Optimal proportional reinsurance and dividend pay-out for insurance companies with switching reserves

Abstract: This paper presents a model for an insurance company that controls its risk and dividend payout. The financial reserve of this company is modeled as an Ito process with positive drift. While the diffusion coefficient can be interpreted as the risk exposure, the drift can be understood as the potential profit. The new feature of this paper is to consider that the potential profit and risk exposure of this company depends on the dynamic state of the economy. Thus, in order to take into account the state of the economy, the drift process and the diffusion coefficient are modeled as a continuous time Markov chain. The aim is to maximize the dividend payout of this insurance company whose manager is risk averse. This problem is formulated as a Markovian optimal control problem, so the Hamilton-Jacobi-Bellman equation is solved to yield the solution.

Keywords: Dividends, insurance, optimal control, reinsurance, stochastic control.

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1. Introduction

Recently, many works have dealt with diffusion models for insurance companies with controllable risk exposure (H JGAARD and TAKSAR, 1998a, 1998b, 1999; TAKSAR, 2000; CANDENILLAS et al., 2006; LUO et al., 2008). This paper, as in the previous ones, considers that the financial reserve of the insurance company is modeled as an Ito process with positive drift. While the diffusion coefficient can be interpreted as the risk exposure, the drift can be associated to the potential profit when the number of sold policies is sufficiently large. The new feature of this paper is to consider that the potential profit and the risk exposure of the insurance company depend on the dynamic state of the economy. In order to take this idea into account, the drift process and diffusion coefficient are modeled as a continuous time Markov chain, i.e., the reserve of the company is modeled as a switching diffusion model. This type of model intends to consider a large class of changes which can affect the potential profit of the insurance company. Public insurance system changes, Legislation changes and population income changes are examples. In fact, these changes are very common in emergent markets.

Actually, the idea of modeling by using switching models is not new in the financial literature. The seminal paper (HAMILTON, 1989), which is set in the context of discrete time models, studies discrete shifts that may occur in the growth rate of nonstationary series. On the other hand, in the continuous time set, switching diffusion models have been used to value European options (NAIK 1993; CHOURDAKIS, 2000; GUO, 2001), to choose the correct time to buy and sell stocks (ZHANG, 2001), to deal with portfolio selection in a mean variance sense (YIN and ZHOU, 2004) and portfolio selection (CAJUEIRO and YONEYAMA, 2004; BÄUERLE and RIEDER, 2004). Additionally, a simpler result of a similar problem may be found in David (1997).

This paper is organized as follows. In section 2, the problem is formulated and motivated. In section 3, the Hamilton-Jacobi-Bellman equation and the verification theorem are stated. In section 4, following the same lines of Cajueiro and Yoneyama (2004), the problem is solved. Finally, section 5 presents some conclusions of this work.

2 Problem Statement

In order to give a precise mathematical formulation for the problem, one should consider the following statistically mutually independent processes on a suitable probability space (Ω, F, P) :

a) A standard Brownian motion $B = \{B(t), F_t; s \leq t \leq T\}$;

b) A homogeneous continuous time Markov chain $\theta = \{\theta(t), F_t; s \leq t \leq T\}$, with right continuous trajectories, and taking values on the finite set $S = \{1, 2, \ldots, n\}$, characterized by $p(t) = \{p_1(t), p_2(t), \ldots, p_n(t)\}$, with $p_i(t) = P(\theta(t) = i)$, for $i \in S$. Moreover, $p(t)$ satisfies the following Kolmogorov's forward equation $dp/dt = p(t)\Lambda$ where $\Lambda = [\lambda_{ij}]$ is the stationary $n \times n$ transition rate matrix of θ (the infinitesimal generator of a continuous time Markov chain) with $\lambda_{ij} \geq 0$ for $i \neq j$, and $\lambda_{ii} = -\sum_{i \neq j} \lambda_{ij}$. Thus, the process is conservative.

Remark 2.1. The filtration F_t represents the information available at time t and any decision should be made based on it.

One may assume that in the case of no risk or dividend control, the reserve of the company (i.e. the liquid assets of the company or the wealth of the company) evolves according to

$$dW(t) = \mu(\theta)dt + \sigma(\theta)dB(t) \tag{1}$$

where $\mu(\theta) \stackrel{\Delta}{=} \sum_{i=1}^{n} \delta(i,\theta)\mu_i$ is the potential profit, $\sigma(\theta) \stackrel{\Delta}{=} \sum_{i=1}^{n} \delta(i,\theta)\sigma_i$ is the risk exposure of the insurance company and $\delta(i,j)$ is the Kronecker's symbol.

Example 2.2. (Legislation Change) In Brazil, one may point out a good example of a recent change in the federal legislation that had a strong impact on the health insurance companies' potential profit. Until 1998, there was no clear legislation to deal with health insurance in Brazil, but from this date on the situation has completely changed. Therefore, since $\mu(\theta)$ is a measure of the potential profit of the insurance company, it was clearly affected. In most cases, this effect was negative and $\mu(\theta)$ became smaller, since the main idea of this law was to protect insured people from the excesses of insurance companies.

Example 2.3. (Lack of consistency of public policies) Since there is no continuity in public health policies in emergent markets, the return of previously eradicated contagious diseases is common. The return of these diseases may cause a negative impact on the potential profit $\mu(\theta)$ and also cause an increase in the risk exposure $\sigma(\theta)$ of insurance companies.

On the other hand, if one considers that the company controls its risk and its dividend policy, the resulting reserve process (liquid assets of the company or wealth of the company) $(W(t),\theta(t)) \in R \times S$ defined on a suitable probability space (Ω, F, P) may be described by

$$dW(t) = (\pi\mu(\theta) - D)dt + \pi\sigma(\theta)dB(t) \tag{2}$$

and

$$P(\theta(t+\Delta t) = j / \theta(t) = i) = \begin{cases} \lambda_{ij}\Delta t + o(\Delta t) & for \quad i \neq j \\ 1 + \lambda_{ij}\Delta t + o(\Delta t) & for \quad i = j \end{cases} \tag{3}$$

where $\lim_{\Delta t \to 0} \frac{o(\Delta t)}{\Delta t} = 0$.

While $0 \leq \pi \leq 1$ is the retention level, which is the fraction of all incoming claims that the insurance company will insure by itself (it has been considered that the reinsurance company has the same safety loading as the insurance company, an assumption that is known as "cheap reinsurance"), D is the rate where the dividends are paid out.

Let Π and D , respectively, be the set of admissible policies $\pi : R^{+} \times R \times S \to \Pi$ and $D : R^{+} \times R \times S \to D$. One can ensure that equation (2) has a unique strong solution if these sets of admissible policies are restricted with the usual conditions for growing and Lipschitz functions. For details, see theorem 4.6 on page 128 and also section 9.1.2 on page 329 in Liptser and Shiryayev (1977).

Although, the conditions for $\pi \in \Pi$ and $D \in D$ ensure that equation (2) has a unique strong solution, one must not allow the solution of equation (2) to assume negative values. Hence, this problem requires a more restrictive class of admissible controls which is presented as follows:

Definition 2.4. Let $\overline{\Pi}$ and $\overline{D}$ be the set of admissible controls.

Assumption 2.5. $\pi \in \overline{\Pi}$ and $D \in \overline{D}$, if $\pi \in \Pi$, $D \in D$, $\pi(t) = 0$ and $D(t) = 0$ for all $t \in [\tau, T]$, where τ , if it exists, is the time of bankruptcy, i.e., $\tau = \inf_t \{W(t) = 0\}$.

Assumption 2.6. In this work, the jump sizes are considered predictable in $\theta = \{\theta(t), F_t; s \leq t \leq T\}$, i.e., one does not know if a jump will occur, but if it does, its magnitude is known.

Now one may define the so-called value function $\Phi:[s,T]\times R\times S\to R$. Thus, the problem that will be solved in this paper is formulated bellow:

Problem 2.7. Suppose that, starting with the initial reserve $W(s)=w$ at time $t=s$, the manager of an insurance company wants to maximize the performance index given by $E[\int_s^T exp(-\gamma t)U(D)dt+exp(-\gamma T)U(W(T))\mid W(s)=w,\theta(s)=i]$, where T is a known fixed date in the future. If the utility function U of the manager is increasing and concave (the manager is risk-averse), then the problem is to find the value function $\Phi(s,w,i)$ and a Markov policy $\{\pi,D\}$ such that $\Phi(s,w,i)$, for $i=1,\text{L} ,n$, is given by

$$\Phi(s,w,i)=\sup_{\substack{\pi\in\overline{\Pi}\\ D\in\overline{D}}} J(s,w,i,\pi,D)=J(s,w,i,\pi^*(s,w,i),D^*(s,w,i)) \tag{4}$$

where

$$J(s,w,i,\pi,D)=$$

$$E\left[\int_s^T exp(-\gamma t)U(D)dt+exp(-\gamma T)U(W(T))\mid W(s)=w,\theta(s)=i\right] \tag{5}$$

Assumption 2.8. In this paper the utility functions of the manager are assumed to belong to the class of Constant Relative Risk Aversion (CRRA) utility functions, i.e., $U(W)=\frac{W^r}{r}$, where $0<r<1$.

Remark 2.9. The assumption that the manager of the insurance company is risk-averse is a significant difference between this work and Taksar (2000) and many references in it. Moreover, this assumption ensures that the associated Hamilton-Jacobi-Bellman partial differential equation considered here has separable variables as in Taksar (2000), as well as many references within it and also in Merton (1969), Fragoso and Hemerly (1991) and Cajueiro and Yoneyama (2004).

3 The Hamilton-Jacobi-Bellman equation and the verification theorem

Suppose now that Φ is a function that belongs to the class of functions with continuous derivatives of first order in t on $[s,T]$ and first and second order in $W\in R$, almost everywhere. So, relying on Bellman's principle of optimality and Dynkin's formula,[3] one can show that Φ satisfies the following Hamilton-Jacobi-Bellman equation

$$\sup_{\pi\in[0,1],D}\{L^{\pi,D}\Phi(t,W,i)+exp(-\gamma t)U(D)\}=0 \tag{6}$$

with boundary conditions $\Phi(T,W,i)=\exp(-\gamma T)U(W)$, for $i=1,\ldots,n$ and where $L^{\pi,D}\Phi$ is the infinitesimal generator of $\{(W(t),\theta(t));s\le t\le T\}$ which is given by[4]

$$L^{\pi,D}\Phi(t,W,i)=\frac{\partial\Phi(t,W,i)}{\partial t}+(\pi\mu_i-D)\frac{\partial\Phi(t,W,i)}{\partial W}$$

$$+\frac{1}{2}\pi^2\sigma_i^2\frac{\partial^2\Phi(t,W,i)}{\partial W^2}+\sum_{j=1}^n\lambda_{ij}\Phi(t,W,j) \tag{7}$$

[3] Details of this procedure may be found in chapter VI in Fleming and Rishel (1975).

Theorem 3.1. (Dynamic Programming Verification Theorem) Let Φ be the solution of the dynamic programming equation $\sup_{\pi\in[0,1],D}\{L^{\pi,D}\Phi(t,W,i)+exp(-\gamma t)U(D)\}=0$ with boundary conditions $\Phi(T,W,i)=\exp(-\gamma T)U(W)$, for $i=1,\ldots,n$. Then:

a) $J(t,W,i,\pi,D)\le\Phi(t,W,i)$ for any admissible policy $\{\pi,D\}$ and any initial data;

b) If $\{\pi^*,D^*\}$ is an admissible policy such that

$$L^{\pi^*,D^*}\Phi(t,W,i)+exp(-\gamma t)U(D^*)=\sup_{\pi\in[0,1],D}\{L^{\pi,D}\Phi(t,W,i)+exp(-\gamma t)U(D)\}=0$$

c) Thus $\{\pi^*,D^*\}$ is optimal;

Proof: This proof is analogous to the proof of the theorem 4.1 on page 159 in Fleming and Rishel (1975).

4 Problem Solution

This section aims at solving problem 6. However, before considering the main theorem, the following lemmas are required.

Lemma 4.1. The system

$$\frac{dg(t,i)}{dt}+M_{ii}g(t,i)+\sum_{\substack{j=1\\i\neq j}}^{n}M_{ij}g^{1-r}(t,j)g^{r}(t,i)+C=0 \quad for\ i=1,\ldots,n \tag{8}$$

where M is a real matrix and C is a constant, has a unique solution.

Proof: One should note that the system (8) can be rewritten as

$$\frac{d\bar{g}(t,i)}{dt}+\bar{M}_{ii}\bar{g}(t,i)+\sum_{\substack{j=1\\i\neq j}}^{n}\bar{M}_{ij}\bar{g}(t,j)+\bar{C}\bar{g}^{\frac{-r}{1-r}}(t,i)=0$$

where $\bar{g}(t,i)=g(t,i)^{1-r}$, $\bar{M}=(1-r)M$ and $\bar{C}=(1-r)C$

The above equation satisfies the Lipschitz condition and, therefore, has a unique solution (for details, see the theorem that deals with the existence of a solution for nonlinear ordinary differential equations on page 205 in Hochstadt (1963).

Lemma 4.2. Suppose that $M_{ij}\ge 0$ for $i\neq j$ and C is positive constant. Then the system described by

$$\frac{dg(t,i)}{dt}+M_{ii}g(t,i)+\sum_{\substack{j=1\\i\neq j}}^{n}M_{ij}g^{1-r}(t,j)g^{r}(t,i)+C=0 \quad for\ i=1,\ldots,n \tag{9}$$

with final condition $g(T,i)=g_T>0$ has a real positive solution.

Proof: Letting $\tau=T-t$ and defining $\bar{g}(\tau,i)=g(T-\tau,i)$ with initial condition $\bar{g}(0,i)=g_T>0$, one gets

$$\frac{d\bar{g}(\tau,i)}{d\tau}=M_{ii}\bar{g}(\tau,i)+\sum_{\substack{j=1\\i\neq j}}^{n}M_{ij}\bar{g}^{1-r}(\tau,j)\bar{g}^{r}(\tau,i)+C \tag{10}$$

One should note that the variable $\bar{g}(\tau,i)$ starts at $g_T>0$ and if for any reason $\bar{g}(\tau,i)\to 0$ then its derivative $\frac{d\bar{g}(\tau,i)}{d\tau}$ becomes positive and, therefore, $\bar{g}(\tau,i)$ will become more positive. One should also note that system (10) is the forward form of the system (9) which is in the backward form.

Theorem 4.3. The optimal policy $\{\pi^*,D^*\}$ that solves the problem 2.7 is given by

[4] For details, see Fragoso and Hemerly (1991) (and the references therein) which deals with a LQG problem in this Markovian jumping parameter setting.

$$\pi^*(W,i) = min\left(\frac{\mu_i W}{\sigma_i^2(1-r)}, 1\right) \quad for \quad \theta(t) = i \tag{11}$$

and

$$D^*(t,W,i) = \frac{W}{g(t,i)} \quad for \quad \theta(t) = i \tag{12}$$

and the value function is given by $\Phi(t,W,i) = exp(-\gamma t) g(t,i)^{1-r} \frac{W^r}{r}$ with boundary conditions $\Phi(t,0,i) = 0$, for $i = 1,\dots,n$ and where $g(t,i)$, for $i = 1,\dots,n$ is the solution of the real positive system of

$$\frac{dg(t,i)}{dt} + \frac{1}{1-r}\left(\frac{\mu_i^2 r}{2\sigma_i^2(1-r)} - \gamma\right) g(t,i) + \frac{1}{1-r}\sum_{j=1}^{n} \lambda_{ij} g^{1-r}(t,j) g^r(t,i) + 1 = 0 \tag{13}$$

with boundary conditions $g(T,i) = 1$, for $i = 1,\dots,n$.

Proof: If one starts ignoring the constraint on π , from equations (6) and (7), the Hamilton-Jacobi-Bellman equation can be written as

$$\frac{\partial\Phi(t,W,i)}{\partial t} + \sup_{\pi,D}\left\{(\pi\mu_i - D)\frac{\partial\Phi(t,W,i)}{\partial W} + \frac{1}{2}(\pi^2\sigma_i^2)\frac{\partial^2\Phi(t,W,i)}{\partial W^2} + \exp(-\gamma t)\frac{D^r}{r}\right\} + \sum_{j=1}^{n} \lambda_{ij}\Phi(t,W,j) = 0$$

with boundary conditions $\Phi(T,W,i) = \exp(-\gamma T)U(W)$ and $\Phi(t,0,i) = 0$, for $i = 1,\dots,n$.

The policy $\{\pi^*, D^*\}$ that maximizes the above equation is given by

$$\pi^* = -\frac{\mu_i \frac{\partial\Phi(t,W,i)}{\partial W}}{\sigma_i^2 \frac{\partial^2\Phi(t,W,i)}{\partial W^2}} \quad and \quad D^* = \left(\exp(\gamma t)\frac{\partial\Phi(t,W,i)}{\partial W}\right)^{\frac{1}{r-1}}$$

Thus, the Hamilton-Jacobi-Bellman becomes

$$\frac{\partial\Phi(t,W,i)}{\partial t} - \frac{1}{2}\frac{\mu_i^2\left(\frac{\partial\Phi(t,W,i)}{\partial W}\right)^2}{\sigma_i^2\frac{\partial^2\Phi(t,W,i)}{\partial W^2}} - \left(\exp(\gamma t)\frac{\partial i(t,W,i)}{\partial W}\right)^{\frac{1}{r-1}}\frac{\partial\Phi(t,W,i)}{\partial W}\frac{1}{r}$$

$$\exp(-\gamma t)\left(\exp(\gamma t)\frac{\partial\Phi(t,W,i)}{\partial W}\right)^{\frac{r}{r-1}} + \sum_{j=1}^{n}\lambda_{ij}\Phi(t,W,j) = 0$$

If $\Phi(t,W,i) = g(t,i)^{1-r}\exp(-\gamma t)\frac{W^r}{r}$, then one gets

$$\frac{dg(t,i)}{dt} + \frac{1}{1-r}\left(\frac{1}{2}\frac{\mu_i^2 r}{\sigma_i^2(1-r)} - \gamma\right) g(t,i) + \frac{1}{1-r}\sum_{j=1}^{n} \lambda_{ij} g^{1-r}(t,j) g^r(t,i) + 1 = 0$$

which has final condition $g(T,i) = 1$, for $i = 1,\dots,n$ and satisfies lemmas 4.1 and 4.2.

The uniqueness of Φ follows from the uniqueness of g in Lemma 4.1.

Due to the restriction $\pi \in [0,1]$, the above solution is valid whenever $\pi^* \leq 1$. However, since the Hamilton-Jacobi-Bellman equation is a concave quadratic function of π , when the Hamilton-Jacobi Bellman above provides $\pi^* > 1$, the optimal policy is $\pi^* = 1$.

Remark 4.4. It is very interesting to interpret equation (11). One may see that there are two different periods in the life of an insurance company. When the insurance company is small as compared to the potential claims, i.e. $W < \frac{(1-r)\sigma_i^2}{\mu_i}$, the manager of this insurance company should think about the possibility of reducing the risk which the company is faced. On the other hand, when the insurance company grows, sharing the risk, i.e. reinsuring, is not an interesting procedure since the risk may not be a threat of bankruptcy anymore and it also means sharing the profit. Moreover, this notion of size of an insurance company introduced above depends strongly on the state of the economy modeled by a continuous time Markov chain.

The behavior of D^* is described with the help of the next example:

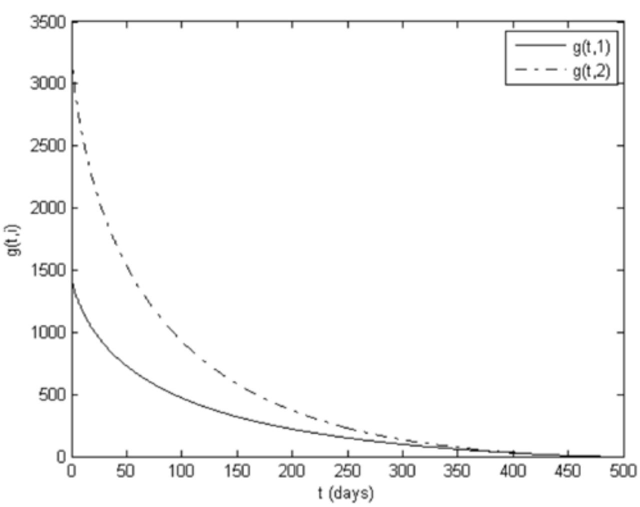
Example 4.5. Consider an economy which can be modeled by a two-state continuous time Markov chain:

a) The state 1 models a state of an economy with parameters $\mu_1 = 0.0010$, $\sigma_1 = 0.1$ and, therefore, $\pi_1 = 0.5$, i.e., the insurance company has to reinsure part of its claims. One should realize that due to the necessity of sharing the risk, this state of this economy is not the most profitable one for an insurance company, since the insurance company also has to share its profit.

b) The state 2 models a state of an economy with parameters $\mu_2 = 0.0022$, $\sigma_2 = 0.07$ and, therefore, $\pi_2 = 1$, i.e., the insurance company insures its claims by itself. Actually, this is a very profitable state of an economy for an insurance company.

Consider also that $\gamma = 0.0001$ and $r = 0.8$ and the situation that the probability of the economy is in state 1 is exactly the same probability of the economy is in state 2, i.e., for example, $\lambda_{11} = -0.005$, $\lambda_{12} = 0.005$, $\lambda_{21} = 0.005$ and $\lambda_{22} = -0.005$.

Figure 1. Plot of $g(t,i)$ of an economy which is modelled by a CTMC with two states which have the same probability of happening.



As one can see in figure 1, the infinitesimal generator of the CTMC has a strong influence on the dividends pay-out. Actually, if one analyses equation (12), one can see that the magnitude of the dividends pay-out depends strongly on the magnitude of the liquid assets W of the insurance company, but the fraction of these liquid assets that is paid as dividends depends on the state of the economy. According to figure 1, when the market is very profitable for insurance companies (state 2), the manager of the company prefers to withhold the biggest part of the holders' dividends to increase the purchase power of the company, i.e., to increase the profitability of the company. On the other hand, when the market is not very profitable for insurance companies (state 1), the manager of the company prefers to pay a bigger fraction of dividends to the holders.

The above results may be interpreted in the following way. Since state 2 is a very profitable state, the number of profitable claims is bigger in this state than in state 1 which is not very profitable. Additionally, it is expected that a well managed company never use its capital to finance non-profitable projects, i.e., projects with negative net value (ROSS et al., 1999). Thus, more money will stay unused to be paid as dividends.

Remark 4.6. The choice of the performance index (5) is not the only option. If one did not want to consider the combined problem of optimizing dividends and reinsurance policies and only the problem of optimizing the reinsurance policy, then one could consider, for instance, the wealth of the company discounted with rate $\gamma > 0$ as the performance index. Thus, the wealth of the insurance company can be described by

$$dW(t) = \pi\mu(\theta)dt + \pi\sigma(\theta)dB(t)$$

and the performance index is given by

$$J(s,w,i,\pi) = E\left[\int_s^T exp(-\gamma t)U(W)dt \mid W(s) = w, \theta(s) = i\right]$$

One can prove that

$$\pi^*(W,i) = min\left(\frac{\mu_i W}{\sigma_i^2(1-r)}, 1\right) \quad for \quad \theta(t) = i$$

and the value function is given by $\Phi(t,W,i) = g(t,i)\frac{W^r}{r}$ where $g(t,i)$ is the unique solution of the backward positive linear system given by

$$\frac{dg(t,i)}{dt} + \frac{\mu_i^2 r}{2\sigma_i^2(1-r)}g(t,i) + \sum_{j=1}^{n}\lambda_{ij}g(t,j) + \exp(-\gamma t) = 0$$

with final condition $g(T,i) = 1$, for $i = 1,\ldots,n$, instead of the nonlinear system given by equation (13). For details, see Cajueiro and Yoneyama (2002a).

5 Conclusions

This paper has studied the combined problem of optimizing the dividend pay-out and the reinsurance policy of an insurance company with switching reserves. The proposed problem has been completely solved by means of dynamic programming arguments and the Hamilton-Jacobi-Bellman equation was solved to provide a closed solution for the problem. As one could see, the optimal dividend pay-out and reinsurance policy in this market depend strongly on the state of an economy which is modeled by a continuous time Markov chain.

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