Self-modulation of electromagnetic waves in non-Maxwellian plasmas

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Recently, a variety of "complex" plasmas has been modelled and analyzed via q / κ distributions and q-statistics

Complex plasmas:

- Dusty plasmas (dust can be strongly interacting)
- Open system
- Laboratory and space plasmas are not very hot: degree of ionization is low
- Nonthermal distributions
- Long-range interactions (≠ ideal plasmas)
- Ex: solar wind, plasma processing, fusion plasmas, ...

M. Maksimovic, V. Pierrard and J. F. Lemaire, Astron. Astrophys. **324**, 725 (1997) – solar wind

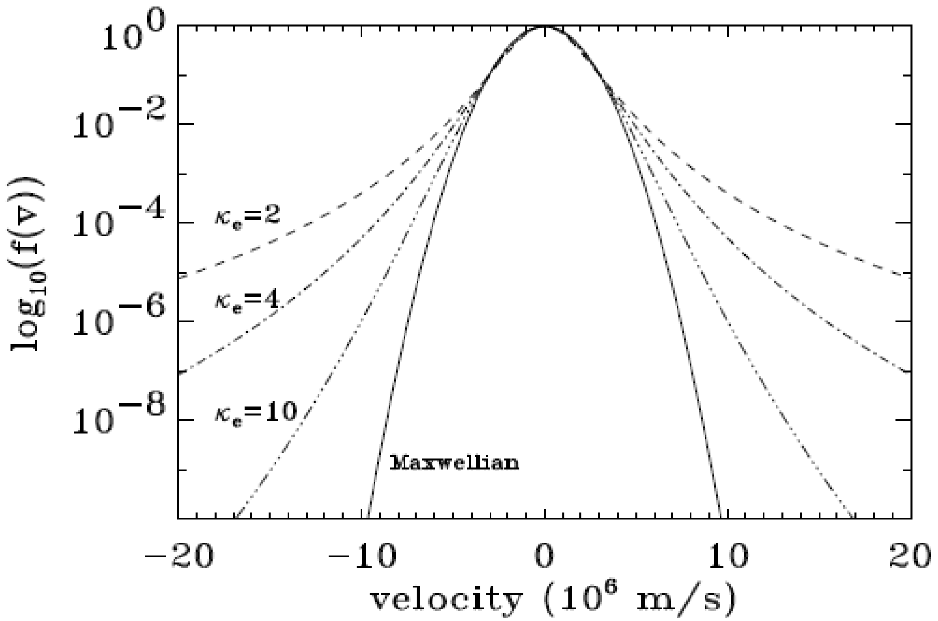


Fig. 2. Different examples of Kappa functions, all normalized to the same value at $v = 0$: $f(0) = 1$. One can see that in the limit $\kappa \to +\infty$, the functions degenerates to a Maxwellian or Gaussian function (solid line).

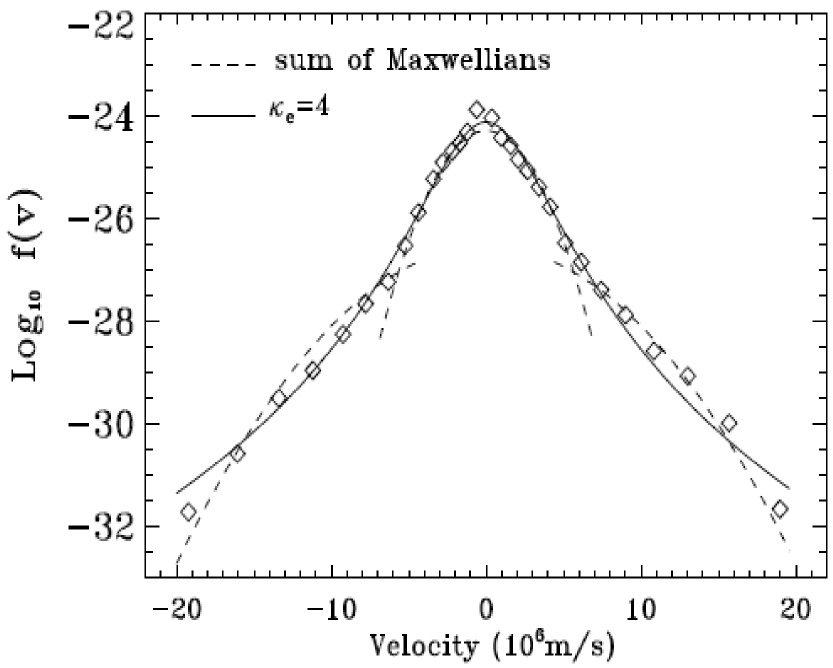
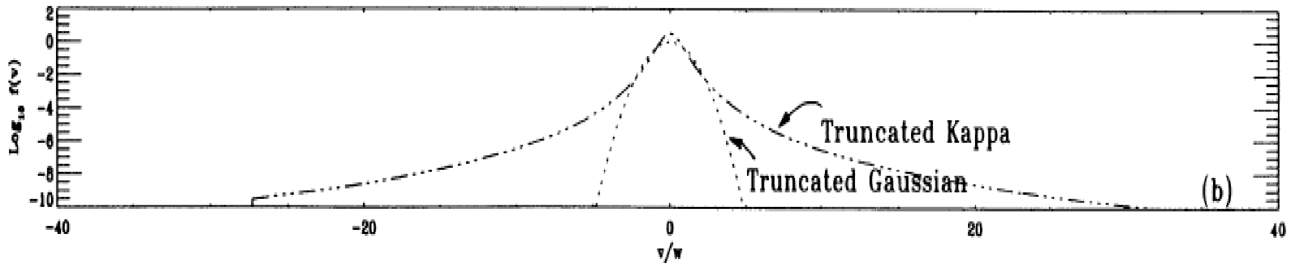


Fig. 3. Electron VDF in the solar wind (diamonds) as reported by Feldman et al. (1975). The dashed lines correspond to the classical model made of the sum of two Maxwellians: a core (n_c = 30.8 cm^{-3} and T_c = 1.6 10^5 K) and a halo (n_h = 2.2 cm^{-3} and T_h = 8.9 10^5 K). The full line represents our fit with a Kappa model (n = 33.9 cm^{-3}, T_κ = 1.9 10^5 K and κ_e = 4). Note that the Kappa model is is a more economic alternative since it needs one parameter less to fit.

$\kappa=4 \to q=1.25$

J. D. Scudder, Astrophys. J. **427**, 446 (1994) – solar corona



$1.37 < q_+ < 1.49$

$1.32 < q_- < 1.38$

B. Liu and J. Goree, Phys. Rev. Lett. **100**, 055003 (2008) – dusty plasma

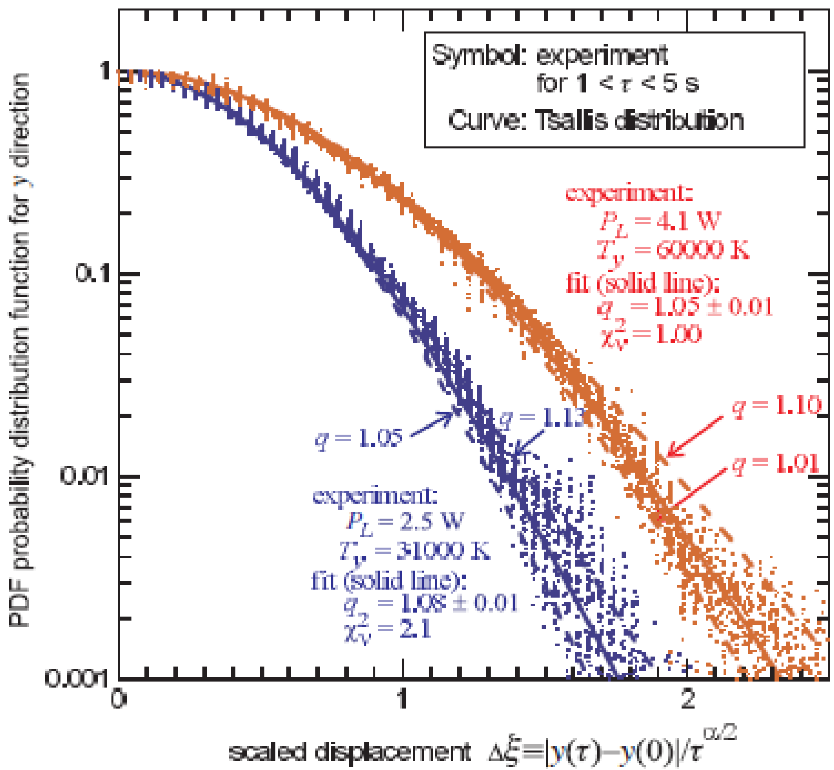


FIG. 3 (color online). PDF for the y direction, and the same conditions as in Fig. 2. Self-similarity is revealed by the collapse of PDFs at successive delays τ, with the scaled displacement $\Delta\xi \equiv |y(\tau) - y(0)|/\tau^{\alpha/2}$. Fitting to Eq. (1) yields the smooth solid curves and the measure of nonextensivity q. Other curves shown bracket the smooth curve for the fit.

A. Esquivel and A. Lazarian, Astrophys. J. **710**, 125 (2010) - ISM

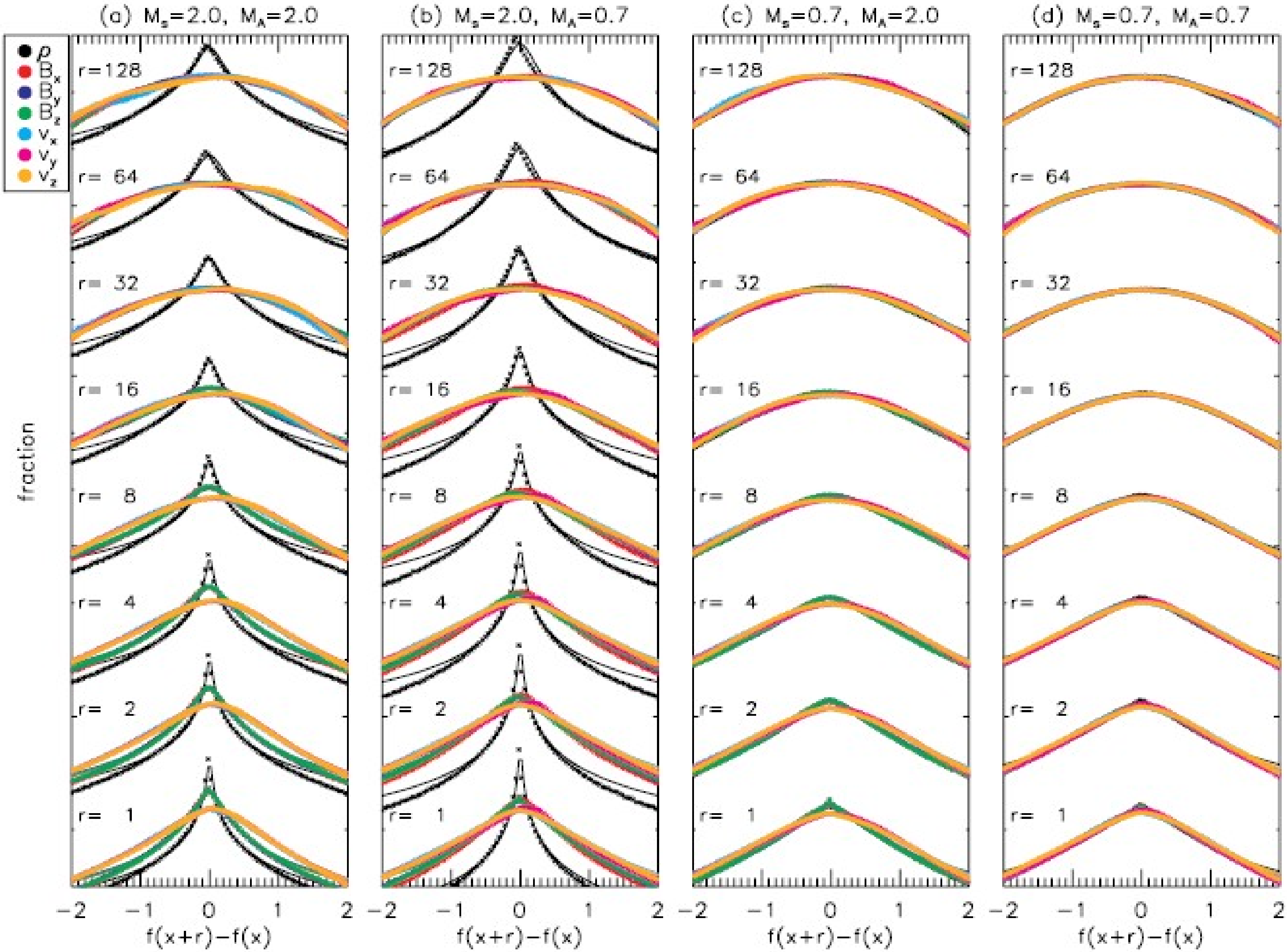


Figure 1. PDFs of fluctuations for different spatial lags r, for all the models. The symbols represent data from the numerical simulations and the lines are the fits with a q-Gaussian (Tsallis) distribution. The different MHD variables are color coded according to the legend at the upper left corner, and the value of the lag is indicated at the left of each set of curves.

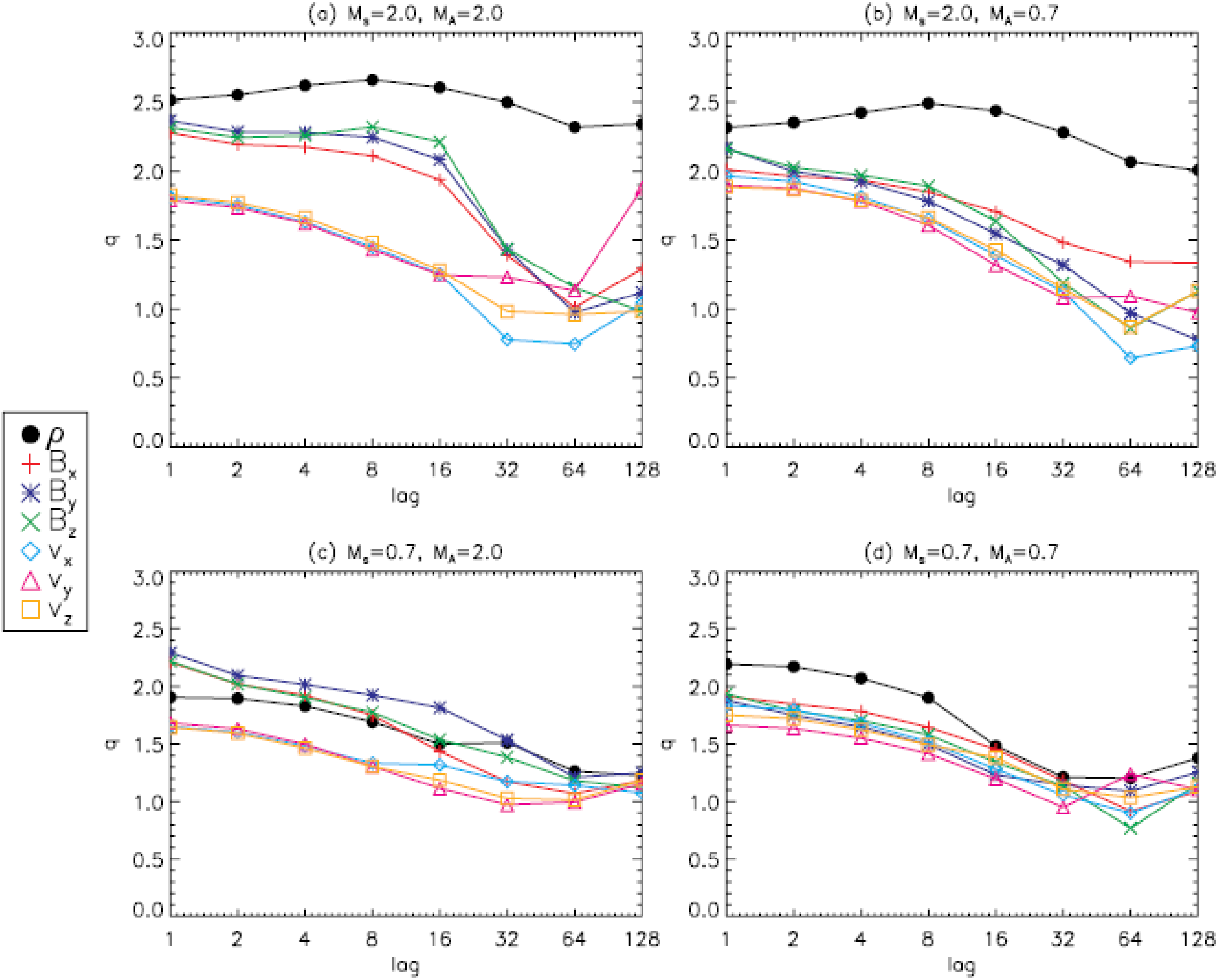
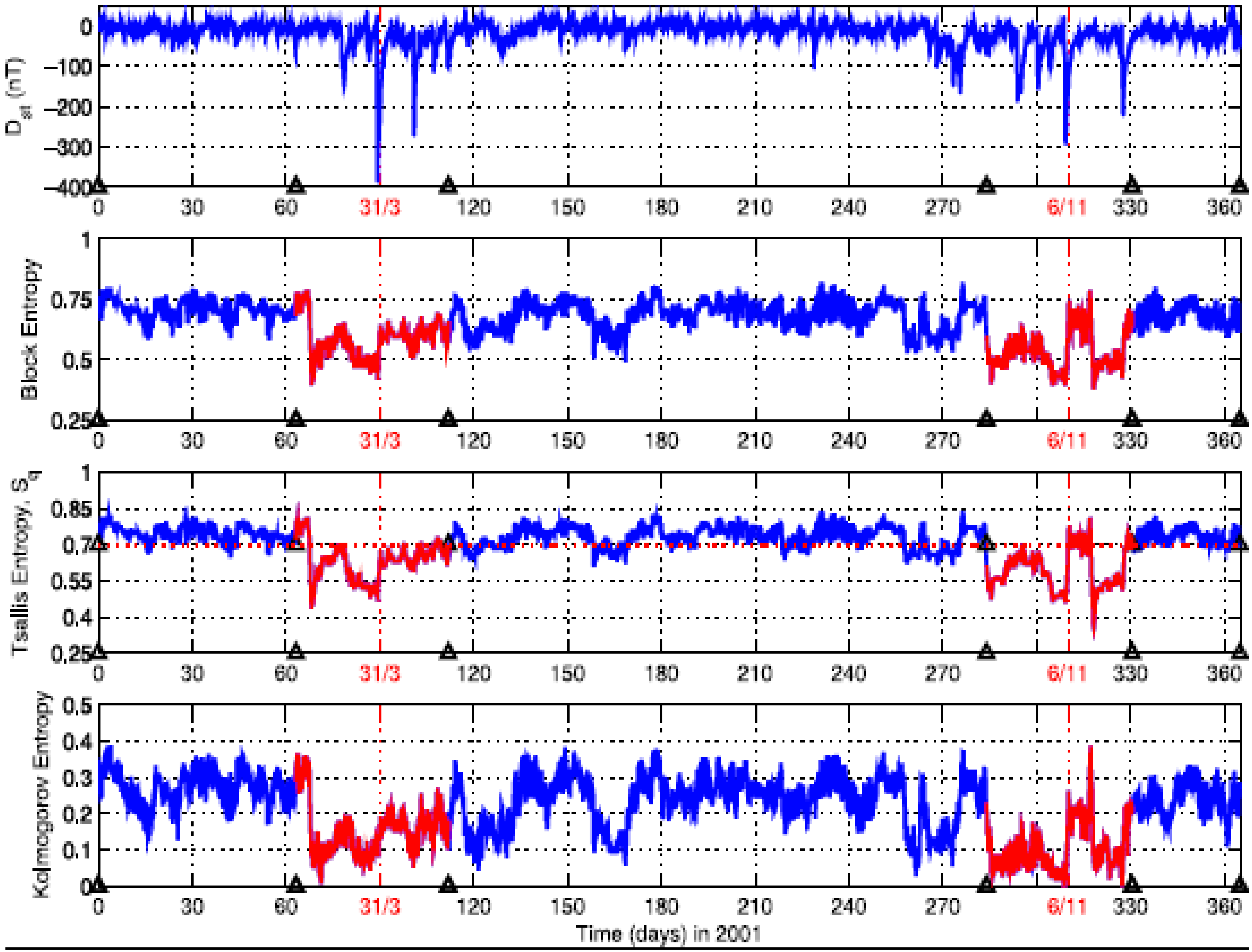


Figure 2. Values of the q parameter from the fits shown in Figure 1, the symbols (and colors) indicate the MHD variable used, according to the legend at the left of the plots.

(A color version of this figure is available in the online journal.)

G. Balasis *et al*., J. Geophys. Res. **114**, A00D06 (2009) - magnetosphere



Points of interest:

- Origin of nonthermal distributions: ???
- Modes/waves and instabilities in nonthermal plasmas

Summary of elementary plasma waves

EM character	oscillating species	conditions	dispersion relation	name
electrostatic	electrons	$\vec{B}_0 = 0$ or $\vec{k} \parallel \vec{B}_0$	$\omega^2 = \omega_p^2 + (3/2)k^2 v_{th}^2$	plasma oscillation (or Langmuir wave)
		$\vec{k} \perp \vec{B}_0$	$\omega^2 = \omega_p^2 + \omega_c^2 = \omega_h^2$	upper hybrid oscillation
	ions	$\vec{B}_0 = 0$ or $\vec{k} \parallel \vec{B}_0$	$\omega^2 = k^2 v_s^2 = k^2 \frac{\gamma_e K T_e + \gamma_i K T_i}{M}$	ion acoustic wave
		$\vec{k} \perp \vec{B}_0$ (nearly)	$\omega^2 = \Omega_c^2 + k^2 v_s^2$	electrostatic ion cyclotron wave
		$\vec{k} \perp \vec{B}_0$ (exactly)	$\omega^2 = [(\Omega_c \omega_c)^{-1} + \omega_i^{-2}]^{-1}$	lower hybrid oscillation
electromagnetic	electrons	$\vec{B}_0 = 0$	$\omega^2 = \omega_p^2 + k^2 c^2$	light wave
		$\vec{k} \perp \vec{B}_0,\ \vec{E}_1 \parallel \vec{B}_0$	$\frac{c^2 k^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2}$	O wave
		$\vec{k} \perp \vec{B}_0,\ \vec{E}_1 \perp \vec{B}_0$	$\frac{c^2 k^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2} \frac{\omega^2 - \omega_p^2}{\omega^2 - \omega_h^2}$	X wave
		$\vec{k} \parallel \vec{B}_0$ (right circ. pol.)	$\frac{c^2 k^2}{\omega^2} = 1 - \frac{\omega_p^2/\omega^2}{1 - (\omega_c/\omega)}$	R wave (whistler mode)
		$\vec{k} \parallel \vec{B}_0$ (left circ. pol.)	$\frac{c^2 k^2}{\omega^2} = 1 - \frac{\omega_p^2/\omega^2}{1 + (\omega_c/\omega)}$	L wave
	ions	$\vec{B}_0 = 0$		none
		$\vec{k} \parallel \vec{B}_0$	$\omega^2 = k^2 v_A^2$	Alfvén wave
		$\vec{k} \perp \vec{B}_0$	$\frac{\omega^2}{k^2} = c^2 \frac{v_s^2 + v_A^2}{c^2 + v_A^2}$	magnetosonic wave

Self-modulation of electromagnetic waves in non-Maxwellian plasmas:

- Particle acceleration (laboratory and space), harmonic generation, etc ...
- Circularly polarized EM waves: SM associated with density fluctuations in the magnetosphere
- Linearly polarized EM waves: propagation is more complex since we have harmonic generation

Propagation of linearly polarized EM waves:

- Harmonics: longitudinal oscillations generated by the ponderomotive force of the EM wave
- Coupling in two frequency scales: electron oscillations and ion-acoustic waves

Simple model to investigate the SM of linearly polarized EM waves in nonthermal plasmas (effect of nonthermality)

- Electron-ion plasma
- Nonthermal velocity distributions (Maxwellian: $\kappa \to \infty$):

$$f_\kappa(v) = \frac{1}{\left(\pi \kappa \theta^2\right)^{3/2}} \frac{\Gamma(\kappa + 1)}{\Gamma(\kappa - 1/2)} \left(1 + \frac{v^2}{\kappa \theta^2}\right)^{-(\kappa+1)}$$

$$\theta = [(\kappa - 3/2)/\kappa]^{1/2} v_T, \kappa > 3/2, -\kappa = 1/(1-q)$$

V. M. Vasyliunas, J. Geophys. Res. **73**, 2839 (1968); C. Tsallis, J. Stat. Phys. **52**, 479 (1988); C. Tsallis, Physica A **221**, 277 (1995).

For the electrons:

$$f_\kappa(v_e) = \frac{n_0}{\left(\pi\kappa_e\theta_e^2\right)^{3/2}} \frac{\Gamma(\kappa_e+1)}{\Gamma(\kappa_e-1/2)} \left(1+\frac{v_e^2-2e\varphi/m_e}{\kappa_e\theta_e^2}\right)^{-(\kappa_e+1)},$$

$$n_e(\phi) = n_0\left[1-\frac{\phi}{(\kappa_e-3/2)}\right]^{-(\kappa_e-1/2)},$$

$$p_e(\phi) = p_0\left[1-\frac{\phi}{(\kappa_e-3/2)}\right]^{-(\kappa_e-3/2)},$$

$$\phi = e\varphi/k_BT_e, p_0 = n_0k_BT_e, \varphi = \varphi_{sc}+\varphi_p$$

For the ions:

$$f_\kappa(v_i) = \frac{n_0}{\left(\pi \kappa_i \theta_i^2\right)^{3/2}} \frac{\Gamma(\kappa_i + 1)}{\Gamma(\kappa_i - 1/2)} \left(1 + \frac{v_i^2 + 2e\varphi_{sc}/m_i}{\kappa_i \theta_i^2}\right)^{-(\kappa_i + 1)},$$

$$n_i(\phi_{sc}) = n_0 \left[1 + \frac{\phi_{sc}}{(\kappa_i - 3/2)}\right]^{-(\kappa_i - 1/2)},$$

$$p_i(\phi_{sc}) = p_0 \gamma_{ie} \left[1 + \frac{\phi_{sc}}{\gamma_{ie}(\kappa_i - 3/2)}\right]^{-(\kappa_i - 3/2)},$$

$$\gamma_{ie} = T_i / T_e, \phi_{sc} = e\varphi_{sc} / k_B T_e$$

We consider weak nonlinear waves (Φ << 1):

$$n_e = n_0(1 + \alpha_0\phi + \alpha_1\phi^2 + \alpha_2\phi^3 \dots),$$

$$p_e = p_0(1 + \phi + \beta_0\phi^2 + \beta_1\phi^3 \dots),$$

$$n_i = n_0\left[1 - \eta_0\frac{\phi_{sc}}{\gamma_{ie}} + \eta_1\left(\frac{\phi_{sc}}{\gamma_{ie}}\right)^2 - \eta_2\left(\frac{\phi_{sc}}{\gamma_{ie}}\right)^3 \dots\right],$$

$$p_i = p_0\gamma_{ie}\left[1 - \frac{\phi_{sc}}{\gamma_{ie}} + \mu_0\left(\frac{\phi_{sc}}{\gamma_{ie}}\right)^2 - \mu_1\left(\frac{\phi_{sc}}{\gamma_{ie}}\right)^3 \dots\right],$$

$$\alpha_0 = (\kappa_e - 1/2)/(\kappa_e - 3/2), \alpha_1 = [(\kappa_e - 1/2)(\kappa_e + 1/2)]/[2(\kappa_e - 3/2)^2],$$

$$\beta_0 = (\kappa_e - 1/2)/[2(\kappa_e - 3/2)], \eta_0 = (\kappa_i - 1/2)/(\kappa_i - 3/2)$$

$$\eta_1 = [(\kappa_i - 1/2)(\kappa_i + 1/2)]/[2(\kappa_i - 3/2)^2], \mu_0 = (\kappa_i - 1/2)/[2(\kappa_i - 3/2)].$$

Fluid and Maxwell´s equations:

- All quantities vary only with z
- EM wave propagating in the z direction with E_x and B_y

$$\frac{\partial E_x}{\partial z} = -\frac{1}{c}\frac{\partial B_y}{\partial t},$$

$$\frac{\partial B_y}{\partial z} = \frac{4\pi e}{c} n_e v_{ex} - \frac{1}{c}\frac{\partial E_x}{\partial t},$$

$$E_x = -(1/c)\partial A_x / \partial t,$$

$$B_y = \partial A_x / \partial z,$$

$$\frac{\partial E_z}{\partial z} = -4\pi e\left(n_e - n_i\right),$$

$$m_e\left(\frac{\partial}{\partial t} + v_{ez}\frac{\partial}{\partial z}\right)v_{ex} = -eE_x + \frac{e}{c}v_{ez}B_y$$

$$m_e n_e\left(\frac{\partial}{\partial t} + v_{ez}\frac{\partial}{\partial z}\right)v_{ez} = -en_eE_z - \frac{e}{c}n_e v_{ex}B_y - \frac{\partial p_e}{\partial z}$$

$$m_i n_i\left(\frac{\partial}{\partial t} + v_{iz}\frac{\partial}{\partial z}\right)v_{iz} = en_iE_z - \frac{\partial p_i}{\partial z}$$

$$\frac{\partial n_e}{\partial t} + \frac{\partial (n_e v_{ez})}{\partial z} = 0, \frac{\partial n_i}{\partial t} + \frac{\partial (n_i v_{iz})}{\partial z} = 0.$$

Perturbation analysis and nonlinear Schrödinger equation:

- Krylov-Bogoliubov-Mitropolsky method for nonlinear wave modulation (T. Kakutani and N. Sugimoto, Phys. Fluids 17, 1617 (1974))
- All physical quantities are considered weakly nonlinear waves:

$$f = f_0 + \varepsilon f_1(a, a^*, \psi) + \varepsilon^2 f_2(a, a^*, \psi) + \varepsilon^3 f_3(a, a^*, \psi) + \ldots ,$$

- For the transverse wave:

$$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) A_x = \frac{\omega_{pe}^2}{c^2} n_e A_x ,$$

$$A_x \equiv eA_x / m_e c^2 , n_e \equiv n_e / n_0$$

- Longitudinal direction (coupling with electron oscillations):

$$\left(\frac{\partial^2}{\partial t^2} - c_{se}^2 \frac{\partial^2}{\partial z^2}\right) n_e + \omega_{pe}^2 (n_e - 1) = \frac{c^2}{2} \frac{\partial^2 A_x^2}{\partial z^2},$$

$$c_{se} = \left[\frac{1}{m_e}\left(\frac{dp_e}{dn_e}\right)_{\phi=0}\right]^{1/2} = \left[\frac{k_B T_e (\kappa_e - 3/2)}{m_e (\kappa_e - 1/2)}\right]^{1/2}.$$

- We start with

$$A_{x1} = a e^{i\psi} + a^* e^{-i\psi},$$

and after a hard work we get ...

$$\left[i\left(A_2 + v_g B_2 \right) + \frac{1}{2}\frac{dv_g}{dk}\left(B_1 \frac{\partial B_1}{\partial a} + B_1^* \frac{\partial B_1}{\partial a^*} \right) - \frac{\omega_{pe}^2}{2\omega} n_{e2} a \right] e^{i\psi}$$

$$+ \frac{c^2}{2\omega}\left(\frac{\partial^2 A_{x3}}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 A_{x3}}{\partial t^2} - \frac{\omega_{pe}^2}{c^2} A_{x3} + \right) + c.c. = 0.$$

with (valid for the coupling with the electron oscillations)

$$n_{e2} = \frac{2c^2k^2}{\left(4\omega^2 - 4k^2c_{se}^2 - \omega_{pe}^2 \right)} a^2 e^{2i\psi} + c.c..$$

- For A_{x3} to be secular-free:

$$i\left(A_2+v_gB_2\right)+\frac{1}{2}\frac{dv_g}{dk}\left(B_1\frac{\partial B_1}{\partial a}+B_1^*\frac{\partial B_1}{\partial a^*}\right)-\frac{\omega_{pe}^2c^2k^2}{\omega\left(4\omega^2-4k^2c_{se}^2-\omega_{pe}^2\right)}|a|^2a=0.$$

With the coordinate transformation

$$\xi=\frac{1}{\varepsilon}\left(z_2-v_gt_2\right)=z_1-v_gt_1=\varepsilon\left(z-v_gt\right),\tau=t_2=\varepsilon t_1=\varepsilon^2t$$

we get the NLS equation,

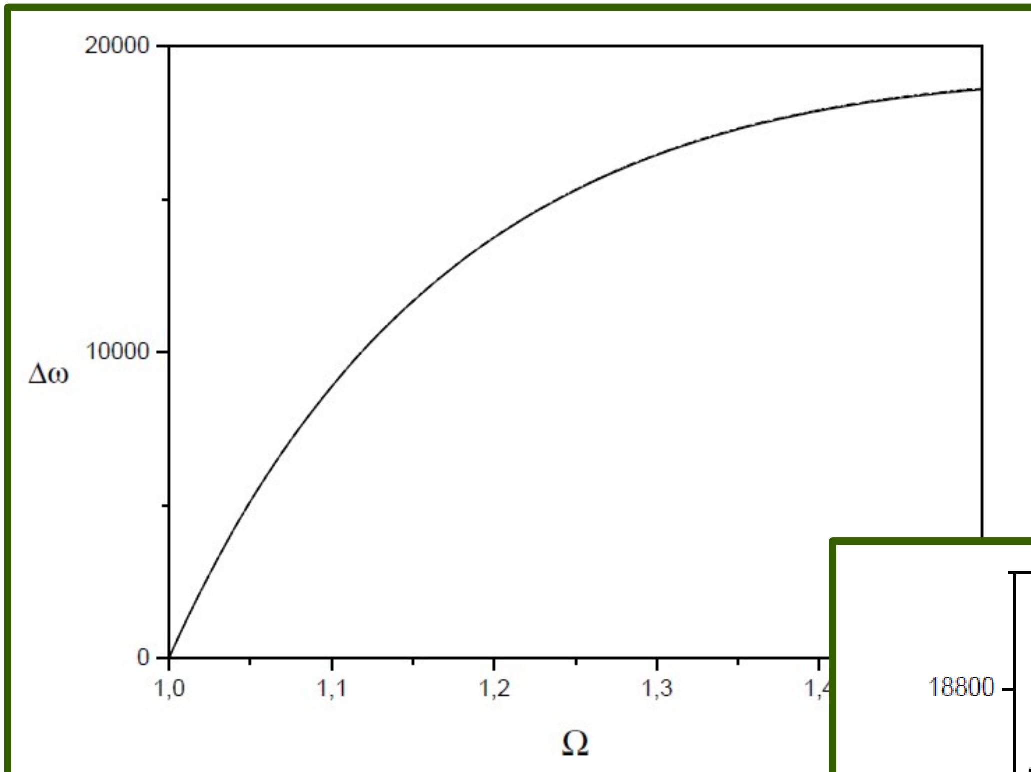
$$i\frac{\partial a}{\partial\tau}+P\frac{\partial^2a}{\partial\xi^2}+Q|a|^2a=0,$$

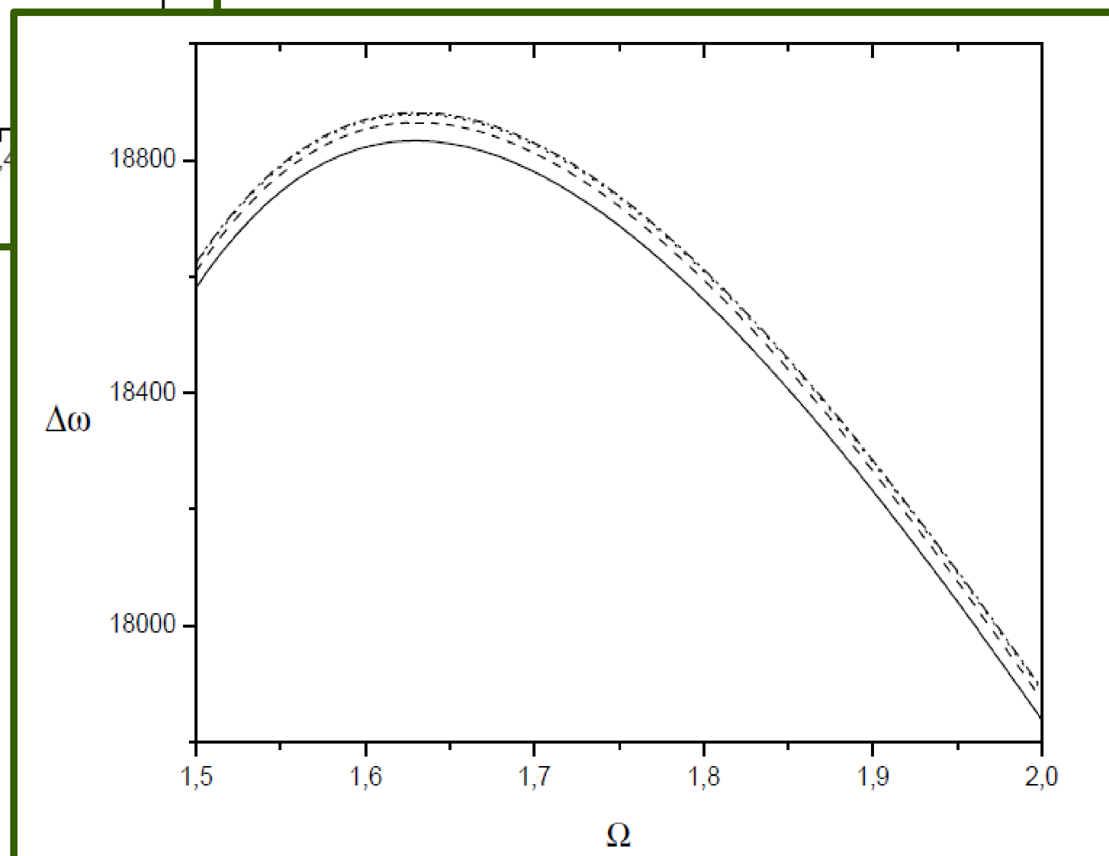
with

$$P = \frac{1}{2}\frac{dv_g}{dk} = \frac{c^2\omega_{pe}^2}{2\omega^3}, Q = -\frac{\omega_{pe}^2 c^2 k^2}{\omega\left(4\omega^2 - 4k^2c_{se}^2 - \omega_{pe}^2\right)}.$$

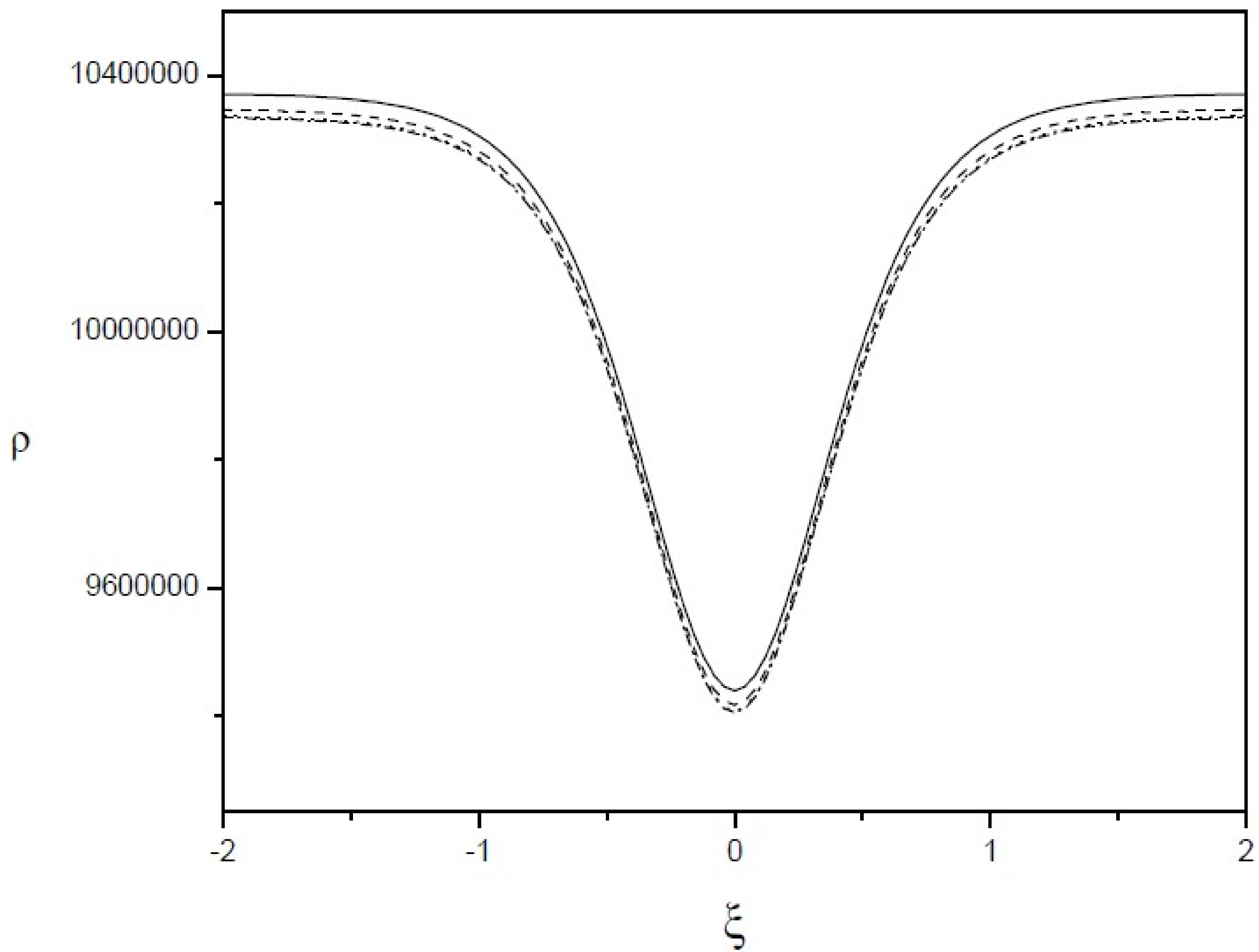
Nonlinear frequency shifts and envelope holes:

- EM wave envelope is always stable: Q/P <0, dispersion and Δω=-Q are always positive
- The effect of superthermal electrons appears in Δω: small effect for high temperatures and intermediate frequencies
- NLS equation admits localized solutions in the form of envelope solitons: Q/P<0 – envelope holes (effect only for gray solitons)
- The effect of the ions is negligible





Maxwellian: dot-dashed line


10400000
10000000
9600000
ρ
-2
-1
0
1
2
ξ

The effect of electron nonthermality in the self-modulation of linearly polarized EM waves appears only for high temperatures and intermediate frequencies. Superthermal electrons have no effect on the stability of the wave envelope, but tend to decrease the nonlinear frequency shift and increase the amplitude of gray solitons. The effect of ion nonthermality is negligible.