Scaling Properties of d-dimensional Complex Networks

S.G.A. Brito[a], T. C. Nunes[b], L.R. da Silva[b,d], C. Tsallis[c,d]

[a] International Institute of Physics, *IIP*, UFRN, Natal, Brazil

[b] Departamento de Física Teórica e Experimental, UFRN, Natal, Brazil

[c] CBPF , Rio de Janeiro, Brazil and Santa Fe Institute, Santa Fe, USA

[d] National Institute of Science and Technology of Complex Systems, Rio de Janeiro, Brazil

Studies in complex networks are quite current and promote the integration of several areas of knowledge. It has been verified in previous research that nonextensive statistical mechanics is a very suitable approach to describe complex networks when there is long-range interactions between their constituents. At the thermodynamic limit the degree distribution is of the form $P(k) \propto e_q^{-k/\kappa}$, where e_q^Z is the q-exponential defined by $e_q^z \equiv [1 + (1-q)z]^{1/(1-q)}$ which optimizes the non-additive entropy S_q (when $q \to 1$, the Boltzmann-Gibbs entropy is recovered). We have performed a study of d-dimensional geographic networks (Natal Model) which grow with preferential attachment involving Euclidean distance by introducing the term $r^{-\alpha_A}$ $(\alpha_A \geq 0)$ into the preferential attachment rule. Given the connection between complex networks and q-statistics, we numerically verified (for $d = 1, 2, 3$ and 4) that the degree distributions exhibit, for both q and κ, universal dependencies with respect to the variable α_A/d. In addition, the limit $q = 1$ is quickly reached when $\alpha_A/d \to \infty$. We also verified that other properties of the network have universal dependencies with respect to α_A/d such as: average shortest path $\langle l \rangle$, dynamic exponent β (from the connectivity time evolution of the sites) and the degree distribution entropy S_q.