Distribuições de Velocidades no Ferromagneto de Heisenberg Inercial

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- Fundamental Hypothesis: Ergodicity

➡ Time Averages $\equiv$ Ensemble Averages

- Maximum Lyapunov Exponent:

$$\lambda_{\max} = \lim_{t\to\infty} \frac{1}{t} \ln\left[\frac{d(t)}{d(0)}\right]$$

$\lambda_{\max} > 0 \;\Rightarrow$ Chaos

- Dynamical Approaches:
(e.g., Numerical Simulations)

- Order of limits $t \to \infty \ , \ N \to \infty$

  is irrelevant for ergodic systems

- Non-Ergodic Systems: ??

Good Systems to Investigate:

- Long-Range Interactions
- Should present their own dynamics
- Examples: Systems of Rotors

➡ Present investigation:

Infinite-Range Heisenberg Rotors

Motivation:
Anomalies in the dynamics of the inertial infinite-range-interaction XY ferromagnet

- Metastable / Quasi-Stationary State (QSS)

$$u \approx u_c \quad :$$

$$t_{QSS} \sim N \;; \quad \lambda_{\max} \sim N^{-\kappa} \;; \quad (\kappa > 0)$$

- The Model:

$$H = K + V$$

$$H = \frac{1}{2}\sum_{i=1}^{N}\sum_{\mu=1}^{n} L_{i\mu}^2 + \frac{1}{2N}\sum_{i,j=1}^{N}(1 - \vec{S}_i.\vec{S}_j)$$

- Normalization: $\sum_{\mu=1}^{n} S_{i\mu}^2 = 1 \quad (\forall i)$

- n=2: XY rotors
- n=3: Heisenberg rotors

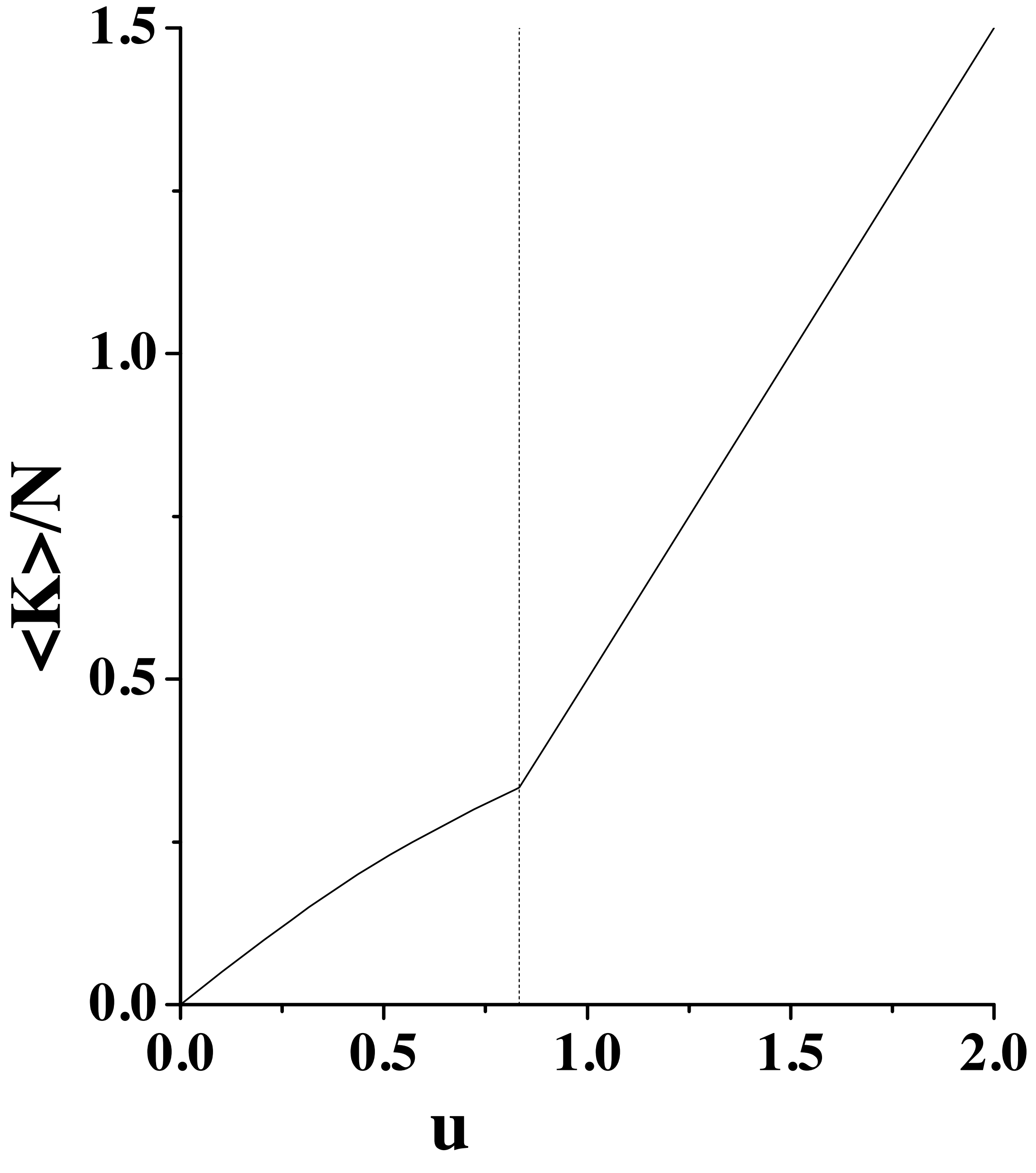
- Canonical Ensemble Solution:

$$u = \frac{n-1}{2\beta} + \frac{1}{2}(1 - \vec{m}^2)$$

$$m \equiv |\vec{m}| = \frac{I_{n/2}(\beta m)}{I_{(n-2)/2}(\beta m)}$$

$$T_c = \frac{1}{n} \; ; \quad u_c = 1 - \frac{1}{2n} \qquad (k_B = 1)$$

$$n = 3 \; \rightarrow \; u_c = \frac{5}{6}$$



1.5
1.0
0.5
0.0
<K>/N
0.0
0.5
1.0
1.5
2.0
u

- Dynamics of the Inertial Heisenberg

$$\dot{\vec{L}}_i = \vec{S}_i \times \left(\frac{1}{N} \sum_{j=1}^{N} \vec{S}_j \right) \qquad (i = 1, 2, \cdots, N)$$

$$\dot{\vec{S}}_i = \vec{L}_i \times \vec{S}_i$$

➡ 6N Equations

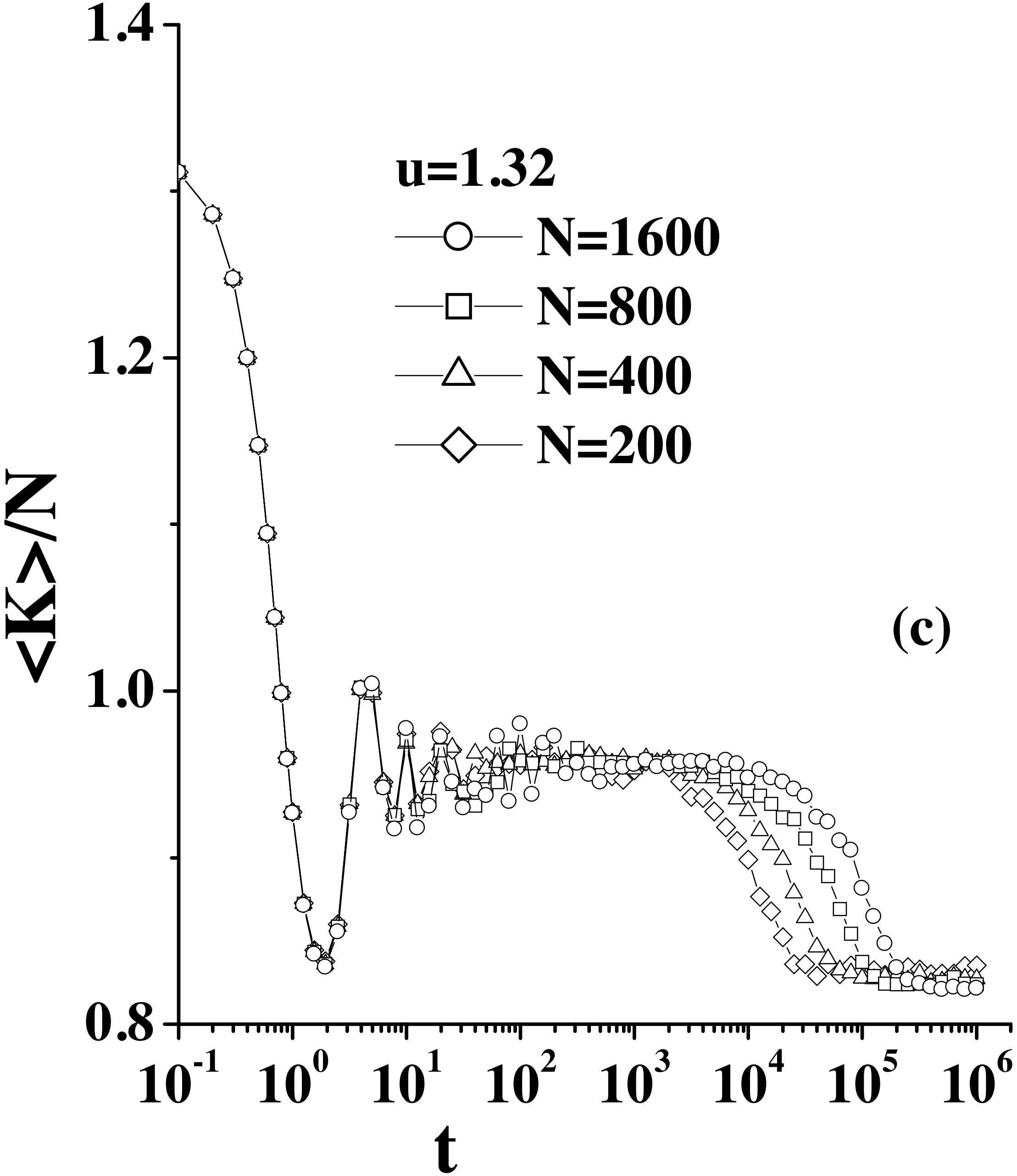
- 4th order Runge-Kutta-Merson Integrator
- Preserve: Total Energy and Spin Norm.

F. D. Nobre and C. Tsallis, Phys. Rev. E 68, 036115 (2003).

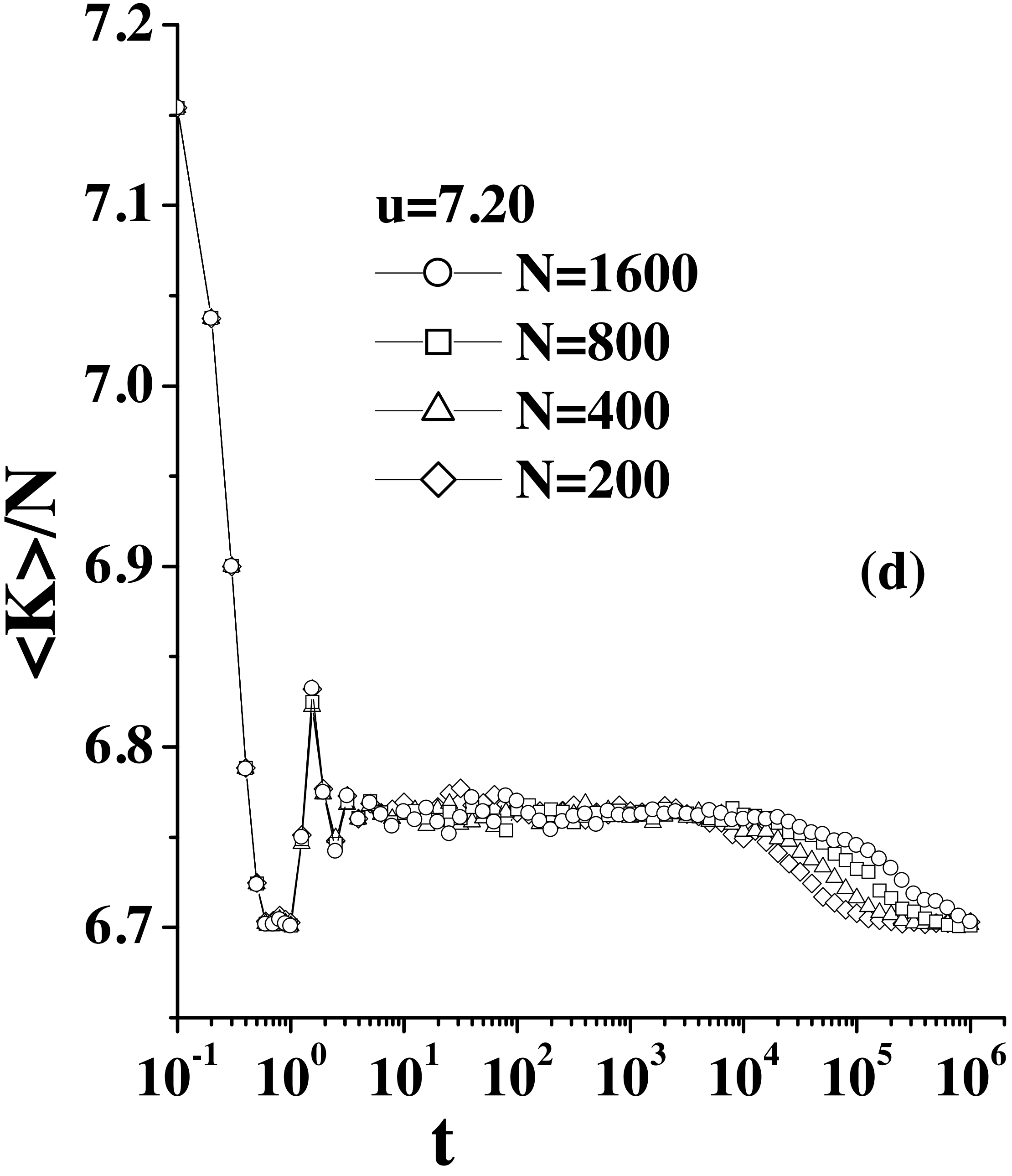
- Initial Magnetization: $m_z(0) = 1$

- Initial Angular Momenta: Equal "Water Bags" for the Components $(\mu = x, y, z)$

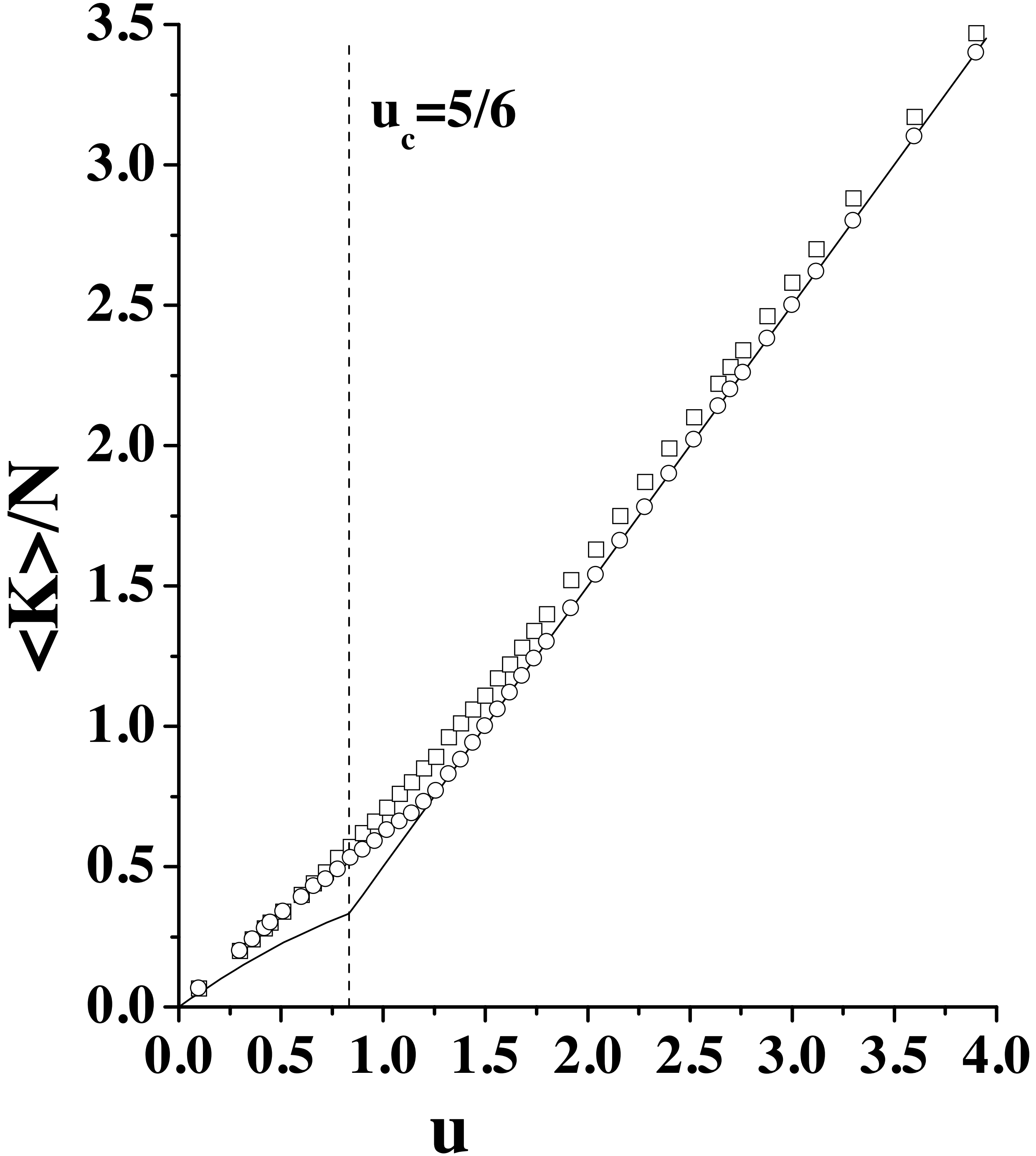
➡ Similar MS for wider range of energies



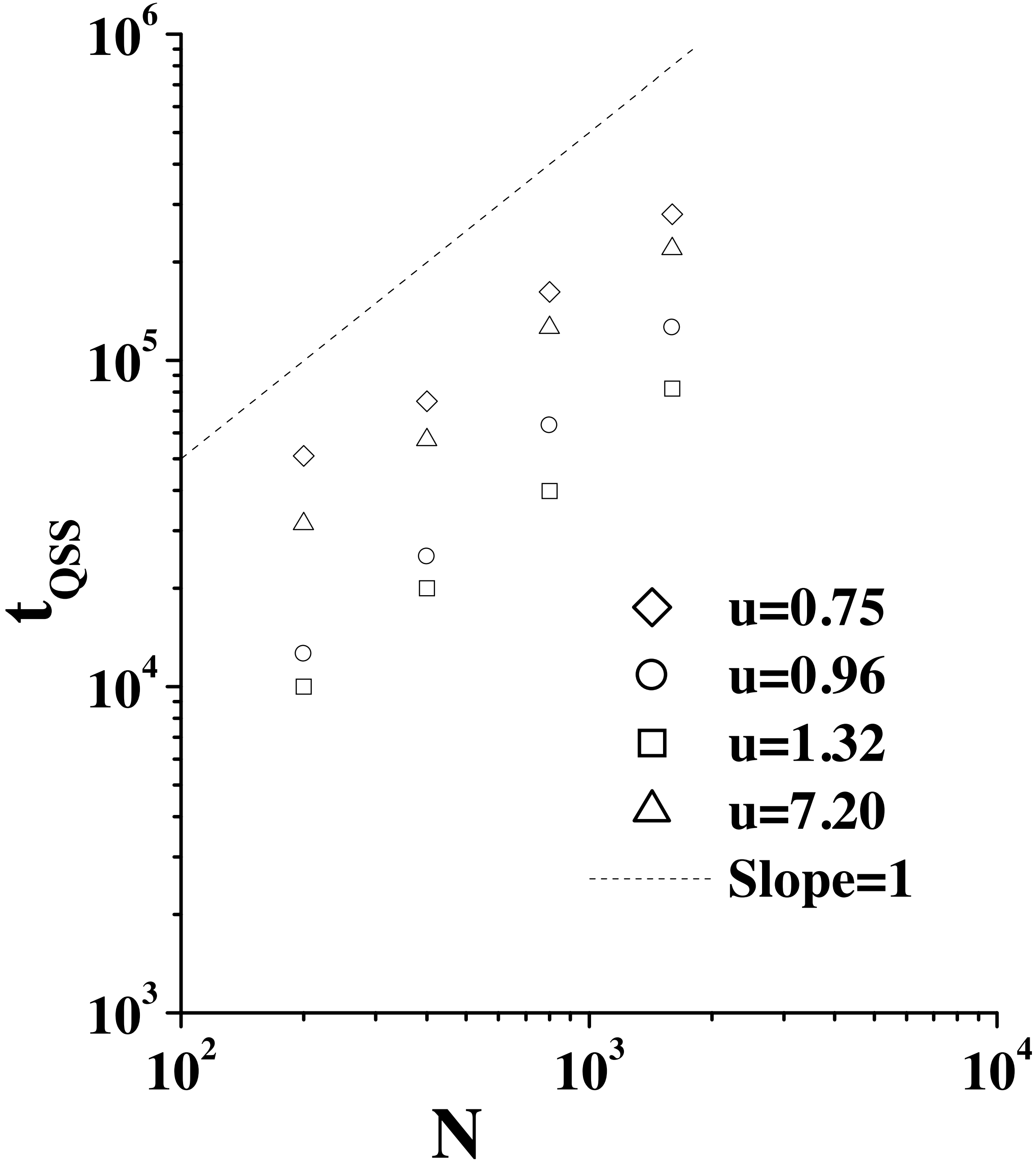
1.4
1.2
1.0
0.8
<K>/N
u=1.32
N=1600
N=800
N=400
N=200
(c)
10^-1 10^0 10^1 10^2 10^3 10^4 10^5 10^6
t



7.2
7.1
7.0
6.9
6.8
6.7
<K>/N
u=7.20
N=1600
N=800
N=400
N=200
(d)
10^{-1} 10^{0} 10^{1} 10^{2} 10^{3} 10^{4} 10^{5} 10^{6}
t



u_c=5/6
<K>/N
u
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0

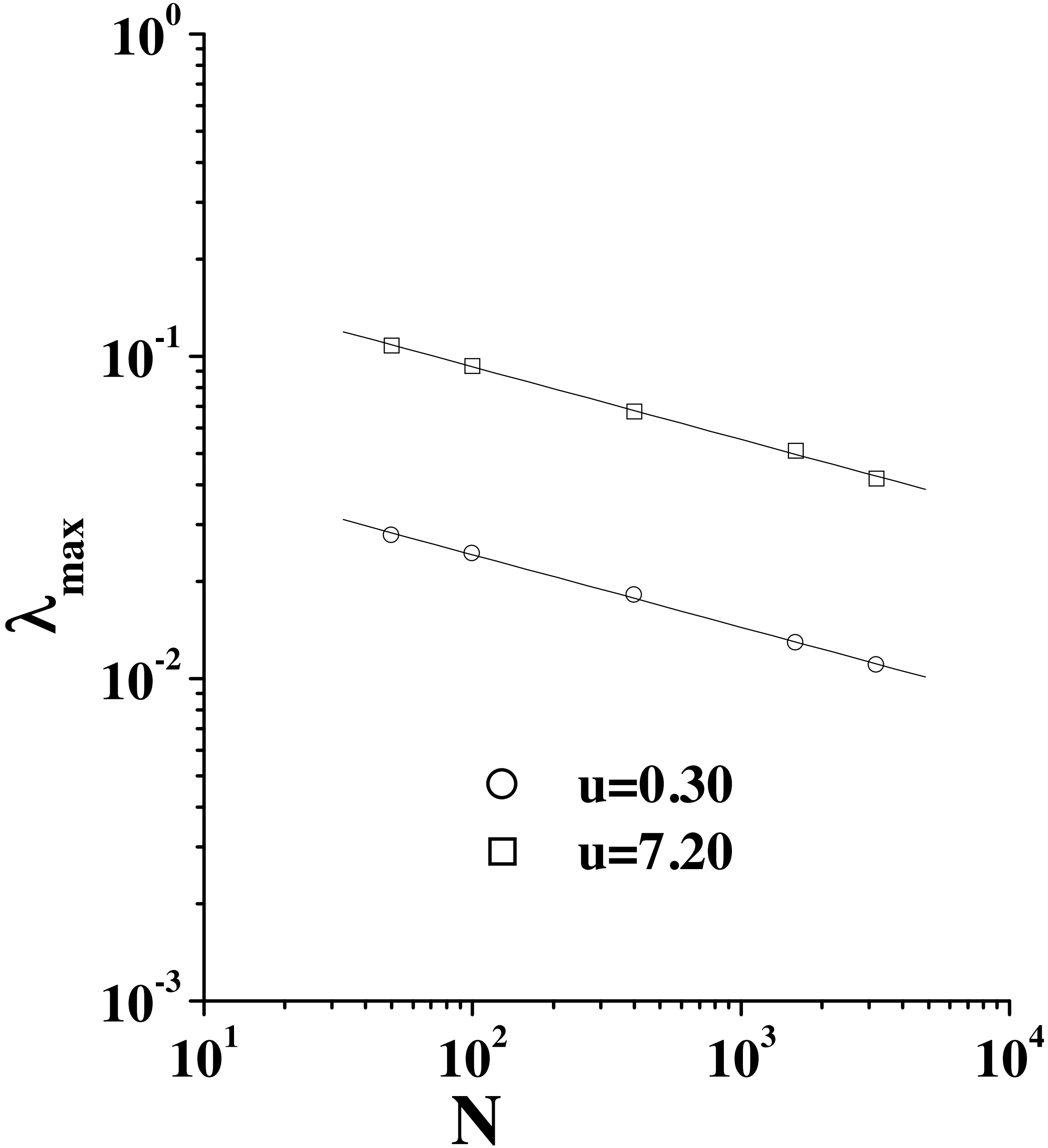


10^6
10^5
10^4
10^3
t_{QSS}
10^2
10^3
10^4
N
u=0.75
u=0.96
u=1.32
u=7.20
Slope=1

- Maximum Lyapunov Exponent

$$\lambda_{\max} = \lim_{t\to\infty} \frac{1}{t} \ln\left[\frac{d(t)}{d(0)}\right]$$

$$d(t) = \left\{ \sum_{i=1}^{N} \sum_{\mu=x,y,z} \left[(\delta L_{i\mu})^2 + (\delta S_{i\mu})^2\right] \right\}^{1/2}$$



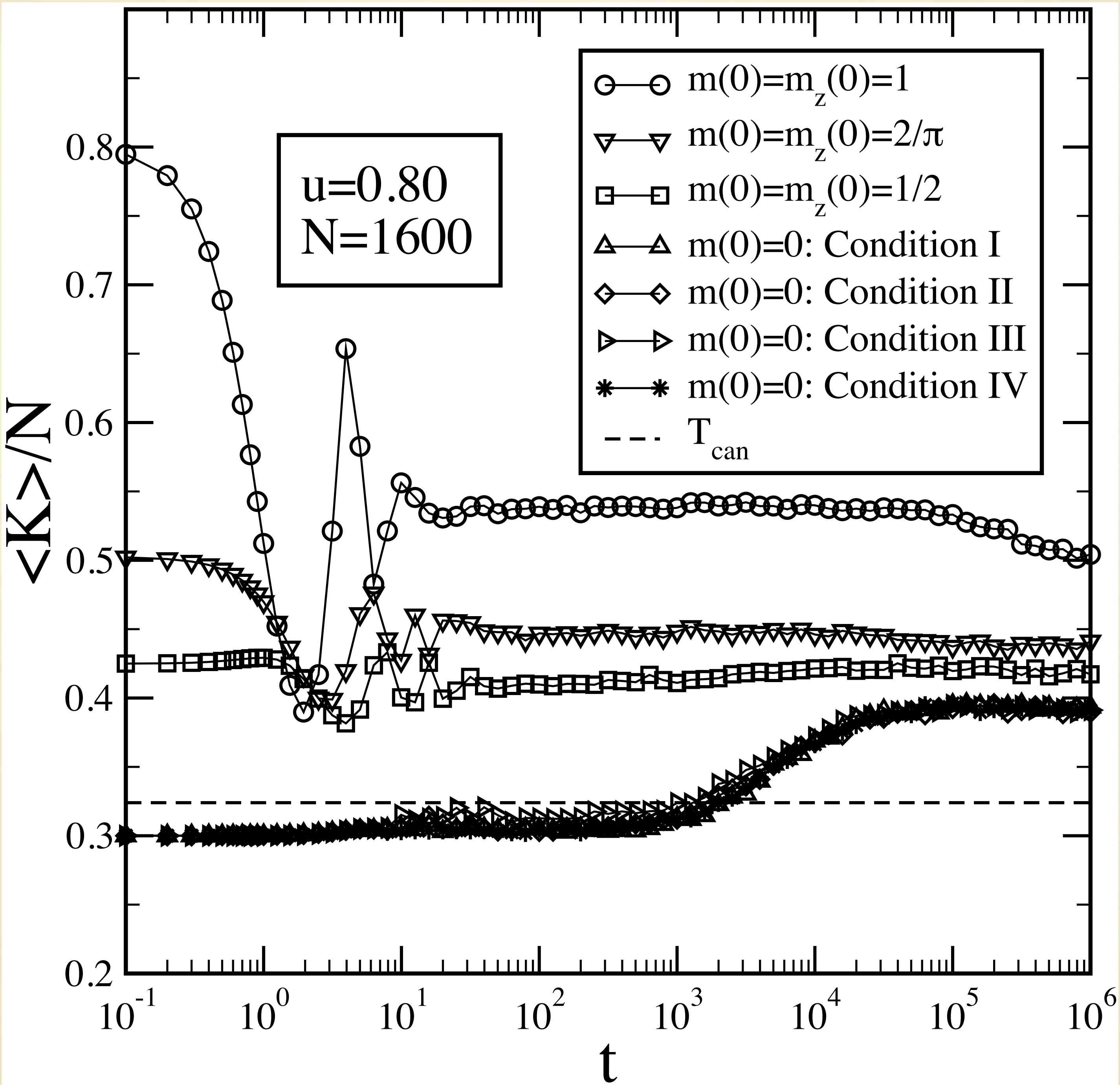
10^0
10^{-1}
10^{-2}
10^{-3}
λ_{max}
u=0.30
u=7.20
10^1
10^2
10^3
10^4
N

F. D. Nobre and C. Tsallis, Physica A 344, 587 (2004).

- Role of Initial Conditions on MS:

i) Different Initial Magnetizations; Equal "Water Bags" for the Angular-Momenta Components

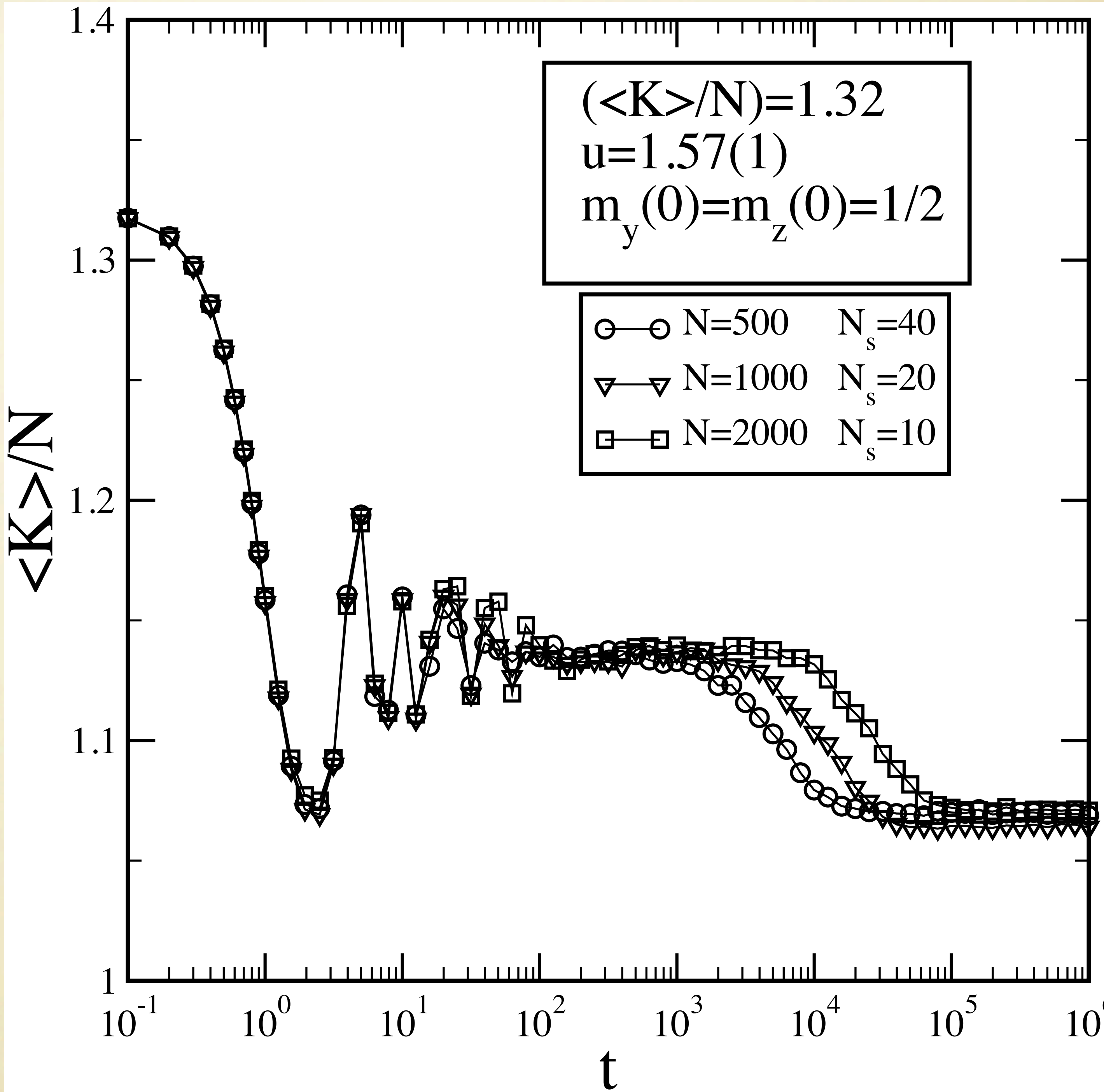
→ Distinct MS



u=0.80
N=1600
m(0)=m_z(0)=1
m(0)=m_z(0)=2/π
m(0)=m_z(0)=1/2
m(0)=0: Condition I
m(0)=0: Condition II
m(0)=0: Condition III
m(0)=0: Condition IV
T_can
<K>/N
0.2
0.3
0.4
0.5
0.6
0.7
0.8
10^-1
10^0
10^1
10^2
10^3
10^4
10^5
10^6
t

$S_y(0) = 1, \quad p = 1/2$

$S_z(0) = 1, \quad p = 1/2$



- PDFs of angular velocities:

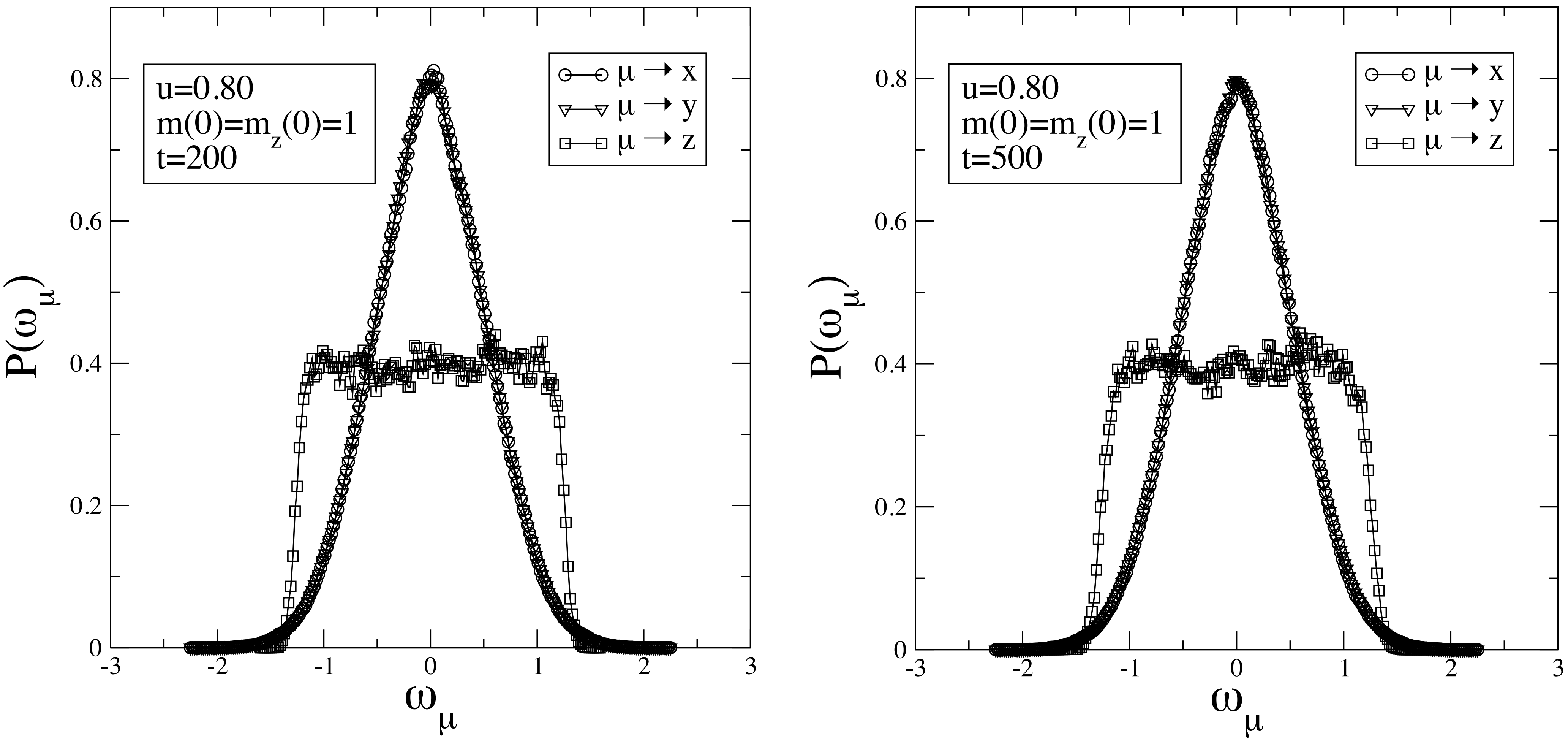
i) Ensemble averages: $P(\omega_\mu)$

ii) Time av. over single trajectories: $P(u_\mu)$

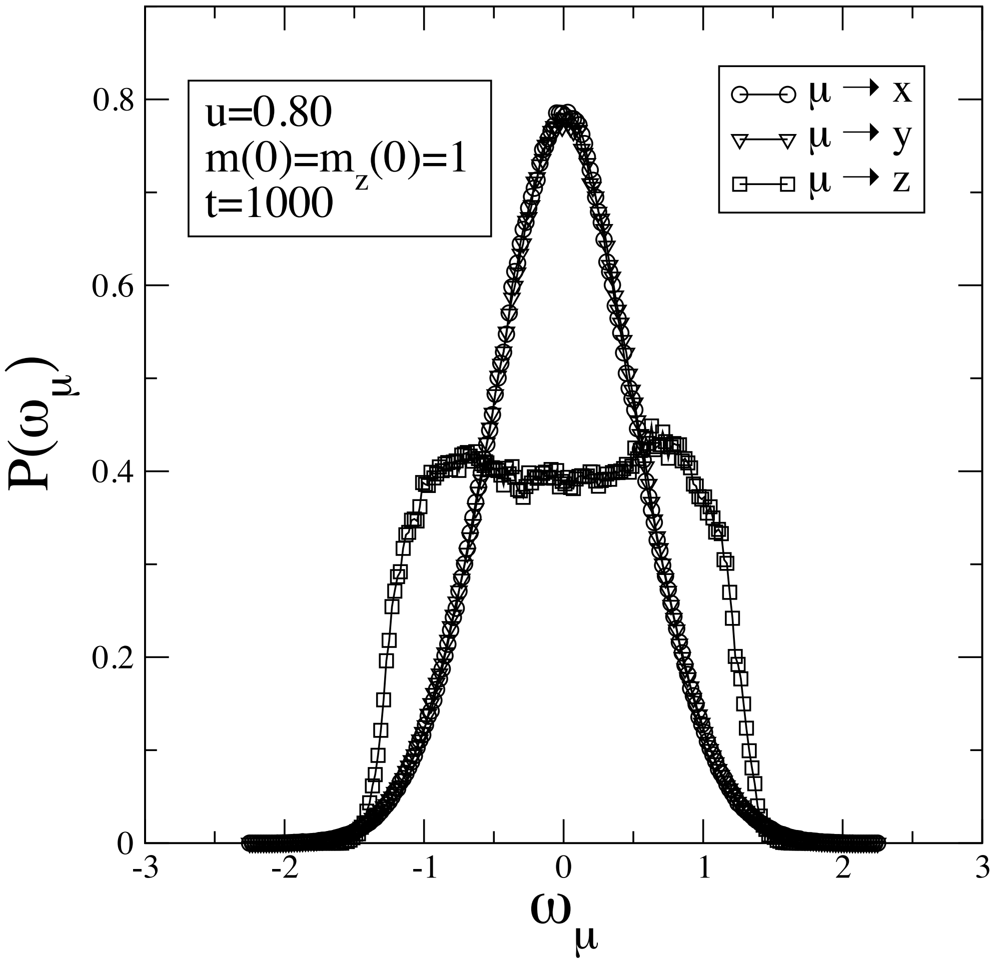
$$u_{i\mu} = \frac{1}{\sqrt{n}} \sum_{k=1}^{n} \omega_{i\mu}(k) \quad (i = 1, \cdots, N;\ \mu = x, y, z)$$

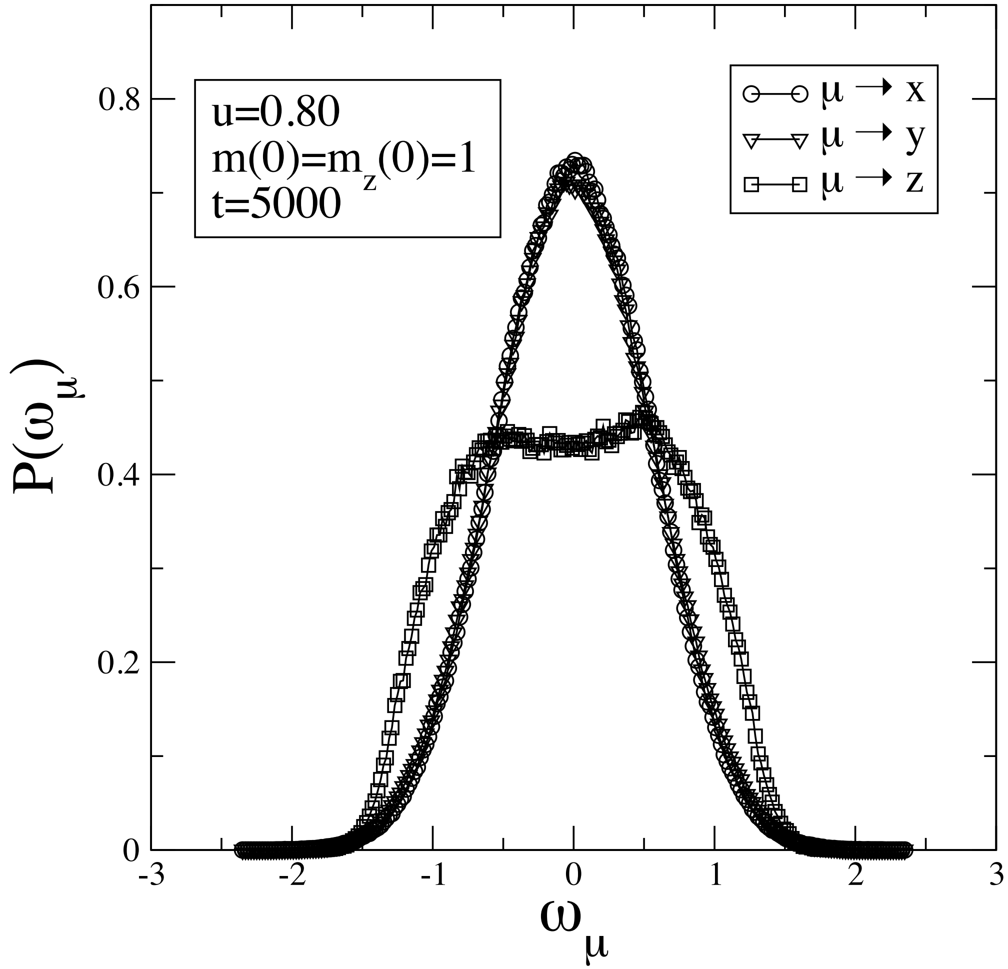
$n:$ no. measures taken at fixed time interval δ

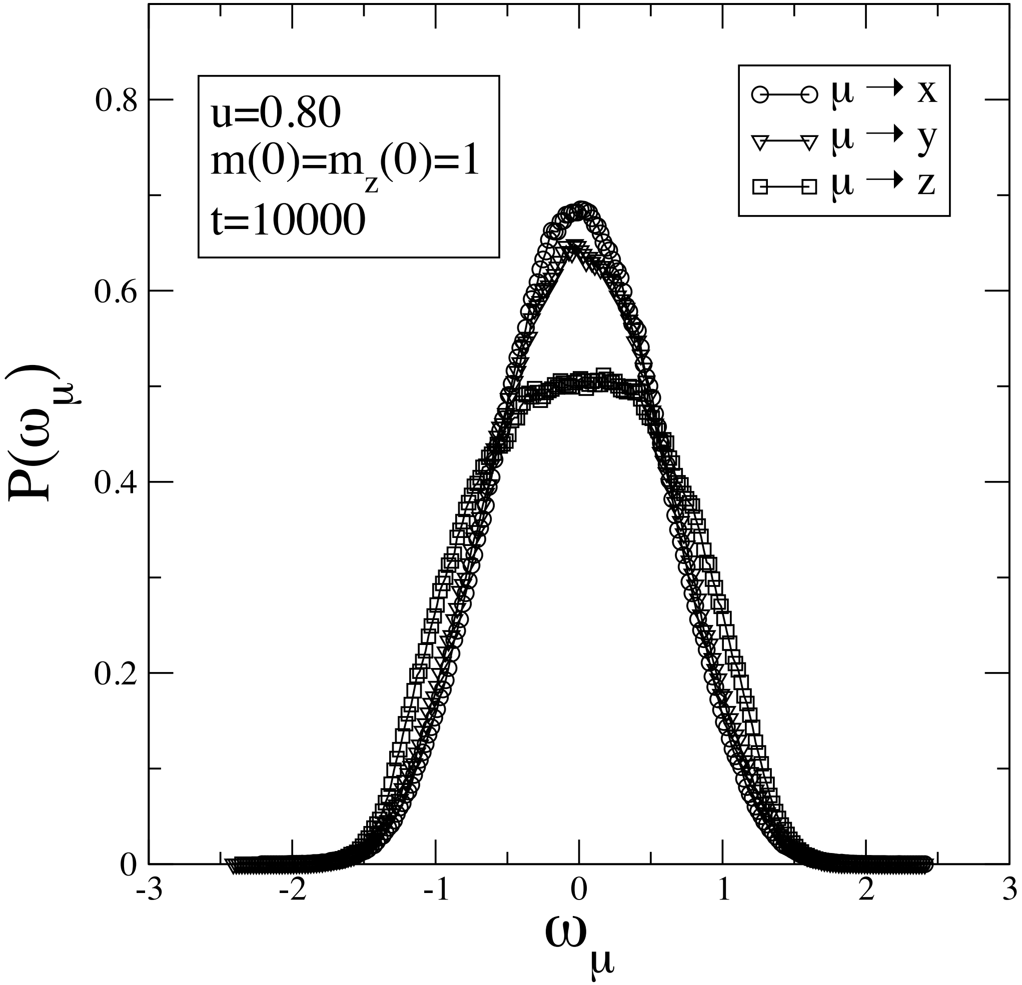
$n\delta:$ total time for measurements



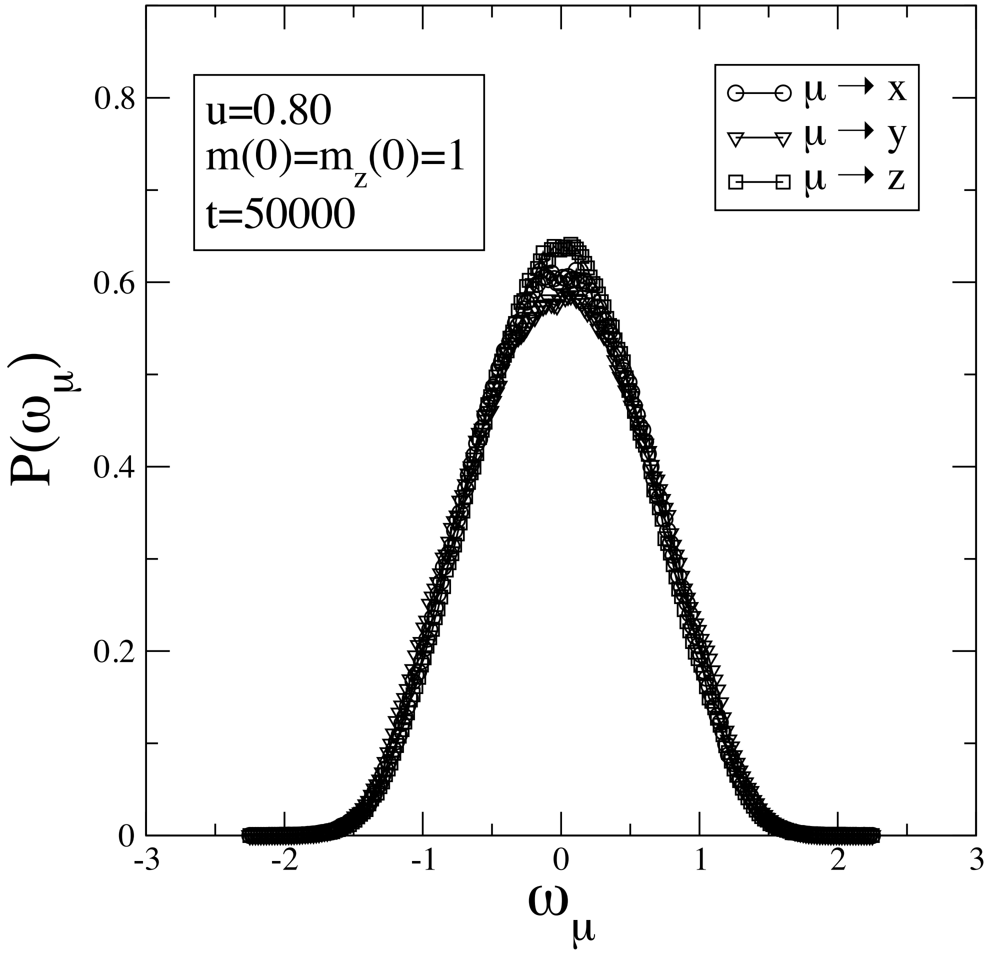
u=0.80
m(0)=m_z(0)=1
t=200
μ → x
μ → y
μ → z
P(ω_μ)
ω_μ
u=0.80
m(0)=m_z(0)=1
t=500
μ → x
μ → y
μ → z
P(ω_μ)
ω_μ


u=0.80
m(0)=m_z(0)=1
t=1000
μ → x
μ → y
μ → z
P(ω_μ)
ω_μ
0.8
0.6
0.4
0.2
0
-3
-2
-1
0
1
2
3

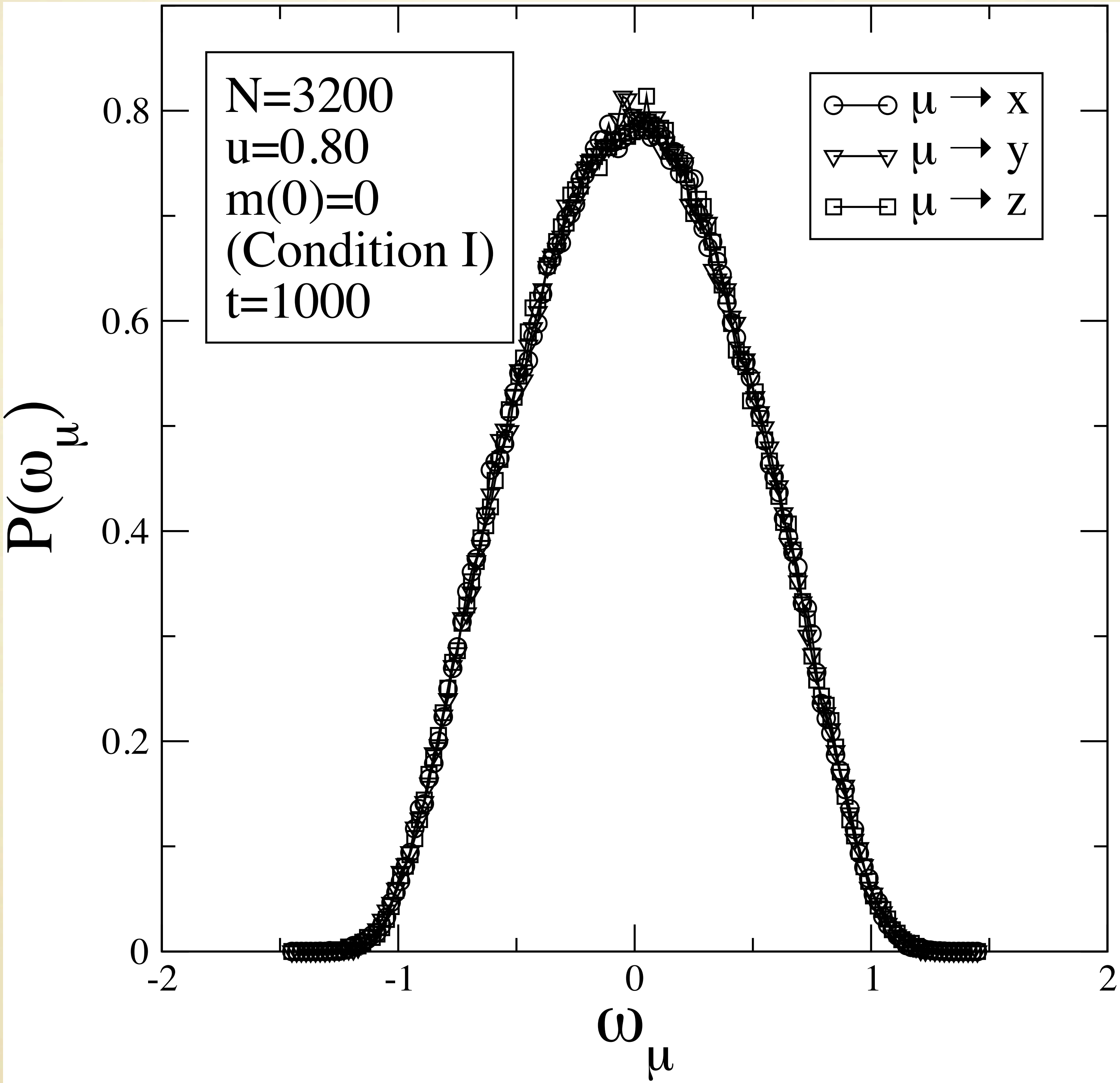

u=0.80
m(0)=m_z(0)=1
t=5000
μ → x
μ → y
μ → z
P(ω_μ)
ω_μ
0.8
0.6
0.4
0.2
0
-3
-2
-1
0
1
2
3

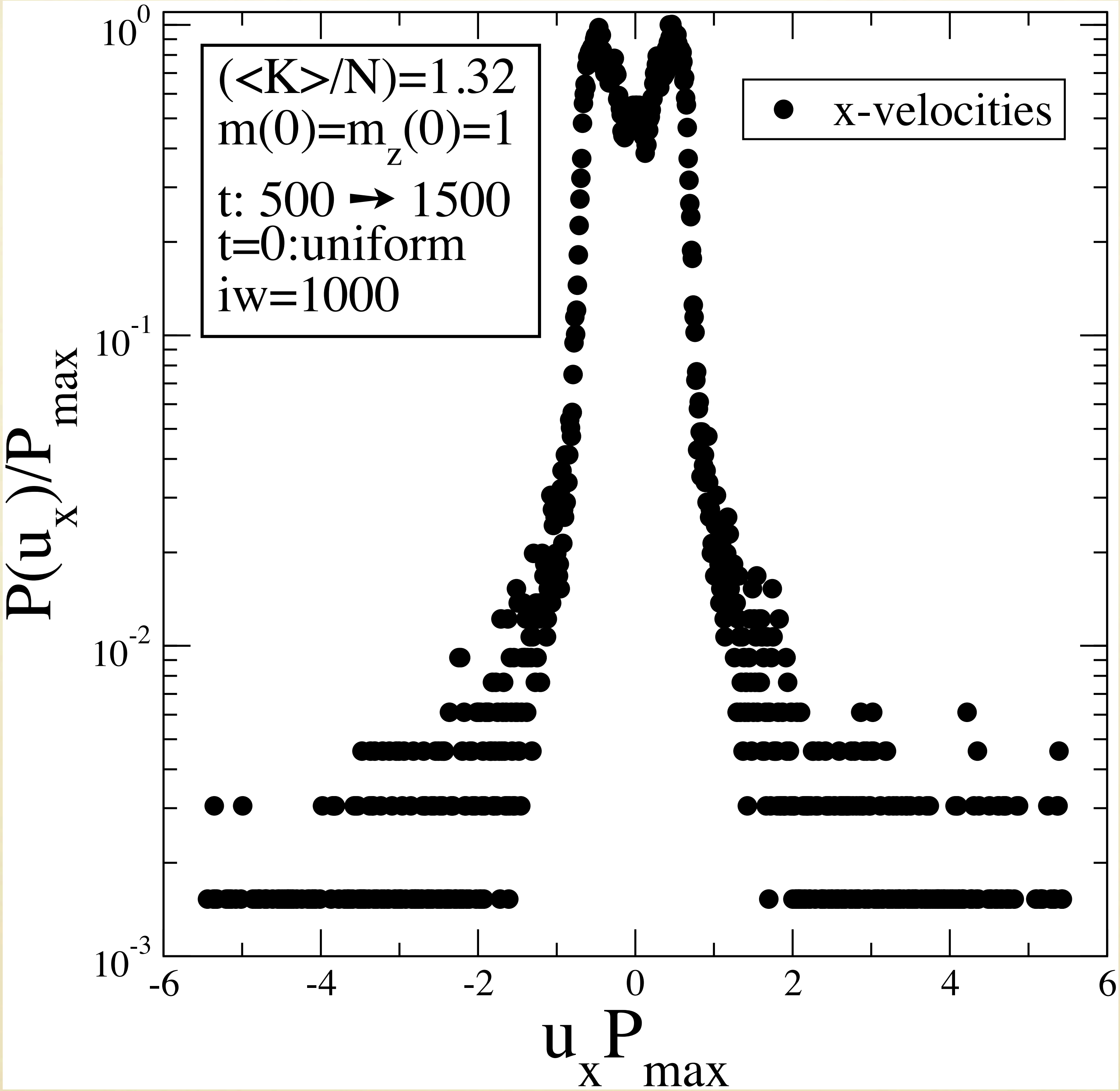


u=0.80
m(0)=m_z(0)=1
t=10000
μ → x
μ → y
μ → z
P(ω_μ)
ω_μ

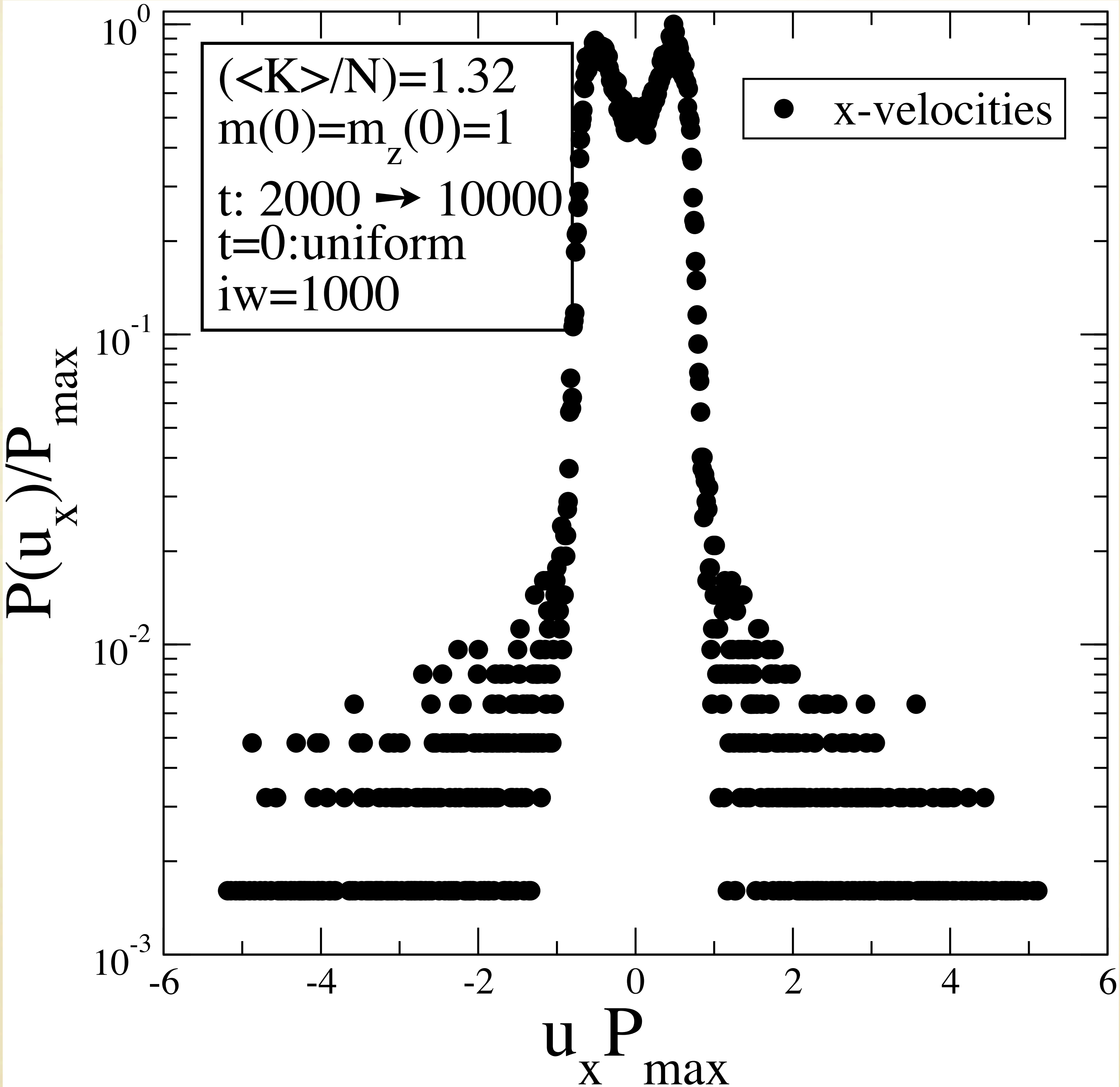


u=0.80
m(0)=m_z(0)=1
t=50000
μ → x
μ → y
μ → z
P(ω_μ)
ω_μ

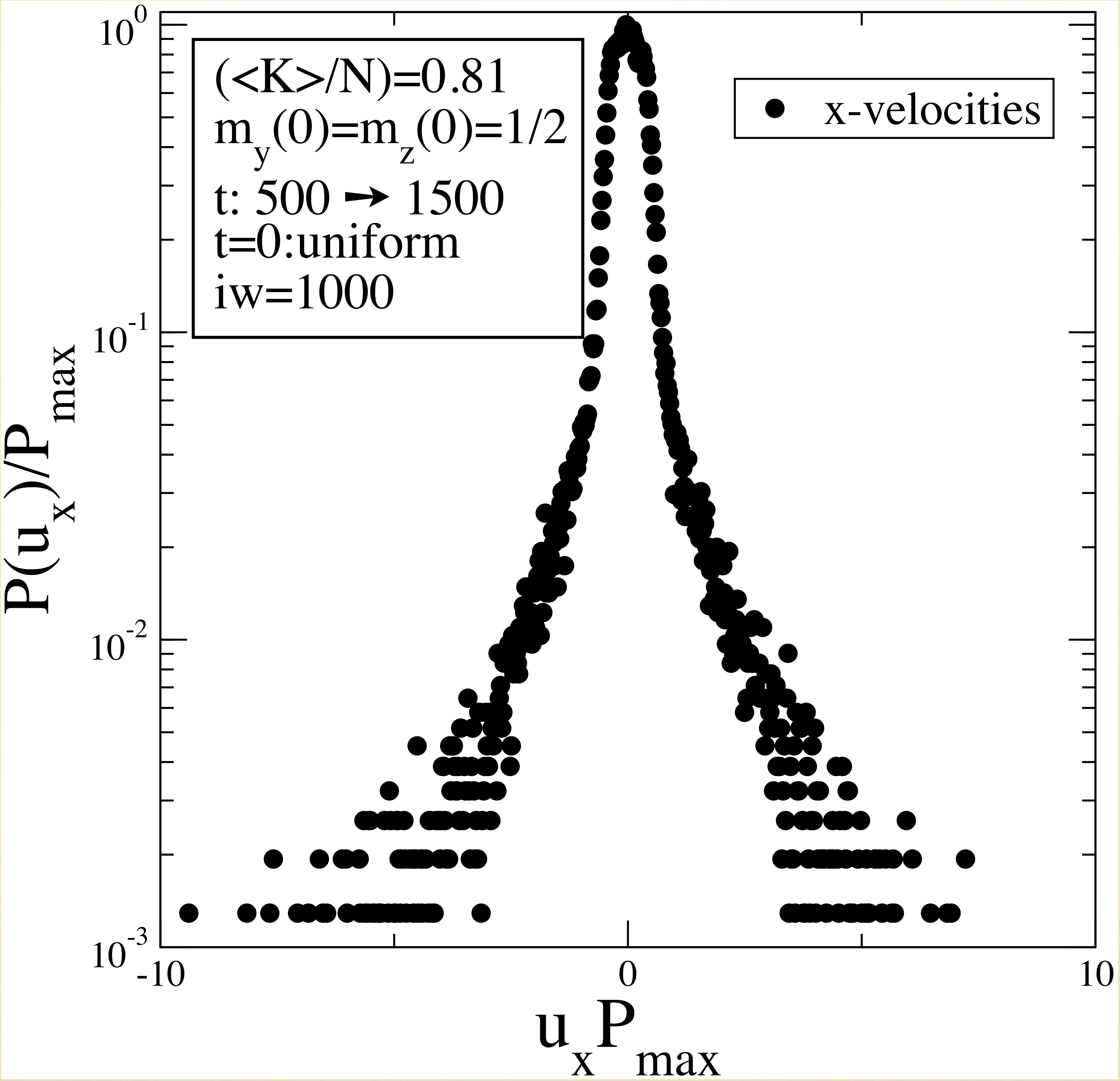

N=3200
u=0.80
m(0)=0
(Condition I)
t=1000
μ → x
μ → y
μ → z
0.8
0.6
0.4
0.2
0
-2
-1
0
1
2
P(ω_μ)
ω_μ



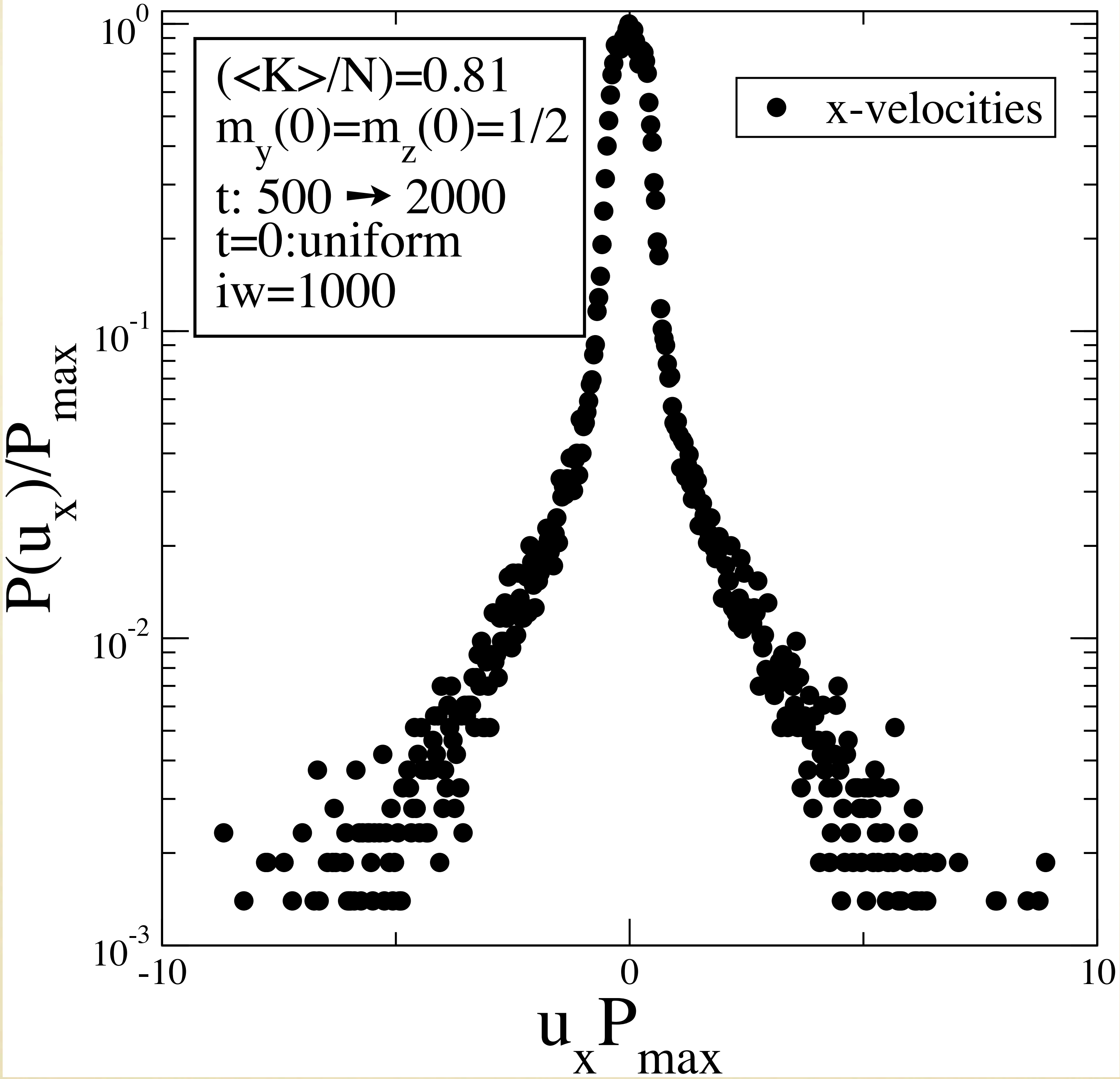
(<K>/N)=1.32
m(0)=m_z(0)=1
t: 500 → 1500
t=0:uniform
iw=1000
x-velocities
P(u_x)/P_max
u_x P_max
10^0
10^-1
10^-2
10^-3
-6
-4
-2
0
2
4
6



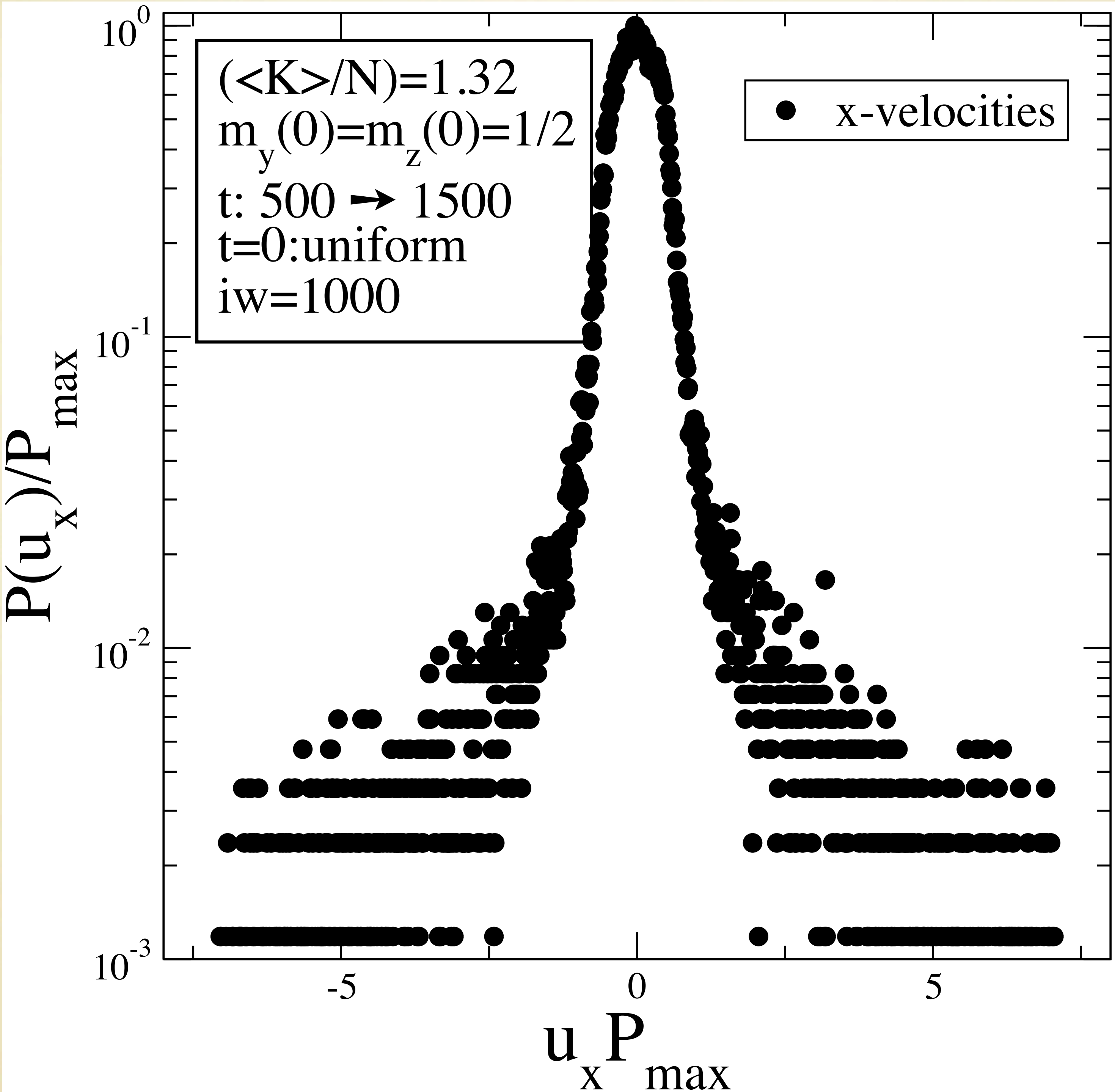
(<K>/N)=1.32
m(0)=m_z(0)=1
t: 2000 ➙ 10000
t=0:uniform
iw=1000
x-velocities
P(u_x)/P_max
u_x P_max
10^0
10^-1
10^-2
10^-3
-6
-4
-2
0
2
4
6



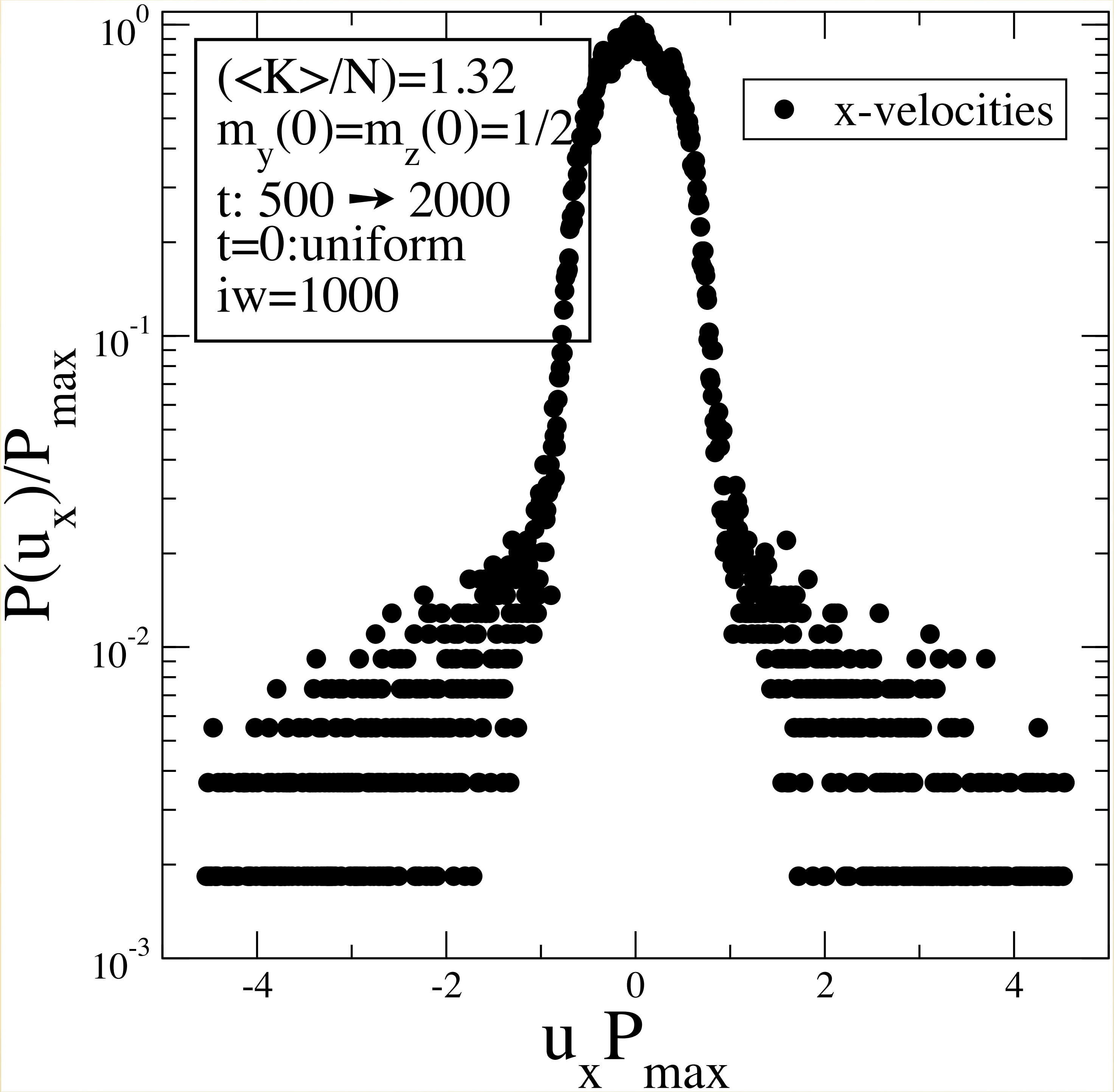
(<K>/N)=0.81
m_y(0)=m_z(0)=1/2
t: 500 ➞ 1500
t=0:uniform
iw=1000
x-velocities
P(u_x)/P_max
u_x P_max
10^0
10^-1
10^-2
10^-3
-10
0
10



(<K>/N)=0.81
m_y(0)=m_z(0)=1/2
t: 500 ➙ 2000
t=0:uniform
iw=1000
x-velocities
P(u_x)/P_max
u_x P_max
10^0
10^-1
10^-2
10^-3
-10
0
10



(<K>/N)=1.32
m_y(0)=m_z(0)=1/2
t: 500 ➞ 1500
t=0:uniform
iw=1000
x-velocities
10^0
10^-1
10^-2
10^-3
-5
0
5
P(u_x)/P_max
u_x P_max


(<K>/N)=1.32
m_y(0)=m_z(0)=1/2
t: 500 ➞ 2000
t=0:uniform
iw=1000
x-velocities
P(u_x)/P_max
u_x P_max
10^0
10^-1
10^-2
10^-3
-4
-2
0
2
4

Conclusions

- Model is Non-Ergodic
- Considering $N \to \infty$ and $t \to \infty$

➡ System remains on QSS forever

- CLT does NOT apply on QSS
- Standard Statistical Mechanics does NOT hold on QSS

- Homogeneous Function (degree 1):

$$S(\lambda V, \lambda U, \lambda N) = \lambda S(V, U, N)$$

→ Extensivity

- Holds **Only** for Short-Range Interactions

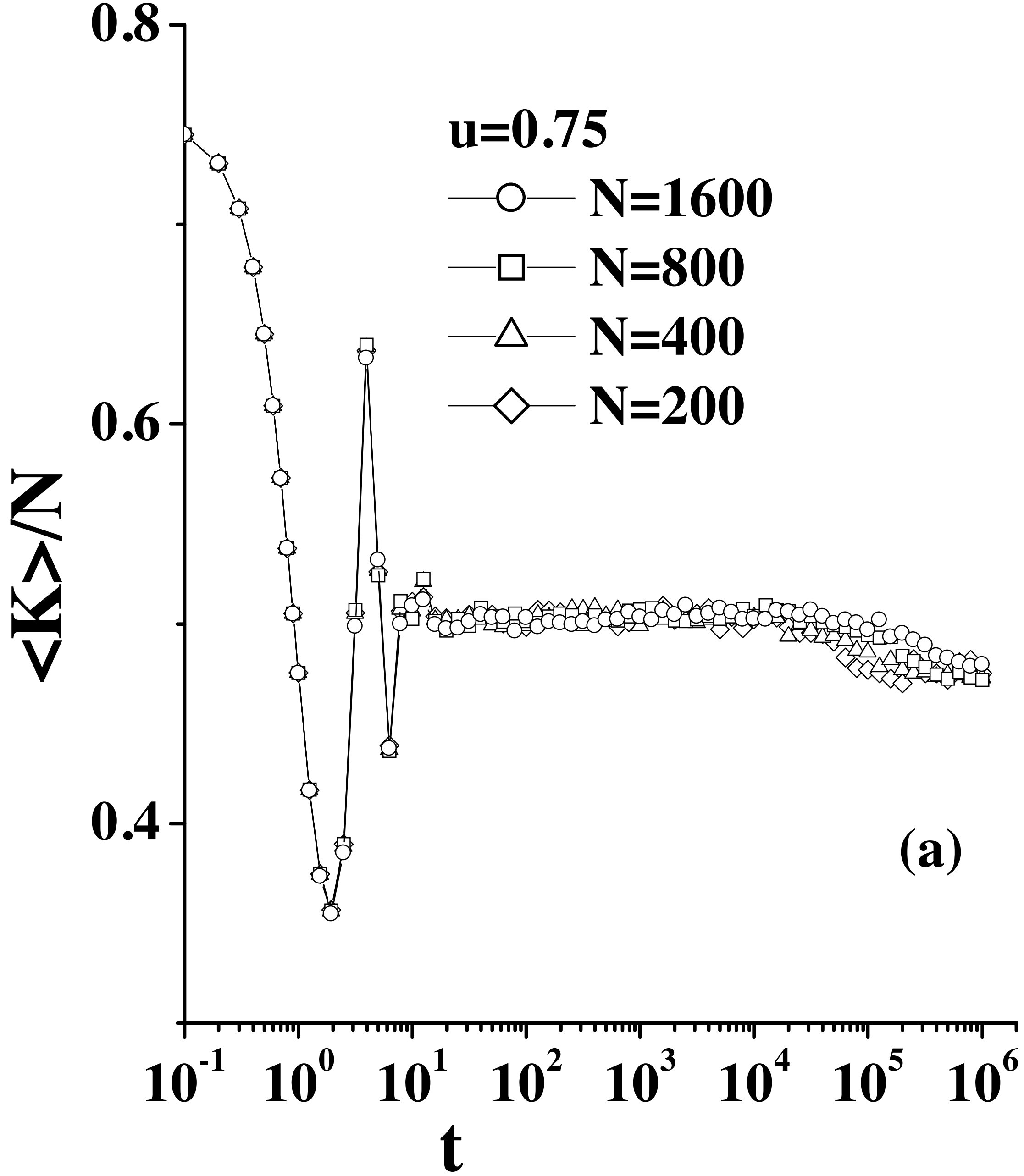
- Possible Alternative: Nonextensive Statistical Mechanics

- Tsallis Entropy:

$$S_q = k\,\frac{1}{q-1}\left(1 - \sum_j p_j^q\right)$$

- Limit $q \to 1$:

$$S_1 = -k_B \sum_j p_j \ln p_j$$

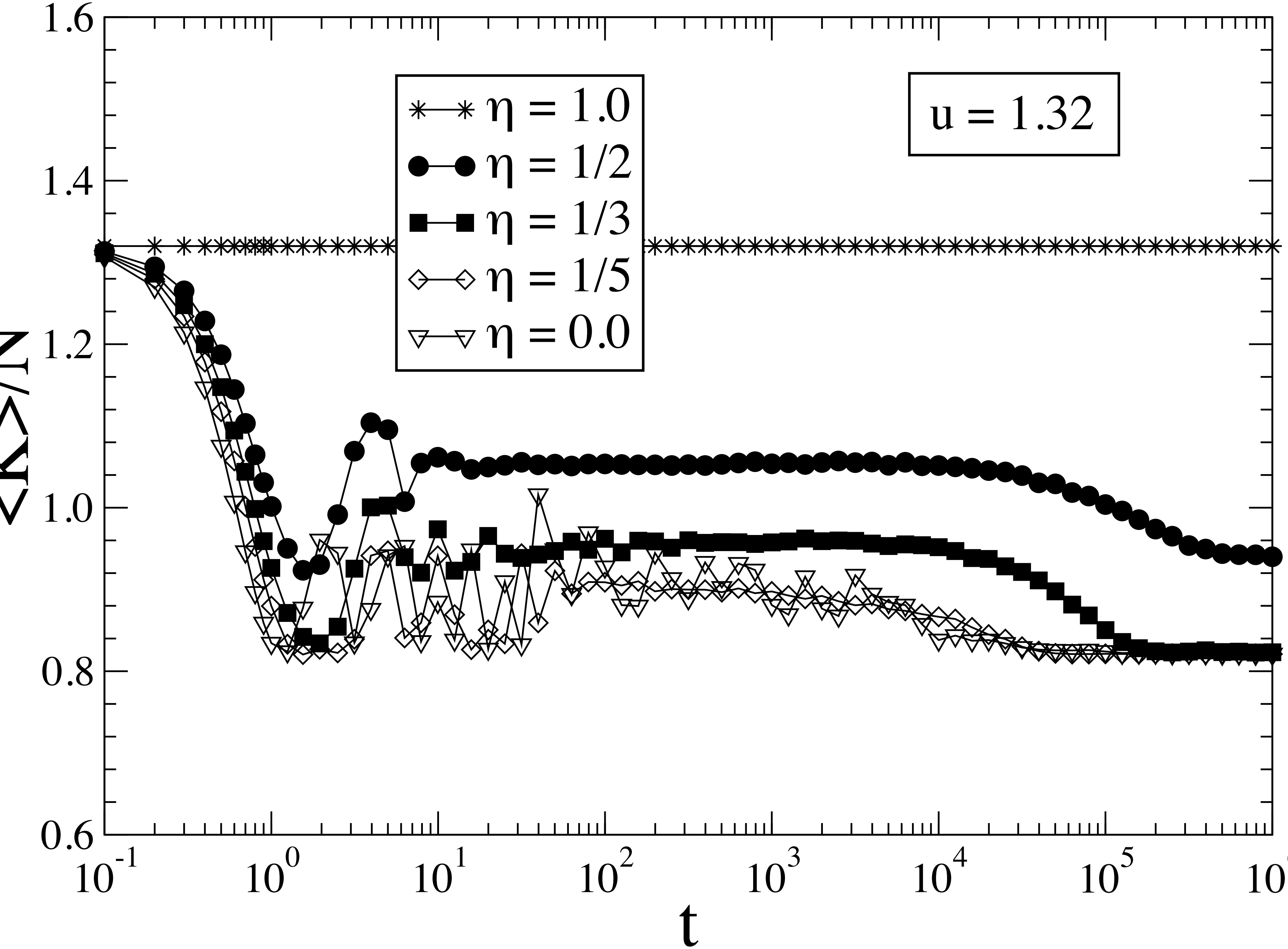


0.8
0.6
0.4
<K>/N
u=0.75
N=1600
N=800
N=400
N=200
(a)
10^{-1} 10^{0} 10^{1} 10^{2} 10^{3} 10^{4} 10^{5} 10^{6}
t

ii) Different Initial "Water Bags"

$$\langle K_z \rangle = \eta \, \langle K \rangle \quad (0 \leq \eta \leq 1)$$

$$\langle K_x \rangle = \langle K_y \rangle = \frac{1}{2}(1-\eta) \, \langle K \rangle$$



1.6
1.4
1.2
1.0
0.8
0.6
<K>/N
10^-1
10^0
10^1
10^2
10^3
10^4
10^5
10^6
t
η = 1.0
η = 1/2
η = 1/3
η = 1/5
η = 0.0
u = 1.32