RESULTADOS, VERIFICAÇÕES E APLICAÇÕES RECENTES NA ESTATÍSTICA NÃO EXTENSIVA

Constantino Tsallis

Centro Brasileiro de Pesquisas Físicas

e Instituto Nacional de Ciência e Tecnologia de Sistemas Complexos

Rio de Janeiro

e

Santa Fe Institute, New Mexico, USA

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TYPICAL SIMPLE SYSTEMS:

e.g., $W(N) \propto \mu^N \;\; (\mu > 1)$

Short-range space-time correlations

Markovian processes (short memory), Additive noise

Strong chaos (positive maximal Lyapunov exponent), Ergodic, Euclidean geometry

Short-range many-body interactions, weakly quantum-entangled subsystems

Linear/homogeneous Fokker-Planck equations, Gausssians

→ Boltzmann-Gibbs entropy (additive)

→ Exponential dependences (Boltzmann-Gibbs weight, ...)

TYPICAL COMPLEX SYSTEMS:

e.g., $W(N) \propto N^\rho \;\; (\rho > 0)$

Long-range space-time correlations

Non-Markovian processes (long memory), Additive and multiplicative noises

Weak chaos (zero maximal Lyapunov exponent), Nonergodic, Multifractal geometry

Long-range many-body interactions, strongly quantum-entangled sybsystems

Nonlinear/inhomogeneous Fokker-Planck equations, *q*-Gaussians

→ Entropy S_q (nonadditive)

→ *q*-exponential dependences (asymptotic power-laws)

- ALGUMAS VERIFICAÇÕES RECENTES

- ALGUNS RESULTADOS NOSSOS RECENTES

QUANTUM SPHERICALLY-SYMMETRIC POTENTIALS WITH *q*-GAUSSIAN GROUND STATES:

Vignat, Plastino, Plastino and Dehesa, 1011.3459 [cond-mat.stat-mech]

$$-\frac{1}{2r^{D-1}}\frac{\partial}{\partial r}\left(r^{D-1}\frac{\partial\psi(r)}{\partial r}\right)+V(r)\ \psi(r)=E\ \ \psi(r) \qquad (h=m=1;\ D\geq 1)$$

where $V_\nu(r)=-\frac{1}{2}\left[1+\frac{D}{2(\nu-1)}\right]\frac{\psi_{\nu-1}(r)}{\psi_\nu(r)}\sim-\frac{2(\nu-1)+D}{r} \qquad \left(\nu\geq 1-\frac{D}{2}\right)$

with $\psi_\nu(r)\equiv\frac{2^{1-\nu}}{\Gamma(\nu)}r^\nu K_\nu(r) \qquad (K_\nu(r)\equiv$ Bessel function of 2nd kind$)$

For the ground state we obtain:

$$\left|\psi(p)\right|^2\propto\frac{1}{\left[1+(q-1)\beta p^2\right]^{\frac{1}{q-1}}}\equiv e_q^{-\beta p^2} \qquad \left(p^2=\sum_{i=1}^{D}p_i^2\right)$$

with $1\leq q=\frac{D+2\nu+1}{D+2\nu}<2$

D=3: Hydrogenoid atom and Newtonian gravitation!

ν	$V_\nu(r)$	q
$\frac{1}{2}$	$-\frac{D-1}{2r}$	$\frac{D+2}{D+1}$
$\frac{3}{2}$	$-\frac{1}{2}\frac{D+1}{r+1} \sim -\frac{D+1}{2r}$	$\frac{D+4}{D+3}$
$\frac{5}{2}$	$-\frac{1}{2}\frac{(D+3)(r+1)}{r^2+3r+3} \sim -\frac{D+3}{2r}$	$\frac{D+6}{D+5}$
$\frac{7}{2}$	$-\frac{1}{2}\frac{(D+5)(r^2+3r+3)}{r^3+6r^2+15r+15} \sim -\frac{D+5}{2r}$	$\frac{D+8}{D+7}$

Vignat, Plastino, Plastino and Dehesa, 1011.3459 [cond-mat.stat-mech]

Journal of Statistical Mechanics: Theory and Experiment
An IOP and SISSA journal

Thermostatistics in the neighbourhood of the π-mode solution for the Fermi–Pasta–Ulam β system: from weak to strong chaos

Mario Leo[1], Rosario Antonio Leo[1] and Piergiulio Tempesta[2]

[1] Dipartimento di Fisica, Università del Salento, Via per Arnesano, 73100—Lecce, Italy
[2] Departamento de Física Teórica II, Facultad de Físicas, Ciudad Universitaria, Universidad Complutense, 28040—Madrid, Spain
E-mail: mario.leo@le.infn.it, leora@le.infn.it and p.tempesta@fis.ucm.es

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Abstract. We consider a π-mode solution of the Fermi–Pasta–Ulam β system. By perturbing it, we study the system as a function of the energy density from a regime where the solution is stable to a regime where it is unstable, first weakly and then strongly chaotic. We introduce, as an indicator of stochasticity, the ratio ρ (when it is defined) between the second and the first moment of a given probability distribution. We will show numerically that the transition between weak and strong chaos can be interpreted as the symmetry breaking of a set of suitable dynamical variables. Moreover, we show that in the region of weak chaos there is numerical evidence that the thermostatistic is governed by the Tsallis distribution.

J. Stat. Mech. (2010) P04021

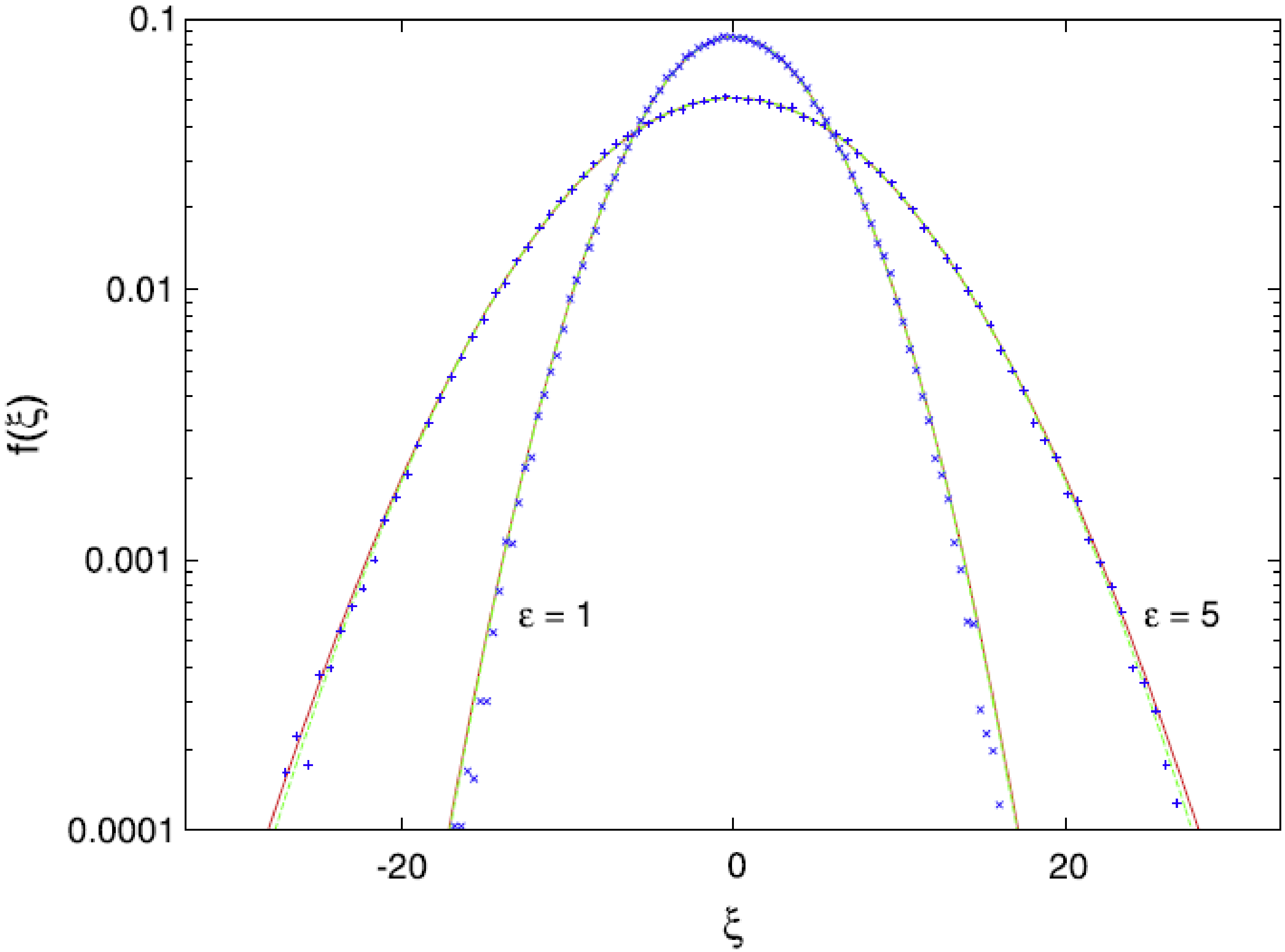


Figure 5. Plot on a linear-log scale of the numerical distribution $f(\xi)$ (blue points) fitted with a Tsallis distribution (red) and a Gauss distribution (green) for $N = 128$, $\epsilon = 1$ and 5. In both cases the Tsallis and Gaussian distributions essentially overlap.

M. Leo, R.A. Leo and P. Tempesta, J Stat Mech P04021 (2010)

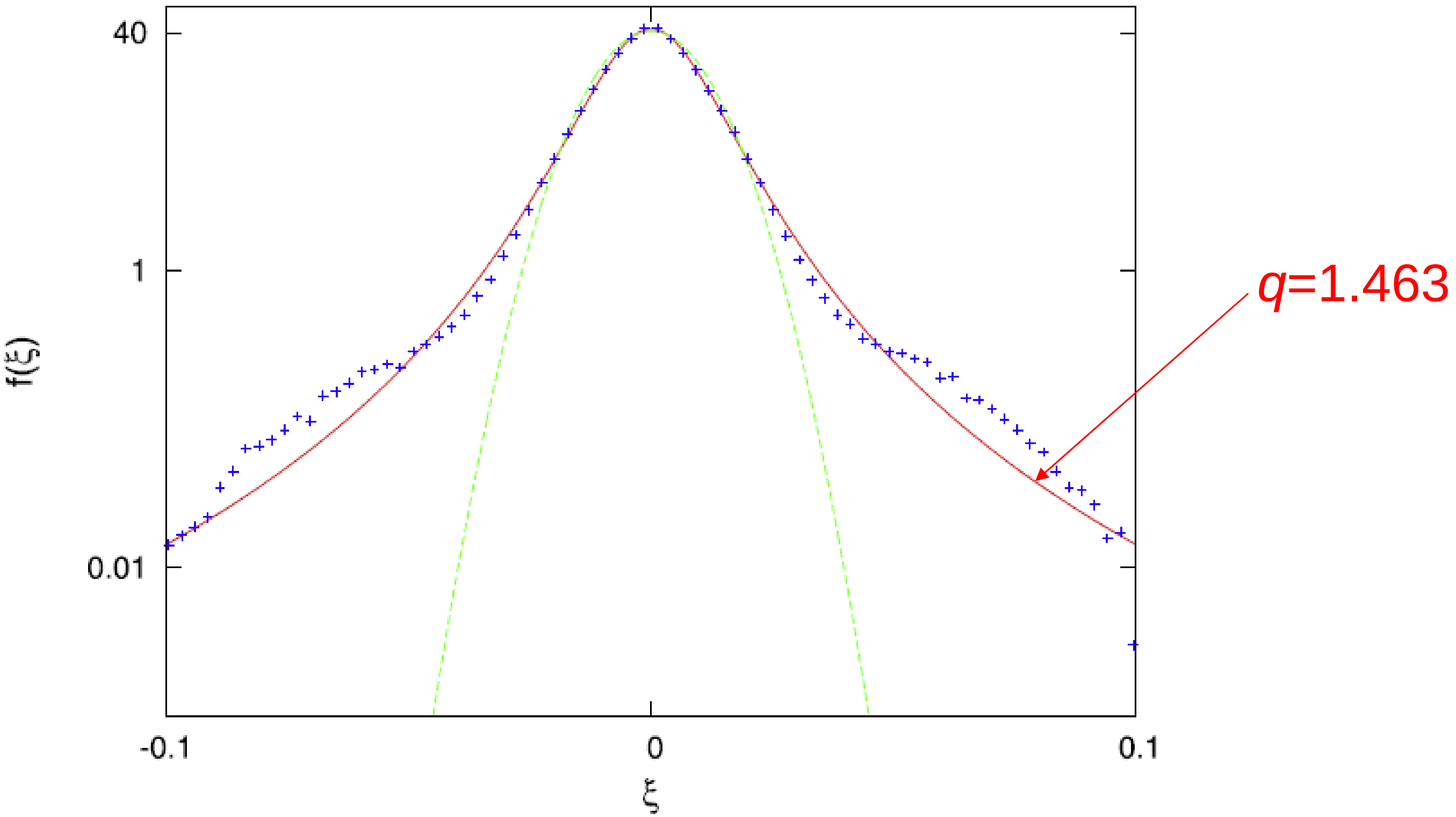


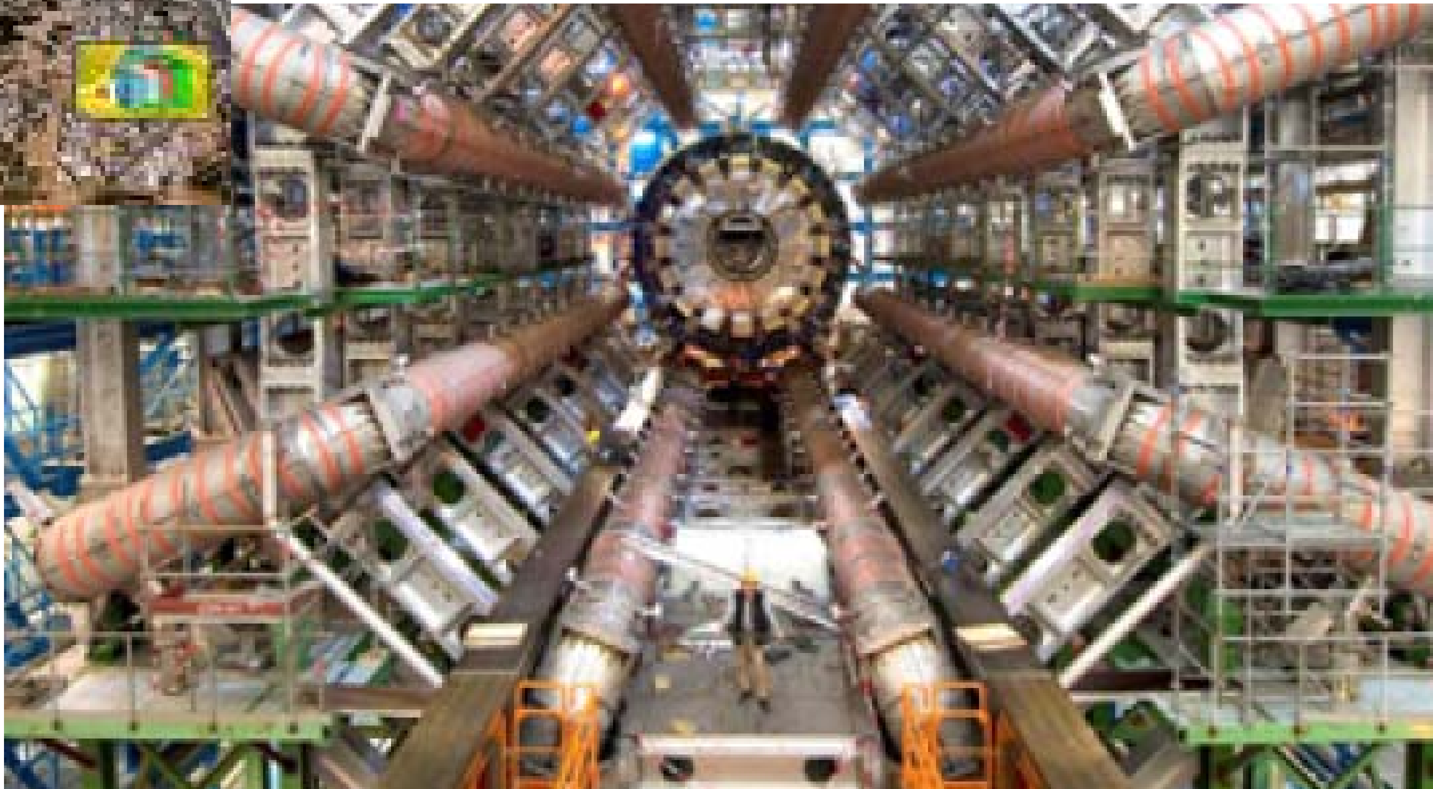
Figure 4. Plot on a linear-log scale of the numerical distribution $f(\xi)$ (blue points) fitted with a Tsallis distribution (red) and a Gauss distribution (green) for $N = 128$ and $\epsilon = 0.006$.

M. Leo, R.A. Leo and P. Tempesta, J Stat Mech P04021 (2010)

LHC (Large Hadron Collider)

CMS (Compact Muon Solenoid) detector

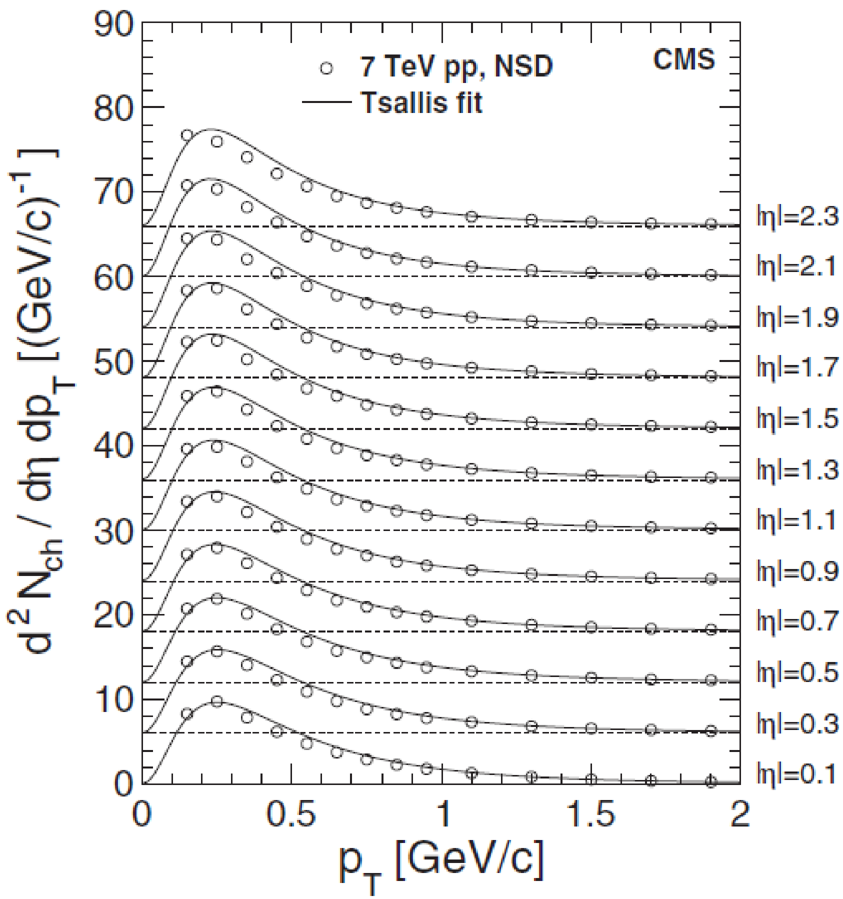
~ 2500 scientists/engineers from 183 institutions of 38 countries

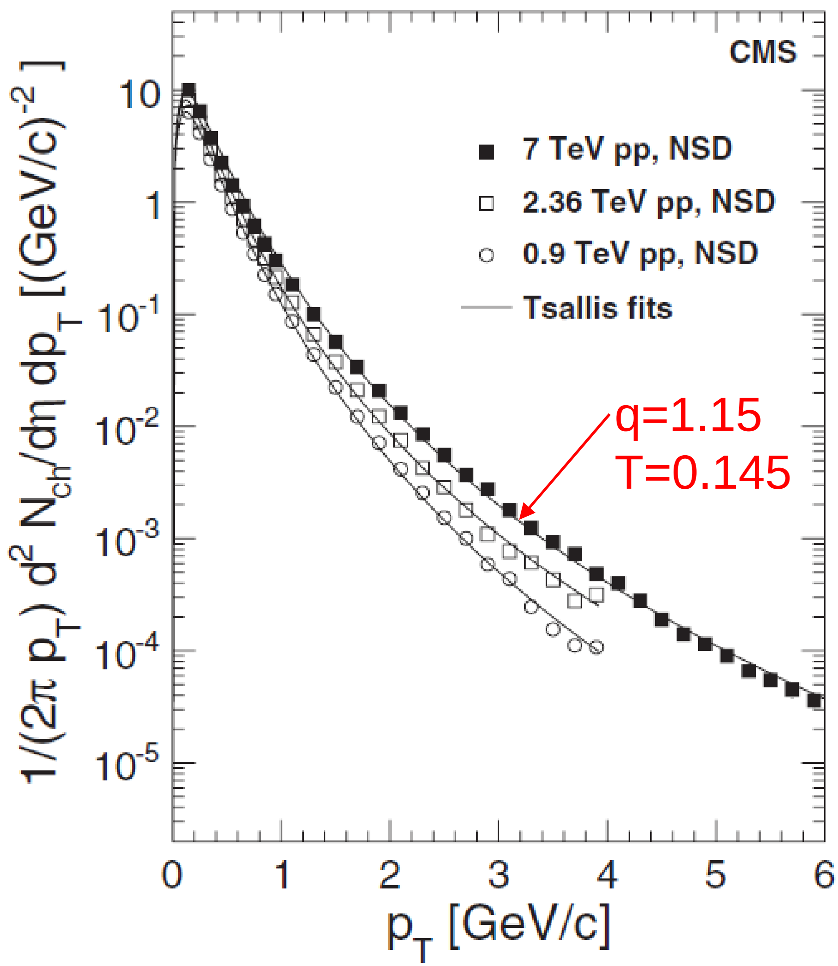


Transverse-Momentum and Pseudorapidity Distributions of Charged Hadrons in *pp* Collisions at $\sqrt{s} = 7$ TeV

V. Khachatryan *et al.**
(CMS Collaboration)

(Received 18 May 2010; published 6 July 2010)



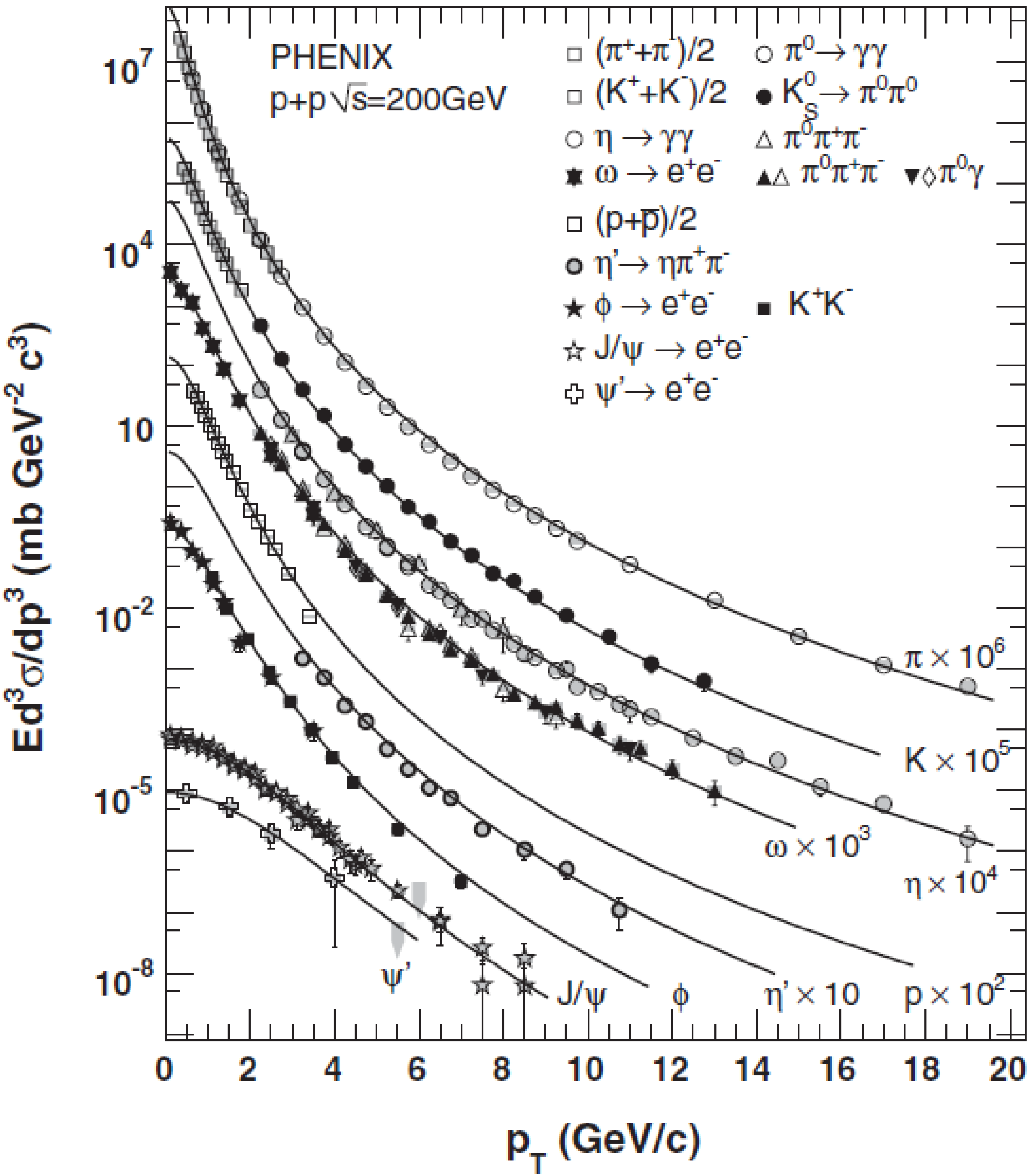


PHYSICAL REVIEW D **83**, 052004 (2011)

Measurement of neutral mesons in $p + p$ collisions at $\sqrt{s} = 200$ GeV and scaling properties of hadron production

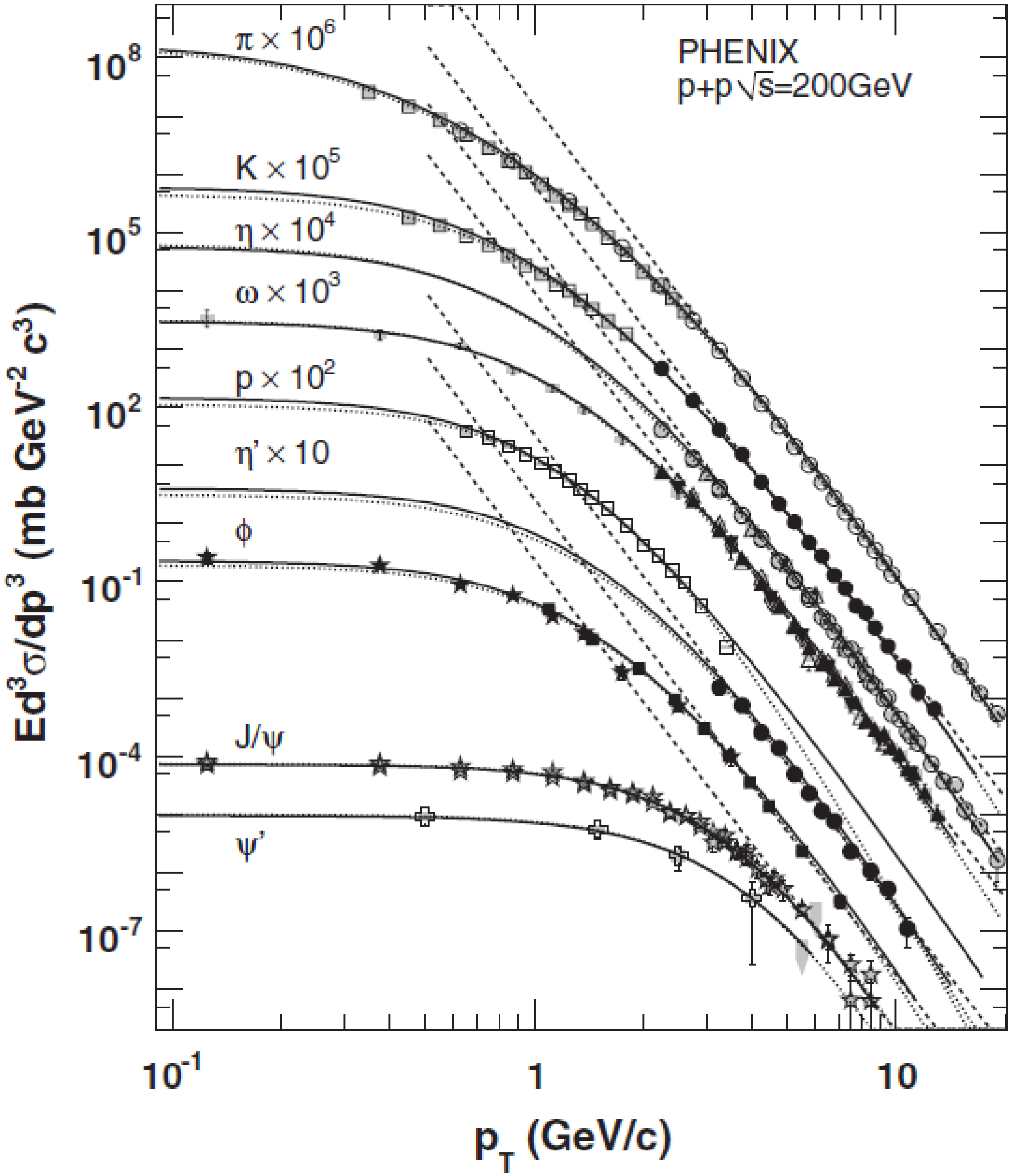
A. Adare,[11] S. Afanasiev,[25] C. Aidala,[12,36] N. N. Ajitanand,[53] Y. Akiba,[47,48] H. Al-Bataineh,[42] J. Alexander,[53] K. Aoki,[30,47] L. Aphecetche,[55] R. Armendariz,[42] S. H. Aronson,[6] J. Asai,[47,48] E. T. Atomssa,[31] R. Averbeck,[54] T. C. Awes,[43] B. Azmoun,[6] V. Babintsev,[21] M. Bai,[5] G. Baksay,[17] L. Baksay,[17] A. Baldisseri,[14] K. N. Barish,[7] P. D. Barnes,[33] B. Bassalleck,[41] A. T. Basye,[1] S. Bathe,[7] S. Batsouli,[43] V. Baublis,[46] C. Baumann,[37] A. Bazilevsky,[6] S. Belikov,[6,*] R. Bennett,[54] A. Berdnikov,[50] Y. Berdnikov,[50] A. A. Bickley,[11] J. G. Boissevain,[33] H. Borel,[14] K. Boyle,[54] M. L. Brooks,[33] H. Buesching,[6] V. Bumazhnov,[21] G. Bunce,[6,48] S. Butsyk,[33,54] C. M. Camacho,[33] S. Campbell,[54] B. S. Chang,[62] W. C. Chang,[2] J.-L. Charvet,[14] S. Chernichenko,[21] J. Chiba,[26] C. Y. Chi,[12] M. Chiu,[22] I. J. Choi,[62] R. K. Choudhury,[4] T. Chujo,[58,59] P. Chung,[53] A. Churyn,[21] V. Cianciolo,[43] Z. Citron,[54] C. R. Cleven,[19] B. A. Cole,[12] M. P. Comets,[44] P. Constantin,[33] M. Csanád,[16] T. Csörgő,[27] T. Dahms,[54] S. Dairaku,[30,47] K. Das,[18] G. David,[6] M. B. Deaton,[1] K. Dehmelt,[17] H. Delagrange,[55] A. Denisov,[21] D. d'Enterria,[12,31] A. Deshpande,[48,54] E. J. Desmond,[6] O. Dietzsch,[51] A. Dion,[54] M. Donadelli,[51] O. Drapier,[31] A. Drees,[54] K. A. Drees,[5] A. K. Dubey,[61] A. Durum,[21] D. Dutta,[4] V. Dzhordzhadze,[7] Y. V. Efremenko,[43] J. Egdemir,[54] F. Ellinghaus,[11] W. S. Emam,[7] T. Engelmore,[12] A. Enokizono,[32] H. En'yo,[47,48] S. Esumi,[58] K. O. Eyser,[7] B. Fadem,[38] D. E. Fields,[41,48] M. Finger, Jr.,[8,25] M. Finger,[8,25] F. Fleuret,[31] S. L. Fokin,[29] Z. Fraenkel,[61,*] J. E. Frantz,[54] A. Franz,[6] A. D. Frawley,[18] K. Fujiwara,[47] Y. Fukao,[30,47] T. Fusayasu,[40] S. Gadrat,[34] I. Garishvili,[56] A. Glenn,[11] H. Gong,[54] M. Gonin,[31] J. Gosset,[14] Y. Goto,[47,48] R. Granier de Cassagnac,[31] N. Grau,[12,24] S. V. Greene,[59] M. Grosse Perdekamp,[22,48] T. Gunji,[10] H.-Å. Gustafsson,[35,*] T. Hachiya,[20] A. Hadj Henni,[55] C. Haegemann,[41] J. S. Haggerty,[6] H. Hamagaki,[10] R. Han,[45] H. Harada,[20] E. P. Hartouni,[32] K. Haruna,[20] E. Haslum,[35] R. Hayano,[10] M. Heffner,[32] T. K. Hemmick,[54] T. Hester,[7] X. He,[19] H. Hiejima,[22] J. C. Hill,[24] R. Hobbs,[41] M. Hohlmann,[17] W. Holzmann,[53] K. Homma,[20] B. Hong,[28] T. Horaguchi,[10,47,57] D. Hornback,[56] S. Huang,[59] T. Ichihara,[47,48] R. Ichimiya,[47] H. Iinuma,[30,47] Y. Ikeda,[58] K. Imai,[30,47] J. Imrek,[15] M. Inaba,[58] Y. Inoue,[49,47] D. Isenhower,[1] L. Isenhower,[1] M. Ishihara,[47] T. Isobe,[10] M. Issah,[53] A. Isupov,[25] D. Ivanischev,[46] B. V. Jacak,[54,†] J. Jia,[12] J. Jin,[12] O. Jinnouchi,[48] B. M. Johnson,[6] K. S. Joo,[39] D. Jouan,[44] F. Kajihara,[10] S. Kametani,[10,47,60] N. Kamihara,[47,48] J. Kamin,[54] M. Kaneta,[48] J. H. Kang,[62] H. Kanou,[47,57] J. Kapustinsky,[33] D. Kawall,[36,48] A. V. Kazantsev,[29] T. Kempel,[24] A. Khanzadeev,[46] K. M. Kijima,[20] J. Kikuchi,[60] B. I. Kim,[28] D. H. Kim,[39] D. J. Kim,[62] E. Kim,[52] S. H. Kim,[62] E. Kinney,[11] K. Kiriluk,[11] Á. Kiss,[16] E. Kistenev,[6] A. Kiyomichi,[47] J. Klay,[32] C. Klein-Boesing,[37] L. Kochenda,[46] V. Kochetkov,[21] B. Komkov,[46] M. Konno,[58] J. Koster,[22] D. Kotchetkov,[7] A. Kozlov,[61] A. Král,[13] A. Kravitz,[12] J. Kubart,[8,23] G. J. Kunde,[33] N. Kurihara,[10]

PHENIX @ RHIC



PHENIX
p+p √s=200GeV
(π⁺+π⁻)/2
π⁰→ γγ
(K⁺+K⁻)/2
K⁰S→ π⁰π⁰
η → γγ
π⁰π⁺π⁻
ω → e⁺e⁻
π⁰π⁺π⁻
π⁰γ
(p+p̄)/2
η'→ ηπ⁺π⁻
φ → e⁺e⁻
K⁺K⁻
J/ψ → e⁺e⁻
ψ'→ e⁺e⁻
π × 10⁶
K × 10⁵
ω × 10³
η × 10⁴
ψ'
J/ψ
φ
η' × 10
p × 10²
Ed³σ/dp³ (mb GeV⁻² c³)
p_T (GeV/c)

$q \simeq 1.10$



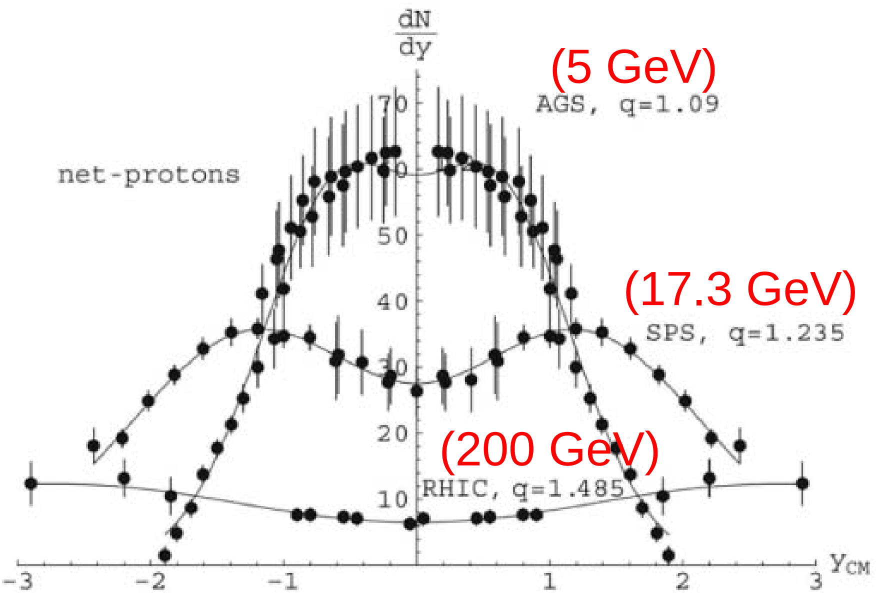
$q \simeq 1.10$

FIG. 13. The p_T spectra of various hadrons measured by PHENIX fitted to the power law fit (dashed lines) and Tsallis fit (solid lines). See text for more details.

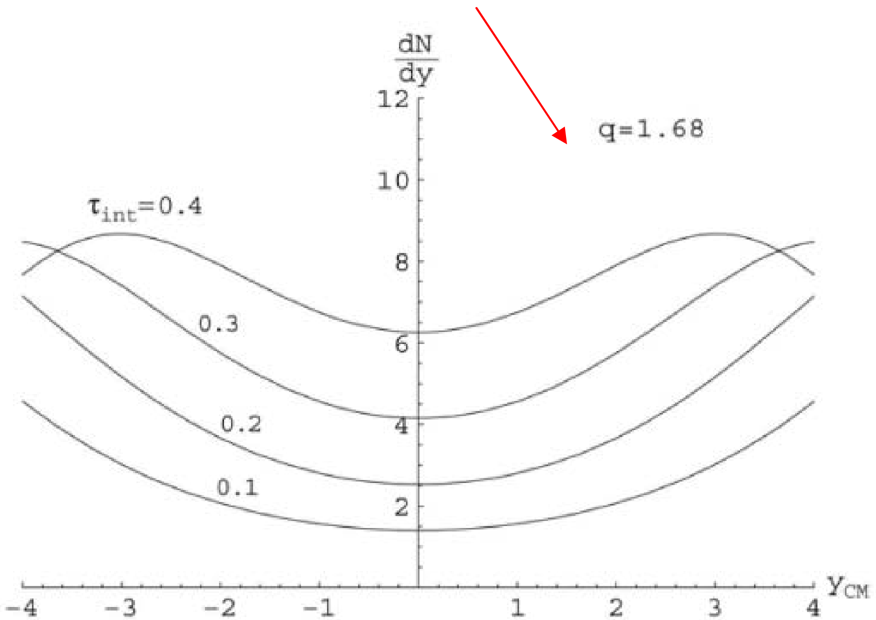
RAPIDITY DISTRIBUTION FOR PROTON PRODUCTION

IN HEAVY-NUCLEI COLLISIONS:

P. Czerski, Int J Mod Phys A **26**, 638 (2011)



prediction for LHC



PHYSICAL REVIEW E **83**, 017401 (2011)

Occupancy of rotational population in molecular spectra based on nonextensive statistics

J. L. Reis Jr.,[1,2] J. Amorim,[3] and A. Dal Pino Jr.[1]

[1] *Departamento de Física, Instituto Tecnológico de Aeronáutica, 12228-900 São José dos Campos, São Paulo, Brazil*

[2] *Universidade Paulista, Rodovia Presidente Dutra, km 156, 12240-420 Pista Sul, São José dos Campos, São Paulo, Brazil*

[3] *Laboratório Nacional de Ciência e Tecnologia do Bietanol–CTBE/CNPEM, Caixa Postal 6170, CEP 13083-970 Campinas, São Paulo, Brazil*

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The procedure to obtain gas temperature in plasmas is to fit the experimental rotational spectrum to a theoretical one based on the Boltzmann distribution. For many systems a single distribution fails to account for the occupation of the levels. Researchers have improved the fitting by coupling two distributions and obtaining two distinct temperatures. They assigned the lowest temperature to the gas. Here, we show that these systems should be described by Tsallis nonextensive statistics and its unique associated temperature. Experimental and simulated spectra are tested and excellent agreement is obtained.

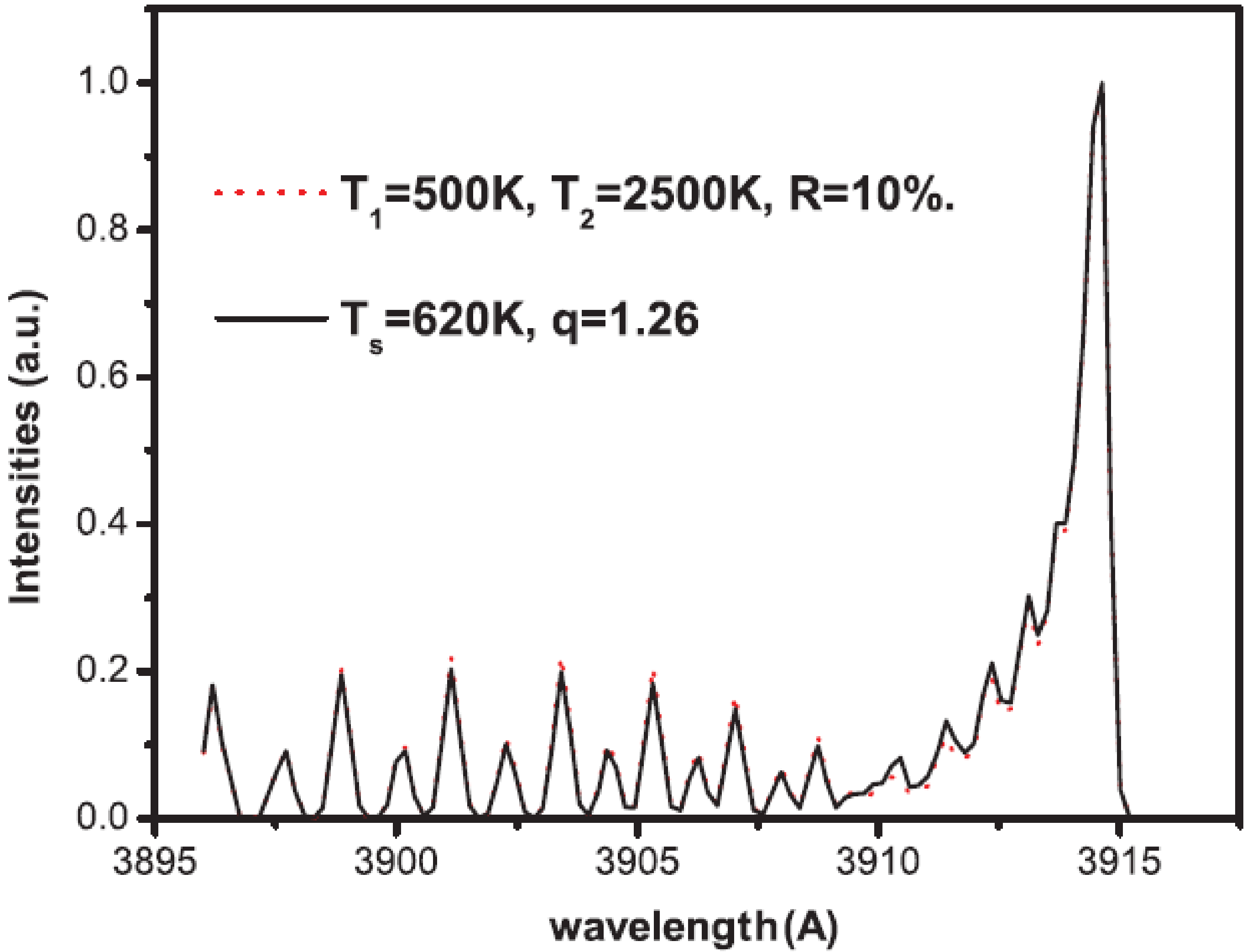

FIG. 1. (Color online) Comparison between two simulated spectra. The dotted line corresponds to a spectrum generated through the two–Boltzmann distribution method and the solid one corresponds to a single Tsallis distribution. Temperatures and the parameters R and q are shown.

Reis, Amorim and Dal Pino
Phys Rev E 83, 017401 (2011)

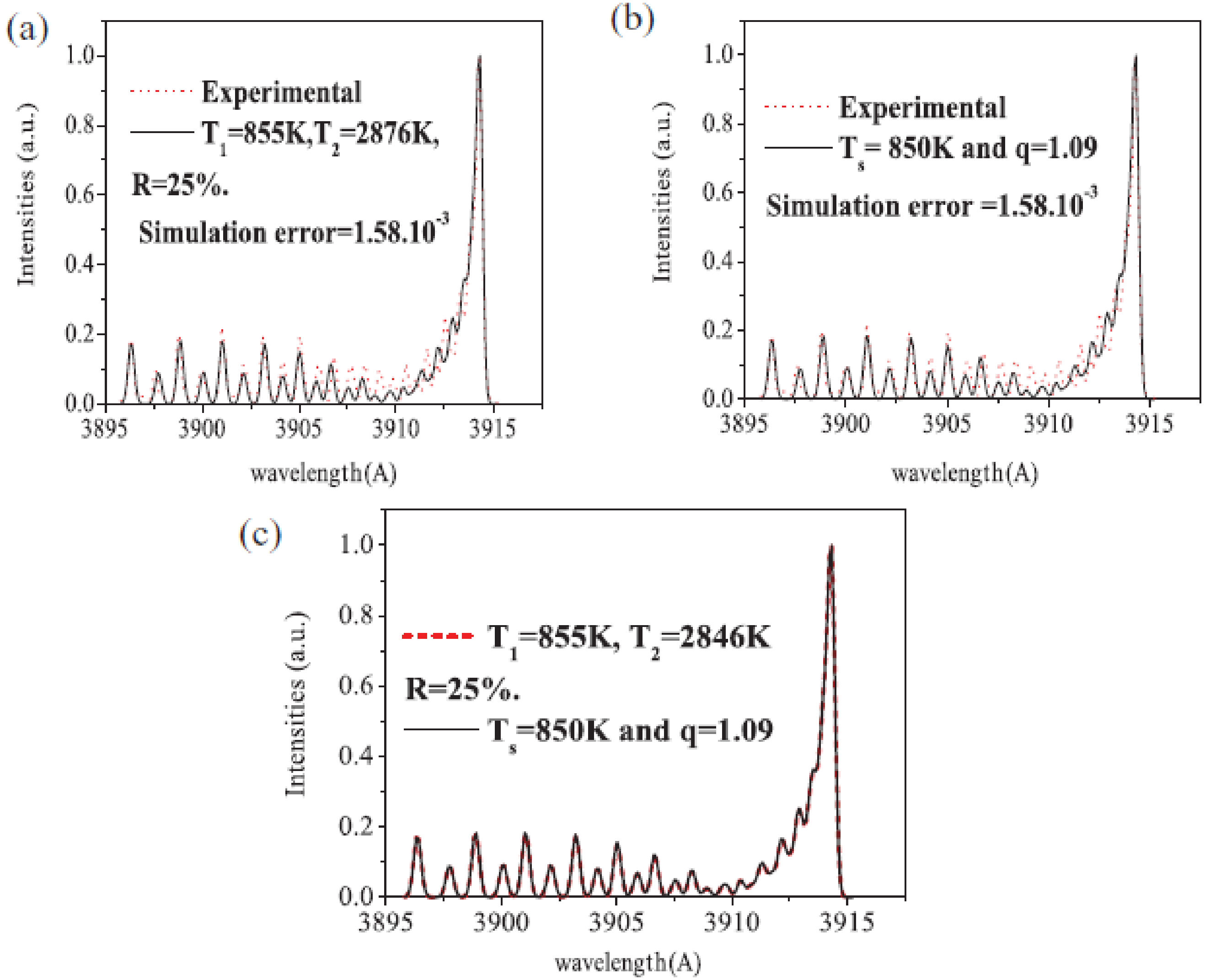

FIG. 2. (Color online) Experimental and simulated spectra for the FNS for 1.0 Torr of pressure and current of 30 mA. (a) Experimental vs the two–Boltzmann distribution method. (b) Experimental vs Tsallis single-temperature distribution. (c) Confrontation of the two simulated spectra.

Reis, Amorim and Dal Pino
Phys Rev E 83, 017401 (2011)

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Strain-profile determination in ion-implanted single crystals using generalized simulated annealing

Alexandre Boulle[a]* and Aurélien Debelle[b]

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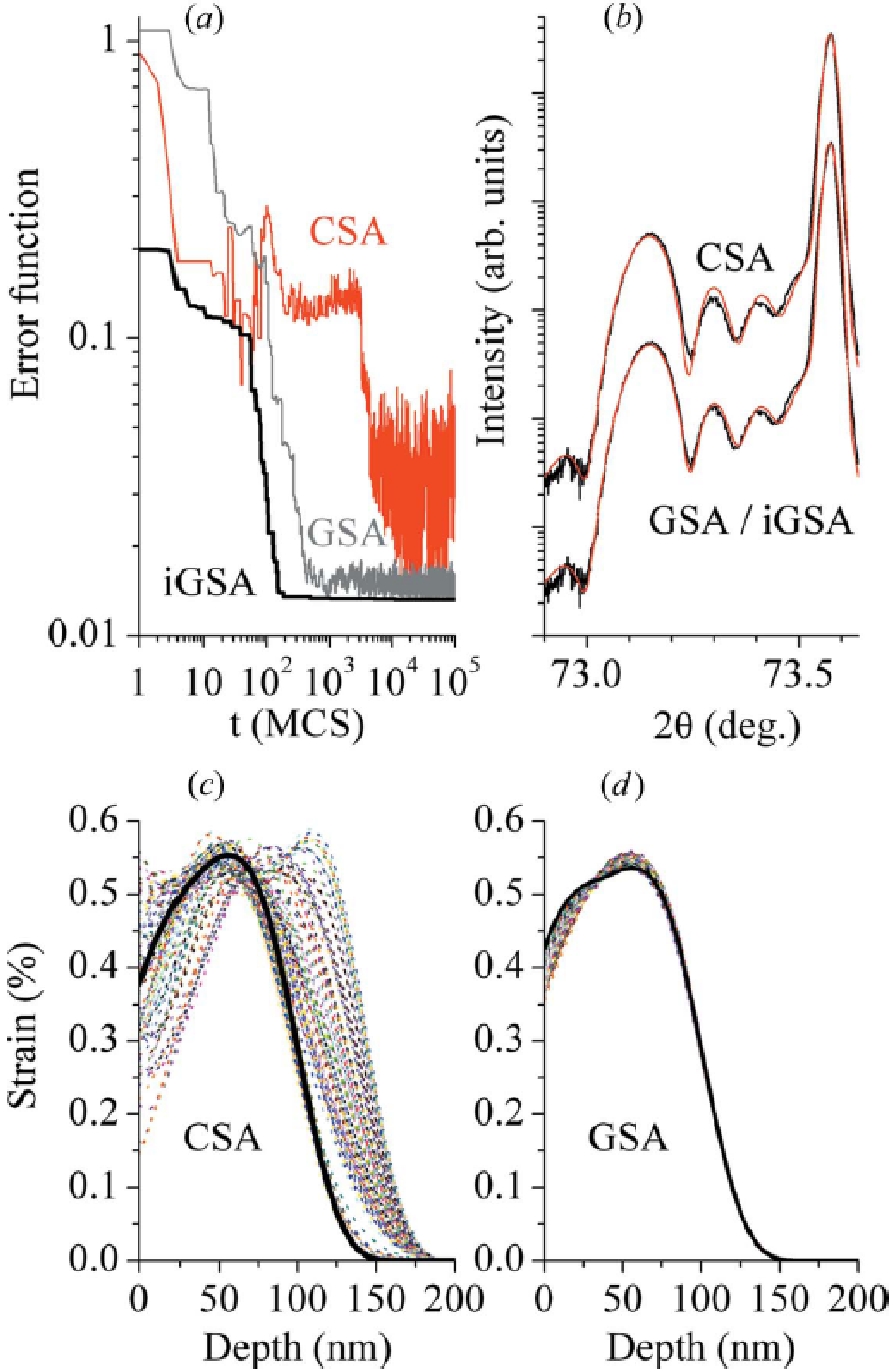
[a]Science des Procédés Céramiques et de Traitements de Surface (SPCTS), CNRS UMR 6638, Centre Européen de la Céramique, 12 rue Atlantis, 87068 Limoges, France, and [b]Centre de Spectrométrie Nucléaire et de Spectrométrie de Masse (CSNSM, UMR 8609), CNRS – IN2P3 – Université Paris-Sud 11, Bâtiment 108, 91405 Orsay Cedex, France. Correspondence e-mail: alexandre.boulle@unilim.fr

A novel least-squares fitting procedure is presented that allows the retrieval of strain profiles in ion-implanted single crystals using high-resolution X-ray diffraction. The model is based on the dynamical theory of diffraction, including a B-spline-based description of the lattice strain. The fitting procedure relies on the generalized simulated annealing algorithm which, contrarily to most common least-squares fitting-based methods, allows the global minimum of the error function (the difference between the experimental and the calculated curves) to be found extremely quickly. It is shown that convergence can be achieved in a few hundred Monte Carlo steps, *i.e.* a few seconds. The method is model-independent and allows determination of the strain profile even without any 'guess' regarding its shape. This procedure is applied to the determination of strain profiles in Cs-implanted yttria-stabilized zirconia (YSZ). The strain and damage profiles of YSZ single crystals implanted at different ion fluences are analyzed and discussed.



(a)
(b)
Error function
CSA
GSA
iGSA
t (MCS)
Intensity (arb. units)
CSA
GSA / iGSA
2θ (deg.)
73.0
73.5
(c)
(d)
Strain (%)
CSA
GSA
Depth (nm)
Depth (nm)

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journal homepage: www.elsevier.com/locate/camwa

A novel automatic microcalcification detection technique using Tsallis entropy & a type II fuzzy index

Mohanalin *, Beenamol, Prem Kumar Kalra, Nirmal Kumar

Department of Electrical Engineering, IIT Kanpur, UP-208016, India

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Keywords:
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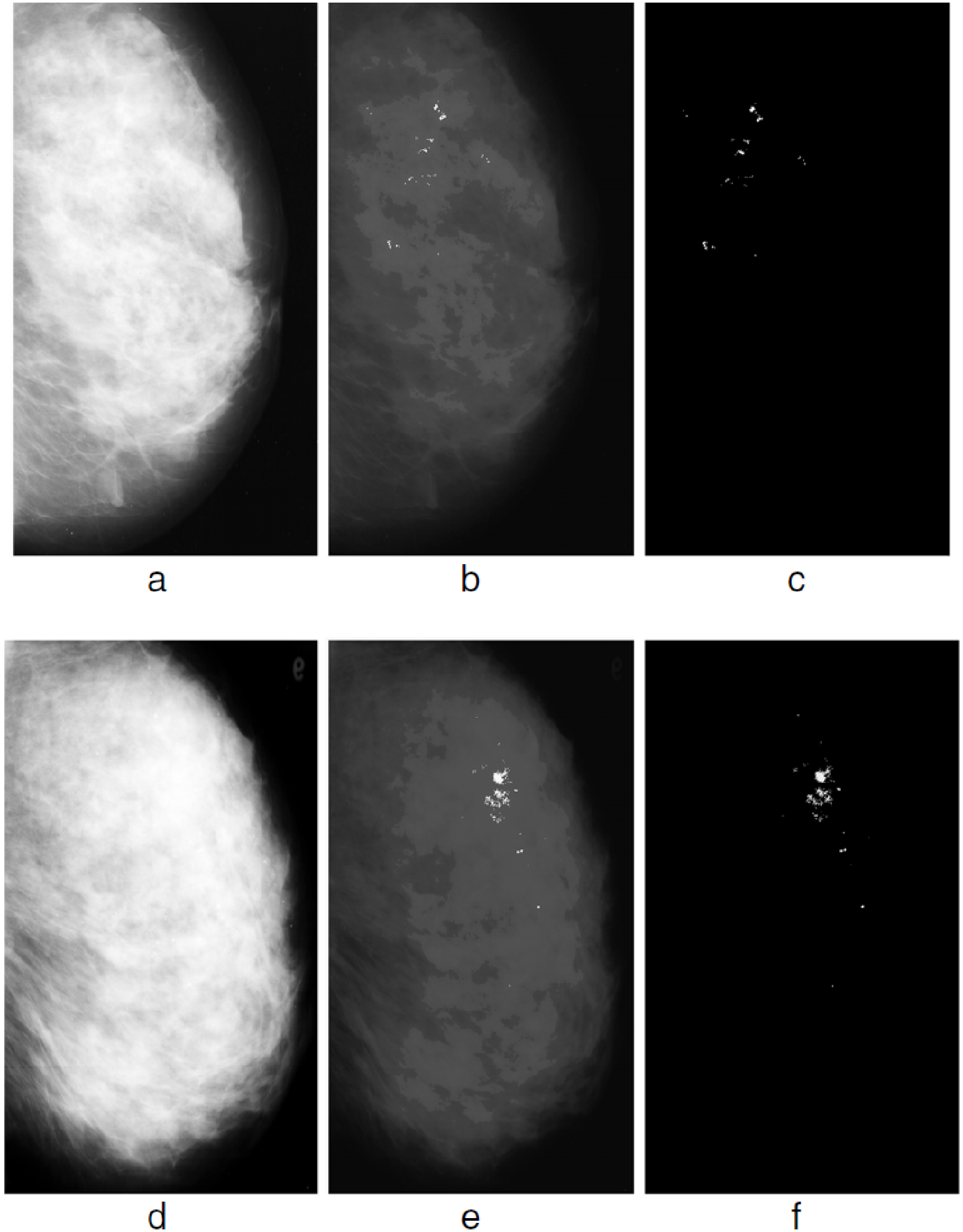
Type II fuzzy index
Shannon entropy
Mammograms
Microcalcification

ABSTRACT

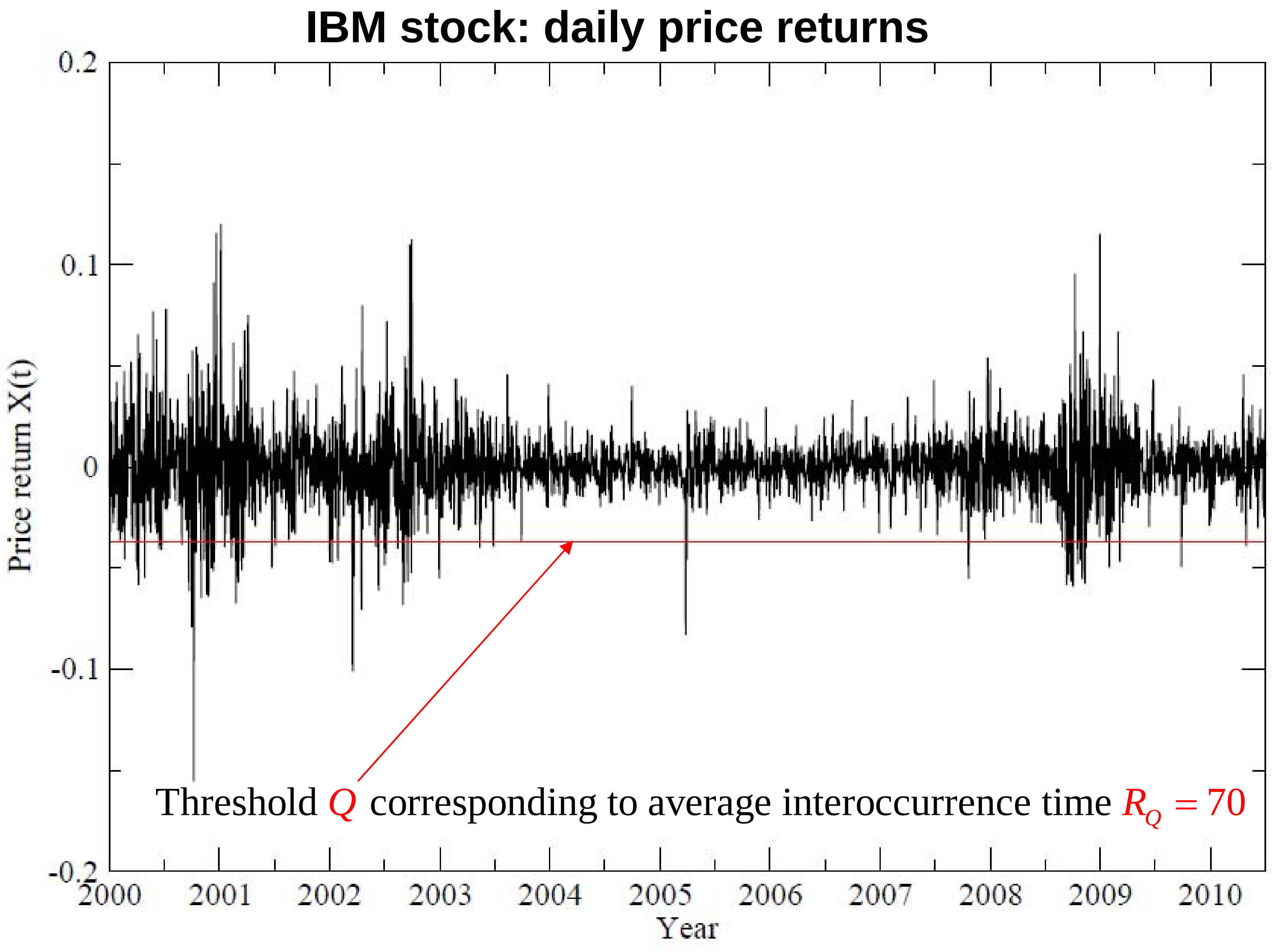
This article investigates a novel automatic microcalcification detection method using a type II fuzzy index. The thresholding is performed using the Tsallis entropy characterized by another parameter '*q*', which depends on the non-extensiveness of a mammogram. In previous studies, '*q*' was calculated using the histogram distribution, which can lead to erroneous results when pectoral muscles are included. In this study, we have used a type II fuzzy index to find the optimal value of '*q*'. The proposed approach has been tested on several mammograms. The results suggest that the proposed Tsallis entropy approach outperforms the two-dimensional non-fuzzy approach and the conventional Shannon entropy partition approach. Moreover, our thresholding technique is completely automatic, unlike the methods of previous related works. Without Tsallis entropy enhancement, detection of microcalcifications is meager: 80.21% Tps (true positives) with 8.1 Fps (false positives), whereas upon introduction of the Tsallis entropy, the results surge to 96.55% Tps with 0.4 Fps.


a
b
c
d
e
f

- ALGUMAS VERIFICAÇÕES RECENTES

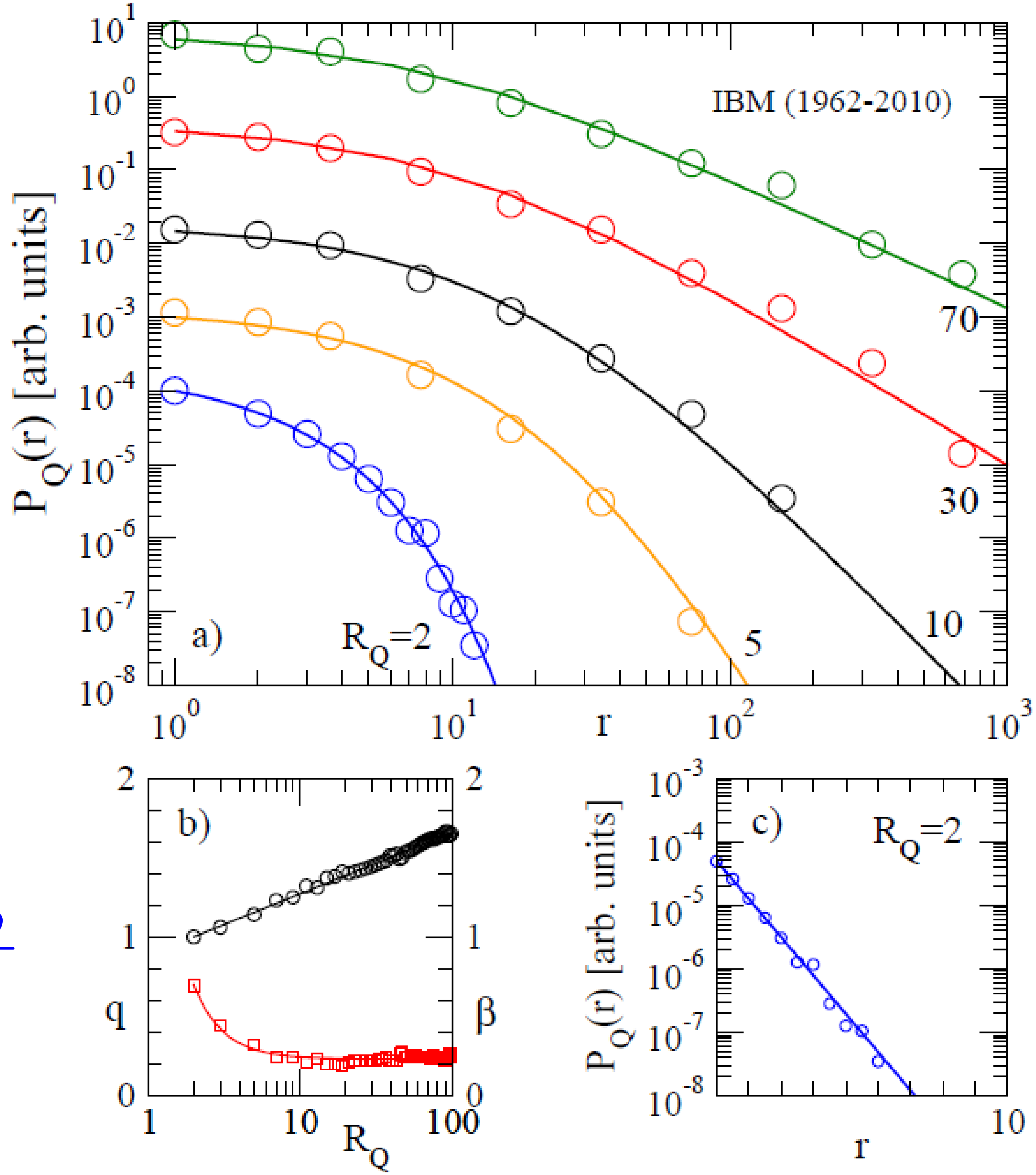
- ALGUNS RESULTADOS NOSSOS RECENTES

EXTREME EVENTS IN FINANCIAL RECORDS:



J. Ludescher, C. T. and A. Bunde (2011)

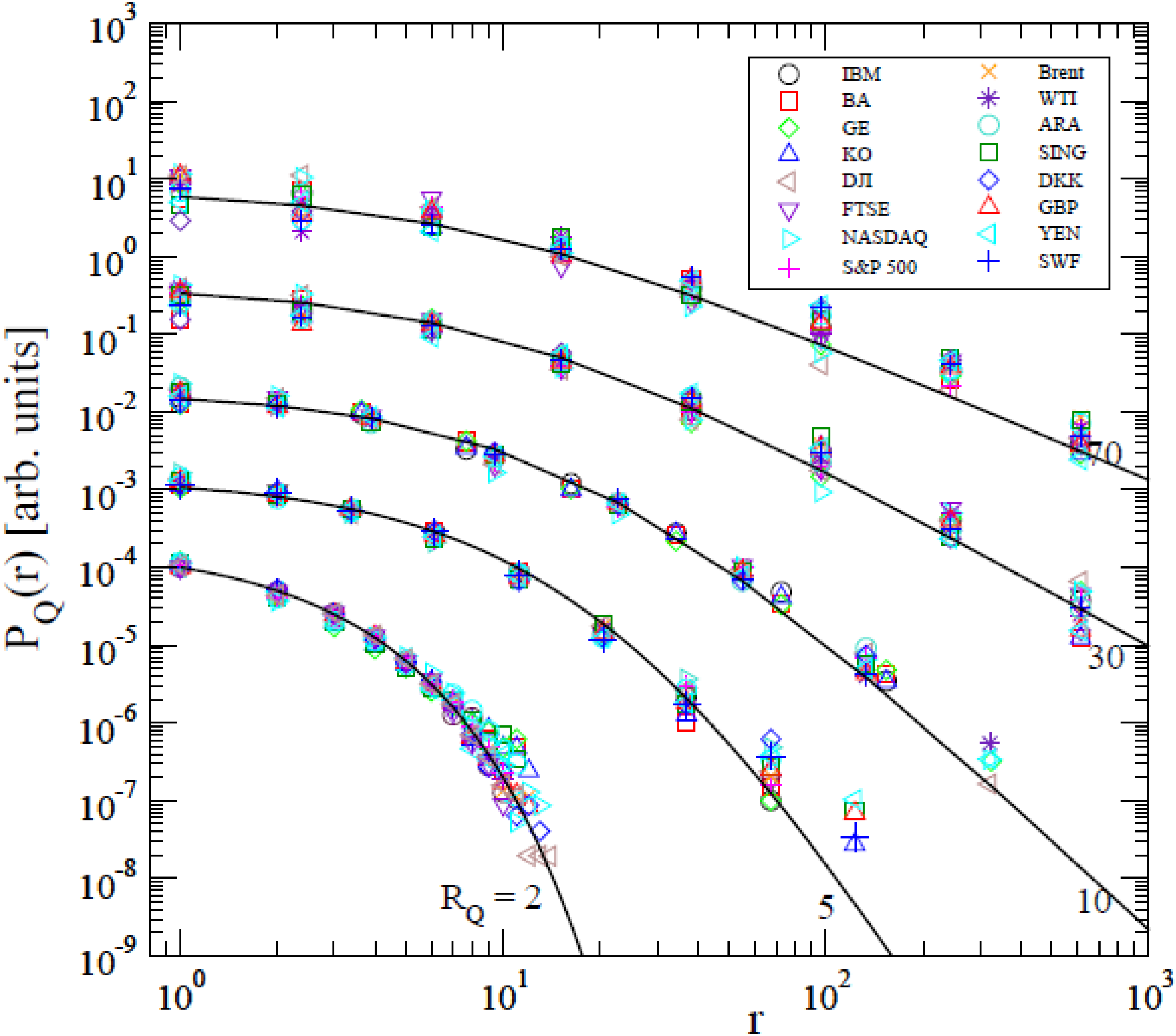
Stocks, stock indices, commodities, currency rates



$$q = 1 + q_0 \ln \frac{R_Q}{2}$$

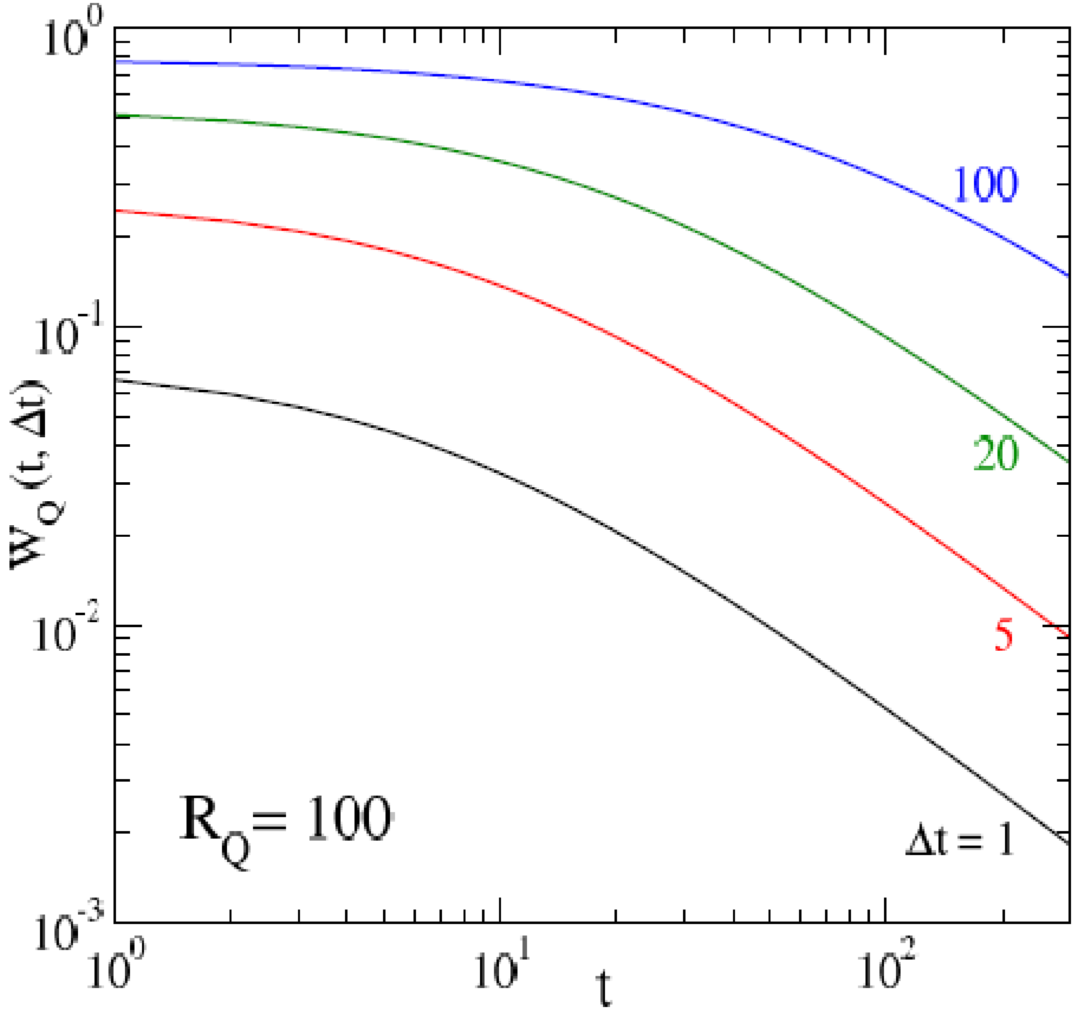
$$q_0 \simeq 0.168$$

J. Ludescher, C. T. and A. Bunde (2011)



J. Ludescher, C. T. and A. Bunde (2011)

Risk function



$$W(t;\Delta t) = 1 - \left[1 + \frac{\beta(q-1)\Delta t}{1+\beta(q-1)t}\right]^{\frac{q-2}{q-1}}$$

J. Ludescher, C. T. and A. Bunde (2011)

Milan j. math. 76 (2008), 307–328
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Milan Journal of Mathematics

On a q-Central Limit Theorem Consistent with Nonextensive Statistical Mechanics

Sabir Umarov, Constantino Tsallis and Stanly Steinberg

JOURNAL OF MATHEMATICAL PHYSICS **51**, 033502 (2010)

Generalization of symmetric α-stable Lévy distributions for $q>1$

Sabir Umarov,[1,a)] Constantino Tsallis,[2,3,b)] Murray Gell-Mann,[3,c)] and Stanly Steinberg[4,d)]

[1]*Department of Mathematics, Tufts University, Medford, Massachusetts 02155, USA*
[2]*Centro Brasileiro de Pesquisas Fisicas and National Institute of Science and Technology for Complex Systems, Rua Dr. Xavier Sigaud 150, 22290-180 Rio de Janeiro, Brazil*
[3]*Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, New Mexico 87501, USA*
[4]*Department of Mathematics and Statistics, University of New Mexico, Albuquerque, New Mexico 87131, USA*

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CENTRAL LIMIT THEOREM

$N^{1/[\alpha(2-q)]}$ *- scaled attractor* $\mathbb{F}(x)$ *when summing* $N \to \infty$ q*-independent identical random variables*

with symmetric distribution $f(x)$ *with* $\sigma_Q \equiv \int dx\, x^2 [f(x)]^Q / \int dx\, [f(x)]^Q$ $\left(Q \equiv 2q-1,\ q_1 = \frac{1+q}{3-q} \right)$

	$q=1$ [*independent*]	$q \neq 1$ (*i.e.*, $Q \equiv 2q-1 \neq 1$) [*globally correlated*]
$\sigma_Q < \infty$ $(\alpha = 2)$	$\mathbb{F}(x) =$ *Gaussian* $G(x)$, *with same* σ_1 *of* $f(x)$ Classic CLT	$\mathbb{F}(x) = G_q(x) \equiv G_{(3q_1-1)/(1+q_1)}(x)$, *with same* σ_Q *of* $f(x)$ $G_q(x) \sim \begin{cases} G(x) & \text{if } \lvert x \rvert << x_c(q,2) \\ f(x) \sim C_q / \lvert x \rvert^{2/(q-1)} & \text{if } \lvert x \rvert >> x_c(q,2) \end{cases}$ *with* $\lim_{q \to 1} x_c(q,2) = \infty$ S. Umarov, C. T. and S. Steinberg, Milan J Math 76, 307 (2008)
$\sigma_Q \to \infty$ $(0 < \alpha < 2)$	$\mathbb{F}(x) =$ *Levy distribution* $L_\alpha(x)$, *with same* $\lvert x \rvert \to \infty$ *behavior* $L_\alpha(x) \sim \begin{cases} G(x) \\ \quad \text{if } \lvert x \rvert << x_c(1,\alpha) \\ f(x) \sim C_\alpha / \lvert x \rvert^{1+\alpha} \\ \quad \text{if } \lvert x \rvert >> x_c(1,\alpha) \end{cases}$ *with* $\lim_{\alpha \to 2} x_c(1,\alpha) = \infty$ Levy-Gnedenko CLT	$\mathbb{F}(x) = L_{q,\alpha}$, *with same* $\lvert x \rvert \to \infty$ *asymptotic behavior* $L_{q,\alpha} \sim \begin{cases} G_{\frac{2(1-q)-\alpha(1+q)}{2(1-q)-\alpha(3-q)},\, \alpha}(x) \sim C^*_{q,\alpha} / \lvert x \rvert^{\frac{2(1-q)-\alpha(3-q)}{2(1-q)}} \\ \quad (\textit{intermediate regime}) \\ G_{\frac{2\alpha q - \alpha + 3}{\alpha+1},\, 2}(x) \sim C^L_{q,\alpha} / \lvert x \rvert^{(1+\alpha)/(1+\alpha q - \alpha)} \\ \quad (\textit{distant regime}) \end{cases}$ S. Umarov, C. T., M. Gell-Mann and S. Steinberg J Math Phys 51, 033502 (2010)

q - GENERALIZED CENTRAL LIMIT THEOREM:

S. Umarov, C.T. and S. Steinberg, Milan J Math **76,** 307 (2008)

q-Fourier transform:

$$F_q[f](\xi) \equiv \int_{-\infty}^{\infty} e_q^{ix\xi} \otimes_q f(x)\, dx = \int_{-\infty}^{\infty} e_q^{ix\xi[f(x)]^{q-1}} f(x)\, dx$$

$(q \geq 1)$

(nonlinear!)

For q<1 see K.P. Nelson and S. Umarov, Physica A **389**, 2157 (2010)

ON THE INVERSE *q*-FOURIER TRANSFORM:

$$f(y)=\left[\frac{2-q}{2\pi}\int_{-\infty}^{+\infty} F_q[f(x+y)](\xi,y)\,d\xi\right]^{1/(2-q)} \qquad (1\le q<2)$$

Particular case *q* = 1:

$$f(y)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} F[f(x+y)](\xi,y)\,d\xi=\frac{1}{2\pi}\int_{-\infty}^{+\infty} F[f(x)](\xi)\,e^{-i\xi y}\,d\xi$$

M. Jauregui and C. T., Phys Lett A (2011), in press

Hilhorst function:

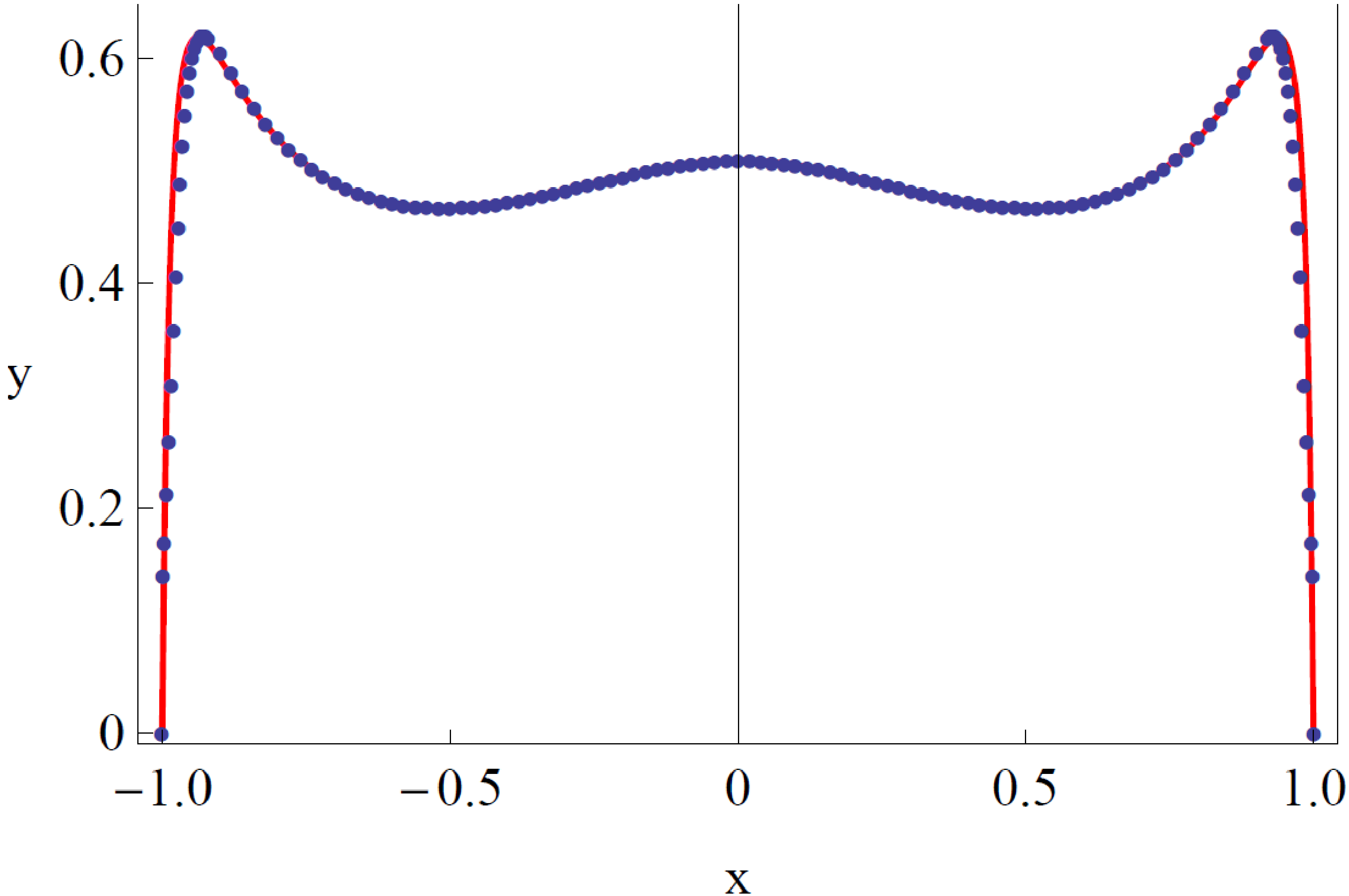
[H.J. Hilhorst, JSTAT P 10023 (2010)]

$$f_A(x) = \begin{cases} \dfrac{\left[\ |x|^{(q-2)/(q-1)} - A\right]^{1/(q-2)}}{C_q\ |x|^{1/(q-1)}\ \left\{1+(q-1)\left[\ |x|^{(q-2)/(q-1)} - A\right]^{2(q-1)/(q-2)}\right\}^{1/(q-1)}} & \text{if } 0 \le A < |x|^{(q-2)/(q-1)} \\ 0 & \text{if } 0 \le |x|^{(q-2)/(q-1)} \le A \\ \text{with} \quad \int_{-\infty}^{\infty} dx\, f_A(x) = 1 & \end{cases}$$

Particular case: $A = 0$

$$f_0(x) = \frac{1}{C_q\ [1+(q-1)x^2]^{1/(q-1)}}$$

Hilhorst function (*q=5/4; A=1*)



M. Jauregui and C. T., Phys Lett A (2011), in press

***q*-PLANE WAVES:**

1) New representation of Dirac delta:

$$\delta(x) = \frac{2-q}{2\pi} \int_{-\infty}^{\infty} dk \; e_q^{-ikx} \quad (1 \le q < 2)$$

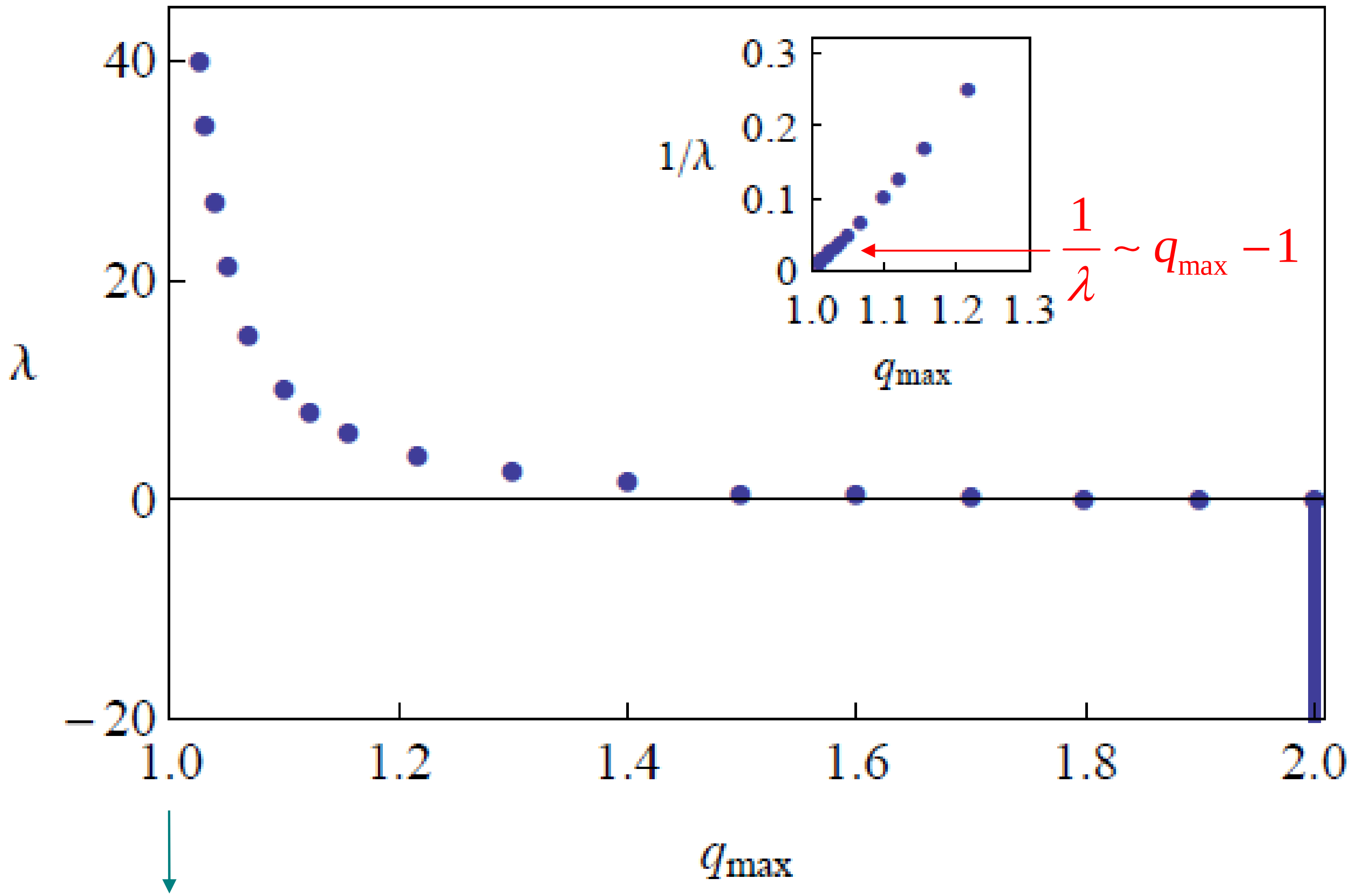
i.e.,

$$\int_{-\infty}^{\infty} dx \; \delta(x - x_0) f(x) = f(x_0)$$

M. Jauregui and C. T., J Math Phys **51**, 063304 (2010)

standard
Dirac delta

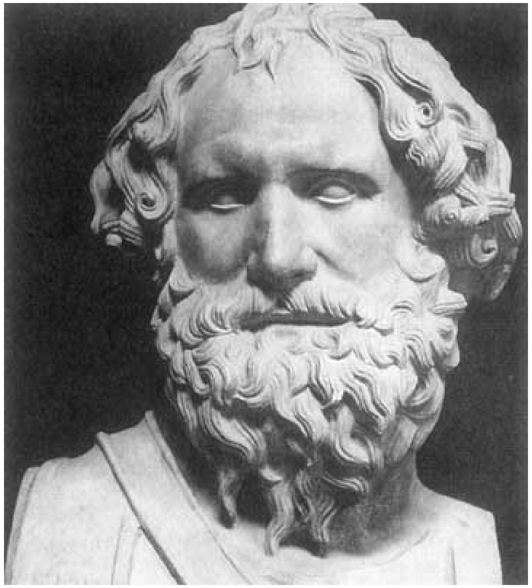
$$f(x) \sim A|x|^{\lambda} \quad (|x| \to \infty;\ \lambda \in R)$$



A. Chevreuil, A. Plastino and C. Vignat, J Math Phys **51**, 093502 (2010)
M. Mamode, J Math Phys **51**, 123509 (2010)
A. Plastino and M.C. Rocca, 1012.1223 [math-ph]

M. Jauregui and C. T., Phys Lett A (2011), in press

2) New representation of π :



Archimedes

(c. 287 BC – c. 212 BC)

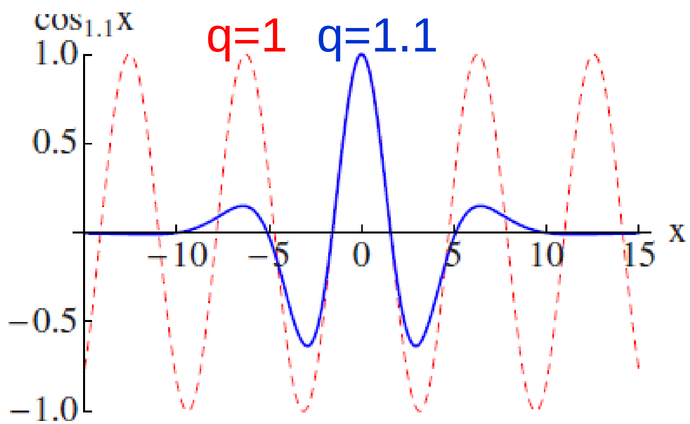
$$\pi = n \sum_{k=0}^{\left\lfloor \frac{n+1}{2} \right\rfloor - 1} (-1)^k \frac{\Gamma\left(n-k-\frac{1}{2}\right)\Gamma\left(k+\frac{1}{2}\right)}{\Gamma(2k+2)\Gamma(n-2k)}, \quad \forall n \in \mathbb{N}$$

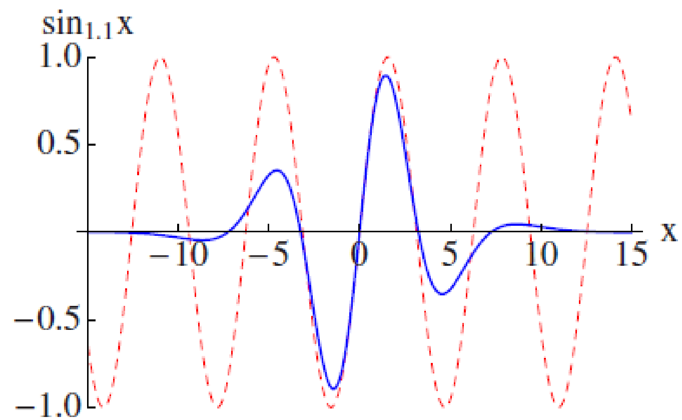
M. Jauregui and C. T., J Math Phys **51**, 063304 (2010)

3) q-plane waves are square integrable ($0 < q < 3$):

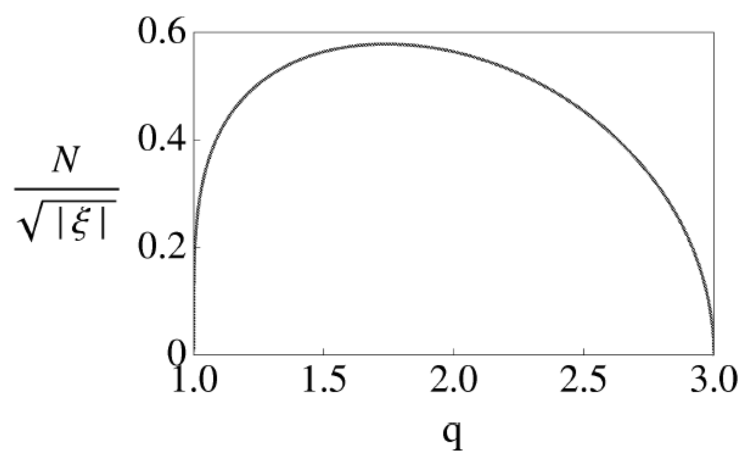
$$\psi(x,t) = e_q^{i(kx-\omega t)} \text{ satisfies } \frac{\partial^2 \psi(x,t)}{\partial t^2} = c^2 \frac{\partial^2 \psi(x,t)}{\partial x^2} \text{ with } \omega = ck$$

$$\Psi(x) \equiv N\ e_q^{i\xi x} = N(\cos_q \xi x + i \sin_q \xi x) \text{ with } \int_{-\infty}^{\infty} dx\, |\Psi(x)|^2 = 1$$





$$\frac{N}{\sqrt{|\xi|}} = \left[\frac{(q-1)\, \Gamma\left(\frac{1}{q-1}\right)}{\sqrt{\pi}\, \Gamma\left(\frac{3-q}{2(q-1)}\right)} \right]^{1/2}$$



M. Jauregui and C. T., J Math Phys **51**, 063304 (2010)

Nonlinear Relativistic and Quantum Equations with a Common Type of Solution

F. D. Nobre,[1,*] M. A. Rego-Monteiro,[1] and C. Tsallis[1,2]

[1]*Centro Brasileiro de Pesquisas Físicas and National Institute of Science and Technology for Complex Systems, Rua Xavier Sigaud 150, 22290-180 Rio de Janeiro–RJ Brazil*

[2]*Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, New Mexico 87501, USA*

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Generalizations of the three main equations of quantum physics, namely, the Schrödinger, Klein-Gordon, and Dirac equations, are proposed. Nonlinear terms, characterized by exponents depending on an index q, are considered in such a way that the standard linear equations are recovered in the limit $q \to 1$. Interestingly, these equations present a common, solitonlike, traveling solution, which is written in terms of the q-exponential function that naturally emerges within nonextensive statistical mechanics. In all cases, the well-known Einstein energy-momentum relation is preserved for arbitrary values of q.

4) q – generalized Schroedinger equation

(quantum non-relativistic spinless free particle)

$$i\hbar \frac{\partial}{\partial t}\left[\frac{\Phi\left(\vec{x},t\right)}{\Phi_0}\right] = -\frac{1}{2-q}\frac{\hbar^2}{2m}\nabla^2\left[\frac{\Phi\left(\vec{x},t\right)}{\Phi_0}\right]^{2-q} \qquad (q \in R)$$

Its exact solution is given by

$$\Phi\left(\vec{x},t\right) = \Phi_0 \; e_q^{i\left(\vec{p}\,.\,\vec{x}-Et\right)/\hbar} = \Phi_0 \; e_q^{i\left(\vec{k}\,.\,\vec{x}-\omega t\right)}$$

with

$$E = \frac{p^2}{2m} \quad \text{(Newtonian relation!)}$$

$$E = \hbar\omega$$

$$p = \hbar k$$

$\forall q$

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By defining

$$\text{Energy operator} \equiv \hat{E} \equiv i\hbar D_t$$

$$\text{Momentum operator} \equiv \hat{p}_n \equiv -i\hbar D_{x_n} \qquad (n = x, y, z)$$

with $$D_u f(u) \equiv [f(u)]^{1-q} df(u)/du \ ,$$

We verify

$$\hat{E}\, e_q^{i(kx-\omega t)} = E\ e_q^{i(kx-\omega t)} \quad \text{with} \quad E = \hbar\omega\ , \forall q!$$

$$\hat{p}_n\, e_q^{i(kx-\omega t)} = p_n\ e_q^{i(kx-\omega t)} \quad \text{with} \quad \vec{p} = \hbar\,\vec{k}\ , \forall q!$$

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5) q-generalized Klein-Gordon equation:

(quantum relativistic spinless free particle: e.g., mesons π)

$$\nabla^2 \Phi\left(\vec{x},t\right) = \frac{1}{c^2}\frac{\partial^2 \Phi\left(\vec{x},t\right)}{\partial t^2} + q\,\frac{m^2 c^2}{\hbar^2}\Phi\left(\vec{x},t\right)\left[\frac{\Phi\left(\vec{x},t\right)}{\Phi_0}\right]^{2(q-1)} \qquad (q \in R)$$

Its exact solution is given by

$$\Phi\left(\vec{x},t\right) = \Phi_0\; e_q^{i\left(\vec{p}\,.\,\vec{x}-Et\right)/\hbar} = \Phi_0\; e_q^{i\left(\vec{k}\,.\,\vec{x}-\omega\, t\right)}$$

with

$$E^2 = p^2 c^2 + m^2 c^4 \qquad (\forall q) \qquad \text{(Einstein relation!)}$$

Particular case: $m = 0 \;\Rightarrow\; q$-plane waves

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5) *q*-generalized Dirac equation:

(quantum relativistic spin $1/2$ matter and anti-matter free particles:

e.g., electron and positron)

$$i\hbar \frac{\partial\,\Phi\left(\vec{x},t\right)}{\partial t} + i\hbar c\left(\vec{\alpha}.\vec{\nabla}\right)\Phi\left(\vec{x},t\right) = \beta mc^2 A^{(q)}\left(\vec{x},t\right)\,\Phi\left(\vec{x},t\right) \qquad (q \in R)$$

with

$$\vec{\alpha} \equiv \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}; \quad \beta \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad (4 \times 4 \text{ matrices})$$

$$A^{(q)}_{i\,j}\left(\vec{x},t\right) \equiv \delta_{ij}\left[\frac{\Phi_j\left(\vec{x},t\right)}{a_j}\right]^{q-1} \qquad \left(A^{(1)}_{i\,j}\left(\vec{x},t\right) = \delta_{ij}\right) \qquad (4 \times 4 \text{ matrix})$$

where $\{a_j\}$ are complex constants.

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Its exact solution is given by

$$\Phi\left(\vec{x},t\right) \equiv \begin{pmatrix} \Phi_1\left(\vec{x},t\right) \\ \Phi_2\left(\vec{x},t\right) \\ \Phi_3\left(\vec{x},t\right) \\ \Phi_4\left(\vec{x},t\right) \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} e_q^{i\left(\vec{p}\,.\,\vec{x}-Et\right)/\hbar} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} e_q^{i\left(\vec{k}\,.\,\vec{x}-\omega\, t\right)}$$

with $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix}$ being the same $\forall q$

hence

$$E^2 = p^2c^2 + m^2c^4 \quad (q \in R) \quad \text{(Einstein relation!)}$$

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