Controlling self-organized criticality

D. O. Cajueiro[1,3] R. F. S. Andrade[2,3]

[1]Department of Economics, Universidade de Brasília.

[2]Instituto de Física, Universidade Federal da Bahia.

[3]National Institute of Science and Technology for Complex Systems.

INCT-SC Meeting

Motivation

☺ Why???

- Very large energy dissipating events affects living populations in the environment where they occur.

Introduction

⇒ Control means...

- "Control" can be understood as a series of man-devised actions to interfere in the processes by which the system dissipates energy, in such way as to concentrate dissipation in moderate sized events and reduce the occurrence probability of very large avalanches.

Introduction

⇒ What is possible to do?

- The difficulties to control large events like earthquakes, hurricanes, floods and so on, depend both on the magnitude of the stored energy as well as on the impossibility of interfering, in appropriate way, in the dynamics of energy dissipating events.
- Under certain limits, other events following SOC statistics are already subject to human control, e.g., the series of induced avalanches in restricted hill slides, where the purpose is to warrant safety for ski riders.
- Similar control can reduce crisis caused by the break of large economic bubbles.

Main idea

[Cajueiro and Andrade. Controlling self-organized criticality in sandpile models. Phys. Rev. E 81, 015102(R), 2010.]

The control scheme, devised to avoid large avalanches in a pre-selected restricted area of the system, is divided into two different stages:

- First stage: The control just learns about the dynamics of the system and acquires a global estimate of avalanche risk in the pre-selected area.
- Second stage: The control scans the preselected region and identifies potentially large events whenever the avalanche risk is high enough. Once a threat is detected, an externally induced avalanche is triggered.

The system

- We consider the two dimensional system Γ schematically represented by the array

$$\Gamma = \begin{bmatrix} O & O & O & O & O & O & O \\ O & O & O & O & O & O & O \\ O & O & T_L & T & T_R & O & O \\ O & O & L & X & R & O & O \\ O & O & B_L & B & B_R & O & O \\ O & O & O & O & O & O & O \\ O & O & O & O & O & O & O \end{bmatrix}.$$

  where each element of Γ indicated by O, T_L, T, T_R, L, X, R, B_L, B and B_R represents by itself a fixed size square region of sites, corresponding to smaller arrays of order $N_R \times N_R$.
- Avalanche size control takes place inside region X only.

Some important definitions

- Controlled $\times$ uncontrolled avalanches
- Internal $\times$ external avalanches

The control scheme: first stage

- In the first control stage, one has to estimate the conditional probability $p_{K/J}(t+1/t)$ of occurring the addition of mass in region $K \in \mathcal{R}$ at time $t+1$ assuming mass was added on a site in region $J \in \mathcal{R}$ at time t.

The control scheme: second stage

- In the second stage, such estimates lead to the definition of a threshold value p_c that decides whether the control should be activated whenever a new mass unit is deposited in a given region of Γ. If at time t, mass is added on the region $J \in \mathcal{R}$ and $p_{X/J}(t+1/t) \geq p_c$, then the control should be activated, where

$$\begin{aligned} p_c \quad = \quad & \min(p_{X/T_L}(t+1/t), p_{X/T}(t+1/t), \\ & p_{X/T_R}(t+1/t), p_{X/L}(t+1/t), \\ & p_{X/R}(t+1/t), p_{X/B_L}(t+1/t), \\ & p_{X/B}(t+1/t), p_{X/B_R}(t+1/t)). \end{aligned}$$

The control scheme: second stage

- Such activation requires to follow any virtual avalanche that would occur inside the region X if any of the sites in X were actually chosen at random.
- In order to follow the virtual avalanches, we consider an ☹ *internal replica* Γ_X of the system, i.e., a restricted copy of the model that describes its dynamics inside the region X, as if it was isolated from the rest of Γ.
- Based on this replica of X, if any added particle in site $(i, j) \in X$ generates a virtual avalanche of size $a \geq a_c$, the control "explodes" the corresponding site of Γ.
- This means that a real avalanche is triggered by emptying the site (i, j), which amounts to topple the single unit mass with 50% of probability to the site $(i + 1, j)$ or to the site $(i, j + 1)$.

Modification of the deposition process

- We have changed the nature of the mass deposition process in order to consider a weighted deposition: if at time t, a particle was deposited on the site (i,j), the probability to select the site (k,ℓ) to add the particle at $t+1$ is

$$P[(k,\ell)/(i,j)] = \frac{A}{(\delta[(i,j),(k,\ell)]/B)^{\gamma}}, \tag{1}$$

  where $\delta[(i,j),(k,\ell)]$ is the Euclidian distance between sites (i,j) and (k,ℓ), while A and B are constants related to the normalization of P and to the largest distance between any two sites on the system.
- The correlated deposition rules can be justified by the existence of a natural time correlation in rain, snow, social and financial events.

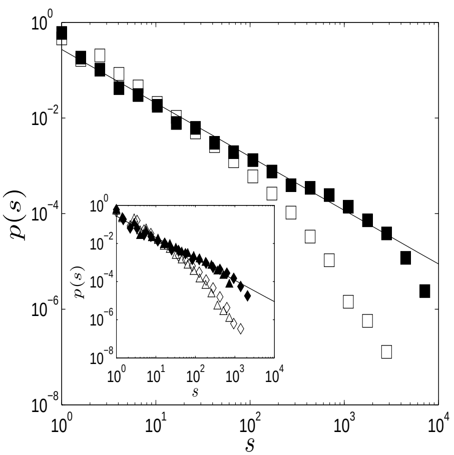
The selection of a target size

- It is necessary to select a target size a_c, which is a choice for the largest natural avalanche that might occur in the system.
- Of course $a_c > 1$, otherwise we would have to release down hill the added mass grain at each time unit.

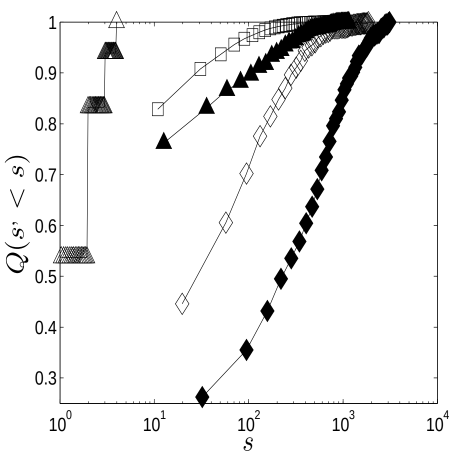
The algorithm

for all time: **do**
  Assume that the deposition process is working at a region $J \in \mathcal{R}$;
  if $p_{X/J}(t+1/t) \geq p_c$ **then**
    for all $(i,j) \in X$ **do**
      A virtual avalanche is triggered in $(i,j) \in X$ using the replica model of the region X;
      The size of the virtual avalanche s is evaluated;
      if $s > a_c$ **then**
        A real avalanche is triggered;
      end if
    end for
  end if
end for

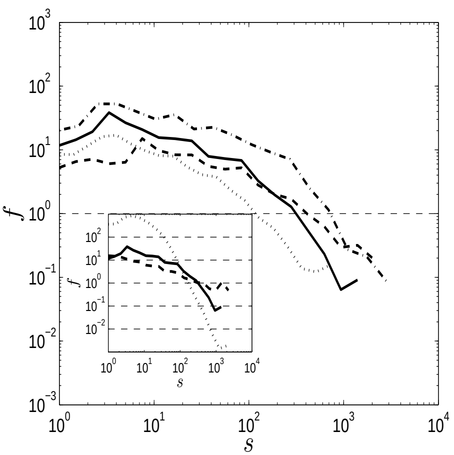
Results



Results



Results



Final remarks

☺ Controlling SOC works! However:

- The control scheme assumes a perfect model of the reality. How to design robust control schemes that can work well even when one does not have a good model of the reality?
- How to define controllability and observability in these classes of controlled systems?

Final remarks

- How to use information of the system such as the mass cumulated in the system or the data related to the internal structure to predict the time of the intervention? ☺ In complex networks, we can do that!
- It would be fascinating to reply this kind of methodology in real systems!
- How to choose optimally the sites to be triggered? ☺ For tiny systems, we can do that!
- Analytical results are welcome!!!!!!!!