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Fraturas de Caminho Ótimo

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Optimal path in disordered media: some definitions and previous related studies

1) On an n-dimensional lattice, we assign to each site i a given "energy" value εi according to a given probability distribution P(ε). The energy of a path is the sum of all energies of its sites.

2) The optimal path (OP) is defined here as the one among all paths connecting the bottom to the top of the lattice that has the smallest energy [Kirkpatrick & Toulouse, *J. Phys. Lett.* (1985); Kertesz, Horvath & Weber, *Fractals* (1992); Barabasi, *Phys. Rev. Lett.* (1996)].

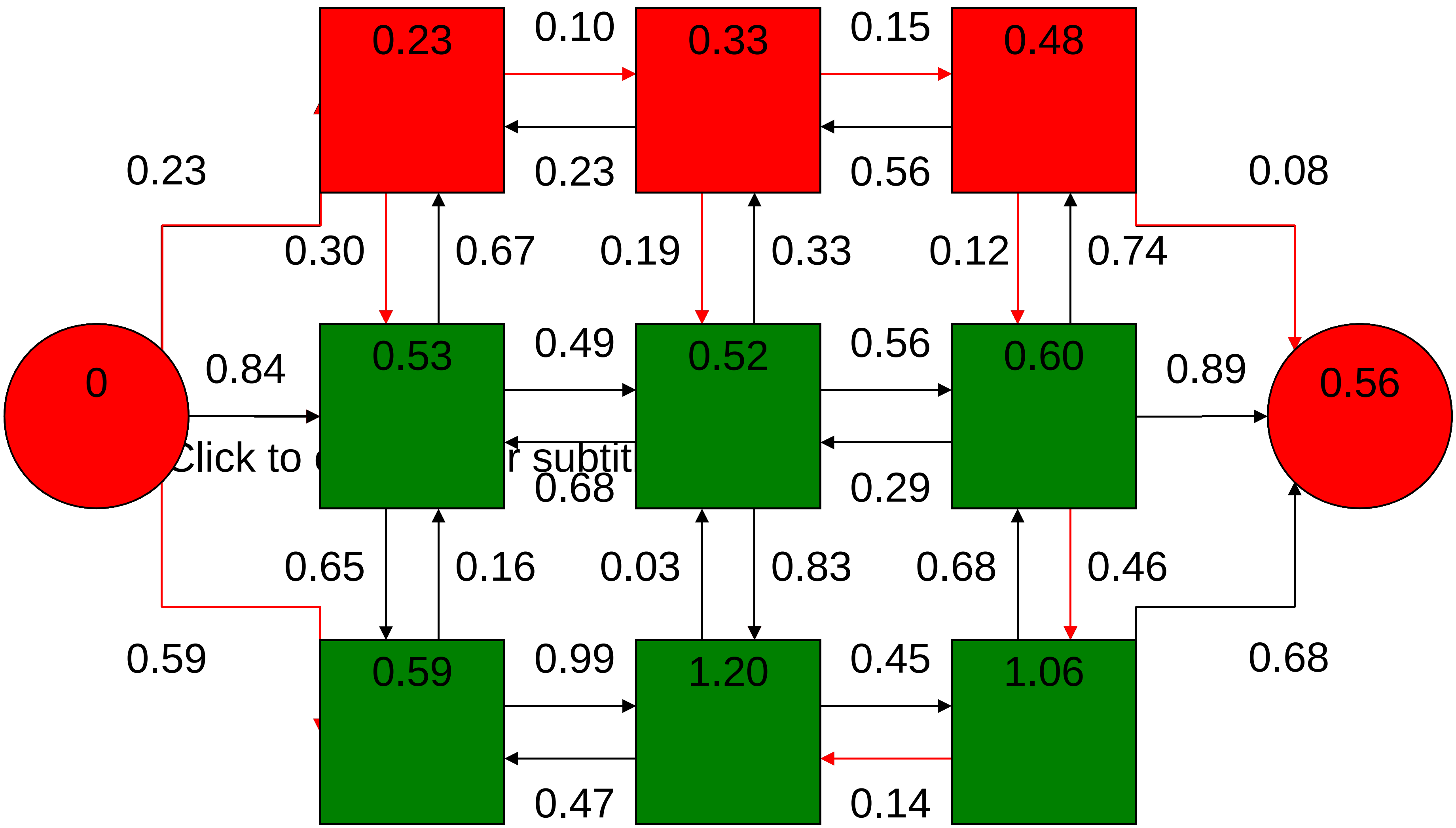
3) Optimal paths extracted from energy landscapes generated with weak disorder are self-affine and belong to the same universality class of directed polymers [Schwartz, Nazaryev & Havlin, *Phys. Rev. E* (1998)].

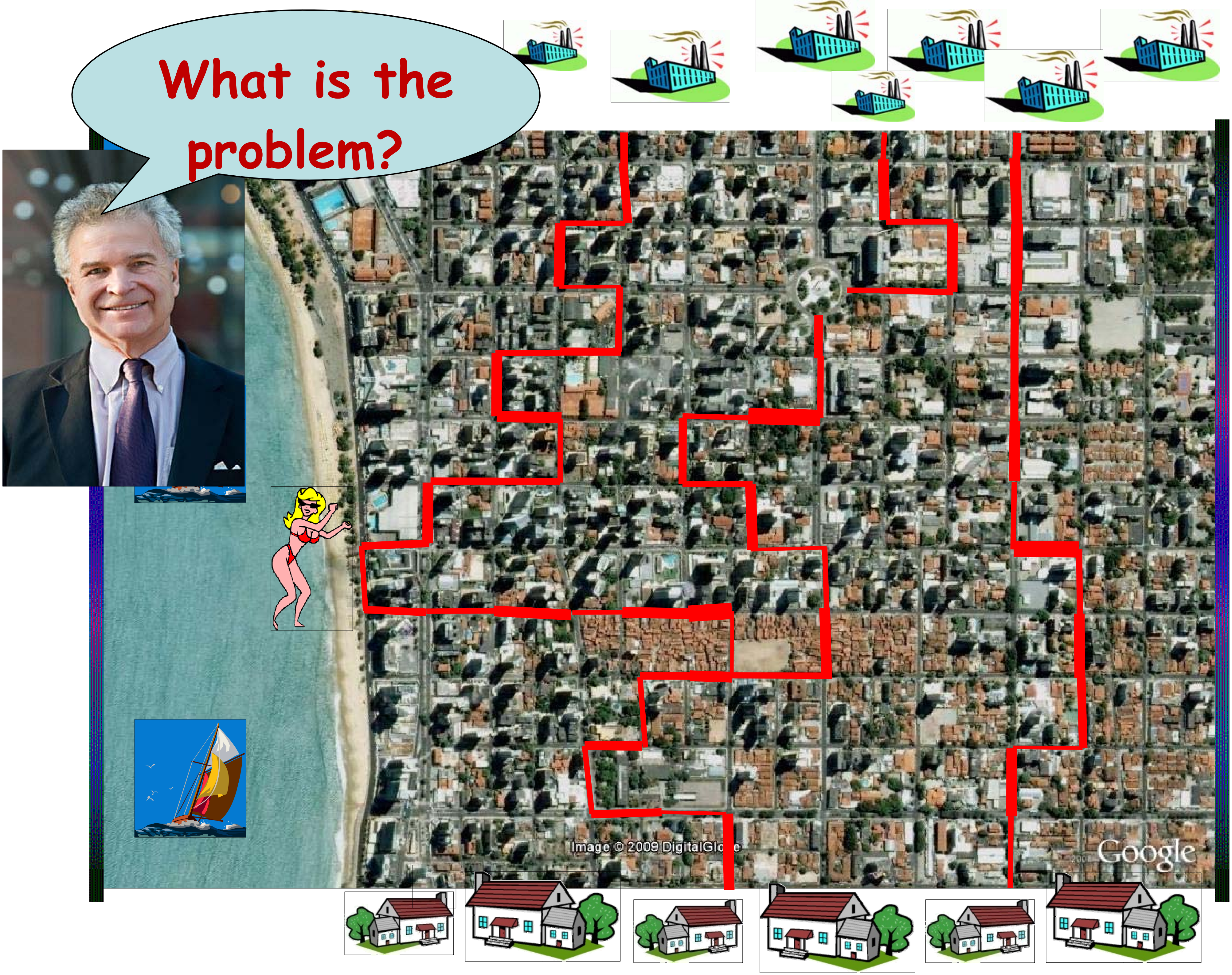
4) In the strong disorder limit, optimal paths are self-similar with fractal dimensions given by Df≈1.22 and 1.43 in two and three-dimensions, respectively [Cieplak, Maritan & Banavar, *Phys. Rev. Lett.* (1994), (1996); Porto, Havlin, Schwarzer & Bunde, *Phys. Rev. Lett.* (1997)].

5) In complex networks, the role of disorder is to destroy the essential small-world behavior of the system [Albert & Barabasi, *Rev. Mod. Phys.* (2002); Braunstein, Buldyrev, Cohen, Havlin & Stanley, *Phys. Rev. Lett.* (2003)].

The Dijkstra Algorithm

Dijkstra, *Num. Math.* (1959)




What is the problem?
Image © 2009 DigitalGlobe
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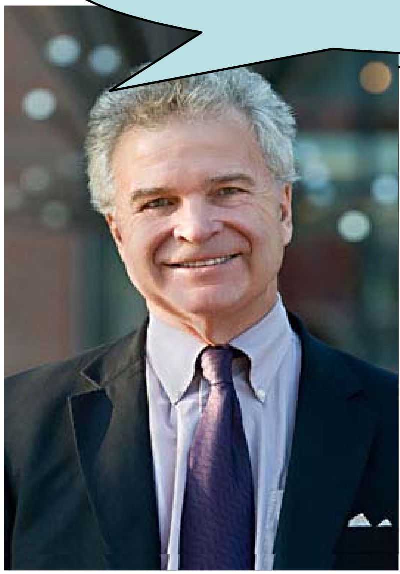
What do you do?



Two important questions arise:

1) How and when will the transportation network collapse?

2) What is the role of disorder on the performance of the transportation network?

We perform numerical simulations:

- Square lattice of size **L** with fixed BC's at the top and bottom and periodic BC's in the transversal direction.

- Disorder is introduced by assigning to each site **i** an energy **εi** given by:

$$\varepsilon_i = \exp[\beta(p_i - 1)]$$

where **pi** is a random variable uniformly distributed in **[0,1]**. This is

$$P(\varepsilon_i) \sim 1/\varepsilon_i$$

$$\varepsilon_{\max} = e^{\beta}$$

Algorithm

Oliveira, Moreira, Herrmann & JSA, submitted (2009)

1) The Dijkstra algorithm [Dijkstra, *Num. Math.* (1959)] is used to calculate the first OP connecting the bottom to the top of the network;

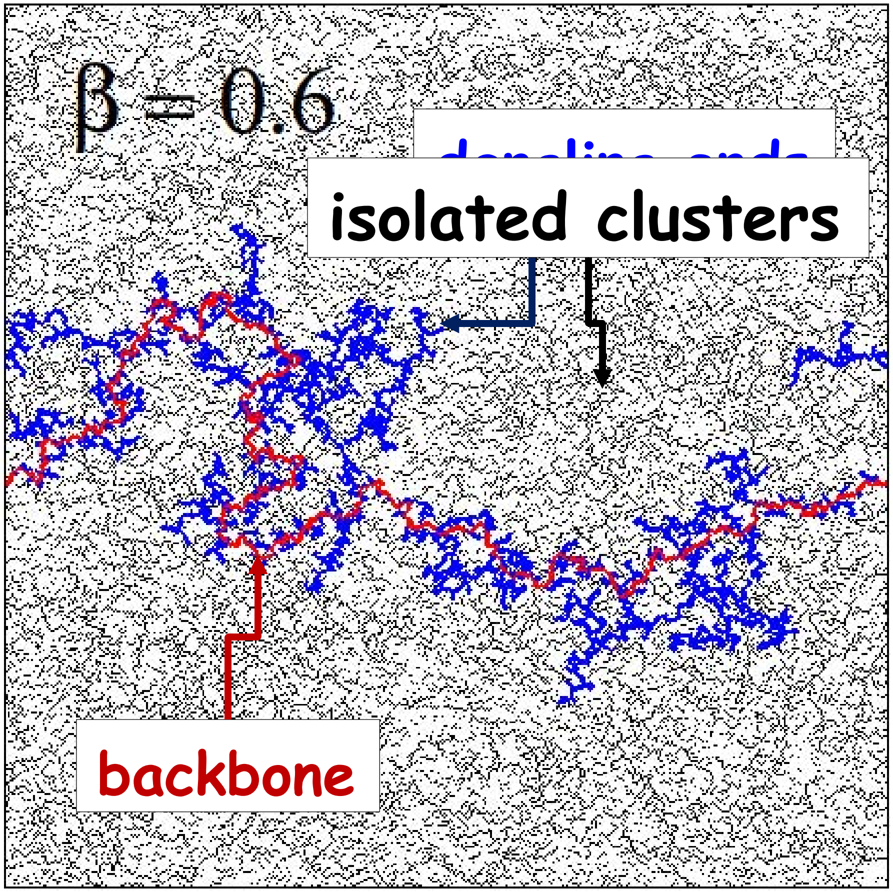
2) The site in the OP having the highest energy is permanently blocked (i.e., a "micro-crack" is formed);

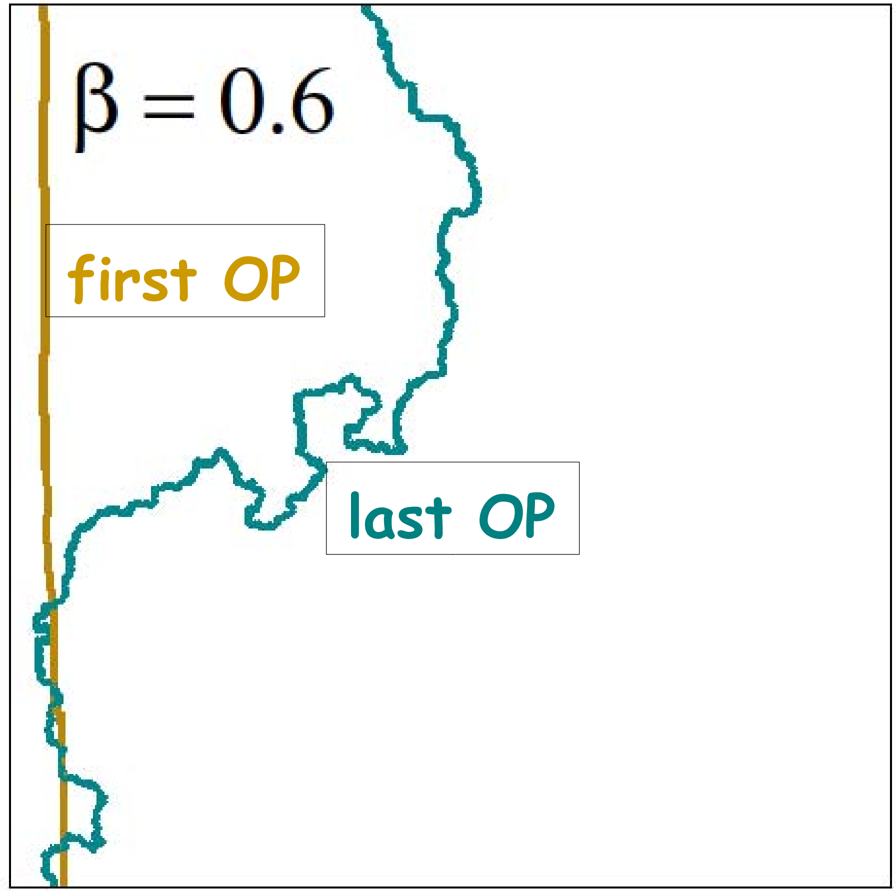
3) The next OP is calculated, from which the highest energy site is again removed and so on, and so forth;

4) The process continues iteratively until the system is disrupted, i.e., we can no longer find any path connecting bottom to top.

$\beta = 0.6$ $\beta = 6.0$ $\beta = 60.0$

Results: weak disorder

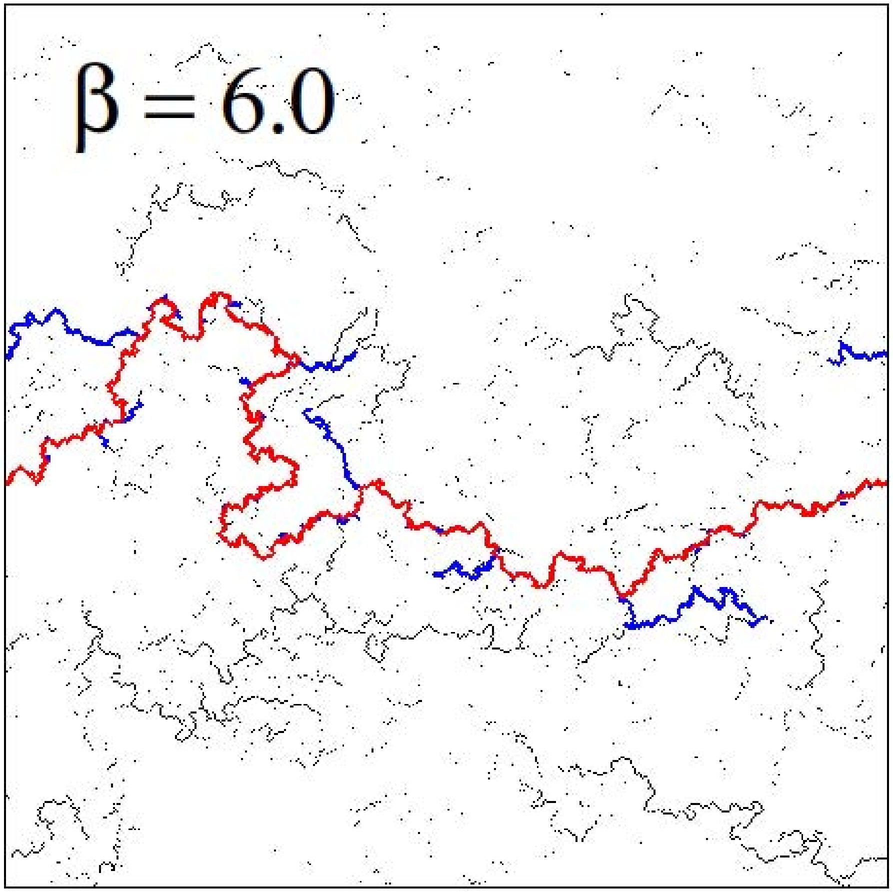


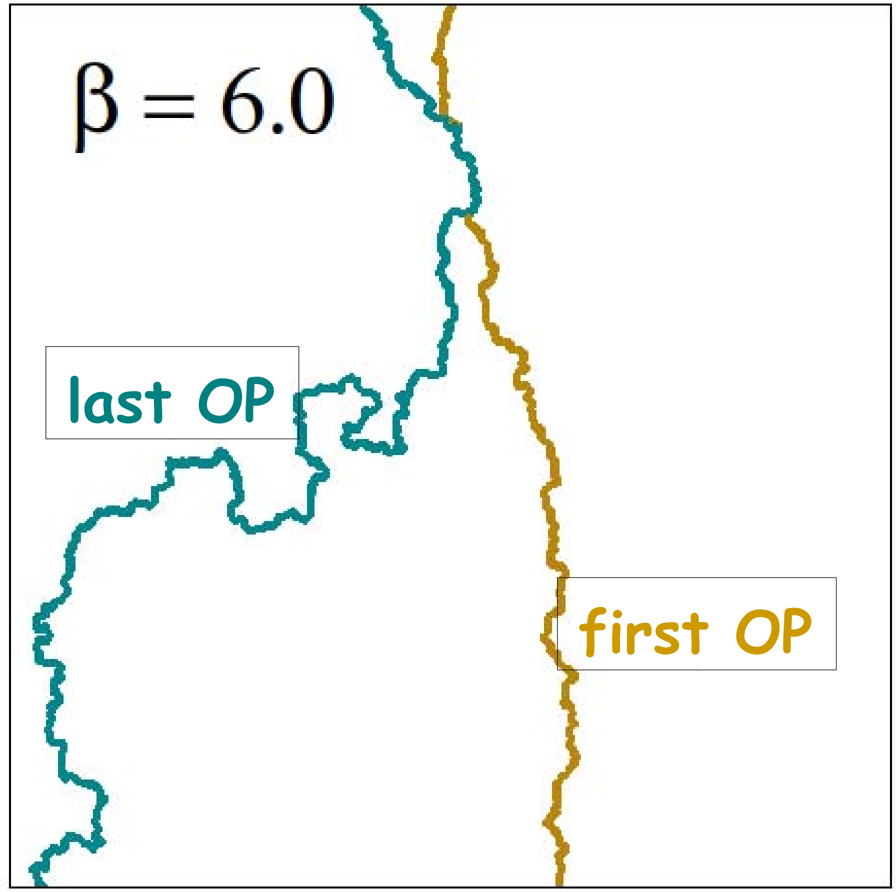


OPC elements

OPC fracture

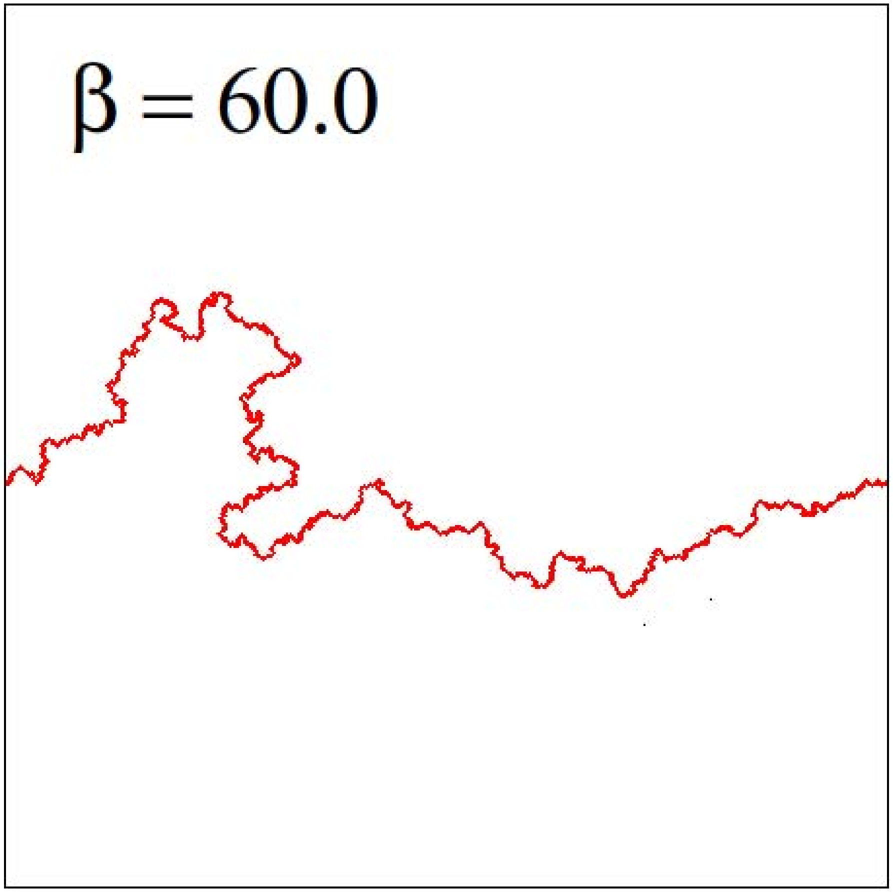
Results: intermediate disorder

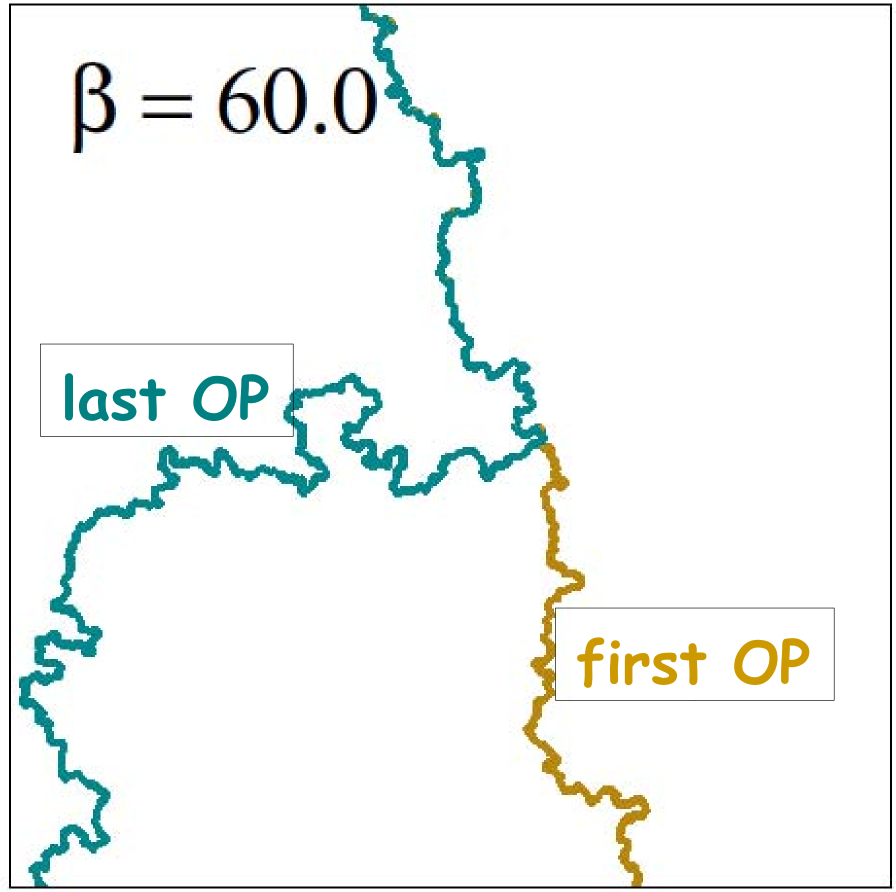




OPC elements
- OPC fracture
  - backbone (preserved!!)
  - dangling ends (reduced)
- isolated clusters (reduced)

Results: strong disorder





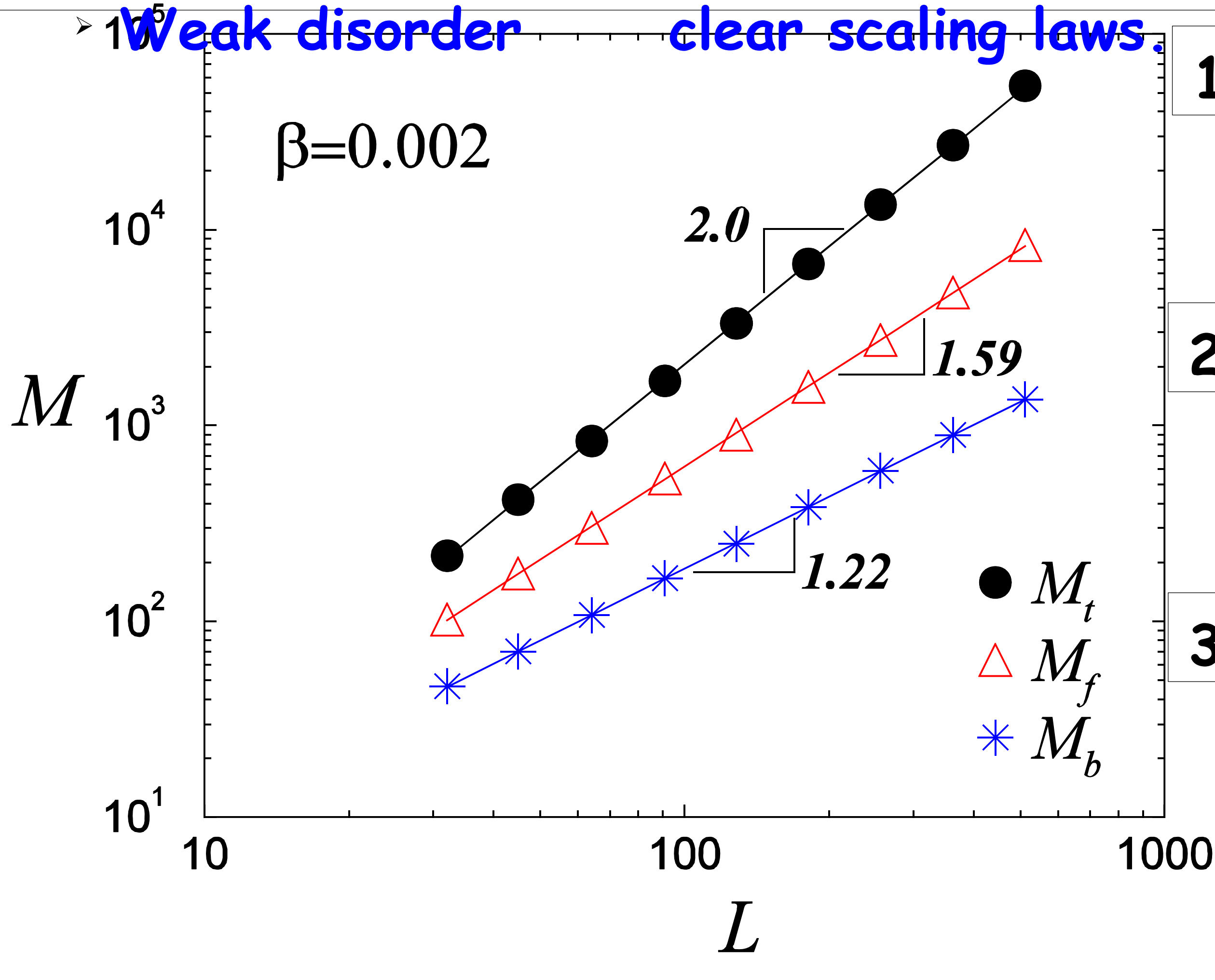
OPC elements

- OPC fracture
  - backbone (preserved!!)
  - dangling ends (disappeared)
- isolated clusters (practically disappeared)

Quantitative Results

- Simulations with 1000 realizations of lattices for each different size $32 \leq L \leq 512$ and distinct values of the disorder parameter β.

- Weak disorder $\Rightarrow$ clear scaling laws.



1) $M_b \sim L^{D_b}$ (backbone)

$$with \quad D_b = 1.22 \pm 0.02$$

2) $M_f \sim L^{D_f}$ (OPC fracture)

$$with \quad D_f = 1.59 \pm 0.02$$

3) $M_t \sim L^{D_t}$ (all cracks)

$$with \quad D_t = 2.00 \pm 0.01$$

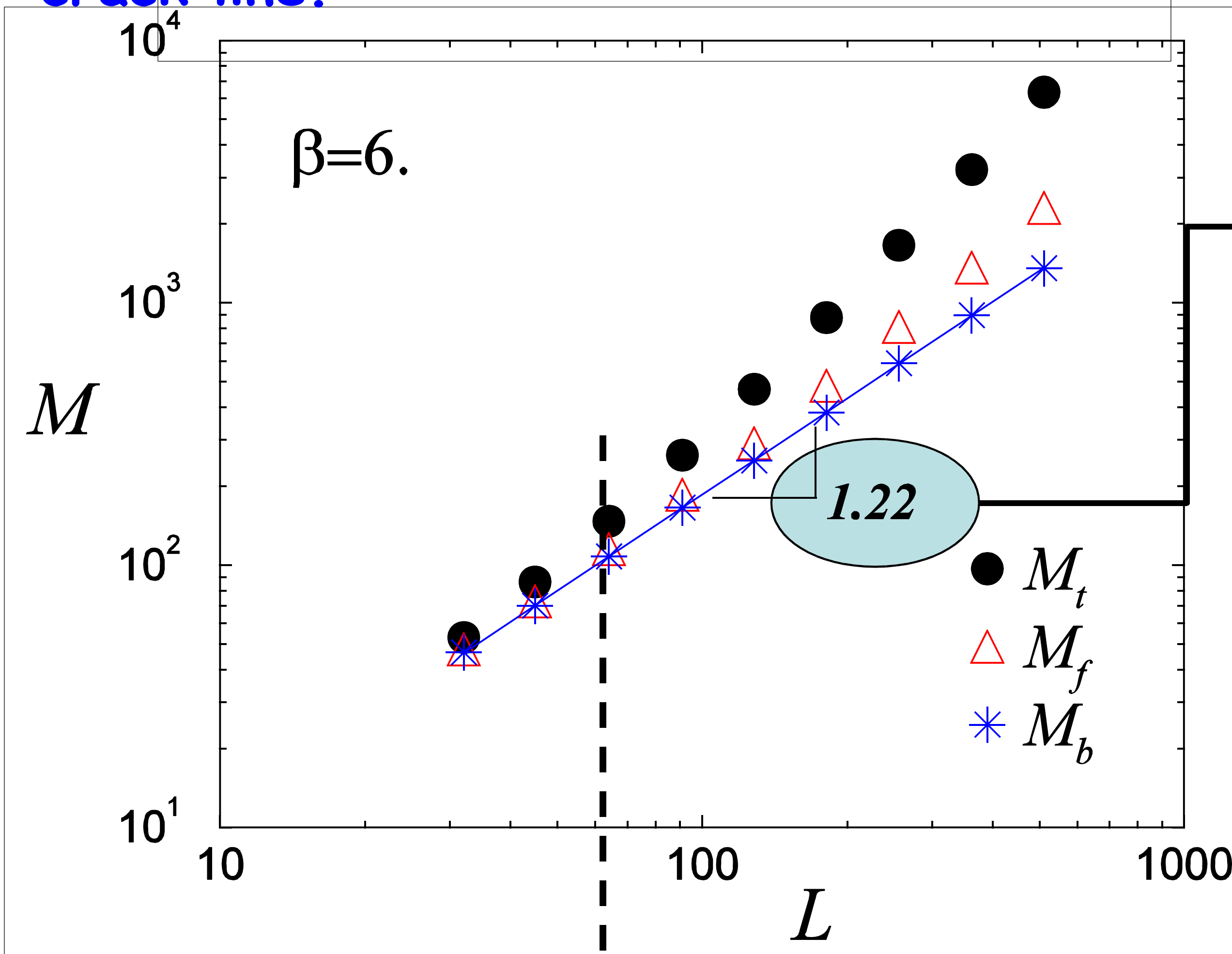
Quantitative Results

- Transition from weak to strong disorder.

$\Rightarrow$

- The stronger the disorder (small L or high β), the smaller is the number of final blocked sites, more localized in a singly-connected crack line.

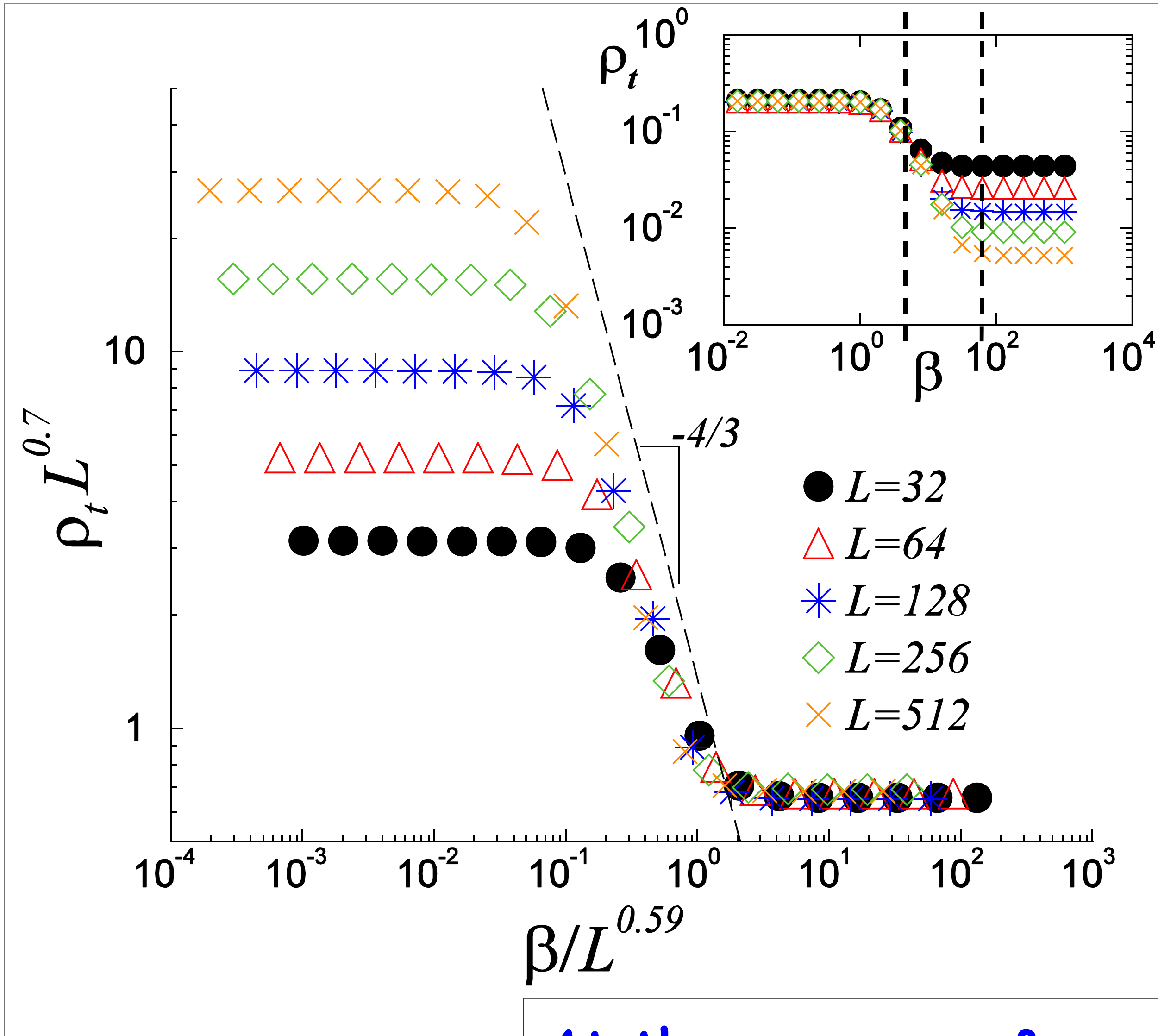
weak disorder strong disorder



1) OP's under strong disorder
2) Disordered Polymers
3) Strands in IP
4) paths on MST's

Transition from weak to strong disorder

weak disorder | strong disorder



Three Regimes

i) small β

$\rho_t \sim const.$

ii) intermediate β

$\rho_t(\beta) \sim \beta^{-\theta}$

$\theta \approx 4/3$

iii) large β

with

$\rho_t(L) \sim L^{D_b-2}$

$D_b \approx 1.22$

At the crossover βx we obtain, with

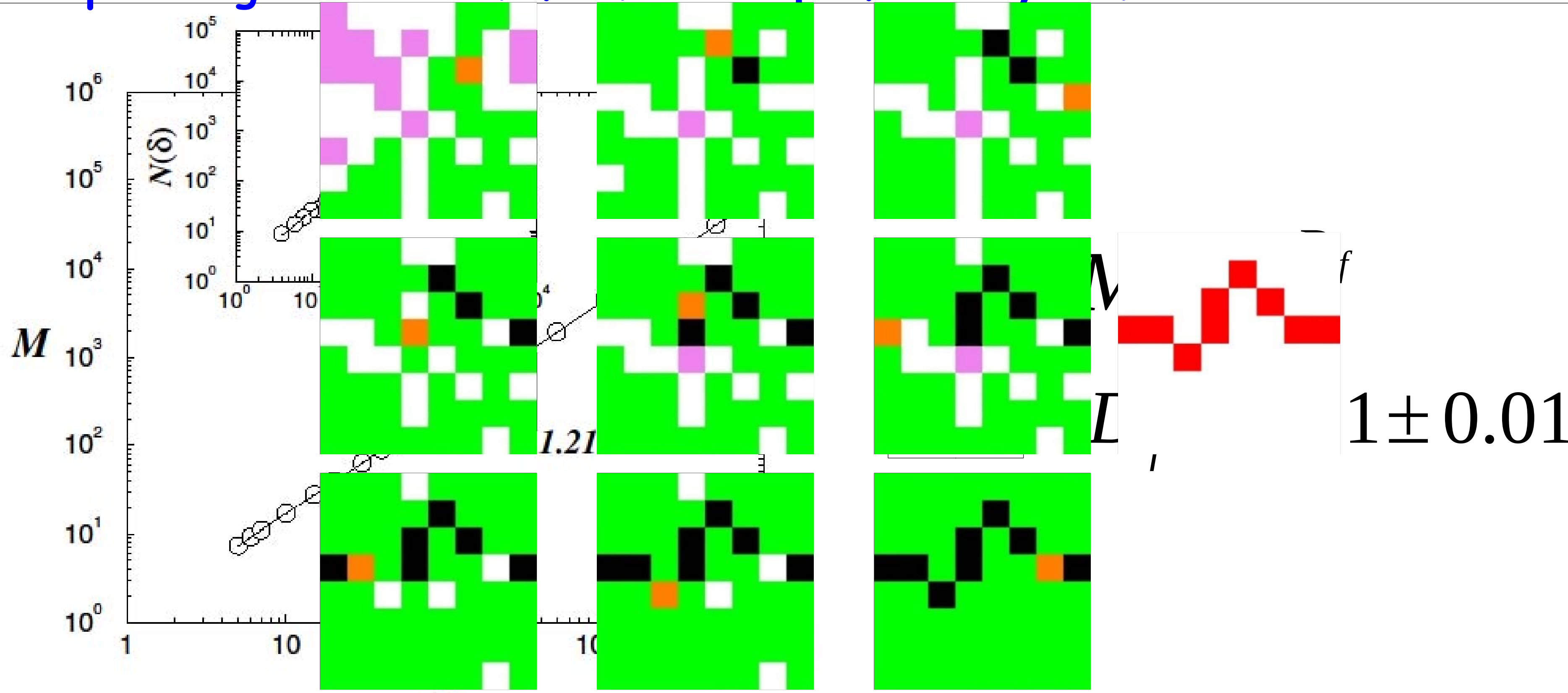
$$\beta_\times^{-\theta} \sim L^{D_b-2} \quad \Rightarrow \quad \beta_\times \sim L^{(2-D_b)/\theta}$$

A closely related problem: Critical Path Cracks (CPC)

Fehr, JSA, Cunha, daSilva, Herrmann, Kadau, Moukarzel & Oliveira, submitted (2008)

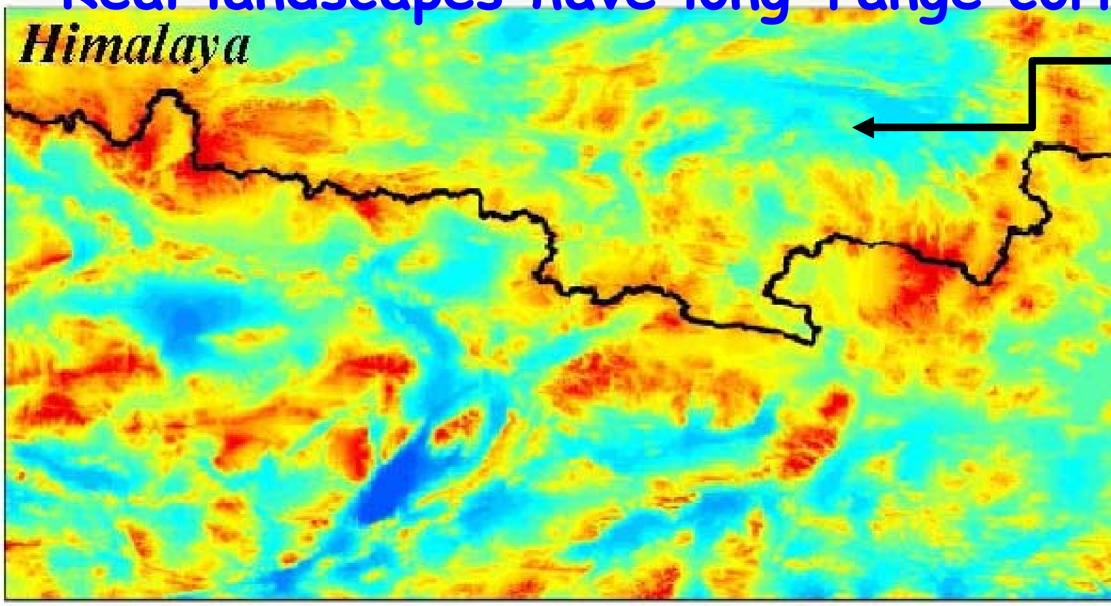
Algorithm:

1) As in percolation, sites are sequentially allocated in a lattice up to the point in which there is a "spanning cluster" connecting bottom to top;
2) The last added site, which is a red site, is permanently blocked;
3) More (non-visited) sites are sequentially added until another "spanning cluster" is formed. The last added site is again permanently blocked and so on, and so forth;
4) The process continues until there is connected cluster of blocked sites separating the bottom from the top of the system.

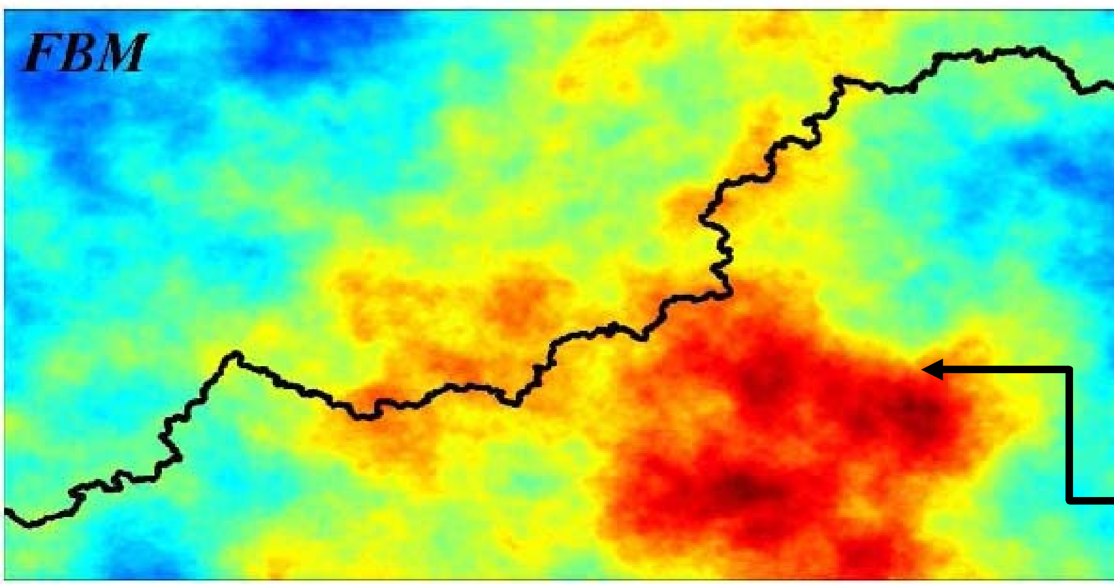


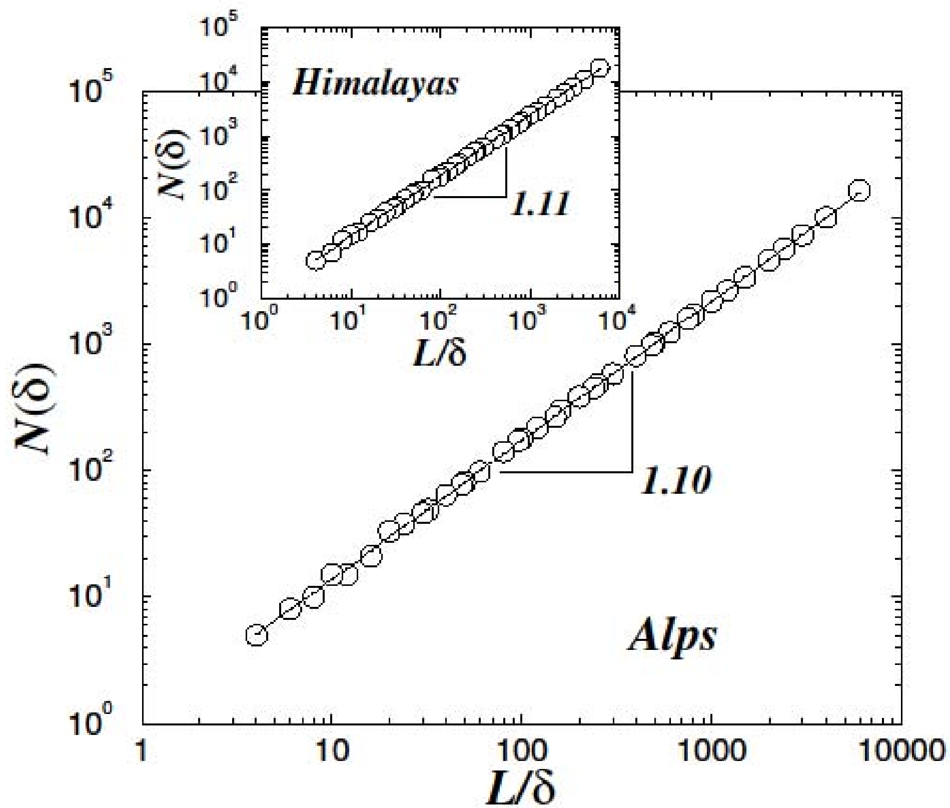
Watersheds in Real Landscapes

- CPC's and water drainage divides (i.e., watersheds) are identical.
- Real landscapes have long-range correlati



Real Landscape
Digital Elevation Maps





Synthetic Landscape
Fractional Brownian Motion

Conclusions

- The backbone of the fracture constituted of OPC's is apparently (not proved) disorder independent. It is also a self-similar object with fractal dimension Db≈1.22.
- This dimension is (statistically) similar to the ones obtained for OP's under strong disorder [Schwartz et al., *PRE* (1998)], Disordered Polymers [Cieplak et al., *PRL* (1994)], strands in Invasion Percolation [Cieplak et al., *PRL* (1996)], and paths on Minimum Spanning Trees [Dobrin et al., *PRL* (2001)].
- The fracture generated with CPC's has a fractal dimension of Df≈1.21.
- The role of disorder is to dramatically reduce the total number of blocked sites before the system collapses:

weak disorder $\Rightarrow \quad M_t \sim L^2$

strong disorder $\Rightarrow \quad M_t \rightarrow M_b \sim L^{1.22}$

- This information can be used to improve a given transportation network or in the design of systems with enhanced performance.

Obrigado!