes

☺D. O. Cajueiro. Optimal navigation for characterizing the role of the nodes in complex networks. Physica A, v. 389, p. 1945-1954, 2010.

- We use the approach of optimal navigation to evaluate the centrality of a node and to characterize its role in a network.
- We show that the centrality measures inherited from the approach of optimal navigation may be considered if one desires to evaluate the centrality of the nodes using other pieces of information beyond the geometric properties of the network.

Navigation in complex networks

Optimal navigation and the centrality of the nodes

- Evaluating the correlations between these inherited measures and classical measures of centralities such as the degree of a node and the characteristic path length of a node, we have found that: (1) in some cases the centrality of the nodes may be explained by the other measures of centrality; (2) in other cases, we have found non-trivial results.

Navigation in complex networks

Learning path in complex networks

☺ D. O. Cajueiro and R. F. S. Andrade. Learning paths in complex networks Europhysics Letters 87, 58004, 2009.

- The approach has been partially motivated by recent progress in characterizing navigation problems in networks, having as extreme situations the completely ignorant (random) walker and the rich directed walker, which can pay for information that will guide to the target node along the shortest path.
- A learning framework based on a first-visit Monte Carlo algorithm is implemented, together with four independent measures that characterize the learning process: Average path length (and normalized), velocity of learning (and normalized)
- The results indicate that the navigation difficulty and learning velocity are strongly related to the network topology.

Controlling self-organized criticality. Why?

- Although self-organized systems organize by themselves in a state of lower energy, this reorganization is very costly for society.
- SOC reorganization generates avalanches of all sizes with power-law like distributions.

How do we intend to do it?

- "Control" can be understood as an interference in the processes by which the system dissipates energy.
- We intend to avoid large avalanches generating avalanches of small and moderate size.

What cannot be done?

- We cannot shake the planet. Therefore, it is not possible to control earthquakes.

What can we do?

- We can induce avalanches in restricted hill slides in order to warrant safety for ski riders.
- We can enforce behavior in self-organized driven (population) systems such as highway traffic.

What can we do?

- We can ensure portfolios using some kind of stop loss strategy.
- We can reduce crises caused by the break of large economic bubbles:

The Fed has no explicit mandate under the law to try to contain a stock-market bubble. Indirectly we had the authority to do so, if we believed stock prices were creating inflationary pressures. (...) All the same, we agreed that trying to avoid a bubble was consistent with our mission, and that it was our duty to take the chance. (...) Then we met again on March 25 and raised short-term rates by 0.25 percent (...). Alan Greenspan – The age of turbulence.

Other related questions

- Are there control schemes behind the self-organized critical avalanches that take place in the brain?
- Is it possible to control economic fluctuations?
- How about fluctuations that arise in economic and business chains?
- How may one efficiently control self-organized driven systems?

Some useful definitions

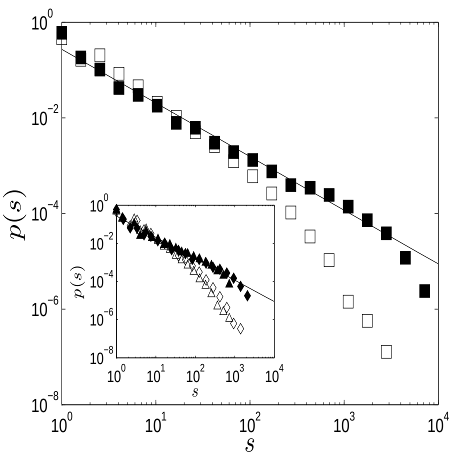
- Controlled avalanches: avalanches generated by the control system
- Uncontrolled avalanches: usual avalanches caused by the natural deposition process

Controlling SOC in two-dimensional squares

☺ Cajueiro and Andrade. Controlling self-organized criticality in sandpile models. Phys. Rev. E 81, 015102(R), 2010.

- The main assumption is that we have a replica model of the system ☹.
- Using the replica model of the system, the control scans the system and identifies potentially large events whenever the avalanche risk is high enough.
- Once a threat is detected, an externally induced avalanche is triggered. [There is a threshold to decide whenever a controlled avalanche should be generated!]

Results



How far from optimal are these heuristics to control SOC?

☺ Cajueiro and Andrade. A dynamical programming approach for controlling the directed abelian Dhar-Ramaswamy model. Forthcoming in Physical Review E (2010)

- We try to minimize a combined cost index that includes the cost of avalanches and the cost of intervention.
- When large avalanches take place in the system, the system is strongly penalized.
- We solve this problem using a dynamical programming approach (for tiny systems ☹).
- We show that the heuristics that are being considered are close to the optimal rules ☺.

Controlling self-organized criticality in complex networks

☺ Cajueiro and Andrade. Controlling self-organized criticality in complex networks. Forthcoming in European Physical Journal B (2010).

- In each node of a complex network, there is a demand for limited recourses.
- Since the resources are limited, if the demand exceeds a given threshold in a node, an avalanche happens, i.e., the provider of the resource in this node is closed and the demand is guided to the neighbors of this node.
- Being a system that exhibit self-organized criticality, it is clear that a local avalanche in one node may trigger avalanches in the node's neighbors, transforming small avalanches into large avalanches.

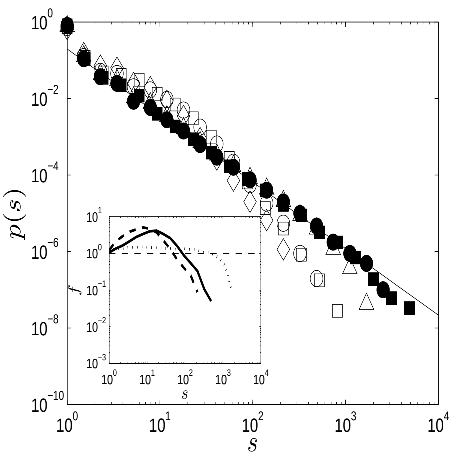
Controlling self-organized criticality in complex networks

- Besides, large avalanches are undesired in the system, since it may destabilize several providers of resources simultaneously.
- Therefore, one way of avoiding this kind of phenomenon is, before it happens in one node with full mass, to close for an instant of time the provider of the resource in the node and to move the demand to the node's neighbors.
- Instantaneous closures can avoid big avalanches.

Control scheme

- We choose the percentage of highest degree nodes of the network that will be controlled.
- To be controlled here means that if a node is close to become critical, the control system triggers an explosion on this node.
- This means that a real controlled avalanche is triggered by emptying the node and the mass available in this site goes randomly to some of the neighbors of this node.
- When it is the case, first we trigger an avalanche in the highest degree node, then we trigger in the second highest degree node and so on.

Results (Free scale networks)



Controlling SOC works, but

- The spatial nature of the problem of controlling self-organized makes the problem very difficult and challenging.
- It is very difficult to deal with the "coupled nonlinear difference equations" that describe the SOC behavior.
- Pure optimization principles with perfect knowledge of the system (until now) only work for tiny systems (based on numerical solutions and the Banach fixed point theorem).

Research agenda

- Analytical results are welcome!
- Some interesting applications or connections with real problems (such as the problem of insurance portfolio) are also welcome.
- It is also interesting to test these ideas em real SOC systems such as rice piles.

Research agenda

- The focus of the work until now is on variations of sandpiles models, where the interesting variable is the size of avalanches. What can be done in the other, for instance, self-organized driven systems.
- Is it possible to build a control system based on some kind of optimization principle that could be in some sense reduce the concentration of mass in the network.
- How to define controllability for this class of controlled systems?

Main conclusion

- Interesting issues can arise if we put together ingredients of Complex systems, optimization theory and control theory.