Catastrophic regime in the discharge of a granular pile

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We present a molecular-dynamics study of discharges in a granular pile evincing a catastrophic regime depending on the outlet size. The avalanche size distribution function suggests a phase transition where the height of the remaining pile is taken as the order parameter. Our results indicate that there is a critical outlet size beyond which discharges become catastrophic and the initial pile is split in two minor piles. As the system size increases, finite-size analysis indicates that the critical orifice width converges to a finite value.

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I. INTRODUCTION

In the last three decades, there was a great impulse in the granular matter research field as a variety of peculiar phenomena have been observed. In particular, there has been an intense effort to verify the existence of self-organized criticality (SOC) in real granular systems since Bak, Tang, and Wiesenfeld [1] proposed a model that resembles the behavior of a stack of macroscopic grains. The search for SOC in granular systems has been carried out either by means of experiments or numerical simulations.

Jaeger, Liu, and Nagel [2] put forward experiments on a rotating drum filled with granular matter. They have not observed scale invariance either in space or in time for the avalanches, the signature of SOC. One year later, Held *et al.* [3] published conflicting experimental results from very slowly evolving sand piles, showing the existence of critical behavior for small piles. A study of avalanches on rice piles [4] corroborated the SOC point of view and raised other issues in the discussion of the problem. In a recent paper, Ramos, Altshuler, and Måløy [5] showed very strong evidence of SOC in two-dimensional (2D) piles. Some people tried to explain these observations using finite-size effect arguments [6] or distinguishing the dynamics of small from big avalanches [7]. Another cause for these discrepancies, pointed out by Buchholtz and Pöschel [8], is the method used to measure the avalanches.

Motivated by the work of Frette *et al.* [4], some researchers adopted a different strategy, changing focus to internal avalanches, i.e., rearrangement of grains in the bulk of the pile in response to some external perturbation [5]. Lattice and pseudodynamic models were proposed in order to investigate numerically the discharge of grains through an orifice at the bottom of a bidimensional silo and a power law was found for the internal avalanche size distribution [9–11]. In opposition, the experiments of Zuriguel *et al.* [12] and Janda *et al.* [13] have shown an exponential tail both in 2D and three-dimensional (3D).

They were also concerned about the jamming transition, that is, the passage from a state where the flow of granular particles eventually gets blocked to a state in which the flow continues forever. Janda *et al.* [13] concluded, in response to the work of Perez [14], that there is not a critical outlet size beyond which the probability of jamming becomes zero. In this paper, we study avalanches in open boundary piles and the results seem to contradict this statement.

We present here a molecular-dynamics study of the discharging process through a single orifice in the substrate of a granular sandpile. Besides the main task of providing additional data for the SOC debate, we address the jamming transition issue in order to shed some light on this subject. This paper is organized as follows: first, we present the model and a detailed description of the numerical simulation; next, we show the results obtained for several system sizes and make a discussion; finally, we present some conclusions and sketch our perspectives out.

II. SIMULATION DESCRIPTION

We shall describe here a simulation of the discharge from an orifice localized at the bottom of a two-dimensional granular pile consisting of frictional disks, using the molecular-dynamics method. The numerical simulations can be divided in two steps. First, we prepare the pile by pouring grains on a substrate constituted of small fixed grains; afterwards we open a hole at the substrate through which the grains may flow.

The simulation method used was the velocity-verlet molecular dynamics algorithm [15] and the particles interact only when they are in contact. A spring-dashpot model is used to simulate the contact in the normal direction, and a spring simulates Coulomb friction in tangential direction-Cundall and Strack model [16]. The particles are subject to a constant gravitational field g that points downward in the vertical direction. Thus, the normal and tangential forces upon the particle i due to the particle j are given, respectively, by

$$\vec{F}_i^{(n)} = H(\Delta_{ij})(k_n\Delta_{ij} - \gamma\vec{v}_{ij}\cdot\vec{n}_{ij})\vec{n}_{ij}, \tag{1}$$

$$\vec{F}_i^{(\tau)} = H(\Delta_{ij})\min(|\vec{f}_r|,\mu|\vec{F}_i^{(n)}|)\operatorname{sgn}(\vec{f}_r\cdot\vec{\tau}_{ij})\vec{t}_{ij}, \tag{2}$$

where H is the step function, i.e., $H(x)=1$ if $x\geq 0$ and $H(x)=0$ otherwise, and

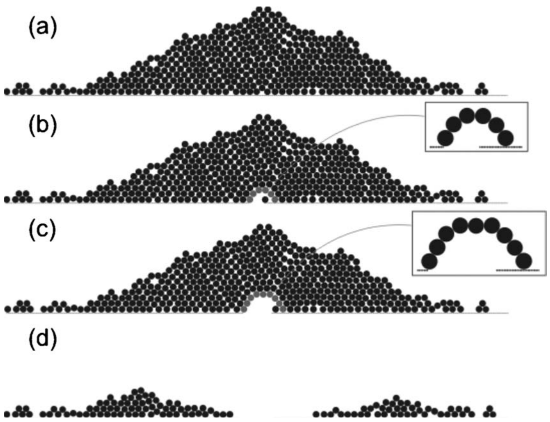

FIG. 1. A granular pile before (top) and after the discharge process for $w=2.2d$, $w=4.2d$, and $w=5.2d$ in sequence. The base length is $L=75d$. We show the blockage arch isolated and amplified for the two cases where the grain flux was interrupted.

$$\Delta_{ij} = d_i/2 + d_j/2 - |\vec{r}_i - \vec{r}_j| \tag{3}$$

is the penetration depth. The tangential spring restoration force at the instant t, $\vec{f}_r(t)$, evolves following the equation:

$$\vec{f}_r(t+\delta t) = \vec{f}_r(t) + k_t \delta t \left(\frac{d_i \omega_i}{2} - \frac{d_j \omega_j}{2} - \vec{v}_{ij} \cdot \vec{\tau}_{ij} \right) \vec{\tau}_{ij}. \tag{4}$$

The other parameters and variables are the normal and tangential spring stiffnesses k_n and k_t, the normal damping constant γ, the friction coefficient μ, the particle diameter d_i, the normal and tangential unitary vectors $\vec{n}_{ij}$ and $\vec{\tau}_{ij}$, the particle position $\vec{r}_i$, the relative velocity vector $\vec{v}_{ij}$, the angular velocity ω_i, and the integration time step δt. As previously stated, the particles are homogeneous disks whose masses are defined in terms of their areas, $m_i = d_i^2 / d_{\max}^2$, where $d_{\max}$ stands for the diameter of the largest grain. The diameter is uniformly distributed around the average grain diameter d with 5% dispersion.

Based on Mindlin [17], who demonstrated that $2/3 < k_t/k_n < 1$ for elastic bodies in contact, we used $k_n=1000$, $k_t=750$; we also chose $\gamma=\gamma_c$ (critical dumping) in order to speed up the equilibrating process and $\mu=0.9$ a typical value for metallic surfaces [18]. The integration time step is much lower than the lowest collision time between two grains, $t_c = \sqrt{m_{\min}/k_n}$, where $m_{\min}$ is the mass of the lightest particle. The units for the quantities mentioned above as well the other dynamical variables, such as position and force, are normalized following the scheme used by Atman *et al.* [19]. In the present case, because the lateral boundaries are open, the total weight is normalized by a complete stack ordered in a triangular lattice constituted of equally massive grains with $m=m_{\max}$.

We start the preparation procedure by defining a flat granular monolayer made of fixed juxtaposed grains whose diameters are equal to one tenth of the average diameter of the pile grains, $\bar{d}$. Then, the deposition process begins: the particles are released from rest, from a height equal to the base length, layer by layer, and are free to leave the pile through the open lateral boundaries. In each layer, the particles are randomly disposed and cover half of the substrate length. The deposition frequency is the maximum possible in order to avoid particle superposition, $f=\sqrt{g/2d_{\max}}$, mimicking a dense rain.

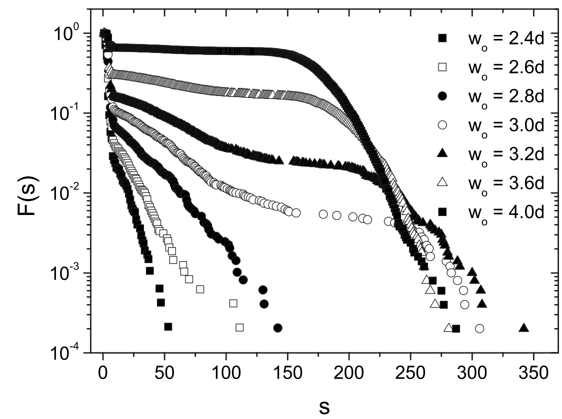

FIG. 2. Accumulated distribution functions of the discharge avalanche size for various w values on $L=75d$ piles. The distributions were built from a set of 5000 samples.

The end of the deposition process occurs when a stationary state for the total number of particles is reached. After the deposition process, we wait for the pile to reach an equilibrium state, and then its final configuration is recorded. An equilibrium state is achieved when the pile satisfies the following conditions: mechanical stability, slipping absence, vertical force balance, and vanishingly small kinetic energy [20].

Once equilibrium is reached, we initiate the second step, that is, we open an outlet of width w at the center of the substrate by removing a certain amount of grains from the substrate and wait until another equilibrium state is reached. The grains that fall out of the pile are eliminated from the system. We compute the number of grains that pass through the orifice as one realization to measure the discharge avalanche size. After that, we perform the same procedure with another pile sample from the set deposited in the first step, discarding the pile after an outlet was opened. This same set of deposited piles is used to perform all discharge realizations for different outlet sizes.

In Fig. 1, we show the equilibrium configurations of the pile before and after the discharge process for three different values of the parameter w. The panel (a) is the configuration of a sample just after the deposition process. The subsequent panels, (b)–(d), represent the final states after the discharging process is over. In (b) and (c), the opening of the orifice does not affect too much the shape of the pile. The initial flux of grains is soon ceased by the formation of an arch right above the orifice. When the orifice is large enough, as in panel (d), a quite different scenario arises, where almost all particles flow through the orifice destroying completely the pile. Thus, there are two well distinct phases for the discharged pile, i.e., the remaining pile after the discharge has ceased, depending on the orifice width which could be characterized by the average height at the center of the pile.

III. RESULTS AND DISCUSSION

In order to verify the existence of SOC state and to investigate the role of the outlet width on discharge properties, we

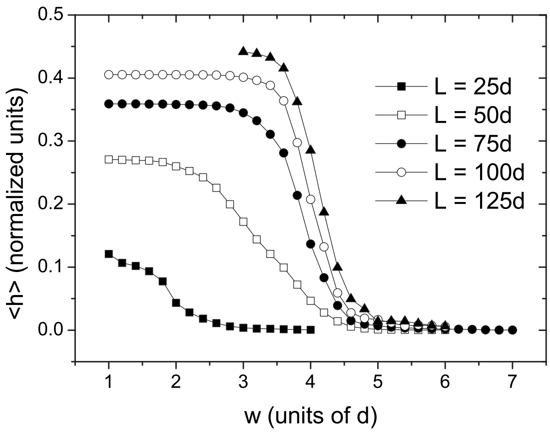

FIG. 3. The order parameter as a function of the orifice width for various system sizes. The order parameter is defined as the average height of the discharged pile at its center. The function is generated from a set of 10k samples for L=25d and L=50d, from 5k for the L=75d and L=100d, and from 1k samples for the L=125d. The variable $\langle h \rangle$ is normalized by the substrate size.

have made discharge simulations on piles with five different substrate sizes: L=25d,50d,75d,100d,125d. The average number of particles remaining in the pile after the deposition is, respectively: $N_{25}=(3\pm1)\times10$, $N_{50}=(1.9\pm0.3)\times10^2$, $N_{75}=(5.0\pm0.5)\times10^2$, $N_{100}=(9.5\pm0.7)\times10^2$, and $N_{125}=(1.6\pm0.1)\times10^3$. The parameter w ranges from 1.0d to 7.0d, in steps of 0.2d. In Fig. 2, we show the graphs of the accumulated distribution for discharge avalanche sizes, $F(s)$, for some values of w on L=75d piles. The function $F(s)$ represents the probability that a discharge of size s or greater than s will occur. The graphs are in semilogarithmic scale to emphasize their exponential tail, a result that agrees with experimental data [13]. Examining the function $F(s)$, we can clearly identify three different regimes. Below w=2.9d, the distribution follows a single exponential law over the entire range of the simulational data. From w=2.9d to w=4.0d, the distribution starts to flatten for middle values of s but keeps

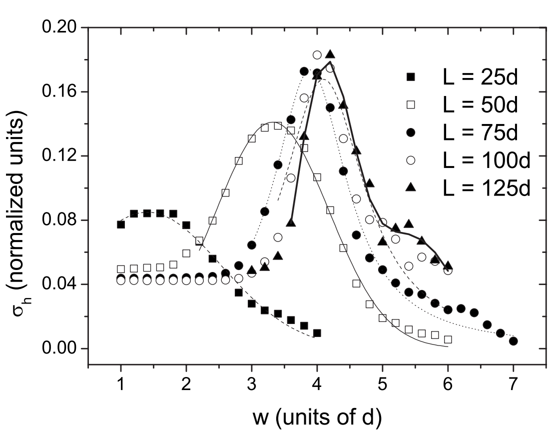

FIG. 4. Fluctuation of the order parameter as a function of the orifice width. The symbols represent the fluctuations obtained from a set of 10k (L=25d,50d), 5k (L=75d,100d), or 1k (L=125d) samples. The lines are peak function fits of these data. As well as the variable $\langle h \rangle$ in graph 3, σ_h is normalized by the substrate size.

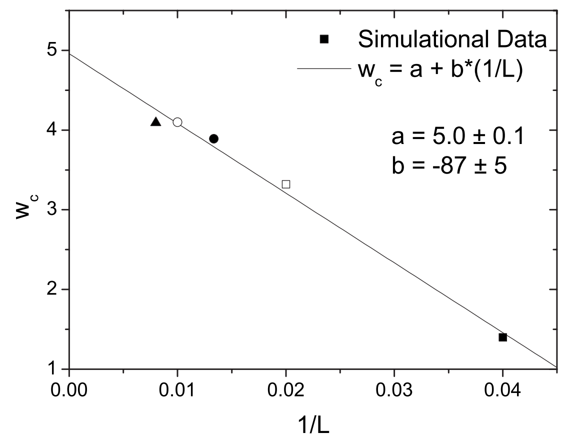

FIG. 5. Critical outlet size dependence on system size. The square symbol represents the simulational data and the line is a linear fit. As $L\rightarrow\infty$ the critical outlet size tends to some limiting value near $w_c=(5.0\pm0.1)d$.

its exponential shape, revealing the existence of two characteristic sizes for the avalanches. In this intermediate regime, the pile either collapse or the granular flow is readily blocked by an arch after a few grains passed through the orifice. Above w=4.0d, the flatten region extends to the left, and the first exponential decay is no longer observed. This indicates the existence of a single characteristic avalanche size, which is big enough to systematically destroy the pile realizations.

We identify this abrupt change as a transition to a catastrophic regime. A good order parameter seems to be the average height of the discharged pile at its center $\langle h \rangle$. For small w, when generally just a few particles pass through the orifice, the pile height will hardly be modified and $\langle h \rangle$ is essentially the height of the unperturbed pile. On the other hand, for large values of w, when in most cases the pile is split in two, $\langle h \rangle$ is nearly zero. Therefore, the two extremes we are concerned about are well captured by this parameter. In Fig. 3, we plot $\langle h \rangle$ versus w and the two regimes are evident. It can be noted that the graphs approximate to the step function as the system size increases, which points toward the existence of a flowing regime and indicates that there is a finite critical arch size. By flowing regime, we mean a state of continuous flow through the orifice without arching formation. The transition from one regime to the other occurs within a finite interval, in which there is a coexistence of the two phases, as shown in Fig. 2. The coexistence of these two phases indicates that the transition could be better described in first order phase transition framework.

In Fig. 4, we present the order parameter fluctuations—defined as the root mean square of the deviations—as a function of w for the different system sizes. The fluctuations present a peak around the value w_c, which depends on the system size. As the system size increases, the peak center moves to the right but tends to a finite value, as shown in Fig. 4. We observe that the w_c grows with L and assumes the value of $w_c=(5.0\pm0.1)d$ when L tends to infinity, as shown in Fig. 5. It suggests the existence of a finite value for w_c as the system approaches the thermodynamic limit. This conclusion is the opposite of that stated by Janda *et al.* [13]. The difference might be only due to geometry but we wonder whether there is a kind of crystallization effect due to low size dispersion; we intend investigate this point in a future work. In fact, the nonexistence of jamming transition in the thermodynamic limit is quite a counterintuitive idea since it requires unrealistically long arches to block the flux through large orifices. To [21] investigated the arches formed in hoppers and found that their size decays exponentially with the outlet size, i.e., long arches is very improbable to occur. Besides, the probability of an arch to be created may depend on the intensity of the flow.

IV. CONCLUSION

In this paper, we have shown that there is no evidence of SOC in the statistics of discharge avalanches on granular piles. Instead, we found that the discharge avalanches have a characteristic size which depends on the orifice width. Nevertheless, there is a transition, as the parameter w varies, which we have characterized by the average height of the discharged pile at its center $\langle h \rangle$. Beyond a threshold value w_c, the pile, in most cases, is completely destroyed after the opening of an orifice. Our data show that w_c tends to a finite value as the system size increases, which made us doubt the nonexistence of a flowing state.

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