Influence of aperiodic modulations on first-order transitions: Numerical study of the two-dimensional Potts model

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We study the Potts model on a rectangular lattice with aperiodic modulations in its interactions along one direction. Numerical results are obtained using the Wolff algorithm and for many lattice sizes, allowing for a finite-size scaling analyses to be carried out. Three different self-dual aperiodic sequences are employed, which leads to more precise results, since the exact critical temperature is known. We analyze two models, with 6 and 15 number of states: both present first-order transitions on their uniform versions. We show that the Harris-Luck criterion, originally introduced in the study of continuous transitions, is obeyed also for first-order ones. Also, we show that the new universality class that emerges for relevant aperiodic modulations depends on the number of states of the Potts model, as obtained elsewhere for random disorder, and on the aperiodic sequence. We determine the occurrence of log-periodic behavior, as expected for models with aperiodic modulated interactions.

I. INTRODUCTION

Nonuniform systems are ubiquitous in nature. The nonuniformity may be a consequence of random disorder or of a deterministic construction of materials from two different atoms, for example. Experimentally, there are already several techniques of surface growth [1–3] that let one control the layout of the layers in order to follow, for example, an aperiodic sequence. One of the main theoretical issues is to what extent the introduction of nonuniformities affects the critical behavior of a system, when compared with its uniform counterpart.

For random quenched disorder and continuous transitions on the uniform model, this question is partially answered by the Harris criterion [4]. According to this criterion, the random model will have the same critical behavior of the uniform one if the specific heat critical exponent α of the latter is negative. The disorder is said to be irrelevant in this case. If α is positive, the critical exponents of the random model are different from the exponents of its uniform counterpart, and the disorder is said to be relevant. For $\alpha = 0$ (the marginal case), logarithm corrections appear [5]. A generalization of this criterion is available also for short-range [6–8] and long-range [9,10] correlated disorder. When the transition in the uniform model is a first-order one, the scenario is qualitatively different: in two dimensions, even an infinitesimal amount of disorder changes the nature of the transition to a continuous one [11–14], while in three dimensions a finite amount of disorder is necessary to change the order of the transition [12].

We are mainly interested here in the critical behavior of models with aperiodic modulation of their interaction parameter(s). For the case of continuous transition on the uniform model, the Harris-Luck criterion determines when the introduction of a given aperiodic modulation changes the universality class of a model, compared with its uniform

version [15]. This change is determined by the crossover exponent Φ, given by

$$\Phi = 1 + d_a \nu(\omega - 1), \qquad (1)$$

where ω is the exponent describing the behavior of geometrical fluctuations of the sequence (see below), d_a is the number of dimensions upon which the aperiodic sequence acts, and ν is the correlation-length critical exponent of the uniform model. When $\Phi > 0$ the sequence is relevant, and when $\Phi < 0$ the sequence is irrelevant.

This criterion applies to continuous transitions in the uniform model, but numerical results [16,17] indicate that it holds true also when the transition in the original model is a first-order one (if this is the case, the introduction of aperiodic modulations on the interaction parameters leads to a scenario totally different from the one for random disorder). This result was established through a study of the ferromagnetic Potts model [18] with $q = 8$ states in Refs. [16] and [17]; in this model a dynamical variable with q states is assigned to each site on a given lattice, and first-neighbor sites tend to be in the same state (see below). For low enough q, the transition is continuous, while it is first order for q above a given value q_c (in two-dimensional lattices, $q_c = 4$). For $q = 8$, the Harris-Luck criterion is obeyed and the order of the transition changes for relevant aperiodic modulations [16,17].

Here we will address two issues. First, we want to provide an independent check of the Harris-Luck criterion for first-order transitions. Second, we test the possibility that the new universality class for relevant modulations depends on the number of states of the Potts model, as is the case for random disorder [19,20].

This paper is organized as follows. In the next section, we review properties of aperiodic sequences and define the model we use. In Sec. III we discuss details of the simulation, and in Sec. IV we present and discuss our results. Finally, in the last section we summarize our findings.

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II. APERIODIC SEQUENCES AND MODEL

The aperiodic sequences we use in this work are constructed in a deterministic way, using a substitution rule on a two-letter alphabet, such that no primitive cell is present. As an example of a two-letter sequence, A and B, we have

$$s(A) \rightarrow AB, \quad s(B) \rightarrow BA. \qquad (2)$$

This notation means that we choose an arbitrary initial letter (usually A) and, from the above rule, we build our sequence in the following way. The following generation of a sequence is obtained by replacing every letter A and B in the previous generation by AB and BA, respectively. In this example, we obtain $ABBABAAB\ldots$ as our sequence.

Some properties of the sequences can be characterized by its substitution matrix $\mathcal{M}$, which is defined as

$$\mathcal{M} = \begin{pmatrix} n_i^{s(i)} & n_i^{s(j)} \\ n_j^{s(i)} & n_j^{s(j)} \end{pmatrix}, \qquad (3)$$

where $n_i^{s(j)}$ is the number of letters i that are generated by applying the rule $s(j)$. Several features of the sequences are determined by the eigenvalues of $\mathcal{M}$. The largest eigenvalue (λ_1) determines the rate of growth of the number of letters $\mathcal{N}$, such that $\mathcal{N} \sim \lambda_1^n$, $n \gg 1$, where n is the number of the generations in the construction of the sequence. The second largest eigenvalue (λ_2) determines the *wandering exponent ω* [see Eq. (1)] through

$$\omega = \frac{\ln |\lambda_2|}{\ln \lambda_1}, \qquad (4)$$

such that the fluctuation in one of the letters, g, is given by [21]

$$g \sim \mathcal{N}^\omega. \qquad (5)$$

In order to study the effects of the introduction of aperiodic modulations on models with first-order transitions, we use the Potts model with q states, which is described by the Hamiltonian

$$\mathcal{H} = -\sum_{\langle i,j \rangle} J_{ij} \delta_{\sigma_i \sigma_j}, \qquad (6)$$

where the sum is made over all first neighbors, J_{ij} is the coupling constant between sites i and j, $\sigma_i (=1,2,\ldots,q)$ represents the state of site i, and $\delta_{\sigma_i \sigma_j}$ is the Kronecker delta. The total magnetization of the Potts model for a lattice of linear size L is defined as

$$m_L(T) = \frac{q\rho_{\max} - 1}{q - 1}, \qquad (7)$$

where $\rho_{\max}$ is the density of the most populous state and T is the temperature. The susceptibility is obtained as the fluctuation of the magnetization, namely,

$$\chi_L(T) = \beta N(\langle m^2 \rangle - \langle m \rangle^2), \qquad (8)$$

where N is the total number of sites.

In the model we study, the interactions in one direction (e.g., horizontal) follow an aperiodic sequence, such that an interaction on a given position assumes a value (J_A or J_B), according to the letter (A or B, respectively) occupying the same position on the aperiodic sequence. In the vertical
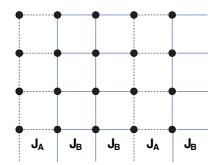

FIG. 1. (Color online) Interactions with strength J_A and J_B are represented by dashed and continuous lines, respectively. Note that the aperiodicity is introduced into the system in only one direction (horizontal, in this figure). In the vertical direction, all interactions have the same value as for the following horizontal bonds. The aperiodic sequence used is the one defined in Eq. (2).

direction, the interactions have the same value as for the following horizontal bonds [see Fig. 1 for an example of the sequence defined in Eq. (2)]. Thus, all models we study have $d_a = 1$ [see Eq. (1)].

The three sequences used in this work are presented in Table I. Two of them are relevant (three-folding, TF, and paper-folding, PF), assuming the Harris-Luck criterion holds true for first-order transitions as well. Therefore, they are expected to change the critical behavior when compared with the uniform model. The third sequence (Thue-Morse, TM) is expected to be irrelevant and should not change the universality class with respect to its uniform counterpart. These sequences were chosen because they are self-dual, which allows for the exact calculation of their critical temperature, through the relation [16]

$$(e^{\beta_c} - 1)(e^{r\beta_c} - 1) = q, \qquad (9)$$

where $\beta_c = J_A/k_B T_c$, k_B is Boltzmann's constant, T_c is the critical temperature, and r is the ratio between the coupling constants, $r = J_A/J_B$. Therefore, r describes the amount of aperiodicity of the model: for $r = 1$ we regain the uniform model, and for very small (or very large) r we obtain a series of independent one-dimensional-like systems (we will comment on this issue below).

TABLE I. Aperiodic sequences, their substitution rules, and wandering exponents ω. $pL_{\max}$ is the largest size of the simulated lattice for $q = 6$. Note the abbreviations for the sequences. The quantity p is the rate of growth of the sequence, after one application of the substitution rule.

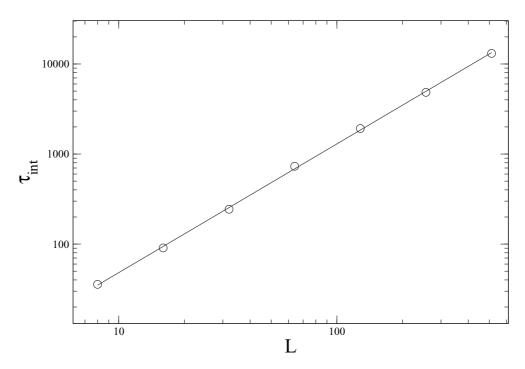| Sequence	Substitution rule	ω	p	$pL_{\max}$
Three-folding (TF)	$A \rightarrow ABA$ $B \rightarrow ABB$	0	3	19 683
Paper-folding (PF)	$AA \rightarrow BAAA$ $AB \rightarrow BAAB$ $BA \rightarrow BBAA$ $BB \rightarrow BBAB$	0	2	16 384
Thue-Morse (TM)	$A \rightarrow AB$ $B \rightarrow BA$	$-\infty$	2	8192

FIG. 2. Log-log graph of the autocorrelation time τ as a function of the linear size of the lattice, L, for the PF sequence with $q = 6$ and $r = 0.1$.

III. DETAILS OF THE SIMULATION

The models we treat are invariant under a duality transformation, and the exact critical temperature $(k_B T_c / J_A)$ is known [16,17]. This knowledge allows for a more precise calculation of critical exponents, using finite-size scaling, but the simulation at T_c also leads to a severe critical-slowing down if a single-spin algorithm, such as Metropolis [22], for example, is used. Therefore, we resort to the Wolff algorithm [22], which was proved to reduce, by a great amount, critical-slowing down. A rectangular lattice of size $pL \times L$ is used, with periodic boundary conditions in both directions. The parameter p varies according to the rate of growth of the aperiodic sequence. As previously mentioned, the aperiodic interactions are introduced in only one dimension (the one with linear size pL). We choose rectangular lattices to be able to introduce larger aperiodic sequences.

We first checked that the equilibrium values for some thermodynamic quantities were independent of the initial configuration. As expected, the time to reach equilibrium increased with lattice size but, for the worse cases, were approximately 100 times the autocorrelation time τ. This quantity was evaluated through the integral of the autocorrelation functions for the magnetization and the energy, and the larger value was considered. In order to calculate thermal averages, configurations 2τ apart (in Wolff steps: one Wolff step corresponds to building and flipping one island) were taken, which guarantees that the errors may be calculated using the standard deviation, for both the magnetization and the energy [22]. All averages are calculated using a sample of at least 10^4 independent configurations (where two configurations are considered independent if they are at least 2τ apart).

For the largest lattices, we parallelized our simulation, such that 20 processes run in different CPUs. Since the time to reach equilibrium was, for these lattices, approximately 100 times the autocorrelation time, this procedure allows for an economy in computational time. On the other hand, it prevents us from calculating the autocorrelation time. To overcome this difficulty, we have simulated long time series for the smallest lattices and have extrapolated the autocorrelation times for the largest ones. See Fig. 2 for an example of this procedure for the PF sequence, for $q = 6$ and $r = 0.1$; the extrapolation to $L = 8192$, in this case, leads to $\tau = 710\,000$, in Wolff steps. If this task was made in only one processor, it would take more than eight months to obtain 1000 independent configurations.

IV. RESULTS

Our first goal is to determined the order of the transition and, as a consequence, provide an independent test of the Harris-Luck criterion. In order to do this, we use the so-called Lee-Kosterlitz method [23], which works as follows. The quantity $F_L(E) = -\ln P_L(E)$ is calculated for various linear lattice sizes, L, where $P_L(E)$ is the probability distribution for energy E. It is expected that, for first-order transitions, $P_L(E)$ will present two peaks of the same height in the thermodynamic limit, one for the disordered phase (high E) and other for the ordered one (low E). For continuous transitions, on the other hand, the presence of only one peak is expected. One then studies the dependence of $\Delta F \equiv \ln(P_L(E_p)/P_L(E_b))$ with L, where E_p and E_b are the values of energy where $P_L(E)$ has a maximum and a minimum, respectively. More precisely, a plot of ΔF as a function of $1/L$ is made, in order to obtain the tendency of this barrier in the thermodynamic limit $(L \to \infty)$. If ΔF goes to zero in this limit, the transition is a continuous one; otherwise, it is a first-order transition.
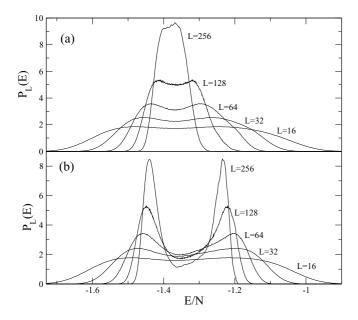
FIG. 3. Plot of $P_L(E)$ (see text) as a function of the energy per spin for the PF sequence (a) and for the TM sequence (b), for various values of the linear lattice size L, for $r = 0.9$ and $q = 6$, at $T_c(L)$.

In Fig. 3 we plot examples of the behavior of $P_L(E)$ for what we expect to be continuous (a) and first-order (b) transitions, for $r = 0.9$ and $q = 6$, for the TM and PF sequences. Note that the two-peak structure is present for the TM sequence,

whereas for the PF one a crossover effect takes place and only one peak is observed for the largest value of L.

In order to put our results on a more quantitative ground, the behavior of the energy barrier as a function of $1/L$ is plotted in Fig. 4, for a homogeneous model and for the three sequences studied here with $r = 0.9$ and $q = 6$. Note the distinct behavior for the homogeneous model and for the TM sequence, when compared with the PF and TF sequences. While ΔF increases for $L \to \infty$ for the former two models, it goes to zero, in the same limit, for the PF and TF sequences, after a subtle increase for small values of L. This indicates that the transitions is continuous for the PF and TF sequences and first order for the TM sequence. This result is consistent with the Harris-Luck criterion. In order to test if this behavior is a consequence of a value of r close to unity (homogeneous model), we have also performed the same procedure for $r = 0.7$ (inset of Fig. 4) for the TM sequence, comparing with the behavior for $r = 0.9$: it is still evident that ΔF increases with L, and no sign of a crossover is seen, for the range of L studied.

Note that the Harris-Luck criterion depends on the homogeneous model only through the value of ν [see Eq. (1)]. For models with first-order transitions, one expects that $\nu = 1/d$ [24], where d is the dimension of the system. Specifically for the Potts model, the value of ν will not depend on the number of states q, for $q > 4$ in two dimensions. So it is expected that our conclusions will hold true for other values of q such that the phase transition is a first-order one. Although not shown here, our results for $q = 15$ support this picture.
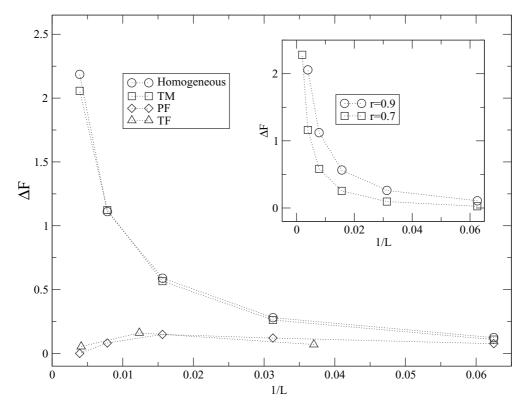

FIG. 4. The energy barrier ΔF as a function of $1/L$ for the homogeneous model and the TM, PF, and TF sequences, for $r = 0.9$ and $q = 6$ (main graph). In the inset, a comparison of the behavior of the energy barrier is made for the TM sequence and $r = 0.9$ and 0.7.
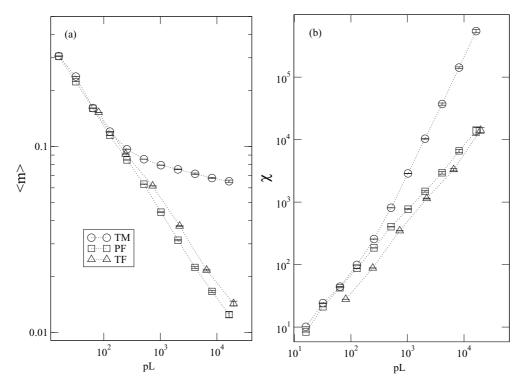
FIG. 5. Log-log graph of the magnetization (a) and susceptibility (b) vs. pL for the three aperiodic sequences, for $q = 6$ and $r = 0.1$. Full lines correspond to power-law fittings, and dotted lines are guides for the eye. Error bars are smaller than the points.

Therefore, our calculations suggest that the Harris-Luck criterion is also obeyed when the homogeneous model presents a first-order transition. In fact, for $\nu = 1/d = 1/2$ and $d_a = 1$, the crossover exponent Φ reads [see Eq. (1)]

$$\Phi = \tfrac{1}{2}(\omega + 1). \qquad (10)$$

Looking at the values of ω from Table I, we see that Φ is negative (irrelevant sequence), $\Phi = -\infty$, for the TM sequence and positive (relevant sequences), $\Phi = 1/2$, for the TF and PF sequences. Our findings agree with the results of Refs. [16] and [17].

We now turn to the question of the new universality classes that emerge when the aperiodic sequence is a relevant one, which is the case for the TF and PF sequences. We will also address the possibility that the new universality class depends on the number of states of the Potts model. In order to calculate the critical exponents, we make simulations at the exact critical temperature T_c (these two sequences are self-dual, which allows for the exact calculation of T_c). The quantities we calculate are the magnetization m and the susceptibility χ, which are expected to behave, at $T = T_c$, as

$$m \sim L^{-\beta/\nu}, \quad \chi \sim L^{\gamma/\nu}, \qquad (11)$$

where L is the linear size of the lattice and ν is the critical exponent of the correlation length. Log-periodic corrections to the above behavior are expected, for aperiodic models and, as we will see shortly, are found in our results. We have calculated the two quantities cited above for $q = 6$ and $q = 15$ in order to test the dependence of the new critical exponents with the number of states.

An example of the results we obtain is presented in Fig. 5: the log-log graphs of the magnetization (a) and susceptibility (b) for all three sequences, for $r = 0.1$ and $q = 6$, are depicted. The exponents β/ν and γ/ν are the slope of the curves $m_L(T_c) \times pL$ and $\chi_L(T_c) \times pL$, respectively. Note the evident crossover behavior for the TM sequence, which is an irrelevant one. The slope of the magnetization curve initially follows the behavior of the curves for the PF and TF sequences but eventually presents a curvature, and the slope tends to values closer to zero. Although this is not our stronger evidence for a first-order transition, the behavior is consistent with the one for ΔF. The same trend is obtained for the susceptibility, and a crossover to a different behavior is obtained for the TM sequence.

One possible way to obtain more precise values for the critical exponents is to make fittings for different ranges of pL. More precisely, we initially make fittings for $pL \geqslant 32$, which defines our first estimate. The second estimate is obtained removing the point with the smallest value of pL used in the first estimate. The procedure is repeated until only three points remain, which defines our last estimate [16,17,25]. We then plot these values versus $1/pL_{\min}$, where $pL_{\min}$ is the smallest value of pL used in a given fitting. The results for β/ν and γ/ν are depicted in Fig. 6: it appears that the values for the TM sequence converge to $\beta/\nu = 0$ and $\gamma/\nu = 2$, which are the expected values for a first-order transition [24]. For the PF and TF sequences, our results are analogous to the ones in Refs. [16,17]. However, a detailed look at Figs. 6(b) and 6(c) for small values of $1/pL_{\min}$ shows that the convergence to the thermodynamic limit is not the usual one: oscillations are present for both β/ν and γ/ν, and they do not die out for the largest lattices. This behavior maybe a sign of log-periodic
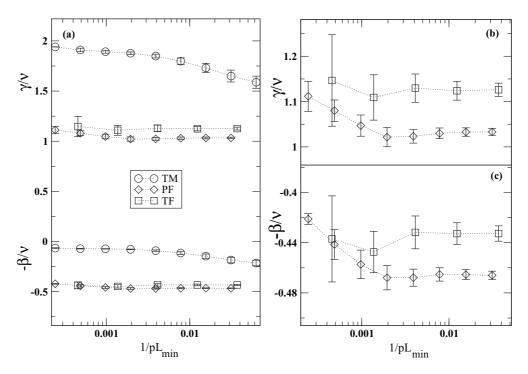
FIG. 6. Semilog graph of the exponents vs. $1/pL_{min}$ for the three-folding sequence, for $r = 0.1$, $q = 6$, and the three sequences studied.

oscillations, expected to be present in models with aperiodic modulations [26,27].

In order to test this possibility, we have calculated the ratio of the exponents in the following way. For each three successive points of $m(T_c)$ or $\chi(T_c)$ we fit a straight line and plot its slope as a function of $1/p\tilde{L}$, where $p\tilde{L}$ is the smallest value of the three used to make a given fit. For the "usual" behavior, the value of these slopes should converge to the thermodynamic value of β/ν or γ/ν, respectively [25]. In Fig. 7 we show the results of this procedure for the ratio β/ν, for the PF sequence, $r = 0.1$, and $q = 6$ and 15. As clearly seen in this figure, our results show no sign of this expected convergence. In fact, our data are well fitted by a log-periodic function, in the form

$$\Theta(L) = \Theta + A\cos[B\log(L) + C], \quad (12)$$

where $\Theta(L)$ stands for the slope of the straight line at a given L and Θ stands for γ/ν or β/ν, depending on whether we use the data for $\chi(T_c)$ or $m(T_c)$, respectively. This shows the expected behavior for systems with aperiodic modulations [26,27].

Note that Eq. (12) has four fitting parameters, and this requires a reasonable amount of data points. We can accomplish this for the PF sequence, and good fittings are obtained in this case. However, two issues should be addressed here. First, note that fewer data points are available for the TF sequence, in comparison with the PF one (this is due to the larger rate of growth of the former sequence). Therefore, one should anticipate that a fitting using Eq. (12) might not be satisfactory. We have, therefore, applied a second procedure to obtain γ/ν and β/ν for the PF sequence: we ignore the log-periodic oscillation and fit the data to a straight line. In this case, only two parameters are necessary, and one hopes that fewer points are needed to provide a good fitting. In Table II

we compare β/ν and γ/ν calculated using the two procedures explained above, for the PF sequence and $q = 6$ and 15. One can note that, within error bars (assumed here as one standard deviation), the values for straight-line fitting and with the use of a log-periodic function [Eq. (12)] coincide. This corroborates our strategy for the TF sequence: since for this sequence a precise fitting using a log-periodic function is not possible, we will rely on the straight-line fitting, to obtain γ/ν and β/ν.

Also from Table II, we notice that the values for γ/ν and β/ν depend on the number of estates q. This is the same result as for random disorder [19,20,28]. Note also that although our values for β/ν are not equal to those for random disorder [28], for a given q, the percentage difference between the values for $q = 6$ and 15 are the same for our model and for the model studied in Ref. [28].

The second issue we would like to address is the presence of crossover effects, for values of r close to 1 and close to 0. For the former, one expects "contamination" from the uniform behavior, with a crossover effect and effective exponents, not representing the behavior of the aperiodic model. On the other hand, for $r = 0$ the lattice breaks into isolated strips, which are one-dimensional. Therefore, one has to take into consideration

TABLE II. Values for the ratios β/ν and γ/ν for the PF sequence, calculated through a straight-line fitting (S) and using a log-periodic function (LP). q is the number of states. Each entry is the average over three values of r (see text).

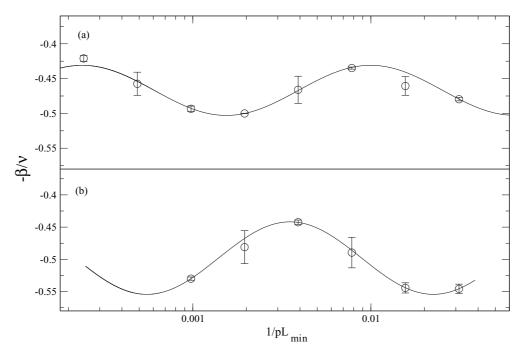| q	β/ν		γ/ν	
	S	LP	S	LP
6	0.469(5)	0.470(4)	1.032(12)	1.03(1)
15	0.509(16)	0.499(10)	0.93(6)	0.93(2)

FIG. 7. Graphs of β/ν versus $1/p\tilde{L}$ (see text) for the PF sequence, $r = 0.1$, and $q = 6$ (a) and $q = 15$ (b).

the possibility that a crossover from the one-dimensional model to the aperiodic behavior may take place. This would be shown as a dependence of the critical exponents on r [25]; in fact, from our results obtained for r ranging from 0.001 to 0.5, we notice that reasonable stable values for β/ν and γ/ν are obtained for r between 0.05 and 0.1. This means that the aperiodic-model universality class is located in this interval. In Fig. 8 we show the behavior of the two ratios mentioned

above as functions of r, for the PF sequence and $q = 6$ (our results for the TF sequence or for $q = 15$ follow this same behavior). Since we cannot be more precise in locating the value of r that represents the behavior of the aperiodic model, we take our values for γ/ν and β/ν as the average of the values calculated for $r = 0.1, 0.067$, and 0.05. Note that γ/ν increases for small r and for r close to 1. This is the expected behavior, since for $r = 1$ γ/ν is expected to assume the value 2
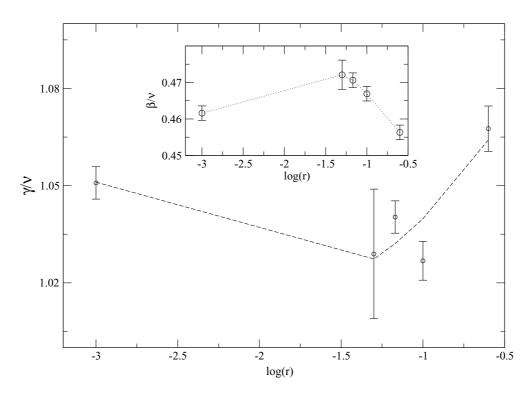

FIG. 8. Graphs of the ratios γ/ν (main graph) and β/ν (inset) as a function of $\log(r)$ for the PF sequence and $q = 6$.

TABLE III. Values for the ratios β/ν and γ/ν for PF and TF sequences, calculated through a straight-line fitting. q is the number of states. Each entry is the average over three values of r (see text). Numbers between parentheses represent the error in the (two) last digit(s). Results for $q = 8$ are from Ref. [17], for $r = 10$ (equivalent to our $r = 0.1$).

	PF			TF		
q	6	8	15	6	8	15
β/ν	0.469(5)	0.49(1)	0.509(16)	0.433(4)	0.44(2)	0.47(2)
γ/ν	1.032(12)	1.01(1)	0.93(6)	1.13(1)	1.09(3)	1.00(3)

in two dimensions [16,24]. The same is true for r close to zero, since in this limit the one-dimensional character of the model should lead to an exponential divergence of the susceptibility. For β/ν, the decrease in its value for r close to 1 and 0 reflects the fact that in these two limits the transition is first order, such that $\beta = 0$ [16,24]. Note that our estimate for the location of the aperiodic-model universality class coincides with the result for random-disorder Potts-model universality class [28].

Finally, we turn to the TF sequence. As mentioned above, we do not have enough points to make a reliable fitting with a log-periodic function. Therefore, we resort to straight-line fittings to obtain β/ν and γ/ν. We have made simulations for many values of r and noticed that the stable region is between $r = 0.05$ and 0.1 (the exact point where the aperiodic-model universality class is present may vary from one sequence to another and from $q = 6$ to 15, but we do not have enough precision to pinpoint the correct value of r). So we have taken as our results averages over $r = 0.1$, 0.067, and 0.05. In Table III we summarize our results for both sequences and for $q = 6$ and 15 for straight-line fittings.

Our results strongly suggest that the new universality class that emerges when aperiodic modulations are introduced in the Potts model depends on the number of states q and on the aperiodic sequence. This latter conclusion is supported by the calculations in Ref. [17]. Together with the results of Refs. [16] and [17], we can establish that β/ν (γ/ν) increases (decreases) with q, but we are not able to propose a dependence law. Finally, we would like to mention that our results, shown in Table III, for both sequences and both values of q satisfy the equality $2\beta/\nu + \gamma/\nu = 2$, within error bars.

V. CONCLUSION

We have performed a numerical simulation of Potts models on the square lattice with aperiodic modulations of the interaction parameter in one direction. The Wolff algorithm was used, and three different aperiodic sequences were studied, for $q = 6$ and 15 number of states. For these cases the transition on the uniform model is a first-order one. Using the Lee-Kosterlitz method, we verified that the Thue-Morse sequence does not change the universality class of the transitions, whereas the Paper-Folding and Three-Folding sequences turn the transition into a continuous one. These results are in accordance with the Harris-Luck criterion and confirm and extend the results of Refs. [16] and [28]. Therefore, we expect that this criterion applies to first-order transitions, although it was introduced in the context of models with continuous transitions.

For the two sequences that are relevant, we calculate the universality class, simulating the systems on their exact critical temperature. Using finite-size scaling arguments, we were able to show that this new universality class depends on the number of states q and on the aperiodic sequence. This latter result is in accordance with earlier calculations [16,28].

We have also made simulations to calculate the exponent of the correlation length ν, studying quantities such as the fourth-order cumulant and logarithmic derivatives of the magnetization [29]. However, strong oscillations, already seen in Refs. [16] and [28], prevented us from getting precise results.

In order to calculate ν, obtain more precise values for the value of r that represents the critical behavior of the aperiodic model, and go to larger values of q, we are now using a transfer matrix matrix approach. Mapping the Potts model onto a Whitney polynomial [30], the number of states q enter as a parameter, and the matrix involved grows much slower than q^N, where N is the number of rows of the finite system. This allows for more precise values, for large q, than using Monte Carlo simulation.

Note added in proof. Our attention was drawn to the work by Turban and Igloi [31] where the authors calculate critical parameters for a Potts model with a free surface, for large number of states q. When the system follows a Fibonacci-type modulation, which is marginal, in the context of the Harris-Luck criterion, the exponents vary with the strength of the aperiodic perturbation.

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