Dynamic Ising Model: Reconstruction of Evolutionary Trees

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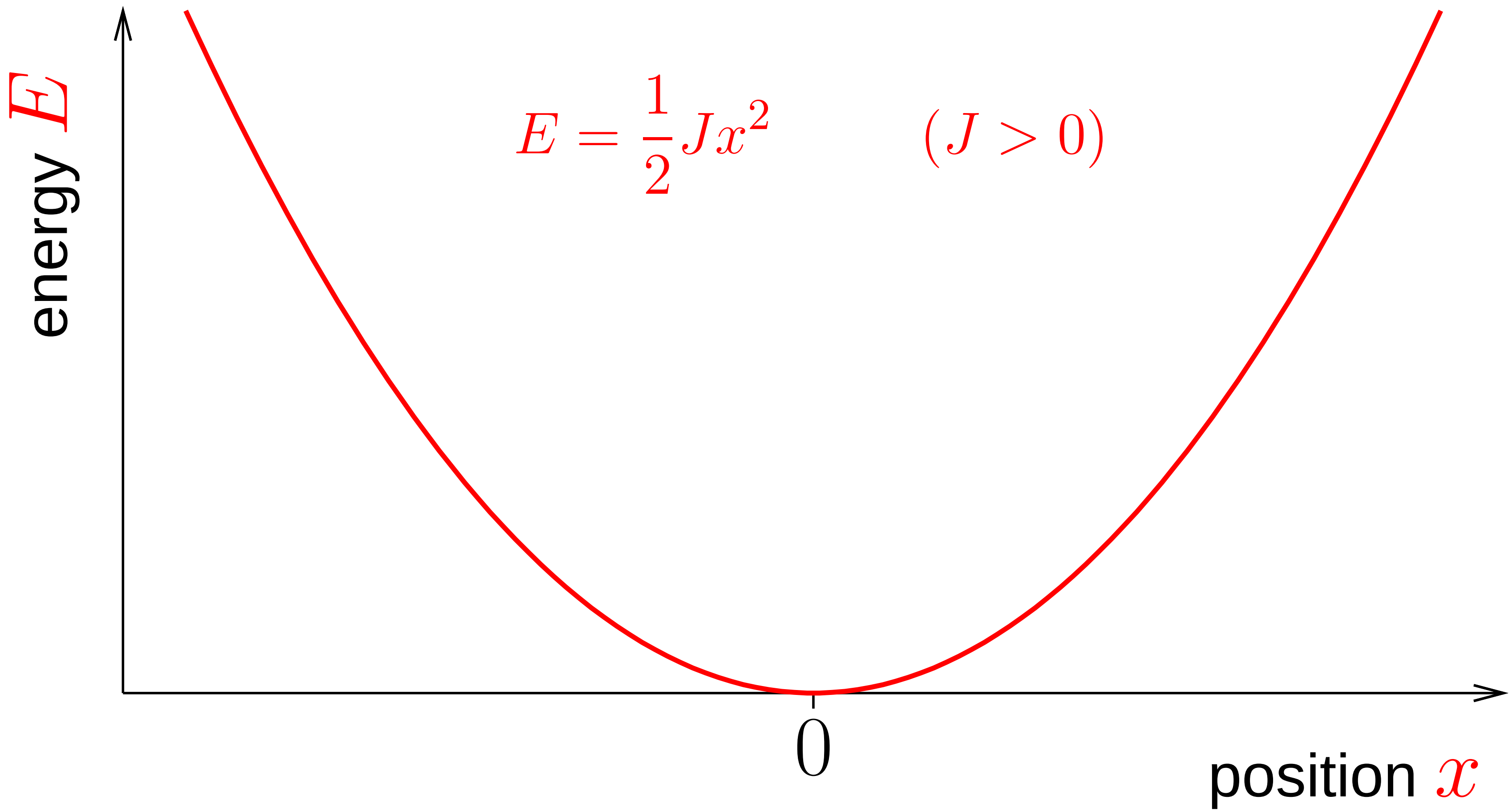
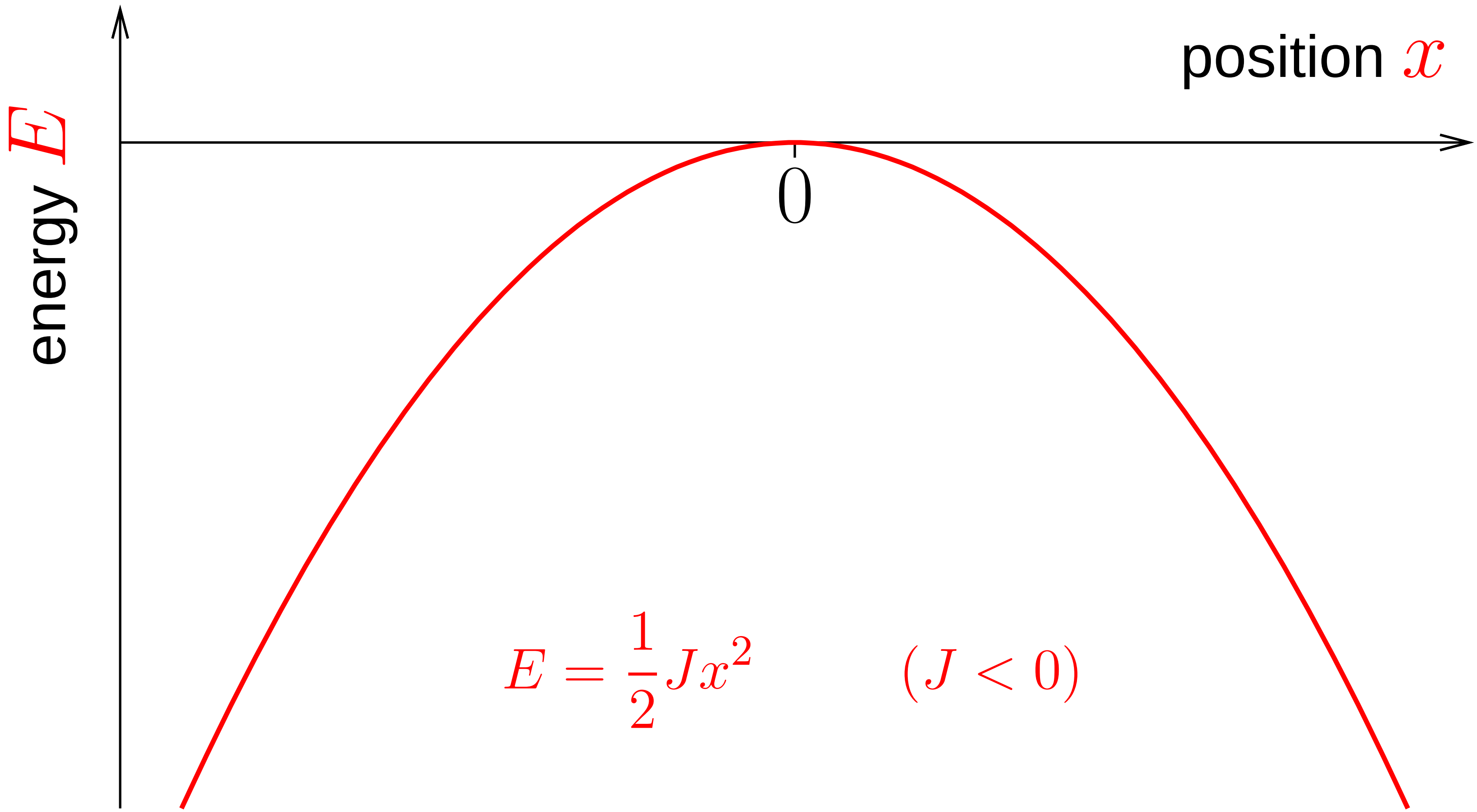
One particle (confinement)



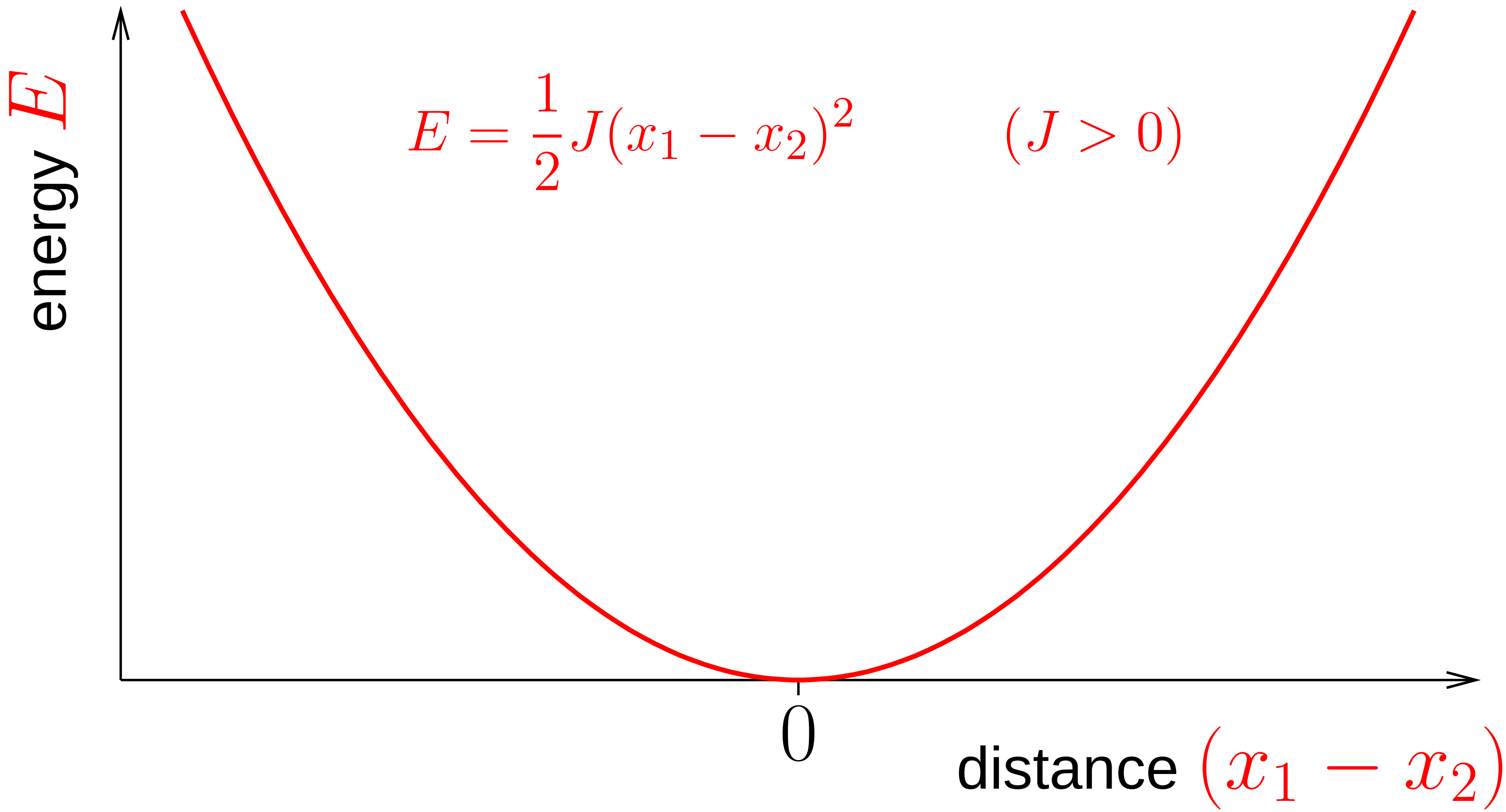
One particle tied to the origin by a spring.

One particle (runaway)



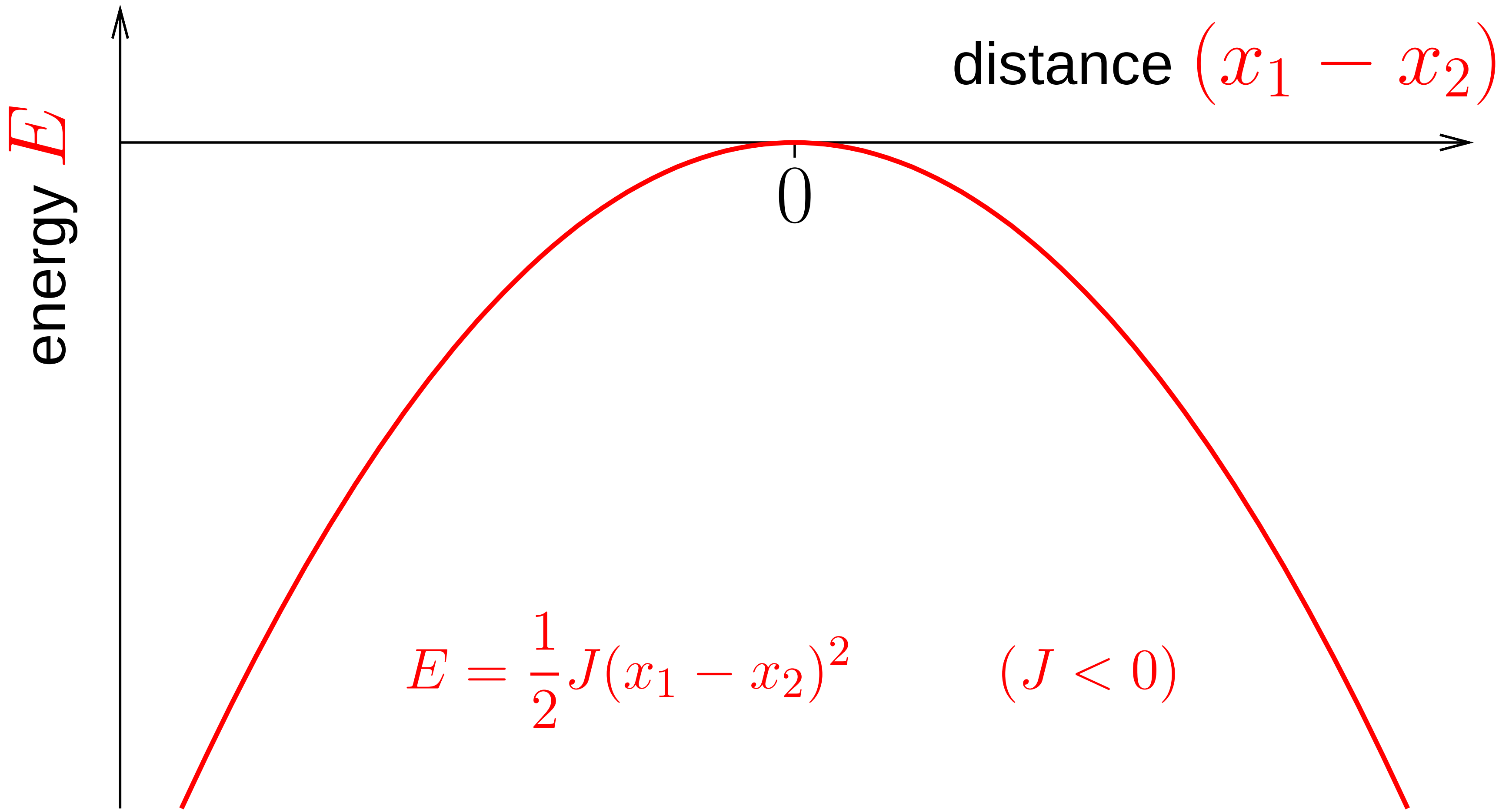
One particle repelled away from the origin by a anti-spring.

Two attracting particles



Two particles tied to each other by a spring.

Two repelling particles



Two particles repelling each other by a anti-spring.

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$$\text{update}: \quad \begin{cases} x_i(t+\Delta t) = x_i(t) + v_i(t)\Delta t + \frac{1}{2} \frac{F(x_i)+F(\bar{x}_i)}{2} \Delta t^2 \\ \\ v_i(t+\Delta t) = v_i(t) + \frac{F(x_i)+F(\bar{x}_i)}{2} \Delta t \end{cases}$$

Verlet

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Due to universality, many different systems can be described by this model: ferromagnetism or liquid gas transition for $J_{ij} > 0$; antiferromagnetism for $J_{ij} < 0$ in bipartite lattices; spin glasses for both positive and negative, random J_{ij}; and Boolean systems in general.

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Onsager's exact solution (1944) for the thermodynamic behaviour of the uniform ferromagnet in two dimensions is a paradigmatic scientific achievement.

Back to the current model

$$\frac{1}{2}J_{ij}(x_i - y_j)^2 = \frac{1}{2}J_{ij}x_i^2 + \frac{1}{2}J_{ij}x_j^2 - J_{ij}\,x_i x_j$$

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$$\frac{1}{2}J_{ij}(x_i - y_j)^2 = \overbrace{\frac{1}{2}J_{ij}x_i^2 + \frac{1}{2}J_{ij}x_j^2}^{\text{“external” potential}} \underbrace{- \ J_{ij}\,x_i x_j}_{\text{interaction}}$$

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"external" potential

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interaction

The traditional <u>discrete</u> Ising model ($-J_{ij} S_i S_j$) lacks a proper dynamics. Artificial rules based on thermodynamic equilibrium (Metropolis, etc) are generally adopted.

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Now, the classical Newton's dynamics applies to the current continuous version.

That is why I called it Dynamic Ising Model.

The first evolutionary tree

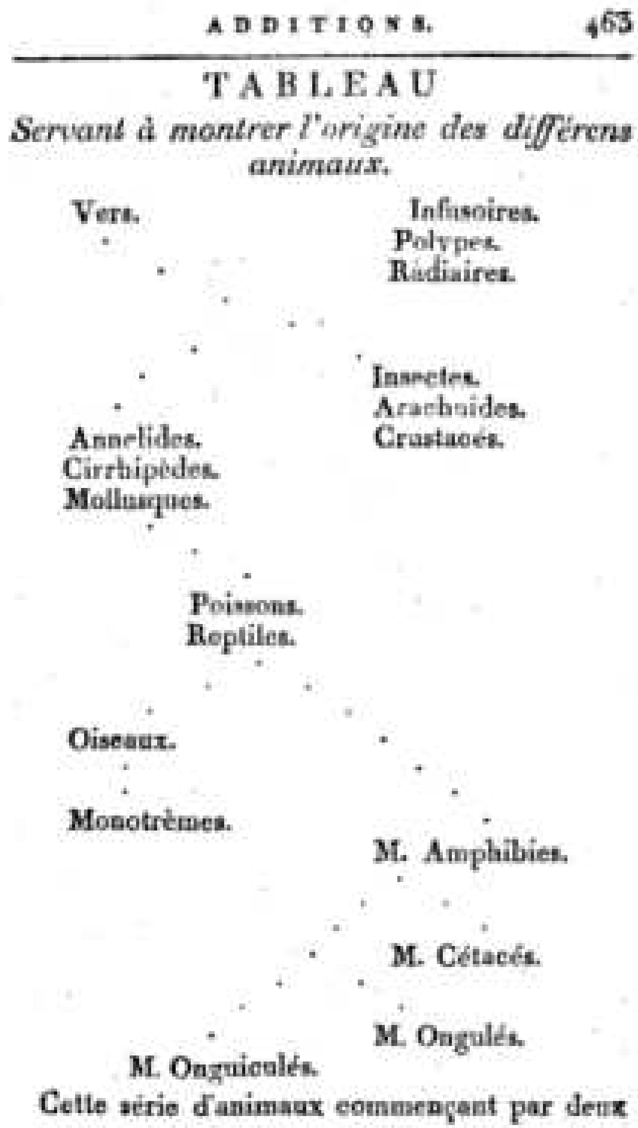

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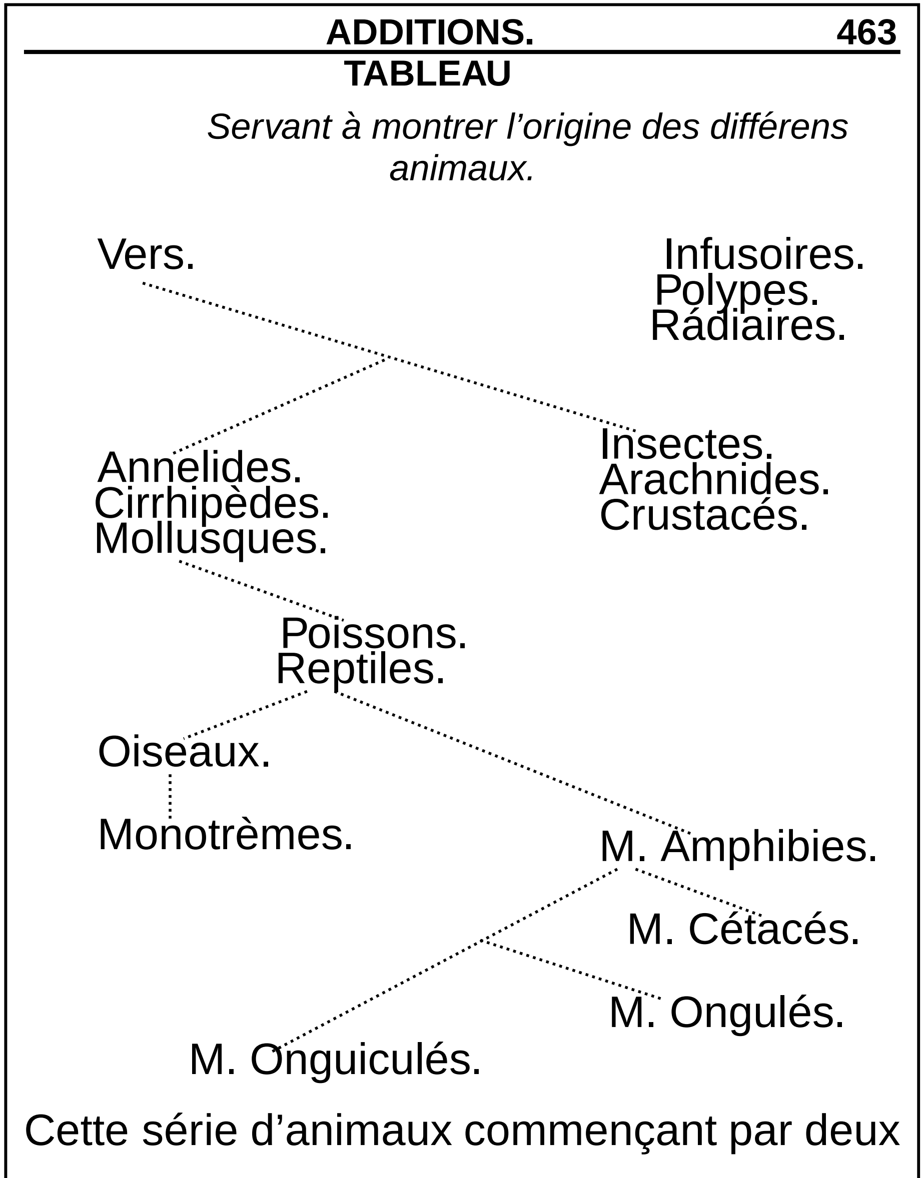
TABLEAU

Servant à montrer l'origine des différens animaux.

Cette série d'animaux commençant par deux

The first evolutionary tree



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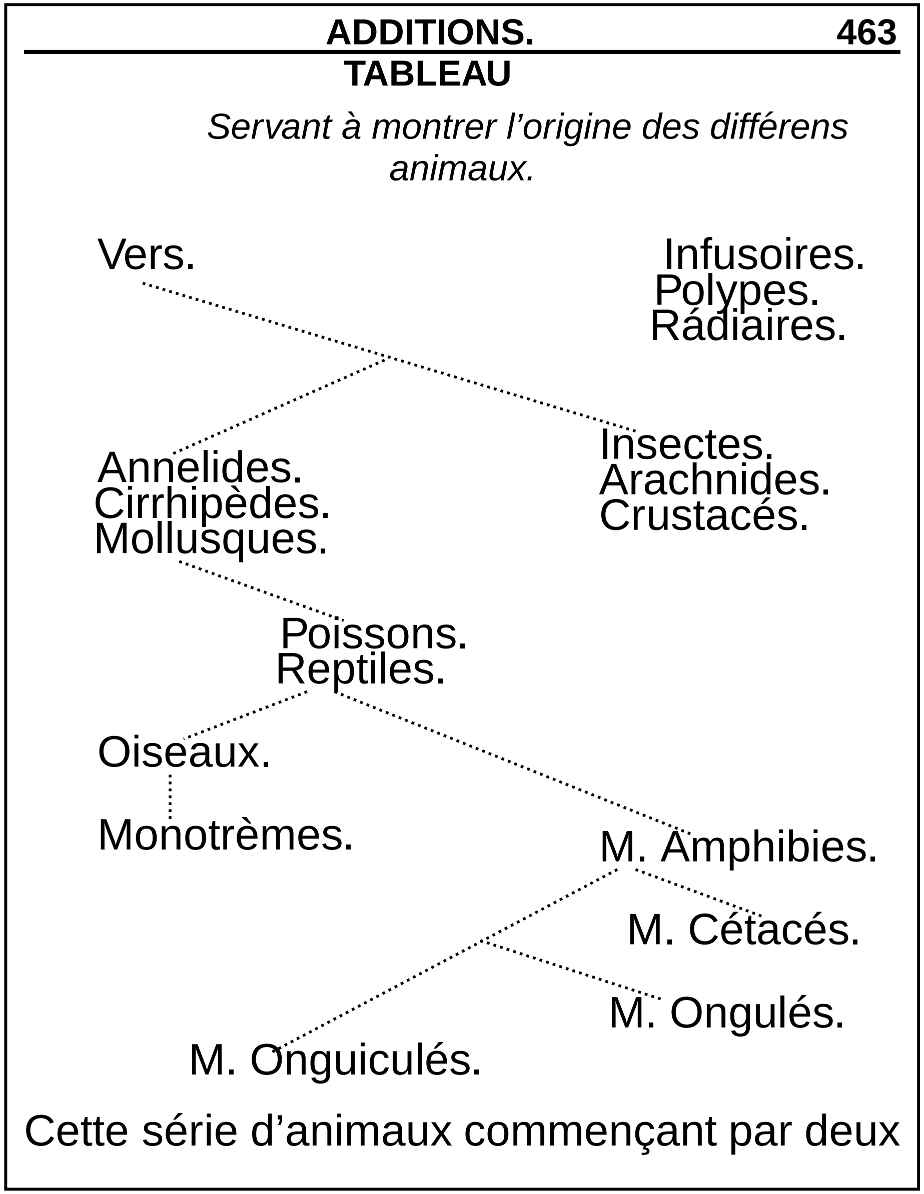
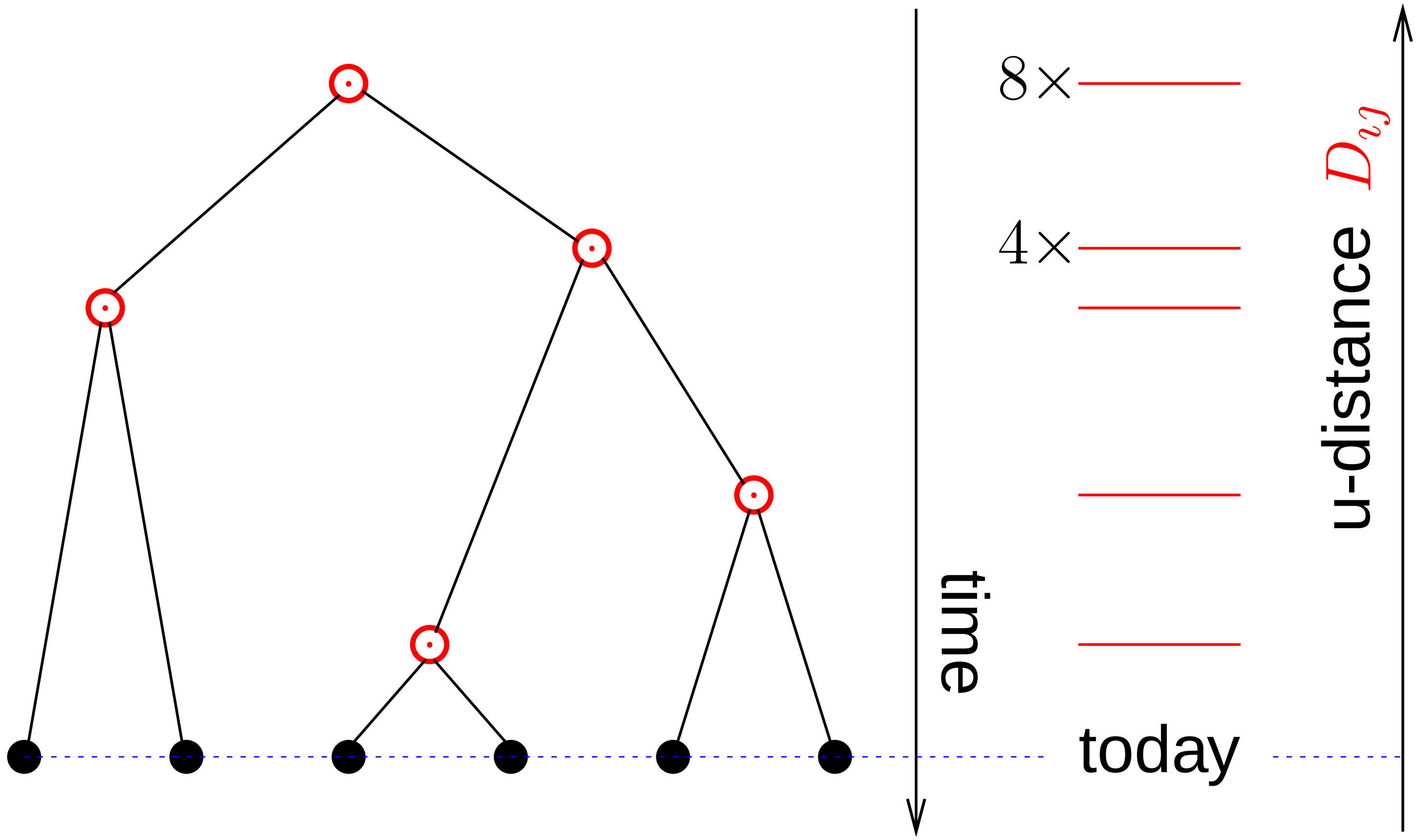


La Phylosophie Zoologique

Lamarck (1809)

Darwin's birth year

Another evolutionary tree



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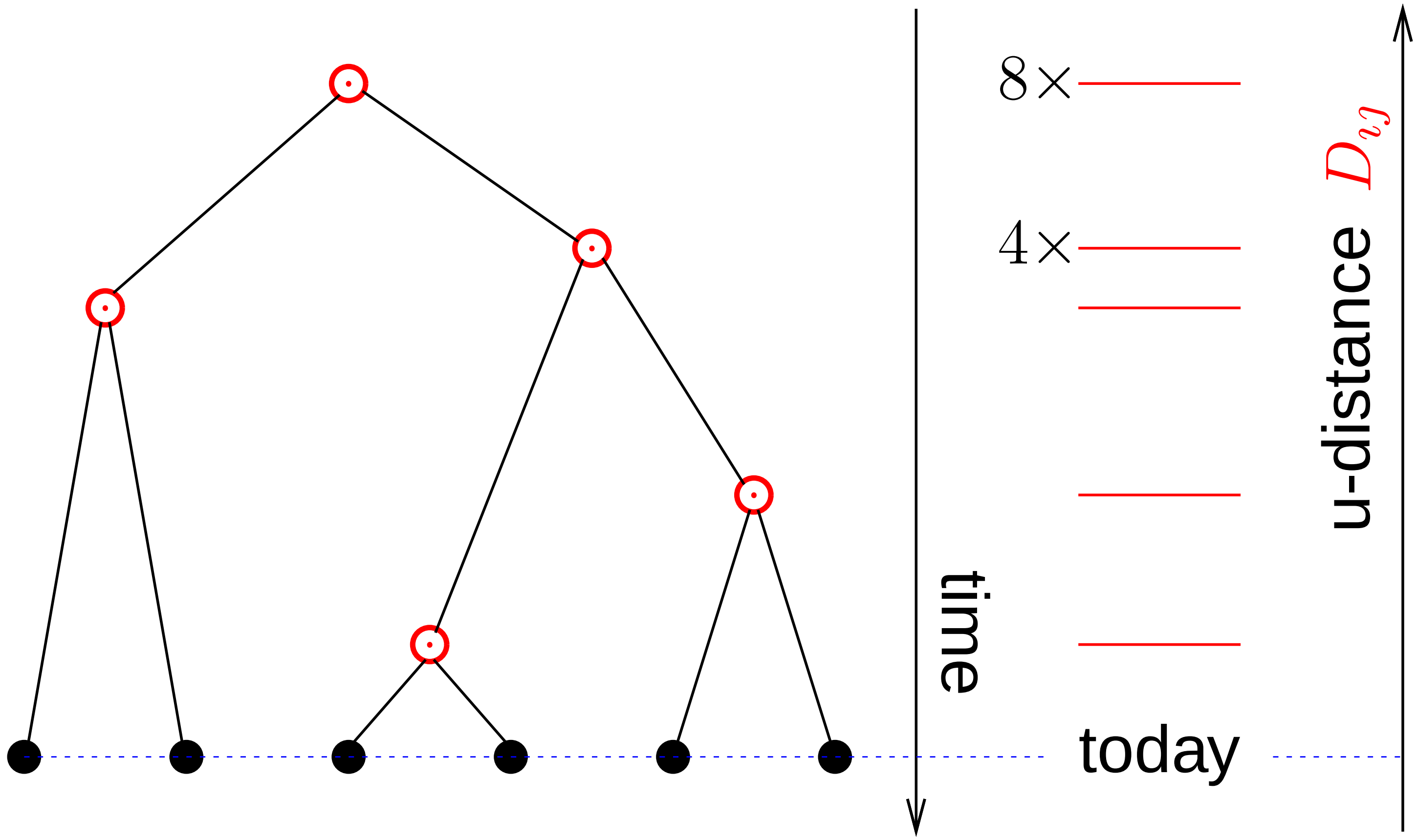
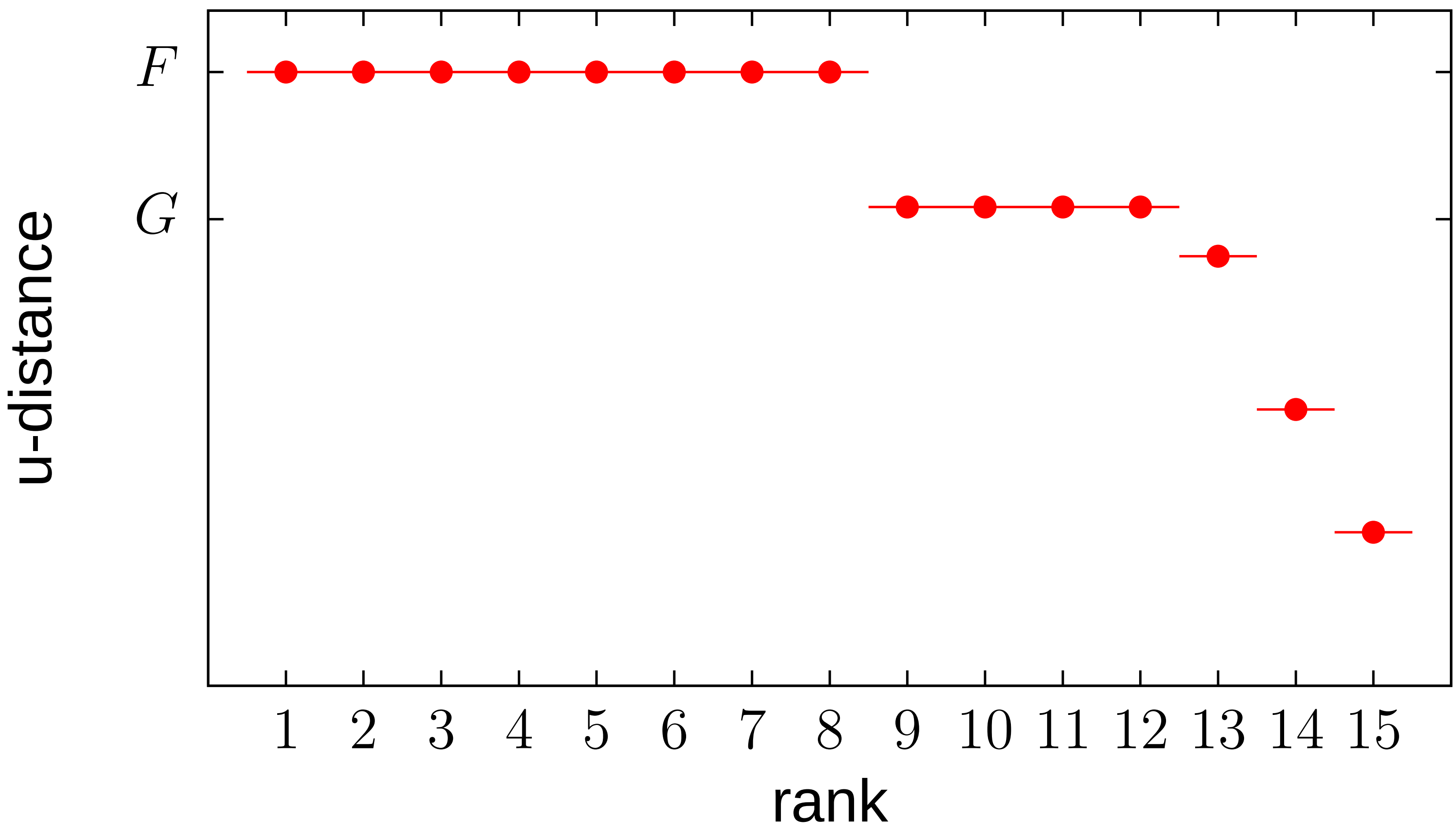


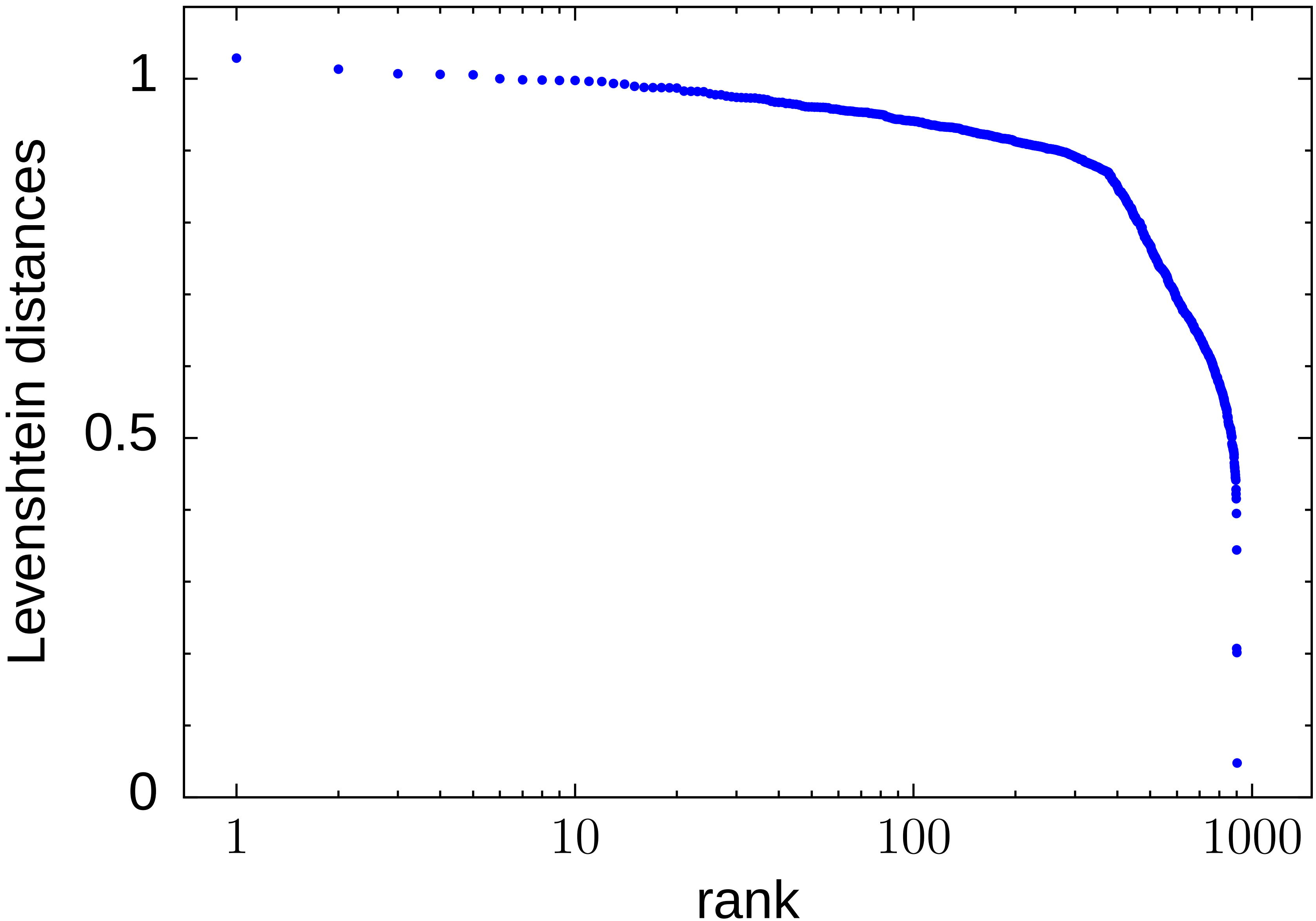
$$J_{ij} = D - D_{ij}$$

Parameter D is adjusted in between the two higher levels.

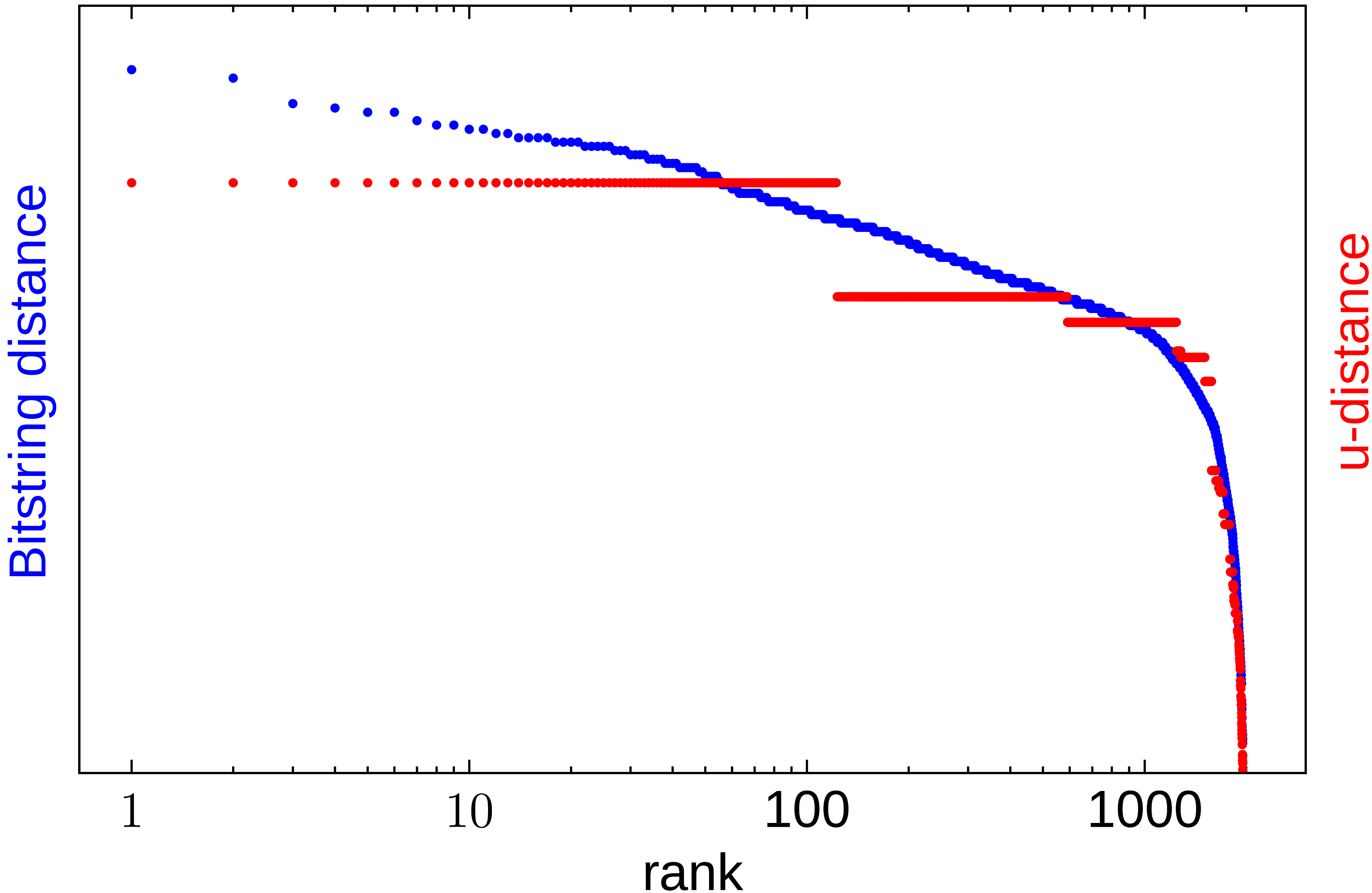
Rank plot, same tree



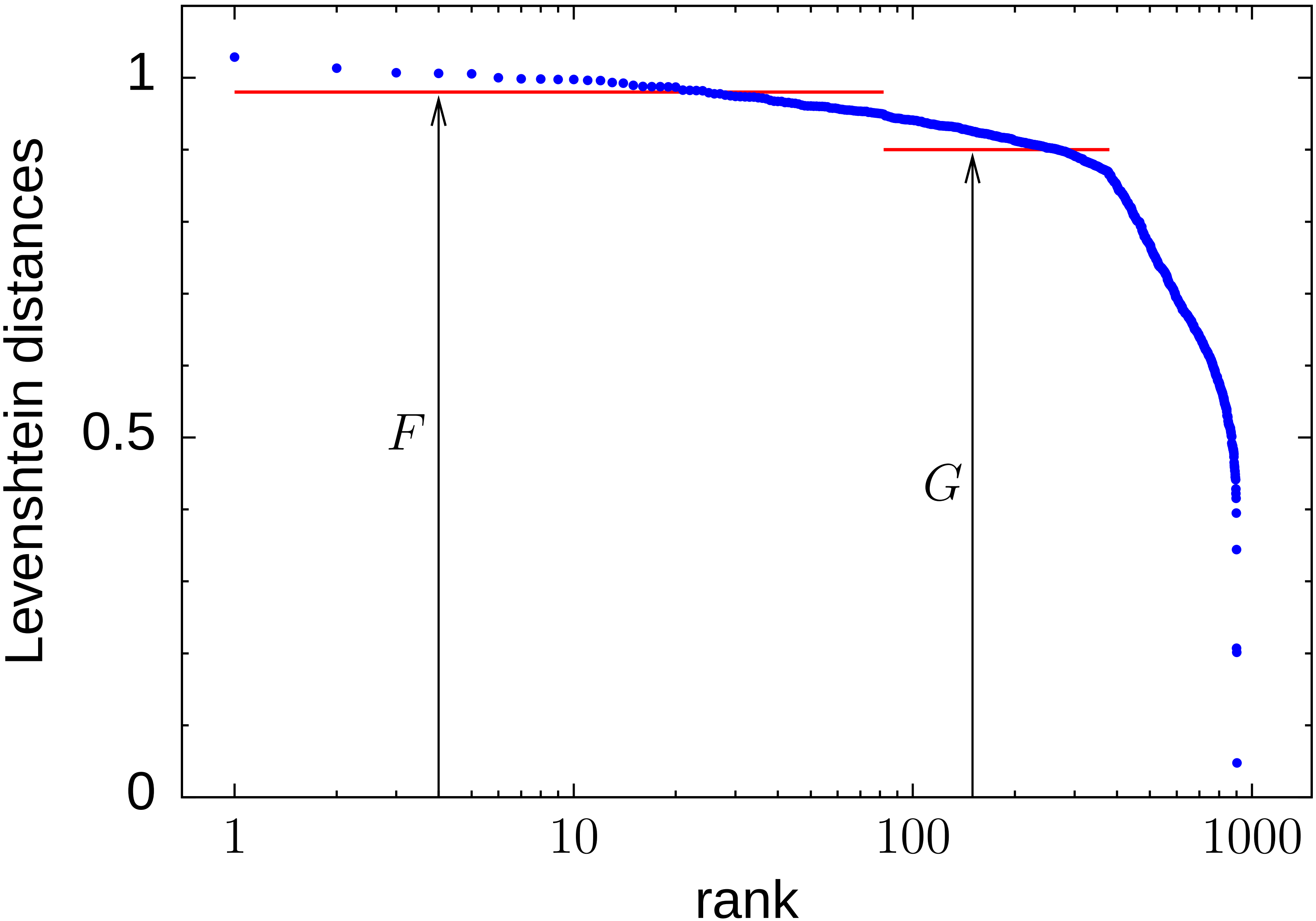
Tupian family, 43 languages



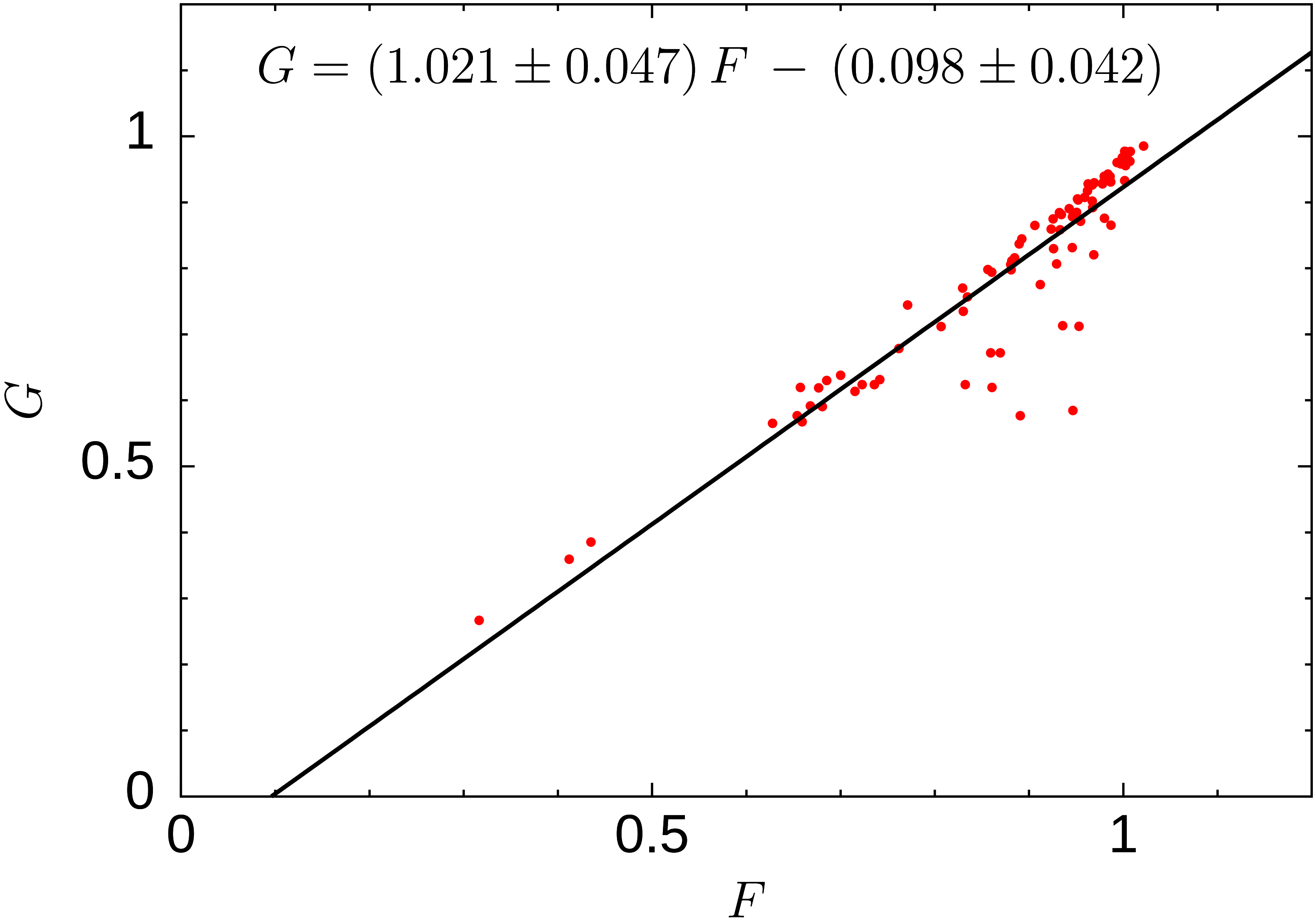
Simulated tree, 63 languages



Measures for each real family



89 real families



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Veit Schwaemmle (Stüttgart)
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Paulo Victor Santos Souza