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Role of arches in the generation of shear bands in a dense 3D granular system under shear

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Abstract. A model for propagation of arches on cubic lattices, to simulate the internal mobility of grains in a dense granular system under shear is proposed. In this model, the role of the arches in granular transportation presents a non-linear dependence on the local values of the stress components that can be modeled geometrically. In particular, we study a modified Couette flow and were able to reproduce qualitatively the experimental results found in the literature.

1. Introduction

Granular Systems (GS) have been the object of many studies recently, due to the many applications they present [1, 2] and to the large quantity of unusual equilibrium and nonequilibrium physical phenomena they exhibit. One of these interesting phenomena is a new model on the role played by the elastic stresses in such systems [3, 4, 5].

The internal stresses of GS are, in comparison with most continuous systems, quite unusual: the stresses are propagated via the contact forces between the grains, which are distributed unevenly in these systems. Therefore, one can see arches of forces: lines of high stress contacts which are responsible for the transmission of the internal forces. Therefore, the grains in any GS can be classified as being either part of these arches or part of the mass of relatively loose grains, the so-called soft phase [2].

The importance of arches in GS can be clearly seen in many of their unusual phenomena, like the Janssen effect [6]. In a recent work, based on the strong influence of arches in dense granular media [7], a phenomenological model for a bi-dimensional granular system, sufficiently dense so that no fluidized phase is present and thus only the stress effects of the solid phase are relevant, was presented [8]. When a shear stress was applied, the transport of granular material and the formation of shear bands were observed as expected. In the present work, we expand this phenomenological model to a simplified 3D model, in which we reproduce qualitatively some experimental results obtained by Fenistein *et al* [9, 10] and find them compatible with theoretical predictions of Török *et al* [11, 12].

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2. The 2D Model

The 2D model is described in detail in a previous work [8]. There, the system is represented by a rectangular lattice, constituted of cells and their edges. Each cell corresponds to a volume of grains and is assigned a local parameter $\psi(r) = \psi_{i,j}$ (with i, j integers) associated with the local distribution of grains. The grains are not represented individually but only through the cell's granular mass, via this parameter $\psi_{i,j}$.

It is assumed there (as well as in this work) that the grains inside a given cell are part of the soft phase, i.e., their interaction is weak. On the other hand, the arches are represented by the edges of these cells, interacting with the granular material inside them and with the adjacent cells. The forces, which represent the arches, evolve with time by phenomenological rules and their magnitude is a stochastic function of time, and are given an orientation associated with the direction of local mass motion. Two possible mechanisms arise for momentum transport: via collisions and frictional contact forces (in the soft phase), and via direct normal contact (mostly along the arches).

When external shear stress was applied for long times, the appearance of a stationary state that closely resembles experimental data was obtained, including shear bands formation.

3. The 3D Model

Based on the success of our 2D model in describing the behavior of a dense granular system under shear, in particular the appearance of shear bands, we applied the same concepts in formulating a lattice-based 3D model. Now our square cells are transformed into cubic cells, trying to maintain the same properties for the formation and breaking up of the arches. We used the experimental setup of Fenistein *et al.* [9, 10] as the modeled experimental setup in these simulations, namely a modified cylindrical Couette cell with radius R and filling height H_s. Instead of letting an inner cylinder rotate with respect to an outer one, their cell has no inner wall, but exerts the shear deformation via its base, which is split into a rotating inner disk of radius R_s and an outer annulus rotating in the opposite direction. Both the cubic cell of our model and the experimental setup are shown in figure 1. This setup has attracted considerable interest because it provides access into the fundamental problem of shear band formation.

Fenistein *et al.* observed the behavior of the topmost granular layer in their experiment, with interesting results: they noticed that a shear band was formed, but the rotating disk at the topmost granular layer was smaller than the one at the base. More interestingly, the radius of the shear band at the top did not vary linearly with the height of the granular column, and it ceased to exist altogether at the topmost layer after a certain height. Török *et al.* [11, 12] proposed theoretical models to describe these results, based on minimal dissipation principles. They accounted for the experimental results, and their model proposed an explanation for the behavior of the granular system after that certain height: the shear band collapsed, forming a dome − hence the impossibility of observing the shear band looking only at the topmost layer.

Therefore we used this cubic cell lattice, with the arches propagating along the edges of the cells, to model this experimental setup, in order to verify if we could obtain, qualitatively, both the behavior observed experimentally and the theoretical predictions (in the case of the dome-like shear band formation). Following the phenomenological equations obtained for our 2D model [8], we used, for the stress propagation, equations for the forces applied by the arches on the loose grains inside the cells (soft phase) and how they change over time, both for the strengthening and softening of the arches and, in consequence, the forces they apply. 1 shows the force equation for the edge uf (see figure 1), along the x axis, of the cell i, j, k, which is the address of the cell in the lattice. It should be noted that this orienting of the arches comes from the apparent conflict of trying to describe a strictly local quantity (the stress) by means of a variable associated with a finite range in length. In fact, the granular mobility will depend on the gradients of stress, hence the necessity of orienting the edge-arches.

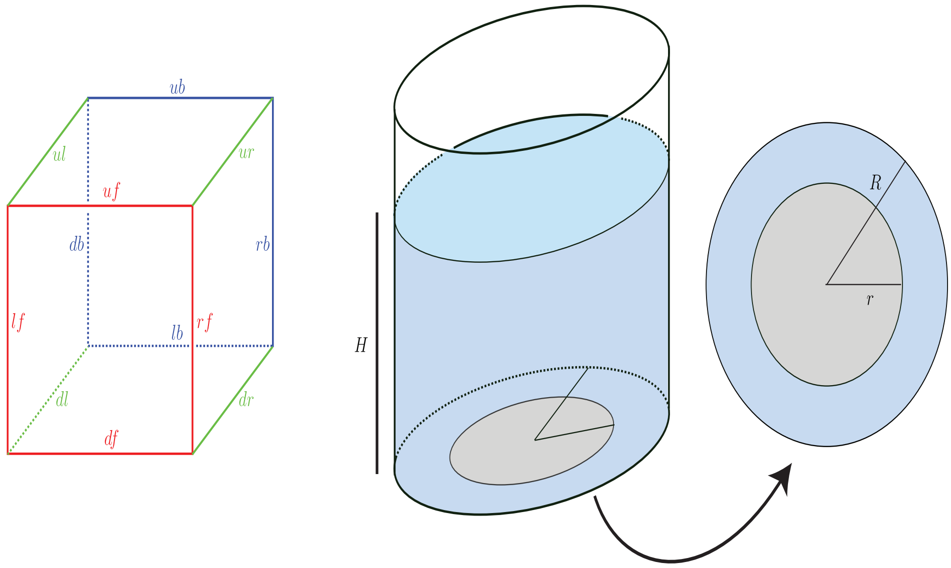

Figure 1. On the left, representation of a cubic cell of our 3D model, with the "address" of each cell edge. On the right, schematic representation of our experimental model: a cylinder filled granular material up to a height $H = H_s$ and with a rotating internal disk of radius $r = R_s$ at the base.

$$
\begin{aligned}
&F^{x(uf)}_{i,j,k}(t+\tau) = \\
&= \frac{1}{\alpha+\beta+\gamma+\lambda}\left[\alpha\left(\frac{F^{x(uf)}_{i,j-1,k}(t)+F^{x(uf)}_{i,j+1,k}(t)}{2}\right)+\right. \\
&\left.+\beta\left(\frac{F^{x(uf)}_{i,j-2,k}(t)+F^{x(uf)}_{i,j+2,k}(t)}{2}\right)+\gamma\left(\frac{F^{x(df)}_{i,j,k}(t)+F^{x(ub)}_{i,j,k}(t)}{2}\right)+\lambda F^{x(db)}_{i,j,k}(t)\right]+ \\
&+c_2\left\{\left[\frac{(\psi_{i,j+1,k}-\psi_{i,j,k})}{|\psi_{i,j+1,k}-\psi_{i,j,k}|+\epsilon}\right]e^{(|\psi_{i,j+1,k}-\psi_{i,j,k}|)}+\right. \\
&\left.+\left[\frac{(\psi_{i,j-1,k}-\psi_{i,j,k})}{|\psi_{i,j-1,k}-\psi_{i,j,k}|+\epsilon}\right]e^{(|\psi_{i,j-1,k}-\psi_{i,j,k}|)}\right\}+ \\
&+\eta^{x(uf)}_{i,j,k}(t),
\end{aligned}
\tag{1}
$$

where each term can be analyzed individually. The first term represents the contributions of the first and second edges immediately before and after the edge we are looking at (proportion constants α and β) and the other edges of this cell in the same direction x (proportion constants γ and λ). The second term represents the contributions of the soft phase media, since $\psi_{i,j,k}$ is the cell density (this means that if the cell is almost full, it would be much more difficult to continue filling it, and vice-versa; this term takes this into account). And the last term ($\eta^{x(uf)}_{i,j,k}(t)$) is the stochastic term to account for random diffusion. The force equations for the other edges are symmetric to the one above, and can be found accordingly. The transport equation reads:

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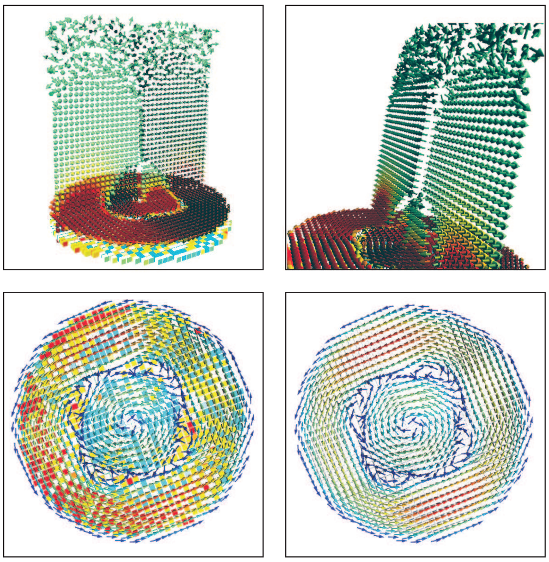

Figure 2. Images of the computer simulations perpetrated with our 3D model. The arrows represent the granular flux at a cross section of cells of the 3D lattice. The granular transport and the formation of the dome-like shear band (expected for filling heights above $H_s > R_s$) can be seen on the upper images. The bottom images show the flux at a horizontal plane of cells. Vortexes can be seen along the shear band profile.

$$\Delta\psi_{ijk} = K \ln(k_f |F_{R_{i,j,k}}| + 1) + \eta^{\psi}_{i,j,k}(t), \tag{2}$$

where K sets the time scale, k_f is the force scale, the η's are randomly generated numbers (through a normal distribution), ψ is the density function of the grain cells and F_R is the resultant of the arch forces applied on the edges of the (i, j, k) cell and calculated for all directions separately.

4. 3D Model Results

We performed the simulations using the equations above for different configurations of radiuses R_s for the rotating base disk and filling heights H_s for the granular media inside the cylinder. In order to make the shear band formation more evident, instead of keeping the cylinder steady while rotating the inner disk at the base, we make it rotate in the other direction. It can be seen, when looking at figure 2, that not only the shear band is – once again – formed, but also that its dome-like shape agrees with the theoretical predictions of the literature. Even more interestingly, when the filling height H_s is reduced to $H_s < R_s$, then the experimental results are reproduced, as one can seen in figure 3.

Thus it can be said that this arch-based model with the cubic lattice and the forces being transmitted to the loose granular media inside is quite good at reproducing qualitatively the physical phenomena at a real dense granular system. The most important thing to point out is that this provides a physical explanation for the transport properties observed in such systems – the arches (and the stresses they introduce in the systems).

Nonetheless, the model, as it is, still presents limitations. Foremost of those, and one that can be seen in figure 2, is the formation of the four vortexes at the shear band. Although it is indeed a turbulent area and vortexes are bound to appear, they should appear randomly and

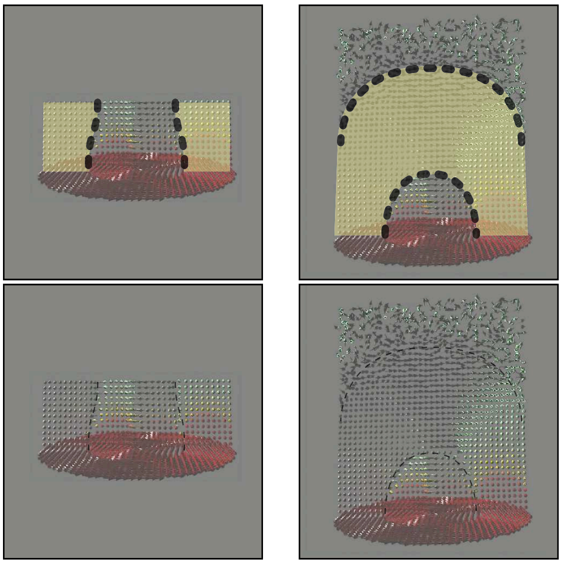

Figure 3. Comparison between lower filling heights $H_s < R_s$ and higher filling heights $H_s > R_s$. The colored shadow on the upper images are there to highlight the shear band pattern. It can clearly be seen that the shear band profile follows qualitatively the ones observed by Fenistein *et al.* [9, 10] and predicted theoretically by Török *et al.* [11, 12].

were not supposed to remain constant throughout the simulation. These, on the other hand, do not only remain constant through the entire simulation, but appear systematically at the same locations. Therefore, it is certain that it is an artificial result of our simulation. Most likely, it is a consequence of simulating a cylindrical system with a cubic (i.e. cartesian) lattice. The rotating disk at the base is not modeled with geometrical precision, and hence the appearance of vortexes at the shear band.

The other limitation that can also be seen from figure 2 is the neglect of friction between the grains and the walls of the cylinder. It was seen in our 2D model [8] that such precaution was crucial to a more accurate physical description of a real system. Without the friction in the walls, the force transmitted from the base of the cylinder to the grains at the upper layers of the filling height (after the collapse of the shear band) is almost negligible. Therefore, without the friction on the walls, the random component of the force equations become predominant and the random flux observed at the top of the simulation (see figures 2 and 3) is produced, which is clearly not physical. A second (and interesting) consequence of this limitation is that, for $R_s > R/2$, the shear band collapses, but not to the center, forming the dome, but in the direction of the walls, as can be seen in figure 4. This is consistent with the principle of minimal dissipation [12] − since there is no friction on the walls, the least energy dissipation will be achieved by collapsing to the walls. If we included the effects of friction on the cylinder walls, we would expect to see the shear band collapsing to the center, forming the dome, since the walls would be dissipating energy.

5. Conclusion

The 3D model expands on the results of our previously published 2D model, in the sense that now the qualitative behavior of experimental results and that predictions of theoretical models found in the literature were reproduced. This shows that arches are, in fact, very likely candidates for being responsible for important granular transportation properties.

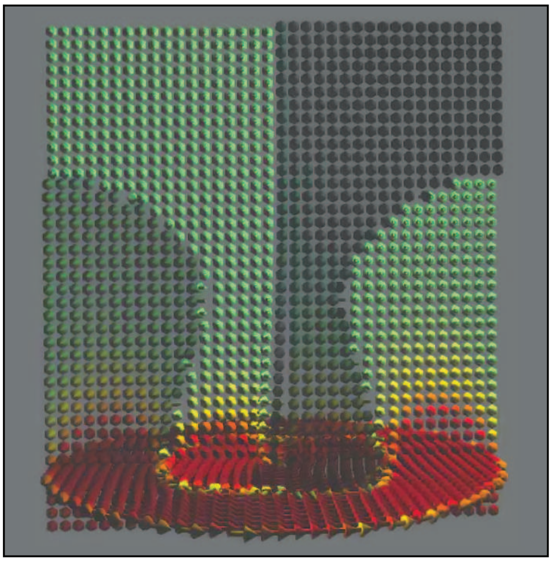

Figure 4. Non-physical result obtained when $R_s > R/2$, due to the absence of friction effects on the walls of the cylinder. The shear band collapses to the wall, instead of forming the dome.

The model has some important limitations that have to be taken into account and need to be improved for obtaining quantitative results, namely the inclusion of friction effects on the walls and a modification of the lattice structure of the model. Probably, a lattice based on cylindrical coordinates would be a more friendly approach than our cubical cells lattice.

The model's limitations do not, however, diminish the significance of the arches for the grains transportation. Quite the opposite – even with such important limitations, the model is able to produce good qualitative results.

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