Stationary probabilistic distributions of the Hénon-Heiles Hamiltonian system

Ernesto P. Borges and Daniel Prado Martins Fernandes
Instituto de Física, Universidade Federal da Bahia, Salvador-BA 40170-115 Brazil

q-Gaussian distributions have been found in both dissipative and conservative low-dimensional maps. We have now observed q-Gaussian distributions in Poincaré maps of the Hénon-Heiles Hamiltonian system. Specification of the Poincaré section and the total energy reduces the four-dimension phase space of the Hamiltonian to a two-dimension problem. The 2-D phase space is composed by strong chaos regions, characterized by positive Lyapunov exponents, and weak chaos regions, with zero Lyapunov exponents. Gaussian distributions of fluctuations of momenta are identified when the initial conditions are taken within the strong chaos regions, while q-Gaussian behavior emerges when the initial conditions are taken within the weak chaos regions. This scenario is similar to the one recently noticed for the standard map.