

# Multiplicative Exchange-Investment Models: Properties and Applications

Cristian F. Moukarzel<sup>1,\*</sup>

<sup>1</sup> *CINVESTAV del IPN, Departamento de Física Aplicada,  
97310 Mérida, Yucatán, México.*

In Econophysics studies, Exchange-Investment models are thought to encompass the two most important ingredients underlying the dynamics of wealth in a collaborative environment of economic agents, namely: a) Conservative Exchange, modeling stochastic commercial interactions among agents, and, b) wealth-modifying Investment, modeling the random evolution of individual wealths due to independent productive processes. Investment is usually represented as a random multiplicative process acting independently on each agent's wealth, and its properties are well understood in isolation. Exchange processes, on the other hand, allow for a more complex set of descriptions. Many of the stochastic exchange models studied so far are additive, that is, a random amount of wealth is added to one agent and subtracted from the other. Additive exchange models usually give rise to near-exponential distributions of wealth, and their use is justified more in terms of analytic solvability than because of their being realistic descriptions of commercial interaction. In recent years, a stochastic exchange model has emerged, under the name of Yard-Sale, that offers an acceptably realistic, yet still simple, description of commercial exchange processes. In Yard-Sale, a random *fraction* of the wealth of one agent is transferred between agents. This entails the interaction with a multiplicative character, which in turn is responsible for several non-intuitive properties. In this talk, recent advances in the field of Yard-Sale models of commercial exchange, both in and out of equilibrium, will be described. Conditions can be analytically given, under which the trade dynamics gives rise to condensation of wealth onto a few agents in the long run, even if each trade favors the poorest agent on average. The resulting time-dependent wealth distribution can be derived asymptotically in a trade-investment context, by solving the Langevin equation for agent wealths under random multiplicative noise and Yard-Sale trade. Comparison of our results against multi-period real-world data for the distribution of per-capita gross domestic products (gdp), suggests that the world-distribution of gdp can be adequately described by our model. Our study suggests that prevailing trade conditions appear to be systematically biased in favor of wealthier countries, thus leading to concentration of wealth in the long run. This implies that wealth distributions are not stationary but converging to a condensed state, under the presently prevailing economic exchange conditions among countries.

---

\* cristian@mda.cinvestav.mx