Intransitivity, coexistence and synchronization in four species cyclic games

Alessandra F. Lütz,[1] Annette Cazaubiel,[2,1] Charlotte Rulquin,[2,1] and Jeferson J. Arenzon[1,*]
[1]*Instituto de Física, Universidade Federal do Rio Grande do Sul, Porto Alegre RS, Brazil*
[2]*École Normale Supérieure, Paris, France*

The Rock-Paper-Scissors (RPS) game and its generalizations with $\mathcal{S} > 3$ species are well studied models for cyclically interacting populations. Four is, however, the minimum number of species that, by allowing other interactions beyond the single, cyclic loop, breaks both the full intransitivity of the food graph and the one predator, one prey symmetry. Ref. [1] have shown the existence, on a square lattice, of two distinct phases, with either four or three coexisting species. In both phases, each agent is eventually replaced by one of its predators but these strategy oscillations remain localized as long as the interactions are short ranged. Distant regions may be either out of phase or cycling through different food web subloops (if any). Upon replacing a minimum fraction Q of the short range interactions by long range ones, there is a Hopf bifurcation and global oscillations become stable [2]. Surprisingly, to build such long distance, global synchronization, the four species coexistence phase requires less long range interactions than the three species phase, while one would naively expect the contrary. Moreover, deviations from highly homogeneous conditions ($\chi = 0$ or 1) increase Q_c and the more heterogeneous is the food web, the harder the synchronization is. By further increasing Q, while the three species phase remains stable, the four species one has a transition to an absorbing, single species state. The existence of a phase with global oscillations for $\mathcal{S} > 3$, when the interaction graph has multiple subloops and several possible local cycles, lead to the conjecture that global oscillations are a general characteristic, even for large, realistic food webs. We exemplify with a prey-predator model whose defensive and attacking behaviors, respectively, are either individual or collective [3,4]. Depending on the parameters of the model, the effective interactions between the four possible strategy combinations (e.g., grouped preys vs single predator) can be mapped on a four strategies cyclic model as described above.

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*Electronic address: arenzon@if.ufrgs.br