Statistical characterization of discrete conservative systems: The web map

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We numerically study the two-dimensional, area preserving, web map. When the map is governed by ergodic behavior, it is, as expected, correctly described by Boltzmann-Gibbs statistics, based on the additive entropic functional $S_{BG}[p(x)] = -k \int dx\, p(x) \ln p(x)$. In contrast, possible ergodicity breakdown and transitory sticky dynamical behavior drag the map into the realm of generalized q-statistics, based on the nonadditive entropic functional $S_q[p(x)] = k\frac{1-\int dx\, [p(x)]^q}{q-1}$ ($q \in \mathcal{R}; S_1 = S_{BG}$). We statistically describe the system (probability distribution of the sum of successive iterates, sensitivity to the initial condition, and entropy production per unit time) for typical values of the parameter that controls the ergodicity of the map. For small (large) values of the external parameter K, we observe q-Gaussian distributions with $q = 1.935\ldots$ (Gaussian distributions), like for the standard map. In contrast, for intermediate values of K, we observe a different scenario, due to the fractal structure of the trajectories embedded in the chaotic sea. Long-standing non-Gaussian distributions are characterized in terms of the kurtosis and the box-counting dimension of chaotic sea.

References

[1] Ruiz, Tirnakli, Borges, Tsallis; Phys. Rev. E **96**, 042158 (2017).