Switching points in economic series: Asymmetric tendencies and long range correlations

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Physica A 391 170 (2012)
Physica A 391 1489 (2012)

Outline

- Financial markets
- Switching events in time series
- Asymmetries between positive and negative tendencies
- Local roughness exponents
- Smoothing kernels
- Evaluation of tops and bottoms
- Long range correlations and probability distributions
- Conclusions

Financial markets

- Financial markets as typical complex systems (CS):
  - Large number of agents (degrees of freedom)
  - Large amount of information (global fields)
  - Different perspectives (local force rules)
  - Conflicting interests (agent-agent interaction)
- Forces of different nature
- Small disturbances may result in large effects
- Stochastic nature of system outputs
- Large amount of actual data from market records helps measure, understand, and predict CS

Switching events in time series

- Time series ↔ primary information source
- Market fluctuations ↔ infer dynamical behavior
- Trends ↔ specific features of economic dynamics
  - Upward trends (“bubbles”)
  - Downward trends (“financial collapse”)
- Change at most different scale times
  - Macroscopic bubbles persisting for hundreds of days
  - Microscopic bubbles persisting for only seconds

Switching events in time series

- Non-stationary series
- Trends for persistent rise or fall of prices change
- Switching points (SP) concept (Preis and Stanley, PNAS 2011) → change from negative to positive trend
- Typical events in any generic complex system
- How to identify and measure?
- Original definition: SP event identified by a very large value of the return variance

Switching events in time series

- Can other features present in the records be used to detect SP's?
- This presentation: two possible approaches based on previously introduced tools
- Asymmetric detrended fluctuation analysis(A-DFA) (Ramirez, Rodriguez, and Echeverria, Physica A 2009)
- Top-bottom approach with smoothing kernels (Lo, Mamaysky, and Wang, J. Finance 2000)

Asymmetric tendencies

- Upward trends with distinct time scale (slow) as compared to fast market crashes
- Look for measures to detect asymmetries in raising and decaying trends
- Asymmetric detrended fluctuation analysis(A-DFA)
- Separates fluctuation contributions according to local trend character

Asymmetric tendencies

- Fluctuations casted into two groups in all different scales → two new scaling exponents (H^+ and H^-)
- Symmetric series with respect to the trends → $H^+ = H^- = H$ (usual roughness or Hurst exponent H)
- Otherwise, A-DFA assigns asymmetric character
- Upward trends with distinct time scale (slow) as compared to fast market crashes

Asymmetric tendencies

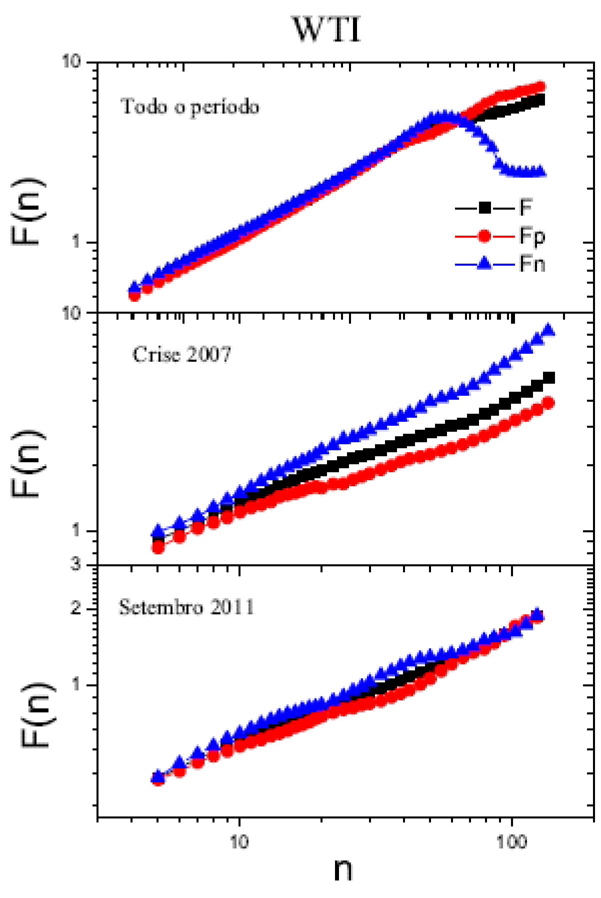
- Series of equidistant increments $\{x(t)\}, t = 1, \dots, N$.
- $y(t) = \sum_{j=1}^{t} x(j)$
- Divide interval $[1, N]$ into a series of M_n boxes of length n labeled by (m,n)
- Evaluate fluctuation $y_s(t) = y(t) - p_1(t; (m, n)$
- Evaluate the residue $f(m, n) = \frac{1}{n} \sum_{j \in (m,n)} {y_s}^2(j)$
- Take the average $F(n) = \left[\frac{1}{M_n} \sum_m f(m, n)\right]^{1/2}$
- Check whether $F(n) \sim n^H$

Asymmetric tendencies

- Evaluate also increment fluctuations
- $x_s(t) = x(t) - r_1(t; (m,n)), \quad r_1(t) = c\,t + d$
- Identify local trend by the sign of c
- Define acordingly two box sets B^+ and B^-
- Two further averages
  $F^{\pm}(n) = \left[\frac{1}{M^{\pm}{}_n} \sum_{m \in B^{\pm}} f(m,n)\right]^{1/2}$
- Check whether $F^{\pm}(n) \sim n^{H^{\pm}}$

Asymmetric tendencies

- Example of the WTI oil price series



Local roughness exponents

- Local A-DFA
- Combines changes to detect trend asymmetry with local dependency of the exponent $H(t)$
- Replace N by window width $L+1$
- Evaluate $H(t)$, $H^{+}(t)$, and $H^{-}(t)$ taking $L/2$ points to the left and $L/2$ to the right of point t
- Existence of a width limit for event localization
- Validity of $F^{\pm}(n) \sim n^{H^{\pm}}$ with n restricted to $L/4$
- Minimum of 5 points $\rightarrow L \geq 40$

Local roughness exponents

- Example for a Weierstrasse function

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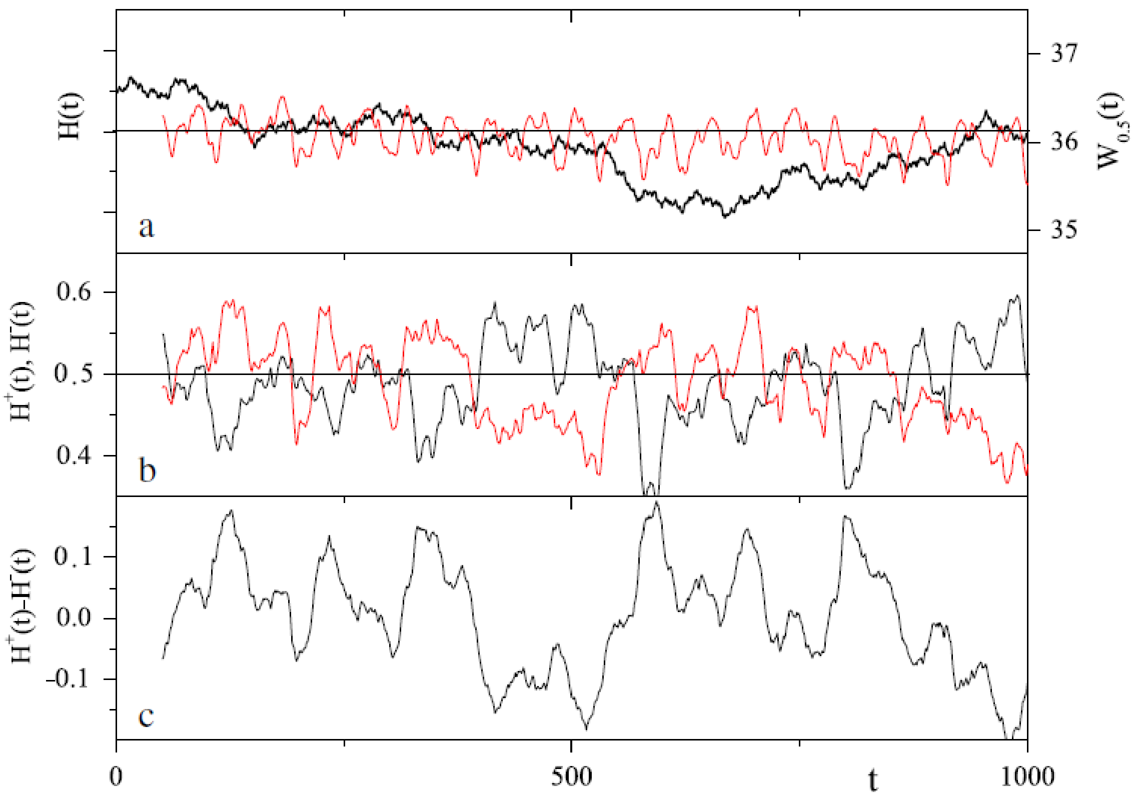

Local roughness exponents

- How precisely $H^{+}(t)$ and $H^{-}(t)$ localize SP's ?

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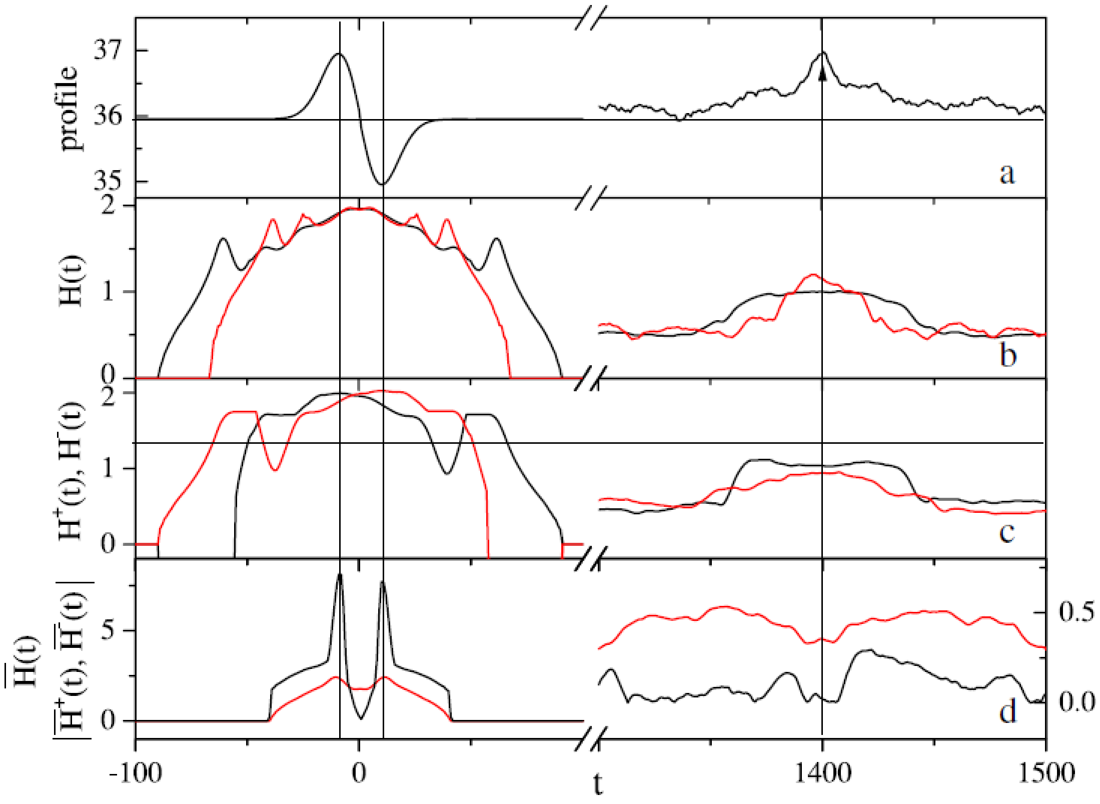


Local roughness exponents

- To coincide extreme of signal and H shift the coordinate of $H(t)$ by $\pm L/2$
- To reduce the production of satellite replace the arithmetic by geometric average
- Define a combination of $H(t)$ values

$$\bar{H}^{\pm}(t) = H^{\pm}(t + L/2)H^{\pm}(t - L/2)$$

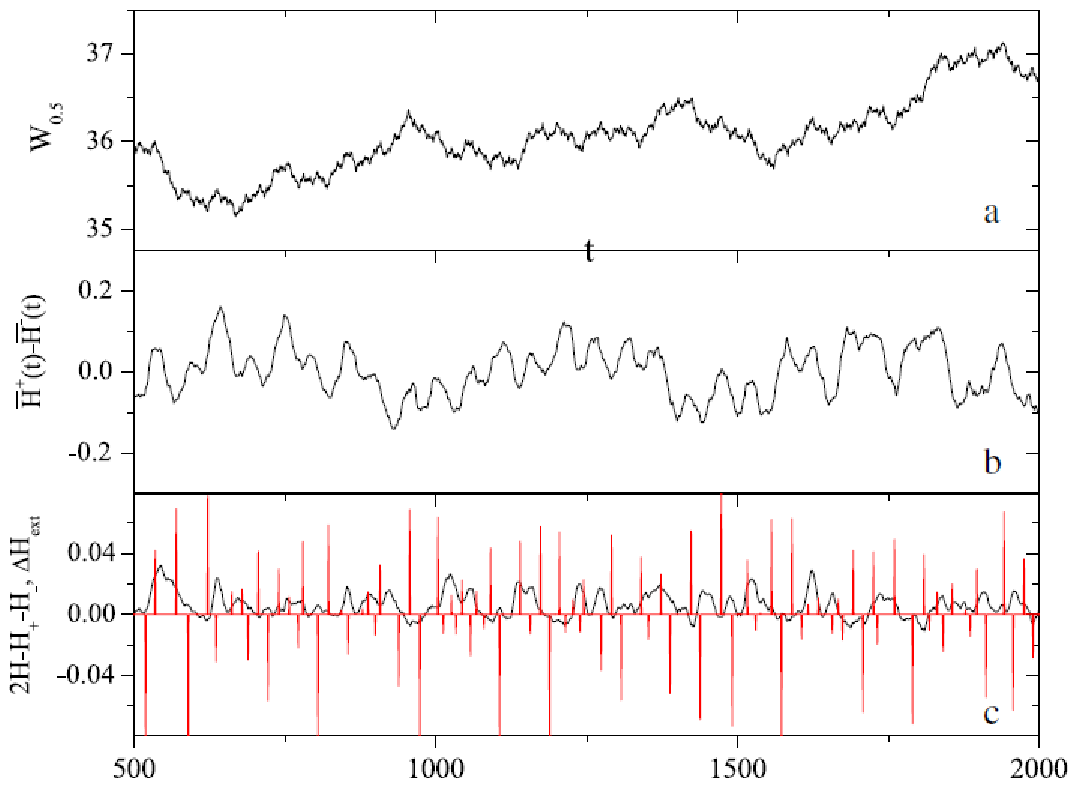
- Compute combination of $\bar{H}$ values

$$\bar{H}^{+}(t) - \bar{H}^{-}(t) \ ,$$

$$2\bar{H}^{\pm}(t) - \bar{H}^{+}(t) - \bar{H}^{-}(t)$$

Local roughness exponents

- How precisely $\overline{H}^{+}(t)$ and $\overline{H}^{-}(t)$ localize SP's ?



- Peaks are single out by $\Delta\overline{H}_{ext}$

Local roughness exponents

- Investigated data sets

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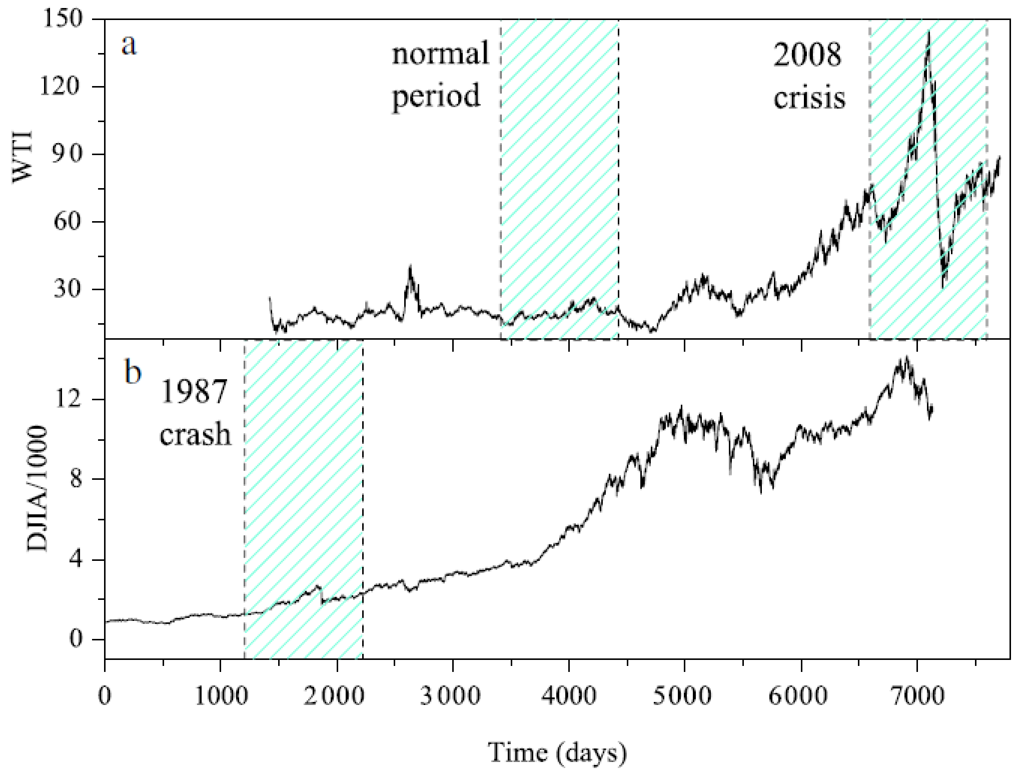

Local roughness exponents

- Identification of SP's for different values of L and threshold values $\Delta\bar{H}_{crit}$

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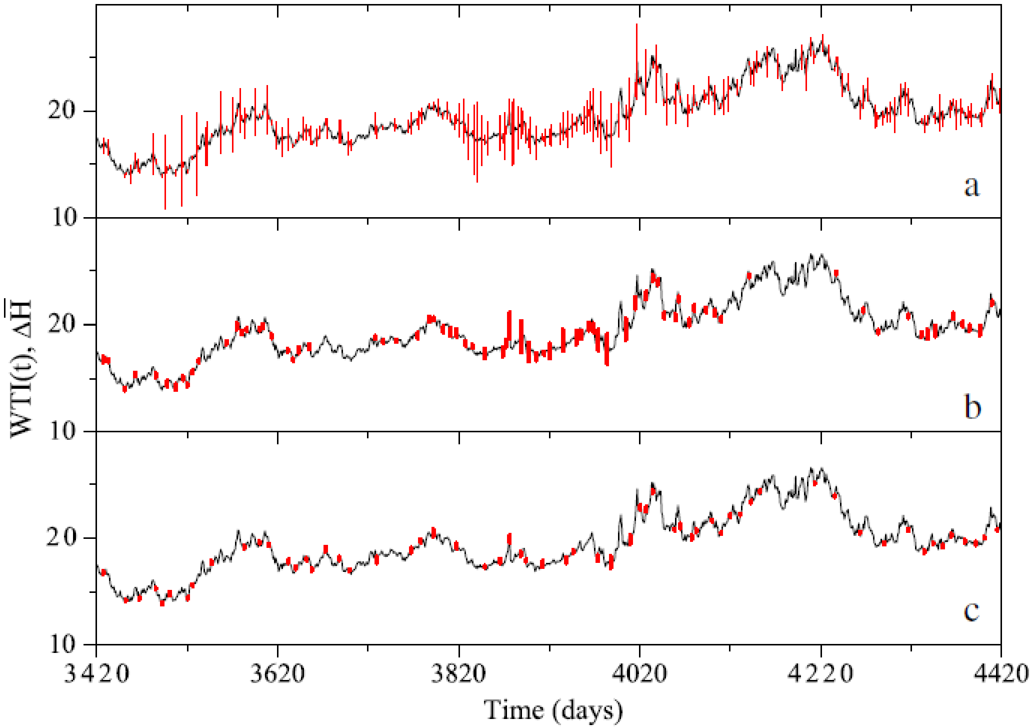

Local roughness exponents

- Magnitude probability distribution of SP events

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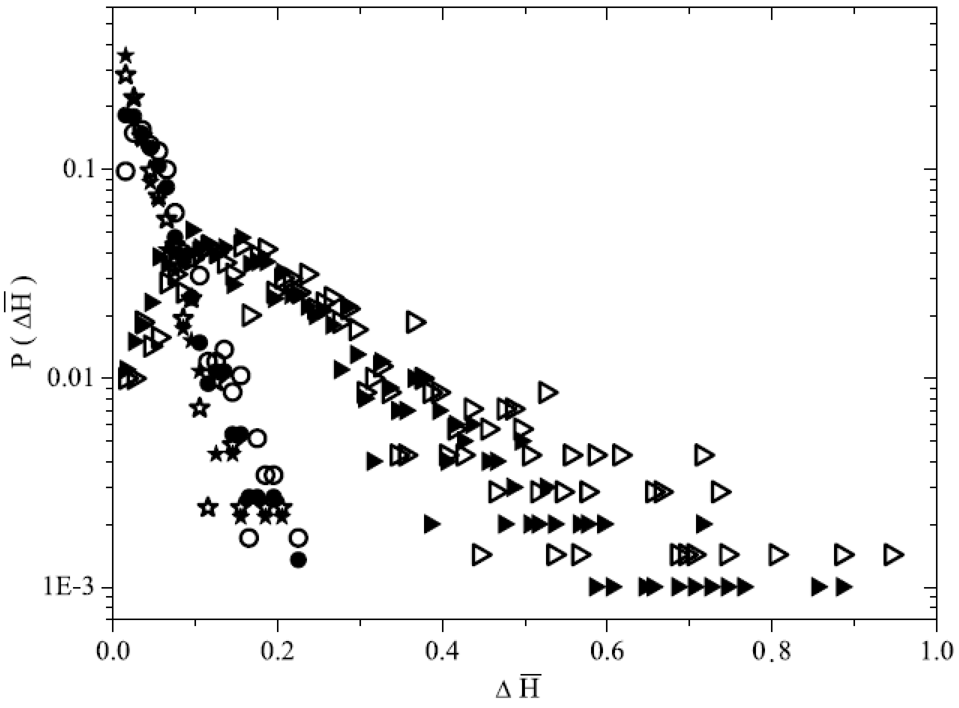


- Peaks are single out by

Local roughness exponents

- Measures $\overline{H}^{\pm}(t)$ are able to identify SPs in actual and deterministic series
- Sufficiently tunable framework by small number of parameters L, ℓ, and ΔH_c
- Choose two independent length scales under which the system can be analyzed
- Number and magnitude of SPs related by the preselected values of the quoted three parameters,
- More relevant events can be filtered accordingly
- Exponential decay of $P(\Delta\overline{H})$

Tops and bottoms

- Top–bottom (TB) price approach
- Local extreme in price series
- Top price → asset is sold by a too high value
- Bottom price → asset is sold by a too low value
- Identify TB ↔ SP
- Actual local extreme ↔ changes of expectations
- Another attempt to identify SP's and corresponding properties without using volatility

Tops and bottoms

- Combine this observation with return intervals approach in financial fluctuations (Wang, Yamasaki, Havlin, and Stanley, Phys. Rev. E 2006)
- Return interval → time interval between two consecutive volatilities above a given threshold
- Investigate properties of TB-return and TB-interval
- TB-return → absolute value of the difference between consecutive T/B prices or B/T prices
- TB-interval → time interval between consecutive T/B or B/T events

Tops and bottoms

- T/B events in financial time series → important pieces of information for several investors
- Patterns of technical analysis based on T/B relative positions
- TB returns and TB intervals strongly related
- TB return ↔ maximal amount of money an investor can make/loose in a given TB interval when the price of the asset rises/falls
- Memory effects with different behaviors correlated with the probability distribution patterns

Smoothing kernels

- T/B in smooth continuous functions → easy task
- T/B in financial time series → awkward task
- Adopt procedure by Lo et al. (2000) → procedure for smoothing series and T/B search
- Sign of the slope of the smoothed curve
- Lo et al. seems to be the first one to use smoothing method in the analysis of financial series.

Smoothing kernels

- Assume the price series of an asset is

$$p(t) = y(t) + \varepsilon(t),$$

- $y(t) \rightarrow$ nonlinear fixed smooth function
- $\varepsilon(t) \rightarrow$ white noise sequence
- Assume the estimator

$$\hat{y}(t) = \frac{1}{T} \sum_{s=1}^{T} \omega_s(t) p(s)$$

- Weights ω_s: larger when s is close to t
- Choice of weights $\rightarrow$ defines the width and form of neighborhood where the average is evaluated

Smoothing kernels

- Gaussian kernel with width h
- Consider

$$\omega_{sh}(t) \equiv K_h(t-s)/g_h(t),$$

$$\text{with } g_h(t) = \frac{1}{T}\sum_{u=1}^{T} K_h(t-u),$$

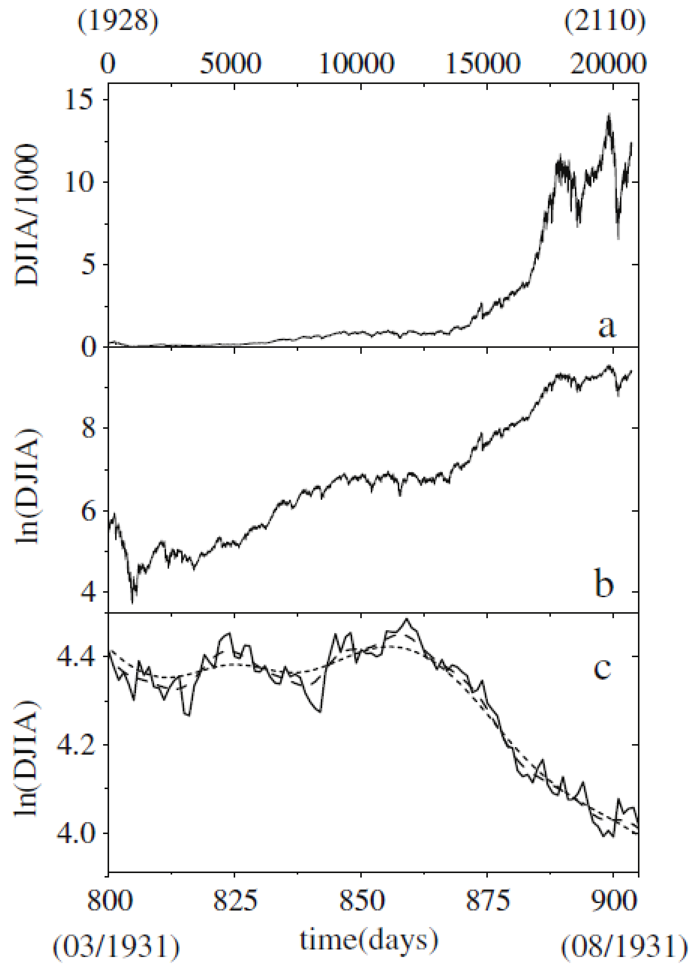
$$\text{and } K_h(t) = \frac{1}{h\sqrt{2\pi}} e^{-t^2/2h^2}$$

- Finally obtain

$$\hat{y}(t) = \frac{\sum_{u=1}^{T} K_h(t-u)p(s)}{\sum_{u=1}^{T} K_h(t-u)}$$

Smoothing kernels

- DJIA in the 1928–2010 interval



Long range correlations

- Search for long range correlation in probability distribution and correlation function
- ${p_h}^R(x)$: distribution of TB log-return with $x = |{\hat{y}_h}^M - {\hat{y}_h}^m|$, ${\hat{y}_h}^M$ and ${\hat{y}_h}^m$ two consecutive extreme values of the smoothed ln(DJIA) series
- ${p_h}^I(x)$: distribution of TB interval with ${\hat{y}_h}^M$ and ${\hat{y}_h}^m$ time corresponding to two consecutive extreme values of the smoothed ln(DJIA) series

Long range correlations

- Integrated probability distribution

$$P_h(x > X) = \int_X^\infty p_h(x)dx$$

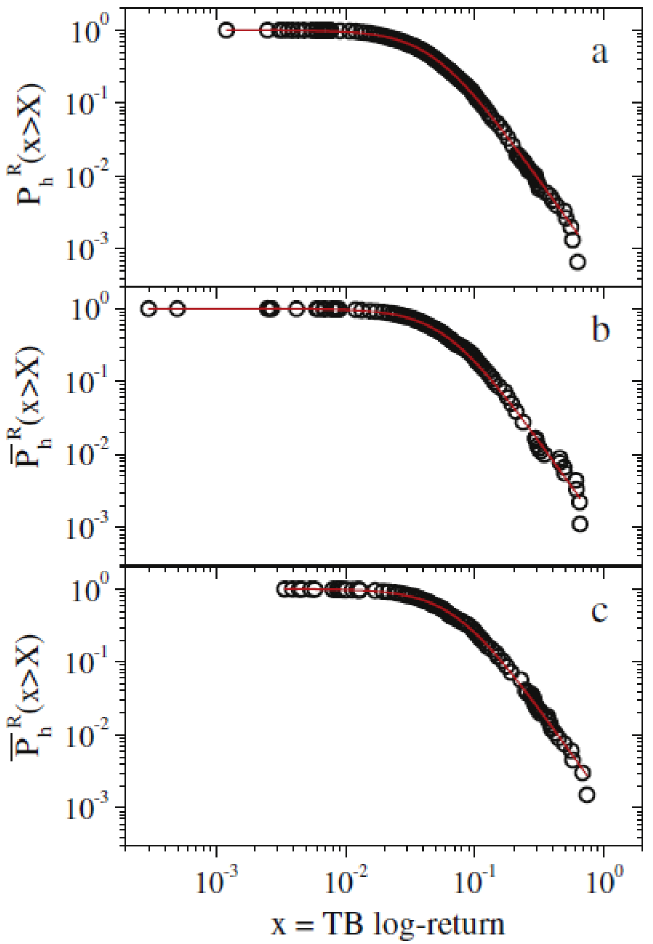
$$\bar{P}_h(x > X) = \frac{1}{X}\int_X^\infty p_h(x)dx$$

- Generalized *q*-exponential functions

$$\exp_q(x) = [1 + (1 - q)x]_+^{1/(1-q)}$$

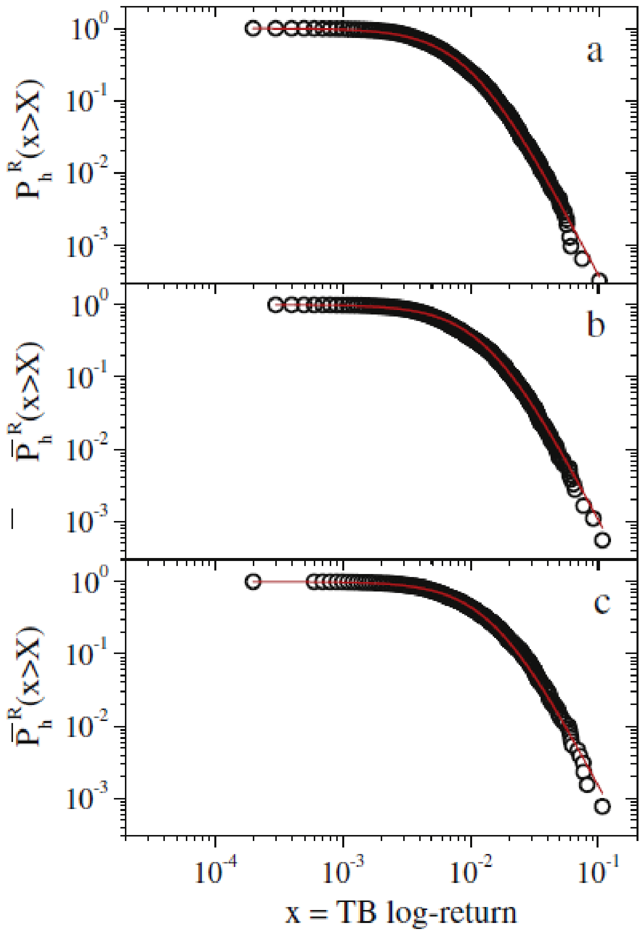
Long range correlations

- DJIA – 1928-2010
- Integrated probability distribution
- TB log-returns
- *h* = *3, 5, 7*
- Circles: data points
- Solid lines: *q*-Gaussian function ~ $\exp_q(-\beta x^2)$
- *q =1.8*, independent of *h*
- Fits the whole interval



Long range correlations

- Intra-day NEI series sampled at 5 min interval
- Integrated probability distribution
- TB log-returns
- $h = 3, 5, 7$
- Circles: data points
- Solid lines: *q*-Gaussian function ~ $\exp_q(-\beta x^2)$
- $q = 1.5$, independent of h



Long range correlations

- Integrated probability distribution
- TB interval
- *h* = *3, 5, 7*
- Functional dependence not clear

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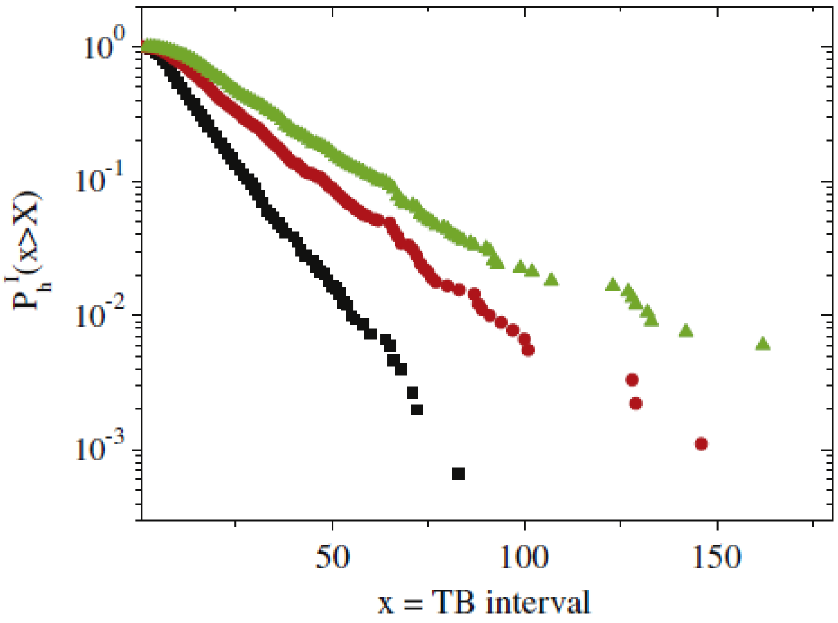

Long range correlations

- Threshold magnitude M_g influences distribution of TB intervals
- Closer analysis to functional dependence
- $M_g \in [0,0.10]$
- Evaluate distribution wrt two distinct variables
- x = Time interval: starts from previous results with very rapid decay
- x = Number of events: starts from a single point

Long range correlations

- *h=3*
- *x* = Time interval
- $exp_q(x)$, where q depends on M_g
  - Exponent starts at 5 and decays to 1.25 with M_g
- *x* = Number of events
- No clear functional dependence

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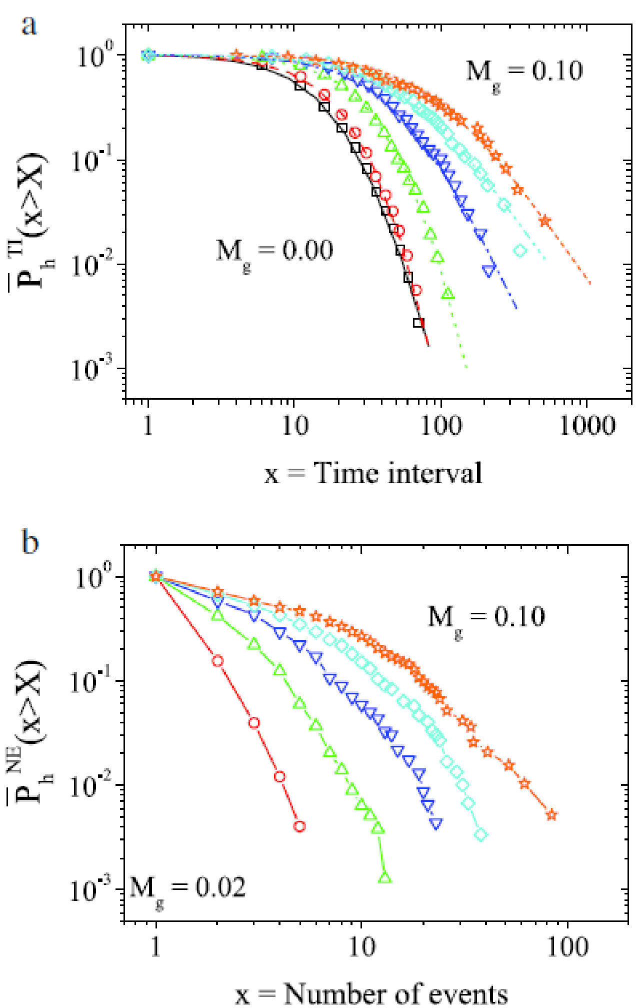

Long range correlations

- Fluctuation correlation function

$$C\,(\tau) = \frac{1}{T}\frac{1}{A}\sum_{e=1}^{T}(x(e) - \bar{x})(x(e+\tau) - \bar{x})$$

$$A = \sum_{e=1}^{T}(x(e) - \bar{x})^2$$

Long range correlations

- Fluctuation correlation function
- TB log-return and TB intervals with distinct properties
- Long range correlation ↔ $\exp_q(x)$ dependency with $q \neq 1$
- NEI and DJIA with different q and characteristic decaying time

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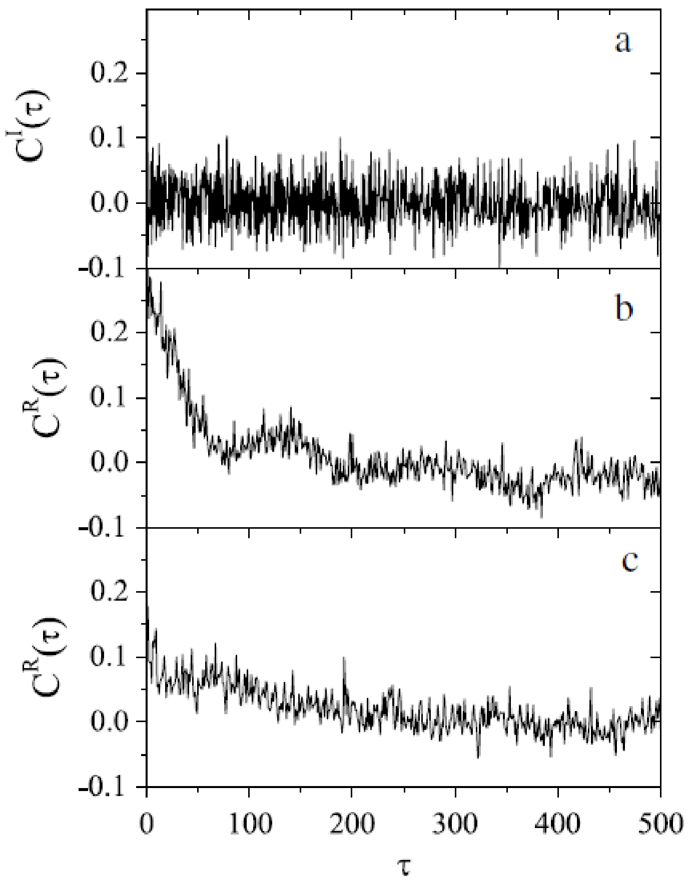

Conclusions

- Two new approaches to detect SP's
- Characterization of statistical distribution of events
- Different measures express SP's in particular form of the distribution
- Actual numerical values of distribution parameters reflect series properties

Thanks for the attention!!!