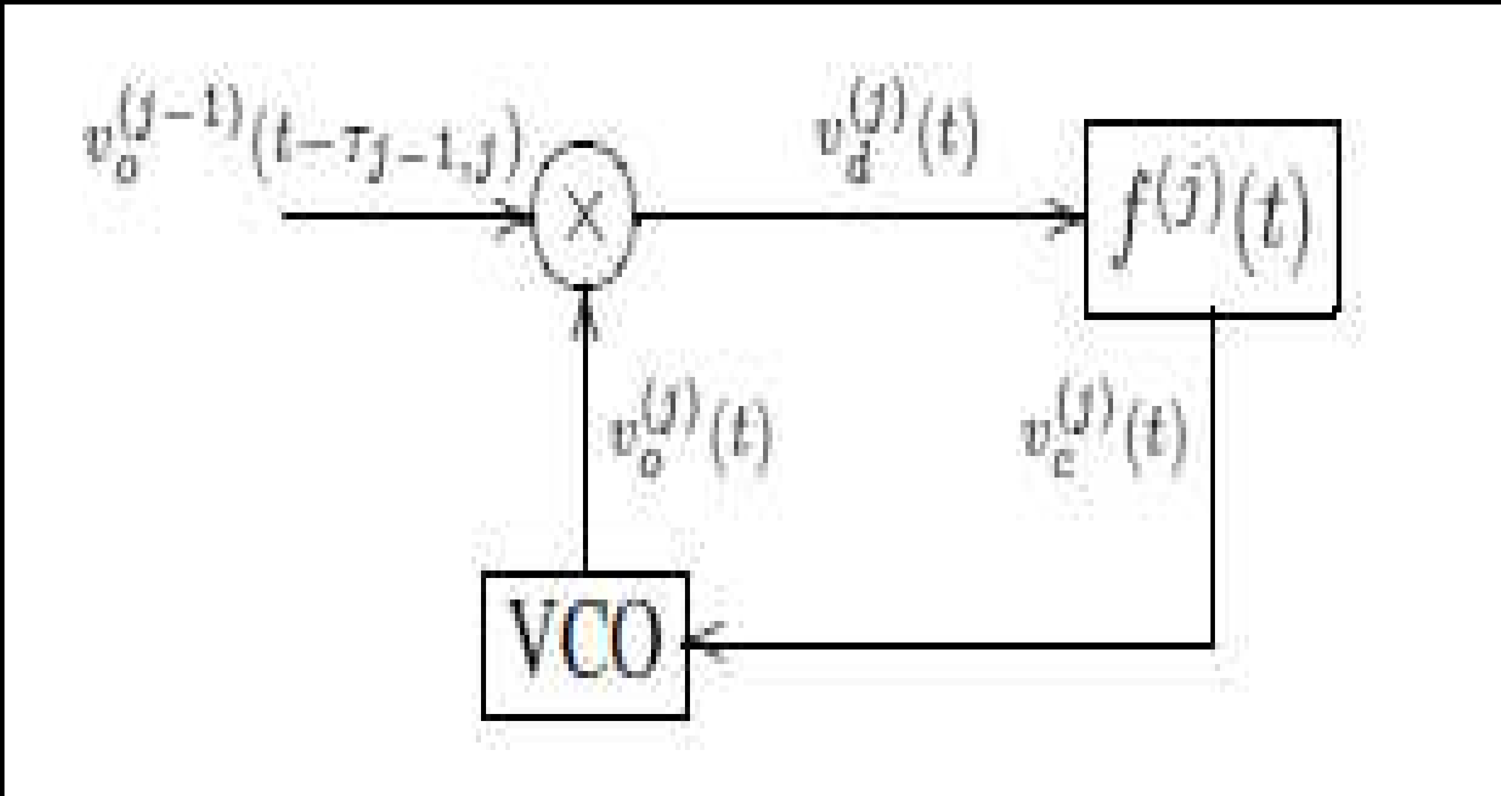
Múltiplos estados síncronos em redes de PLLs

- José Roberto Castilho Piqueira
- Laboratório de Sincronismo
- Departamento de Engenharia de Telecomunicações e Controle
- EPUSP

- Redes de telecomunicações digitais: base de tempo precisa em cada nó.
- Demodulação
- Retirada e colocação de canais
- Leitura e separação de dados (Multiplexação e demultiplexação)

Problema



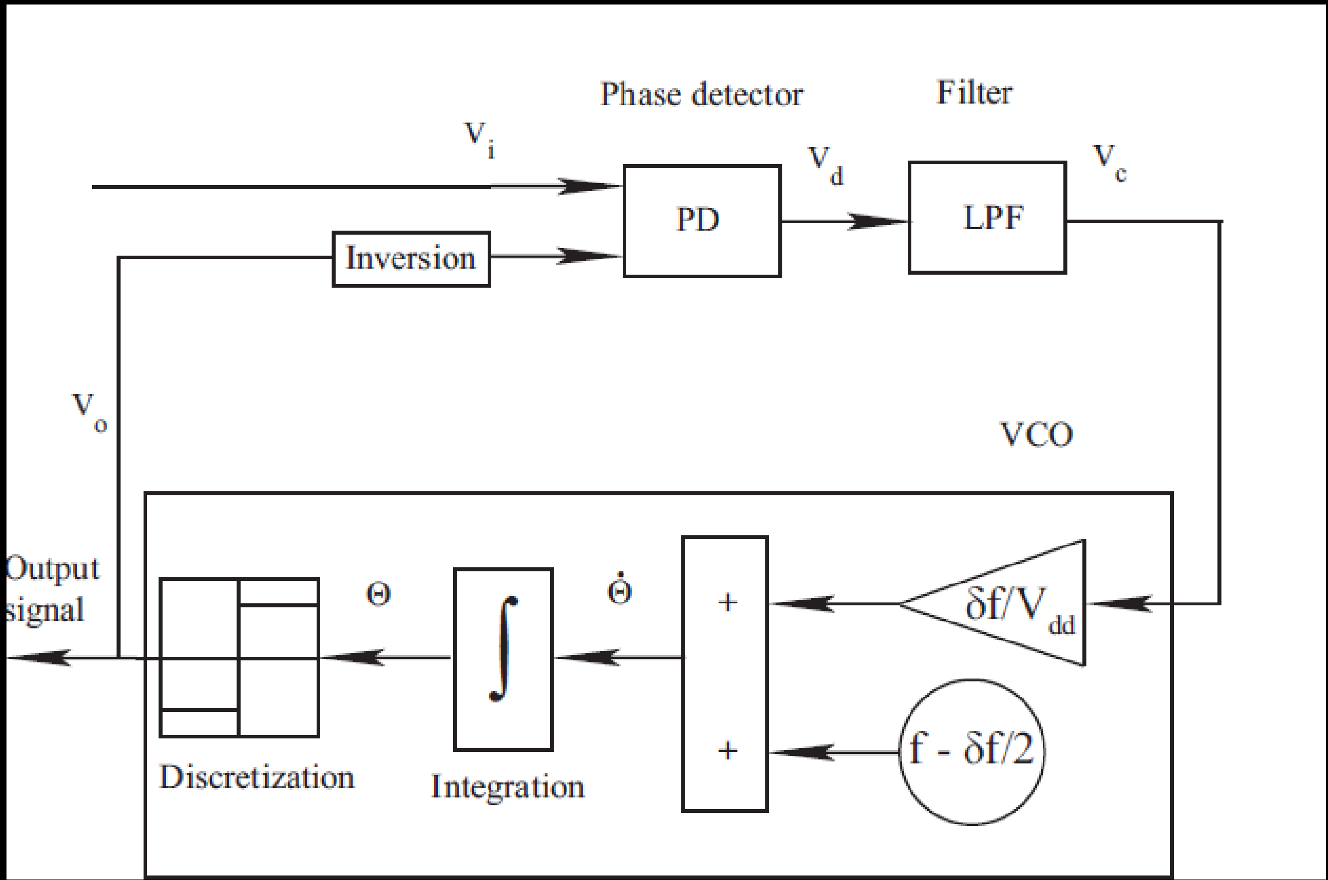
O PLL: sinais de fase e frequência

- Mestre-escravo: via única e via dupla
- cadeia, estrela, enlace e árvore. Digitalização das redes até os anos 80, com PLLs analógicos nos nós e um oscilador de precisão atômica como mestre.

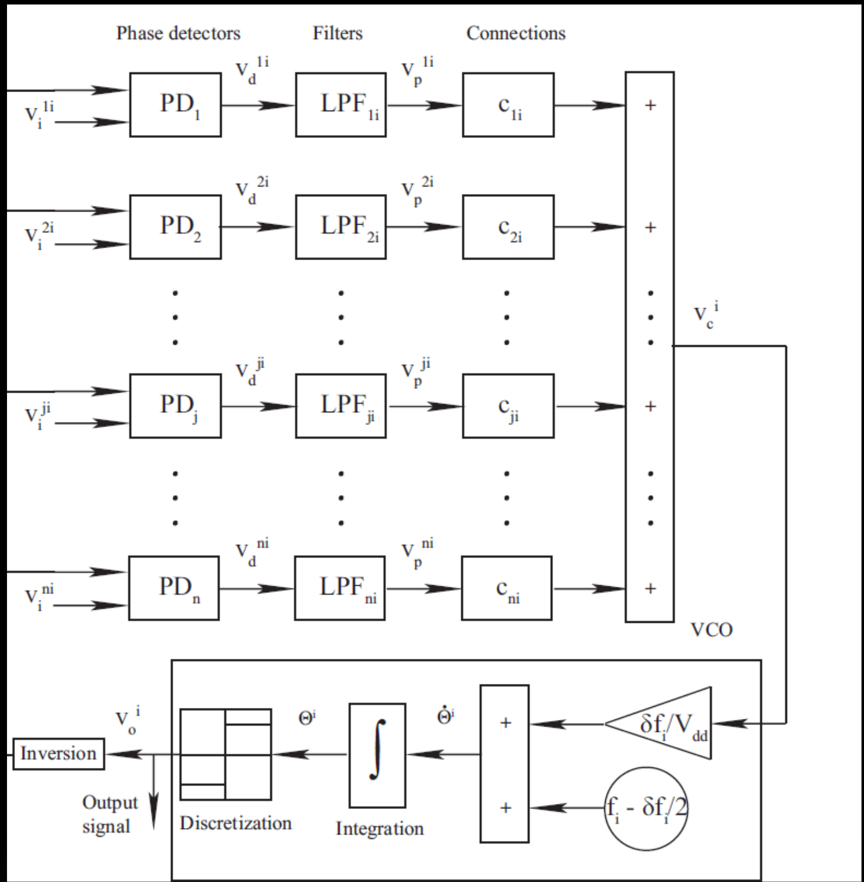
Arquiteturas de redes (I)

- Redes mutuamente conectadas com circuitos definidos ad-hoc e com funções totalmente distribuídas.
- PLLs digitais como alternativa simples, barata e precisa para a deteção dos sinais de tempo nos nós.

Arquiteturas de redes (II)


Phase detector
Filter
V i
V d
V c
PD
LPF
Inversion
V o
VCO
Output signal
Θ
Θ̇
+
+
δf/V dd
f - δf/2
Discretization
Integration

PLL digital



Redes mutuamente conectadas

Hoppensteadt FC, Izhikevich EM. Pattern Recognition via Synchronization in Phase-Locked Loop Neural Networks. IEEE Trans. Neural Networks, 2000; 2(3), 734-738.

Hoppensteadt FC, Izhikevich EM. Synchronization of MEMS Resonators and Mechanical Neurocomputing. IEEE Trans. Circuits and Syst. I: Fundamental Theory and Applications, 2001; 4(2), 133-138.

Piqueira JRC, Orsatti FM, Monteiro LHA. Computing with Phase Locked Loops: Choosing Gains and Delays. IEEE Trans. Neural Networks, 2003; 14(1), 243-247.

O que isso tem a ver com Biologia

$$\dot{\Theta} = f + \delta f \left(\frac{v_c}{V_{dd}} - \frac{1}{2} \right)$$

$$F(s) = \frac{f_c}{s + f_c},$$

VCO e filtro no PLL digital

$$v_c^i = \sum_{j=1}^{n} c_{ji} v_p^{ji}.$$

$$\dot{\Theta}^i = f_i + \delta f_i \left(\frac{\sum_{j=1}^{n} c_{ji} v_p^{ji}}{V_{dd}} - \frac{1}{2} \right)$$

Modelo matemático da rede

$$\dot{\Theta}^{i} = W_i + \delta W_i \left(\frac{\sum_{j=1}^{n} c_{ji} v_p^{ji}}{V_{dd}} - \frac{1}{2} \right).$$

Modelo normalizado

$$\dot{\Theta}^1 = \dot{\Theta}^2 = \ldots = \dot{\Theta}^n = W_s$$

$$W_s = \dot{\Theta}^i = W_i + \delta W_i \left(\sum_{j=1}^{n} c_{ji} \Xi_{ji} \right), \qquad i = 1 \ldots n.$$

$$-0.5 \leq \Xi_{ji} \leq 0.5$$

Estado síncrono da rede

$$\Xi_{ji} = \Xi_{j1} - \Xi_{i1} + \alpha_{ji}, \qquad i,j = 1 \ldots n.$$

$$Ax^T = B$$

Nó 1 como referência

$$A = \begin{bmatrix} 1/\delta W_1 & -c_{21} & \dots & -c_{j1} & \dots & -c_{n1} \\ 1/\delta W_2 & 1 & \dots & -c_{j2} & \dots & -c_{n2} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 1/\delta W_i & -c_{2i} & \dots & 1 & \dots & -c_{ni} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 1/\delta W_n & -c_{2n} & \dots & -c_{in} & \dots & 1 \end{bmatrix}$$

Equação do estado síncrono (I)

Equação do estado síncrono (II)

$$x = \begin{bmatrix} W_s & \Xi_{21} \dots \Xi_{i1} \dots \Xi_{n1} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{W_1}{\delta W_1} \\ \frac{W_2}{\delta W_2} + \sum_{j=1}^{n} \alpha_{j2} c_{j2} \\ \vdots \\ \frac{W_i}{\delta W_i} + \sum_{j=1}^{n} \alpha_{ji} c_{ji} \\ \vdots \\ \frac{W_n}{\delta W_n} + \sum_{j=1}^{n} \alpha_{jn} c_{jn} \end{bmatrix}$$

$$\alpha_{ij} = \begin{cases} 0 & \text{if } |\Xi_{j1} - \Xi_{i1}| \le 0,5; \\ +1 & \text{if } \Xi_{j1} - \Xi_{i1} < -0,5; \\ -1 & \text{if } \Xi_{j1} - \Xi_{i1} > 0,5. \end{cases}$$

$$N_{\alpha} = (n^2 - 3n + 2)/2$$

Condições adicionais

$$\Xi_{vu} = \frac{(n-1)(W_u - W_v)}{n\delta W},$$

$$\delta W_L > \frac{2(n-1)}{n}(W_{max} - W_{min})$$

Critério de sincronismo

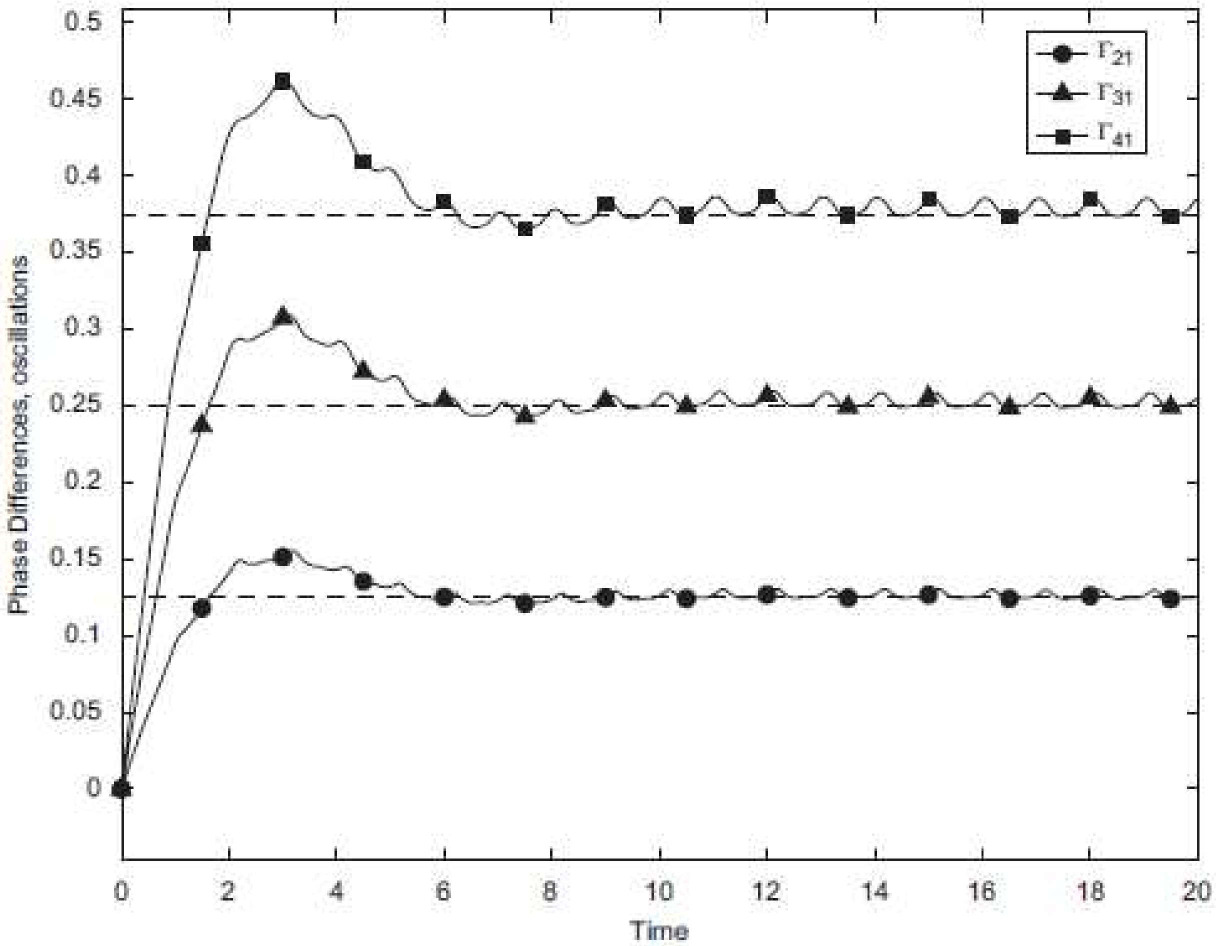


Fig. 3. Phase differences between nodes for a 4-node network with $W_1 = 0.85$, $W_2 = 0.95$, $W_3 = 1.05$ and $W_4 = 1.15$. The values $\Xi_{21} = 0.125$, $\Xi_{31} = 0.25$ and $\Xi_{41} = 0.375$ obtained by analytical methods for the synchronous state are indicated in dotted lines.

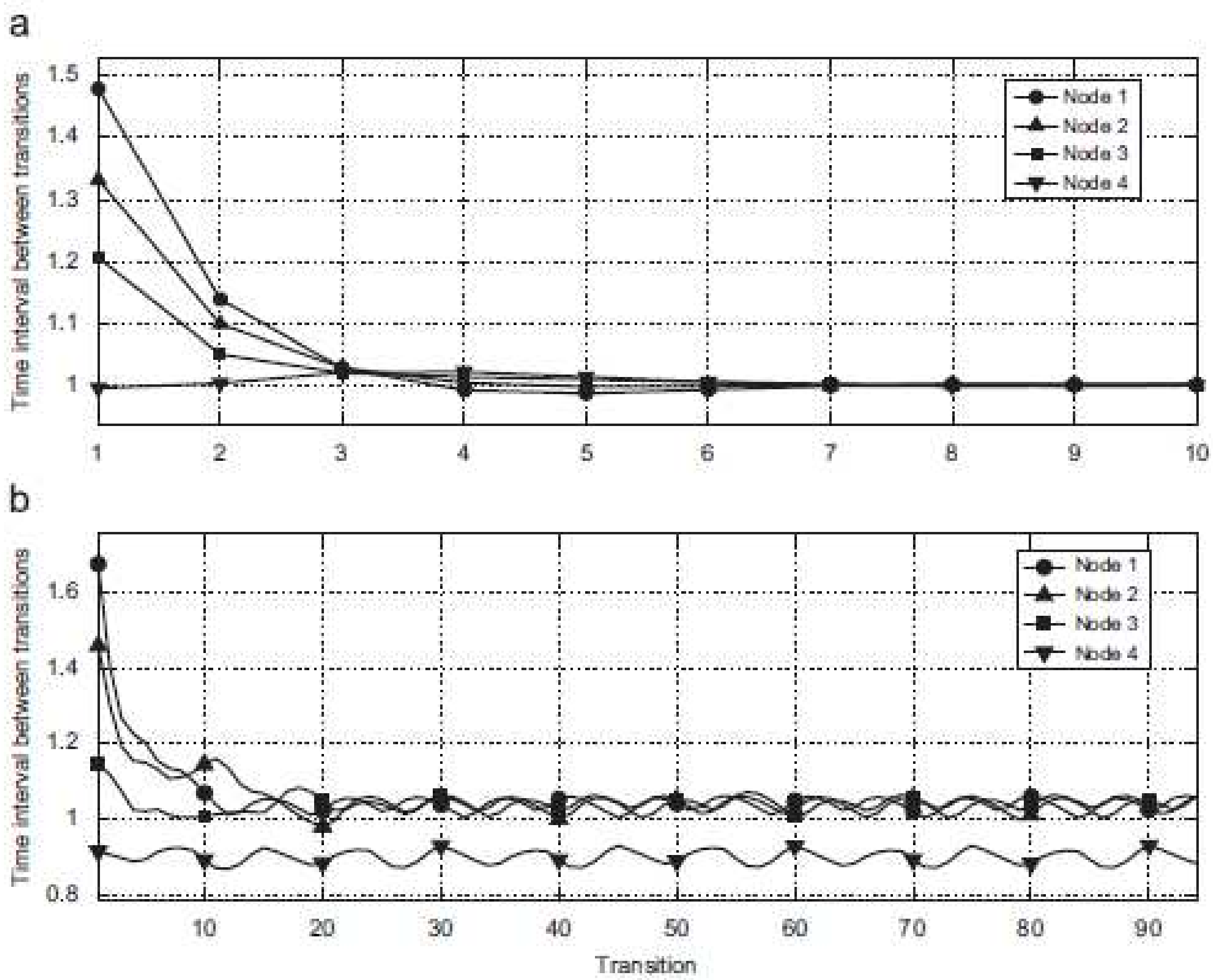


Fig. 4. Time intervals between VCO positive transitions for the 4-node network with $W_1 = 0.85$, $W_2 = 0.95$, $W_3 = 1.05$ and $W_4 = 1.15$. The loop gain of all nodes are set to $\delta W = 0.6$. (a) The cut-off frequencies of all filters are set to $F_c = 1$. (b) The cut-off frequencies of all filters are set to $F_c = 0.25$.

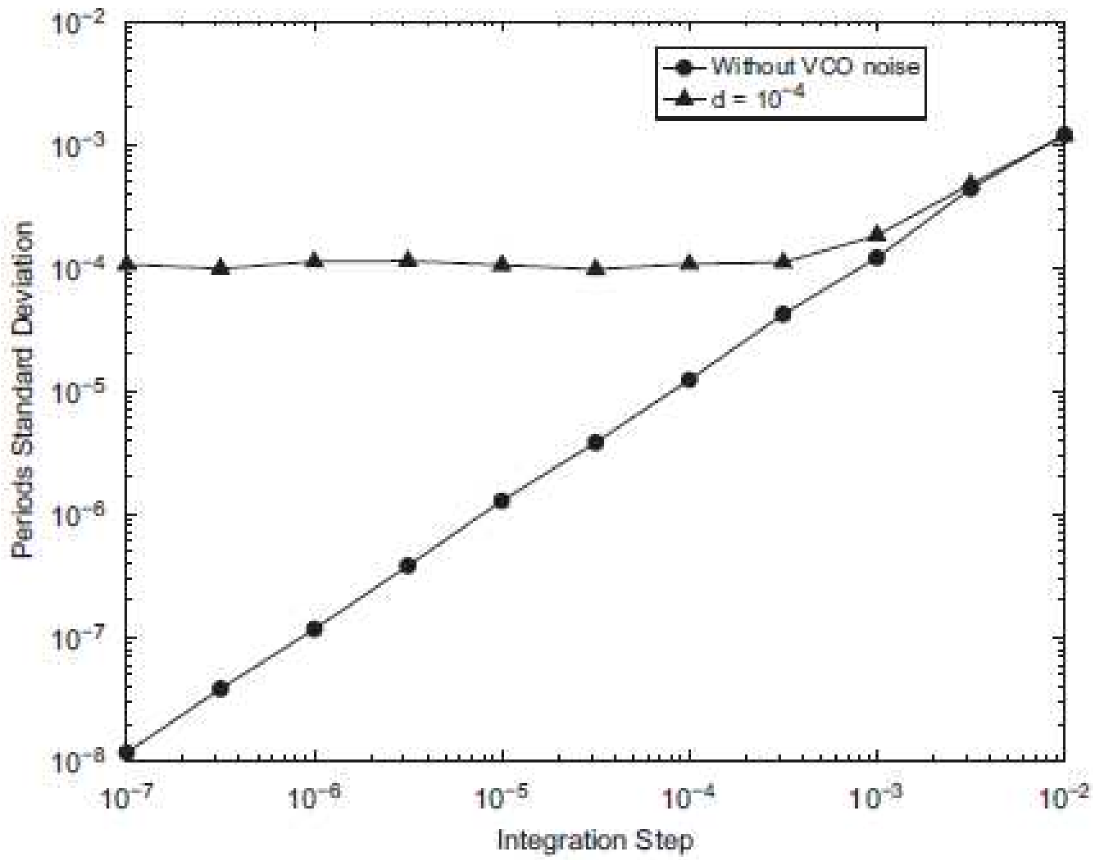


Fig. 5. Standard deviations of the time intervals between transitions for different integration steps. For both cases, the network parameters are $W_1 = 0.85$, $W_2 = 0.95$, $W_3 = 1.05$ and $W_4 = 1.15$, $F_c = 1$ and $\delta W = 0.6$. Simulations were conducted for a total time $\hat{t} = 100$ and the standard deviations were calculated for $50 < \hat{t} < 100$. Results are indicated for the case of noise-free oscillators and for the network having oscillators with precision $d = 10^{-4}$.

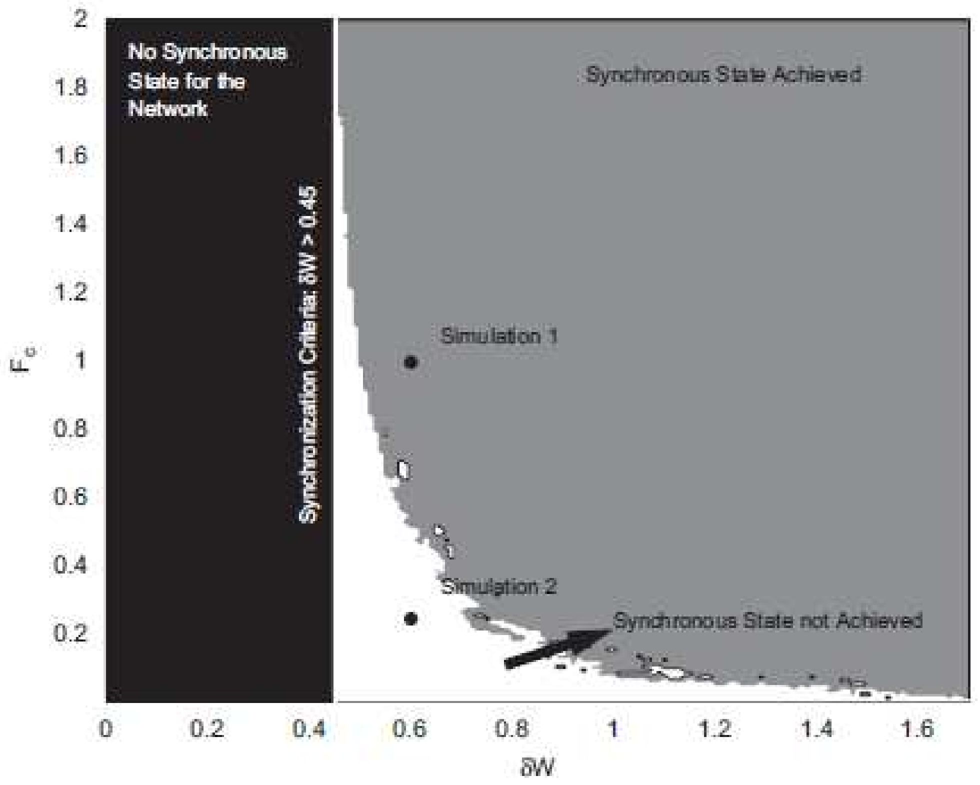


Fig. 6. Verification of whether or not a synchronous state is achieved for a 4-node network with $W_1 = 0.85$, $W_2 = 0.95$, $W_3 = 1.05$ and $W_4 = 1.15$. Each point in the figure corresponds to a simulation conducted for a different pair of parameters δW and F_c. δW changes between 0 and 1.7, while F_c varies between 0.01 and 2, both with step 0.01. The three regions correspond to configurations for which (i) no synchronous state exists; (ii) at least one synchronous state exists, but is not achieved and (iii) at least one synchronous state exists and is achieved.