Nonlinear stationary structures in nonthermal plasmas

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Introduction

It is well known that laboratory and space plasmas can contain distinct populations of hot and cold electrons [1]. In two-electron plasmas, electron-acoustic waves (EAWs) with wave frequency larger than the ion plasma frequency can be generated [2]. In their classical paper of 1978, Bezzerides, Forslund and Lindman [3] investigate the nonlinear regime and analyze the existence of rarefaction waves and shocks in a two-electron temperature isothermal plasma. The study of rarefaction waves (and shocks) is important for a variety of problems in plasma physics, including the so-called current-free double layers [4].

A double layer (DL) consists of a positive/negative Debye sheath, connecting two quasineutral regions of a plasma. It may be regarded as a BGK equilibrium in some cases, for which certain conditions must be fulfilled. The strong Langmuir DL is the best known of these structures. The current-free double layer (CFDL) constitutes a different group, for which there is no trapped ion population. Contrary to the Langmuir DL, the CFDL is weak, with $\varphi < k_B T_h/e$ (φ is the potential drop across the layer and T_h is the temperature of the hot electron population). It is worth to mention that in general the plasma distributions near a DL are strongly non-Maxwellian [5].

As a preparatory step for a deeper investigation of CFDLs, we follow the steps of Ref.[3] and analyze the conditions for the existence of rarefaction waves and shocks in nonthermal plasmas. The dynamics of the plasma is described by the fluid equations, with the cold and hot electrons modeled by the Maxwellian and κ distributions, respectively. Some preliminary results are presented, and the influence of the superthermal electrons present in the long tails of the κ distribution is discussed.

Model equations

We can write the electron number density as

$$N_e(\varphi) = N_c(\varphi) + N_h(\varphi), \qquad (1)$$

where

$$N_c(\varphi) = N_{c0} e^{e\varphi/k_B T_c}, \qquad (2)$$

$$N_h(\varphi) = N_{h0}\left[1 - \frac{e\varphi}{(\kappa - 3/2)\, k_B T_h}\right]^{-(\kappa-1/2)}, \qquad (3)$$

with $\kappa > 3/2$. In the above equations T_c is the temperature of the cold electrons and $N_{c0} + N_{h0} = N_0$. In the limit $\kappa \to \infty$ we obtain the Boltzmann distribution (1) also for the hot electrons .

The ions are assumed to obey the cold hydrodynamic equations, and the ion and electron densities are related through Poisson's equation

$$\frac{\partial^2}{\partial z^2}\left(\frac{e\varphi}{k_B T_h}\right) = \frac{4\pi e^2 N_0}{k_B T_h}\left[n_e(\varphi) - n_i\right], \qquad (4)$$

where $n_e(\varphi) = N_e(\varphi)/N_0$, $n_i = N_i/N_0$ and $\phi = e\varphi/k_B T_h$.

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Rarefaction waves and shocks

Introducing the similarity parameter $\xi = (z/t)/c_h$, where $c_h = (k_B T_h/m_i)^{1/2}$, and following Ref.[3] we obtain the equation that governs the electrostatic potential ϕ, i.e.

$$\frac{d\phi}{d\xi}\left(1 + \frac{1}{2}\frac{dc_s^2}{d\phi}\right) + c_s = 0, \qquad (5)$$

where

$$c_s^2 = \frac{dP_e}{dn_e} \equiv \frac{n_e(\phi)}{\left\{ n_c(\phi)\,\tau + n_h(\phi)\frac{(\kappa-1/2)}{(\kappa-3/2)}\left[1 - \frac{\phi}{(\kappa-3/2)}\right]^{-1}\right\}}, \qquad (6)$$

is the square of the normalized speed of sound. In the above expression $\tau = T_h/T_c$ and $P_e \equiv P_e(\phi)$ is the normalized pressure, which obeys the relation $dP_e(\phi)/d\phi = n_e(\phi)$. From Eq.(5), it is straightforward to see that a necessary condition for rarefaction shocks to exist is

$$\frac{dc_s^2}{d\phi} + 2 \le 0, \qquad (7)$$

with the equality defining the onset of the singularity in the rarefaction wave. As discussed by Bezzerides, Forslund and Lindman, when both electron populations are modeled via Maxwellian distributions, condition (7) reduces to $\tau \ge 5 + \sqrt{24} \approx 9.9$. This can be seen in Fig.(1), where we plot $z = 2 + dc_s^2/d\phi$ as a function of $x = \alpha = N_{h0}/N_0$ and $y = -\phi$ for $\kappa = 500$ (Maxwellian limit). In (a) we have $\tau = 10$ and $z = 0$ (onset of the singularity) for a broad range of α's. For $\tau = 12$ we observe that $z \le 0$ also for a broad range of α's, with the formation of the shock between two extremes of ϕ. However, as κ decreases (Fig.(2), $\kappa = 5$), we notice the singularity starts to appear (for a short range of α's) only for $\tau \approx 11$. As τ increases, we have the formation of the shock for all the values of α. For longer tails (Fig.(3), $\kappa = 2.5$), the singularity appears for an even larger value of τ (≈ 13) . For the same value of τ, we observe that electron nonthermality seems to "disturb" the formation of the shock: as κ decreases, the shock does not appear for all the values of α (Figs.(2b) and (3b)).

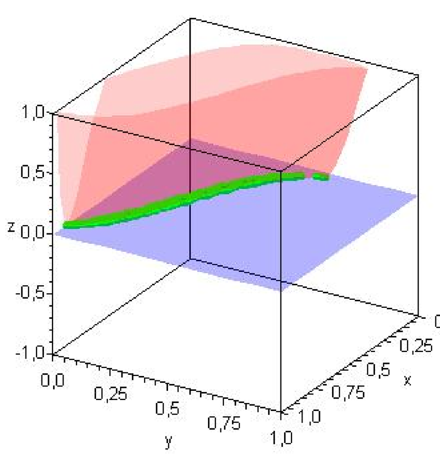

Figure 1: (a) $\kappa = 500$ **and** $\tau = 10$

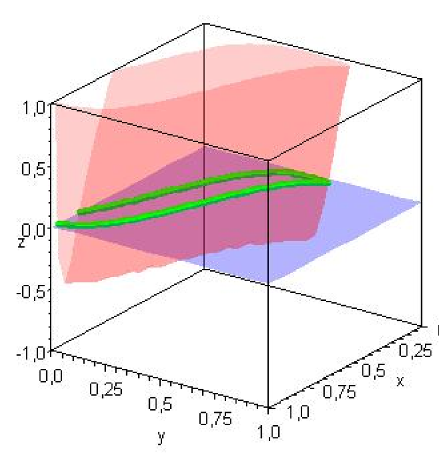

Figure 1: (b) $\kappa = 500$ **and** $\tau = 12$

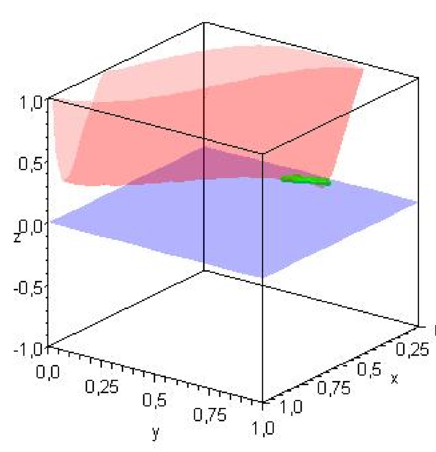

Figure 2: (a) $\kappa = 5$ **and** $\tau = 11$

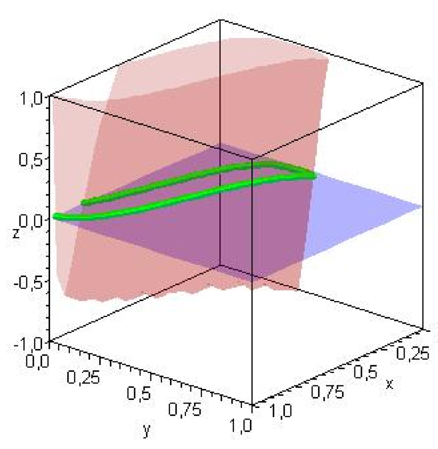

Figure 2: (b) $\kappa = 5$ **and** $\tau = 16$

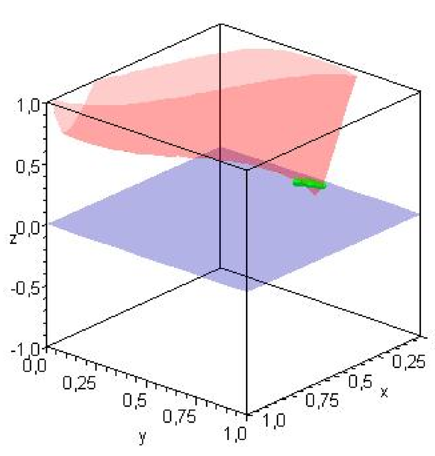

Figure 3: (a) $\kappa = 2.5$ **and** $\tau = 13$

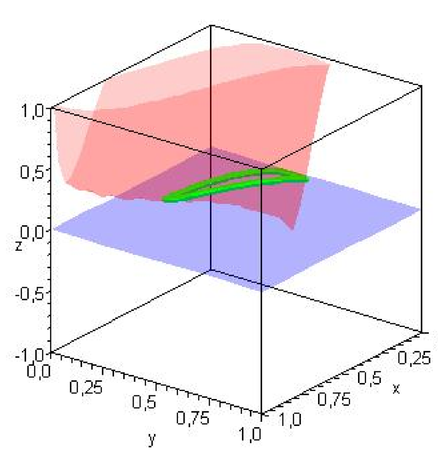

Figure 3: (b) $\kappa = 2.5$ **and** $\tau = 16$

Solving Eq.(5) numerically, we can analyze the different profiles obtained for the electrostatic potential ϕ. In Fig.(4) we present the results for $\kappa = 500$, $\alpha = 0.01$ and $\tau = 9.9$ and 20, respectively. In (a) we notice the eminent formation of the shock, while in (b) it is observed that ϕ is not a single valued function of ξ. This discontinuity in the profile of the electrostatic potential represents the shock formation. For such a small value of α ($= 0.01$) and $\kappa = 2.5$, the onset of the singularity appears for $\tau = 10.2$ (Fig.(5a)). As α grows ($\alpha = 0.2$, Fig.(6)), the shock becomes eminent only for $\tau = 14.2$. Is is also noticed that the shock starts to appear for smaller values of ξ as α becomes larger.

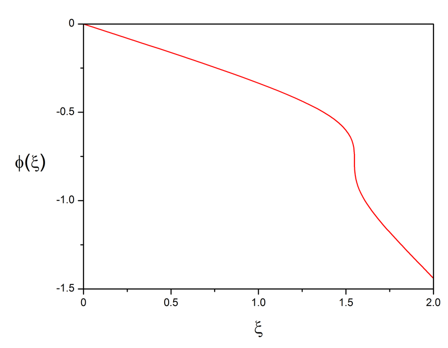

Figure 4: (a) $\kappa = 500$, $\alpha = 0.01$ **and** $\tau = 9.9$

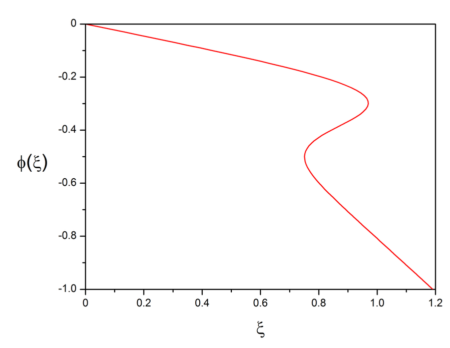

Figure 4: (b) $\kappa = 500$, $\alpha = 0.01$ **and** $\tau = 20$

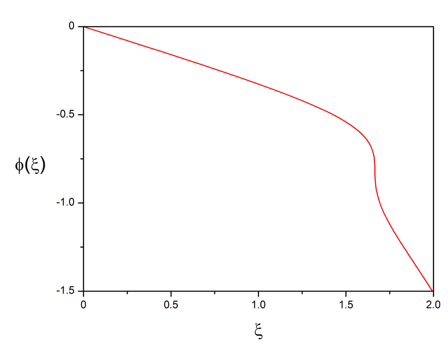

Figure 5: (a) $\kappa = 2.5$, $\alpha = 0.01$ **and** $\tau = 10.2$

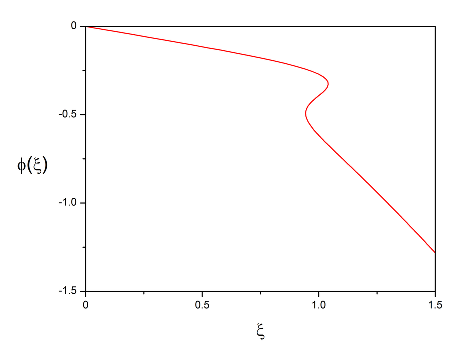

Figure 5: (b) $\kappa = 2.5$, $\alpha = 0.01$ **and** $\tau = 20$

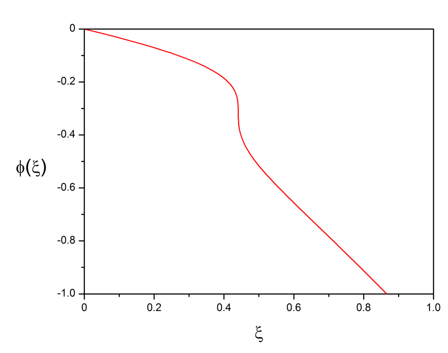

Figure 6: $\kappa = 2.5$, $\alpha = 0.2$ **and** $\tau = 14.2$

Conclusions

The presented results indicate that electron nonthermality (represented by the parameter κ) influences the onset of the singularity in the rarefaction wave and the formation of the shock. For distributions with longer tails (small κ) the formation of the shock becomes eminent only for larger values of τ when compared to the Maxwellian case. It is also noticed that, for a fixed τ, a decrease in κ implies the disappearance of the shock for larger values of α.

Acknowledgments

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