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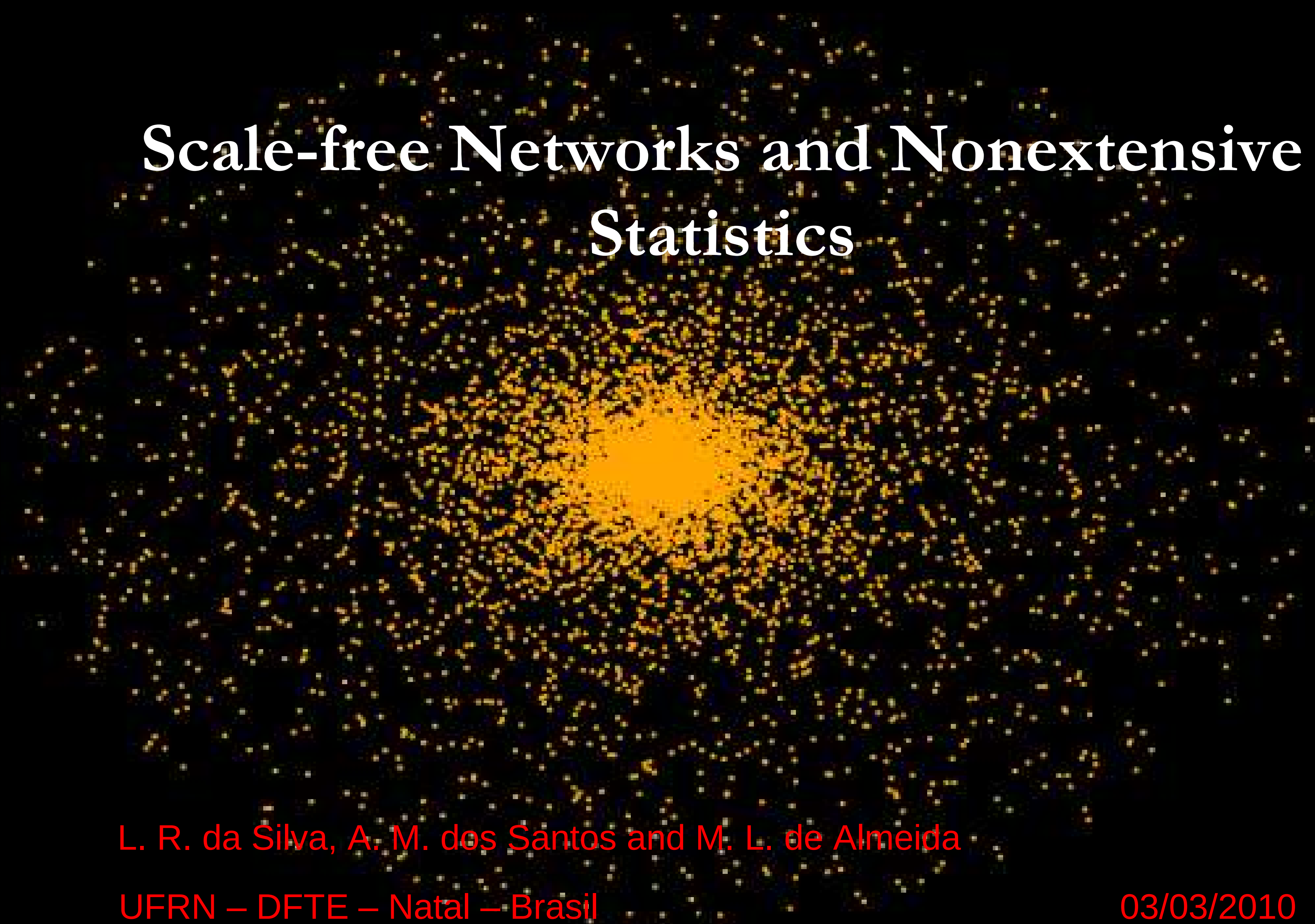

Scale-free Networks and Nonextensive Statistics

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UFRN – DFTE – Natal – Brasil

03/03/2010

OUR GOALS

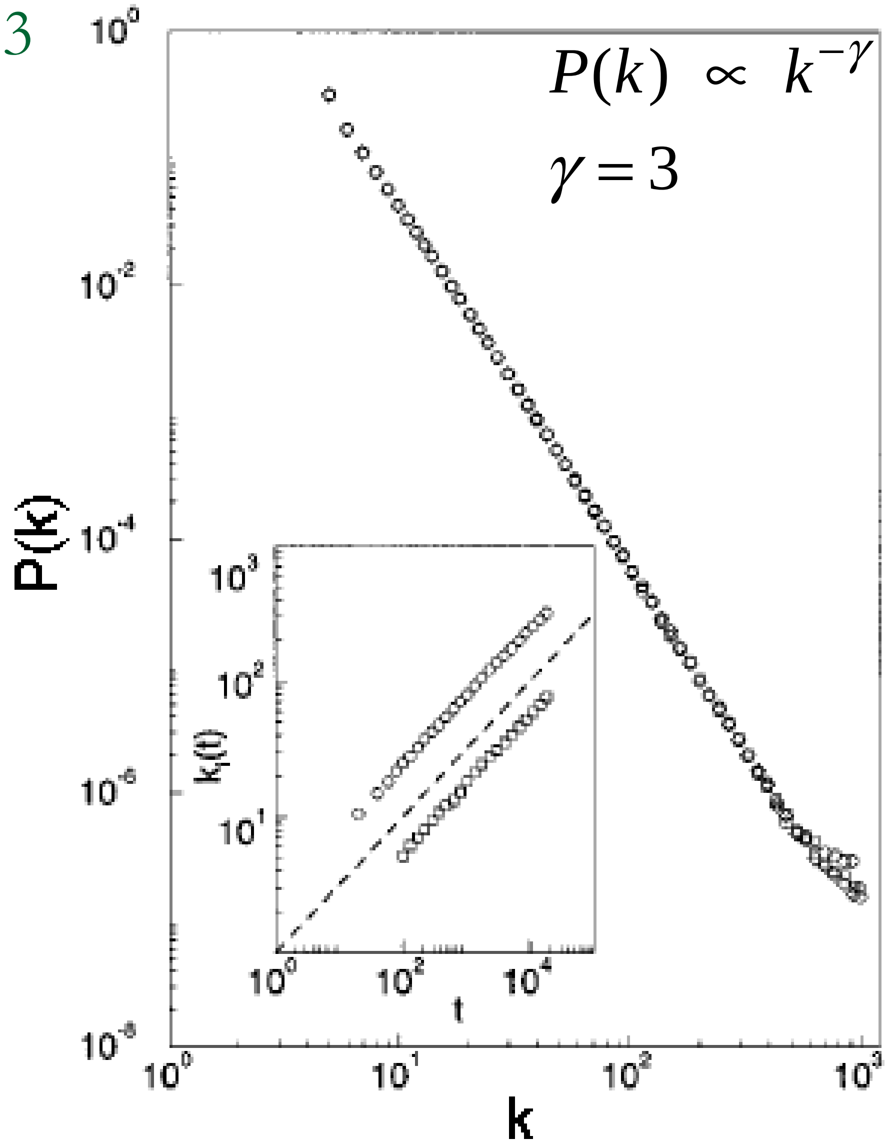
- Growth of an asymptotically scale-free network including metrics.
- Growth of a geographically localized network (around its baricenter).
- To exhibit effects of competition between metrical neighborhood, connectivity, affinity and fitness.
- To analyze the influence of considering a fitness power-law distributed.
- To analyse the influence of the affinity.
- Last but not least, to exhibit the connection between scale-free networks and nonextensive statistics.

Scale-free Networks[1,2,3]

- Barabási and Albert[1];

$$\Pi(k_i) = \frac{k_i}{\sum_{j=1}^{N'} k_j} \qquad (01)$$

$$< k_i > = \left(\frac{t}{i}\right)^{\beta} \qquad (02)$$



[1]Science **286**, 509 (1999) ; Rev. Mod Phys. **74**, 47 (2002)

[2]M. Boguñá and R. Pastor-Satorras, Physical Review E **68**, 036112 (2003)

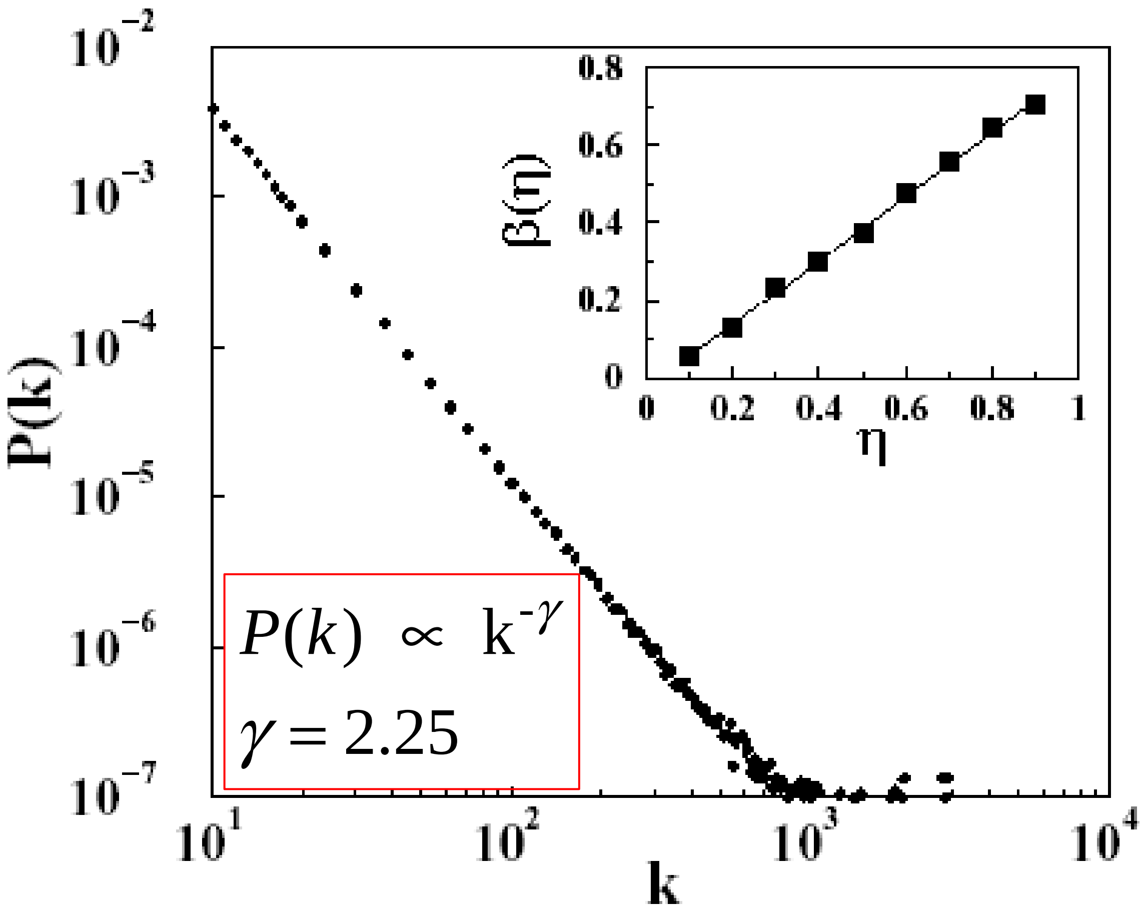
[3]S. Thurner and C. Tsallis, Europhycs Letters **72**, 197 (2005)

Fitness Model

- Bianconi and Barabási[4];
- Albert and Barabási[5];

$$\Pi(k_i) = \frac{k_i \eta_i}{\sum_{j=1}^{N'} k_j \eta_j} \quad (04)$$

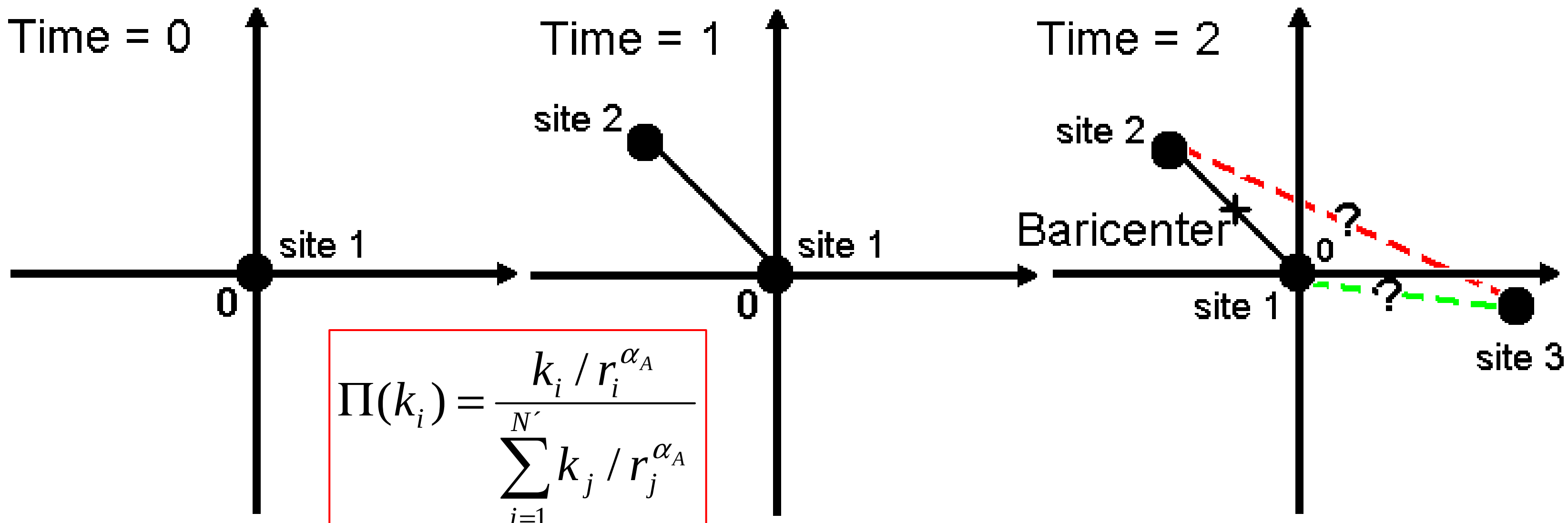
$$< k_i > = \left(\frac{t}{i} \right)^{\beta(\eta_i)} \quad (05)$$



[4]Europhys. Lett. **54**,436 (2001) ; [5]Rev. Mod Phys. **74**, 47 (2002)

Barabási-Albert Model with Euclidean Distance Power-law Distributed

Network Construction:



$$\Pi(k_i) = \frac{k_i / r_i^{\alpha_A}}{\sum_{j=1}^{N'} k_j / r_j^{\alpha_A}}$$

$$P(r) \propto r^{-\gamma_G}$$

$$\gamma_G > 1$$

where

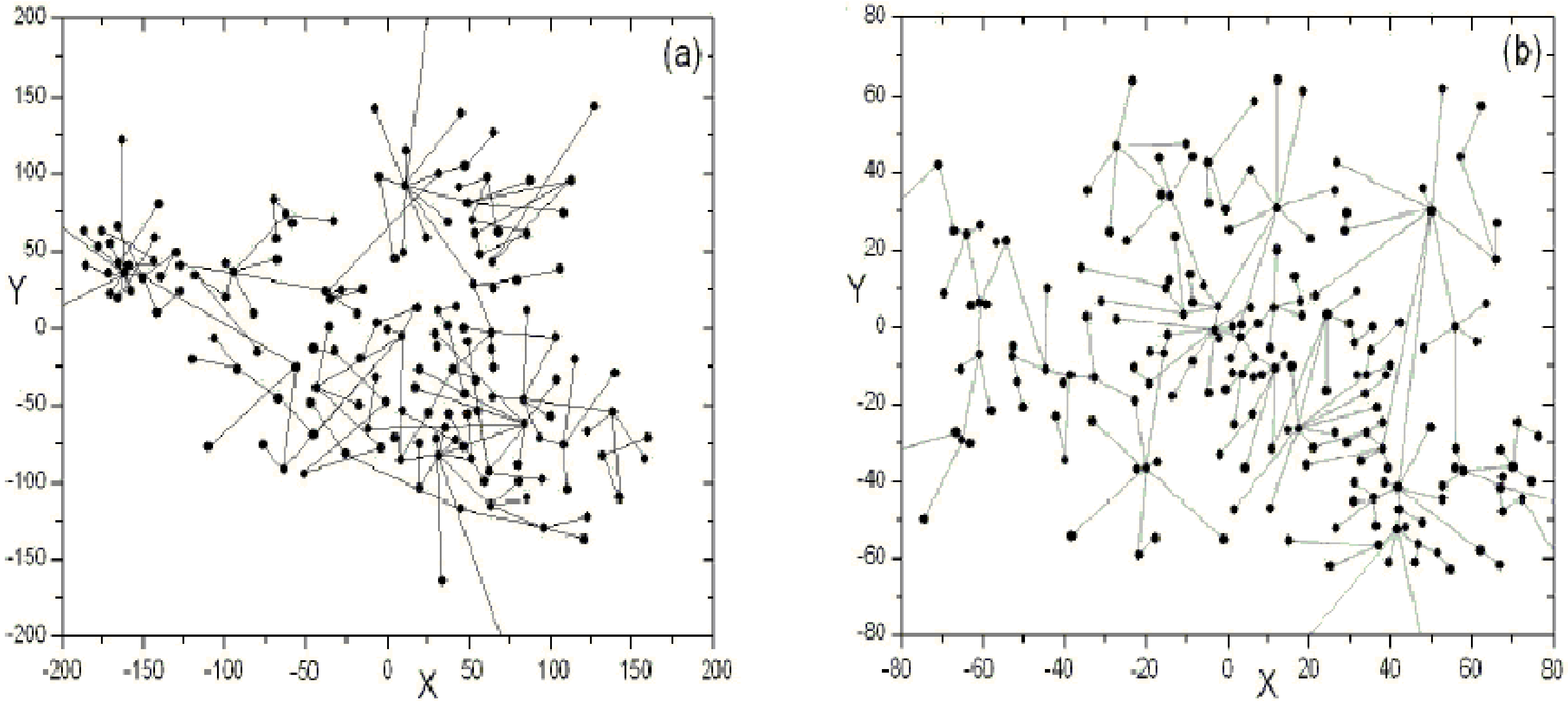
$$\gamma_G = \frac{3+\alpha_G}{2+\alpha_G}$$

$$\alpha_G \geq 0$$

$$r = (1-\xi')^{-(2+\alpha_G)}$$

$$\theta = 2\pi\xi'$$

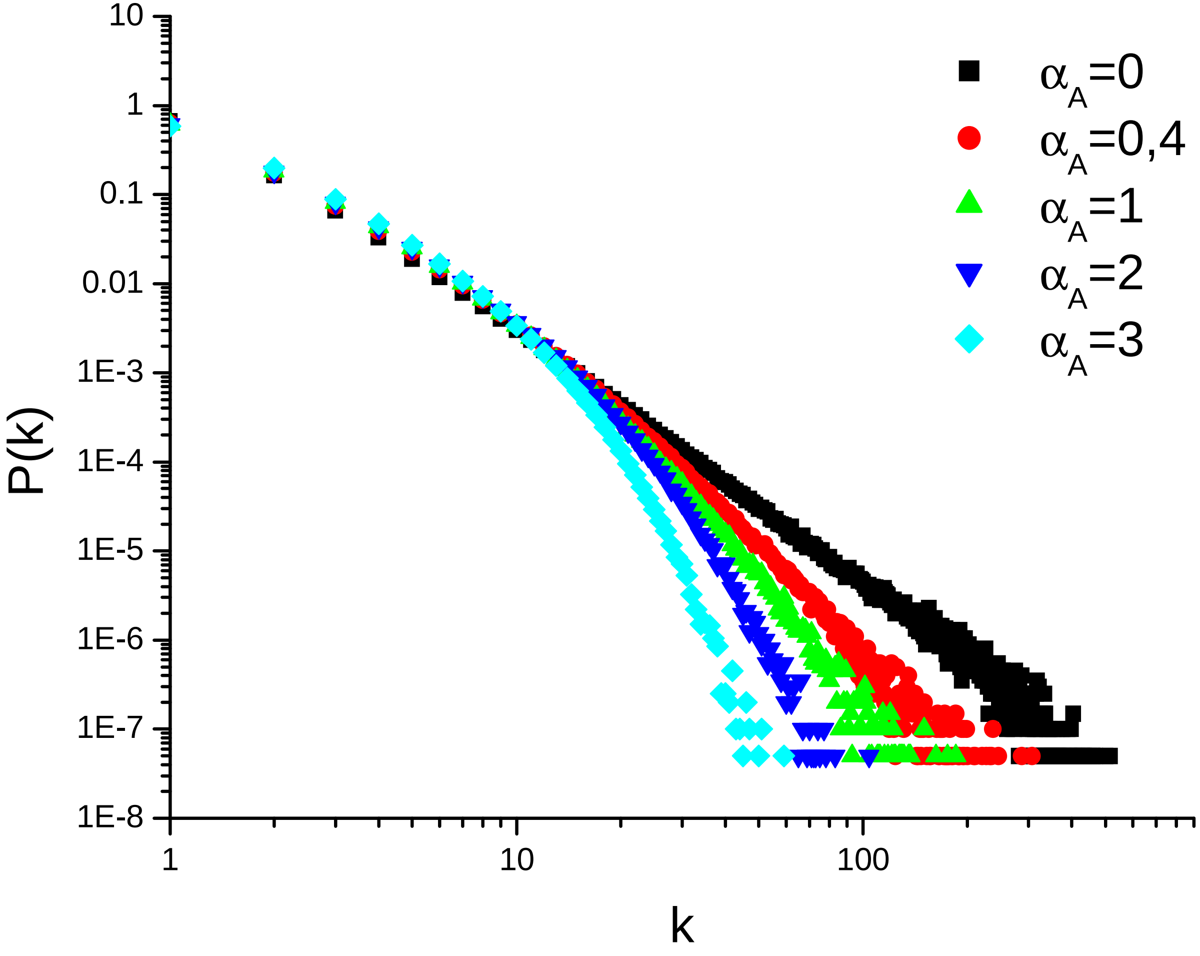
Examples



N = 250 nodes (a) $(\alpha_G, \alpha_A) = (1, 0)$ and (b) $(\alpha_G, \alpha_A) = (1, 4)$.
The starting site is at $(X, Y) = (0, 0)$. Notice the spontaneous emergence of hubs.

Barabási-Albert Model with Euclidean Distance Power-law Distributed

$$\Pi(k_i) = \frac{k_i / r_i^{\alpha_A}}{\sum_{j=1}^{N'} k_j / r_j^{\alpha_A}} \quad (03)$$



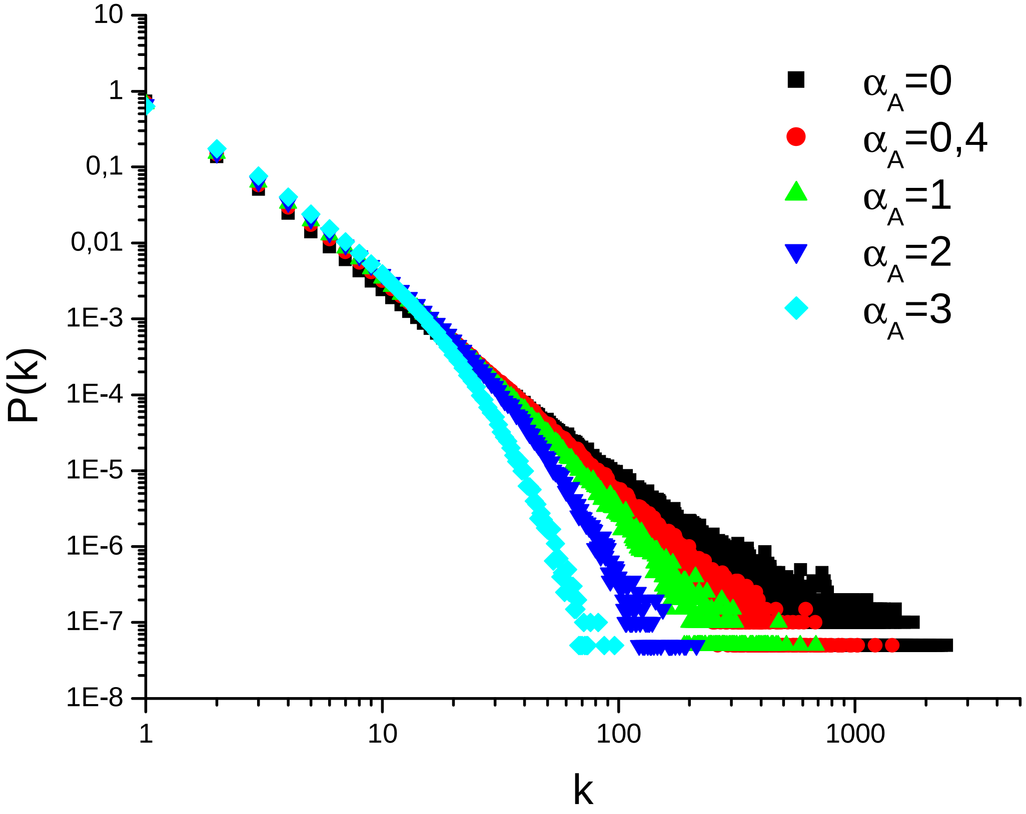
Soares, Tsallis, Mariz and da Silva, Europhys. Lett. **70,** 70 (2005)

Fitness Model with Euclidean Distance Power-law Distributed

- Inspired in the works of Soares, Tsallis, Mariz and da Silva[3],

  and Bianconi and Barabasi[4].

$$\Pi(k_i) = \frac{k_i \eta_i / r_i^{\alpha_A}}{\sum_{j=1}^{N'} k_j \eta_j / r_j^{\alpha_A}} \quad (06)$$



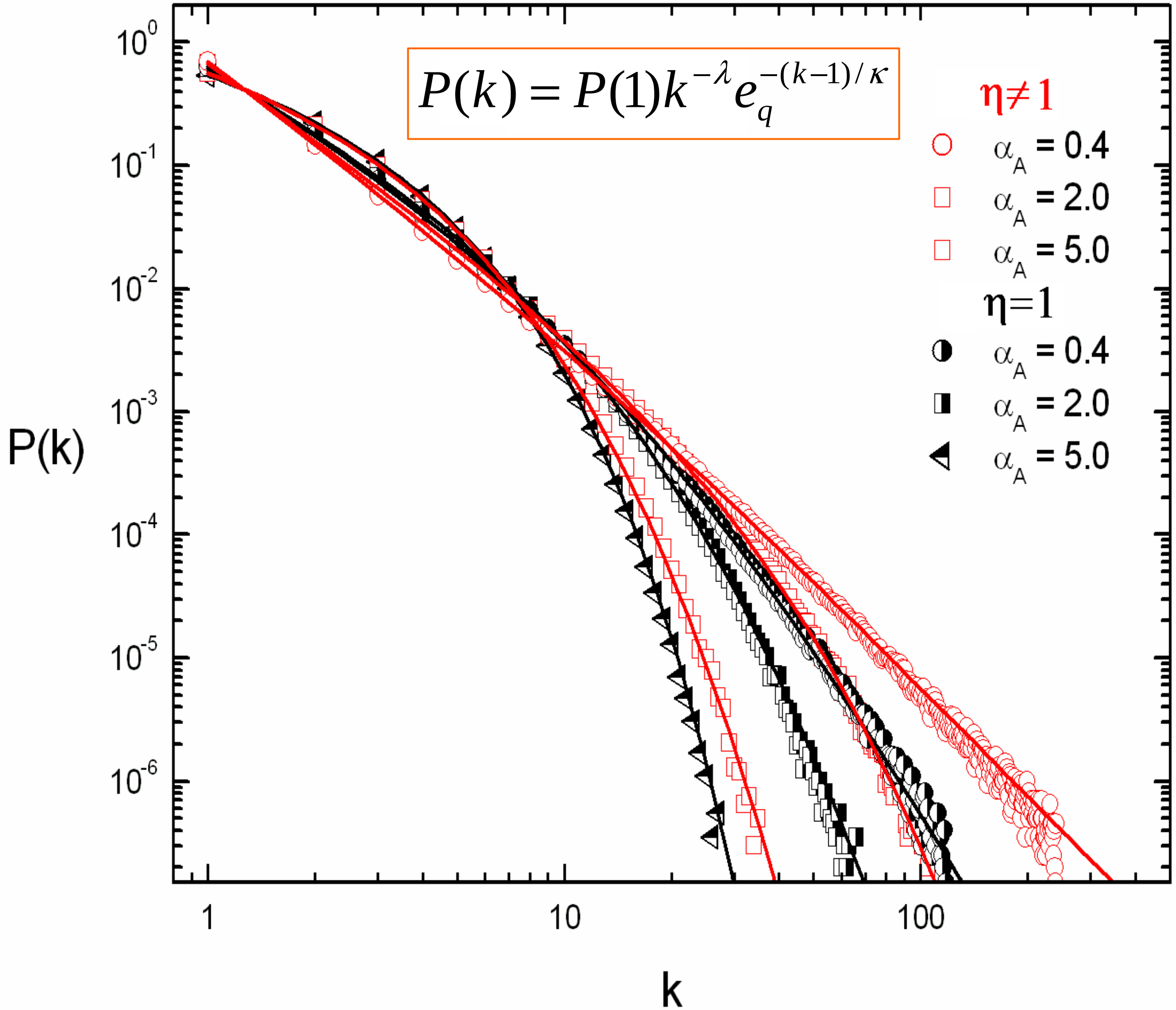
Meneses, Cunha, Soares, and da Silva,
Progress of Theoretical Physics Supplement **162,** 131 (2006)

Tsallis Nonextensive Statistical Mechanics

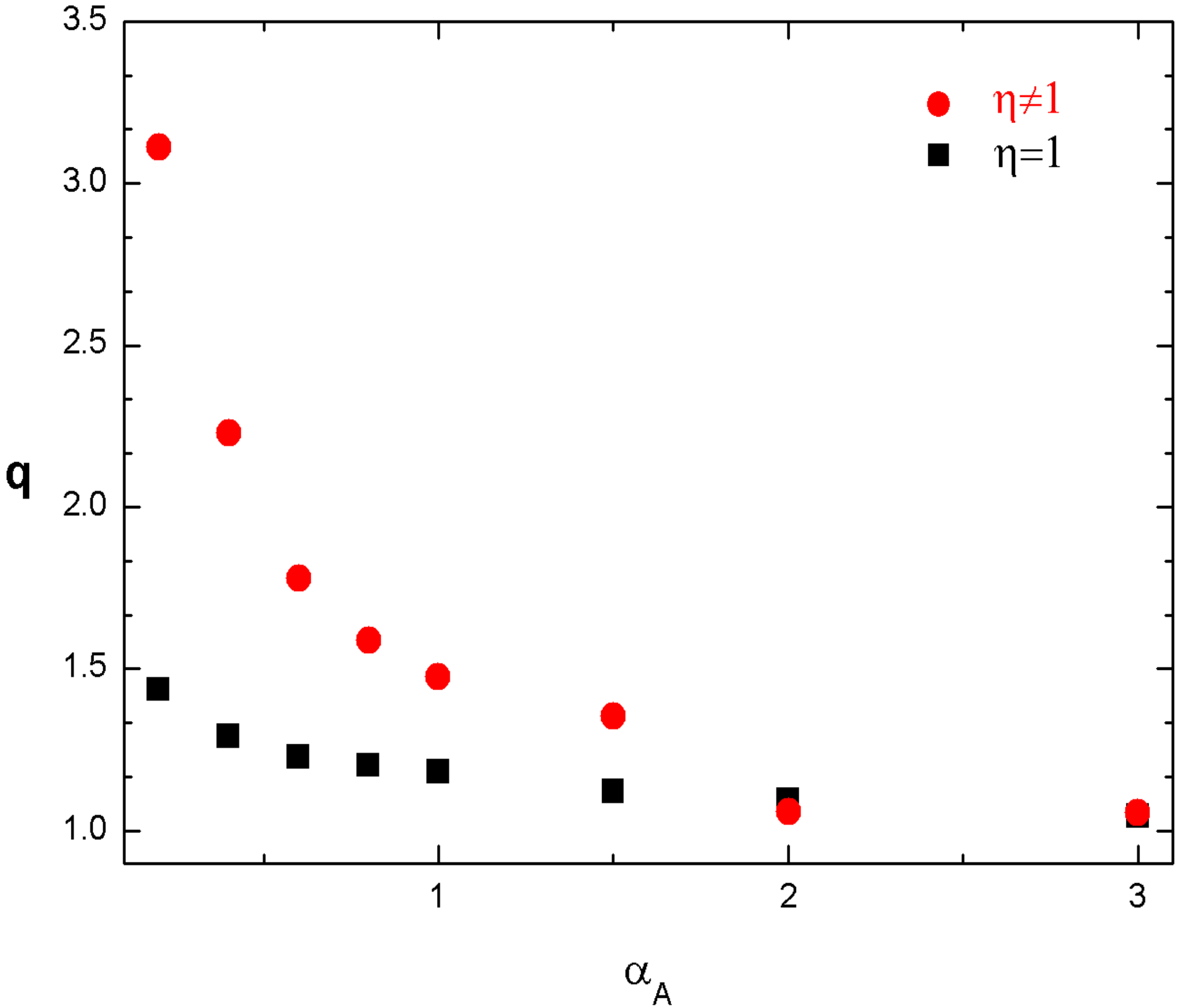
$$s_q = \frac{1 - \int dk [P(k)]^q}{q - 1} \qquad (q \in \Re; S_1 = S_{BG})$$

$$P(k) = P(1) k^{-\lambda} e_q^{-(k-1)/\kappa} \qquad (07)$$

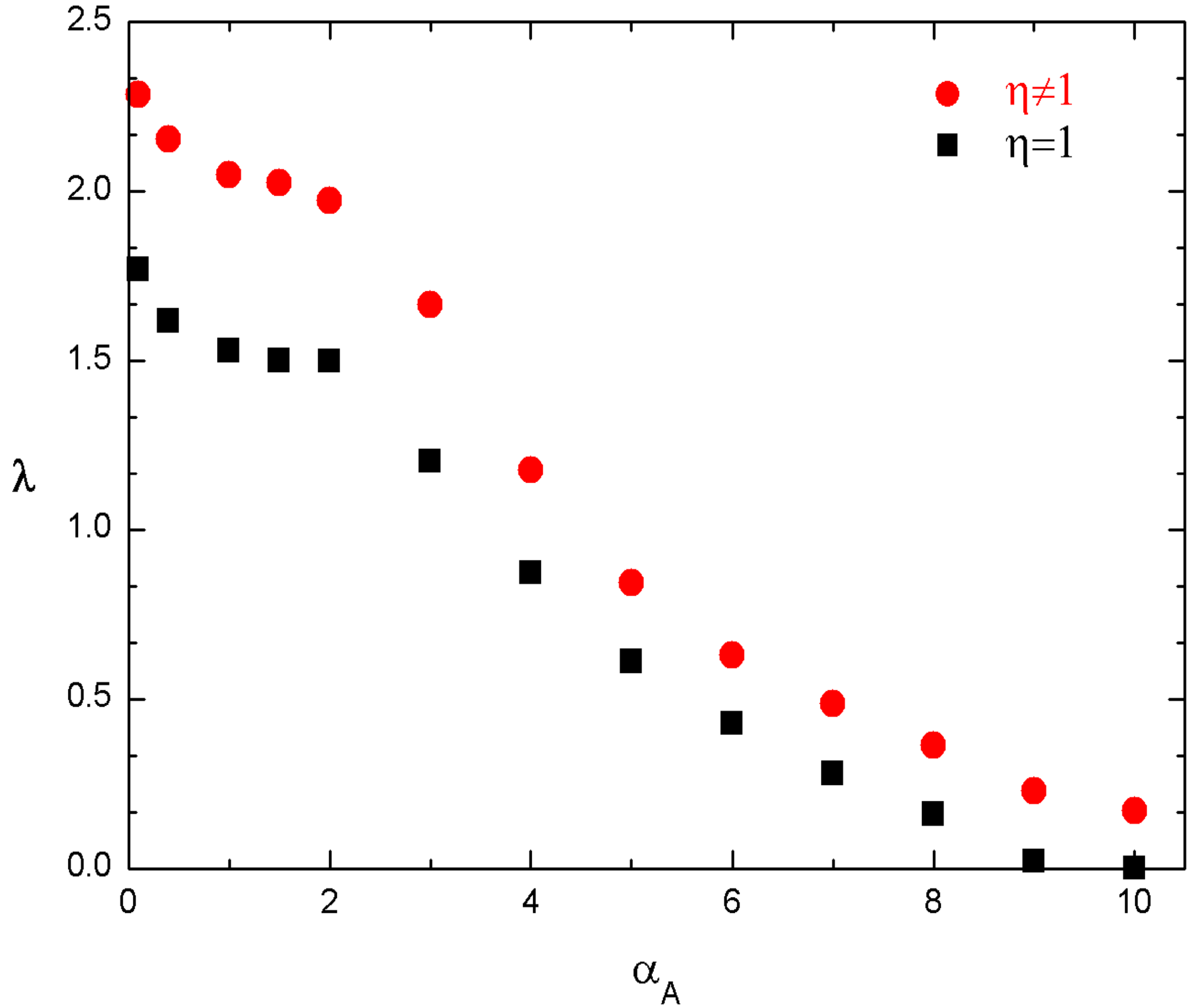
$$e_q^x \equiv [1 + (1-q)x]^{1/(1-q)}$$



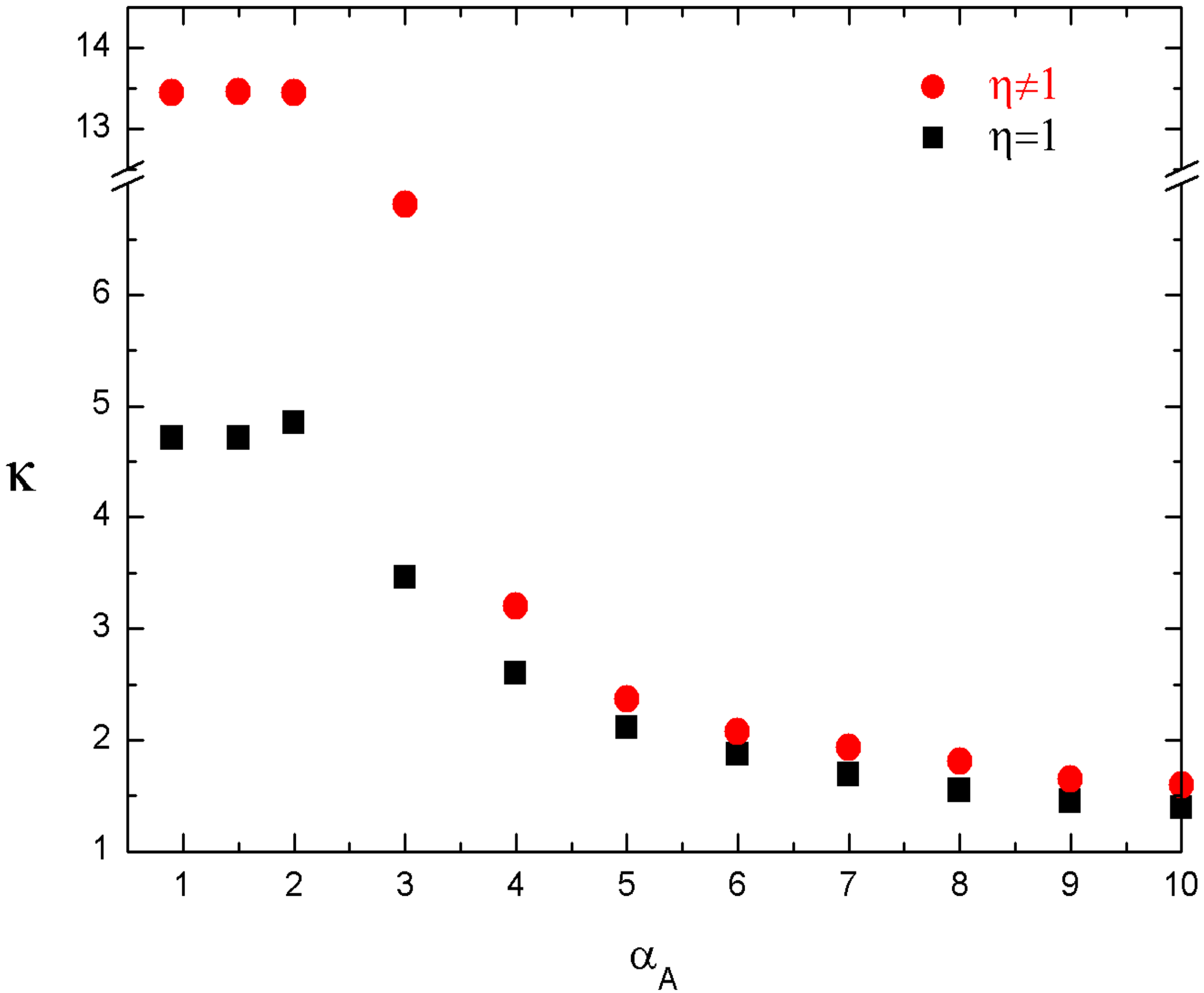
Connectivity distribution *P*(*k*) for typical values α_A for η≠1 and η=1 models. The symbols are numerical results and continuous lines are the best fits according to P(k).



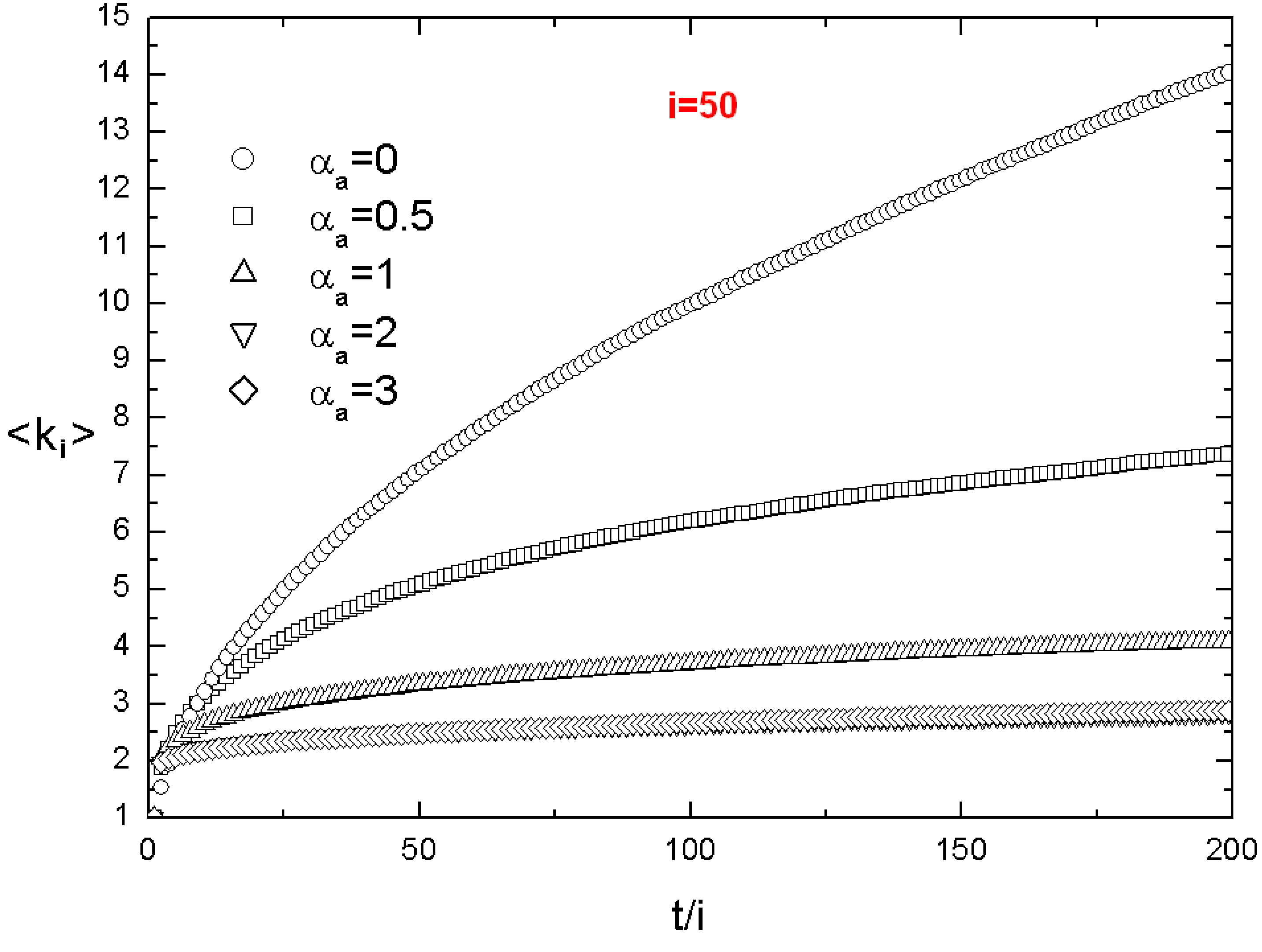
α_A-dependence of q for both η≠1 and η=1 models. In this graph we observe some kinds of changements of regimes at α_A = 2 (which coincides with the space dimensionality).



α_A-dependence of λ for both η≠1 and η=1 models. In this graph we observe some kinds of changements of regimes at $\alpha_A = 2$ (which coincides with the space dimensionality).



α_A-dependence of q for both η≠1 and η=1 models. In this graph we observe some kinds of changements of regimes at α_A = 2 (which coincides with the space dimensionality).



Temporal dependence of the average connectivity for η≠1, in 2000 samples.

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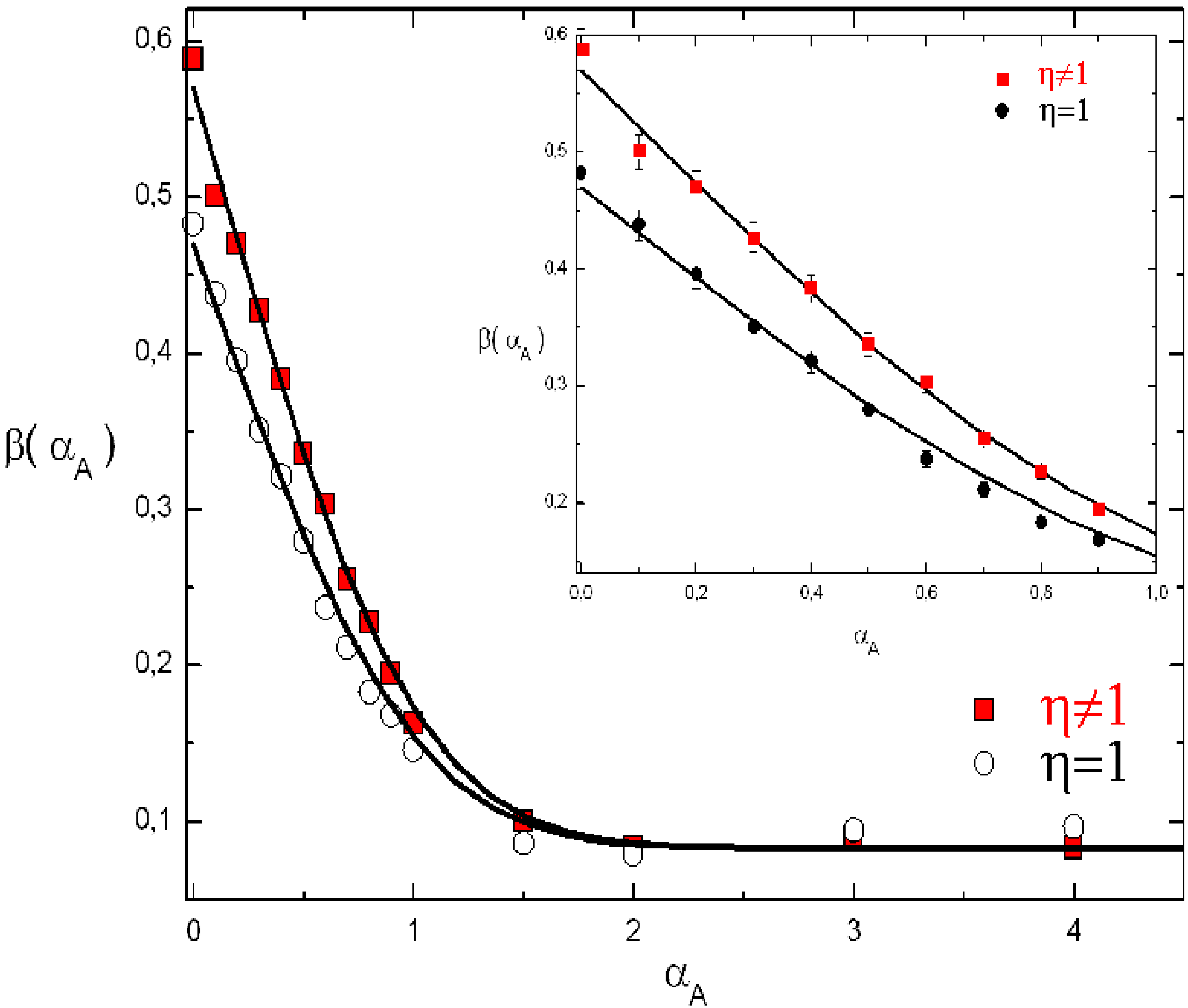
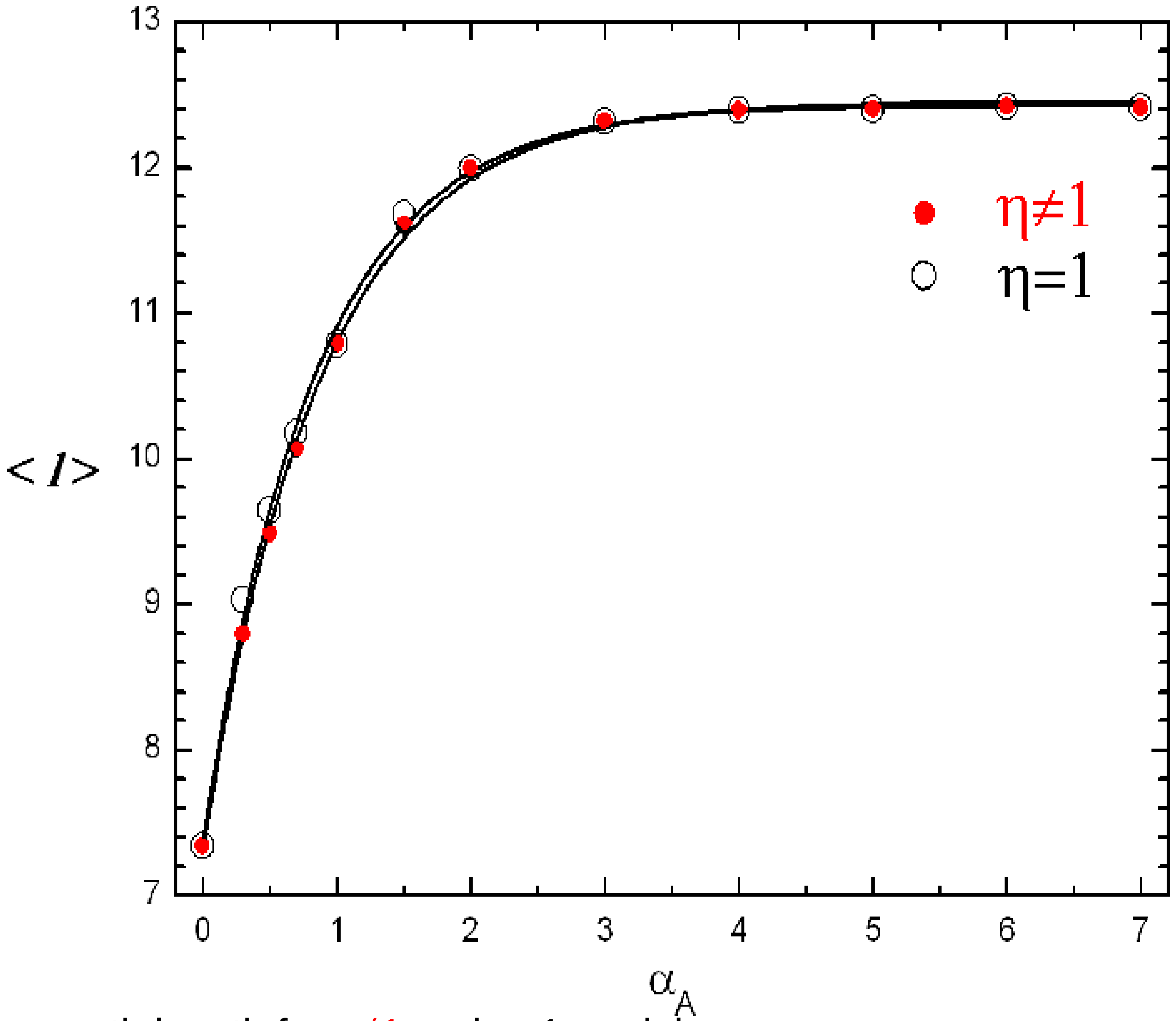
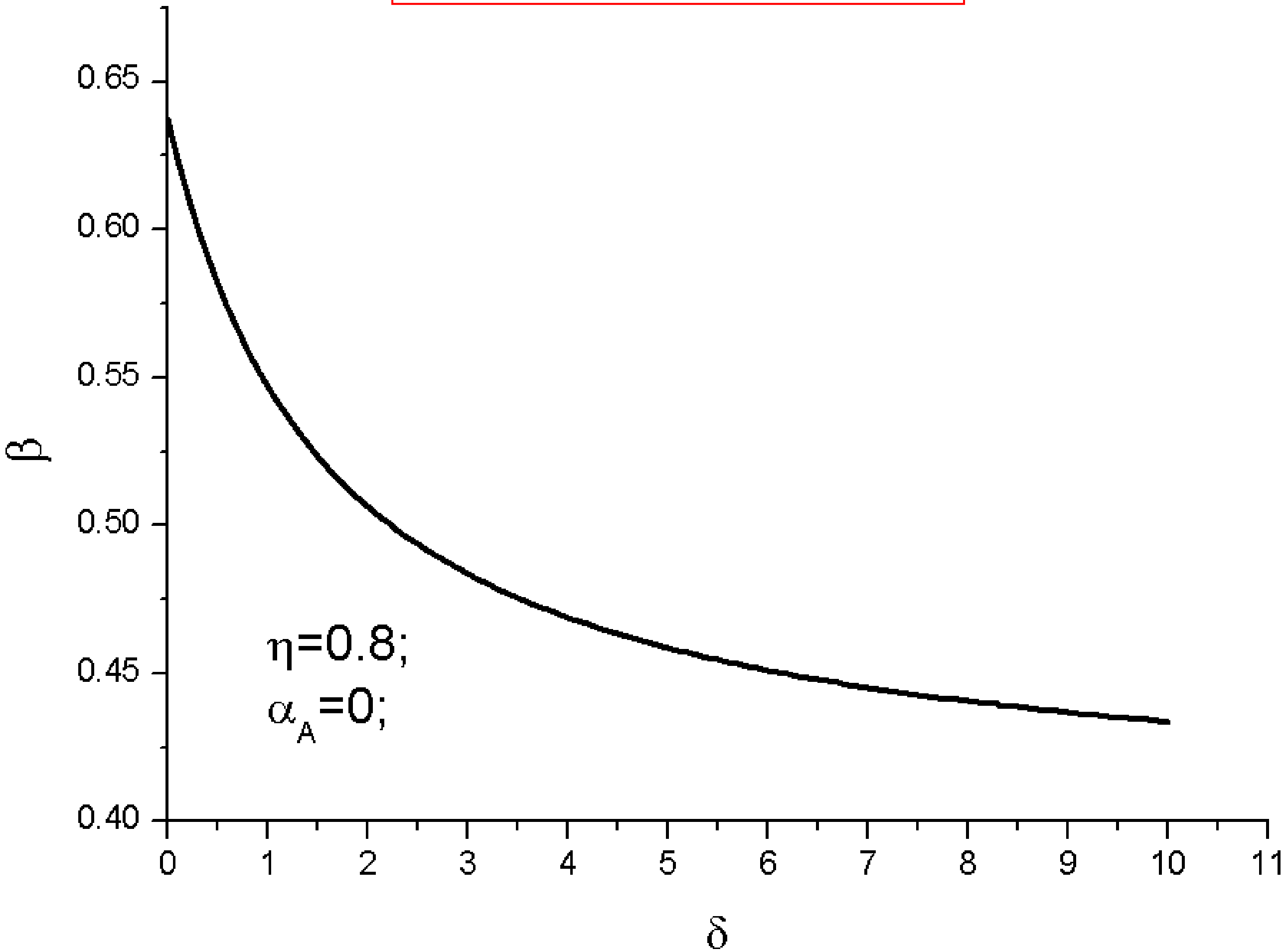


Average connectivity exponent for α_A values relative to measures on node *i* = 50.



Average path length for $\eta \neq 1$ and $\eta = 1$ models.

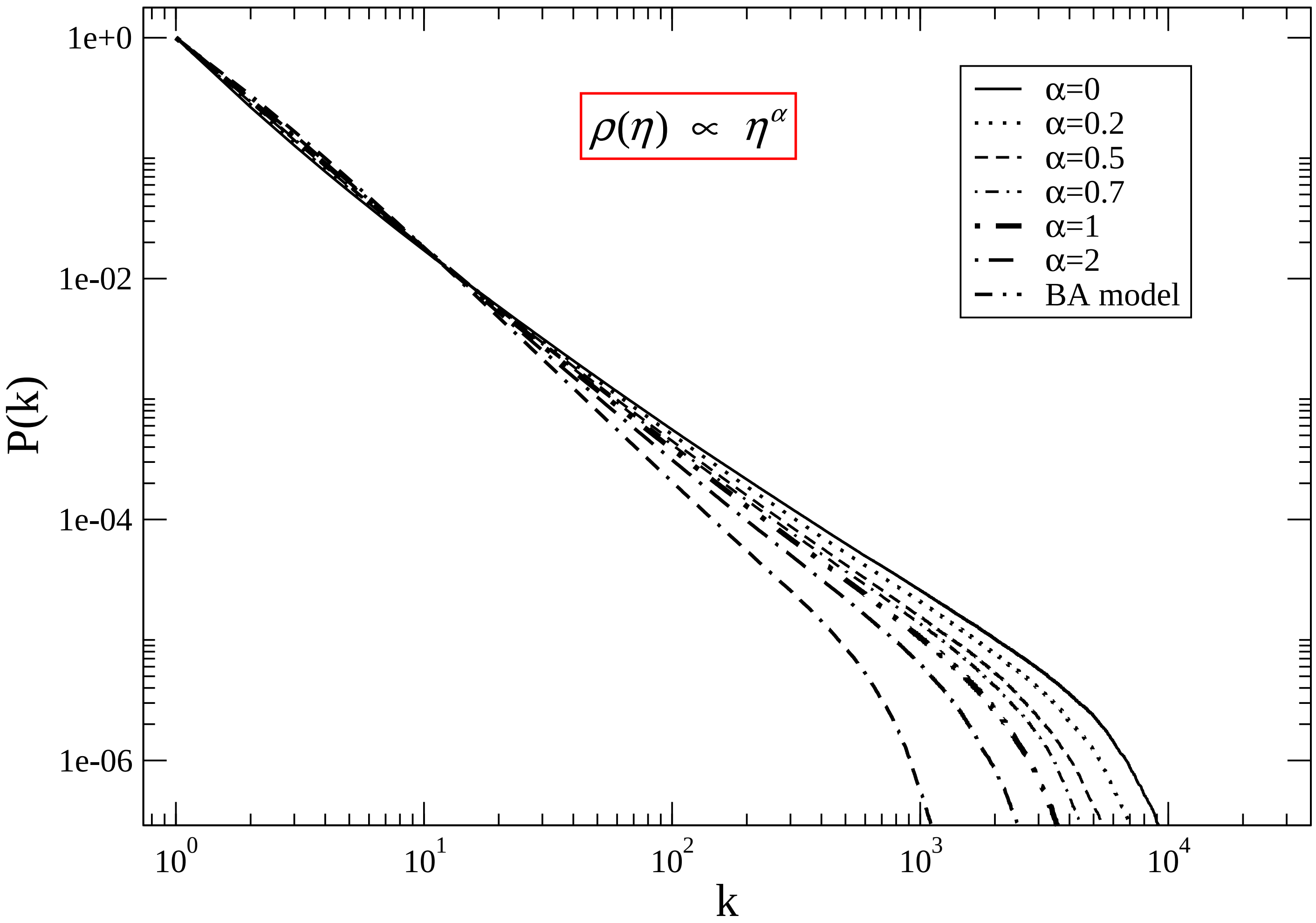
$\alpha_A \neq 0$ in progress



$$\rho(\eta)\,\alpha\,\eta^{\delta}$$

Average connectivity exponent for δ values relative to measures on node i = 20.

Model with Fitness Power-law Distributed



$2.25 < \gamma < 3$

Generalized Model: Fitness and Euclidean Distance Power-law Distributed

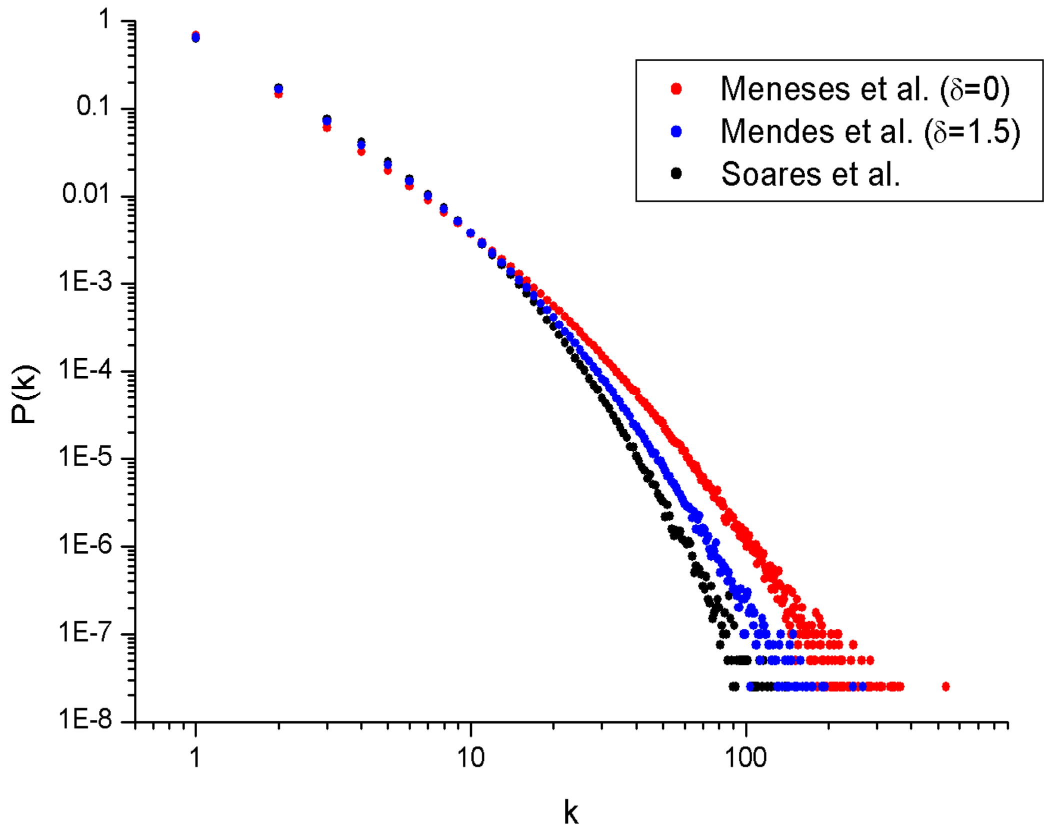
- Inspired in the works of Meneses et al;
- Mendes et al;

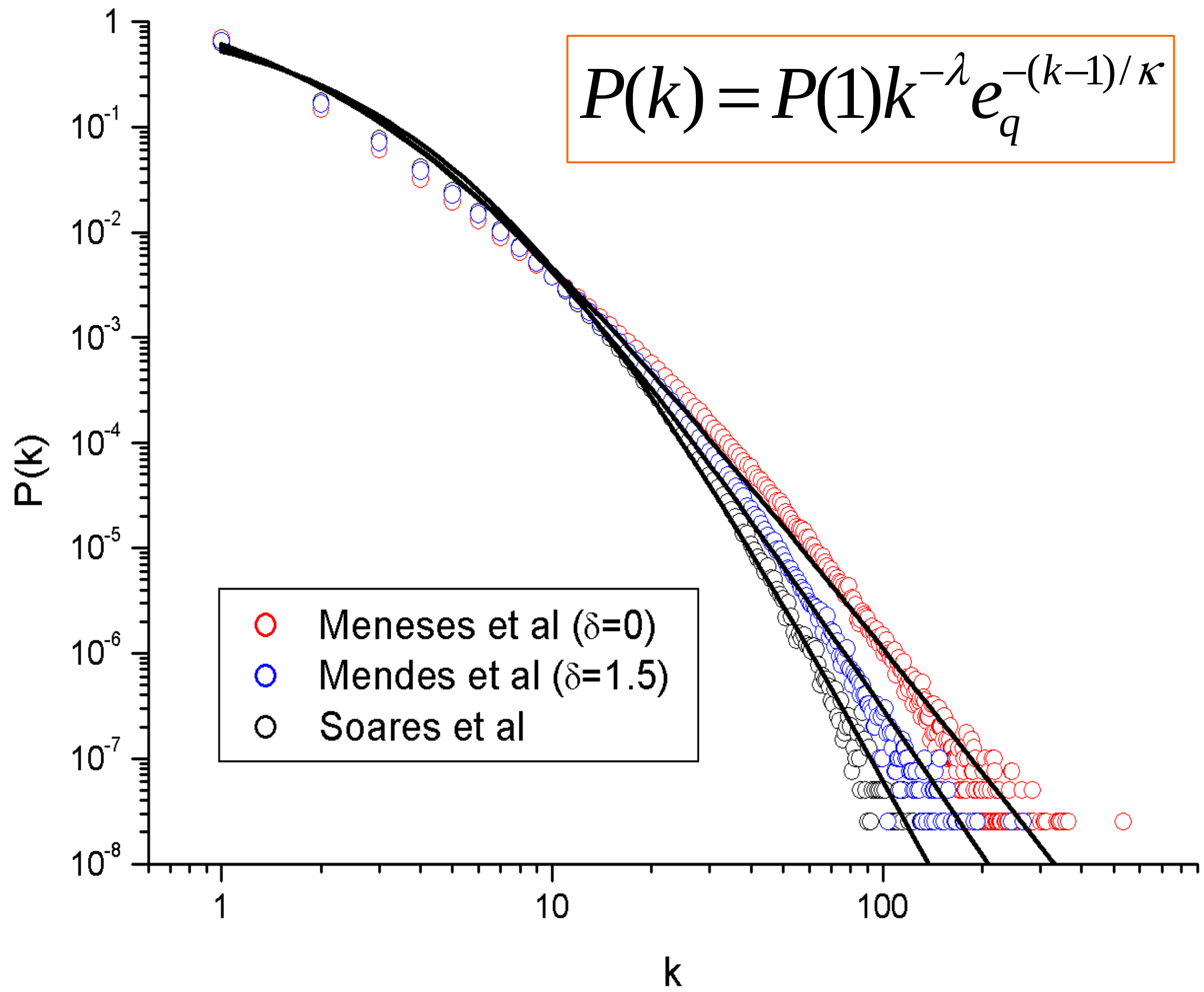
$$\Pi(k_i) = \frac{k_i \eta_i / r_i^{\alpha_A}}{\sum_{j=1}^{N'} k_j \eta_j / r_j^{\alpha_A}} \quad (08)$$

with

$$\rho(\eta) \propto \eta^{\delta}$$

$$\delta \geq 0$$

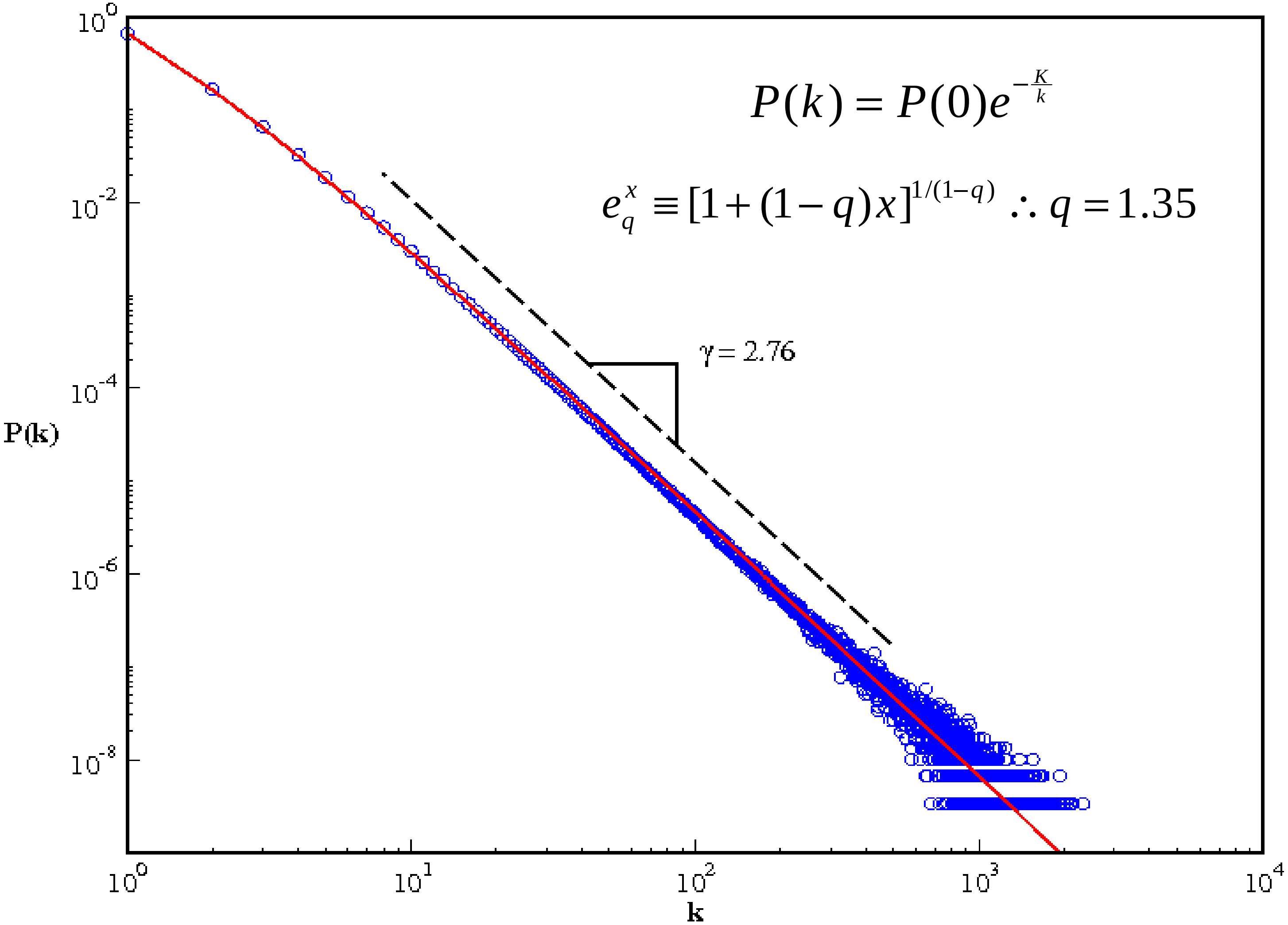


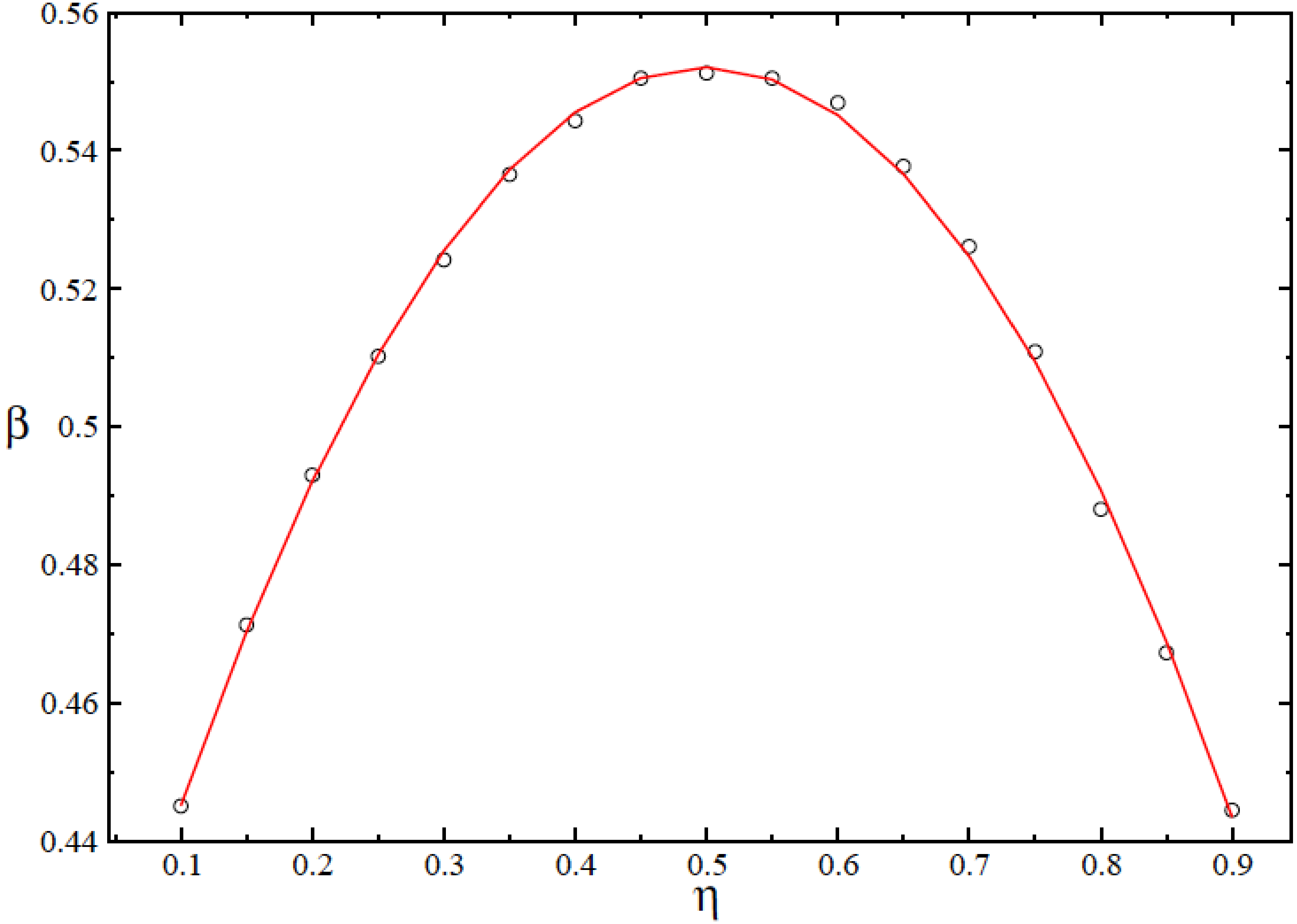


Conectivity distribution *P*(*k*) for α_A=2 for Meneses et al, Mendes et al and Soares et al models. The symbols are numerical results and continuous lines are the best fits in according to P(k).

Acquaintance Model

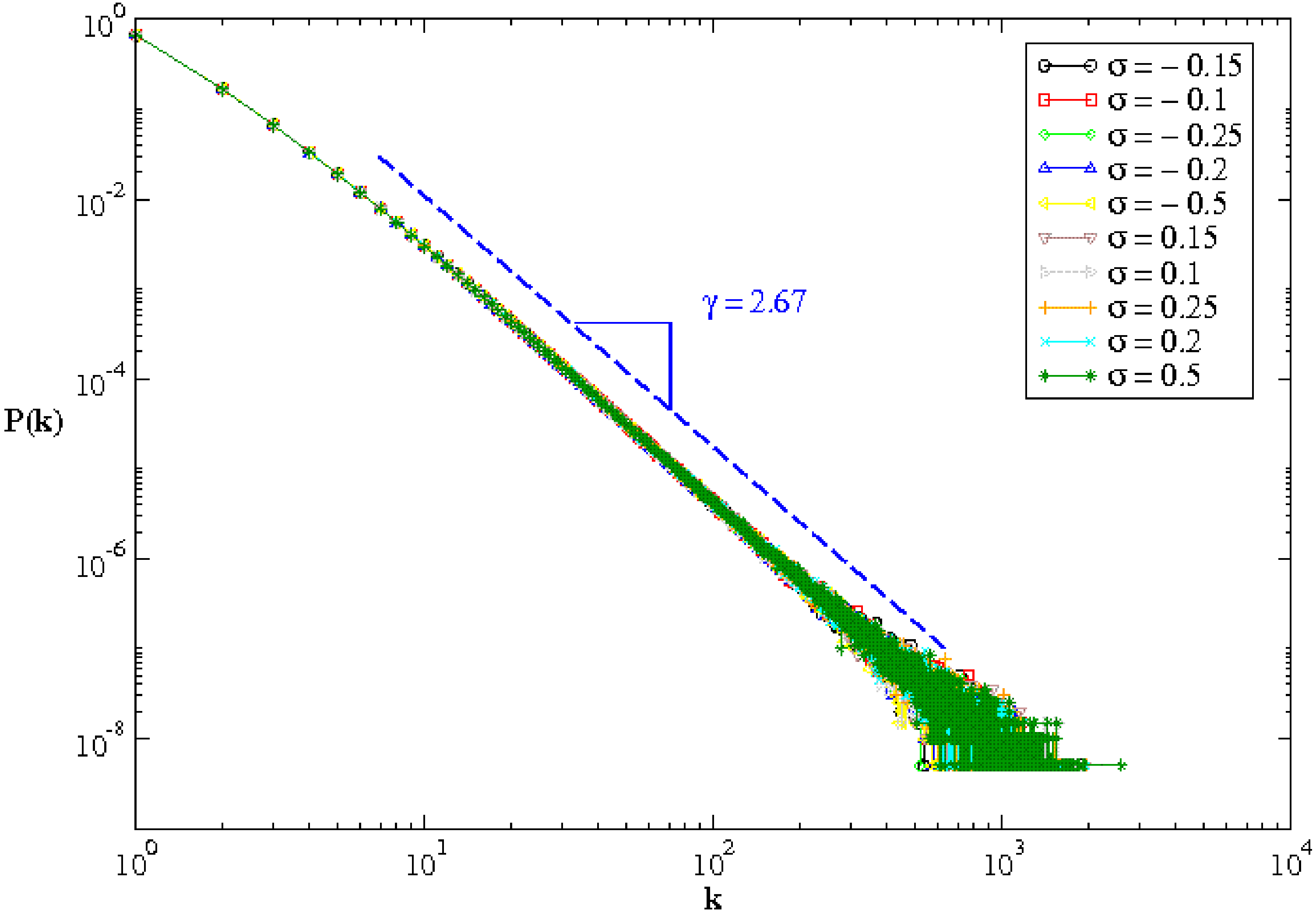
$$\Pi_{i \to j} \propto k_j \left(1 - \left|\eta_i - \eta_j\right|\right)$$




0.56
0.54
0.52
β
0.5
0.48
0.46
0.44
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
η

General Acquaintance Model

$$\Pi_{i\to j} \,\square\, k_j\left(1-|\eta_i-\eta_j|^{\sigma}\right)$$



Summary

(a)

- The present models

MODEL	CONECTIVITY	FITNESS	METRIC	AFFINITY
Barabási-Albert	YES	NO	NO	?
Bianconi et al	YES	UNIFORM	NO	?
Soares et al	YES	NO	POWER-LAW	?
Meneses et al	YES	UNIFORM	POWER-LAW	?
Mendes et al	YES	POWER-LAW	POWER-LAW	?
Almeida et al	YES	?	?	YES
Santos et al	YES	?	?	YES

Summary

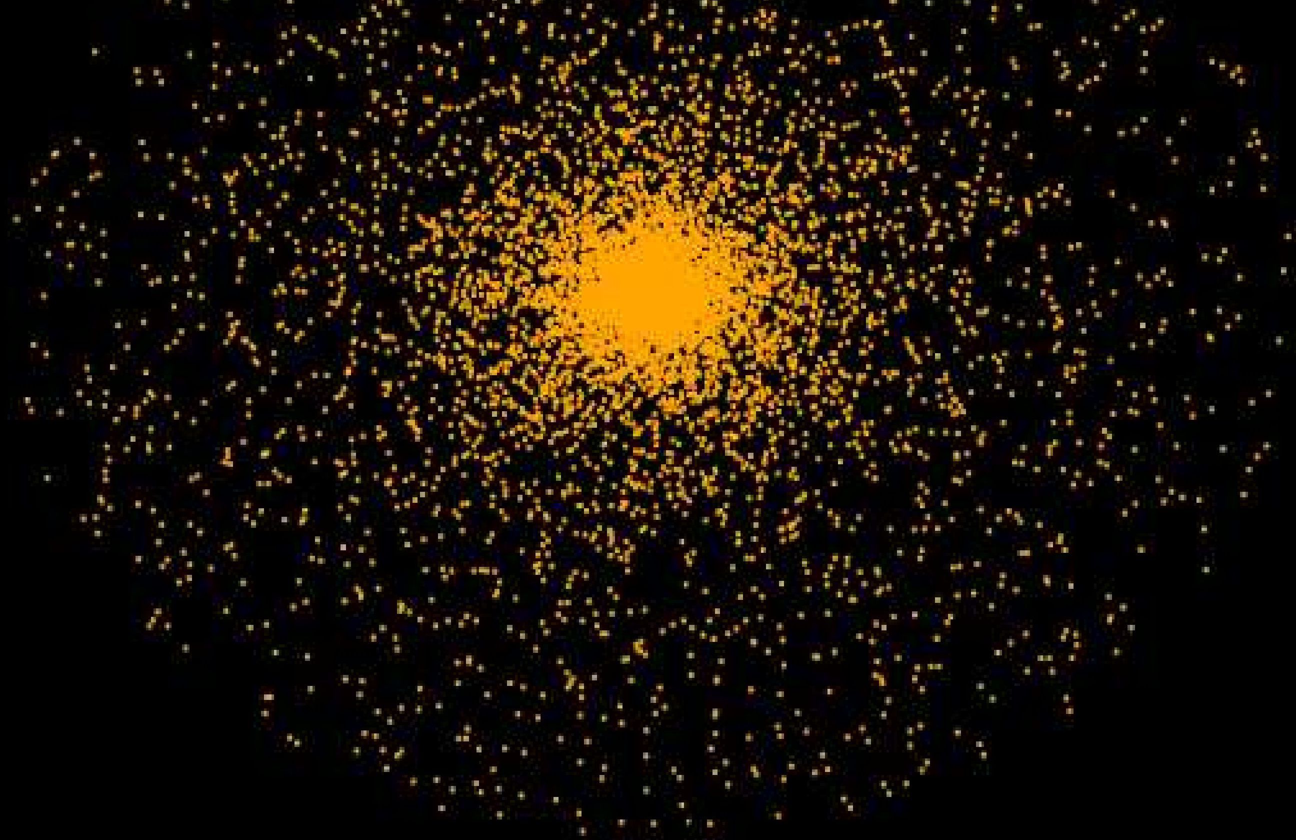
(b)

- We study the effect of competition between the relevant variables: connectivity k, fitness η and metrics r.
- The fitness may give the possibility to the younger nodes to compete equally with the older ones, when the younger node gets a high fitness.
- By including metrics favors the linking between first neighbors.
- The average connectivity $<k_i>$ is appreciably influenced by metrics and by fitness, while the average path length $<l>$ keeps approximatively the same.

Summary

(c)

- The degree distribution $P(k)$ of the present generalized model appears to be the q-exponential function that emerges naturally within Tsallis nonextensive statistics.



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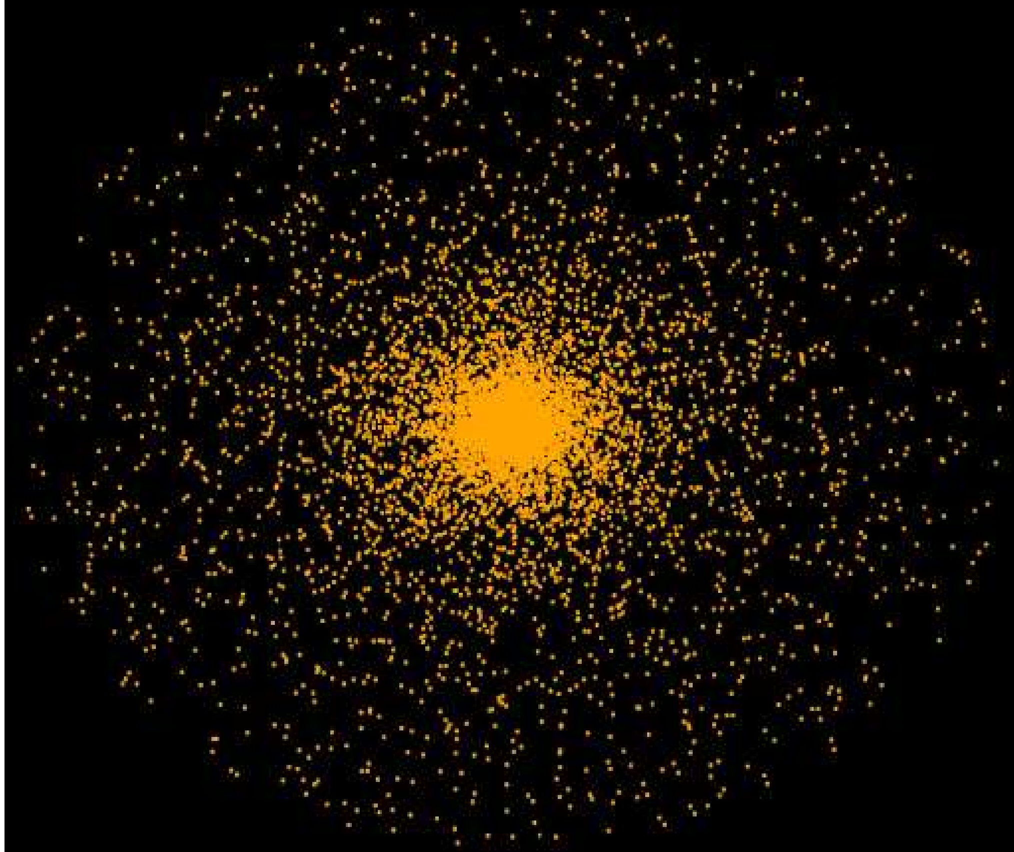

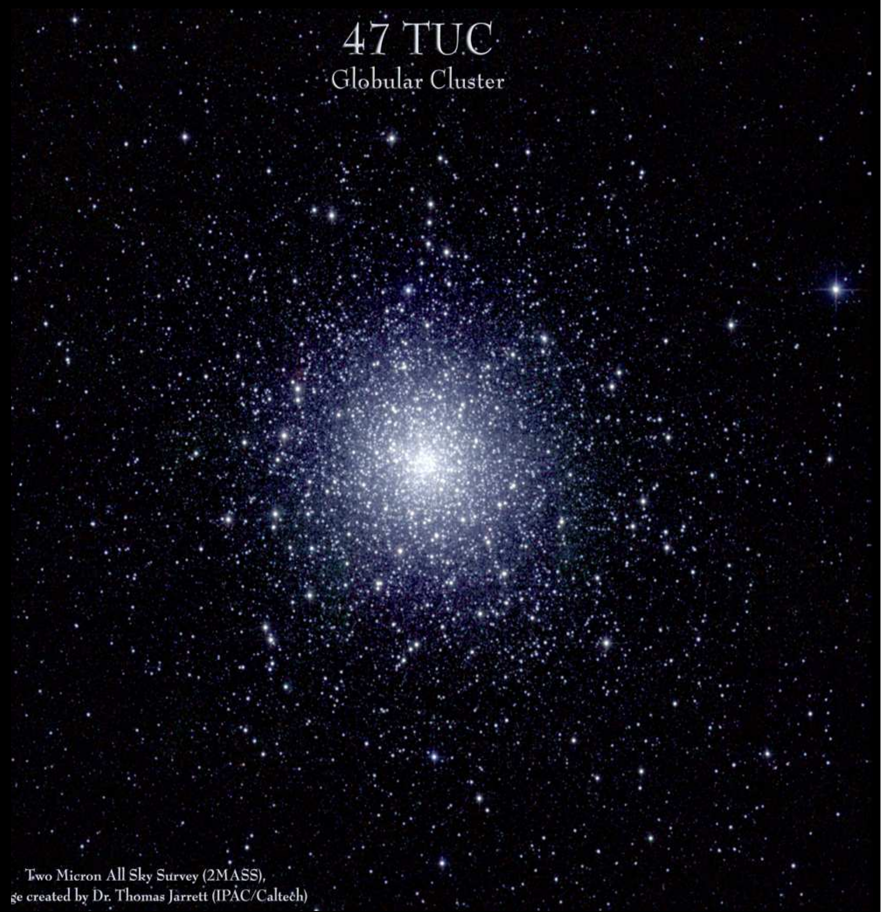

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1. D. J. B. Soares, C. Tsallis, A. M..Mariz, and L. R. da Silva.

"Preferential Attachment Growth Model and Noextensive Statistical Mechanics"

Europhysics Letters **70**, 70 (2005)

2. J. S. Andrade Jr., H. J. Herrmann, R.F Andrade and L. R. da Silva.

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Physical Review Letters **94**, 018702 (2005).

3. M. D. de Meneses, Sharon D. da Cunha, D.J.B. Soares and L. R. da Silva.

"Preferential Attachment Scale-free Growth Model with Random Fitness and Connection with Tsallis Statistics"

Progress of Theoretical Physics Supplement **162** 131 (2006)

4. D. J. B. Soares, J. S. Andrade Jr., H. J. Herrmann and L. R. da Silva

"Three Dimension Apollonian Networks"

International Journal of Modern Physics **17** 1219 (2006)

5. P. G. Lind, L. R. da Silva, J. S. Andrade Jr. and H. J. Herrmann.

"The Spread of Gossip in American Schools"

Europhysics Letters **78**, 68005 (2007)

References

6. P. G. Lind, L. R. da Silva, J. S. Andrade Jr. and H. J. Herrmann.

"Spreading Gossip in Social Networks"

Physical Review E **76**, 036117 (2007)

7. S. B. Jácome, A. A. Moreira, J. S. Andrade L. R. da Silva and H. J. Herrmann

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Generating more realistic complex networks from power-law distribution of fitness

9. A. M. Filho, U. L. Fulco and L. R. da Silva

Epidemic Diffusion System in Scale-free Networks

10. M. L. de Almeida (Dissertação de Mestrado)

Dinâmica e Estrutura de Redes Complexas no Modelo de Afinidade

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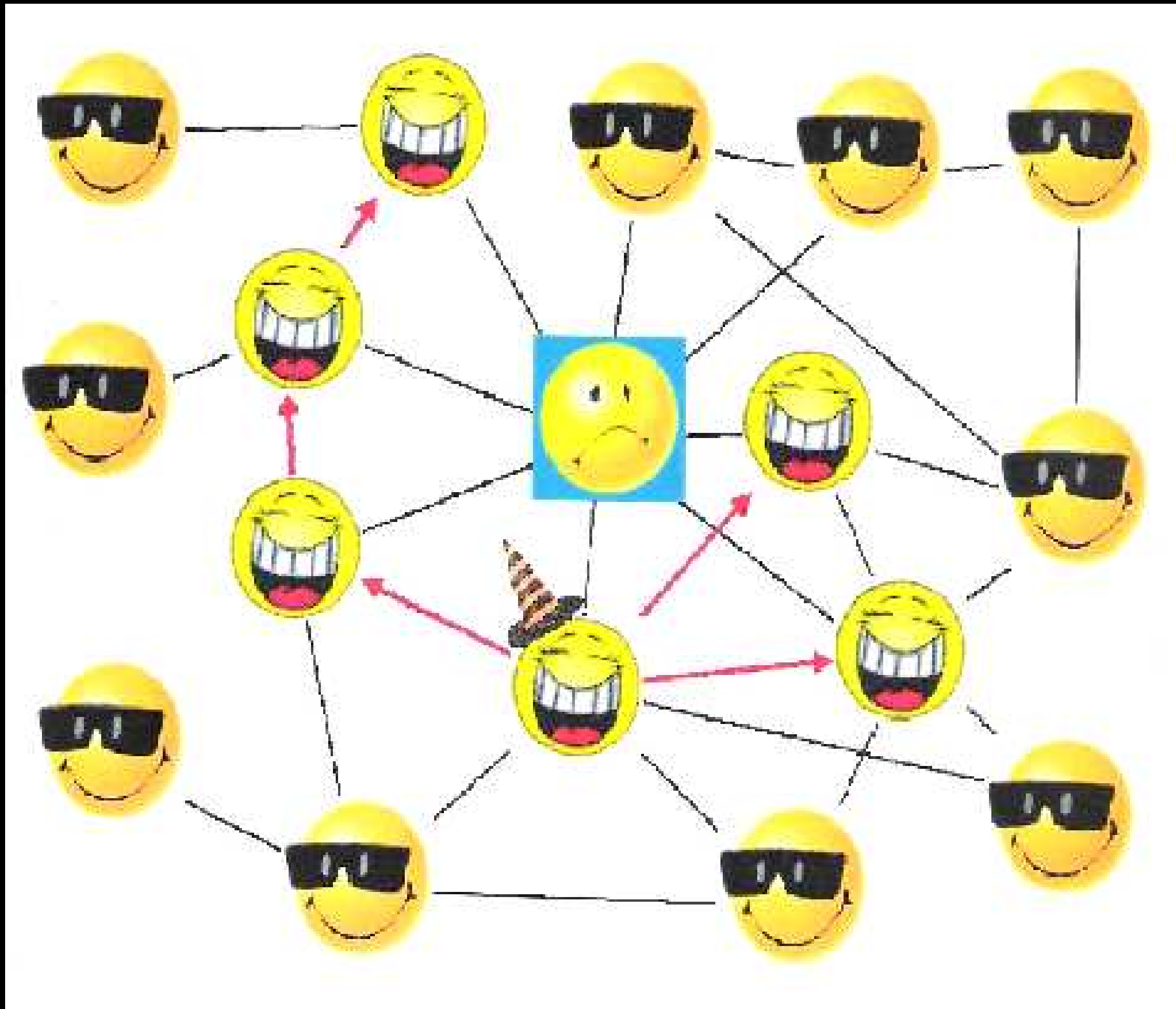
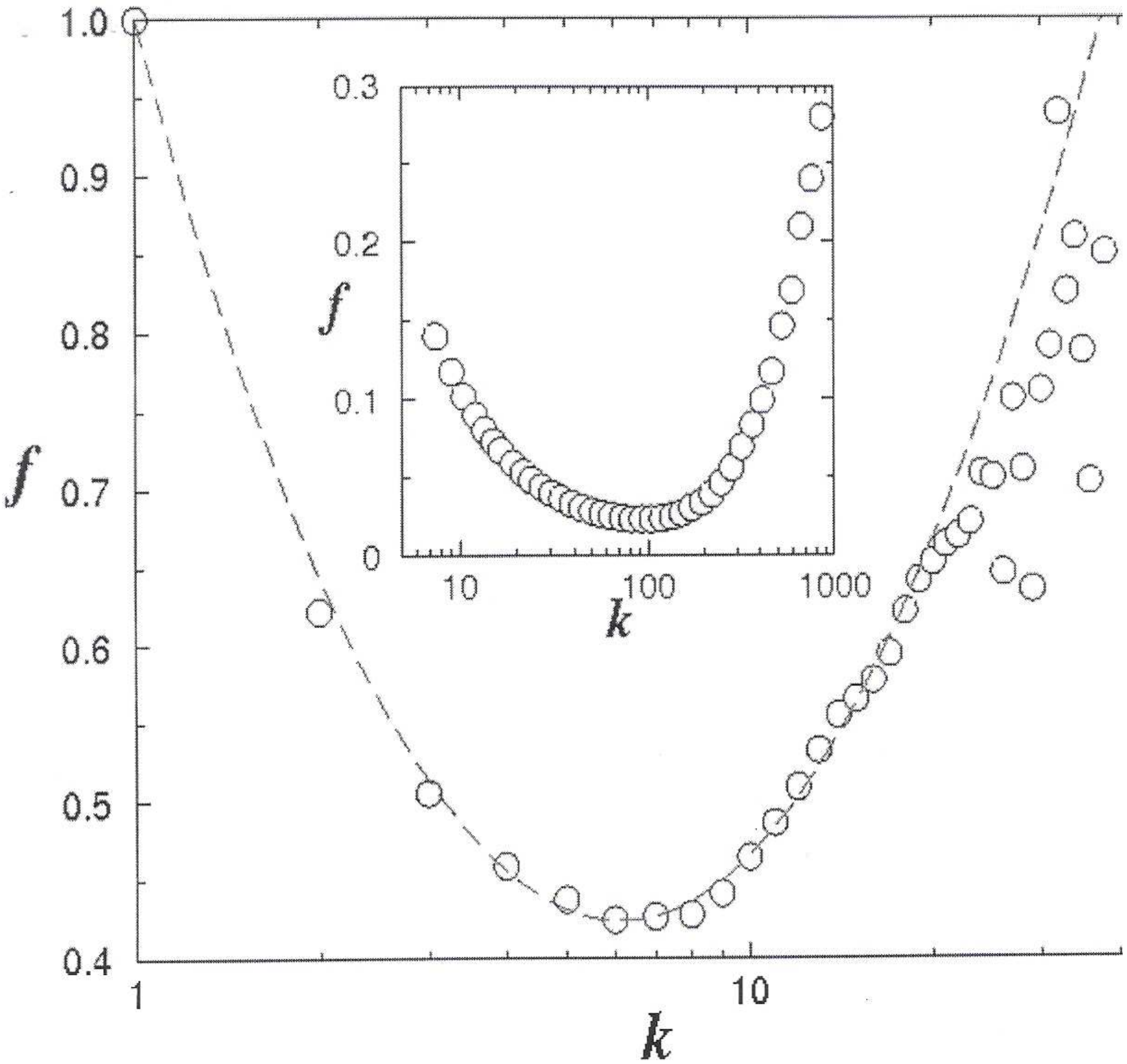
Thank you very much

6. How gossip propagates



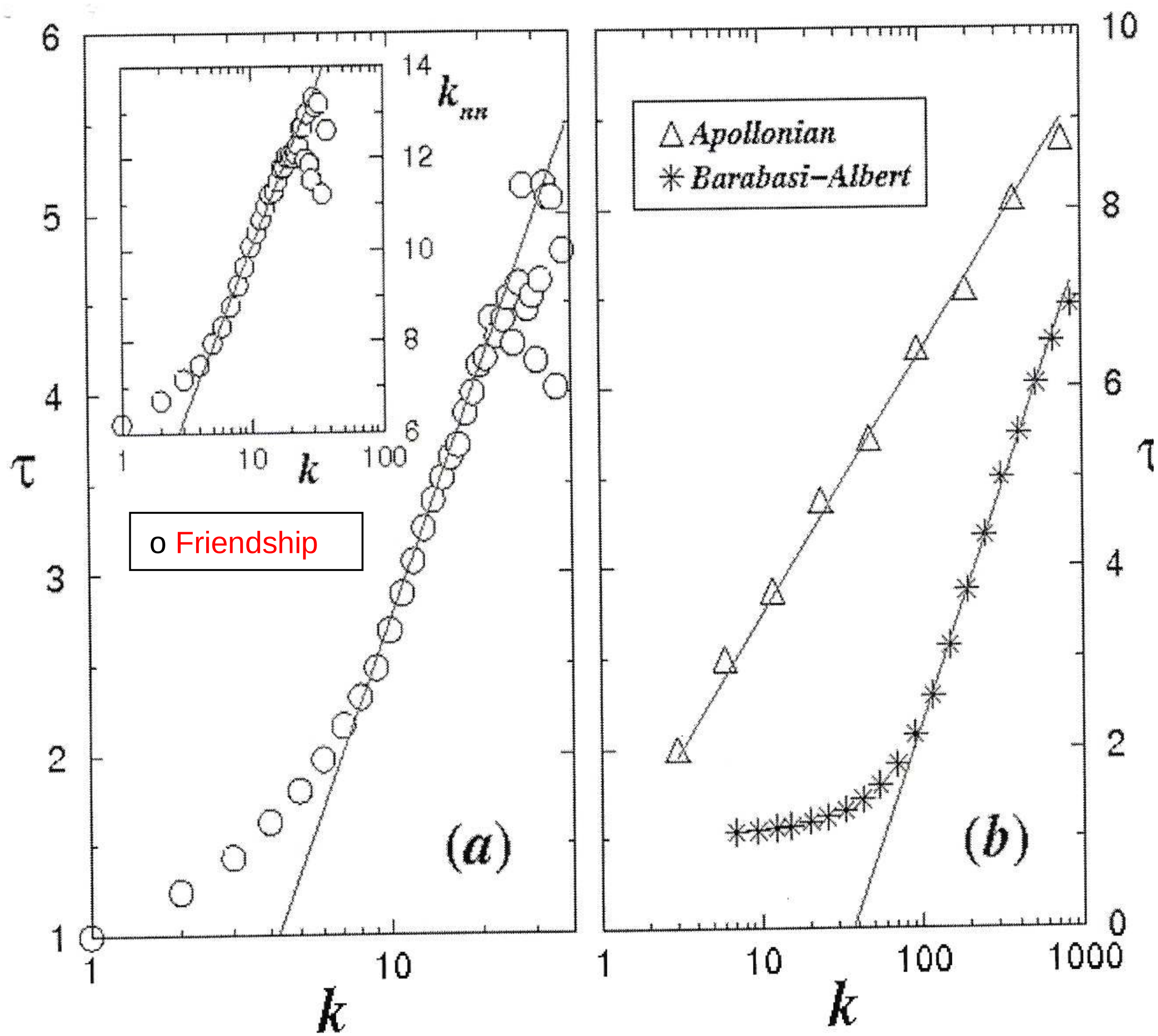
Europhysics Letters **78**, 68005 (2007)

Spread Factor $(f = n_f / k)$

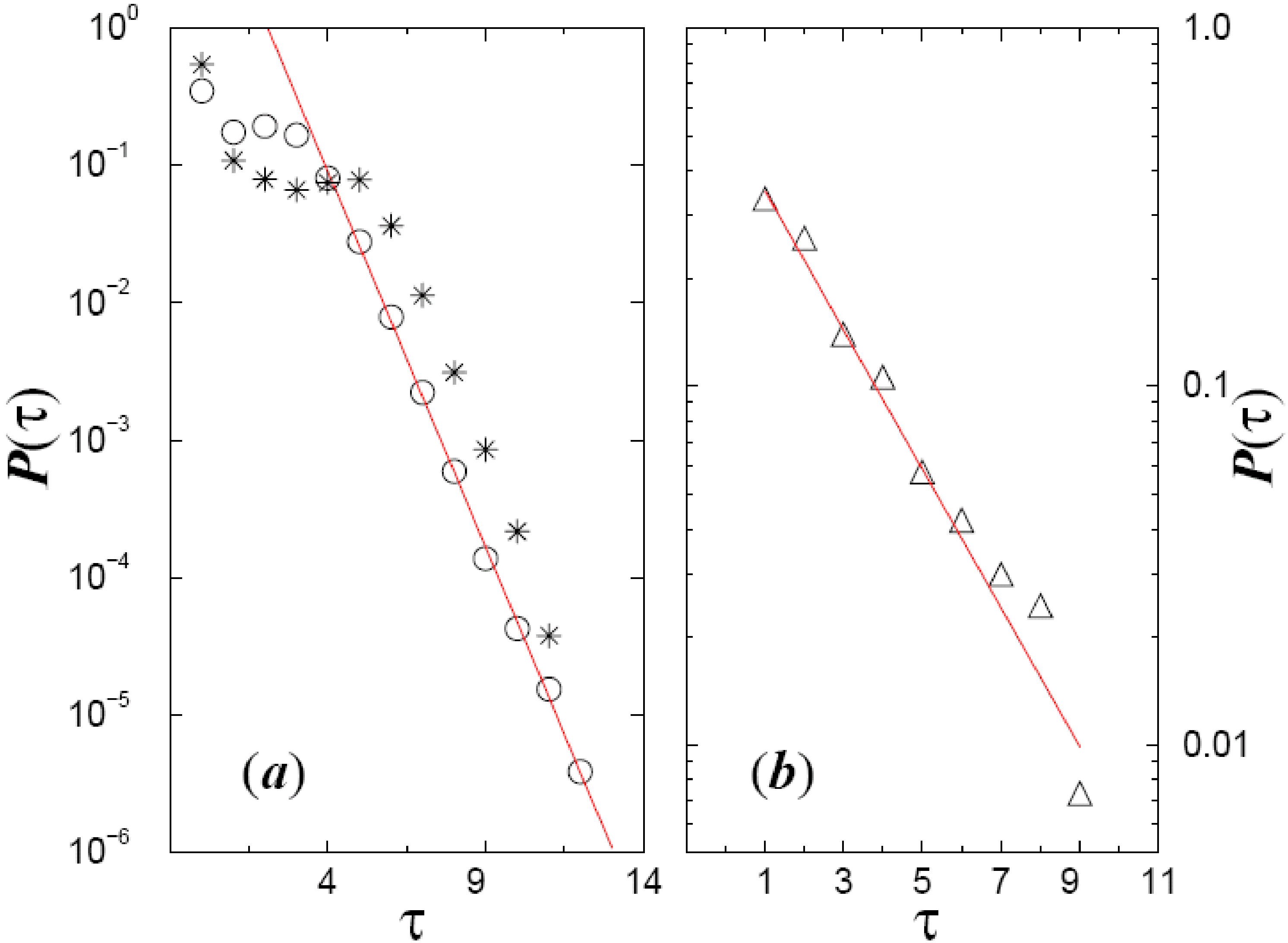


U.S Friend Schools

Inset graph: Barabási-Albert


6
5
4
3
2
1
τ
1
10
k
14
12
10
8
6
k_{nn}
1
10
k
100
o Friendship
(a)
△ Apollonian
✳ Barabasi–Albert
(b)
1
10
100
1000
k
10
8
6
4
2
0
τ

$$\tau = A + B\log(k)$$



o Friendship

∆ Apollonian

* BA; m=7

GAS-LIKE (NODE COLLAPSING) NETWORK:

S. Thurner and C. T., Europhys Lett **72**, 197 (2005)

Number *N* of nodes fixed (*chemostat*); i=1, 2, …, *N*

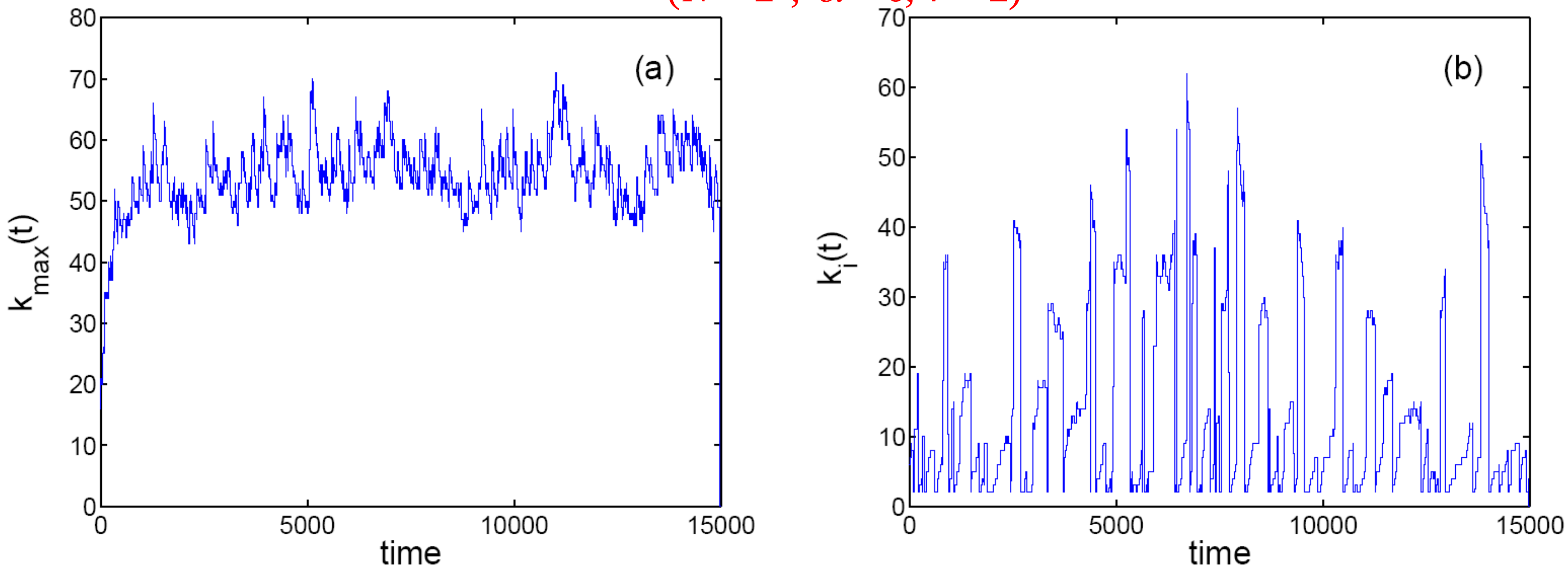
$$\text{Merging probability} \;\; p_{ij} \propto \frac{1}{d_{ij}^{\;\alpha}} \quad (\alpha \geq 0)$$

$d_{ij} \equiv$ *shortest path (chemical distance) connecting nodes i and j on the network*

$\alpha = 0$ *and* $\alpha \to \infty$ *recover the random and the neighbor schemes respectively*

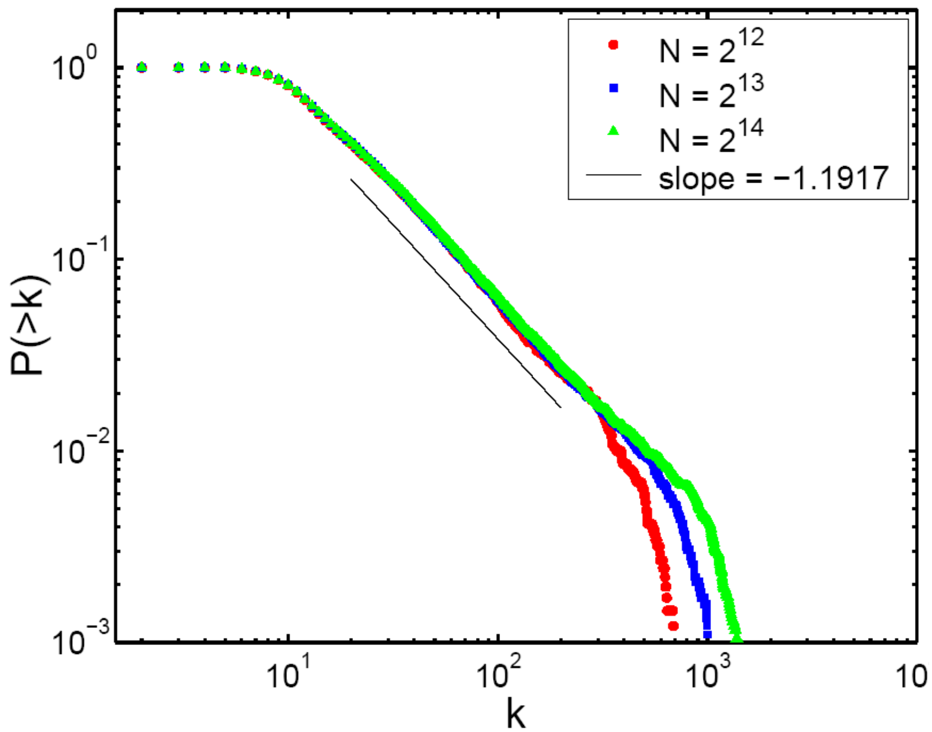
(Kim, Trusina, Minnhagen and Sneppen, *Eur. Phys. J. B* 43 (2005) 369)

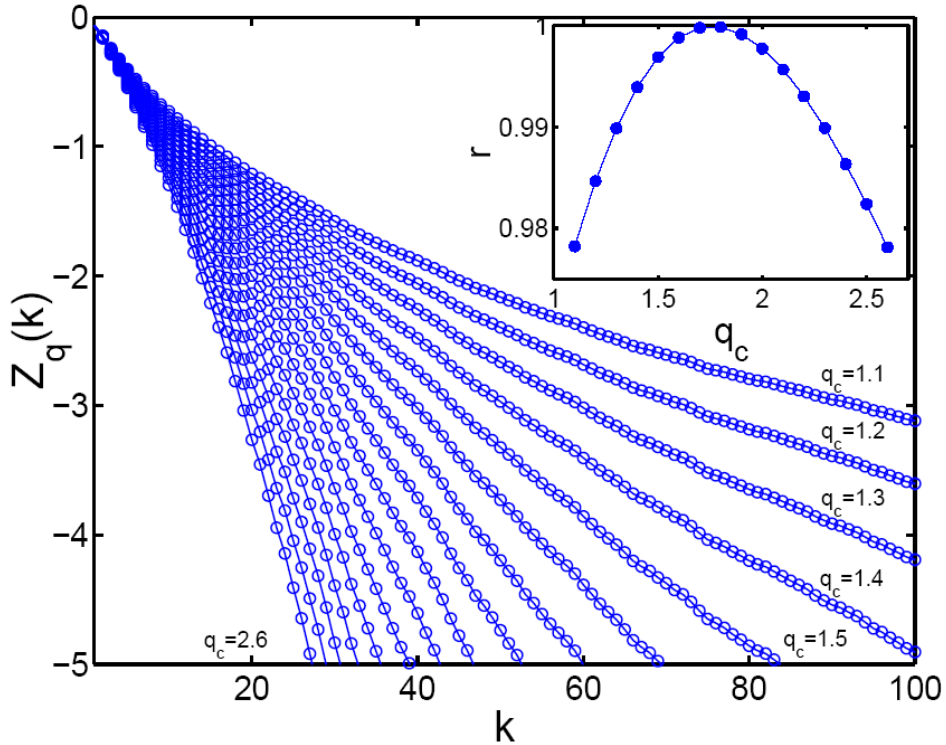
$(N = 2^7;\ \alpha = 0;\ r = 2)$



Degree of the most connected node — Degree of a randomly chosen node

$$(\alpha \to \infty \; ; \;\; <r>=8)$$



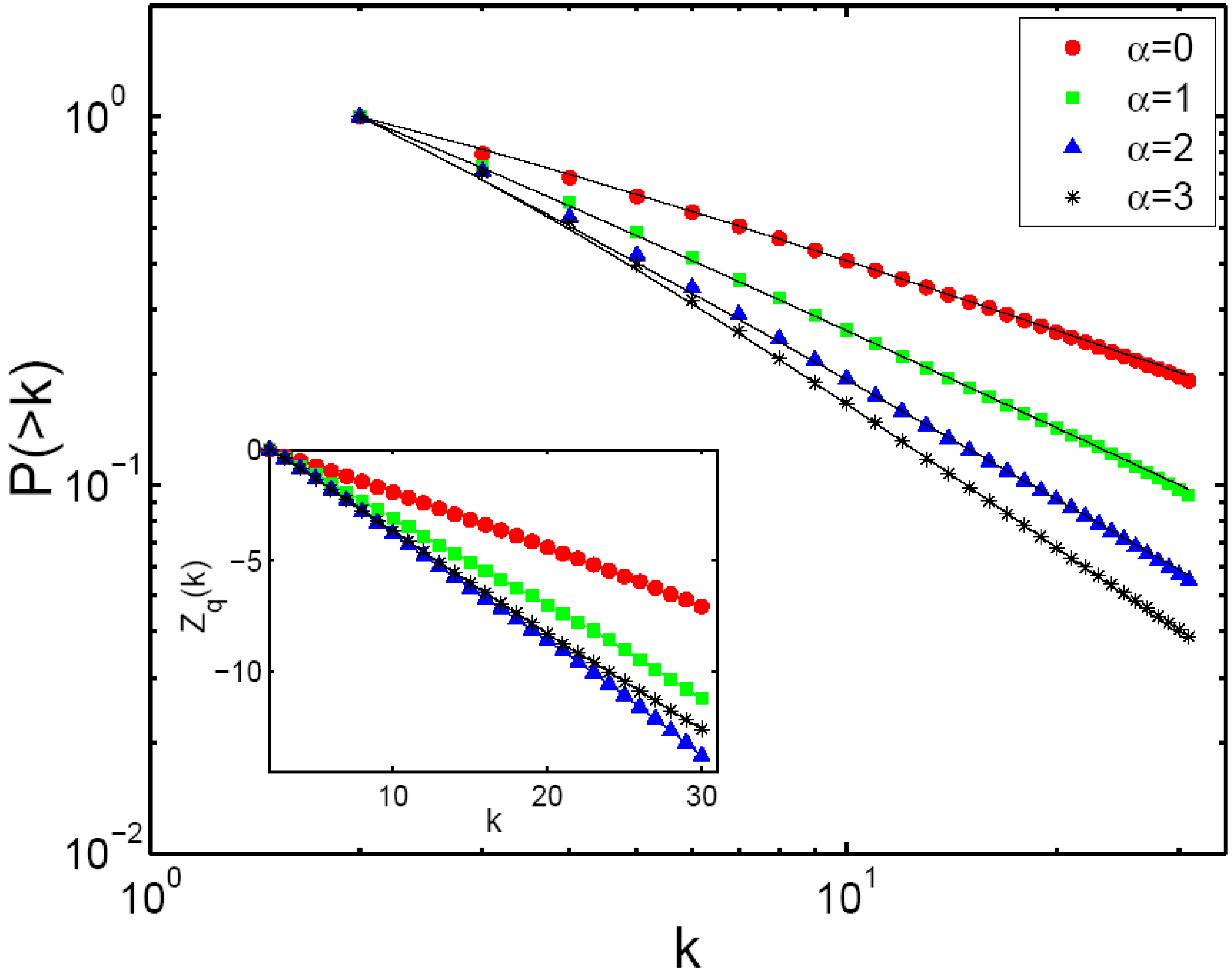


$(\alpha\to\infty;$

$$Z_q(k) \equiv \ln_q\left[P(>k)\right] \equiv \frac{\left[P(>k)\right]^{1-q}-1}{1-q}$$

$$(optimal\ q_c = 1.84)$$

S. Thurner and C. T., Europhys Lett **72**, 197 (2005)



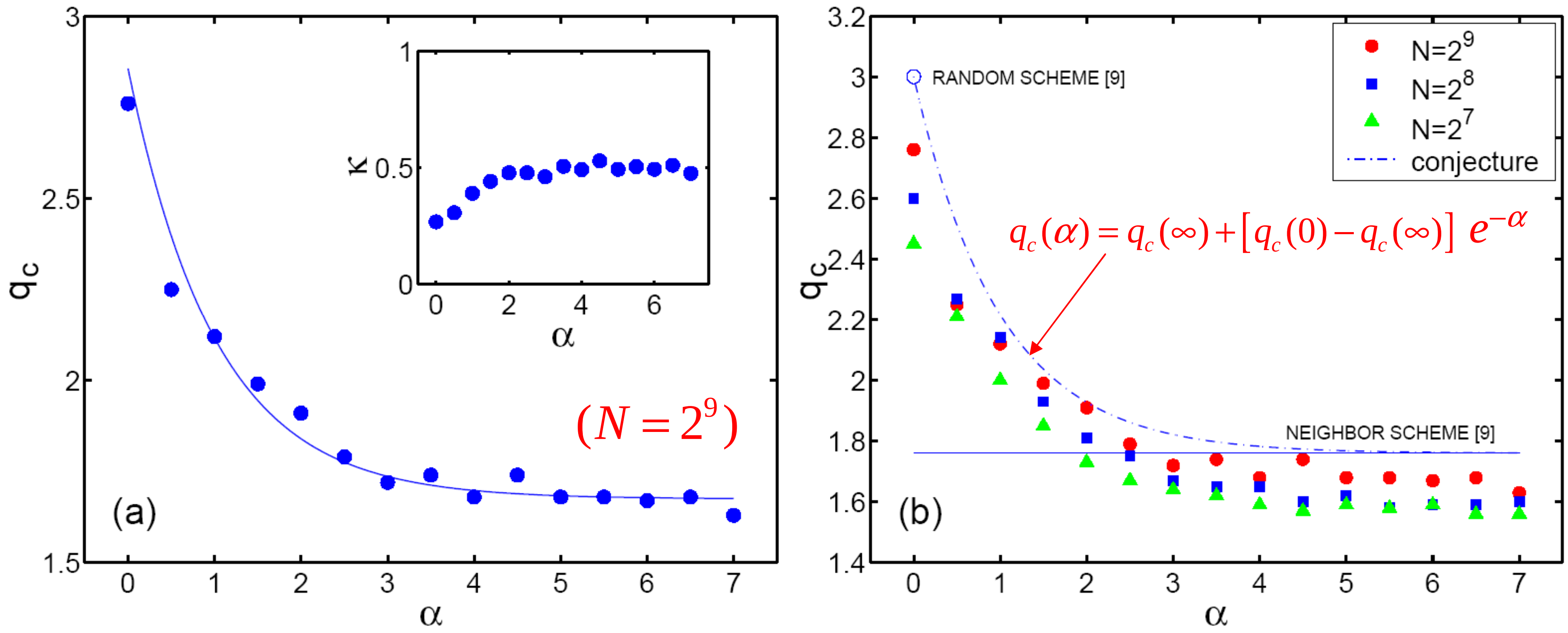
$(N = 2^9;\ r = 2)$

$$P(\geq k) = e_{q_c}^{\ -(k-2)/\kappa} \quad (k = 2,\ 3,\ 4, \ldots)$$

$$linear\ \ correlation \in [0.999901, 0.999976]$$

S. Thurner and C. T., Europhys Lett **72**, 197 (2005)

$(r = 2)$



S. Thurner and C. T., Europhys Lett **72**, 197 (2005)