equações de Fokker-Planck não lineares

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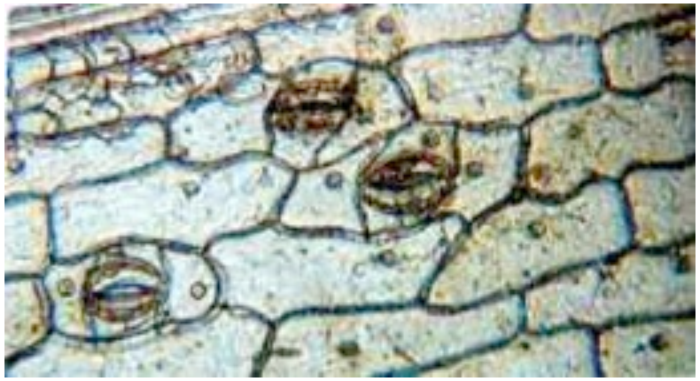
movimento Browniano

Robert Brown

- Robert Brown (botânico) - 1827



http://www.brianjford.com/wbbrownc.htm

A

BRIEF ACCOUNT

OF

MICROSCOPICAL OBSERVATIONS

Made in the Months of June, July, and August, 1827,

ON THE PARTICLES CONTAINED IN THE POLLEN OF PLANTS;

AND

ON THE GENERAL EXISTENCE OF ACTIVE MOLECULES

IN ORGANIC AND INORGANIC BODIES.

BY

ROBERT BROWN,

F.R.S., Hon. M.R.S.E. and R.I. Acad., V.P.L.S.,

MEMBER OF THE ROYAL ACADEMY OF SCIENCES OF SWEDEN, OF THE ROYAL SOCIETY OF DENMARK, AND OF THE IMPERIAL ACADEMY NATURÆ CURIOSORUM; CORRESPONDING MEMBER OF THE ROYAL INSTITUTES OF FRANCE AND OF THE NETHERLANDS, OF THE IMPERIAL ACADEMY OF SCIENCES AT ST. PETERSBURG, AND OF THE ROYAL ACADEMIES OF PRUSSIA AND BAVARIA, ETC.

R. Brown, Edinb. New Philos. J. 5 (1828) 358–371 is only apparent in those drops that are flattened, in consequence of being nearly or absolutely in contact with the stage of the microscope.

That the motion of the particles is not produced by any cause acting on the surface of the drop, may be proved by an inversion of the experiment; for by mixing a very small proportion of oil with the water containing the particles, microscopic drops of oil of extreme minuteness, some of them not exceeding in size the particles themselves, will be found on the surface of the drop of water, and nearly or altogether at rest; while the particles in the centre or towards the bottom of the drop continue to move with their usual degree of activity.

By means of the contrivance now described for reducing the size and prolonging the existence of the drops containing the particles, which, simple as it is, did not till very lately occur to me, a greater command of the subject is obtained, sufficient perhaps to enable us to ascertain the real cause of the motions in question.

Of the few experiments which I have made since this manner of observing was adopted, some appear to me so curious, that I do not venture to state them until they are verified by frequent and careful repetition.

I shall conclude these supplementary remarks to my former Observations, by noticing the degree in which I consider those observations to have been anticipated.

That molecular was sometimes confounded with animalcular motion by several of the earlier microscopical observers, appears extremely probable from various passages in the writings of Leeuwenhoek, as well as from a very interesting Paper by Stephen Gray, published in the 19th volume of the Philosophical Transactions.

Needham also, and Buffon, with whom the hypothesis of organic particles originated, seem to have not unfrequently fallen into the same mistake. And I am inclined to believe that Spallanzani, notwithstanding one of his statements respecting them, has under the head of *Anima-*

appear to have suspected that particles having analogous motions might exist in other organized bodies, and far less in inorganic matter, I consider myself anticipated by this acute observer only to the same extent as by Gleichen, and in a much less degree than by Müller, whose statements have been already alluded to.

All the observers now mentioned have confined themselves to the examination of the particles of organic bodies. In 1819, however, Mr. Bywater, of Liverpool, published an account of Microscopical Observations, in which it is stated that not only organic tissues, but also inorganic substances, consist of what he terms animated or irritable particles.

A second edition of this Essay appeared in 1828, probably altered in some points, but it may be supposed agreeing essentially in its statements with the edition of 1819, which I have never seen, and of the existence of which I was ignorant when I published my pamphlet.

From the edition of 1828, which I have but lately met with, it appears that Mr. Bywater employed a compound microscope of the construction called Culpepper's, that the object was examined in a bright sunshine, and the light from the mirror thrown so obliquely on the stage as to give a blue colour to the infusion.

The first experiment I here subjoin in his own words.

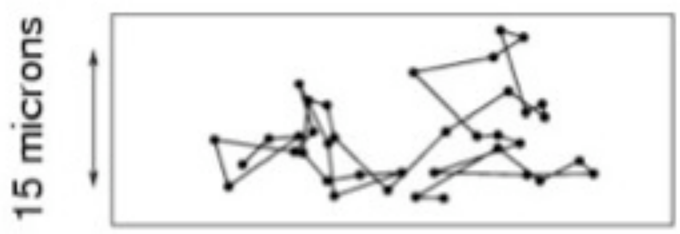
"A small portion of flour must be placed on a slip of glass, and mixed with a drop of water, then instantly applied to the microscope; and if stirred and viewed by a bright sun, as already described, it will appear evidently filled with innumerable small linear bodies, writhing and twisting about with extreme activity."

Similar bodies, and equally in motion, were obtained from animal and vegetable tissues, from vegetable mould, from sandstone after being made red hot, from coal, ashes, and other inorganic bodies.

I believe that in thus stating the manner in which Mr. Bywater's experiments were conducted, I have enabled microscopical observers to judge of the extent and kind of optical illusion to which he was liable, and of which he

microscópio

difusão normal



Einstein

- Albert Einstein (1905) - tese (30 Abril, Julho -> AdP 19 (1906) 289) e artigo sobre movimento Browniano -> (Maio, 11 -> AdP 17 (1905) 549)

$$D = \frac{RT}{N}\frac{1}{6\pi kP}$$

- William Sutherland (Australia) - Março 1905

-> Phil. Mag. 9 (1905) 781

5. *Über die von der molekularkinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen;*
von A. Einstein.

In dieser Arbeit soll gezeigt werden, daß nach der molekularkinetischen Theorie der Wärme in Flüssigkeiten suspendierte Körper von mikroskopisch sichtbarer Größe infolge der Molekularbewegung der Wärme Bewegungen von solcher Größe ausführen müssen, daß diese Bewegungen leicht mit dem Mikroskop nachgewiesen werden können. Es ist möglich, daß die hier zu behandelnden Bewegungen mit der sogenannten „Brownschen Molekularbewegung" identisch sind; die mir erreichbaren Angaben über letztere sind jedoch so ungenau, daß ich mir hierüber kein Urteil bilden konnte.

Wenn sich die hier zu behandelnde Bewegung samt den für sie zu erwartenden Gesetzmäßigkeiten wirklich beobachten läßt, so ist die klassische Thermodynamik schon für mikroskopisch unterscheidbare Räume nicht mehr als genau gültig anzusehen und es ist dann eine exakte Bestimmung der wahren Atomgröße möglich. Erwiese sich umgekehrt die Voraussage dieser Bewegung als unzutreffend, so wäre damit ein schwerwiegendes Argument gegen die molekularkinetische Auffassung der Wärme gegeben.

§ 1. Über den suspendierten Teilchen zuzuschreibenden osmotischen Druck.

Im Teilvolumen V^* einer Flüssigkeit vom Gesamtvolumen V seien z-Gramm-Moleküle eines Nichtelektrolyten gelöst. Ist das Volumen V^* durch eine für das Lösungsmittel, nicht aber für die gelöste Substanz durchlässige Wand vom reinen Lösungs-

A. Einstein, Annalen der Physik 17 (1905) 549–560 und indem wir

$$\frac{1}{\tau}\int_{-\infty}^{+\infty}\frac{\Delta^2}{2}\varphi(\Delta)\,d\Delta = D$$

setzen und nur das erste und dritte Glied der rechten Seite berücksichtigen:

$$\frac{\partial f}{\partial t} = D\frac{\partial^2 f}{\partial x^2}. \tag{1}$$

Dies ist die bekannte Differentialgleichung der Diffusion, und man erkennt, daß D der Diffusionskoeffizient ist.

An diese Entwicklung läßt sich noch eine wichtige Überlegung anknüpfen. Wir haben angenommen, daß die einzelnen Teilchen alle auf dasselbe Koordinatensystem bezogen seien. Dies ist jedoch nicht nötig, da die Bewegungen der einzelnen Teilchen voneinander unabhängig sind. Wir wollen nun die Bewegung jedes Teilchens auf ein Koordinatensystem beziehen, dessen Ursprung mit der Lage des Schwerpunktes des betreffenden Teilchens zur Zeit $t = 0$ zusammenfällt, mit dem Unterschiede, daß jetzt $f(x, t)\,dx$ die Anzahl der Teilchen bedeutet, deren X-Koordinaten von der Zeit $t = 0$ bis zur Zeit $t = t$ um eine Größe *gewachsen* ist, welche zwischen x und $x + dx$ liegt. Auch in diesem Falle ändert sich also die Funktion f gemäß Gleichung (1). Ferner muß offenbar für $x \gtreqless 0$ und $t = 0$

$$f(x, t) = 0 \quad \text{und} \quad \int_{-\infty}^{+\infty} f(x, t)\,dx = n$$

sein. Das Problem, welches mit dem Problem der Diffusion von einem Punkte aus (unter Vernachlässigung der Wechselwirkung der diffundierenden Teilchen) übereinstimmt, ist nun mathematisch vollkommen bestimmt; seine Lösung ist:

$$f(x, t) = \frac{n}{\sqrt{4\pi D}}\frac{e^{-\frac{x^2}{4Dt}}}{\sqrt{t}}.$$

Die Häufigkeitsverteilung der in einer beliebigen Zeit t erfolgten Lagenänderungen ist also dieselbe wie die der zufälligen Fehler, was zu vermuten war. Von Bedeutung aber ist, wie die Konstante im Exponenten mit dem Diffusionskoeffizienten zusammenhängt. Wir berechnen nun mit Hilfe dieser Gleichung die Verrückung λ_x in Richtung der X-Achse, welche ein Teilchen im Mittel erfährt, oder — genauer ausgedrückt — die Wurzel aus dem arithmetischen Mittel der Quadrate der Verrückungen in Richtung der X-Achse; es ist:

$$\lambda_x = \sqrt{\overline{x^2}} = \sqrt{2\,D\,t}\,.$$

Die mittlere Verschiebung ist also proportional der Quadratwurzel aus der Zeit. Man kann leicht zeigen, daß die Wurzel aus dem Mittelwert der Quadrate der *Gesamtverschiebungen* der Teilchen den Wert $\lambda_x\sqrt{3}$ besitzt.

§ 5. Formel für die mittlere Verschiebung suspendierter Teilchen. Eine neue Methode zur Bestimmung der wahren Größe der Atome.

In § 3 haben wir für den Diffusionskoeffizienten D eines in einer Flüssigkeit in Form von kleinen Kugeln vom Radius P suspendierten Stoffes den Wert gefunden:

$$D = \frac{R\,T}{N}\,\frac{1}{6\,\pi\,k\,P}\,.$$

Ferner fanden wir in § 4 für den Mittelwert der Verschiebungen der Teilchen in Richtung der X-Achse in der Zeit t:

$$\lambda_x = \sqrt{2\,D\,t}\,.$$

Durch Eliminieren von D erhalten wir:

$$\lambda_x = \sqrt{t}\cdot\sqrt{\frac{R\,T}{N}\,\frac{1}{3\,\pi\,k\,P}}\,.$$

Diese Gleichung läßt erkennen, wie λ_x von T, k und P abhängen muß.

Wir wollen berechnen, wie groß λ_x für eine Sekunde ist, wenn N gemäß den Resultaten der kinetischen Gastheorie $6\,.\,10^{23}$ gesetzt wird; es sei als Flüssigkeit Wasser von 17° C. gewählt ($k = 1{,}35\,.\,10^{-2}$) und der Teilchendurchmesser sei 0,001 mm. Man erhält:

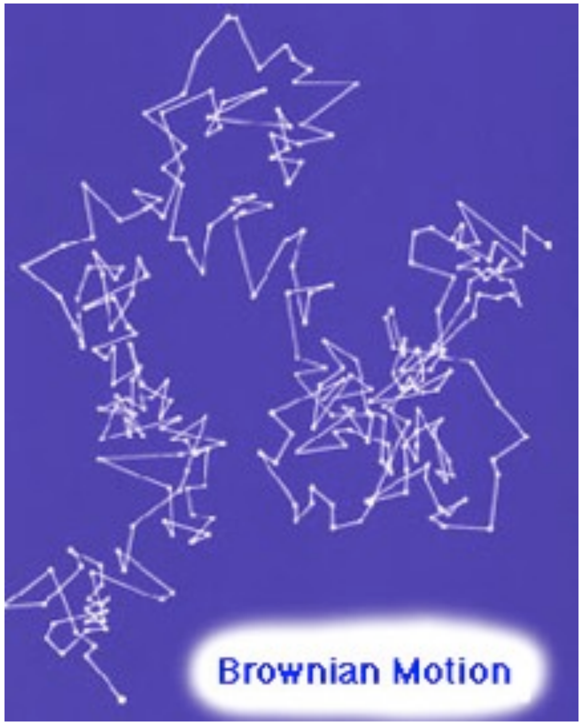
$$\lambda_x = 8\,.\,10^{-5}\,\text{cm} = 0{,}8\ \text{Mikron}.$$

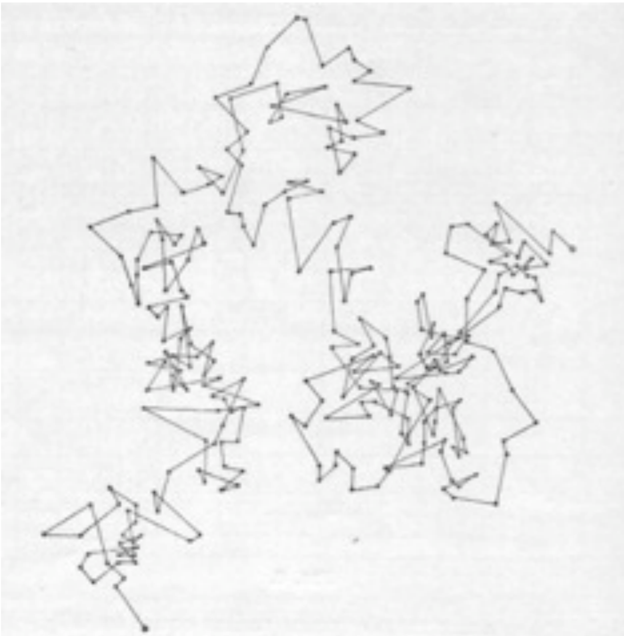
Die mittlere Verschiebung in 1 Min. wäre also ca. 6 Mikron.

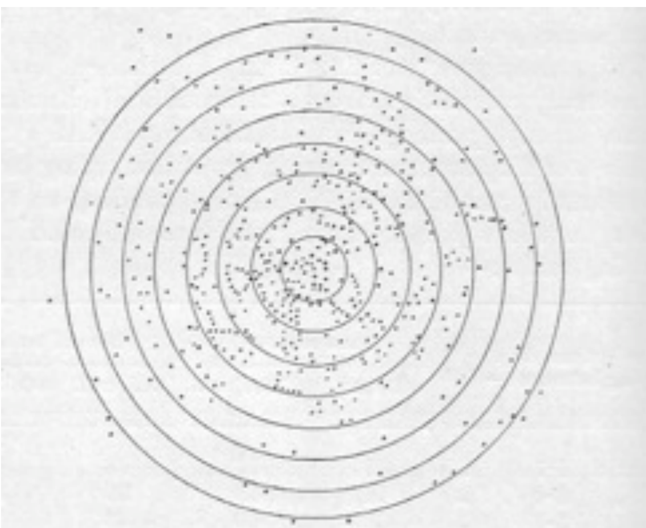
$$\langle (x - \langle x \rangle)^2 \rangle = 2Dt$$

Jean Perrin

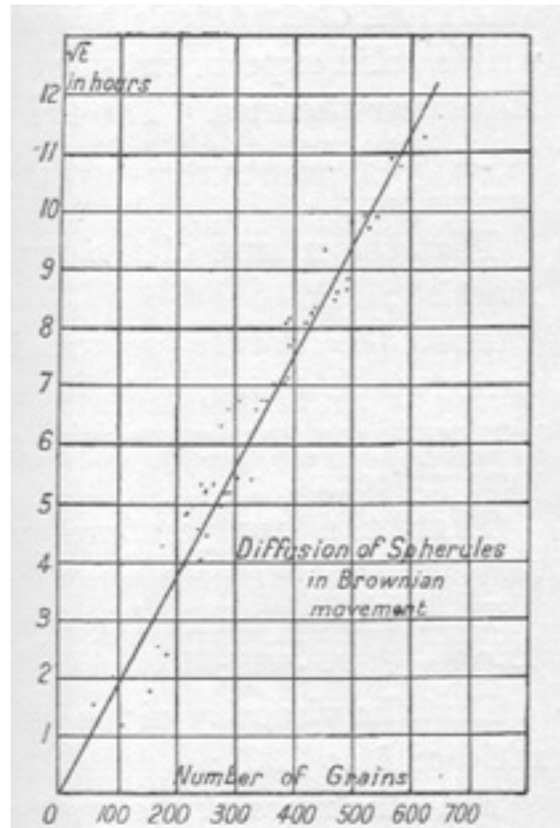
- 1908 J. Perrin, C. R. Acad. Sci. 146, 967
- "Les atomes" (Félix Alcan, Paris, 1913)







Displacement between :—	P for each ring.	n Calculated.	n Found.
0 and $\frac{e}{4}$.	·063	32	34
$\frac{e}{4}$,, $2\frac{e}{4}$.	·167	83	78
$2\frac{e}{4}$,, $3\frac{e}{4}$.	·214	107	106
$3\frac{e}{4}$,, $4\frac{e}{4}$.	·210	105	103
$4\frac{e}{4}$,, $5\frac{e}{4}$.	·150	75	75
$5\frac{e}{4}$,, $6\frac{e}{4}$.	·100	50	49
$6\frac{e}{4}$,, $7\frac{e}{4}$.	·054	27	30
$7\frac{e}{4}$,, $8\frac{e}{4}$.	·028	14	17
$8\frac{e}{4}$,, $9\frac{e}{4}$.	·014	7	9



difusão linear (1 dimensão):

$$\frac{\partial P(x,t)}{\partial t} = D \, \frac{\partial^2 P(x,t)}{\partial^2 x}$$

$$P(x,t) = \frac{1}{(4\pi D t)^{1/2}} \exp\left[-\frac{(x-x_0)^2}{4Dt}\right]$$

$$< x(t) >= x_0 \; ; \quad < (x(t) - x_0)^2 >= 2Dt$$

$$\text{Einstein Relation} : D = \mu k_B T$$

$$P(x,t) = t^{-1/2} g(x^2/t) \Rightarrow \sigma^2 \sim t$$

equação de Fokker-Planck

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial\{F(x)P(x,t)\}}{\partial x} + D\frac{\partial^2 P(x,t)}{\partial x^2}$$

$$F(x) = -\frac{d\phi}{dx}$$

teorema H e equação de FP

BG

$$F = U - TS \; ; \; U = \int_{-\infty}^{\infty} dx\ \phi(x)P(x,t) \; ; \; S = -k_B \int_{-\infty}^{\infty} dx\ P(x,t)\ln P(x,t)$$

$$\frac{dF}{dt} \leq 0$$

válido para EFP "e" entropia deBG

=> relação entre EFP e entropia de BG

solução geral da equação de Fokker-Planck

- dependência no tempo $\longrightarrow$ $F(x) = -kx$

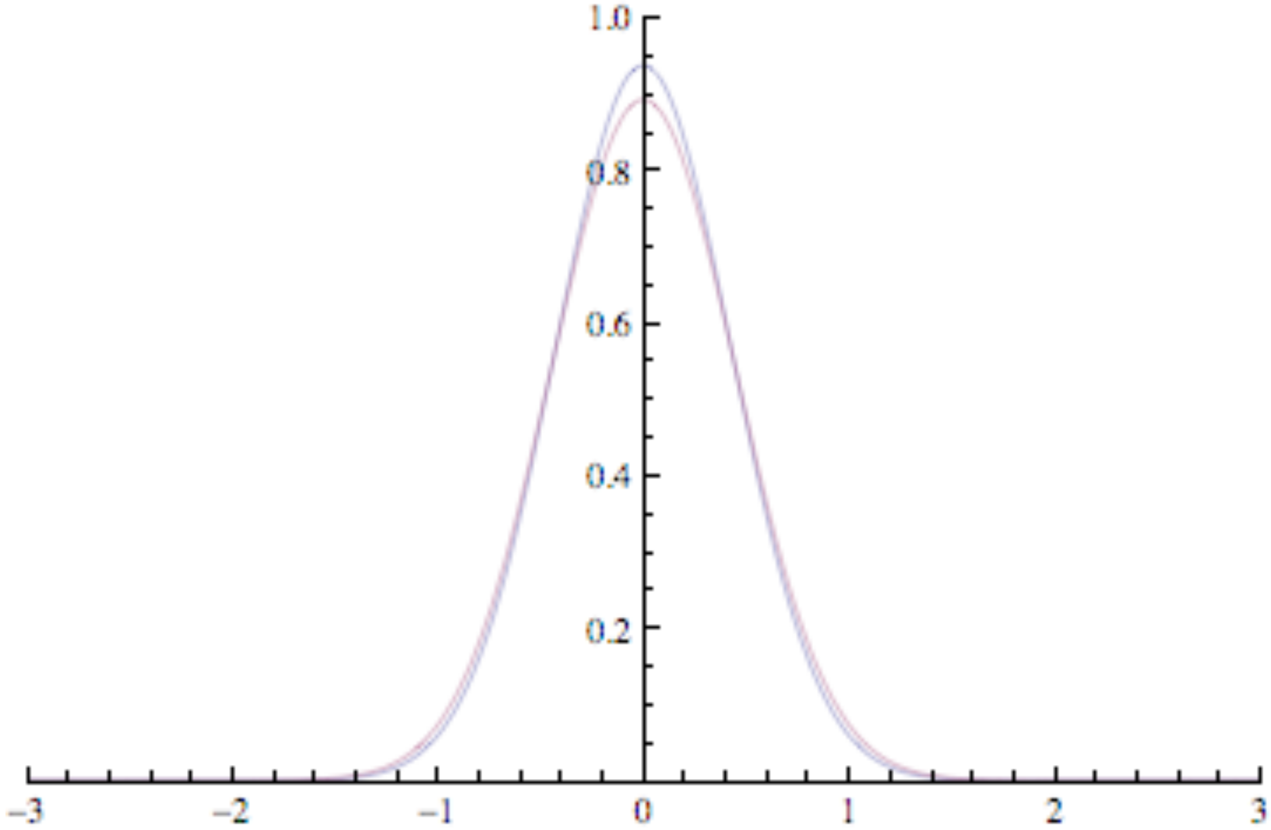
$$P(x,t) = \frac{1}{\sqrt{2\pi D(1-e^{-2t})/k}} e^{-\frac{kx^2}{2D(1-e^{-2t})}}$$

- difusão normal $(t << 1)$

entropia BG

$$P(x,t) = \frac{1}{\sqrt{4\pi Dt/k}} e^{-\frac{kx^2}{4Dt}} \longrightarrow \langle (x - \langle x \rangle)^2 \rangle = \frac{2Dt}{k}$$

- $t >> 1 \longrightarrow \langle (x - \langle x \rangle)^2 \rangle = \frac{D}{k}$ $\left(P(x) = \frac{1}{\sqrt{2\pi D/k}} e^{-\frac{kx^2}{2D}} \right)$


1.0
0.8
0.6
0.4
0.2
−3
−2
−1
0
1
2
3

difusão anômala

difusão anômala

$$< (x(t) - x_0)^2 > \sim \; t^{\gamma} \qquad (\gamma \neq 1)$$

- subdifusivo $(\gamma < 1)$
- superdifusivo $(\gamma > 1)$

comportamento subdifusivo

subdifusão

$$< (x(t) - x_0)^2 > \sim \; t^\gamma \qquad (\, \gamma < 1 : \mathrm{Subdiffusion} \,)$$

- existência de "armadilhas" no espaço, onde as partículas permanecem por um certo tempo, com uma larga distribuição de tempos de escape

- condutividade de cadeias iônicas desordenadas
- fotocopiadoras, impressoras laser
- caminhantes aleatórios em substratos fractais
- difusão em rolos convectivos
- difusão de poluentes em águas subterrâneas
- difusão de proteínas através de membranas celulares

exemplos

- fotocopiadoras, impressoras laser: transporte de elétrons ou buracos em semicondutores amorfos em um campo elétrico

PHYSICAL REVIEW B VOLUME 12, NUMBER 6 15 SEPTEMBER 1975

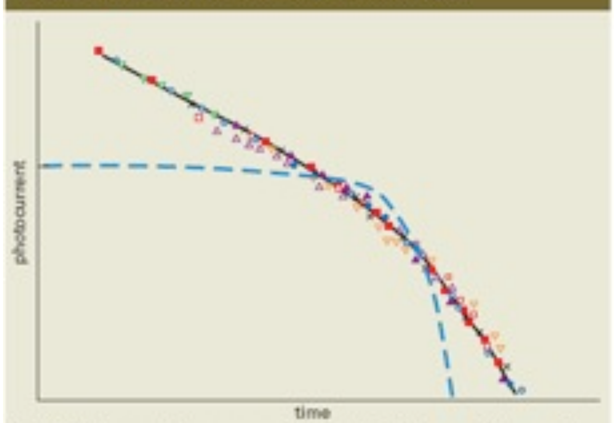
Anomalous transit-time dispersion in amorphous solids

Harvey Scher
Xerox Webster Research Center, 800 Phillips Road, Webster, New York 14580

Elliott W. Montroll
Institute for Fundamental Studies, Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627*

(Received 13 January 1975)

2 Anomalous diffusion in photocopiers

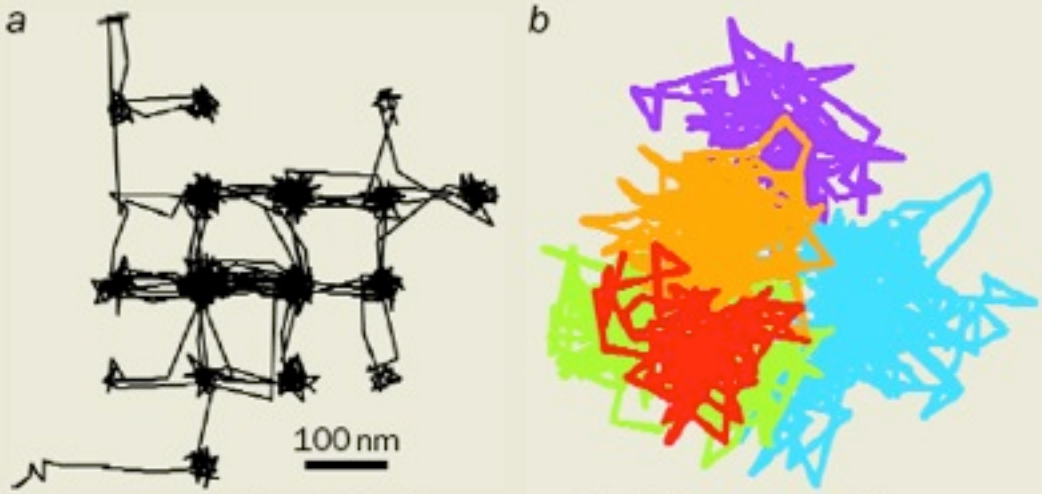


In the 1970s researchers measured the transient photocurrent in amorphous thin films that form the core of photocopier machines (data points). The blue dashed line indicates the expected behaviour if this diffusion process followed Fick's equation, which led Scher and Montroll to describe the process using broad distributions of waiting times. Both axes are logarithmic. This became the best known example of anomalous subdiffusion in nature. From H Scher and E Montroll 1975 *Phys. Rev.* B **12** 2455–2477

subdiffusion

- difusão de proteinas através das membranas celulares

3 Subdiffusion in cells



Researchers have found that the way proteins diffuse across cell membranes can be described by anomalous diffusion that is slower than the normal case. (*a*) This is a simulation of such a random walk, which shows a 2 ms timeframe over which a protein "hops" between 120 nm^2 compartments thought to be formed by the cell's cytoskeleton. (*b*) The experimental trajectories of proteins in the plasma membrane of a live cell (shown in a 0.025 ms timeframe) provide evidence for this trapping nature, as shown by the different colours. The long residence times in these compartments is thought to be the origin of the anomalous behaviour.

Physics World, august 2005

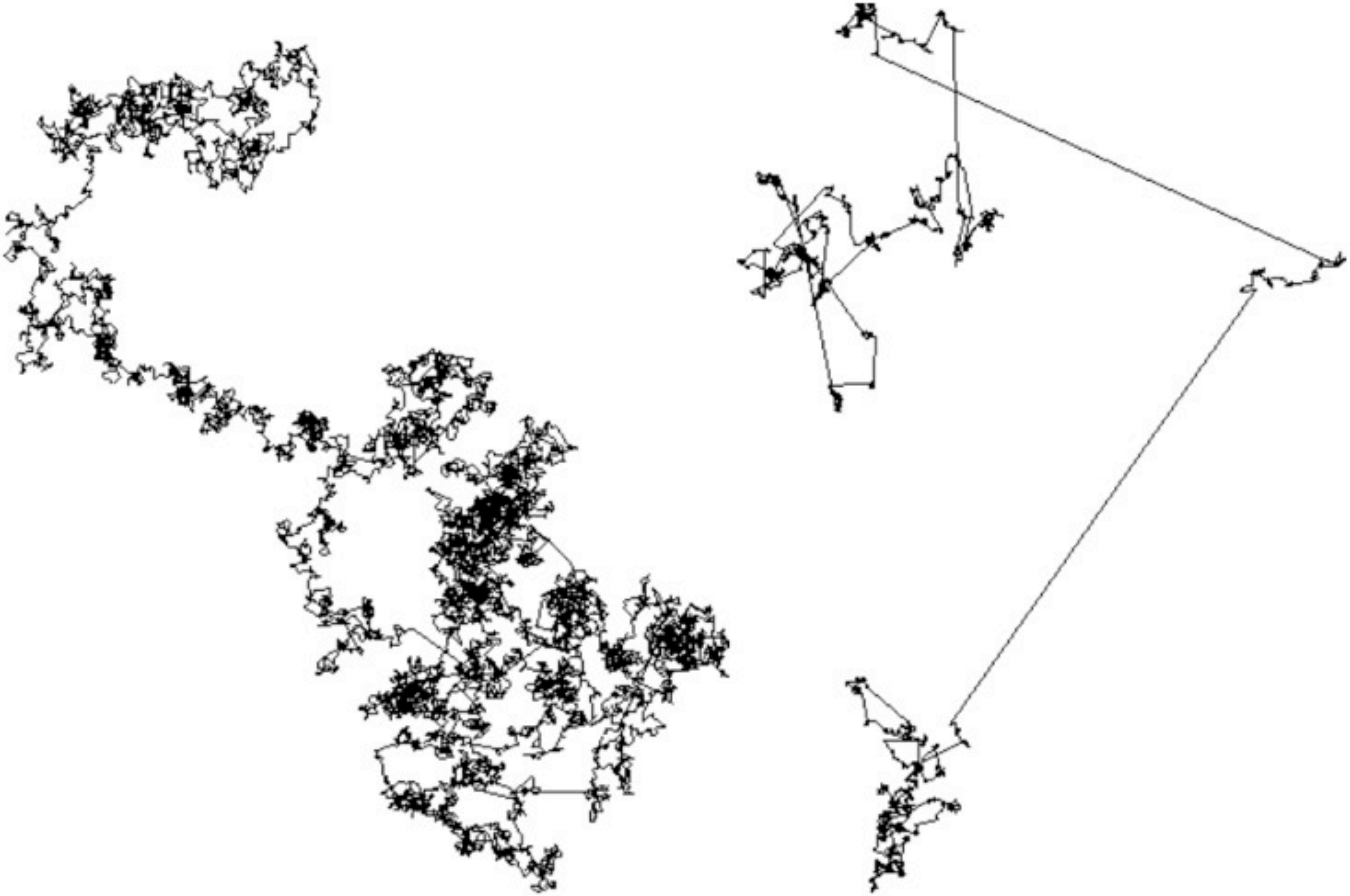
superdifusão

superdifusão

$$< (x(t) - x_0)^2 > \sim \ t^\gamma \qquad (\ \gamma > 1 : \text{Superdiffusion} \ )$$

- existência de correlações de longo alcance (no tempo) presentes na velocidade das partículas traçadoras ou vôos de Levy.

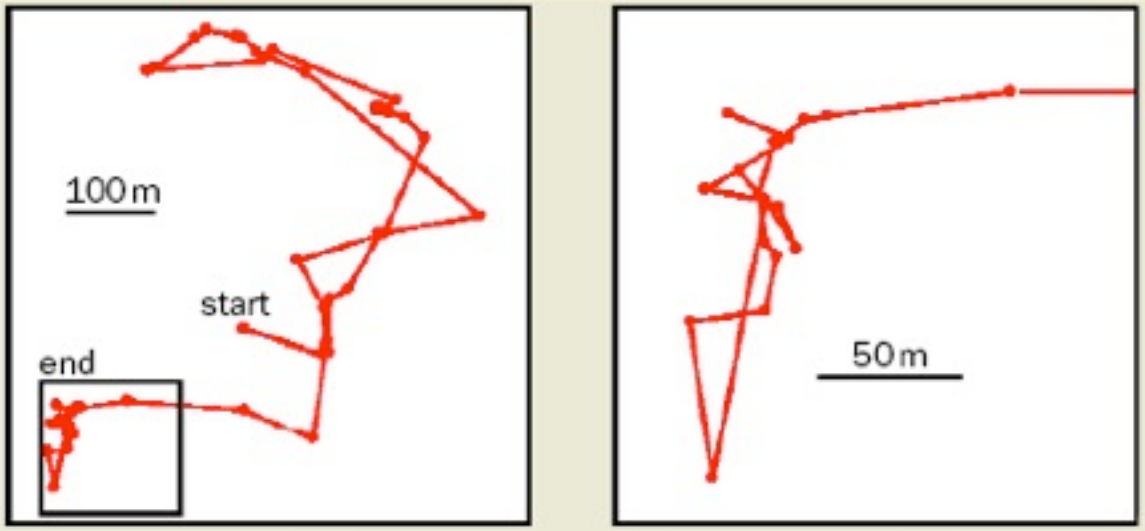
- difusão de Richardson em fluidos turbulentos
- difusão de micelas em água salgada
- vôo de albatroses
- bactéria, plancton, chacais, macacos aranha
- parece que a superdifusão supera a difusão normal (BM) como estratégia para encontrar comida localizada aleatoriamente, etc



difusão anômala -> superdifusão

- spider monkeys

4 Superdiffusion in monkey behaviour



The typical trajectories of spider monkeys in the forest of the Mexican Yucatan peninsula display steps with variable lengths, which correspond to a diffusive process that is faster than that of normal diffusion. An example of such a trajectory is shown on the left. A magnified part of it is shown on the right; this image looks qualitatively similar to the larger-scale trajectory, which is an important property of Lévy walks. Similar behaviour is found in the foraging habits of other animals, and could mean that anomalous diffusion offers a better search strategy than that of normal diffusion.

➔ modificações na eq. de FP linear

teorias fenomenológicas

- equações de Fokker-Planck não lineares
- equações de Fokker-Planck fracionárias
- eq. FP com coeficientes de difusão não-homogêneos
- ...

- equação FP com derivadas fracionárias - memória temporal longa

Use and Abuse of a Fractional Fokker-Planck Dynamics for Time-Dependent Driving

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(Received 14 March 2007; published 21 September 2007)

We investigate a subdiffusive, fractional Fokker-Planck dynamics occurring in time-varying potential landscapes and thereby disclose the failure of the fractional Fokker-Planck equation (FFPE) in its commonly used form when generalized in an *ad hoc* manner to time-dependent forces. A modified FFPE (MFFPE) is rigorously derived, being valid for a family of dichotomously alternating force fields. This MFFPE is numerically validated for a rectangular time-dependent force with zero average bias. For this case, subdiffusion is shown to become enhanced as compared to the force free case. We question, however, the existence of any physically valid FFPE for arbitrary varying time-dependent fields that differ from this dichotomous varying family.

A widely used approach to study subdiffusive processes is based on the fractional Fokker-Planck equation (FFPE) [9,10],

$$\frac{\partial}{\partial t}P(x,t) = {}_0\hat{D}_t^{1-\alpha}\left[-\frac{\partial}{\partial x}\frac{F(x)}{\eta_\alpha} + \kappa_\alpha\frac{\partial^2}{\partial x^2}\right]P(x,t). \quad (1)$$

Here, $F(x)$ is the force, η_α is the fractional friction coefficient, κ_α is the fractional free diffusion coefficient, and ${}_0\hat{D}_t^{1-\alpha}$ denotes the Riemann-Liouville fractional derivative,

$${}_0\hat{D}_t^{1-\alpha}\chi(t) = \frac{1}{\Gamma(\alpha)}\frac{\partial}{\partial t}\int_0^t dt'\frac{\chi(t')}{(t-t')^{1-\alpha}}. \quad (2)$$

- coeficiente de difusão inomogêneo

N·H
ELSEVIER

10 August 1998

Physics Letters A 245 (1998) 67–72

PHYSICS LETTERS A

Ito–Langevin equations within generalized thermostatistics

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Communicated by A.R. Bishop

The standard Fokker–Planck equation has the following form (cf. Ref. [14]),

$$\frac{\delta}{\delta t}P(x) = -\frac{\delta}{\delta x}[K(x)P(x)] + \frac{1}{2}\frac{\delta^2}{\delta x^2}[D(x)P(x)] \tag{3}$$

and describes the temporal evolution of the probability distribution P of the state variable x. The variable

- stariolo PLA (94),
-Kaniadakis&Quarati
 PhysA (97)

- equação de Fokker-Planck não linear

equações fenomenológicas

equação de meios porosos
(M. Muskat - 1937) $\longrightarrow$ $$\frac{\partial P(x,t)}{\partial t} = D\frac{\partial^2 P^{\nu}(x,t)}{\partial x^2}$$

$$\langle (x - \langle x \rangle)^2 \rangle \sim t^{\frac{2}{\nu+1}}$$

- A. R. Plastino e A. Plastino, Physica A 222 (1995) 347;

  C. Tsallis e Bukman D. J., PRE 54 (1996) R2197

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial \{F(x)P(x,t)\}}{\partial x} + D\frac{\partial^2 P^{\nu}(x,t)}{\partial x^2}$$

$$F(x) = -\frac{d\phi}{dx}$$

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial\{F(x)P(x,t)\}}{\partial x} + D\frac{\partial^2 P^\nu(x,t)}{\partial x^2}$$

$$F(x) = -\frac{d\phi}{dx}$$

solução estacionária $(\nu = 2 - q)$

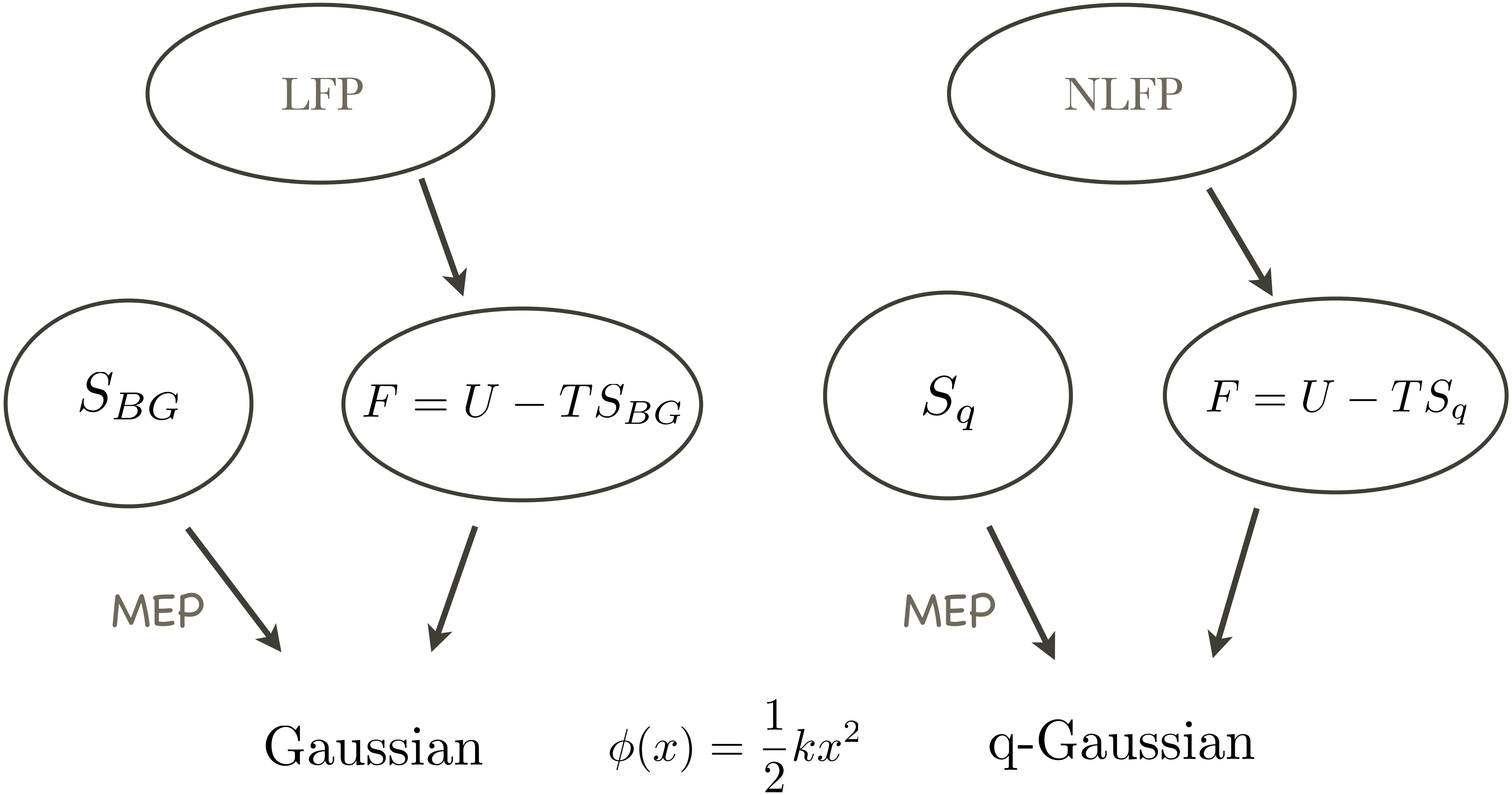
$$P(x) = C[1 - \beta(1-q)\phi(x)]^{1/(1-q)}$$

$$\beta = (1/D)[C^{q-1}/(2-q)] \qquad (C \text{ is a positive constant})$$

mesma distribuição que maximiza a entropia de Tsallis com o vínculo externo $\phi(x)$!

NLFPE <-> entropia de Tsallis!

FPE ⟷ entropia



- equação de Langevin

PHYSICAL REVIEW E VOLUME 57, NUMBER 6 JUNE 1998

Microscopic dynamics of the nonlinear Fokker-Planck equation: A phenomenological model

Lisa Borland*
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(Received 16 December 1997)

We derive a phenomenological model of the underlying microscopic Langevin equation of the nonlinear Fokker-Planck equation, which is used to describe anomalous correlated diffusion. The resulting distribution-dependent stochastic equation is then analyzed and properties such as long-time scaling and the Hurst exponent are calculated both analytically and from simulations. Results of this microscopic theory are compared with those of fractional Brownian motion. [S1063-651X(98)00206-2]

PACS number(s): 66.10.Cb, 05.20.−y, 05.60.+w, 05.40.+j

isting theory. The resulting Ito-Langevin equation has the form

$$\frac{dx}{dt}=K(x,t)+\sqrt{Q}f(x,t)^{(\nu-1)/2}\eta(t), \tag{18}$$

where the evolution of f is given by the Fokker-Planck equation of equation (2). A trajectory of Eq. (18) is determined by both equations simultaneously. It is apparent that there is feedback from the macroscopic level of description of the system in terms of the probability distribution f to the microscopic kinetics.

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Computing the non-linear anomalous diffusion equation from first principles

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Abstract

We investigate asymptotically the occurrence of anomalous diffusion and its associated family of statistical evolution equations. Starting from a non-Markovian process *à la* Langevin we show that the mean probability distribution of the displacement of a particle follows a generalized non-linear Fokker–Planck equation. Thus we show that the anomalous behavior can be linked to a fast fluctuation process with memory from a microscopic dynamics level, and slow fluctuations of the dissipative variable. The general results can be applied to a wide range of physical systems that present a departure from the Brownian regime.

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- equação mestre

equação mestre -> equação de Fokker-Planck NL

$$\frac{\partial P(n,t)}{\partial t} = \sum_{m=-\infty}^{\infty} \left[P(m,t) w_{m,n}(t) - P(n,t) w_{n,m}(t) \right]$$

- taxas de transição não lineares -> equação de Fokker-Planck não linear

$$\omega_{m,n}(t) \longrightarrow \omega_{m,n}(P,t)$$

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial \{F(x)\Psi[P(x,t)]\}}{\partial x} + \frac{\partial}{\partial x}\left\{ \Omega[P(x,t)] \frac{\partial P(x,t)}{\partial x} \right\}$$

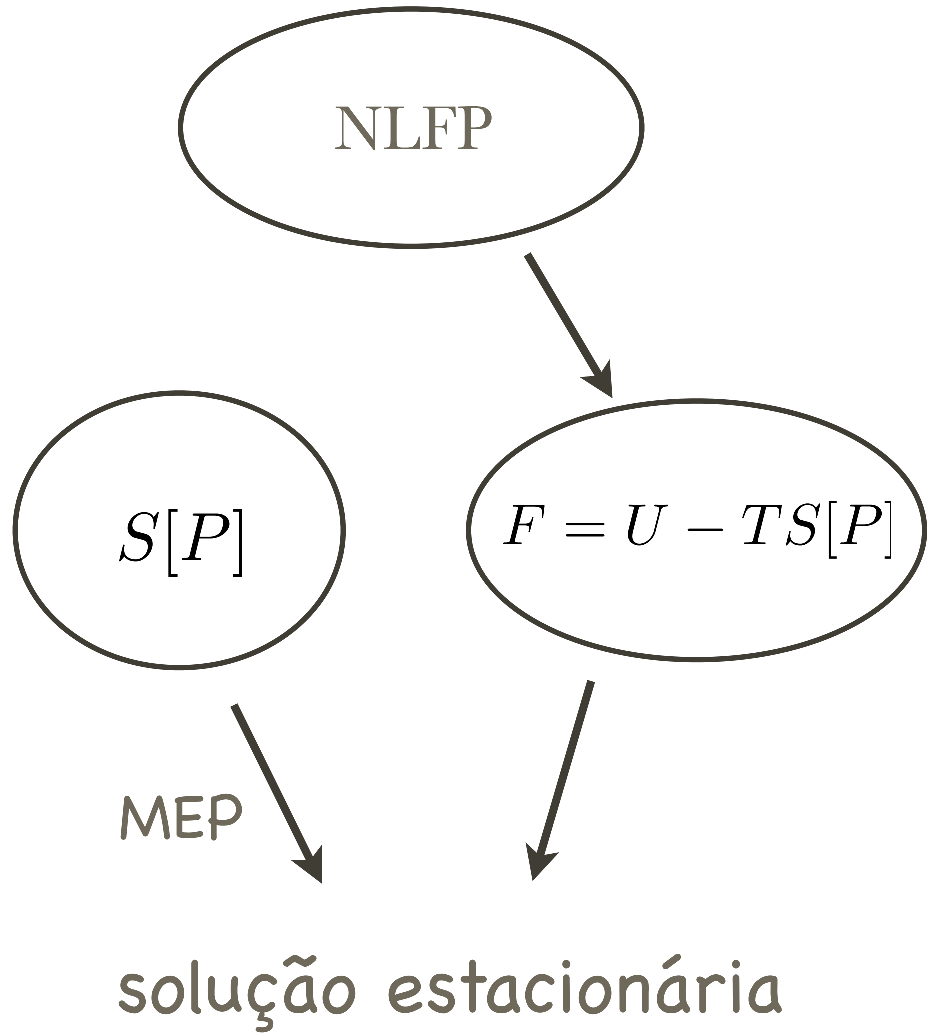
teorema H

$$S[P] = \int_{-\infty}^{\infty} g[P(x,t)]\, dx$$

$$\frac{d^2 g}{dP^2} \leq 0$$

$$-\frac{1}{\beta}\frac{d^2 g[P]}{dP^2} = \frac{\Omega[P]}{\Psi[P]}$$

$$\frac{dF}{dt} \leq 0$$



família de EFPs <-> entropias

$$-\frac{1}{\beta}\frac{d^2g}{dP^2} = \frac{\Omega[P]}{\Psi[P]}$$

relação entropia <-> FPE

mesma razão $\frac{\Omega[P]}{\Psi[P]}$ mesma entropia!

seja

$\Omega[P] = a[P]b[P]\ ; \quad \Psi[P] = a[P]P$ → liberdade do funcional a[P]

$$\frac{d^2g[P]}{dP^2} = -\beta\frac{\Omega[P]}{\Psi[P]} = -\beta\frac{a[P]b[P]}{a[P]P} = -\beta\frac{b[P]}{P}$$

Schwammle V, Nobre FD, EMFC, PRE 2007

Schwammle V, EMFC, Nobre FD, EPJB 2007

- atratores gaussianos - BG

NLFPEs <-> entropia de Boltzmann-Gibbs

$$\frac{d^2 g[P]}{dP^2} = -\beta \; \frac{b[P]}{P}$$

$$b[P] = D$$

$$\beta = k_B/D$$

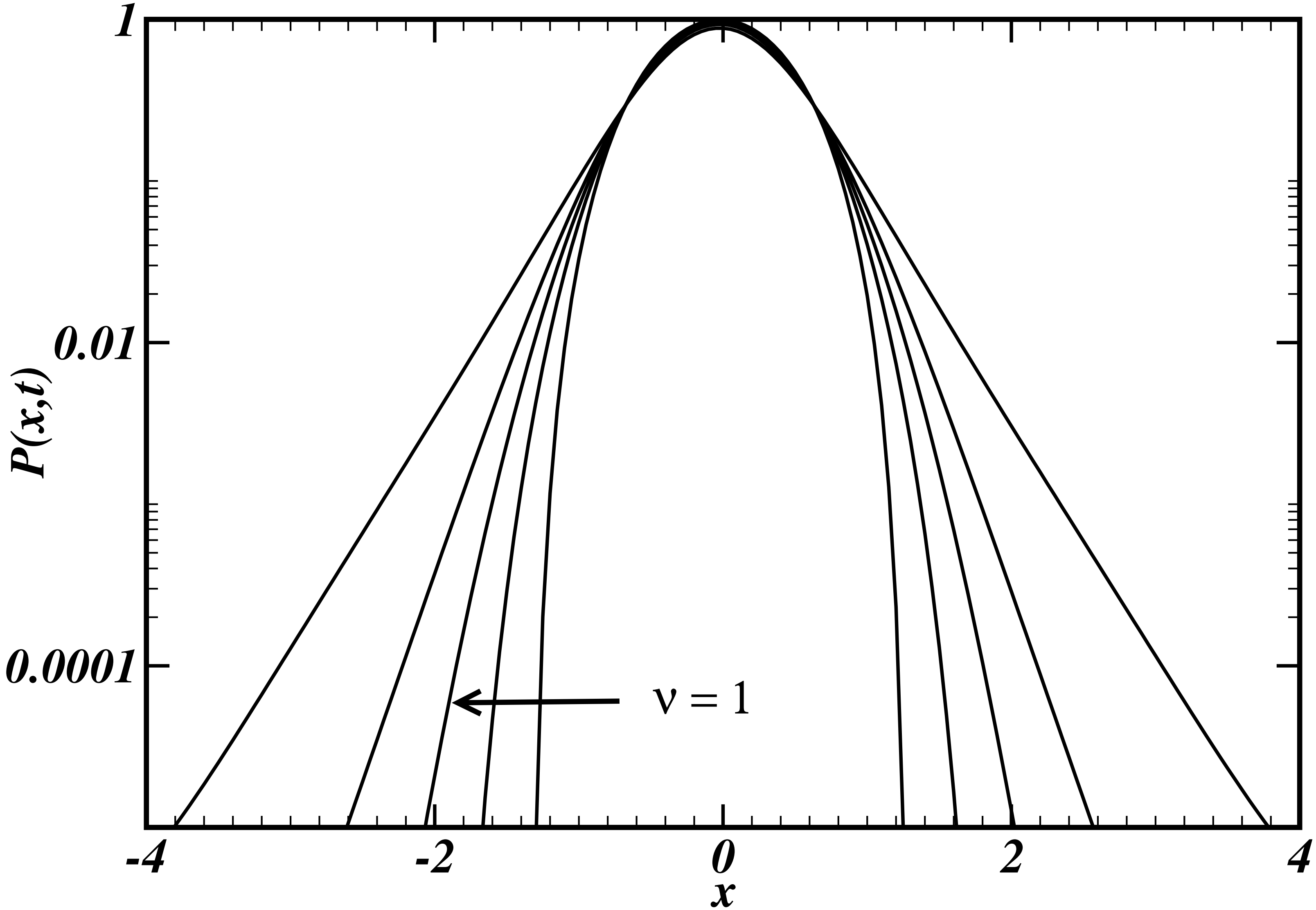
$$\frac{dg}{dP} = -\beta D \ln P + C \quad \Rightarrow \quad g[P] = -k_B P \ln P$$

$$S = -k_B \int P(x) \ln P(x) dx$$

$$\text{if} \quad a[P] \propto P^{\nu-1}$$

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial}{\partial x}\left(F(x)P(x,t)^{\nu}\right) + D\frac{\partial}{\partial x}\left(P(x,t)^{\nu-1}\frac{\partial P(x,t)}{\partial x}\right)$$

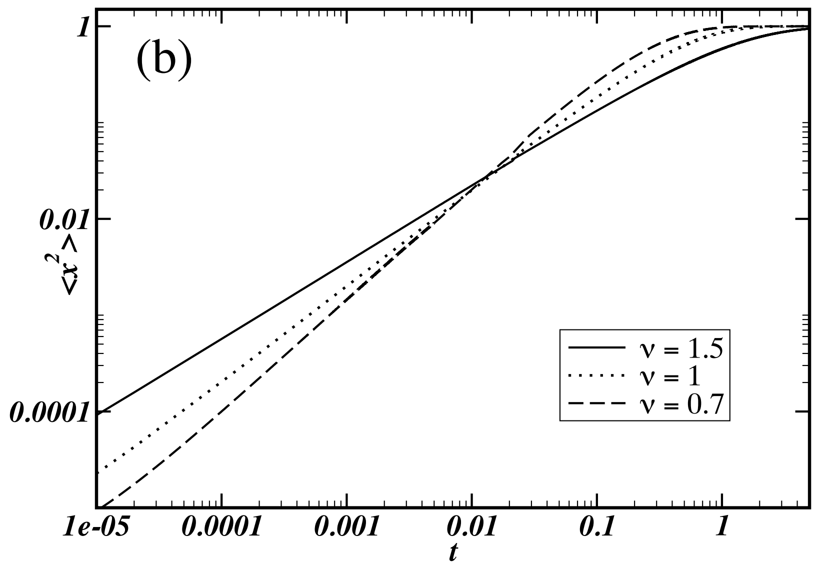
$\nu = 1$ ⟶ equation de Fokker-Planck



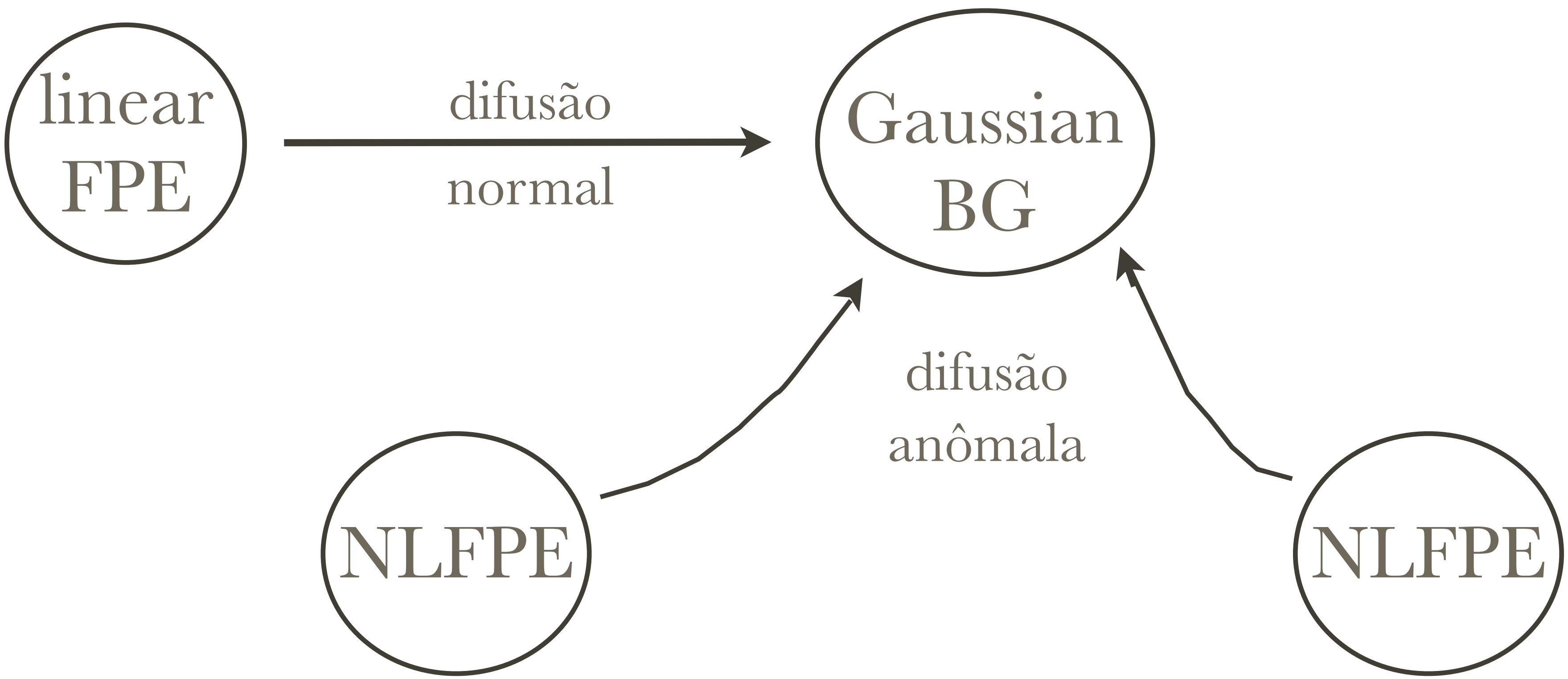
$$F(x) = -kx$$
$$P(x,0) = \delta(x)$$

$$t = 0.1$$
$$\nu = 0.7, 0.9, 1, 1.1, 1.25$$

$$b[P] = D$$



$$\langle x^2 \rangle \sim \left(\frac{2}{\nu^2}\right) t^{\frac{2}{\nu+1}}$$



- atratores q-gaussianos

NLFPEs <-> entropia de Tsallis

$$\frac{d^2 g[P]}{dP^2} = \frac{\Omega[P]}{\Psi[P]} = -\beta \, \frac{b[P]}{P}$$

$$b[P(x,t)] = D\nu P(x,t)^{\nu-1}$$

$$g[P] = k_B \frac{P - P^{\nu}}{\nu - 1}$$

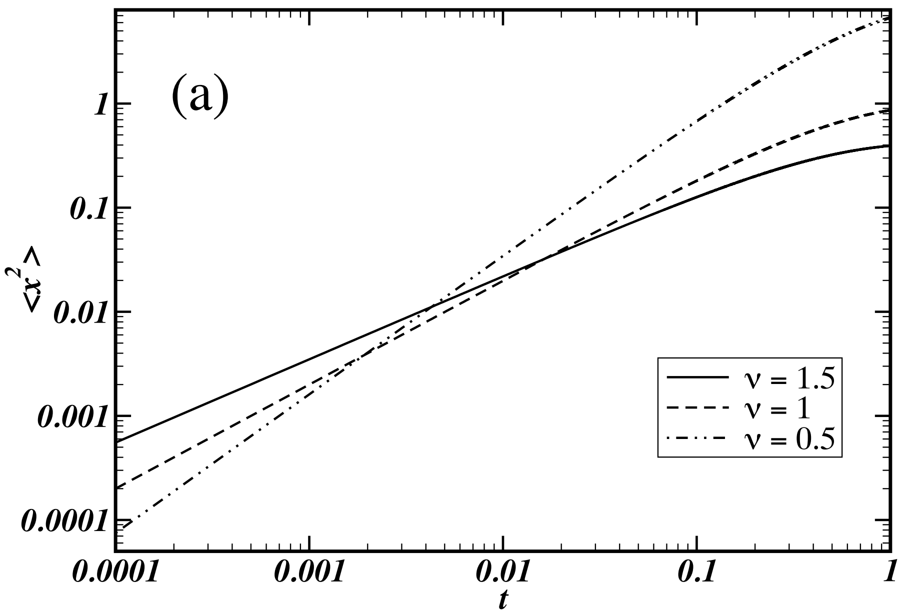
$$\text{if} \quad a[P] \propto P^{\mu-1}$$

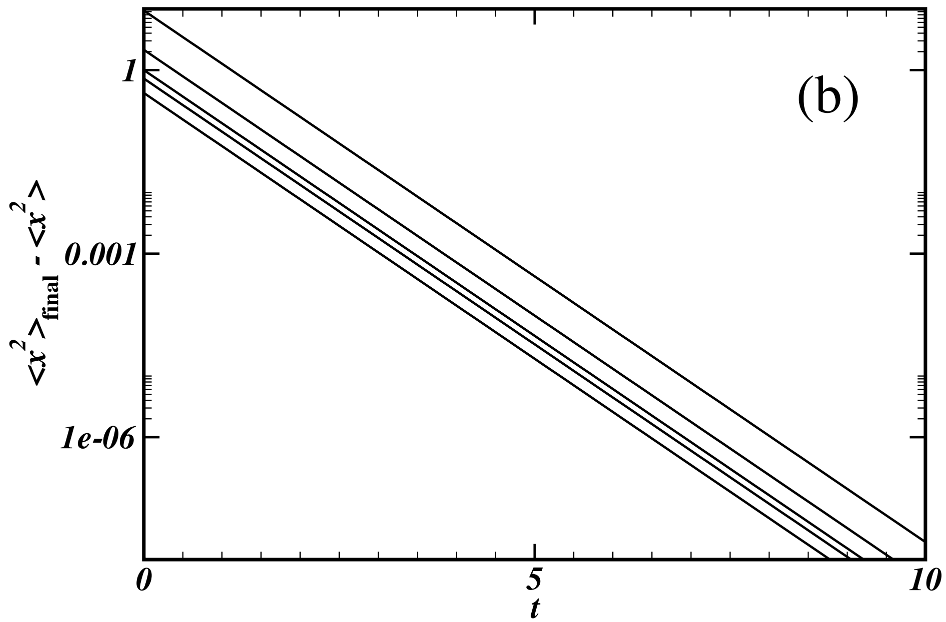
$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial}{\partial x}\left(F(x)P(x,t)^{\mu}\right) + D\frac{\partial}{\partial x}\left(P(x,t)^{\mu+\nu-2}\frac{\partial P(x,t)}{\partial x}\right)$$

$\mu = 1 \longrightarrow$ P&P-PhysA1995

a) $\mu = 1$ (NLFPE - Plastino&Plastino1995)

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial}{\partial x}\left(F(x)P(x,t)\right) + D\frac{\partial}{\partial x}\left(P^{\nu-1}(x,t)\frac{\partial P(x,t)}{\partial x}\right)$$





$$< x^2 > \propto \; t^{2/(\nu+1)}$$

$$< x^2 >_{\mathrm{final}} - < x^2 > \propto e^{-(\nu+1)t}$$

Flux Front Penetration in Disordered Superconductors

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We investigate flux front penetration in a disordered type-II superconductor by molecular dynamics simulations of interacting vortices and find scaling laws for the front position and the density profile

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for short-range attractive pinning forces such as the one we are investigating. In this case, short wavelength modes yield a macroscopic contribution to pinning that cannot be neglected. Consider, for instance, the flow between two coarse-grained regions: short-range microscopic pinning forces give rise to a macroscopic force that should always oppose the motion, while the random force derived above could, in principle, point in the direction of the flow. In other words, $F_c(\vec{r})$ should be considered as a *friction* force [21] whose direction is always opposed to the driving force $\vec{F}_d$ (in our case $\vec{F}_d = a\vec{\nabla}\rho$) and whose absolute value is given by $|g\vec{\nabla}n|$ for $|\vec{F}_d| > |g\vec{\nabla}n|$ and to $|\vec{F}_d|$ otherwise [22].

Collecting all the terms, we finally obtain a disordered nonlinear diffusion equation for the density of flux lines

$$\Gamma \frac{\partial \rho}{\partial t} = \vec{\nabla}(a\rho\vec{\nabla}\rho - \rho\vec{F}_c) + k_B T \nabla^2 \rho . \qquad (5)$$

The boundary conditions representing our MD simulations

crossover length scaling as $\xi_p \sim g_0^{-1/2}$, in agreement with MD simulations (see Fig. 3). In addition, we measure the density profiles and find that they rescale with g_0 in the same way as in MD simulations.

The numerical integration of the diffusion equation allows for a direct analysis of the fluctuations in the front as a function of different internal parameters. Measuring the width W of the fronts as a function of time for different values of g_0, we find that in the initial stage W grows as a power law t^β where $\beta \simeq 0.35$ until it saturates to a value that decreases with g_0. Thus the front crosses over from flat to fractal as it enters into the material. In principle, we can control the strength of the fluctuations and the associated characteristic length ξ^* by tuning g_0, which directly reflects experimentally measurable parameters.

In order to compare the model with experiments, we have to implement appropriate boundary conditions. In Refs. [5,6] the external field was ramped at a constant rate, which corresponds to a constant increase of the boundary

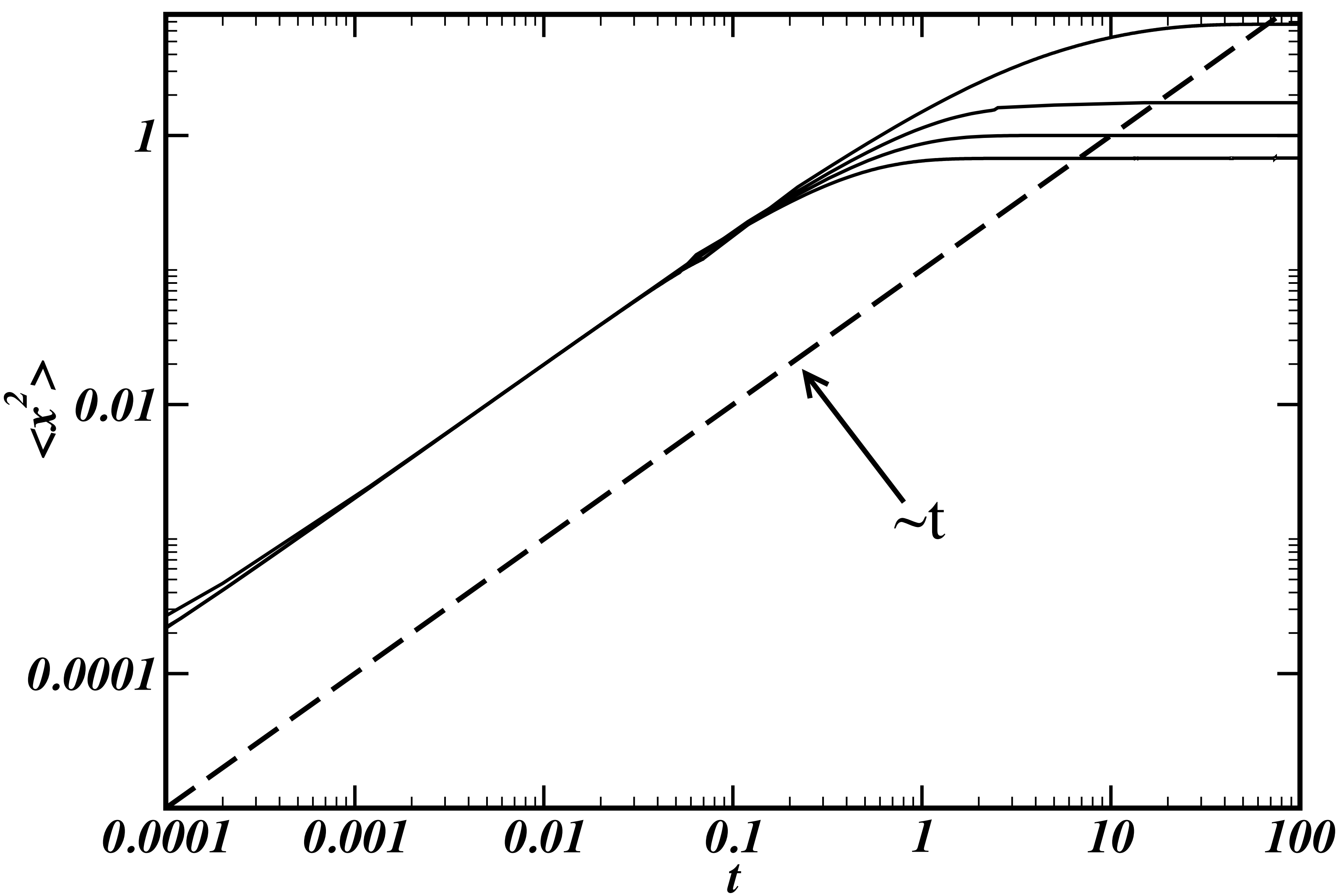
quarta-feira, 10 de março de 2010

EFP não linear -> difusão normal

b) $\nu = 2 - \mu \quad (\mu \neq 1)$

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial}{\partial x}\left(F(x)P(x,t)^{\mu}\right) + D\frac{\partial^2 P(x,t)}{\partial x^2}$$

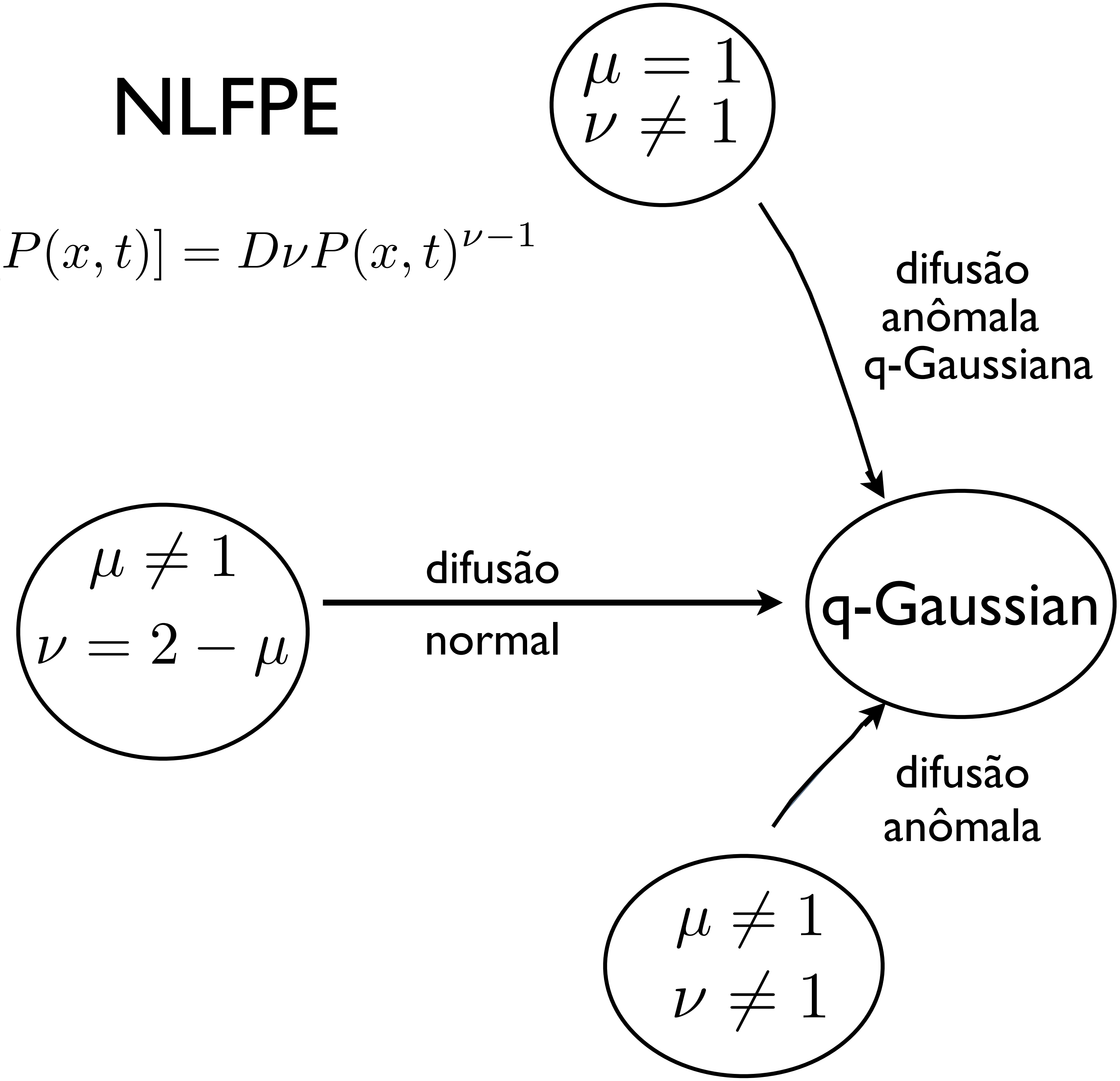
- solução estacionária -> q-Gaussiana



$$\mu = 0.7, 1, 1.2, 1.5$$

NLFPE

$$b[P(x,t)] = D\nu P(x,t)^{\nu-1}$$



observações finais

- NLFPE -> difusão normal e anômala
- NLFPEs <-> entropias <-> MEP
- eq. Langevin -> NLFPE (2)
- eq. mestre -> NLFPE
- NLFPE -> fenômenos físicos diferentes