Exact solution for the Anisotropic Ornstein-Uhlenbeck Process

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Active Matter models commonly consider particles with overdamped dynamics subject to a force (speed) with constant modulus and random direction. Some models include also random noise in particle displacement (Wiener process) resulting in a diffusive motion at short time scales. On the other hand, Ornstein-Uhlenbeck processes consider Langevin dynamics for the particles velocity and predict a motion that is not diffusive at short time scales. However, experiments show that migrating cells may present a varying speed as well as a short-time diffusive behavior. While Ornstein-Uhlenbeck processes can describe the varying speed, Active Mater models can explain the short-time diffusive behavior. Isotropic models cannot explain both: short-time diffusion renders instantaneous velocity ill-defined, hence impeding dynamical equations that consider velocity time-derivatives. On the other hand, both models apply for migrating biological cells and must, in some limit, yield the same observable predictions. Here we propose and analytically solve an Anisotropic Ornstein-Uhlenbeck process that considers polarized particles, with a Langevin dynamics for the particle´s movement in the polarization direction while following a Wiener process for displacement in the orthogonal direction. Our characterization provides a theoretically robust way to compare movement in dimensionless simulations to movement in dimensionful experiments, besides proposing a procedure to deal with inevitable finite precision effects in experiments or simulations.