Scaling properties of *d*-dimensional complex networks

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The area of networks is very interdisciplinary and exhibits many applications in several fields of science. Nevertheless, there are few studies focusing on geographically located *d*-dimensional networks. In this work, we study the scaling properties of a wide class of *d*-dimensional geographically located networks which grow with preferential attachment involving Euclidean distances through $r_{ij}^{-\alpha_A}$ (α_A >=0). We have numerically analyzed the time evolution of the connectivity of sites, the average shortest path, the degree distribution entropy, and the average clustering coefficient for d = 1, 2, 3, 4 and typical values of α_A. Remarkably enough, virtually all the curves can be made to collapse as functions of the scaled variable α_A/d. These observations confirm the existence of three regimes. The first one occurs in the interval $\alpha_A/d \in [0, 1]$; it is non-Boltzmannian with very-long range interactions in the sense that the degree distribution is a q exponential with q constant and above unity. The critical value $\alpha_A/d = 1$ that emerges in many of these properties is replaced by $\alpha_A/d = 1/2$ for the β exponent which characterizes the time evolution of the connectivity of sites. The second regime is still non-Boltzmannian, now with moderately-long-range interactions, and reflects in an index q monotonically decreasing with α_A/d increasing from its critical value to a characteristic value $\alpha_A/d > 5$. Finally, the third regime is Boltzmannian like (with $q \sim= 1$) and corresponds to short-range interactions.