INCT2012

Complex Networks, Long-Range Interactions and Nonextensive Statistics

L. R. da Silva

UFRN – DFTE – Natal – Brazil

04/05/12

OUR GOALS

- Growth of an asymptotically scale-free network including metrics.
- Growth of a geographically localized network (around its baricenter).
- To exhibit effects of competition between metrical neighborhood, connectivity and fitness.
- To analyze the influences of considering a fitness power-law distributed.
- Last but not least, to exhibit the connection between scale-free networks and nonextensive statistics.

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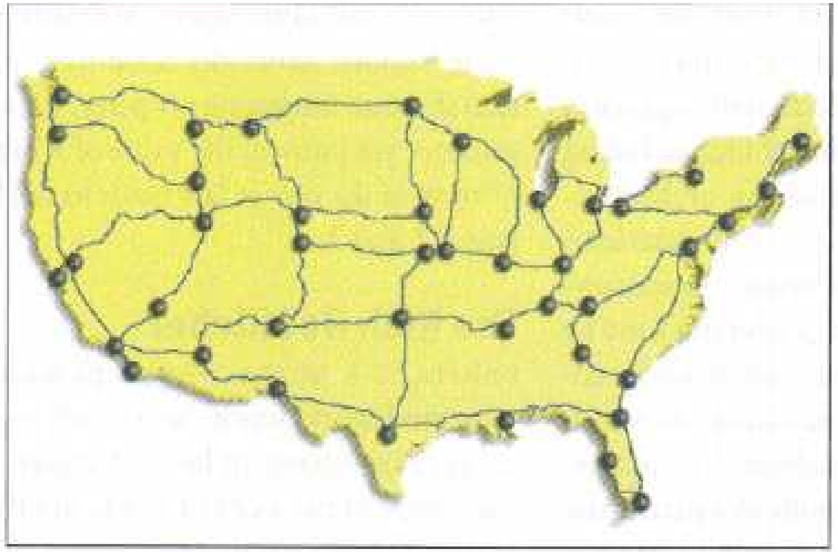
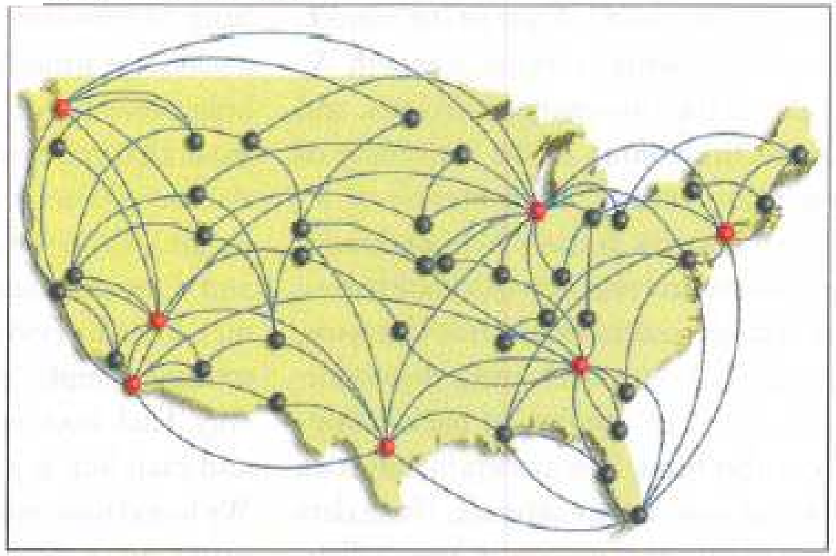
Random Network

Sites have in average the same conectivity.

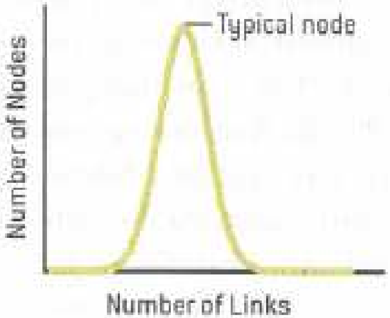


Scale free Network

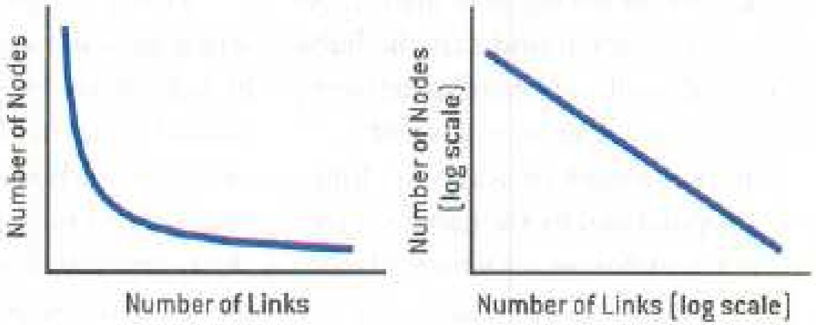
$$P(k) \approx k^{-\gamma}$$

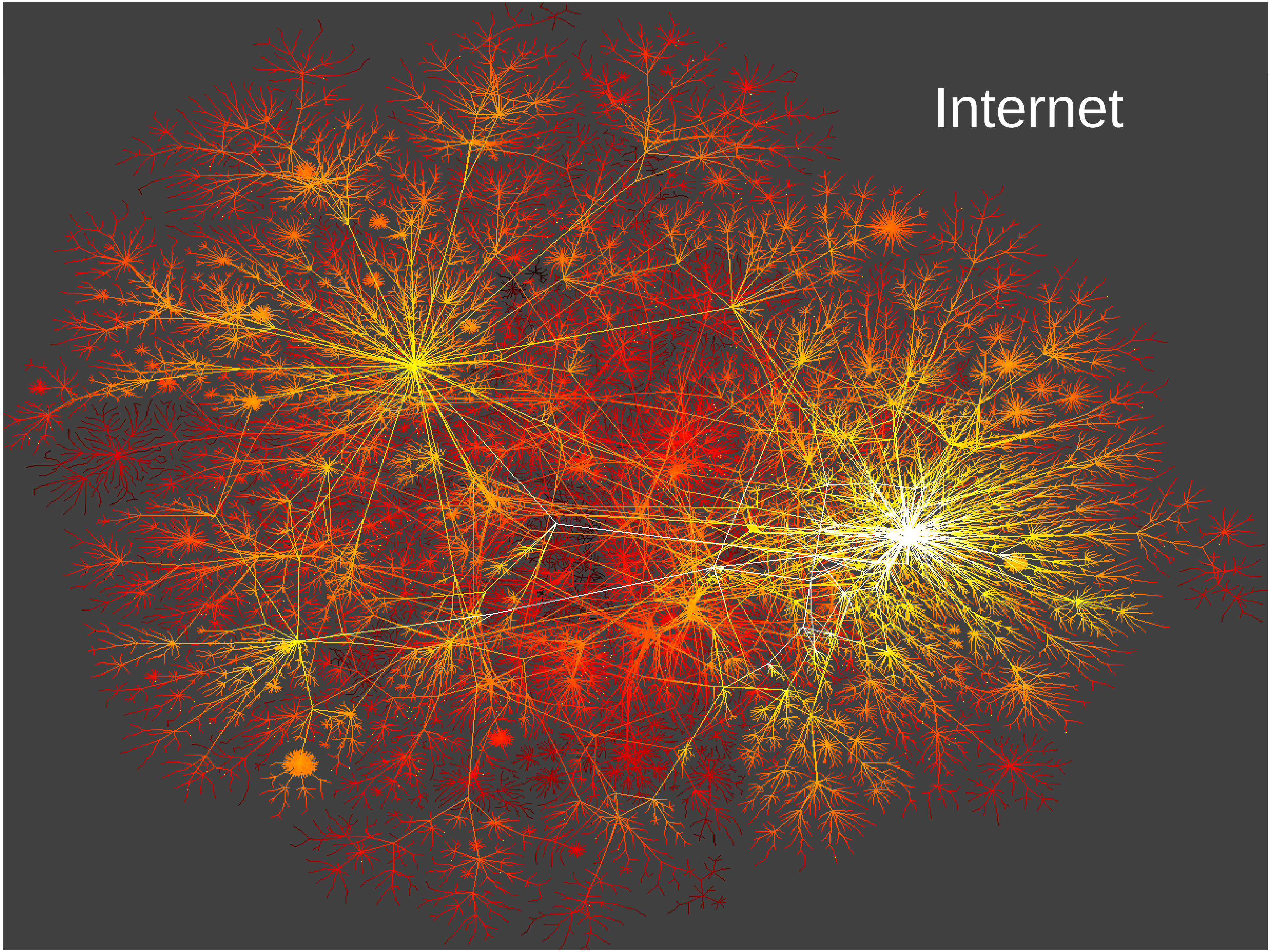


Bell Curve Distribution of Node Linkages

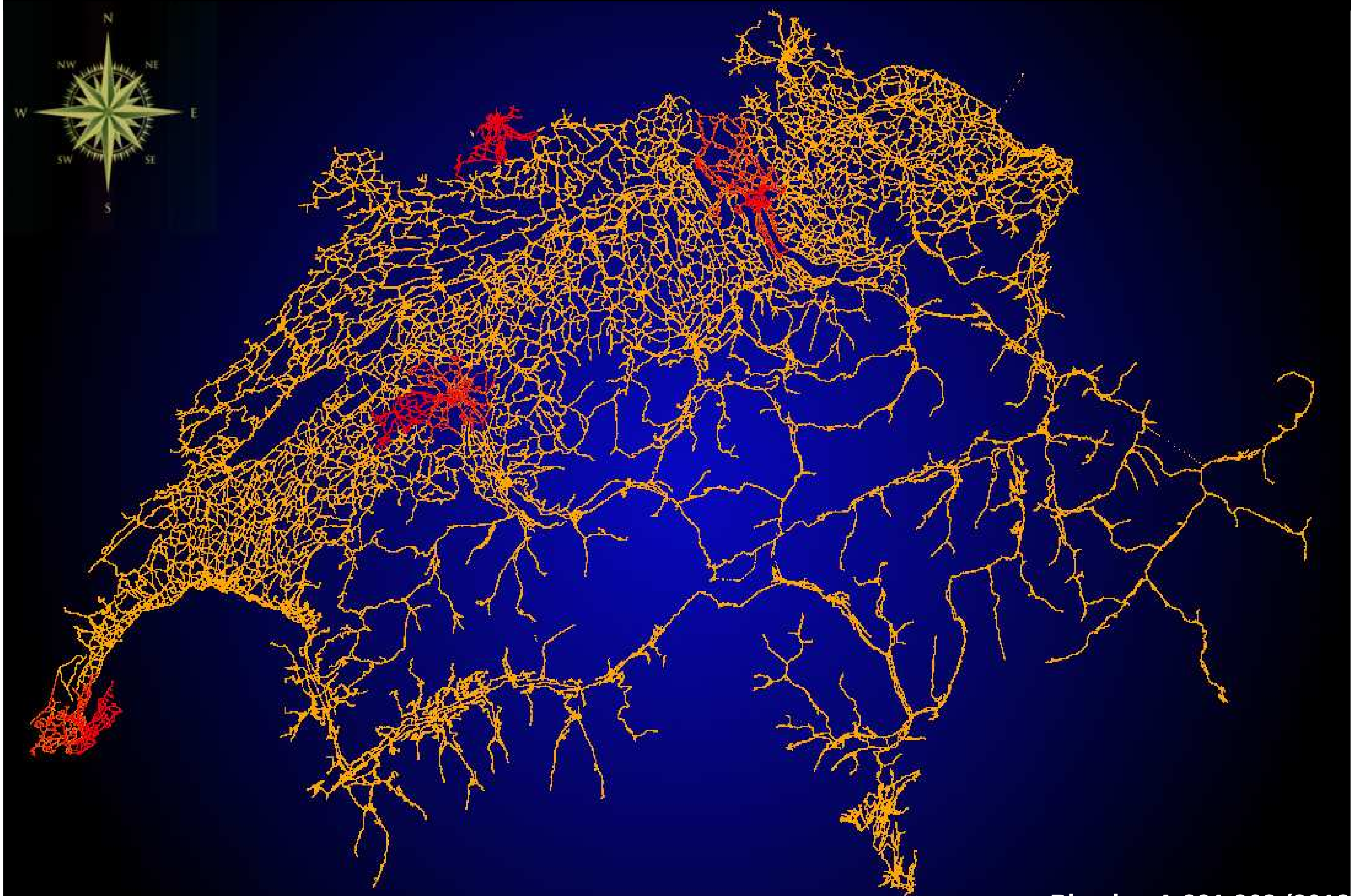


Power Law Distribution of Node Linkages



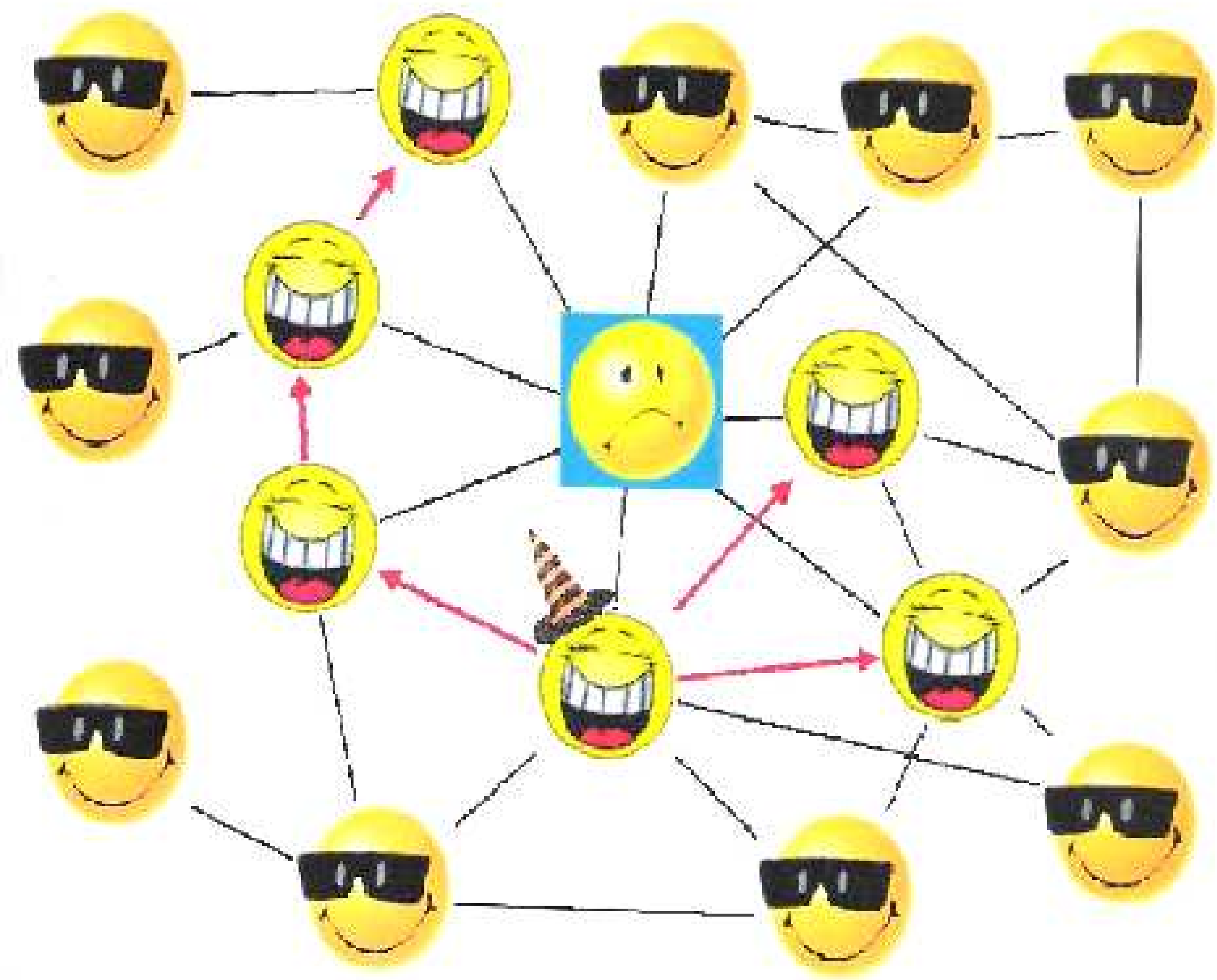

Internet

Swiss road map



Physica A 391 362 (2012

Gossip Network

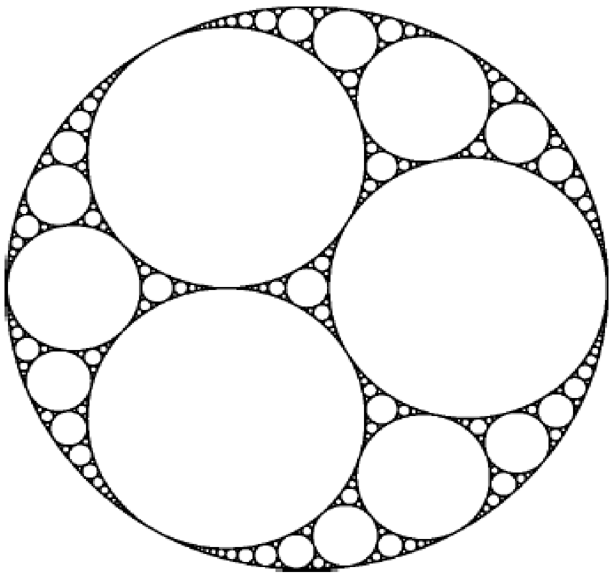


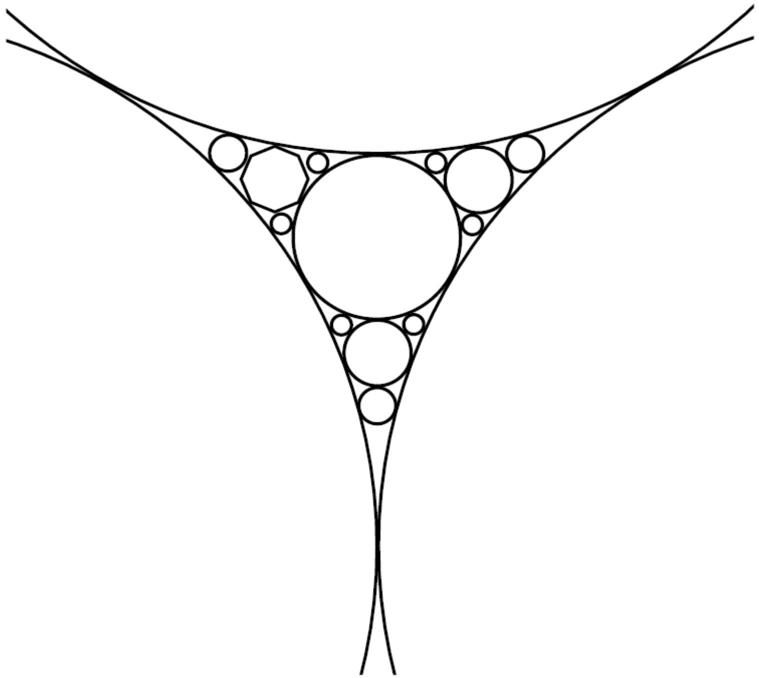
Apollonian Network

PRL 94, 018702 (2005) PHYSICAL REVIEW LETTERS week ending 14 JANUARY 2005

Apollonian Networks: Simultaneously Scale-Free, Small World, Euclidean, Space Filling, and with Matching Graphs

José S. Andrade, Jr.,[1] Hans J. Herrmann,[1,*] Roberto F. S. Andrade,[2] and Luciano R. da Silva[3]

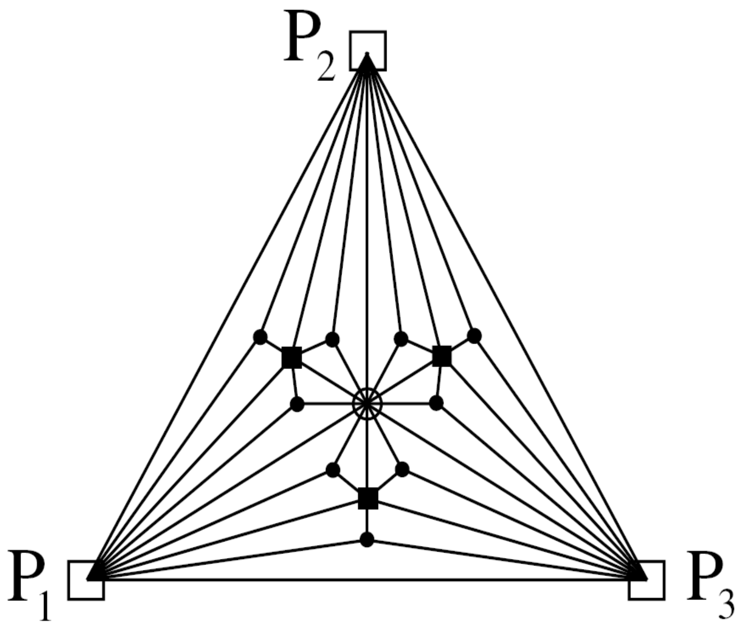




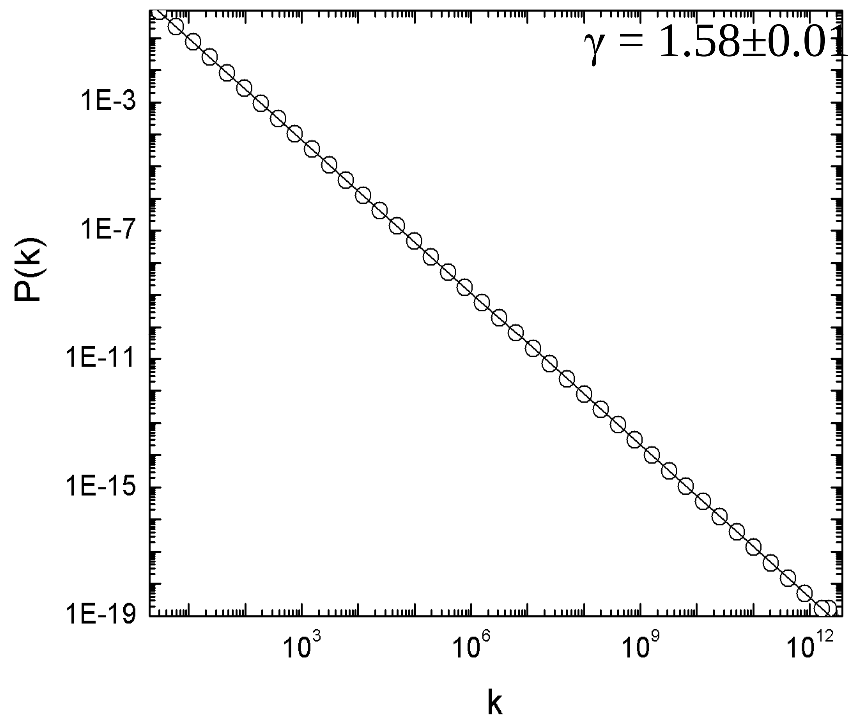
Apollonian Network

Apollonius of Perga
lived from about 262 BC to about 190 BC
Apollonius was known as 'The Great Geometer'.

Apollonian Network: Connectivity Distribution



Sítios	k
3	9
1	12
3	6
9	3



$$P(k) \propto k^{-\gamma}$$

$$\gamma = \ln 3 / \ln 2 = 1.585$$

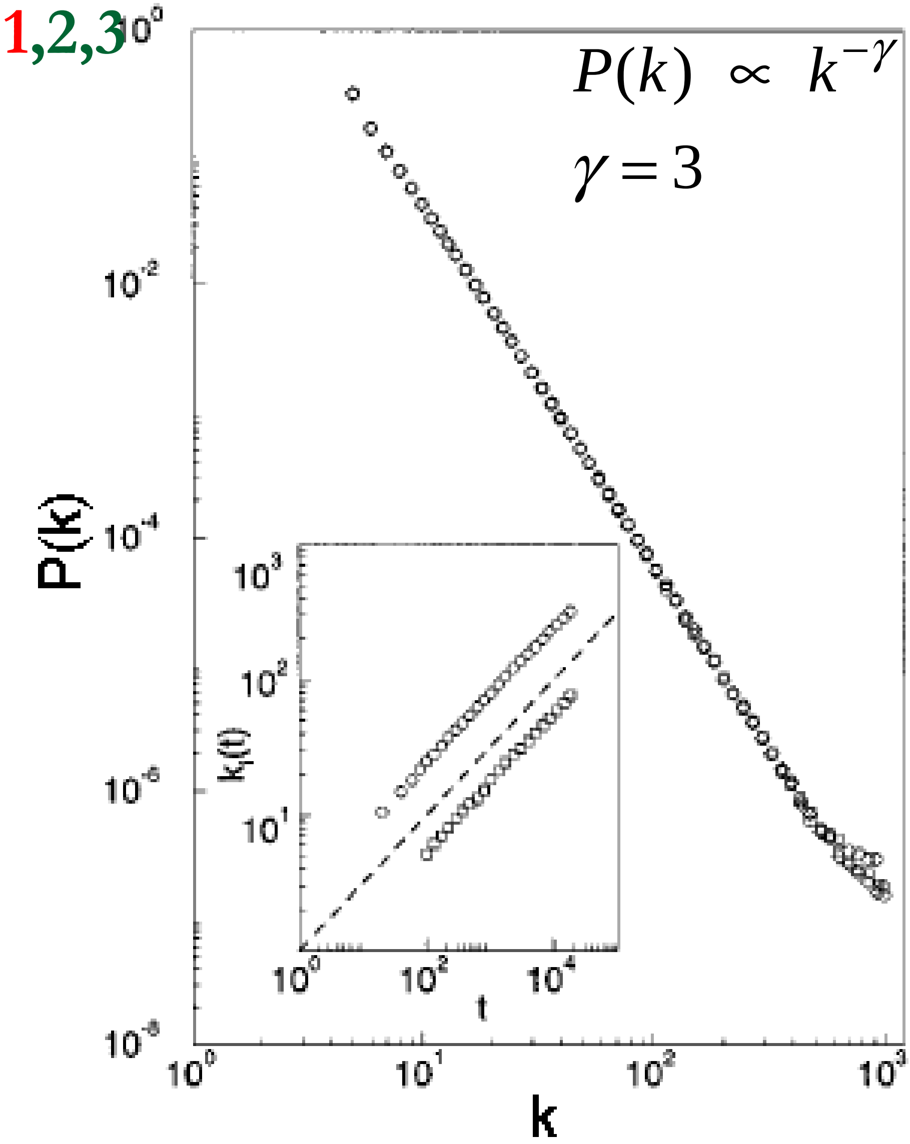
$$\Rightarrow N_n = 3 + \left(3^{n+1} - 1\right) / 2$$

Scale-free Networks[1,2,3]

- Barabási and Albert[1];

$$\Pi(k_i) = \frac{k_i}{\sum_{j=1}^{N'} k_j} \qquad (01)$$

$$< k_i > = \left(\frac{t}{i}\right)^{\beta} \qquad (02)$$



[1]Science **286**, 509 (1999) ; Rev. Mod Phys. **74**, 47 (2002)

[2]M. Boguñá and R. Pastor-Satorras, Physical Review E **68**, 036112 (2003)

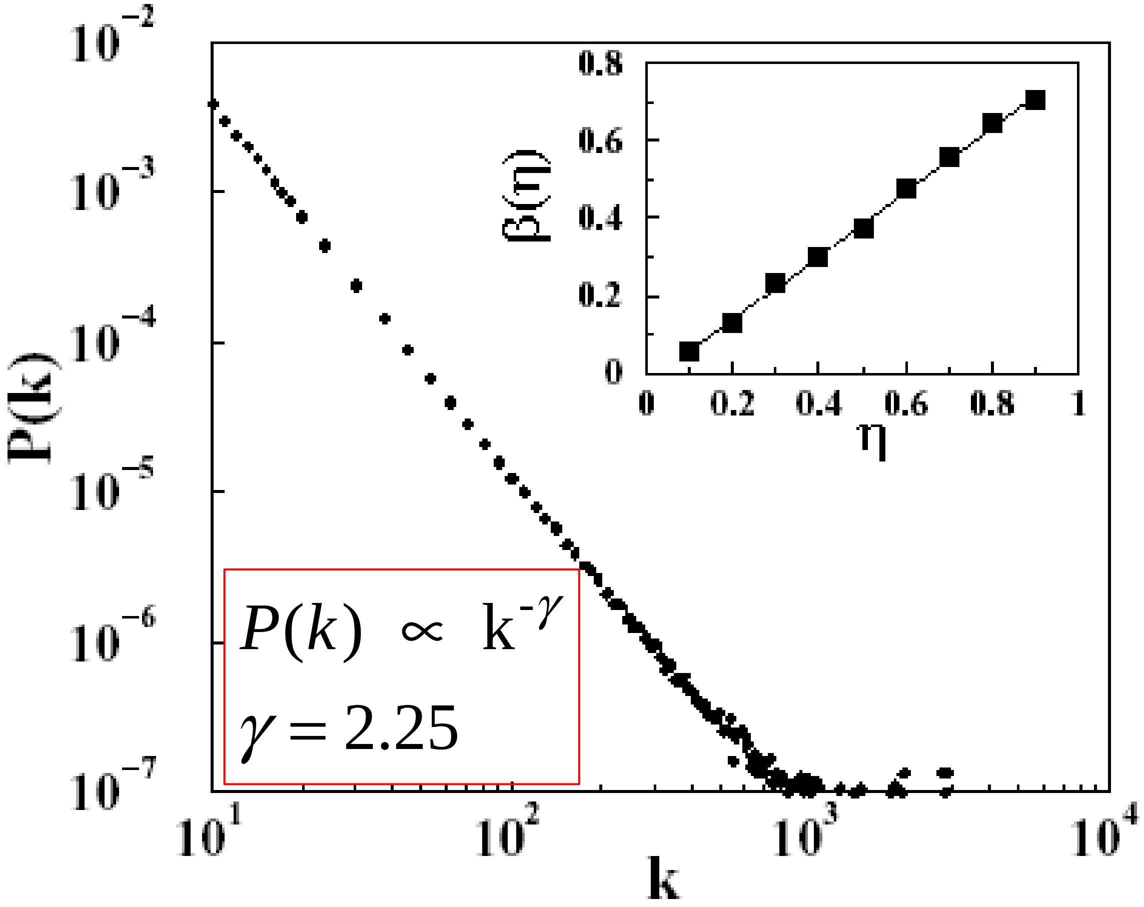
[3]S. Thurner and C. Tsallis, Europhycs Letters **72**, 197 (2005)

Fitness Model

- Bianconi and Barabási[4];
- Albert and Barabási[5];

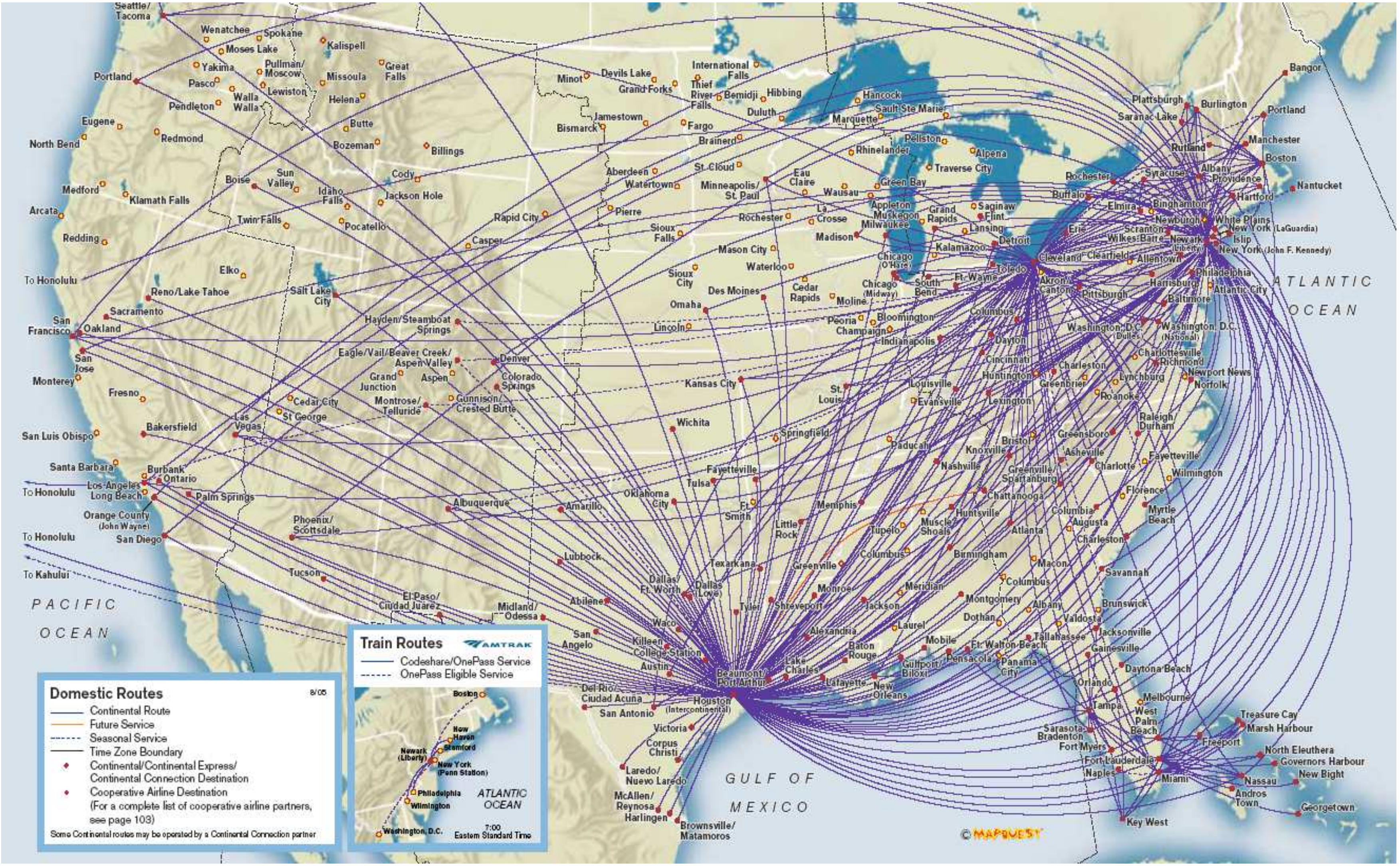
$$\Pi(k_i) = \frac{k_i \eta_i}{\sum_{j=1}^{N'} k_j \eta_j} \quad (04)$$

$$< k_i > = \left(\frac{t}{i}\right)^{\beta(\eta_i)} \quad (05)$$



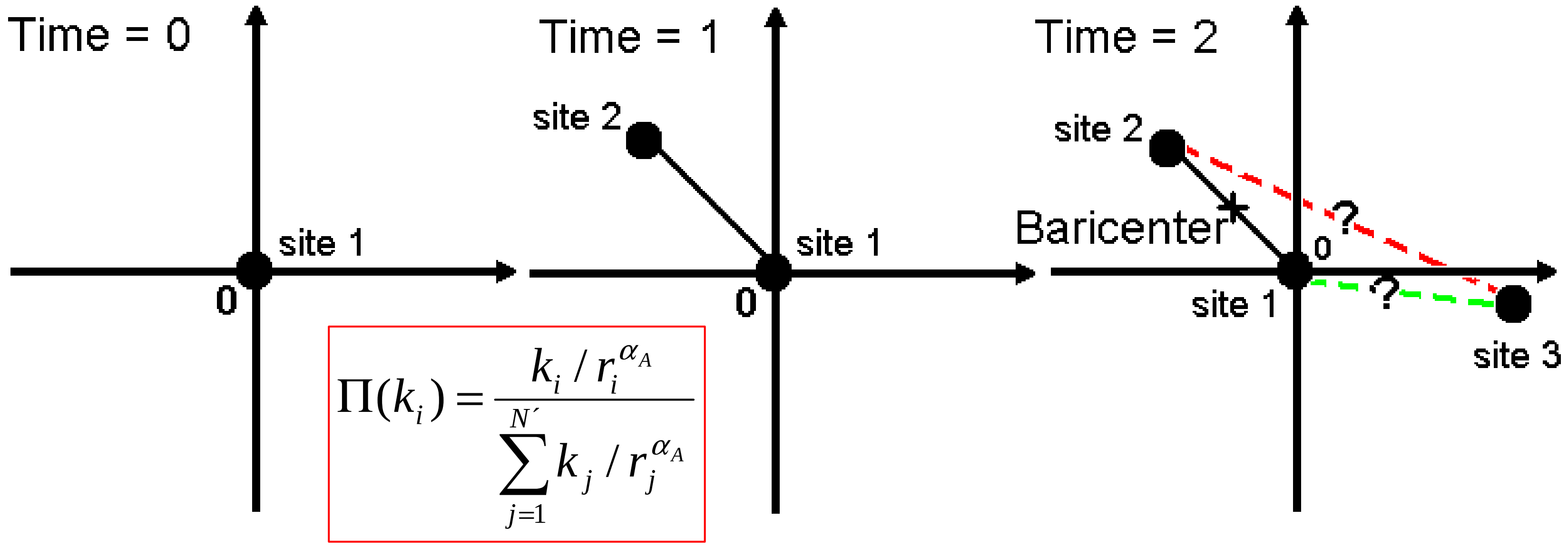
[4]Europhys. Lett. **54**,436 (2001) ; [5]Rev. Mod Phys. **74**, 47 (2002)

Geographic Model



Barabási-Albert Model with Euclidean Distance Power-law Distributed

Network Construction:



$$\Pi(k_i) = \frac{k_i / r_i^{\alpha_A}}{\sum_{j=1}^{N'} k_j / r_j^{\alpha_A}}$$

$$P(r) \propto r^{-\gamma_G}$$

$$\gamma_G > 1$$

where

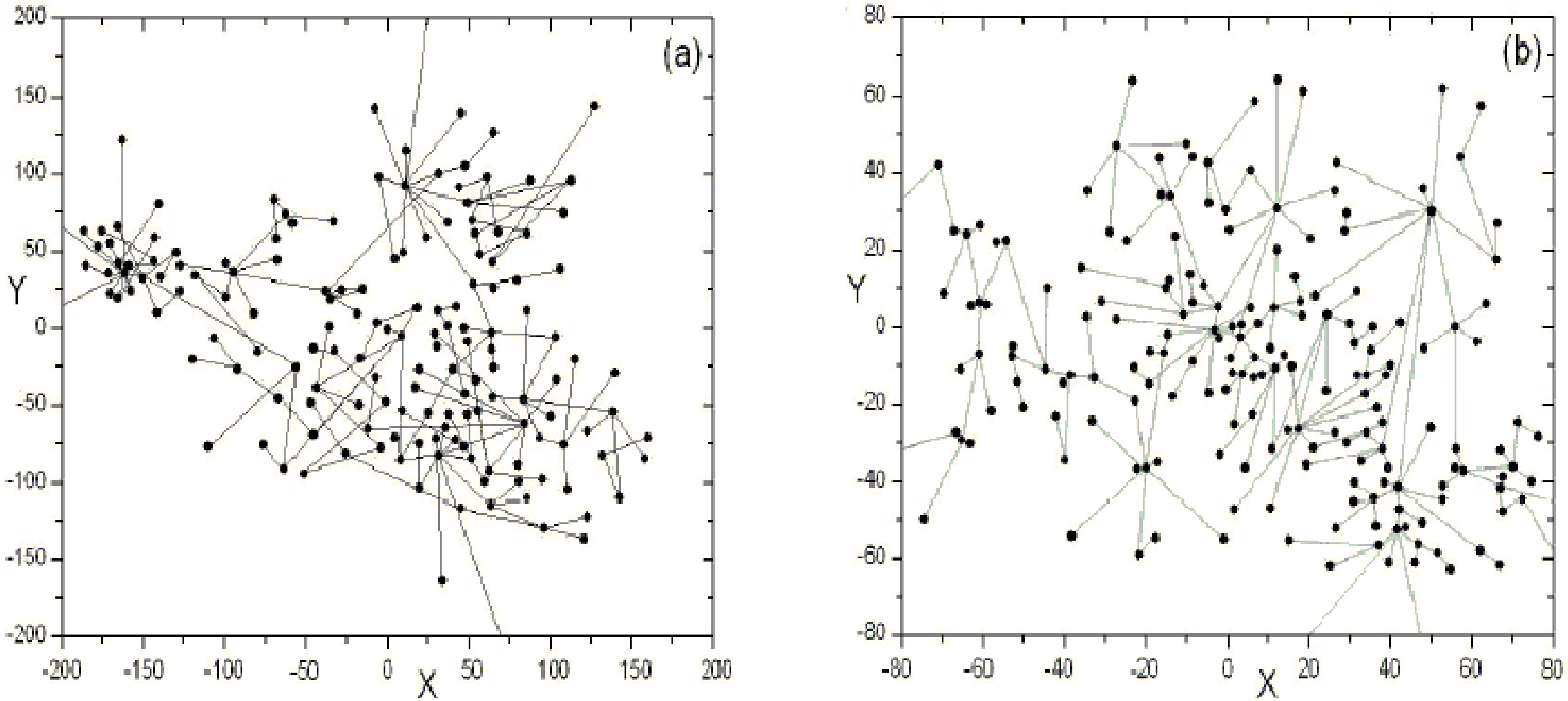
$$\gamma_G = \frac{3+\alpha_G}{2+\alpha_G}$$

$$\alpha_G \geq 0$$

$$r = (1-\xi')^{-(2+\alpha_G)}$$

$$\theta = 2\pi\xi'$$

Examples



N = 250 nodes (a) $(\alpha_G, \alpha_A) = (1, 0)$ and (b) $(\alpha_G, \alpha_A) = (1, 4)$.
The starting site is at $(X, Y) = (0, 0)$. Notice the spontaneous emergence of hubs.

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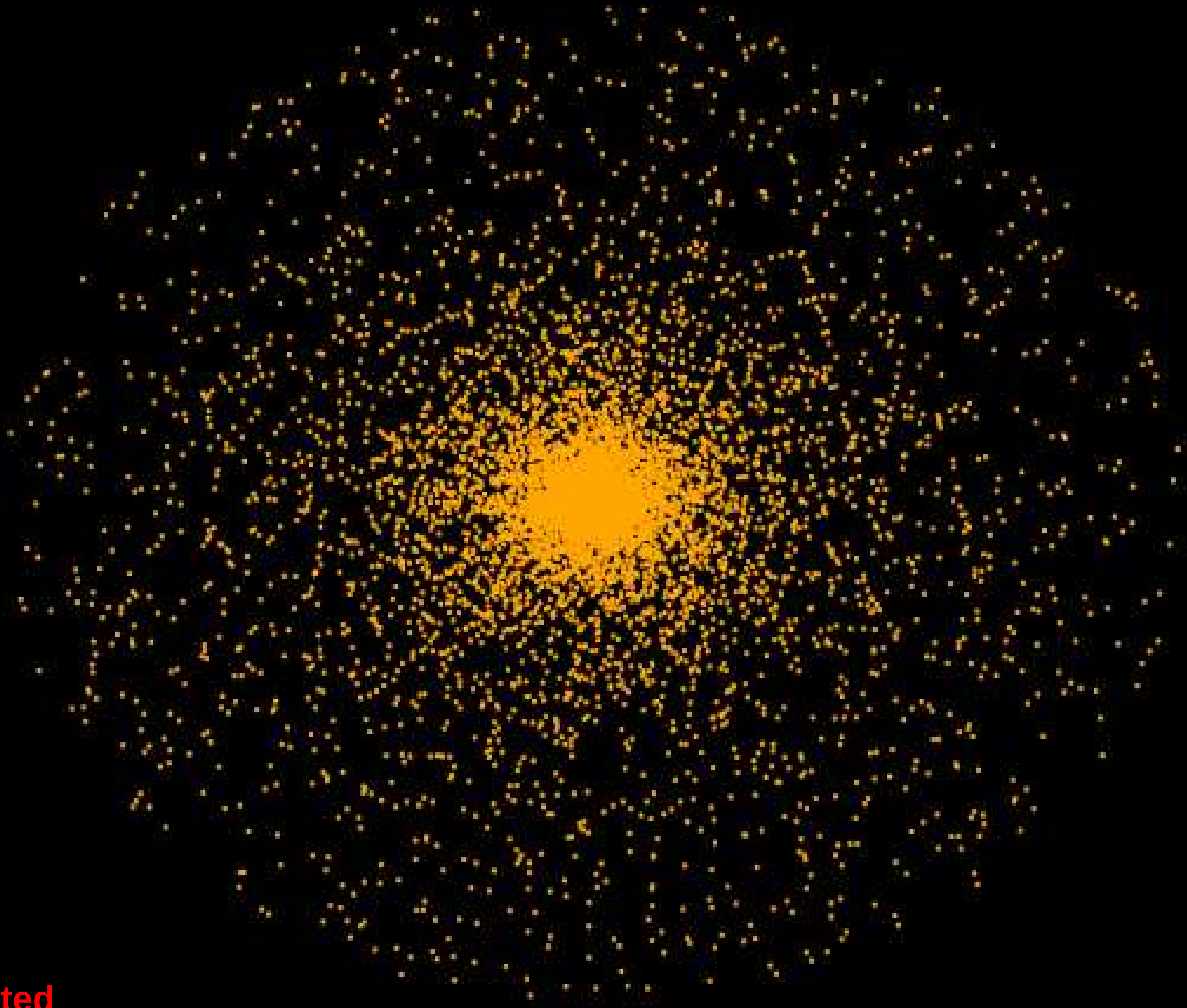
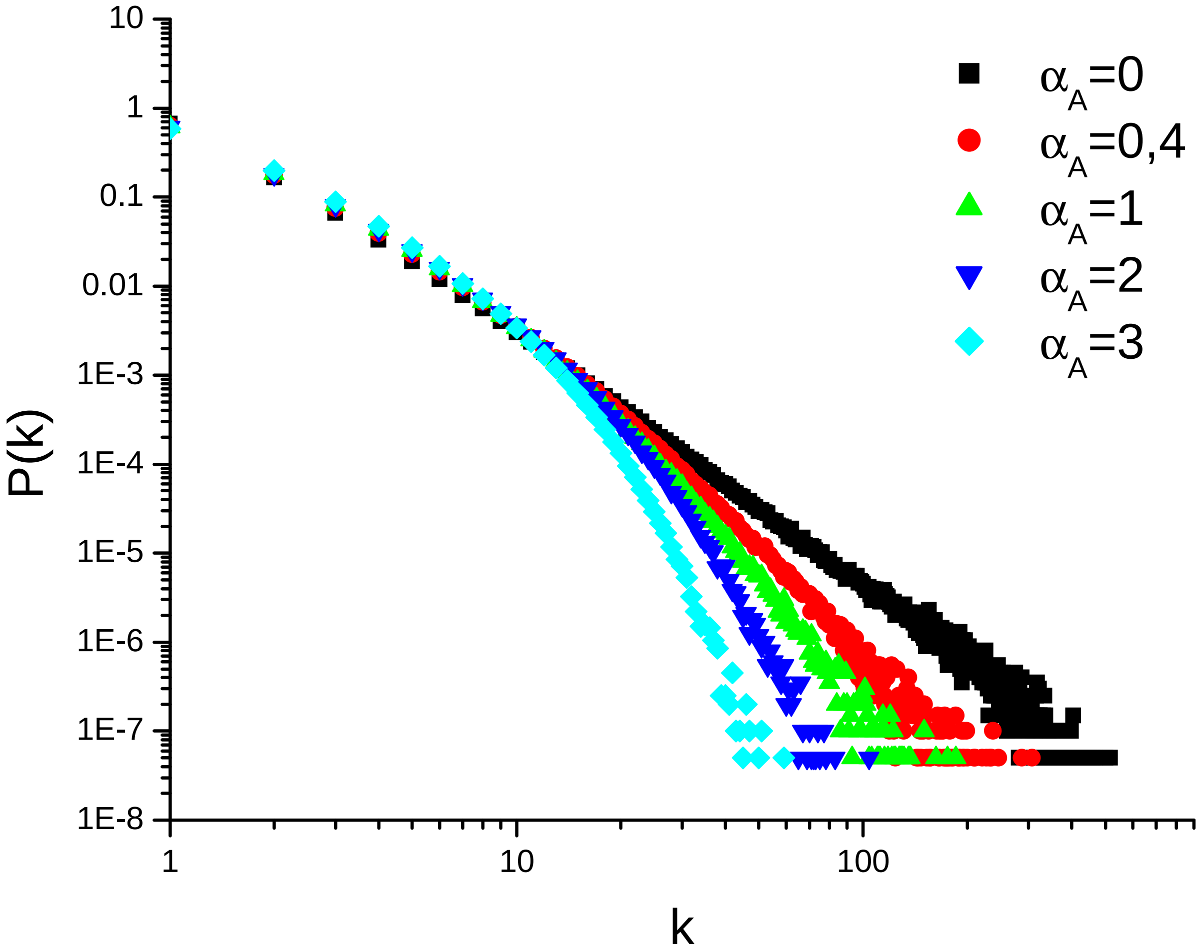


Links omitted

Barabási-Albert Model with Euclidean Distance Power-law Distributed

$$\Pi(k_i) = \frac{k_i / r_i^{\alpha_A}}{\sum_{j=1}^{N'} k_j / r_j^{\alpha_A}} \quad (03)$$



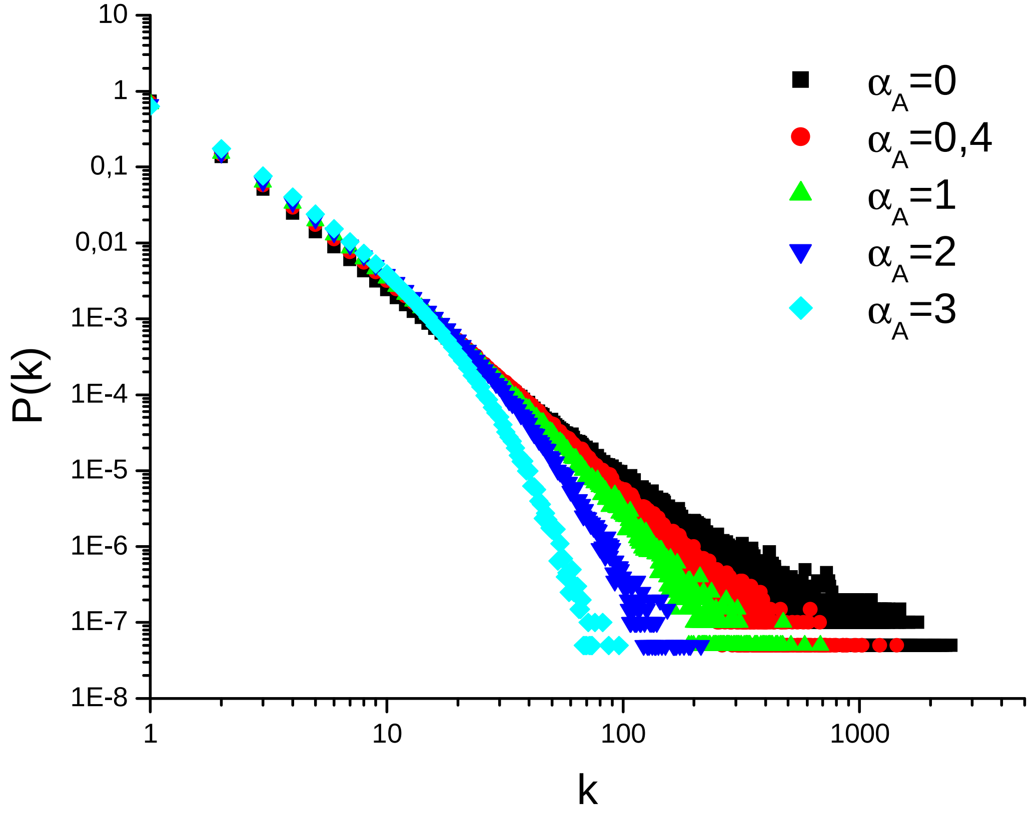
Soares, Tsallis, Mariz and da Silva, Europhys. Lett. **70,** 70 (2005)

Fitness Model with Euclidean Distance Power-law Distributed

- Inspired in the works of Soares, Tsallis, Mariz and da Silva[3],

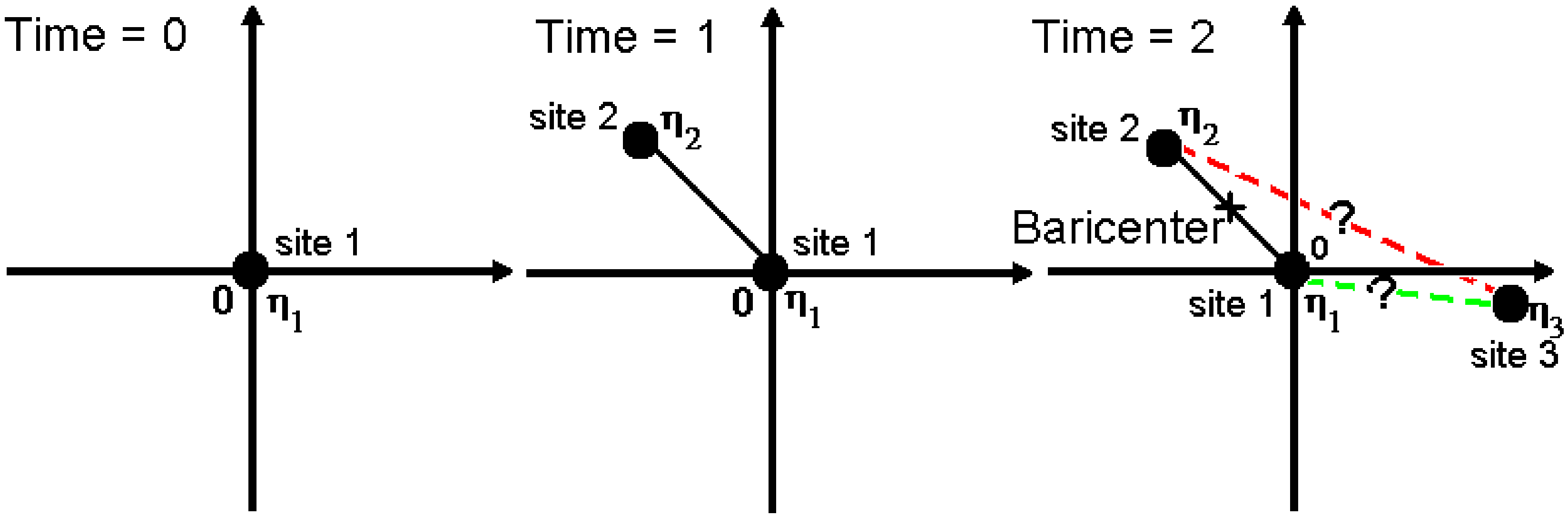
  and Bianconi and Barabasi[4].

$$\Pi(k_i) = \frac{k_i \eta_i / r_i^{\alpha_A}}{\sum_{j=1}^{N'} k_j \eta_j / r_j^{\alpha_A}} \quad (06)$$



Meneses, Cunha, Soares, and da Silva,
Progress of Theoretical Physics Supplement **162,** 131 (2006)

Network Construction



$P(r)\ \alpha\ r^{-\gamma_G}$

$\gamma_G > 1$

where

$$\gamma_G = \frac{3+\alpha_G}{2+\alpha_G}$$

$\alpha_G \geq 0$

$$r = (1-\xi')^{-(2+\alpha_G)}$$

$$\theta = 2\pi\xi'$$

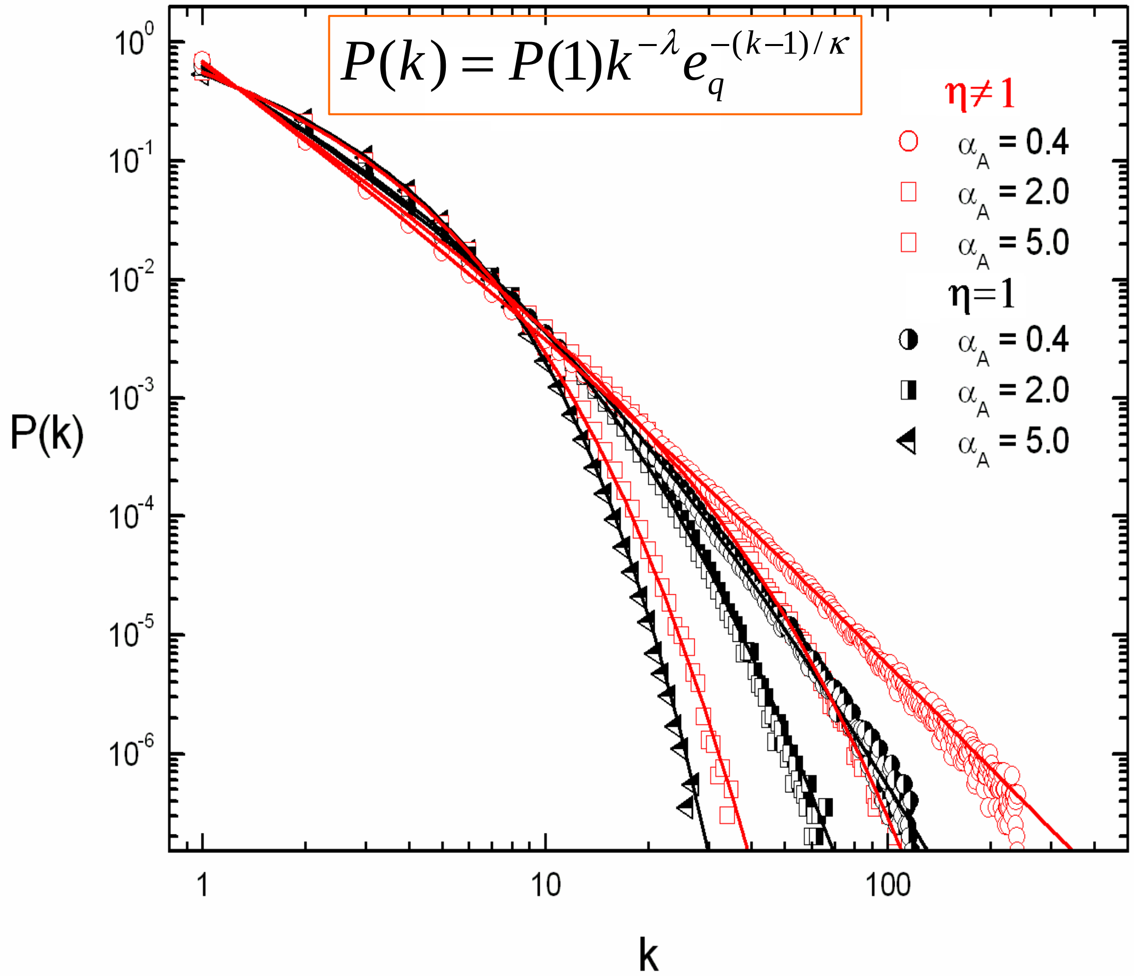
Tsallis Nonextensive Statistical Mechanics

$$s_q = \frac{1 - \int dk [P(k)]^q}{q-1} \qquad (q \in \Re; S_1 = S_{BG})$$

$$\sum_i p_i = 1 \quad \text{and} \quad \frac{\sum_i p_i^q \varepsilon_i}{\sum_i p_i^q} = U_q .$$

$$P(k) = P(1) k^{-\lambda} e_q^{-(k-1)/\kappa}$$

$$e_q^x \equiv [1 + (1-q)x]^{1/(1-q)}$$



Connectivity distribution *P*(*k*) for typical values α_A for η≠1 and η=1 models. The symbols are numerical results and continuous lines are the best fits according to P(k).

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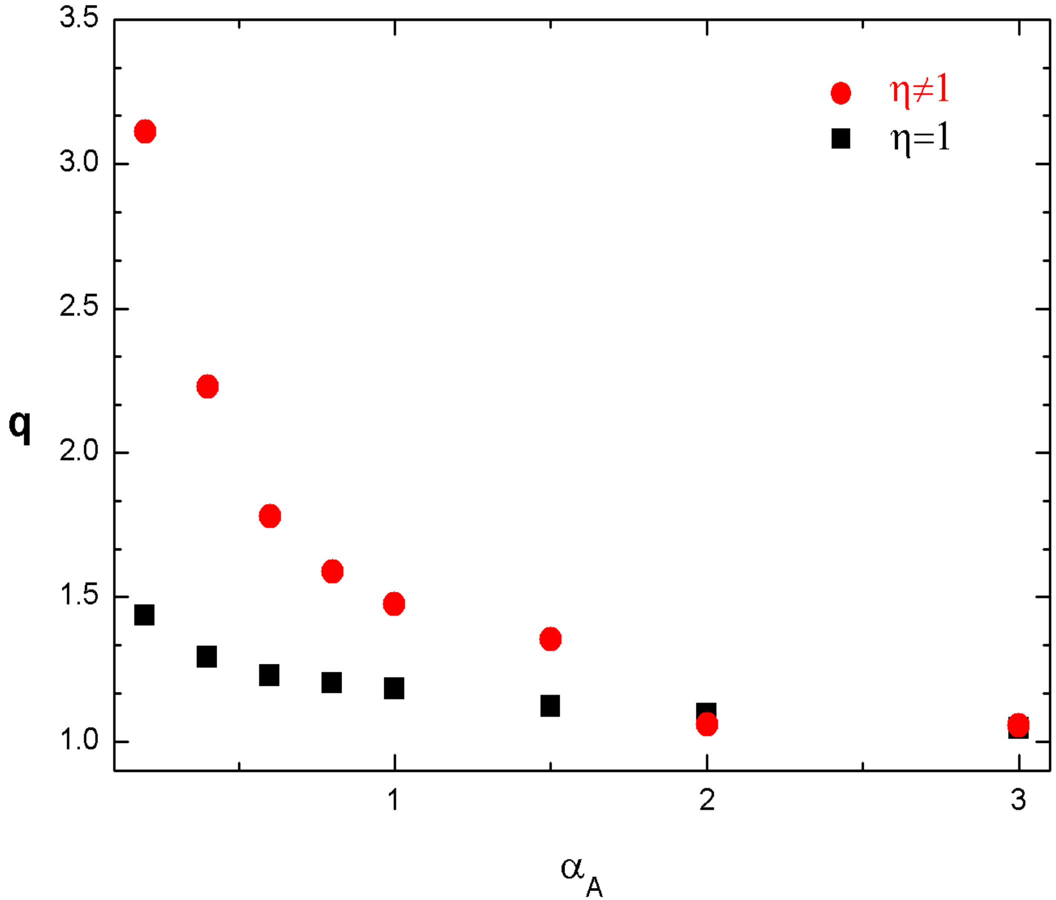

α_A-dependence of q for both η≠1 and η=1 models. In this graph we observe some kinds of changements of regimes at α_A = 2 (which coincides with the space dimensionality).

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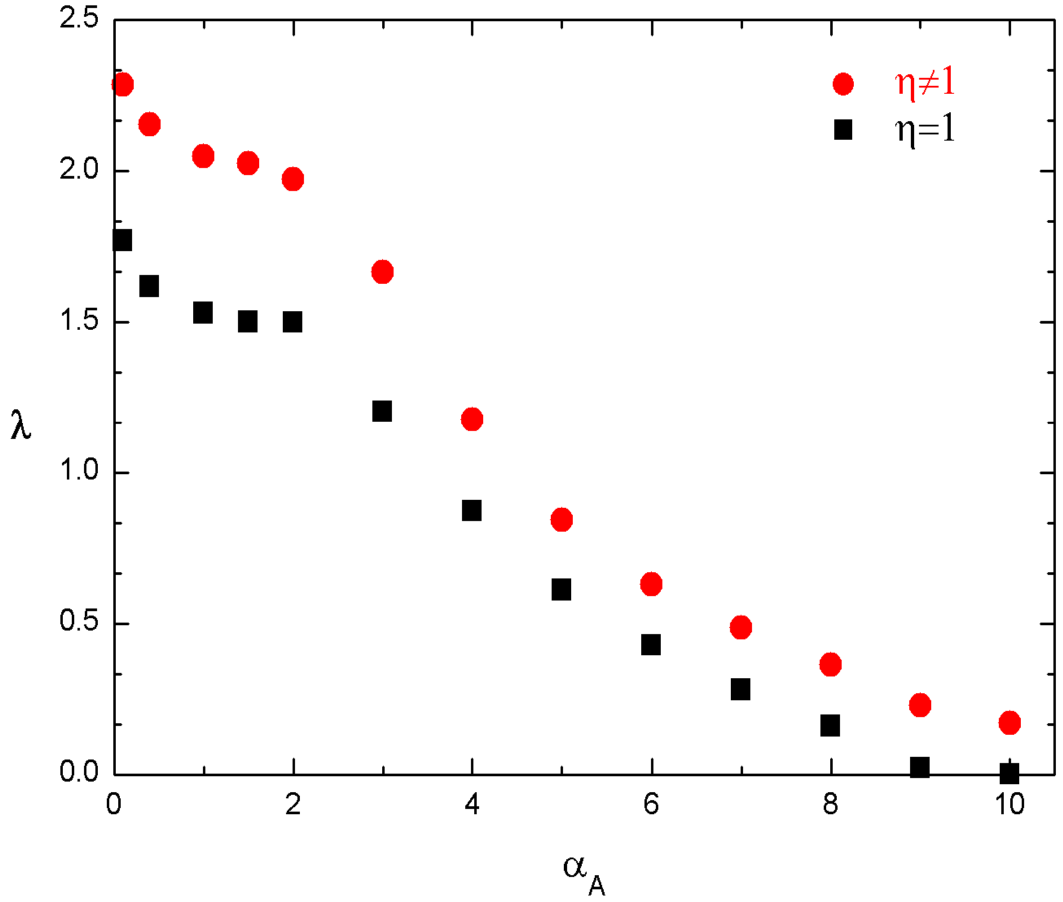
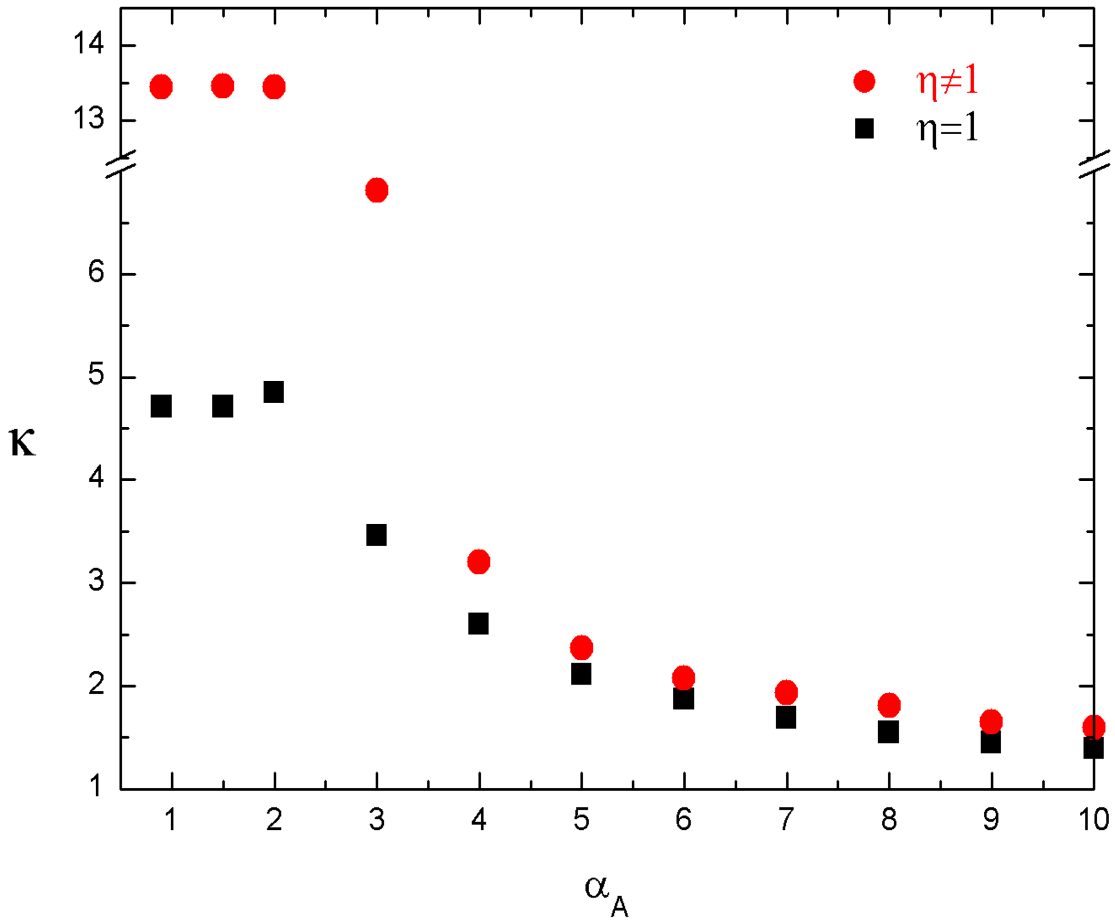
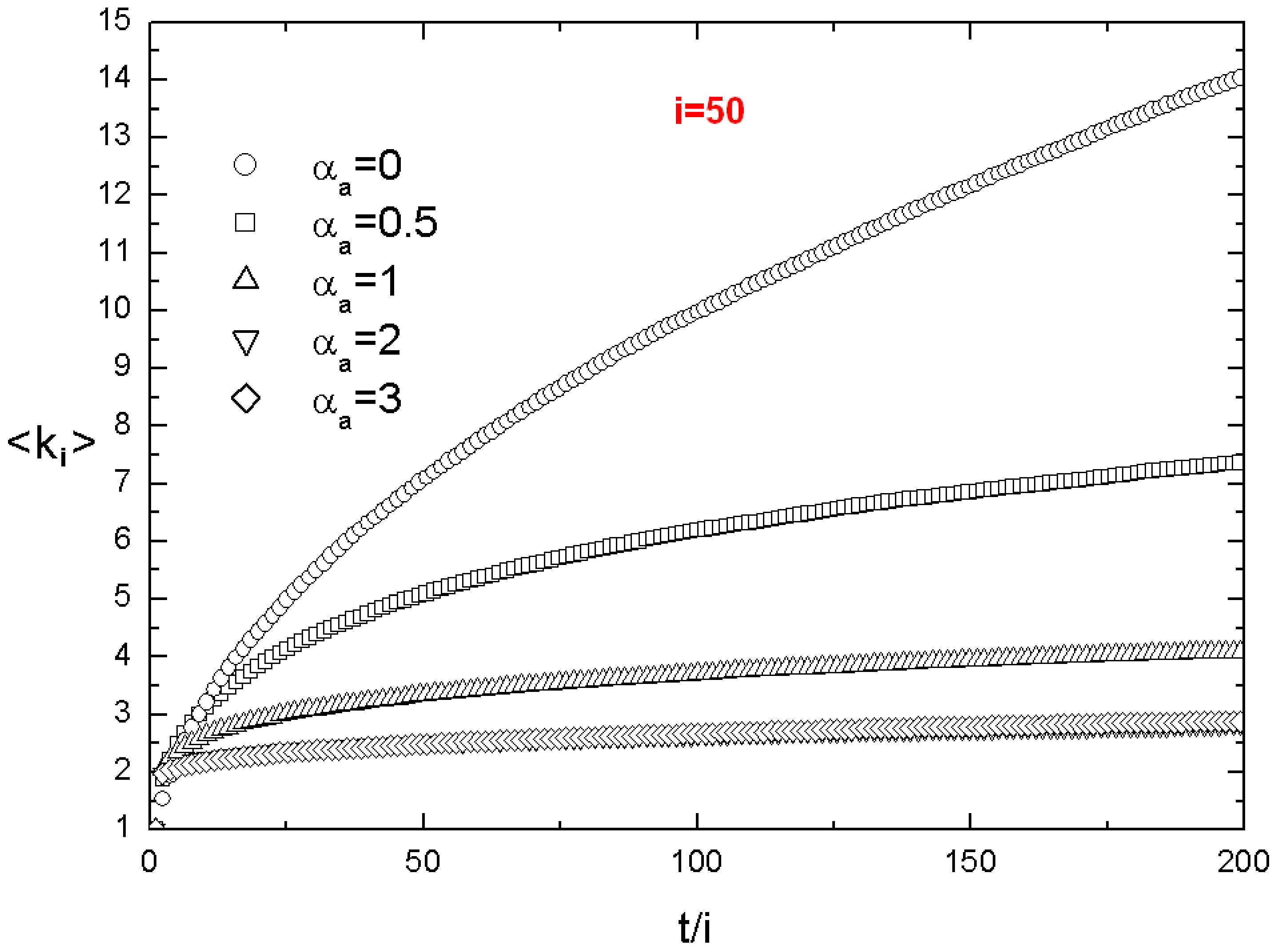


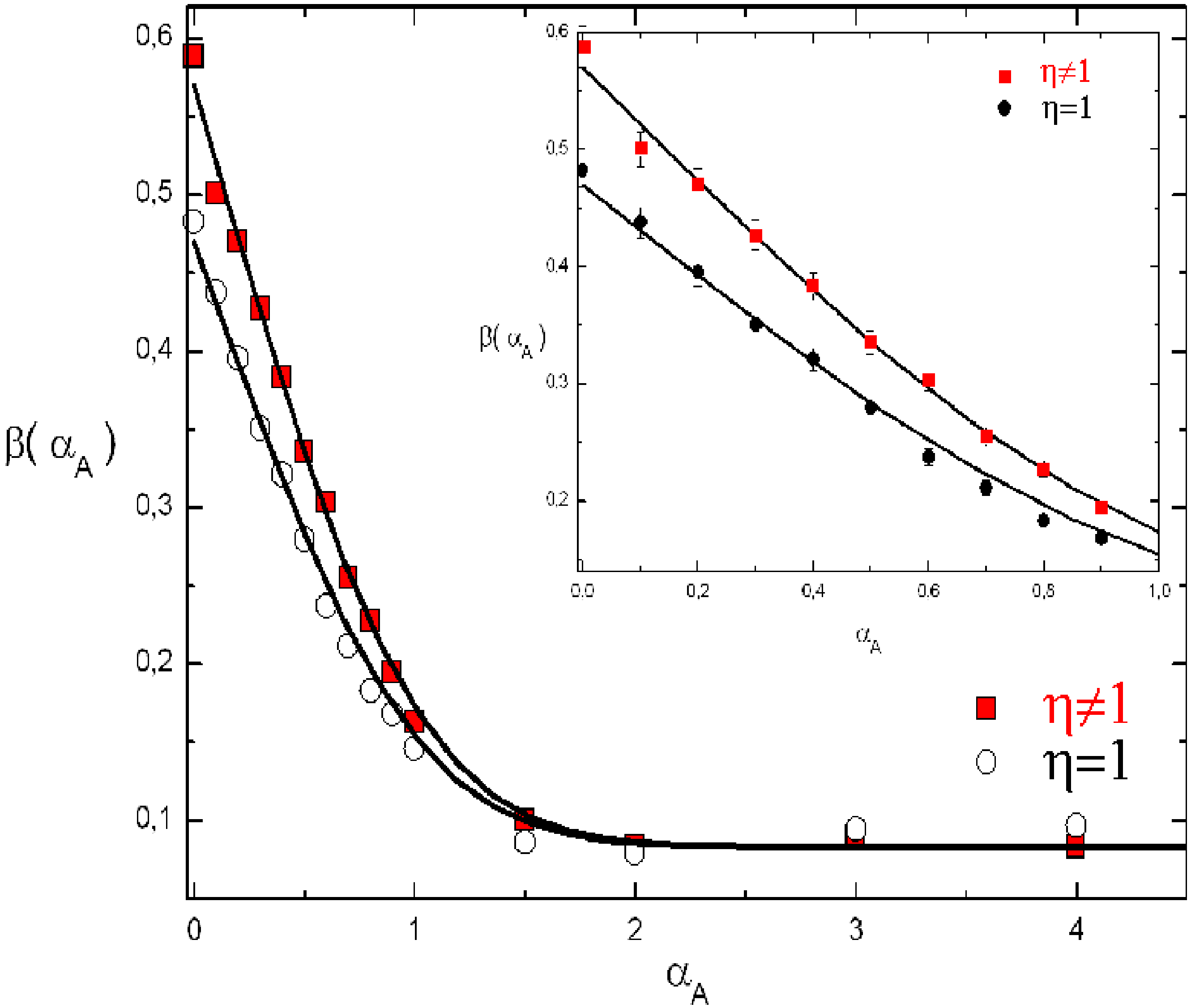
α_A-dependence of λ for both η≠1 and η=1 models. In this graph we observe some kinds of changements of regimes at α_A = 2 (which coincides with the space dimensionality).



α_A-dependence of q for both η≠1 and η=1 models. In this graph we observe some kinds of changements of regimes at α_A = 2 (which coincides with the space dimensionality).



Temporal dependence of the average connectivity for η≠1, in 2000 samples.



Average connectivity exponent for α_A values relative to measures on node $i = 50$.

Generalized Model: Fitness and Euclidean Distance Power-law Distributed

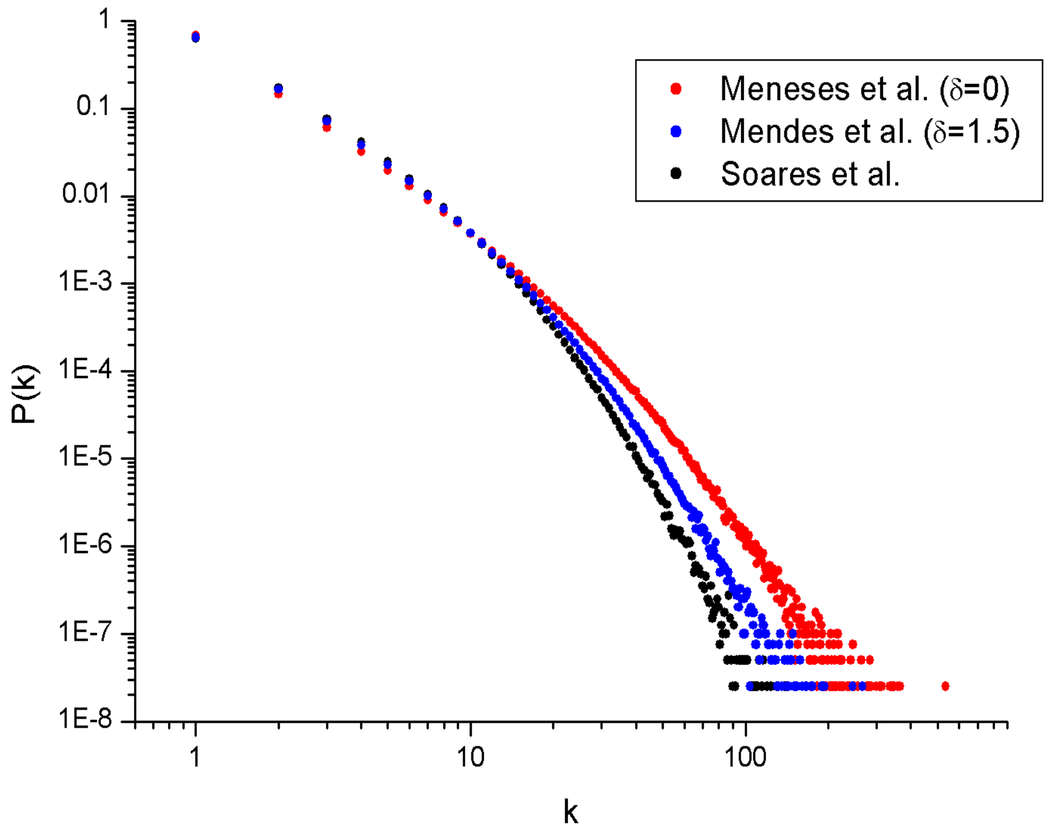
- Inspired in the works of
  Meneses et al;
  Mendes et al;

$$\Pi(k_i) = \frac{k_i \eta_i / r_i^{\alpha_A}}{\sum_{j=1}^{N'} k_j \eta_j / r_j^{\alpha_A}} \quad (08)$$

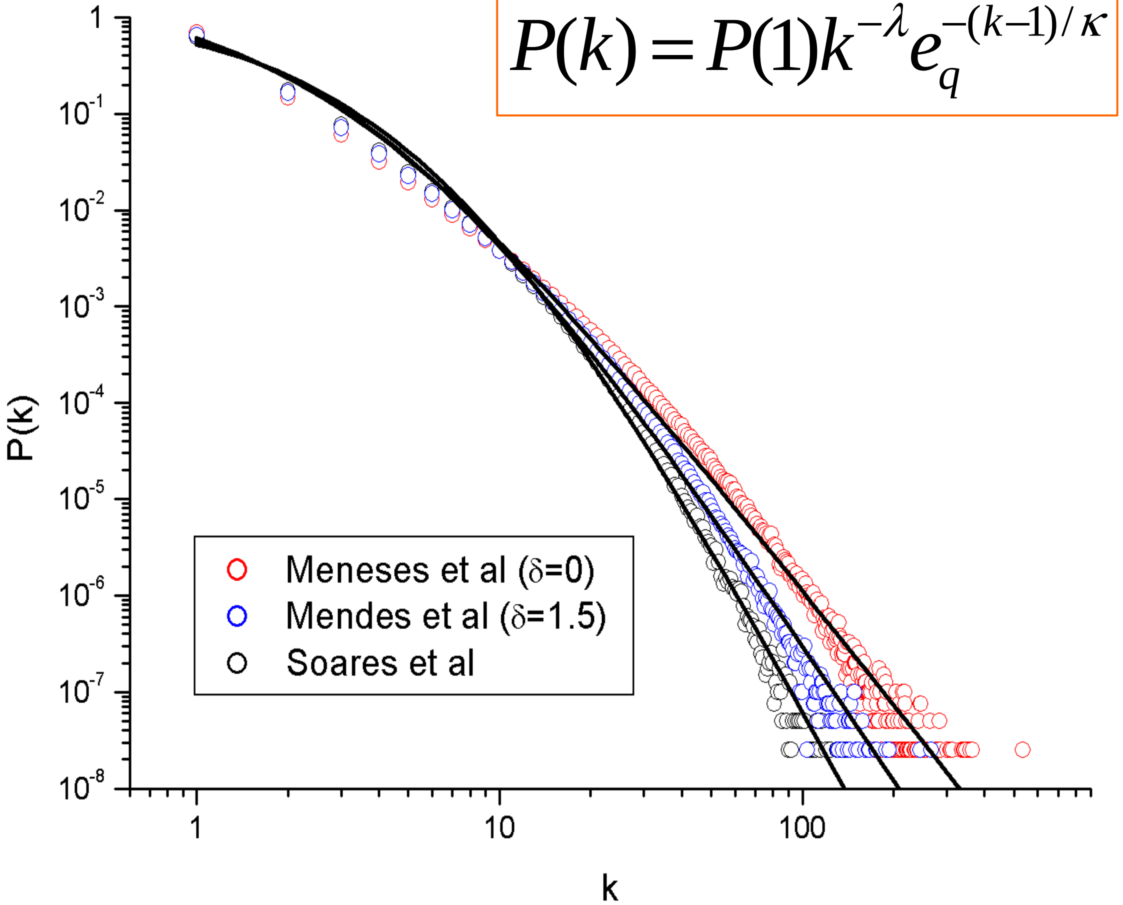
with

$$\rho(\eta) \propto \eta^{\delta}$$

$$\delta \geq 0; \alpha_A = 2$$



$$P(k) = P(1)k^{-\lambda} e_q^{-(k-1)/\kappa}$$



Conectivity distribution $P(k)$ for α_A=2 for Meneses et al, Mendes et al and Soares et al models. The symbols are numerical results and continuous lines are the best fits in according to P(k).

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- The generalized model contains the five previous models:

Model	CONECTIVITY	FITNESS	METRIC
Barabási-Albert	YES	NO	NO
Bianconi et al	YES	UNIFORM	NO
Soares et al	YES	NO	POWER-LAW
Meneses et al	YES	UNIFORM	POWER-LAW
Mendes et al	YES	POWER-LAW	NO
Generalized	YES	POWER-LAW	POWER-LAW

Affinity Model

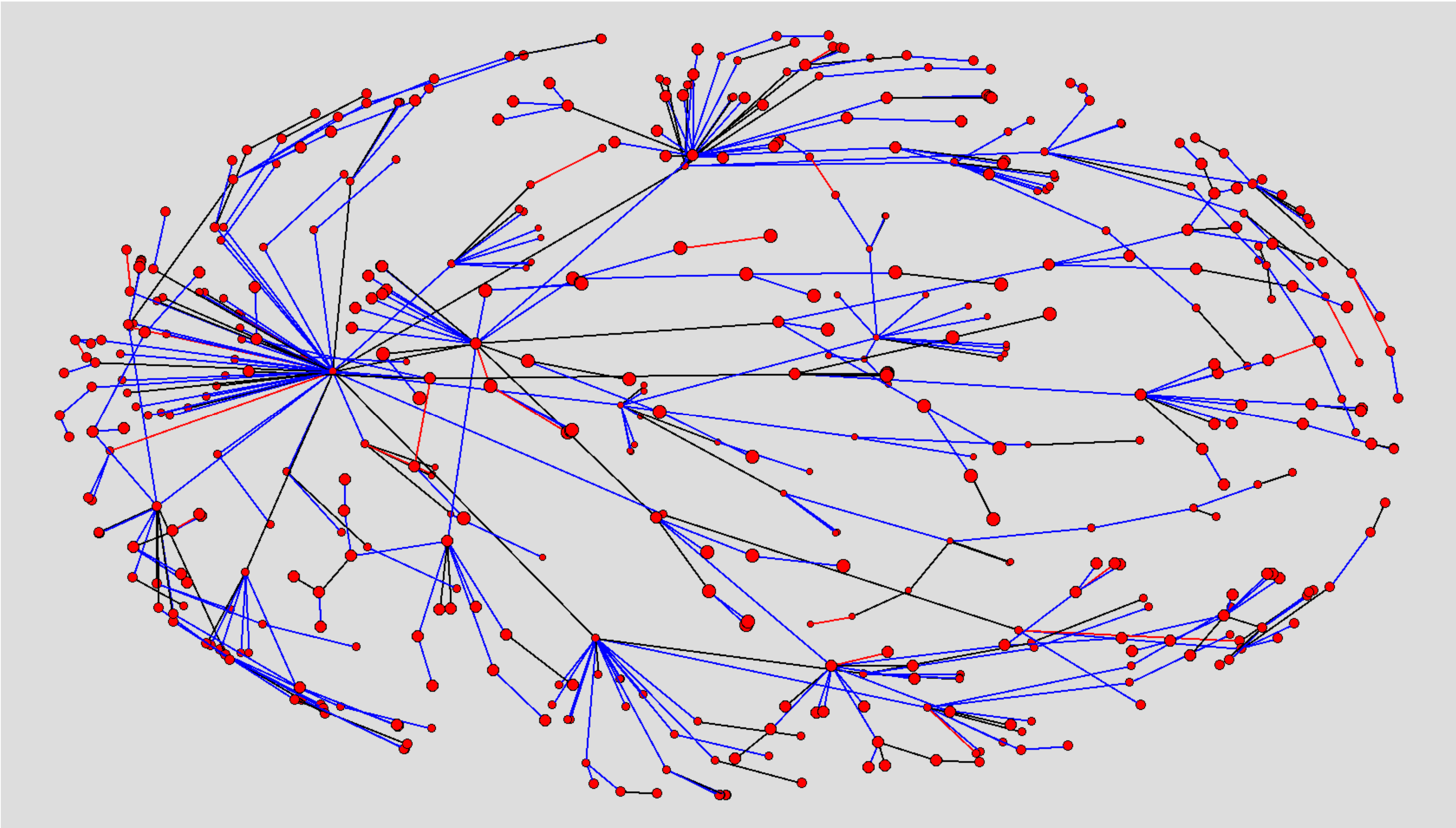
- Inspired in the works of Bianconi and Barabási; G.A. Mendes and L.R. da Silva.

- The links between similar sites are favored.

$$\Pi_{i \to j} \sim k_j \left(1 - \left|\eta_i - \eta_j\right|\right)$$

M.L. Almeida, G.A. Mendes, G.M. Viswanathan A.M. Mariz and L.R. da Silva (Preprint)

blue: big affinity; black: medium affinity; red: small affinity



$$\Pi_{i\to j} \sim k_j\left(1-\left|\eta_i-\eta_j\right|\right)$$

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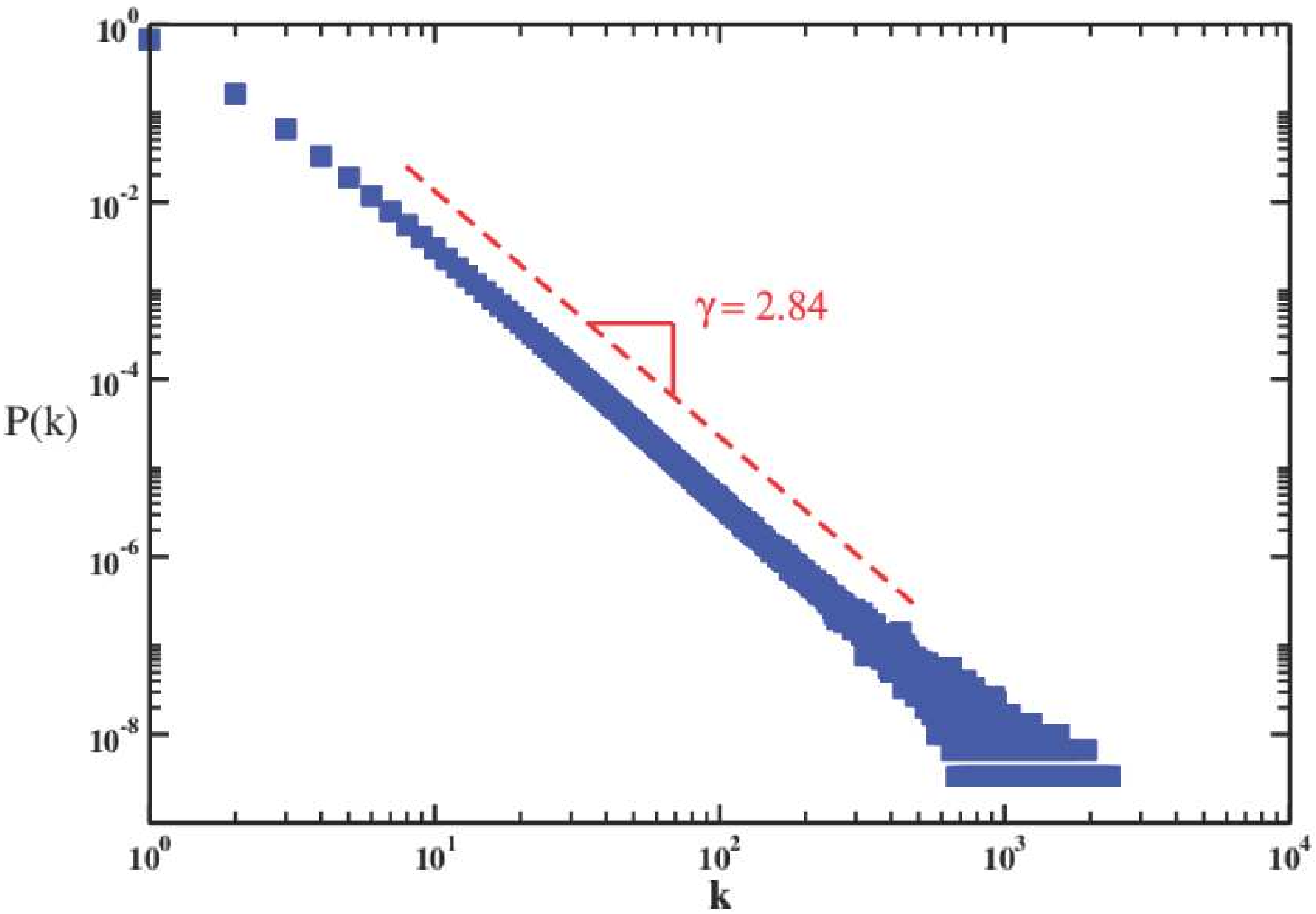

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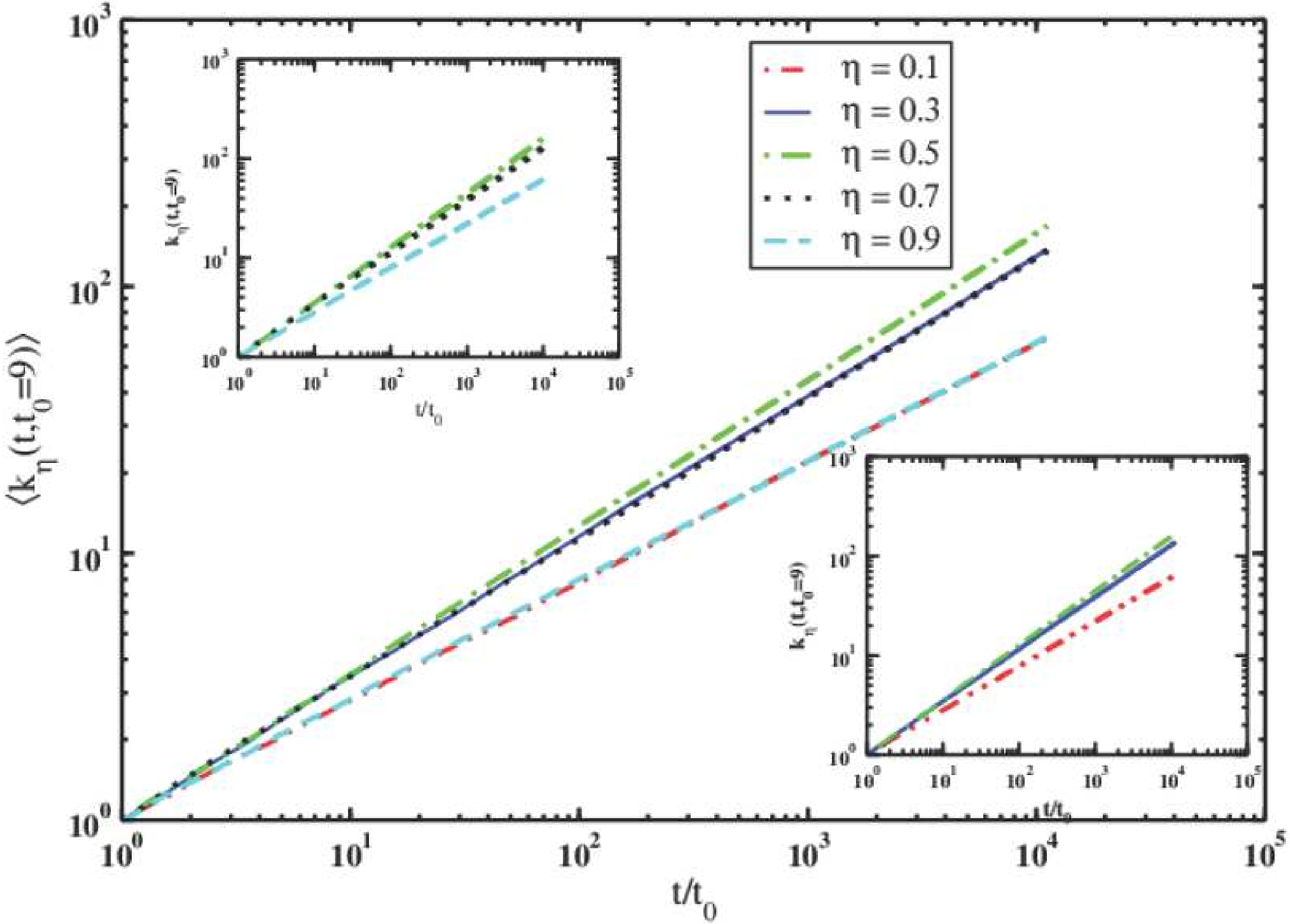
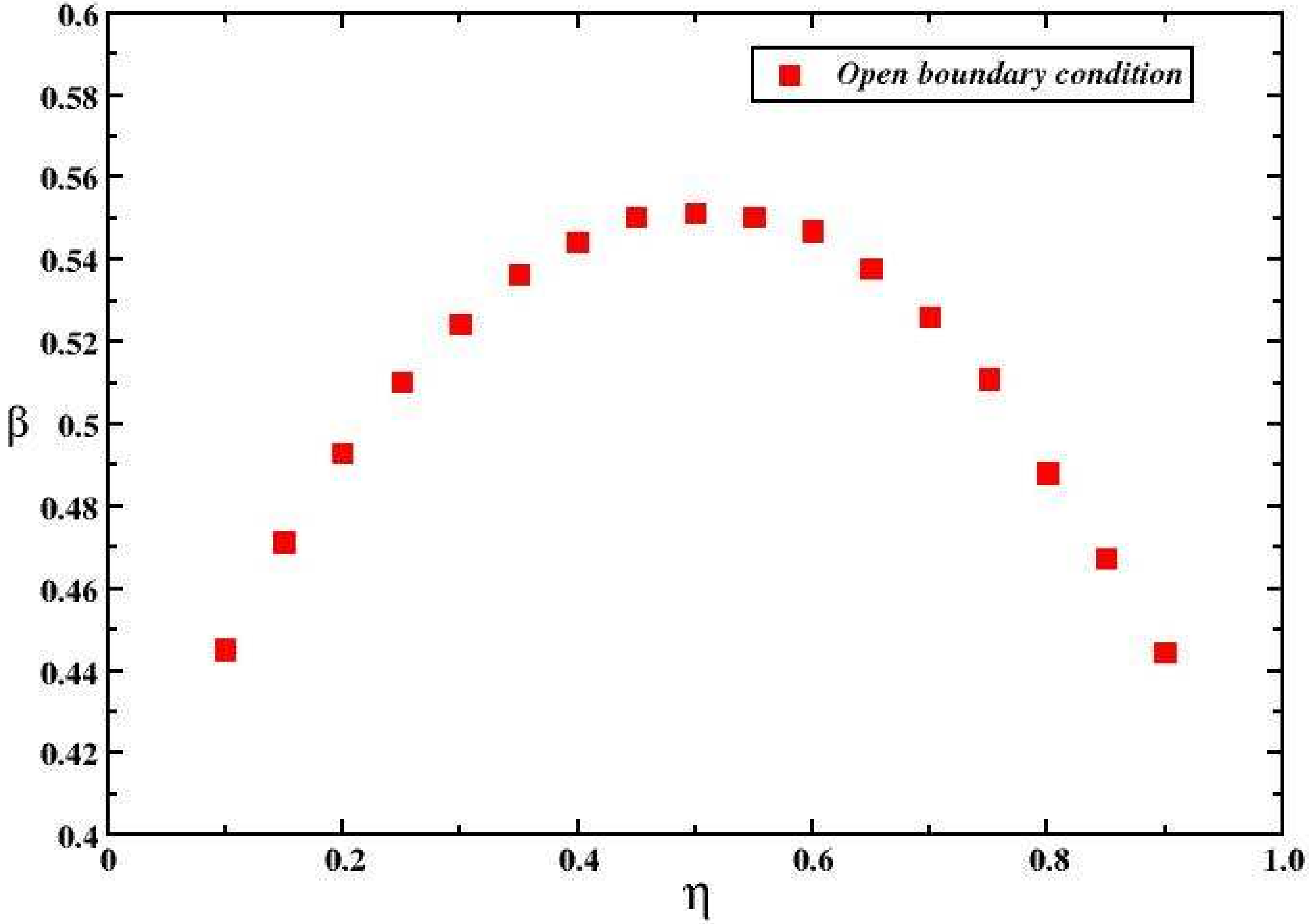
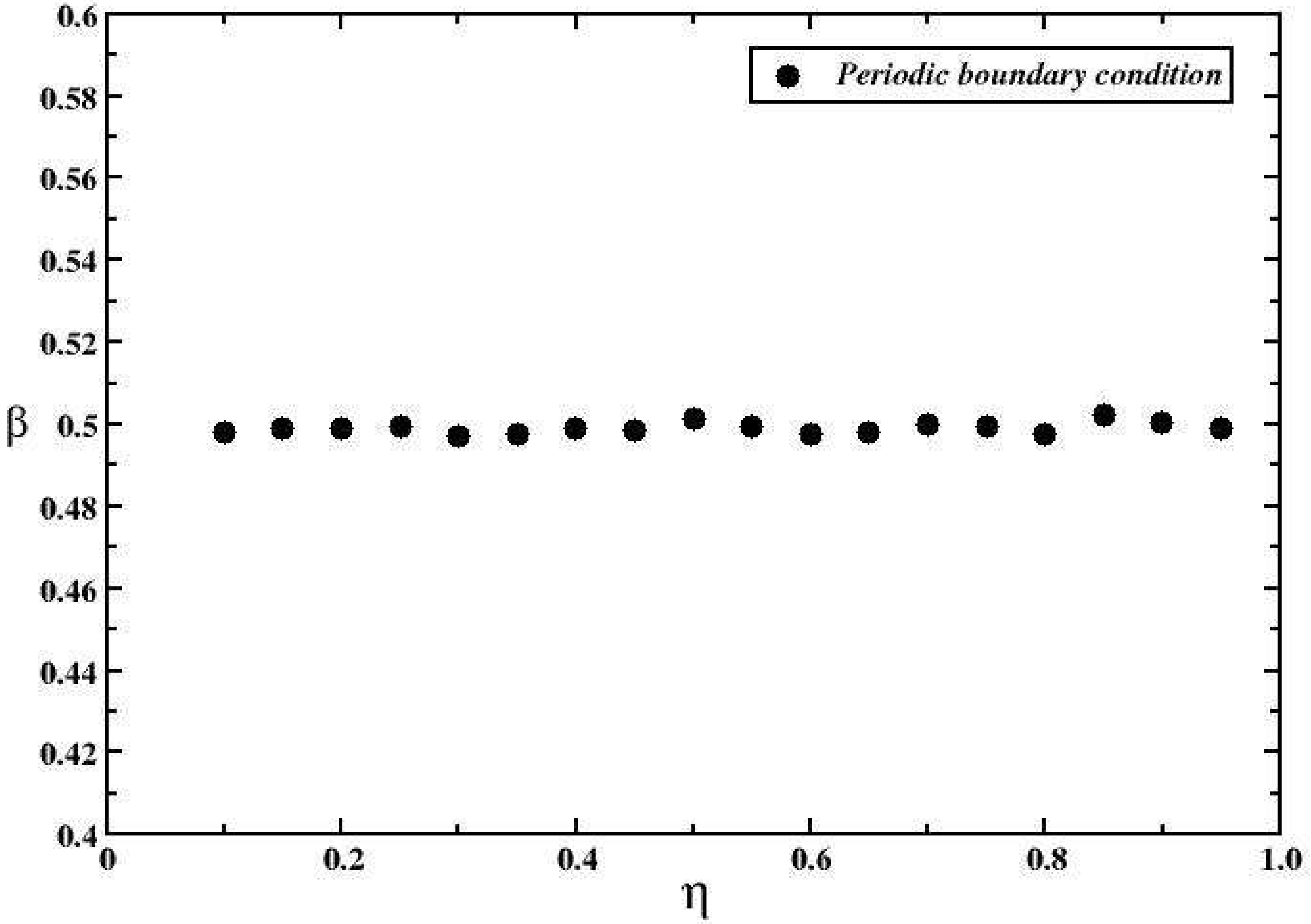
Affinity Model: Connectivity Time Evolution



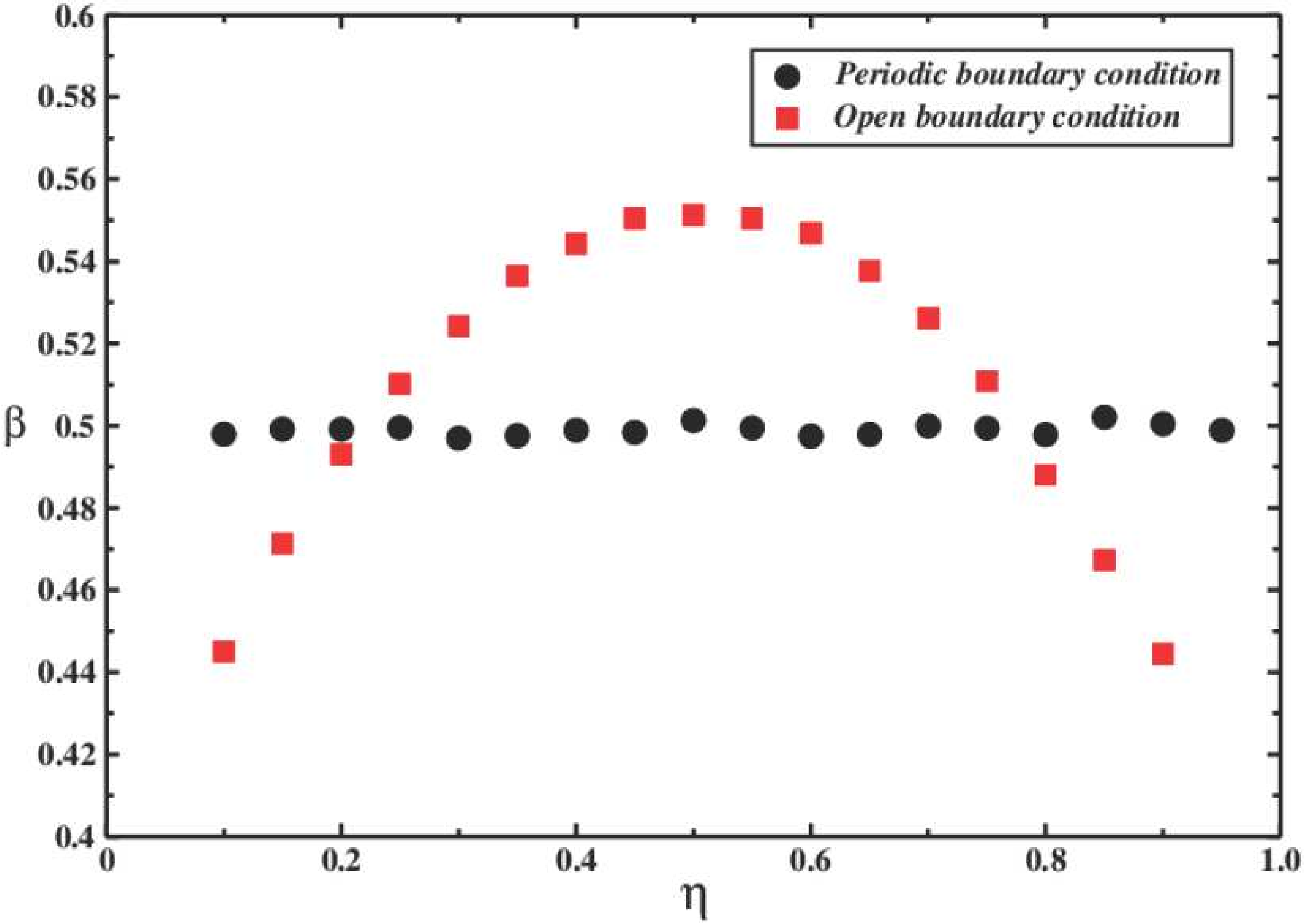
Affinity Model



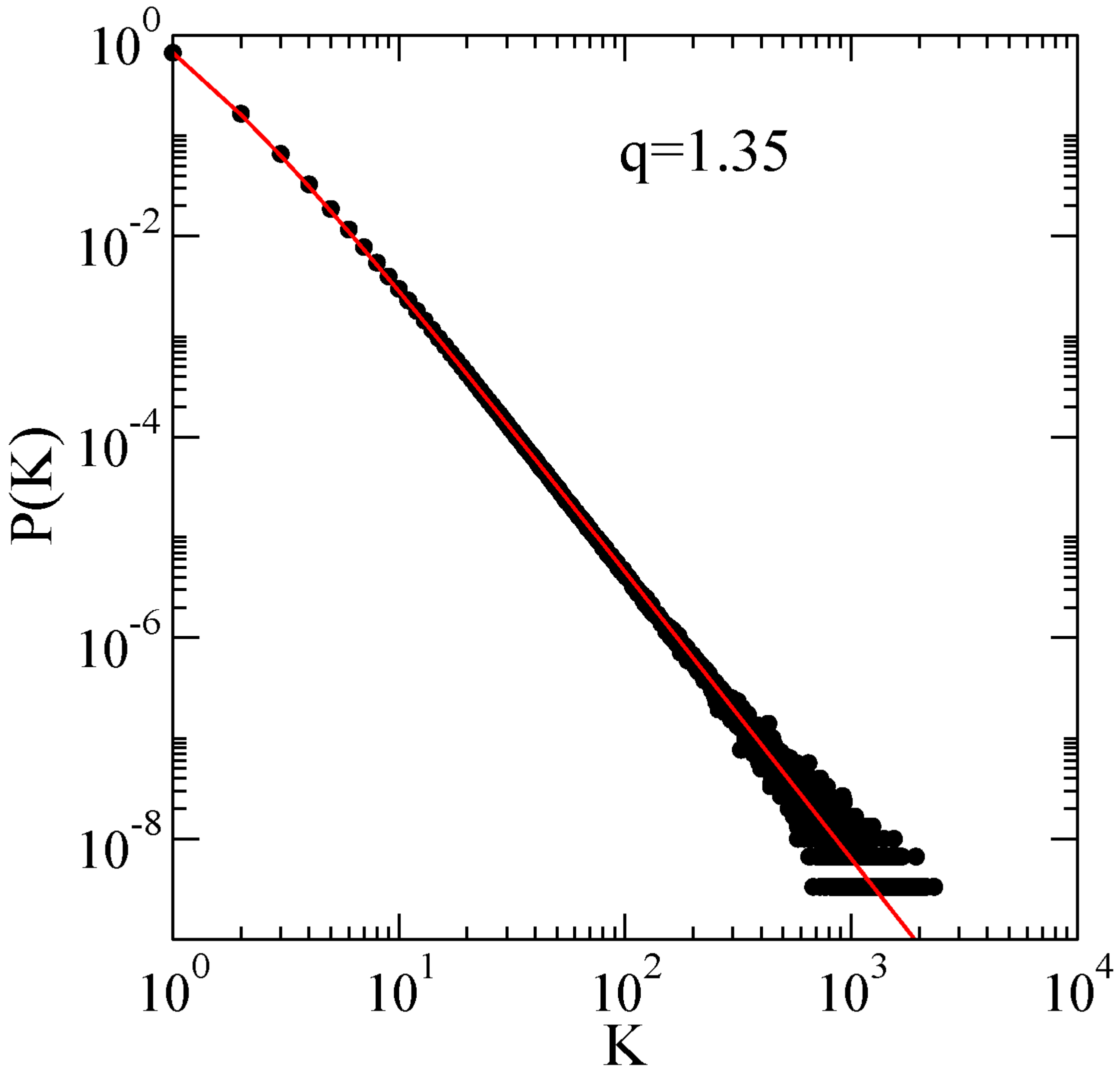
Affinity Model



Affinity Model



Affinity Model: Tsallis Statistics



Summary

(a)

- We study the effect of competition between the relevant variables: connectivity k, fitness η and metrics r.
- The fitness may give the possibility to the younger nodes to compete equally with the older ones, when the younger node gets a high fitness.
- By including metrics favors the linking between first neighbors.
- The average connectivity $<k_i>$ is appreciably influenced by metrics and by fitness, while the average path length $<l>$ keeps approximatively the same.

Summary

(b)

- The degree distribution *P*(*k*) of the present generalized model appears to be the *q*-exponential function that emerges naturally within Tsallis nonextensive statistics.
- We modify the rule of the preferential attachment of the Bianconi-Barabasi model including a factor which represents similarity of the sites.
- The term that corresponds to this similarity is called the affinity and is obtained by the modulus of the difference between the fitness (or quality) of the sites.
- This variation in the preferential attachment generate very unusual and interesting results.

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References

1. D. J. B. Soares, C. Tsallis, A. M..Mariz, and L. R. da Silva.
"Preferential Attachment Growth Model and Noextensive Statistical Mechanics"
Europhysics Letters **70**, 70 (2005)

2. J. S. Andrade Jr., H. J. Herrmann, R.F Andrade and L. R. da Silva.
"Apollonian Networks: Simultaneously Scale-free, Small World, Euclidean, Space Filling and with Matching Graphs."
Physical Review Letters **94**, 018702 (2005).

3. M. D. de Meneses, Sharon D. da Cunha, D.J.B. Soares and L. R. da Silva.
"Preferential Attachment Scale-free Growth Model with Random Fitness and Connection with Tsallis Statistics"
Progress of Theoretical Physics Supplement **162** 131 (2006)

4. D. J. B. Soares, J. S. Andrade Jr., H. J. Herrmann and L. R. da Silva
"Three Dimension Apollonian Networks"
International Journal of Modern Physics **17** 1219 (2006)

5. P. G. Lind, L. R. da Silva, J. S. Andrade Jr. and H. J. Herrmann.
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6. P. G. Lind, L. R. da Silva, J. S. Andrade Jr. and H. J. Herrmann.

"Spreading Gossip in Social Networks"

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7. G. A. Mendes and L.R. da Silva

"Generating more realistic complex networks from power-law distribution of fitness"

Brazilian Journal of Physics **39** 423 (2009)

8. S. S. B. Jácome, **L. R. da Silva**, A. A. Moreira, J. S. Andrade Jr. and H. J. Herrmann

"Iterative Decomposition of the Barabasi-Albert Scale-free Networks"

Physica A **389** 3674 (2010)

9. G. A. Mendes, **L. R. da Silva** and Hans J. Herrmann

"Traffic Gridlock on Complex Networks"

Physica A **391** 362 (2012)

10. M.L Almeida, G.A. Mendes, G. M. Viswanathan A.M. Mariz and L. R. da Silva (Preprint)

"Affinity model in complex networks"

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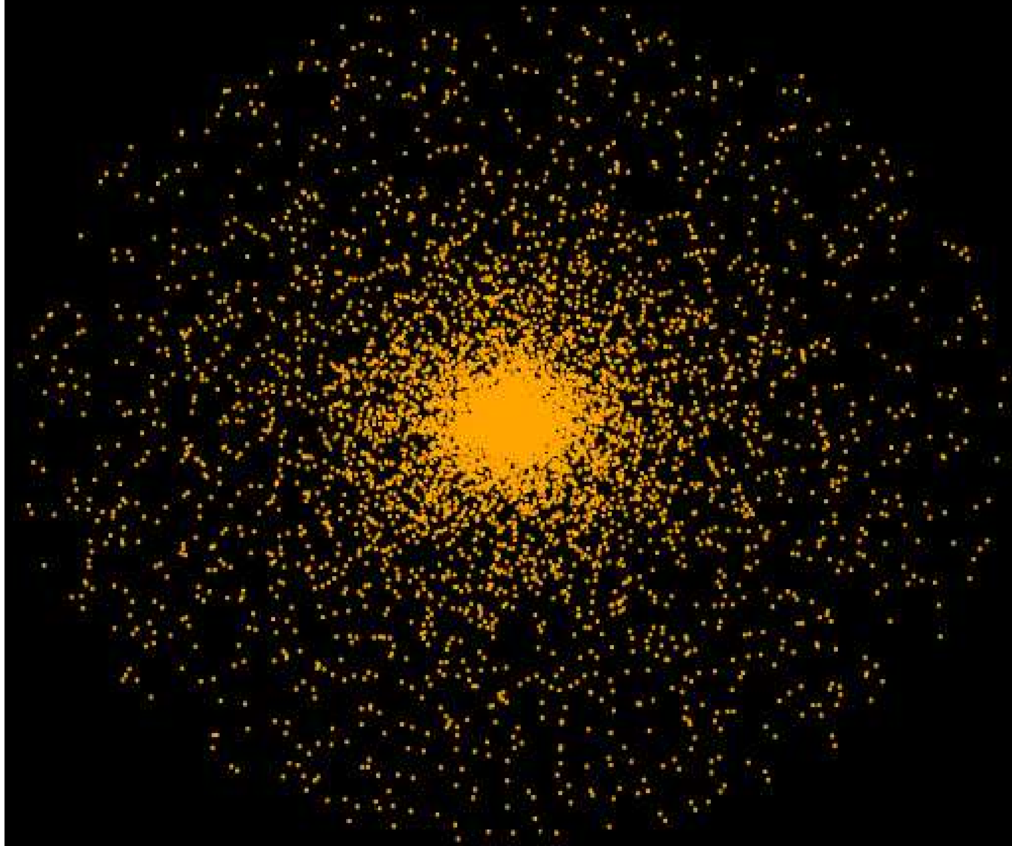

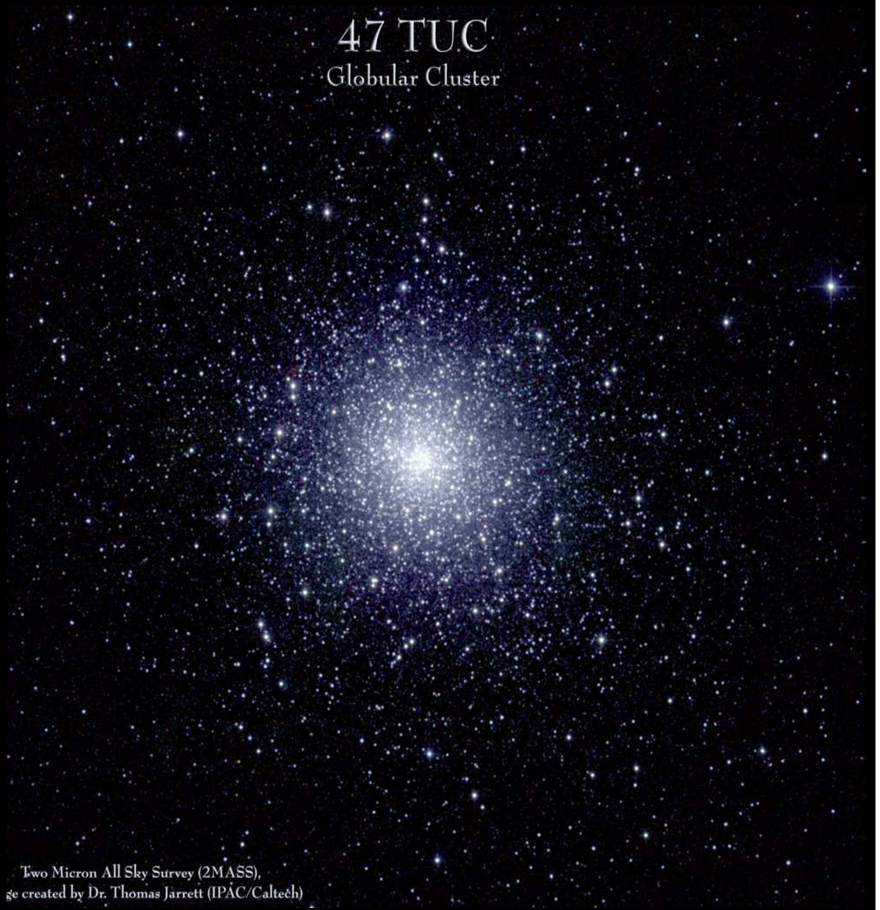
47 TUC
Globular Cluster
Two Micron All Sky Survey (2MASS),
ge created by Dr. Thomas Jarrett (IPAC/Caltech)

ありがとうございました。

THANK YOU VERY MUCH

Rede de Apolônio

PRL 94, 018702 (2005) PHYSICAL REVIEW LETTERS week ending 14 JANUARY 2005

Apollonian Networks: Simultaneously Scale-Free, Small World, Euclidean, Space Filling, and with Matching Graphs

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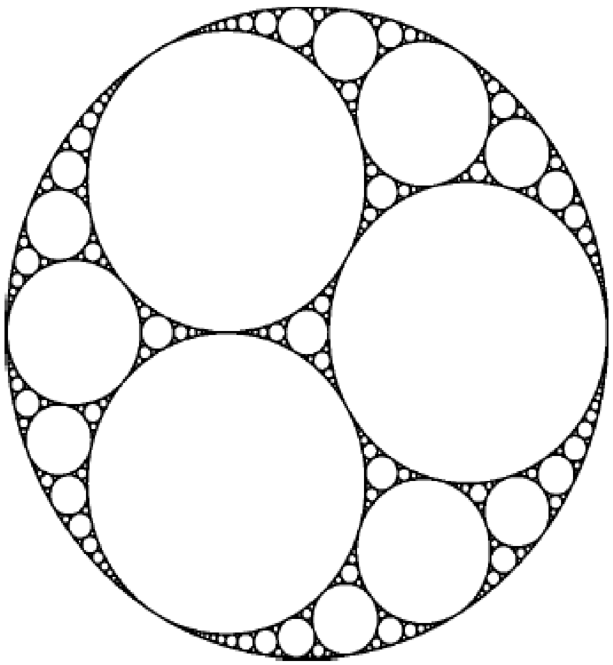
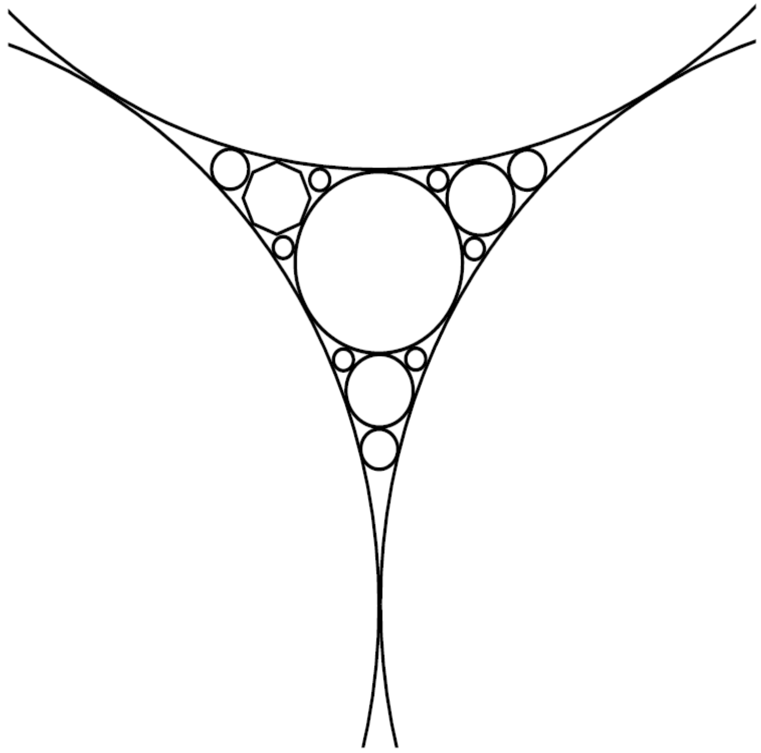
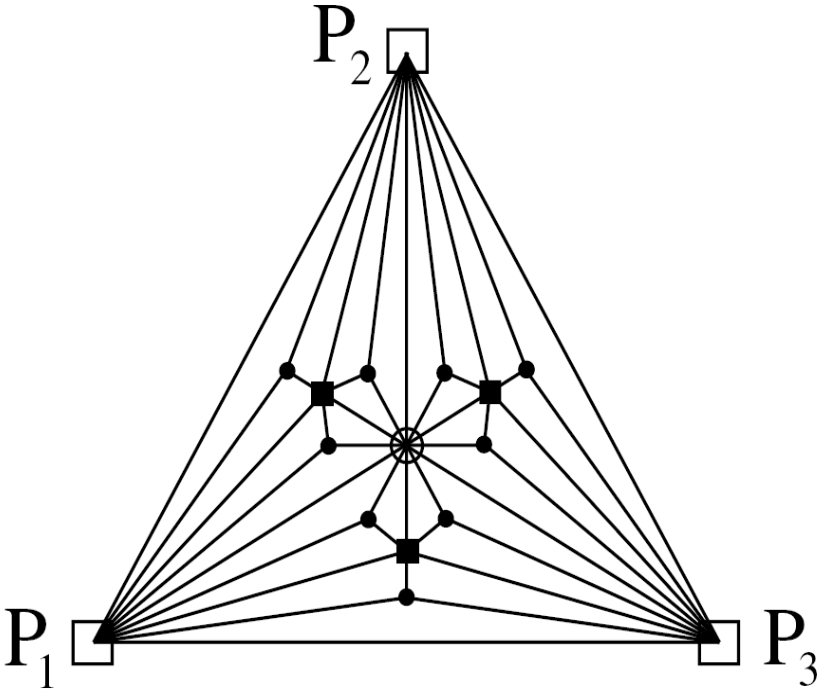


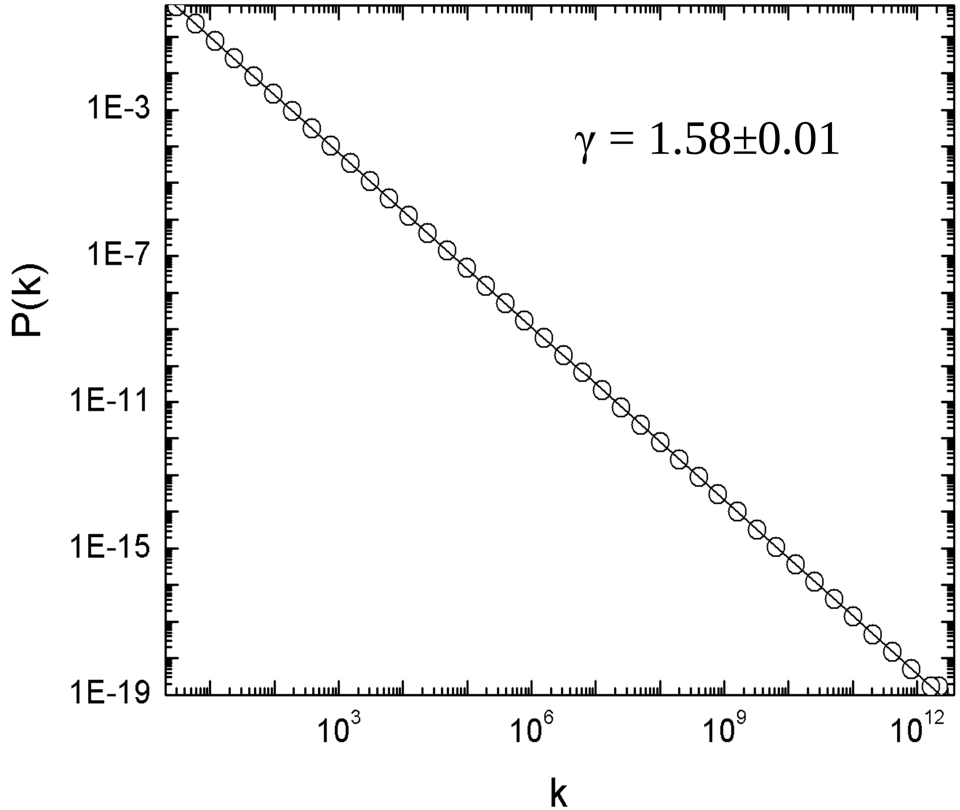


Apollonius of Perga
lived from about 262 BC to about 190 BC
Apollonius was known as 'The Great Geometer'.

Rede de Apolônio

Distribuição de Conectividade





Sítios	k
3	9
1	12
3	6
9	3

$$P(k) \propto k^{-\gamma}$$

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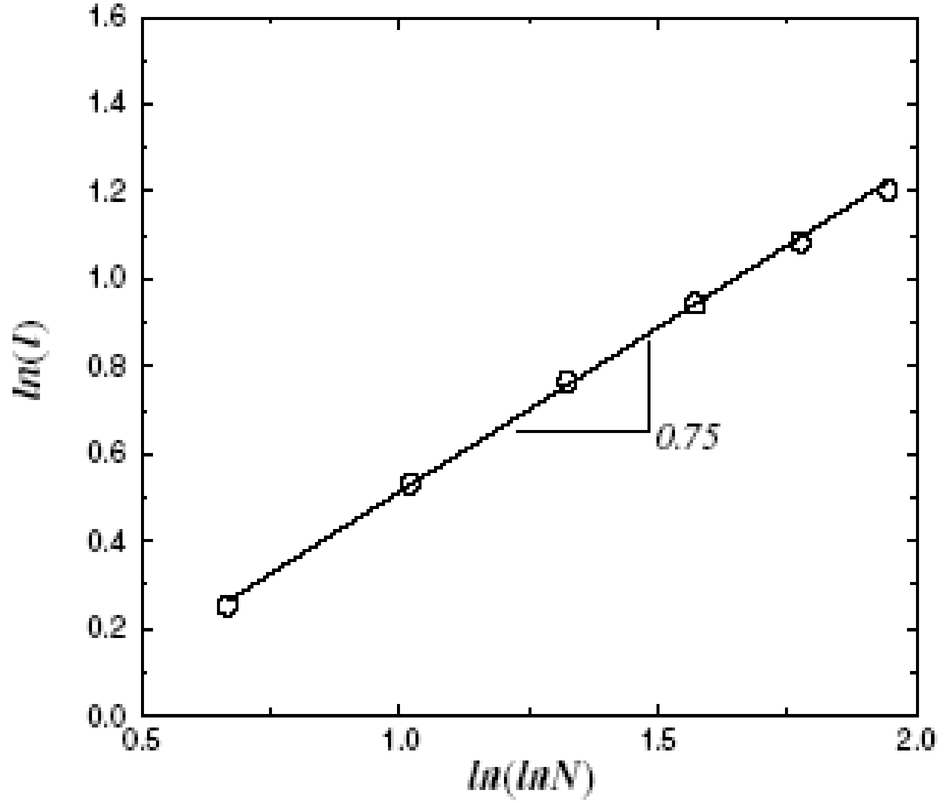
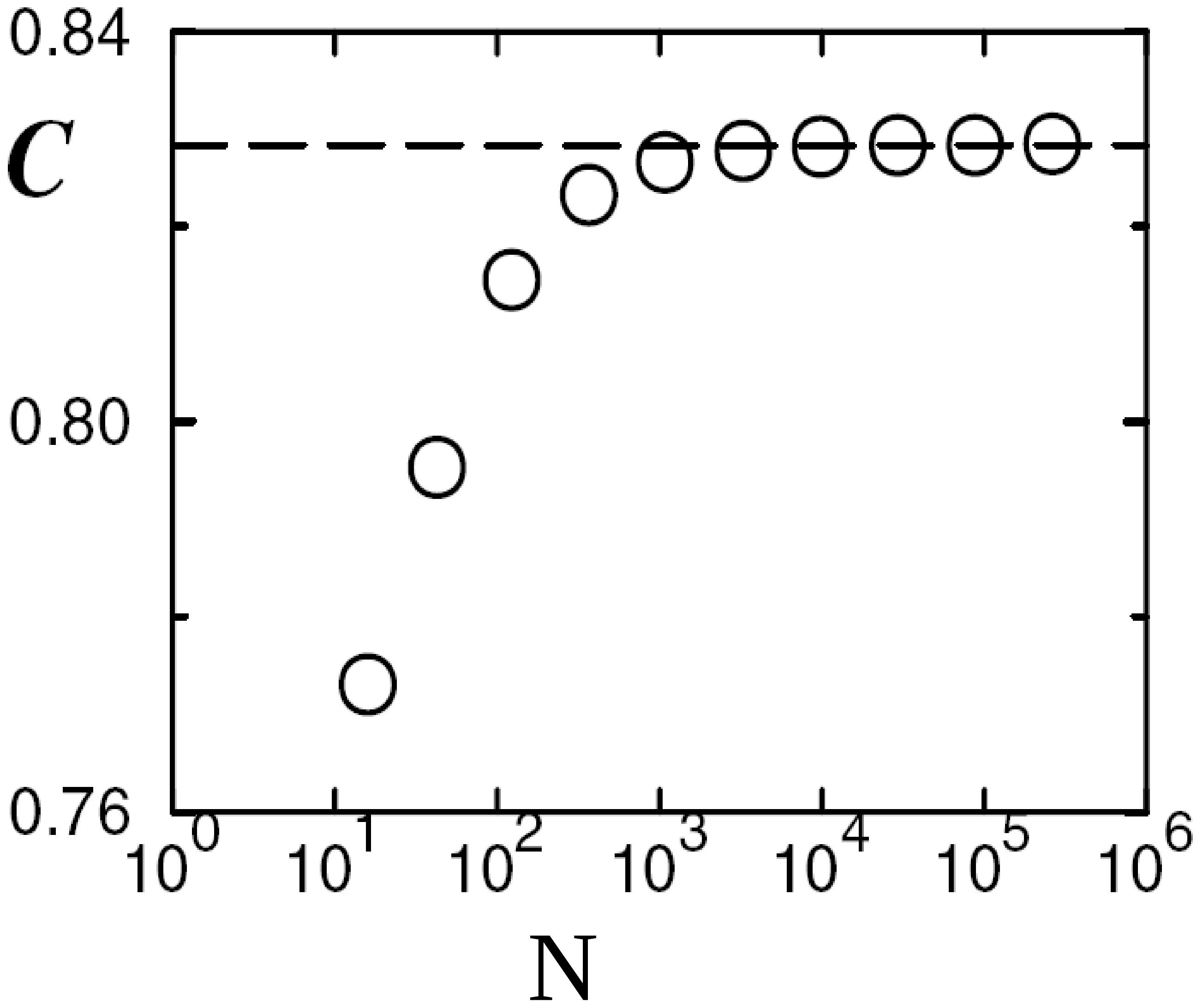
$$\Rightarrow N_n = 3 + \left(3^{n+1} - 1\right) \Big/ 2$$

Menor Caminho Médio

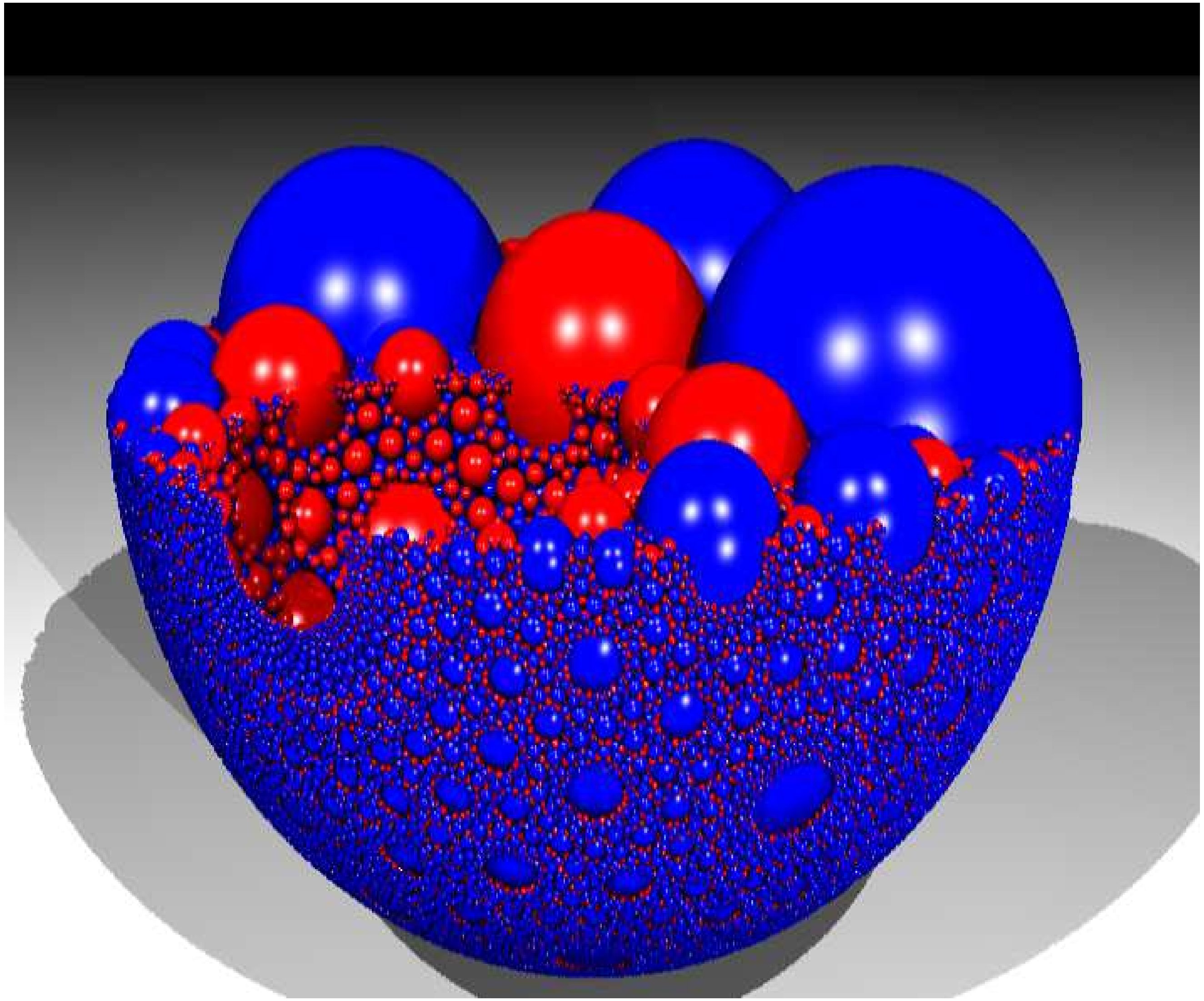


$$l \propto \left(\log\left(N\right)\right)^{\beta} \qquad \beta \approx 3/4$$

Coeficiente de Agregação Médio



Apolonio 3-D

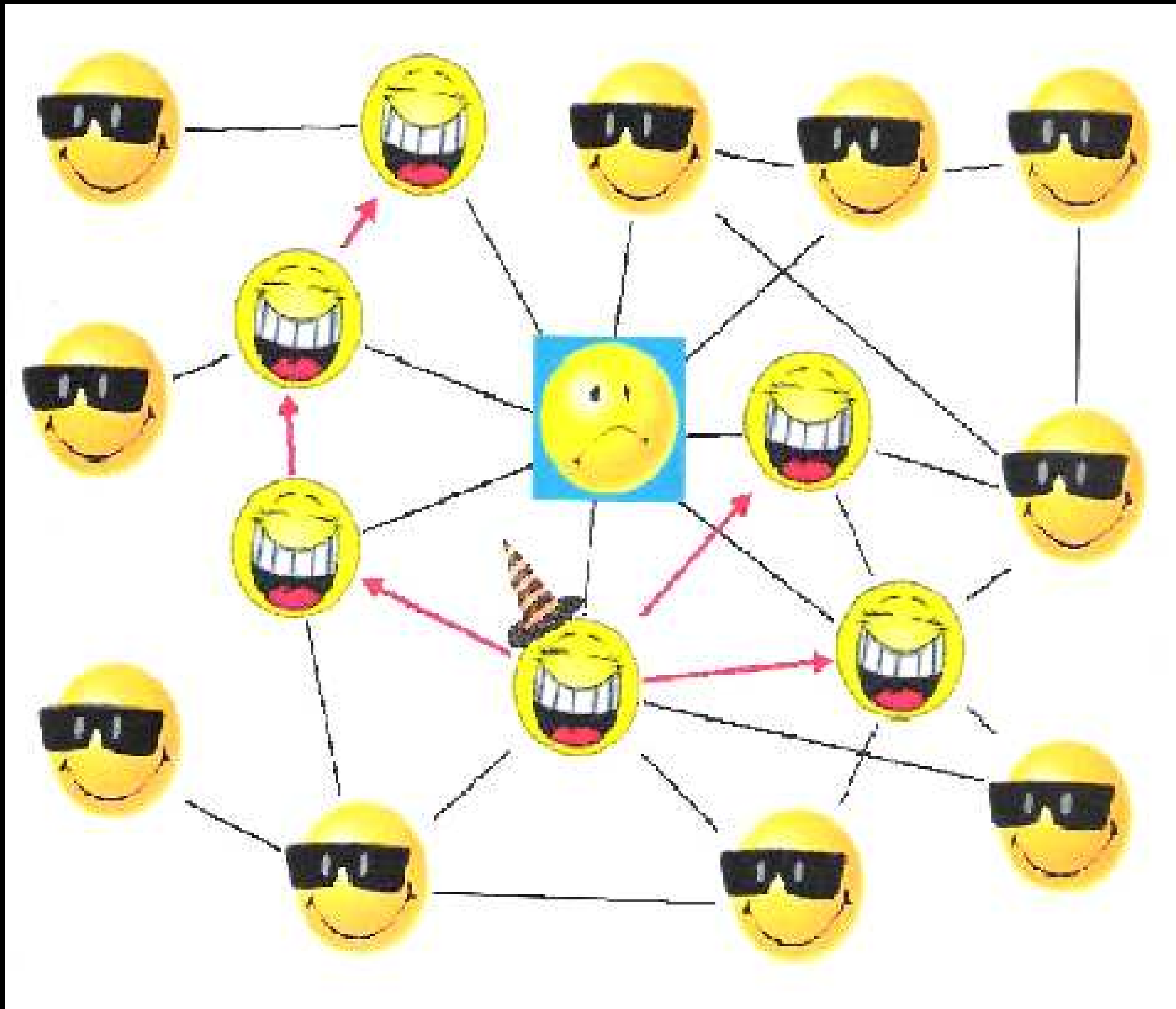


Danyel J. B. Soares, J. S. Andrade Jr, Hans J. Herrmann, **L. R. da Silva[7]**

[7]International Journal of Modern Physics C **17** 1219 (2006)

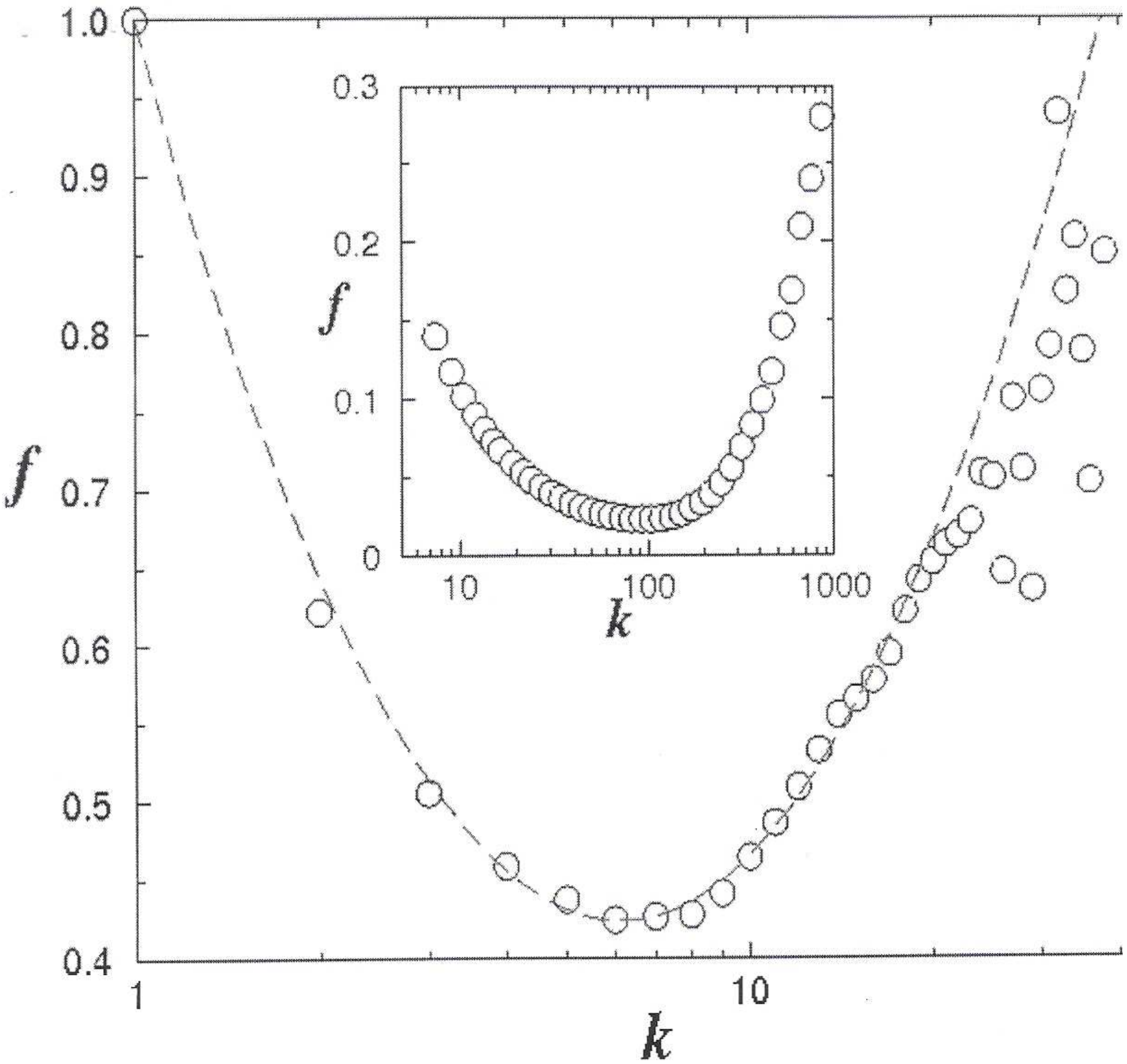
Movie Star

6. How gossip propagates



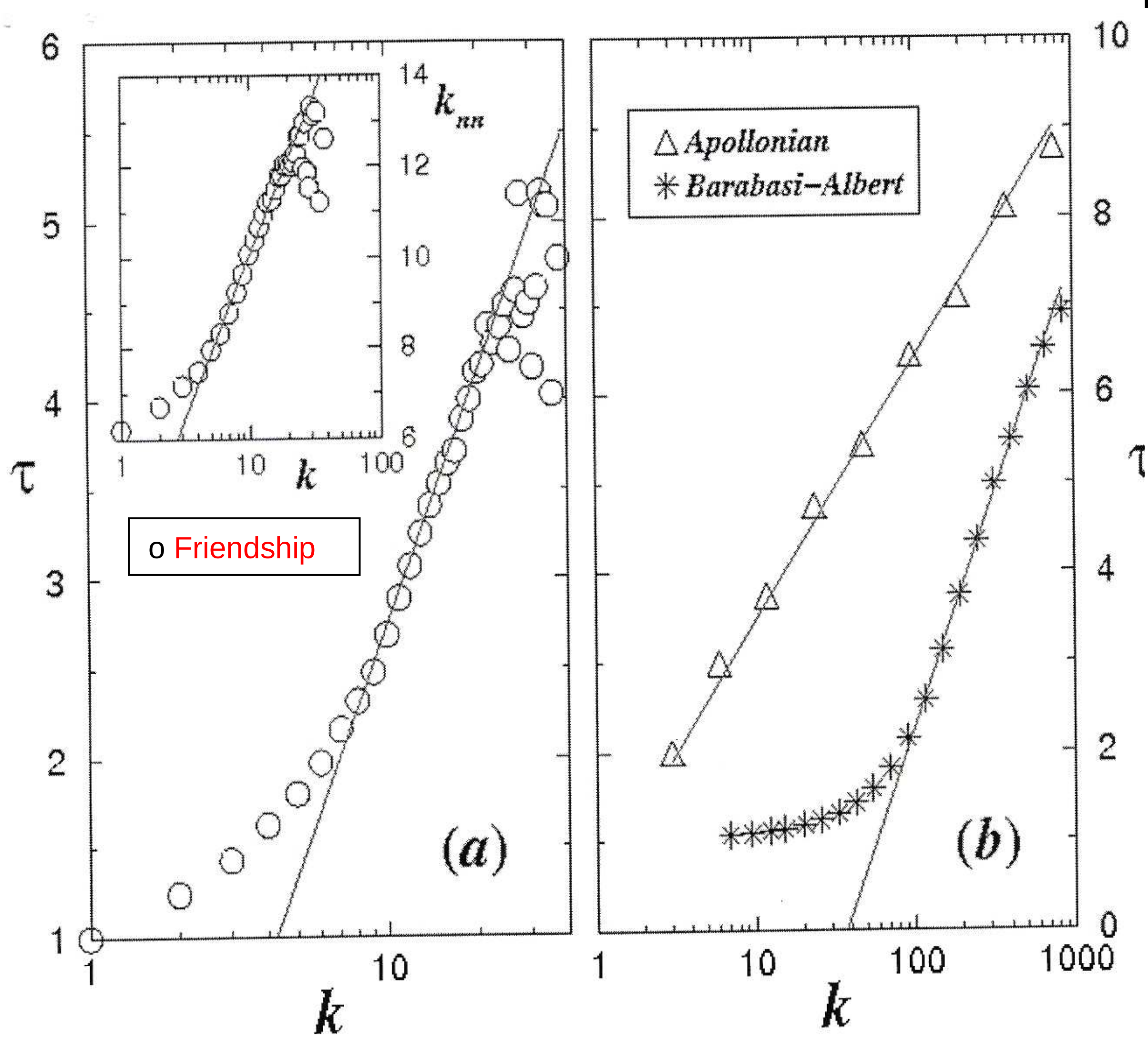
Europhysics Letters **78**, 68005 (2007)

Spread Factor $(f = n_f / k)$



U.S Friend Schools

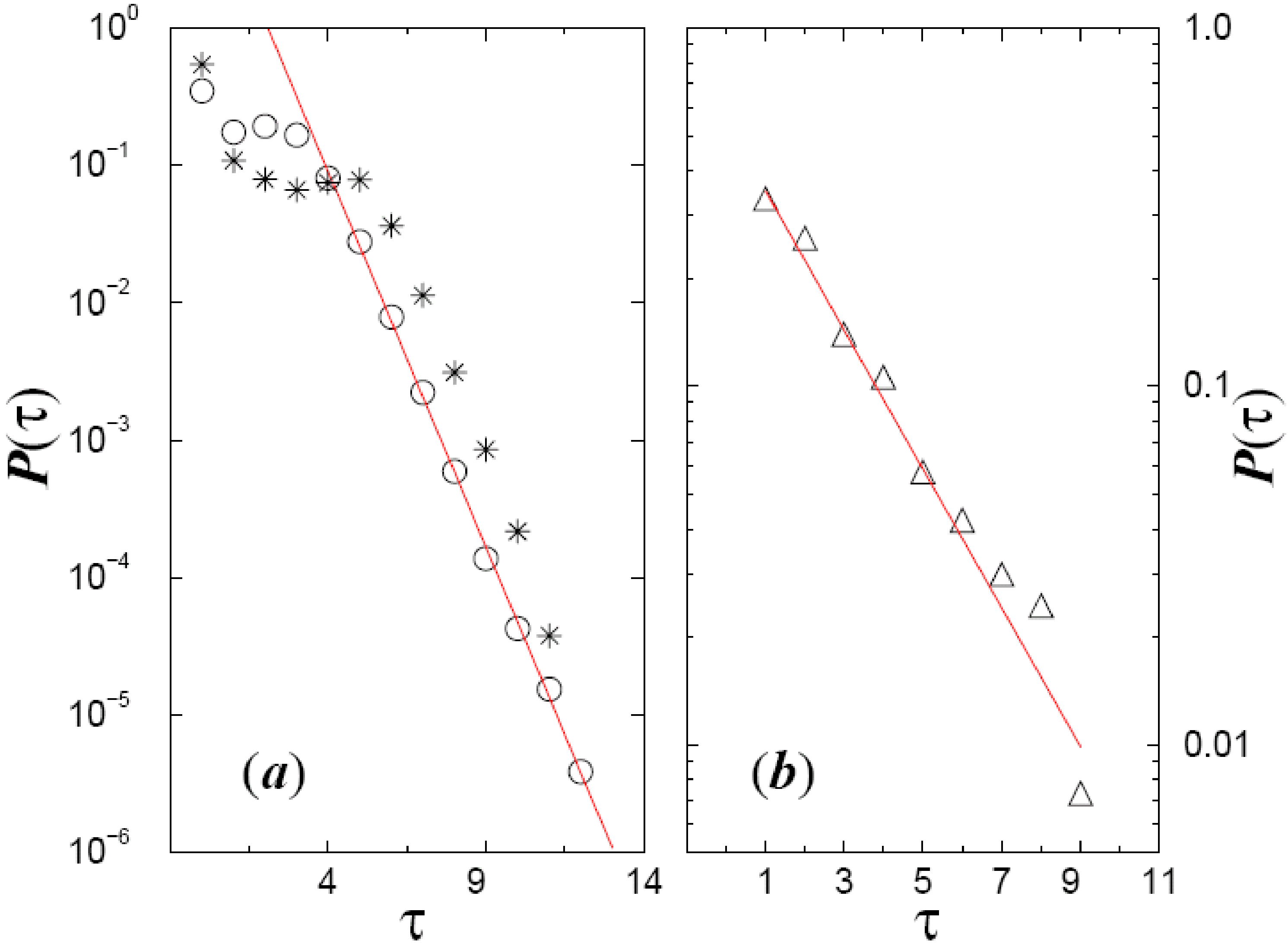
Inset graph: Barabási-Albert



6
5
4
3
2
1
τ
1
10
k
14
12
10
8
6
k_{nn}
1
10
100
k
o Friendship
(a)
Apollonian
Barabasi–Albert
(b)
10
8
6
4
2
0
τ
1
10
100
1000
k

$$\tau = A + B\log(k)$$

INCT2012



o Friendship

Δ Apollonian

* BA; m=7

GAS-LIKE (NODE COLLAPSING) NETWORK:

S. Thurner and C. T., Europhys Lett **72**, 197 (2005)

Number *N* of nodes fixed (*chemostat*); i=1, 2, …, *N*

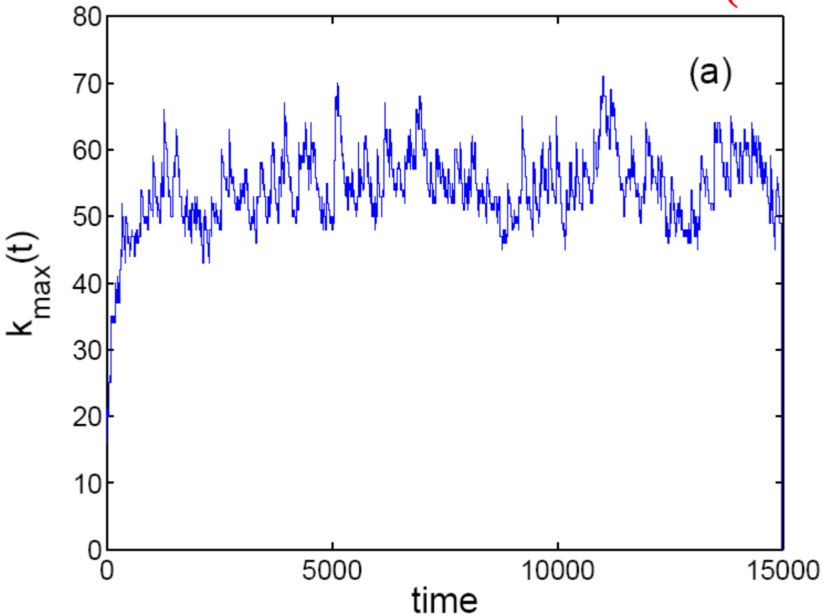
$$Merging\ probability\ \ p_{ij} \propto \frac{1}{d_{ij}^{\ \alpha}} \quad (\alpha \geq 0)$$

$$d_{ij} \equiv shortest\ path\ (chemical\ distance)\ connecting\ nodes\ i\ and\ j\ on\ the\ network$$

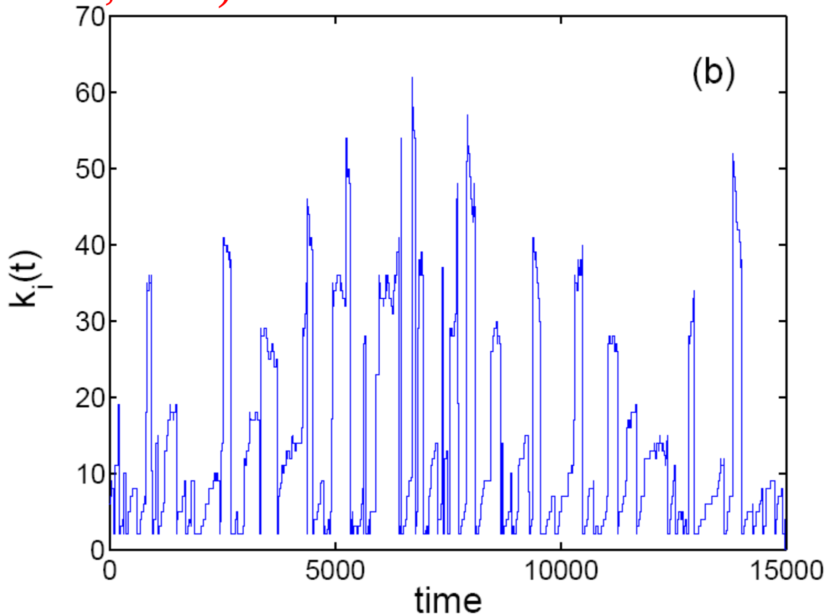
$\alpha = 0$ *and* $\alpha \to \infty$ *recover the random and the neighbor schemes respectively*

(Kim, Trusina, Minnhagen and Sneppen, *Eur. Phys. J. B* 43 (2005) 369)

$(N = 2^7;\ \alpha = 0;\ r = 2)$

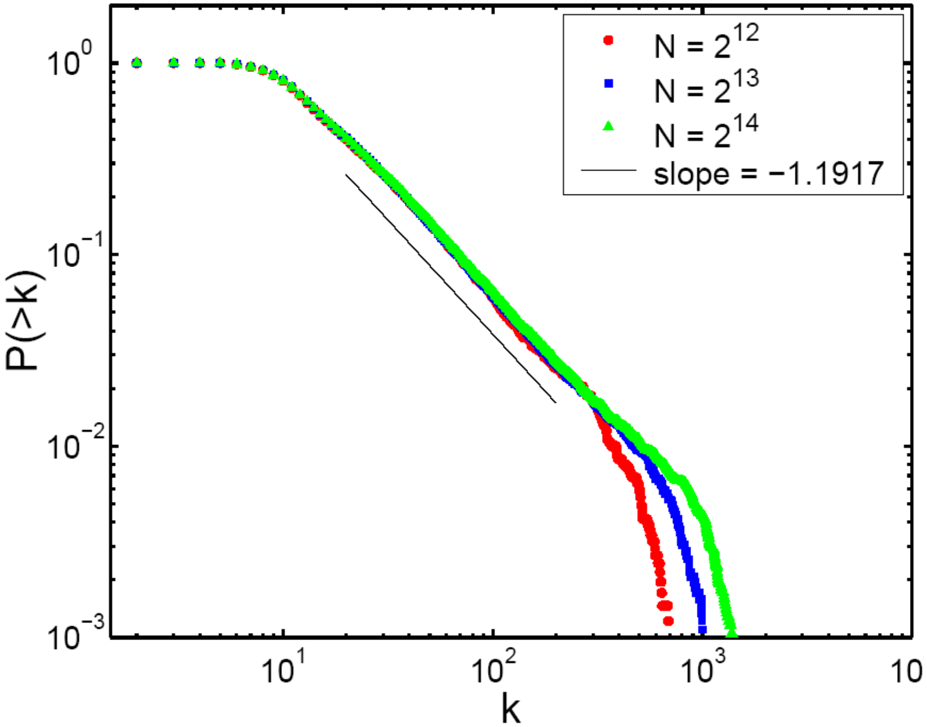


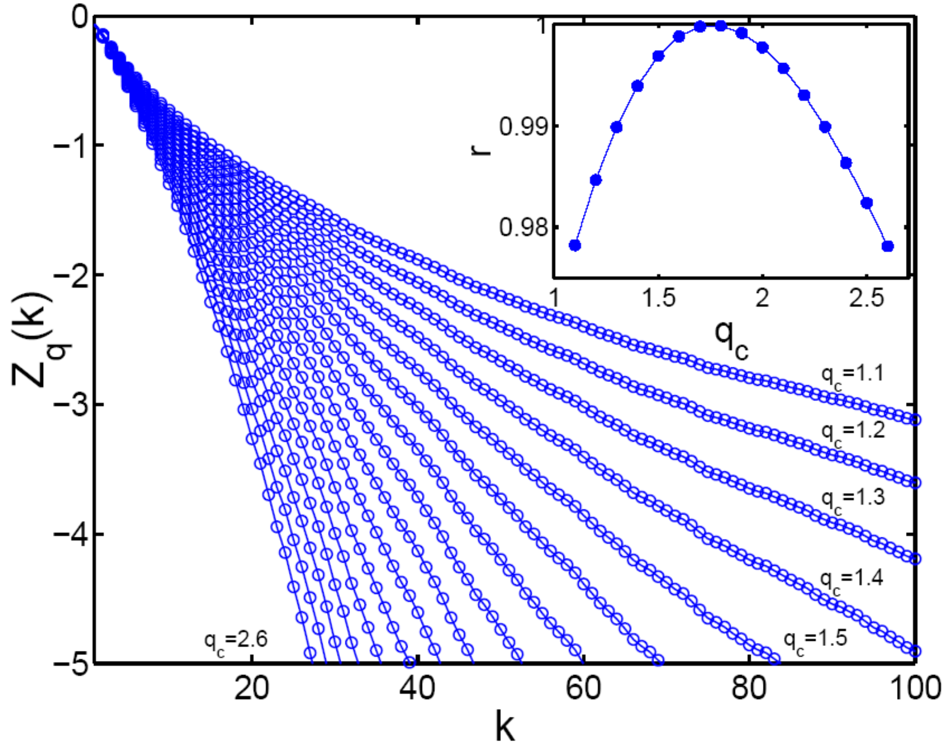
Degree of the most connected node



Degree of a randomly chosen node

$$(\alpha \rightarrow \infty \; ; \; < r >= 8)$$



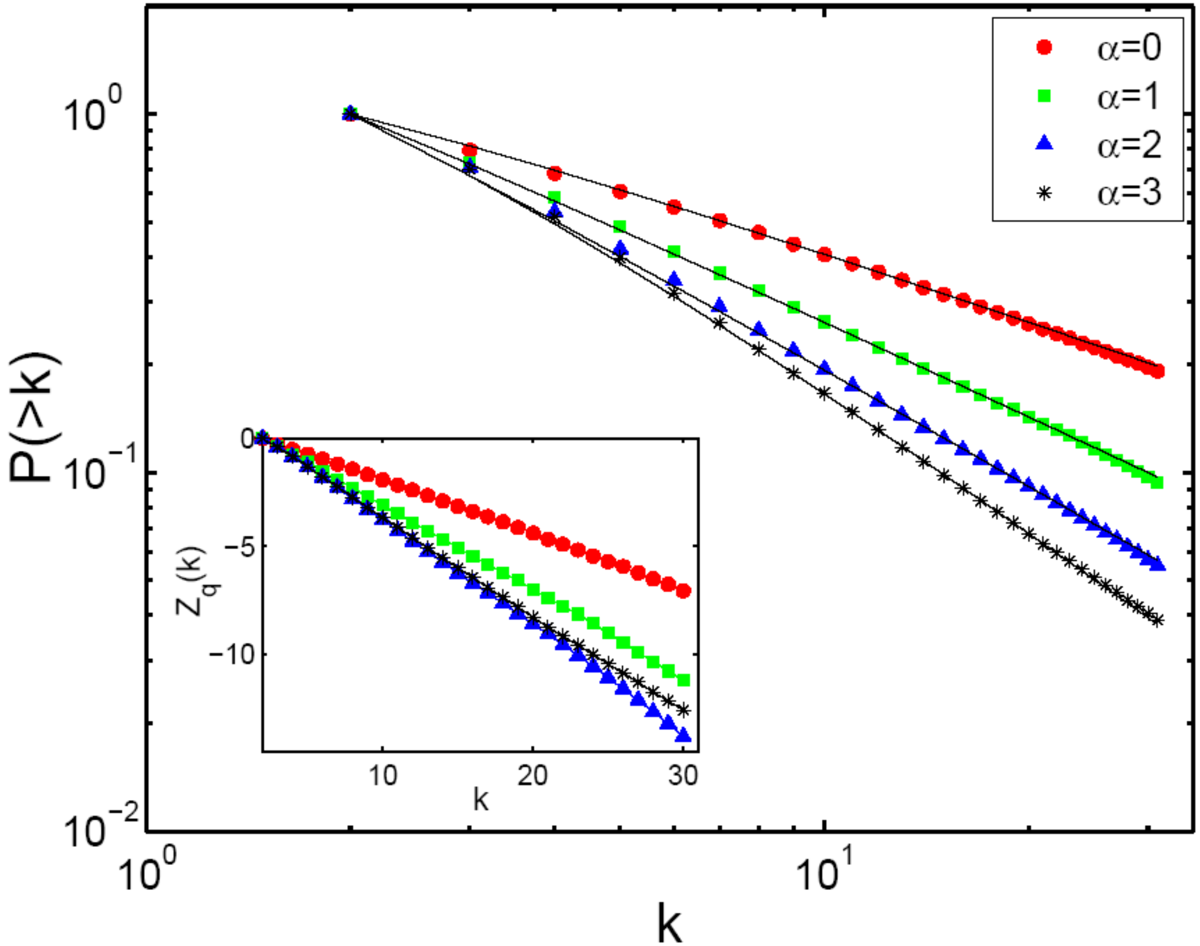


$$Z_q(k) \equiv \ln_q\left[P(>k)\right] \equiv \frac{\left[P(>k)\right]^{1-q} - 1}{1-q}$$

$(optimal \; q_c = 1.84)$

S. Thurner and C. T., Europhys Lett **72**, 197 (2005)

$(N = 2^9;\ r = 2)$

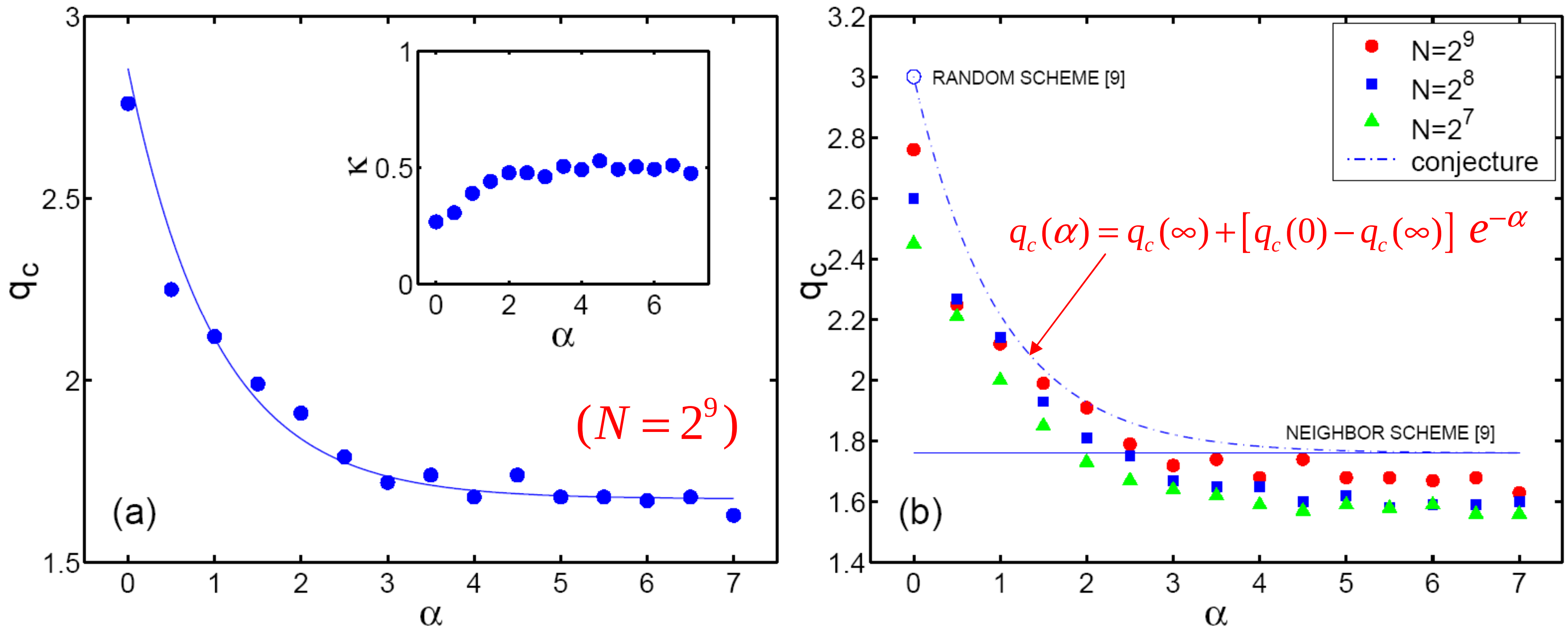


$$P(\geq k) = e_{q_c}^{-(k-2)/\kappa} \quad (k = 2,\ 3,\ 4, \ldots)$$

$$linear\ \ correlation \in [0.999901, 0.999976]$$

S. Thurner and C. T., Europhys Lett **72**, 197 (2005)

$(r = 2)$



S. Thurner and C. T., Europhys Lett **72**, 197 (2005)