MAIS *q*-EXPONENCIAIS E *q*-GAUSSIANAS. E PARA NÃO DIZER QUE NÃO FALEI DE FLORES, A TEORIA DE GRANDES DESVIOS.

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POSTULATED ENTROPIC FUNCTIONALS

	$p_i = \frac{1}{W} \quad (\forall i)$ **equiprobability**	$\forall p_i \; (0 \le p_i \le 1)$ $\left(\sum_{i=1}^{W} p_i = 1 \right)$
BG entropy ($q = 1$)	$k \ln W$	$-k \sum_{i=1}^{W} p_i \ln p_i$
Entropy *Sq* (q *real*)	$k \frac{W^{1-q} - 1}{1-q}$	$k \frac{1 - \sum_{i=1}^{W} p_i^q}{q-1}$

additive

- Concave
- Extensive
- Lesche-stable
- Finite entropy production per unit time
- Pesin-like identity (with largest entropy production)
- Composable
- Topsoe-factorizable
- Amari-Ohara-Matsuzoe conformally invariant geometry
- Van-Barnafoldi-Biro-Urmossy universal thermostat independence

nonadditive (if $q \neq 1$)

Possible generalization of Boltzmann-Gibbs statistical mechanics

[C.T., J. Stat. Phys. **52**, 479 (1988)]

TYPICAL SIMPLE SYSTEMS:

$$\text{e.g.,}\quad W(N) \propto \mu^N \quad (\mu > 1)$$

Short-range space-time correlations

Markovian processes (short memory), Additive noise

Strong chaos (positive maximal Lyapunov exponent), **Ergodic**, Riemannian geometry

Short-range many-body interactions, weakly quantum-entangled subsystems

Linear/homogeneous Fokker-Planck equations, Gausssians

→ Boltzmann-Gibbs entropy (additive)

→ Exponential dependences (Boltzmann-Gibbs weight, ...)

TYPICAL COMPLEX SYSTEMS:

$$\text{e.g.,}\quad W(N) \propto N^\rho \quad (\rho > 0)$$

Long-range space-time correlations

Non-Markovian processes (long memory), Additive and multiplicative noises

Weak chaos (zero maximal Lyapunov exponent), **Nonergodic**, Multifractal geometry

Long-range many-body interactions, strongly quantum-entangled sybsystems

Nonlinear/inhomogeneous Fokker-Planck equations, *q*-Gaussians

→ Entropy S*q* (nonadditive)

→ *q*-exponential dependences (asymptotic power-laws)

ADDITIVITY: O. Penrose, *Foundations of Statistical Mechanics: A Deductive Treatment* (Pergamon, Oxford, 1970), page 167

An entropy is additive if, for any two probabilistically independent systems A and B,

$$S(A+B) = S(A) + S(B)$$

Therefore, since

$$S_q(A+B) = S_q(A) + S_q(B) + (1-q)\, S_q(A)\, S_q(B) \,,$$

S_{BG} and S_q^{Renyi} $(\forall q)$ are additive, and S_q $(\forall q \neq 1)$ is nonadditive .

EXTENSIVITY:

Consider a system $\Sigma \equiv A_1 + A_2 + \ldots + A_N$ made of N (not necessarily independent) identical elements or subsystems A_1 and A_2, ..., A_N.

An entropy is extensive if

$$0 < \lim_{N\to\infty} \frac{S(N)}{N} < \infty \,, \quad i.e., \;\; S(N) \propto N \quad (N \to \infty)$$

EXTENSIVITY OF THE ENTROPY $(N \to \infty)$

If $W(N) \sim \mu^N \quad (\mu > 1)$

$$\Rightarrow S_{BG}(N) = k_B \ln W(N) \propto N \qquad \text{OK!}$$

If $W(N) \sim N^\rho \quad (\rho > 0)$

$$\Rightarrow S_q(N) = k_B \ln_q W(N) \propto [W(N)]^{1-q} \propto N^{\rho(1-q)}$$

$$\Rightarrow S_{q=1-1/\rho}(N) \propto N \qquad \text{OK!}$$

If $W(N) \sim \nu^{N^\gamma} \quad (\nu > 1;\ 0 < \gamma < 1)$

$$\Rightarrow S_\delta(N) = k_B \left[\ln W(N)\right]^\delta \propto N^{\gamma\,\delta}$$

$$\Rightarrow S_{\delta=1/\gamma}(N) \propto N \qquad \text{OK!}$$

IMPORTANT: $\mu^N >> \nu^{N^\gamma} >> N^\rho$ if $N >> 1$

PHYSICAL REVIEW D VOLUME 13, NUMBER 2 15 JANUARY 1976

Black holes and thermodynamics*

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(Received 30 June 1975)

A black hole of given mass, angular momentum, and charge can have a large number of different unobservable internal configurations which reflect the possible different initial configurations of the matter which collapsed to produce the hole. The logarithm of this number can be regarded as the entropy of the black hole and is a measure of the amount of information about the initial state which was lost in the formation of the black hole. If one makes the hypothesis that the entropy is finite, one can deduce that the black holes must emit thermal radiation at some nonzero temperature. Conversely, the recently derived quantum-mechanical result that black holes do emit thermal radiation at temperature $\kappa\hbar/2\pi kc$, where κ is the surface gravity, enables one to prove that the entropy is finite and is equal to $c^3A/4G\hbar$, where A is the surface area of the event horizon or boundary of the black hole. Because black holes have negative specific heat, they cannot be in stable thermal equilibrium except when the additional energy available is less than 1/4 the mass of the black hole. This means that the standard statistical-mechanical canonical ensemble cannot be applied when gravitational interactions are important. Black holes behave in a completely random and time-symmetric way and are indistinguishable, for an external observer, from white holes. The irreversibility that appears in the classical limit is merely a statistical effect.

When entropy does not seem extensive

Earlier speculations about the entropy of black holes has prompted an ingenious calculation suggesting that entropy may (in special circumstances) be the same inside and outside an arbitrary boundary.

Everybody who knows about entropy knows that it is an extensive property, like mass or enthalpy. That, of course, is why the entropy of some substance will be quoted as so much per gram, or mole. If you then take two grams, or two moles, of the same material under the same conditions, the entropy will be twice as much. And there should be no confusion about the units; the simple Carnot definition of a change of entropy in a reversible process is the heat transfer divided by the absolute temperature, so that the units of entropy are simply those of energy divided by temperature, joules per degree (kelvin) in the SI system. The definitions of the Gibbs and Helmholtz free energies would be dimensionally discordant for that reason were it not that entropy (S) always turns up multiplied by temperature T. So much will readily be agreed.

Of course, there is more than that to entropy, which is also a measure of disorder. Everybody also agrees on that. But how is disorder measured? By the number of ways in which the constituents of some material (the atoms and molecules) can be rearranged without changing its properties and without energetic consequences. But now there comes a snag.

Like any extensive property, the combined entropy of two separate chunks of material should be the *sum* of the two entropies, but the number of rearrangements of the combined system must be the *product* of the numbers of ways in which the two parts separately can be rearranged. How to reconcile that with extensivity? By supposing entropy is proportional not to the number of rearrangements (technically called 'complexions'), but with the logarithm thereof. And because entropy decreases as disorder increases, the constant of proportionality must be a negative (real) number.

From that it follows that $S = S_0 - K\log N$, where K is a positive constant with the dimensions of entropy, N is a number (without dimensions) measuring disorder and S_0 is an arbitrary constant entropy. All that is simply a precis of the standard introductory chapter in statistical mechanics textbooks, most of which go on to show how to calculate the properties of assemblages of, say, diatomic molecules from a knowledge of their individual behaviour. Because the number of complexions of a particular state of an assemblage is invariably a function of the number (n) of molecules it contains, usually in the form of $n!$, because n is usually large and because $\log(n!)$ can then be approximated by $n\log n$, the extensive property of entropy then follows simply from the appearance of the leading factor n: entropy is proportional to the number of molecules.

That is what the textbooks say. It also makes sense of what is known of the thermodynamics of the real world. In a sample of a diatomic gas, for example, there are vibrations (one) and rotations (two) as well as three rectilinear degrees of freedom. But the problem is to tell how the energy available is distributed among the different degrees of freedom. The arithmetic simplifies marvellously because (in this case) each molecule and each of its degrees of freedom is independent. The best measure of disorder works out at $N = Z^n$, where n is the number of molecules, and where Z, which must be a

A bit of quantum mechanics goes into the argument as well, notably the notion of the density matrix — an artificially constructed operator (on quantum states) that is well suited to the discussion of systems in which one part (say the black hole) is singled out for attention while the remainder (the Universe outside it) is dealt with in less detail, perhaps because some averaging process is appropriate, or because the whole problem may not be calculable at all. (In Dirac's notation, the density matrix corresponding to some state of the whole Universe would be represented as $|1\rangle\langle 1|$, where "1" is simply the name for a particular state of the Universe.) What matters, where entropy is concerned, is that the density matrix, like all matrices, has eigenvalues from which the entropy can be calculated.

So imagine that the Universe is partitioned into two parts by means of a closed boundary of some kind and filled with a

dealt with explicitly, as other entropy calculations are made. And that could be exceedingly important.

John Maddox

Tackled by
- Jacob D. Bekenstein
- Stephen W. Hawking
- Gary W. Gibbons
- Gerard 't Hooft
- Leonard Susskind
- Michael J. Duff
- Juan M. Maldacena
- Thanu Padmanabhan
- Robert M. Wald

and many others

When entropy does not seem extensive
John Maddox, Nature **365**, 103 (1993)

Everybody who knows about entropy knows that it is an extensive property, like mass or enthalpy. [...] Of course, there is more than that to entropy, which is also a measure of disorder. Everybody also agrees on that. But how is disorder measured? [...] So why is the entropy of a black hole proportional to the square of its radius, and not to the cube of it? To its surface area rather than to its volume?

RAPID COMM

PHYSICAL REVIEW D **73,** 121701(R) (2006)

How robust is the entanglement entropy-area relation?

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We revisit the problem of finding the entanglement entropy of a scalar field on a lattice by tracing over its degrees of freedom inside a sphere. It is known that this entropy satisfies the area law—entropy proportional to the area of the sphere—when the field is assumed to be in its ground state. We show that the area law continues to hold when the scalar field degrees of freedom are in generic coherent states and a class of squeezed states. However, when excited states are considered, the entropy scales as a lower power of the area. This suggests that, for large horizons, the ground state entropy dominates, whereas entropy due to excited states gives power-law corrections. We discuss possible implications of this result to black hole entropy.

The area (as opposed to volume) proportionality of BH entropy has been an intriguing issue for decades.

PHYSICAL REVIEW D **83,** 064034 (2011)

Ideal gas in a strong gravitational field: Area dependence of entropy

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We study the thermodynamic parameters like entropy, energy etc. of a box of gas made up of indistinguishable particles when the box is kept in various static background spacetimes having a horizon. We compute the thermodynamic variables using both statistical mechanics as well as by solving the hydrodynamical equations for the system. When the box is far away from the horizon, the entropy of the gas depends on the volume of the box except for small corrections due to background geometry. As the box is moved closer to the horizon with one (leading) edge of the box at about Planck length (L_p) away from the horizon, the entropy shows an area dependence rather than a volume dependence. More precisely, it depends on a small volume $A_\perp L_p/2$ of the box, up to an order $\mathcal{O}(L_p/K)^2$ where $A_\perp$ is the transverse area of the box and K is the (proper) longitudinal size of the box related to the distance between leading and trailing edge in the vertical direction (i.e. in the direction of the gravitational field). Thus the contribution to the entropy comes from only a fraction $\mathcal{O}(L_p/K)$ of the matter degrees of freedom and the rest are suppressed when the box approaches the horizon. Near the horizon all the thermodynamical quantities behave as though the box of gas has a volume $A_\perp L_p/2$ and is kept in a Minkowski spacetime. These effects are: (i) purely kinematic in their origin and are independent of the spacetime curvature (in the sense that the Rindler approximation of the metric near the horizon can reproduce the results) and (ii) observer dependent. When the equilibrium temperature of the gas is taken to be equal to the horizon temperature, we get the familiar $A_\perp/L_p^2$ dependence in the expression for entropy. All these results hold in a $D+1$ dimensional spherically symmetric spacetime. The analysis based on methods of statistical mechanics and the one

lead to the same result

Thus the extensive property of entropy no longer holds and one can check that it does not hold even in the weak field limit discussed above when $L \gg \lambda$ that is, when gravitational effects subdue the thermal effects along the direction of the gravitational field.

SINCE THE PIONEERING BEKENSTEIN-HAWKING RESULTS, PHYSICALLY MEANINGFUL EVIDENCE HAS ACCUMULATED (e.g., HOLOGRAPHIC PRINCIPLE) WHICH MANDATES THAT

$$\ln W_{black\ hole} \propto \text{AREA}$$

THIS IS PERFECTLY ADMISSIBLE AND MOST PROBABLY CORRECT.

HOWEVER,

IS THIS QUANTITY THE THERMODYNAMICAL ENTROPY???

ENTROPIES

$$S_{BG} = k_B \sum_{i=1}^{W} p_i \ln \frac{1}{p_i} \quad \rightarrow \textit{additive}$$

$$S_q = k_B \sum_{i=1}^{W} p_i \ln_q \frac{1}{p_i} \quad (S_1 = S_{BG}) \quad \rightarrow \textit{nonadditive} \text{ if } q \neq 1$$ C. T. (1988)

$$S_\delta = k_B \sum_{i=1}^{W} p_i \left(\ln \frac{1}{p_i} \right)^\delta \quad (S_1 = S_{BG}) \quad \rightarrow \textit{nonadditive} \text{ if } \delta \neq 1$$ C. T. (2009)

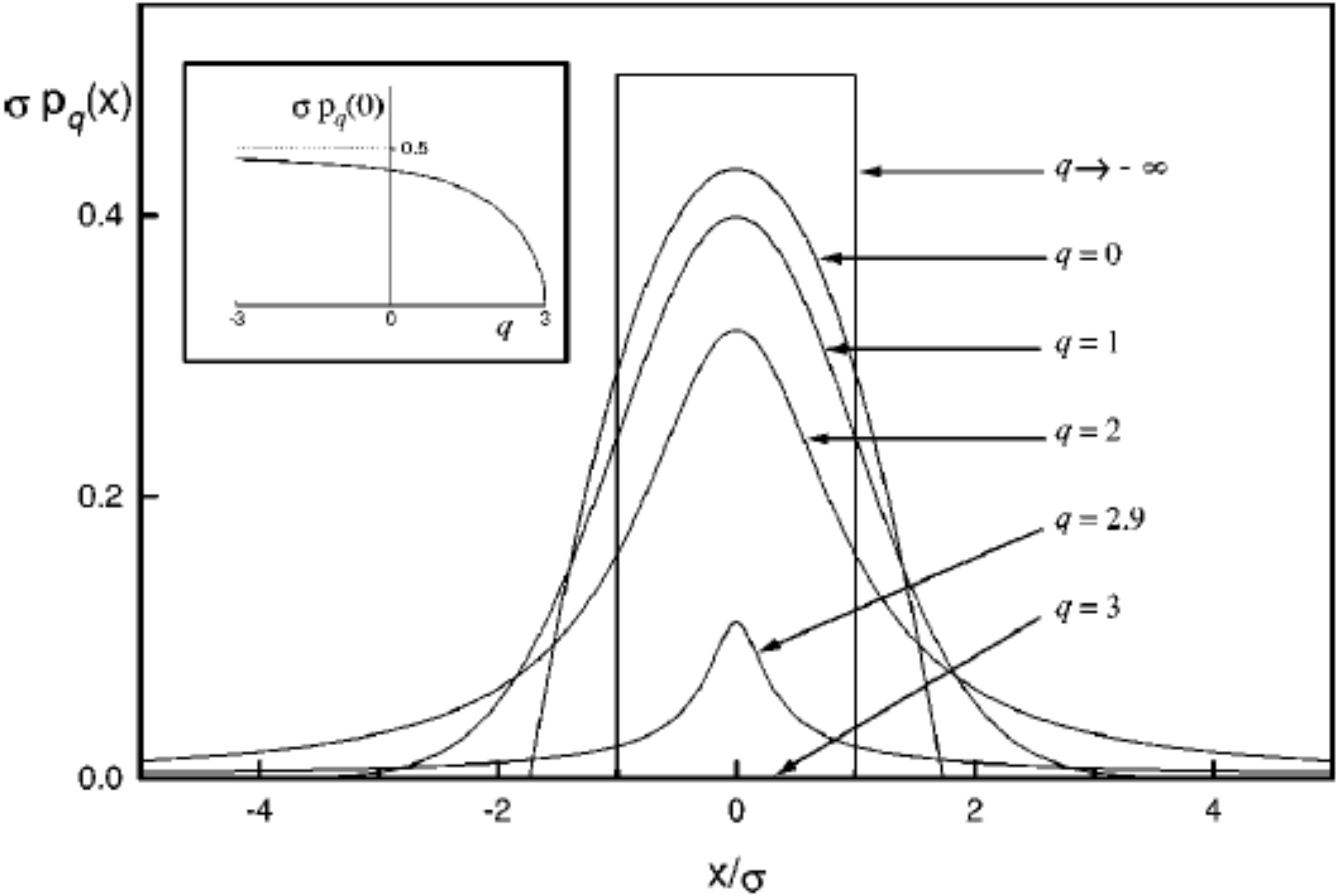
$$S_{q,\delta} = k_B \sum_{i=1}^{W} p_i \left(\ln_q \frac{1}{p_i} \right)^\delta \quad (S_{q,1} = S_q;\ S_{1,\delta} = S_\delta;\ S_{1,1} = S_{BG})$$ C. T. (2011)

$$\rightarrow \textit{nonadditive} \text{ if } (q,\delta) \neq (1,1)$$

C. T. and L.J.L. Cirto (2012), 1202.2154 [cond-mat.stat-mech]

SYSTEMS $W(N)$	ENTROPY S_{BG} (ADDITIVE)	ENTROPY S_q $(q \neq 1)$ (NONADDITIVE)	ENTROPY S_δ $(\delta \neq 1)$ (NONADDITIVE)
$\sim \mu^N$ $(\mu > 1)$	EXTENSIVE	NONEXTENSIVE	NONEXTENSIVE
$\sim N^\rho$ $(\rho > 0)$	NONEXTENSIVE	EXTENSIVE	NONEXTENSIVE
$\sim \nu^{N^\gamma}$ $(\nu > 1;$ $0 < \gamma < 1)$	NONEXTENSIVE	NONEXTENSIVE	EXTENSIVE

q-GAUSSIANS: $p_q(x) \propto e_q^{-(x/\sigma)^2} \equiv \frac{1}{\left[1+(q-1)\,(x/\sigma)^2\right]^{\frac{1}{q-1}}}$ $(q<3)$



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DOI 10.1007/s00032-008-0087-y

Milan Journal of Mathematics

On a q-Central Limit Theorem Consistent with Nonextensive Statistical Mechanics

Sabir Umarov, Constantino Tsallis and Stanly Steinberg

JOURNAL OF MATHEMATICAL PHYSICS **51**, 033502 (2010)

Generalization of symmetric α-stable Lévy distributions for $q>1$

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A. Plastino and M.C. Rocca, Physica A and Milan J Math (2012)

CENTRAL LIMIT THEOREM

$N^{1/[\alpha(2-q)]}$*-scaled attractor* $\mathrm{F}(x)$ *when summing* $N \to \infty$ q*-independent identical random variables*
with symmetric distribution $f(x)$ *with* $\sigma_Q \equiv \int dx\, x^2 [f(x)]^Q / \int dx\, [f(x)]^Q$ $\left(Q \equiv 2q-1,\ q_1 = \frac{1+q}{3-q} \right)$

	$q=1$ [*independent*]	$q \neq 1$ (*i.e.*, $Q \equiv 2q-1 \neq 1$) [*globally correlated*]
$\sigma_Q < \infty$ $(\alpha = 2)$	$\mathrm{F}(x) = \textit{Gaussian } G(x)$, *with same* σ_1 *of* $f(x)$ Classic CLT	$\mathrm{F}(x) = G_q(x) \equiv G_{(3q_1-1)/(1+q_1)}(x)$, *with same* σ_Q *of* $f(x)$ $G_q(x) \sim \begin{cases} G(x) & \text{if } \lvert x \rvert \ll x_c(q,2) \\ f(x) \sim C_q / \lvert x \rvert^{2/(q-1)} & \text{if } \lvert x \rvert \gg x_c(q,2) \end{cases}$ *with* $\lim_{q \to 1} x_c(q,2) = \infty$ S. Umarov, C. T. and S. Steinberg, Milan J Math 76, 307 (2008)
$\sigma_Q \to \infty$ $(0 < \alpha < 2)$	$\mathrm{F}(x) = \textit{Levy distribution } L_\alpha(x)$, *with same* $\lvert x \rvert \to \infty$ *behavior* $\mathrm{L}_\alpha(x) \sim \begin{cases} G(x) \\ \quad \text{if } \lvert x \rvert \ll x_c(1,\alpha) \\ f(x) \sim C_\alpha / \lvert x \rvert^{1+\alpha} \\ \quad \text{if } \lvert x \rvert \gg x_c(1,\alpha) \end{cases}$ *with* $\lim_{\alpha \to 2} x_c(1,\alpha) = \infty$ Levy-Gnedenko CLT	$\mathrm{F}(x) = L_{q,\alpha}$, *with same* $\lvert x \rvert \to \infty$ *asymptotic behavior* $L_{q,\alpha} \sim \begin{cases} G_{\frac{2(1-q)-\alpha(1+q)}{2(1-q)-\alpha(3-q)},\, \alpha}(x) \sim C^*_{q,\alpha} / \lvert x \rvert^{\frac{2(1-q)-\alpha(3-q)}{2(1-q)}} \\ \quad (\textit{intermediate regime}) \\ G_{\frac{2\alpha q-\alpha+3}{\alpha+1},\, 2}(x) \sim C^L_{q,\alpha} / \lvert x \rvert^{(1+\alpha)/(1+\alpha q-\alpha)} \\ \quad (\textit{distant regime}) \end{cases}$ S. Umarov, C. T., M. Gell-Mann and S. Steinberg J Math Phys 51, 033502 (2010)

International Journal of Bifurcation and Chaos, Vol. 22, No. 9 (2012) 1250208 (18 pages)

DOI: 10.1142/S0218127412502082

TIME-EVOLVING STATISTICS OF CHAOTIC ORBITS OF CONSERVATIVE MAPS IN THE CONTEXT OF THE CENTRAL LIMIT THEOREM

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CONSERVATIVE MC MILLAN MAP:

$$x_{n+1} = y_n$$

$$y_{n+1} = -x_n + 2\mu \frac{y_n}{1 + y_n^2} + \varepsilon y_n$$

$\mu \neq 0 \Leftrightarrow$ nonlinear dynamics

G. Ruiz, T. Bountis and C. T., Int J Bifurcat Chaos **22**, 1250208 (2012)

$(\mu, \varepsilon) = (1.6, 1.2)$ $(\lambda_{max} \approx 0.05)$

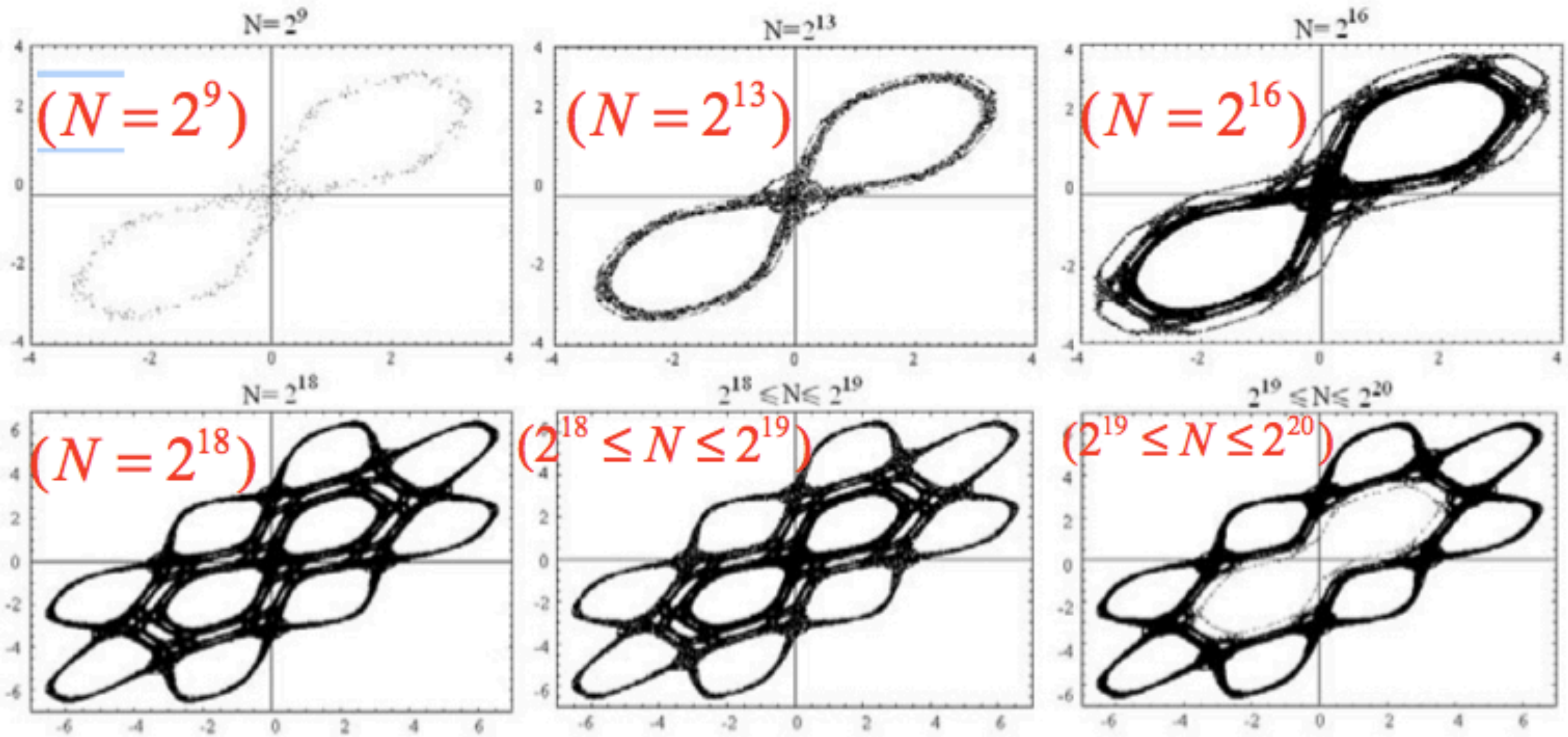
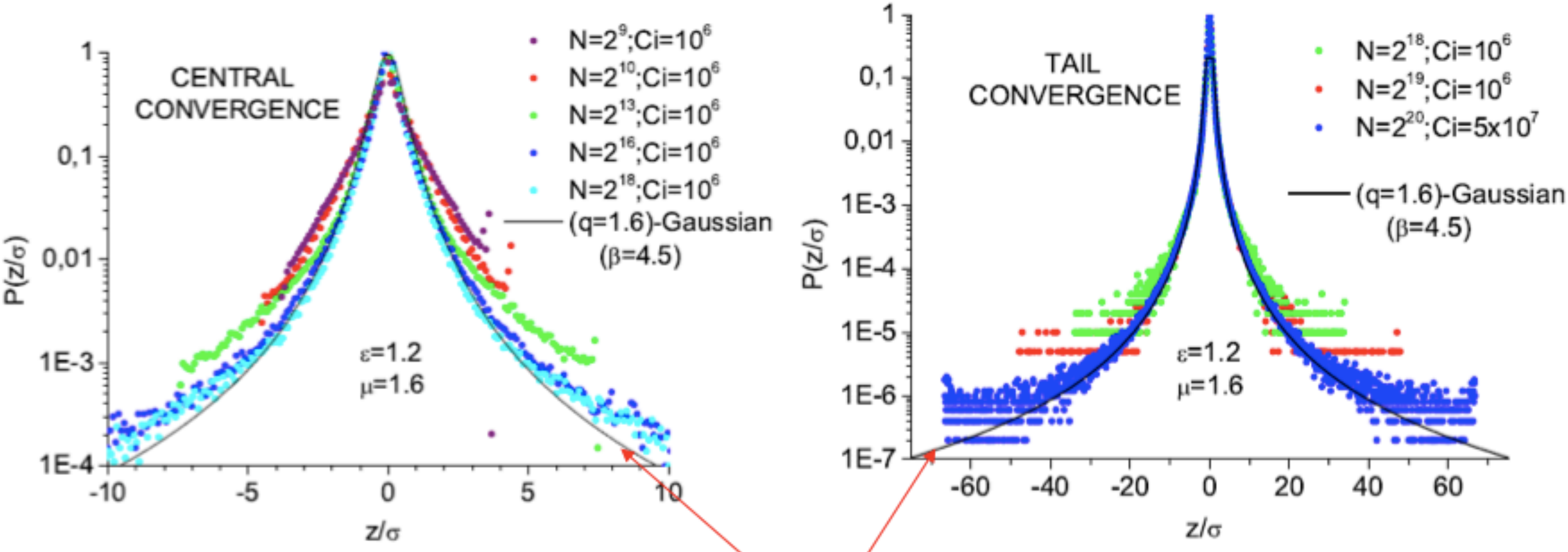


FIG. 10. Structure of phase space plot of Mc. Millan perturbed map for parameter values $\mu = 1.6$ and $\epsilon = 1.2$, starting form a randomly chosen initial condition in a square $(0, 10^{-6}) \times (0, 10^{-6})$, and for $i = 1 \ldots N$ $(N = 2^{10}, 2^{13}, N^{16}, N^{18})$ iterates.

G. Ruiz, T. Bountis and C. T., Int J Bifurcat Chaos **22**, 1250208 (2012)



$$p \propto e_q^{-\beta (z/\sigma)^2}$$

with $(q, \beta) = (1.6, 4.5)$

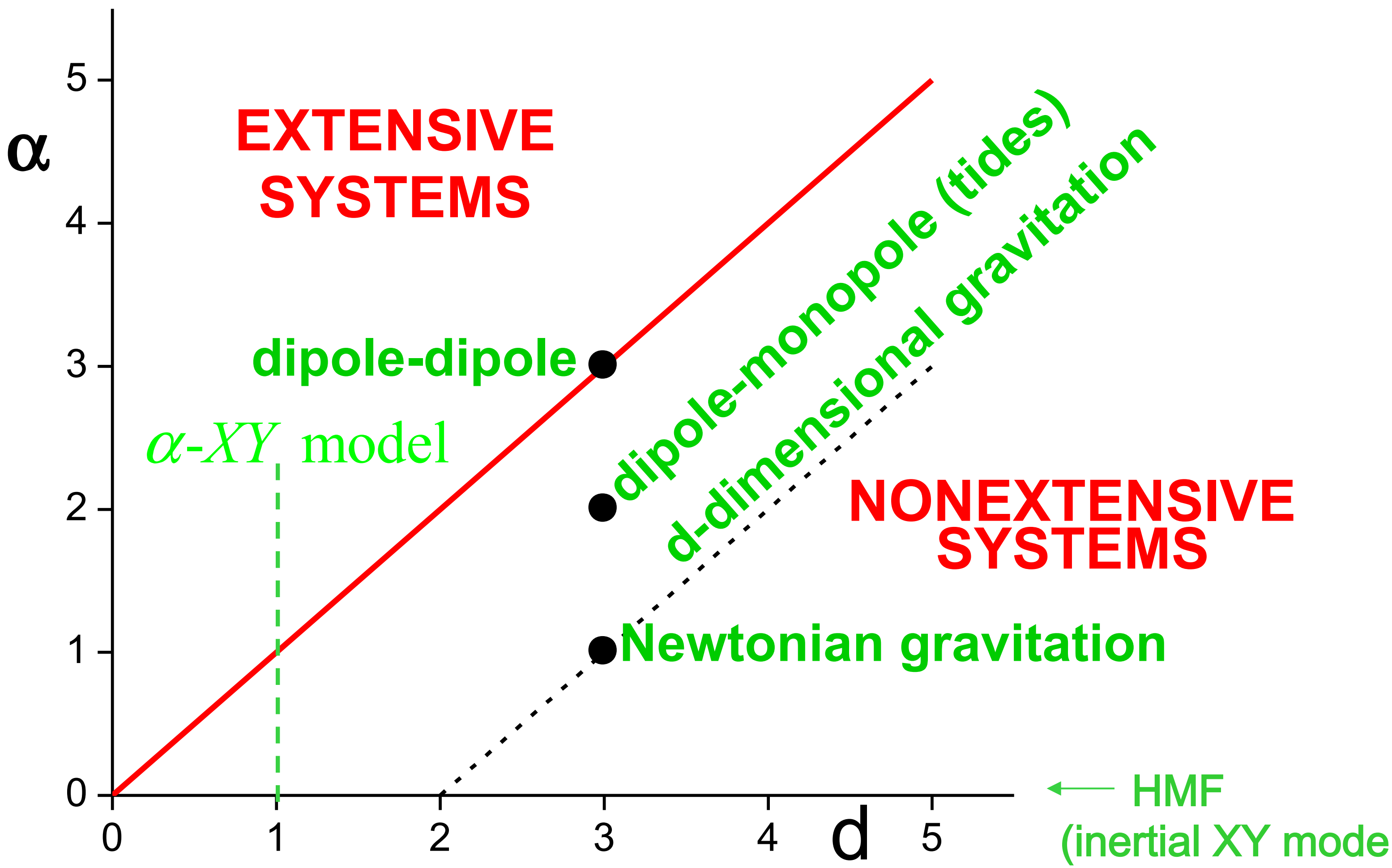
G. Ruiz, T. Bountis and C. T., Int J Bifurcat Chaos **22**, 1250208 (2012)

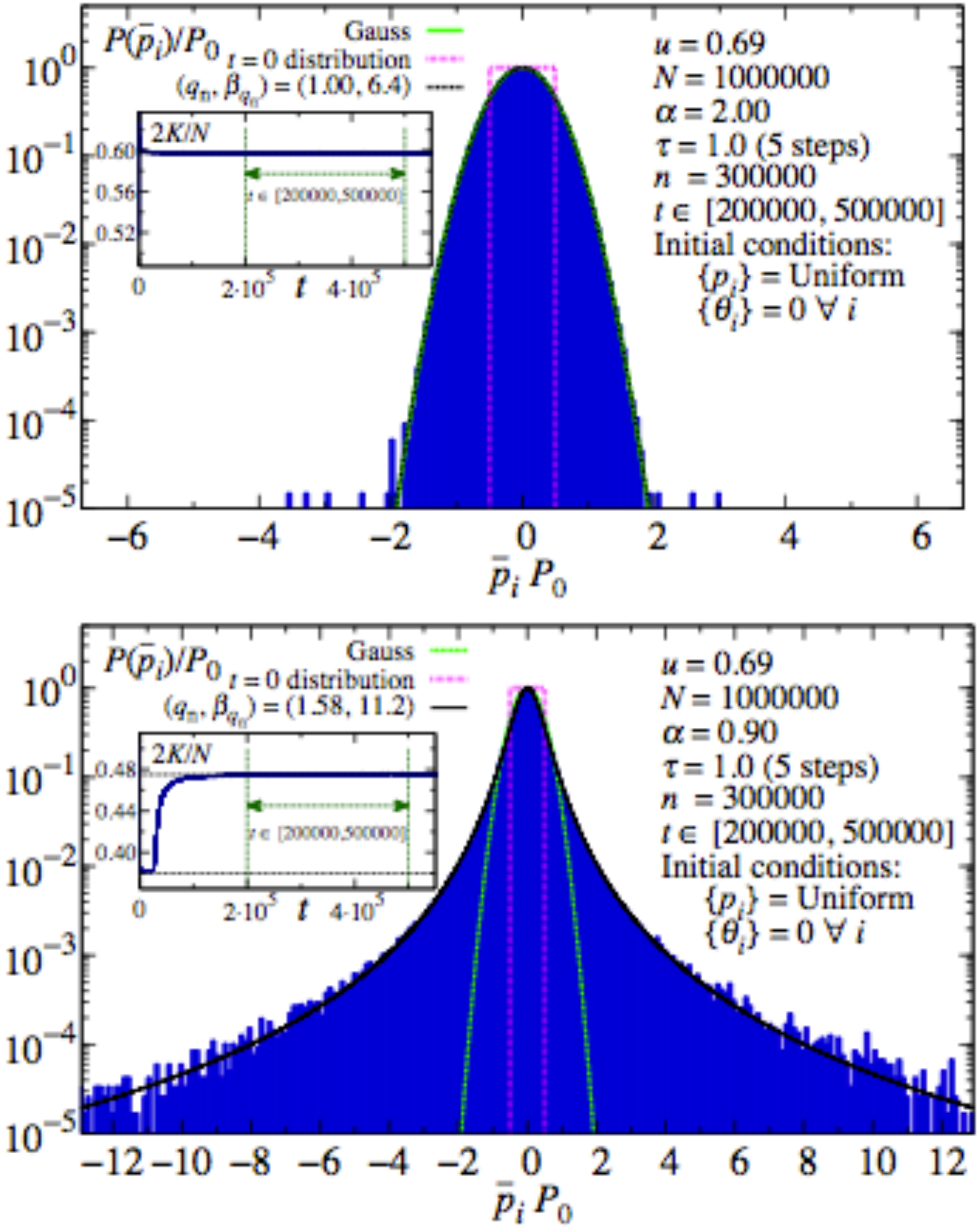
CLASSICAL LONG-RANGE-INTERACTING MANY-BODY HAMILTONIAN SYSTEMS

$$V(r) \sim -\frac{A}{r^{\alpha}} \quad (r \to \infty) \qquad (A > 0,\ \ \alpha \geq 0)$$

$$\textit{integrable if} \quad \alpha / d > 1 \quad (\textit{short-ranged})$$

$$\textit{non-integrable if} \quad 0 \leq \alpha / d \leq 1 \quad (\textit{long-ranged})$$





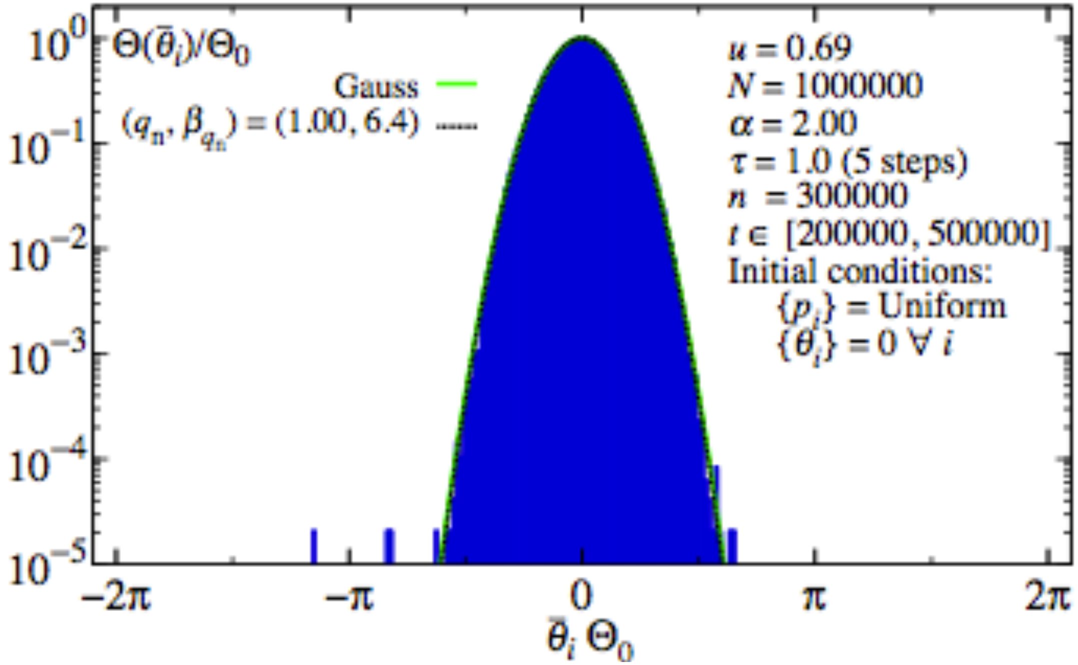
$\alpha = 2$

$q = 1$

$\alpha = 0.9$

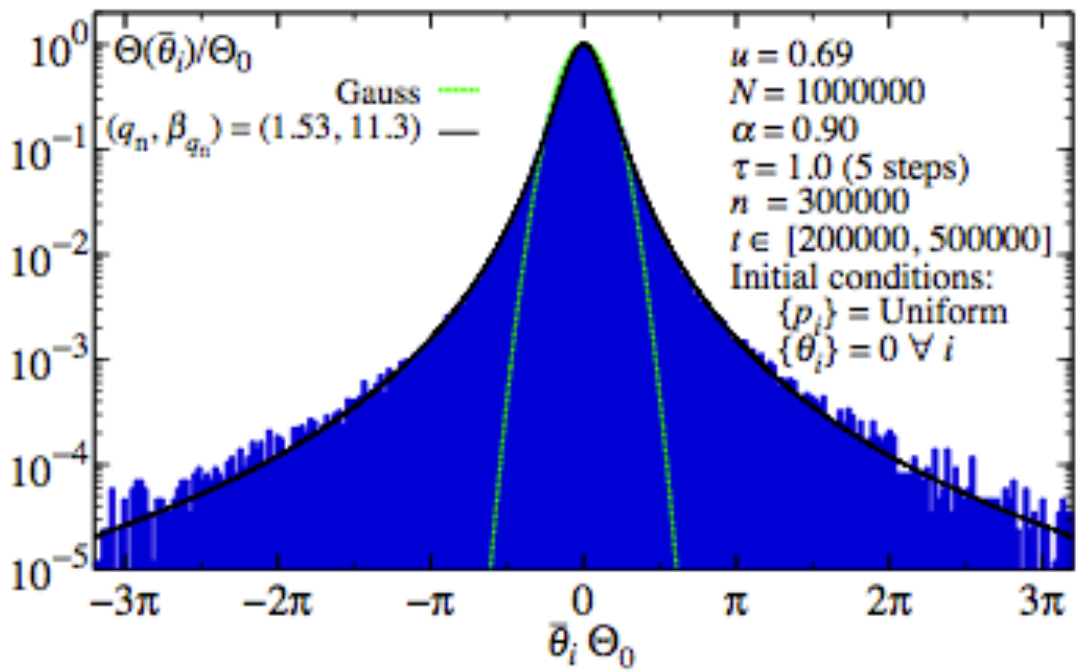
$q = 1.58$

L.J.L. Cirto, V.R.V. Assis and C. T. (2012),1206.6133 [cond-mat.stat-mech]



$\alpha = 2$

$q = 1$



$\alpha = 0.9$

$q = 1.53$

L.J.L. Cirto, V.R.V. Assis and C. T. (2012),1206.6133 [cond-mat.stat-mech]

PHYSICAL REVIEW E **87**, 022910 (2013)

Noise, synchrony, and correlations at the edge of chaos

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We study the effect of a weak random additive noise in a linear chain of N locally coupled logistic maps at the edge of chaos. Maps tend to synchronize for a strong enough coupling, but if a weak noise is added, very intermittent fluctuations in the returns time series are observed. This intermittency tends to disappear when noise is increased. Considering the probability distribution functions (pdfs) of the returns, we observe the emergence of fat tails which can be satisfactorily reproduced by q-Gaussians' curves typical of nonextensive statistical mechanics. The interoccurrence times of these extreme events are also studied in detail. Similarities with the recent analysis of financial data are also discussed.

CORRELATIONS IN COUPLED LOGISTIC MAPS AT THE EDGE OF CHAOS IN THE PRESENCE OF GLOBAL NOISE

We consider a linear chain of N coupled maps with periodic boundary conditions in a noisy environment:

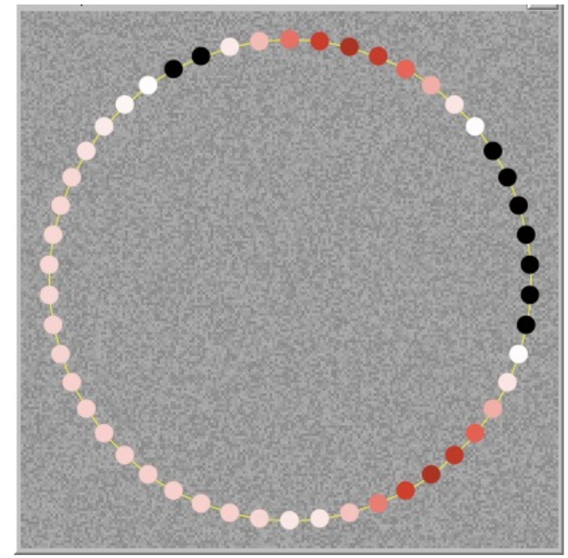
$$x_{t+1}^{i} = (1-\varepsilon) f(x_t^i) + \frac{\varepsilon}{2}[f(x_t^{i-1}) + f(x_t^{i+1})] + \sigma_t$$

$\sigma_t \in [0, \sigma_{max}]$ additive noise

with $\varepsilon \in [0,1]$ coupling strength

and $f(x_t^i) = 1 - \mu (x_t^i)^2 \quad \mu \in [0,2]$ *i*-th logistic map ($i = 1...N$)

[zero noise: N.B. Ouchi and K. Kaneko, Chaos **10**, 359 (2000)]



edge of chaos: $\mu = 1.4011551$

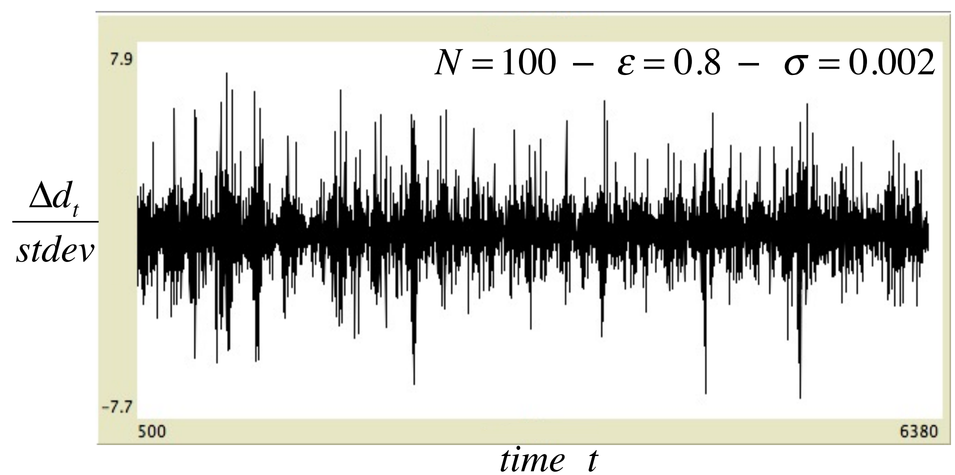
Intermittency in the normalized returns time-series

global parameter

$$d_t = \Sigma_{i=1}^{N} |x_t^i - < x_t^i > |$$

time returns

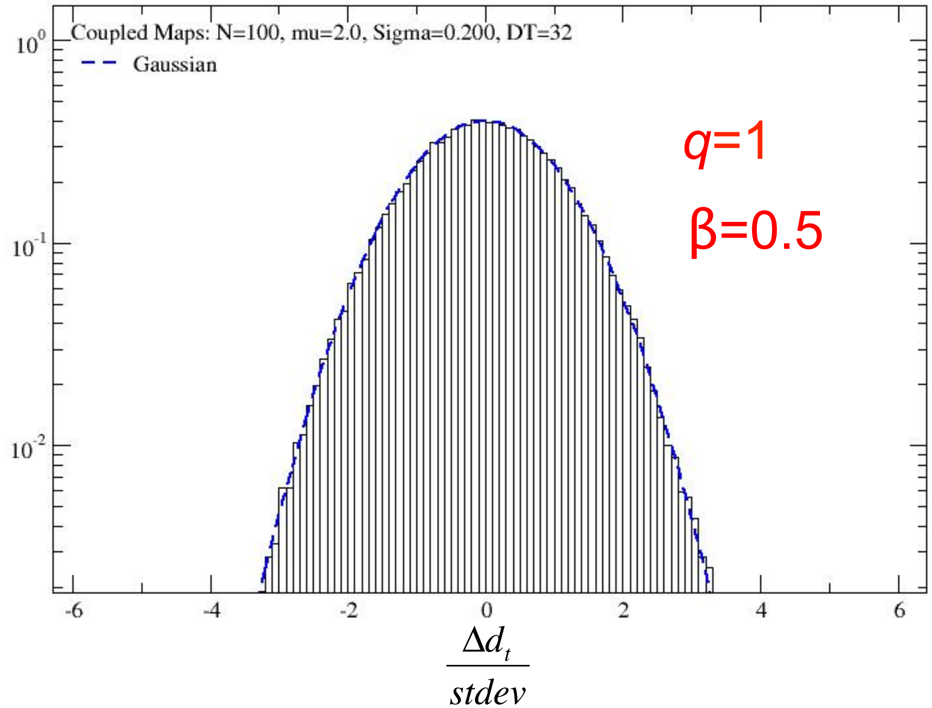
$$\Delta d_t = d_{t+\tau} - d_t$$



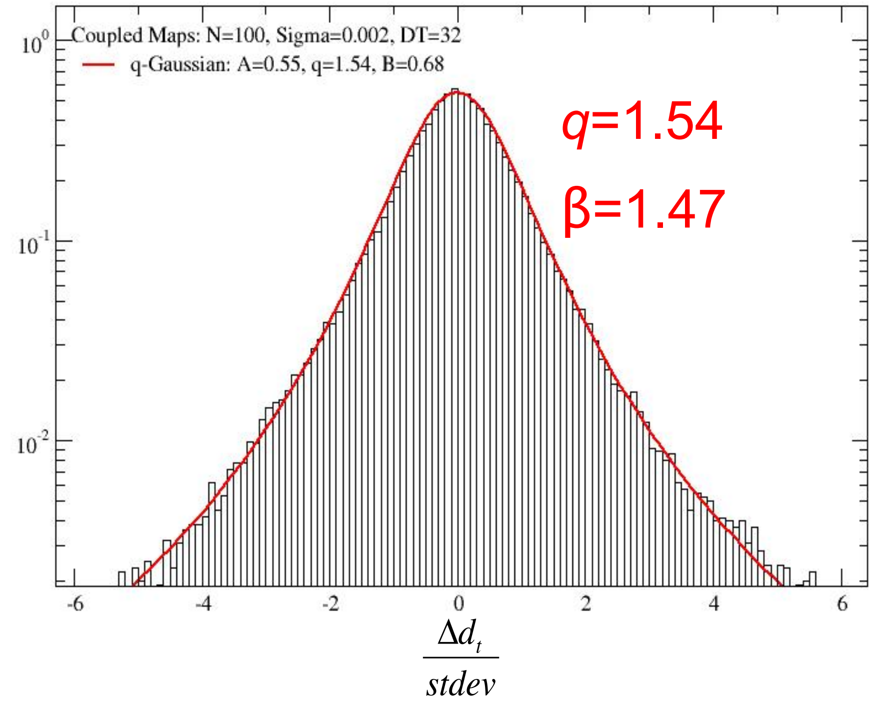
A. Pluchino, A. Rapisarda and C. T., Phys Rev E **87**, 022910 (2013)

$$N = 100;\ \ \varepsilon = 0.8;\ \ \sigma_{\max} = 0.002;\ \ \tau = 32$$

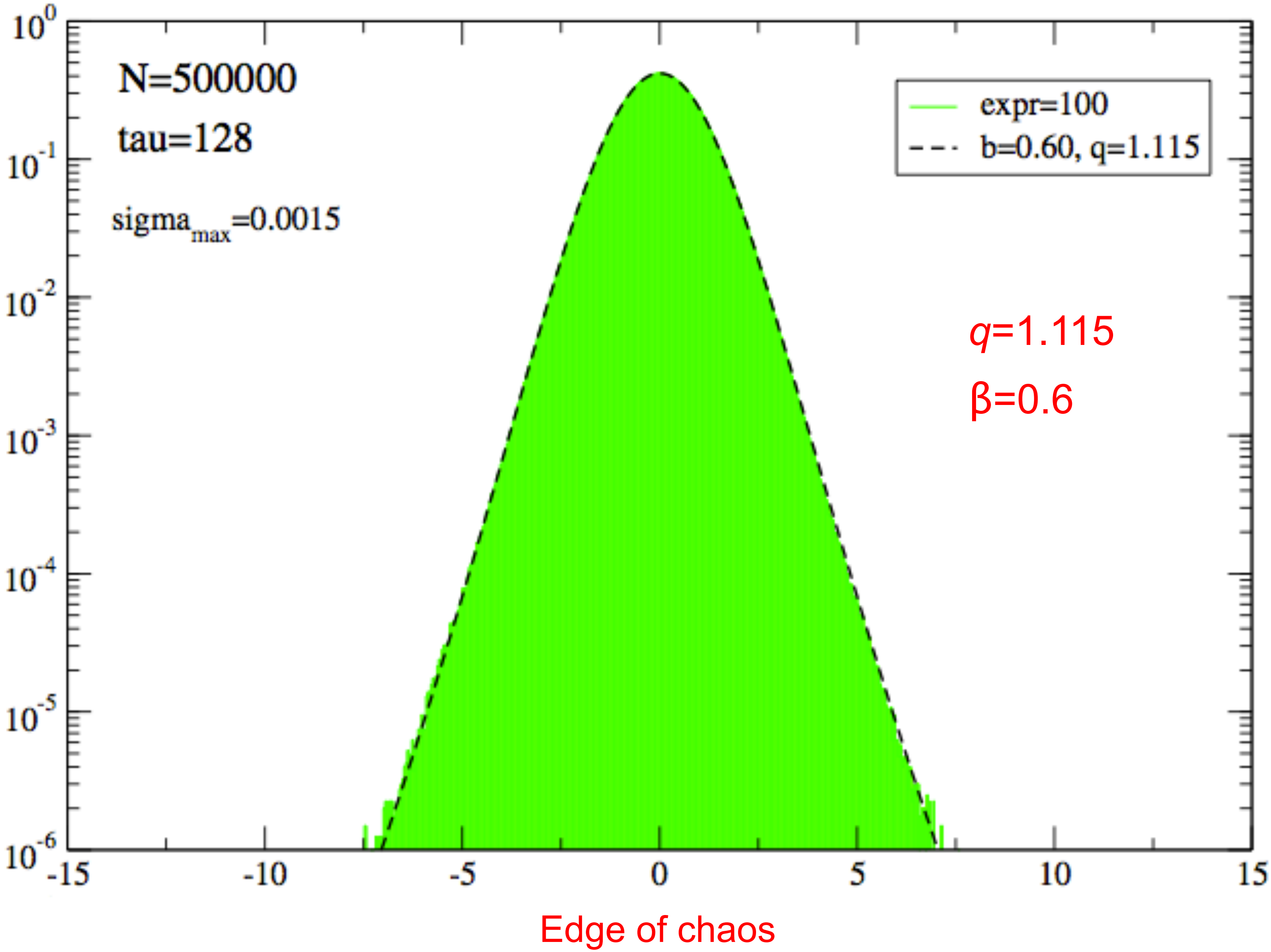
Chaotic Regime: $\mu = 2.0$



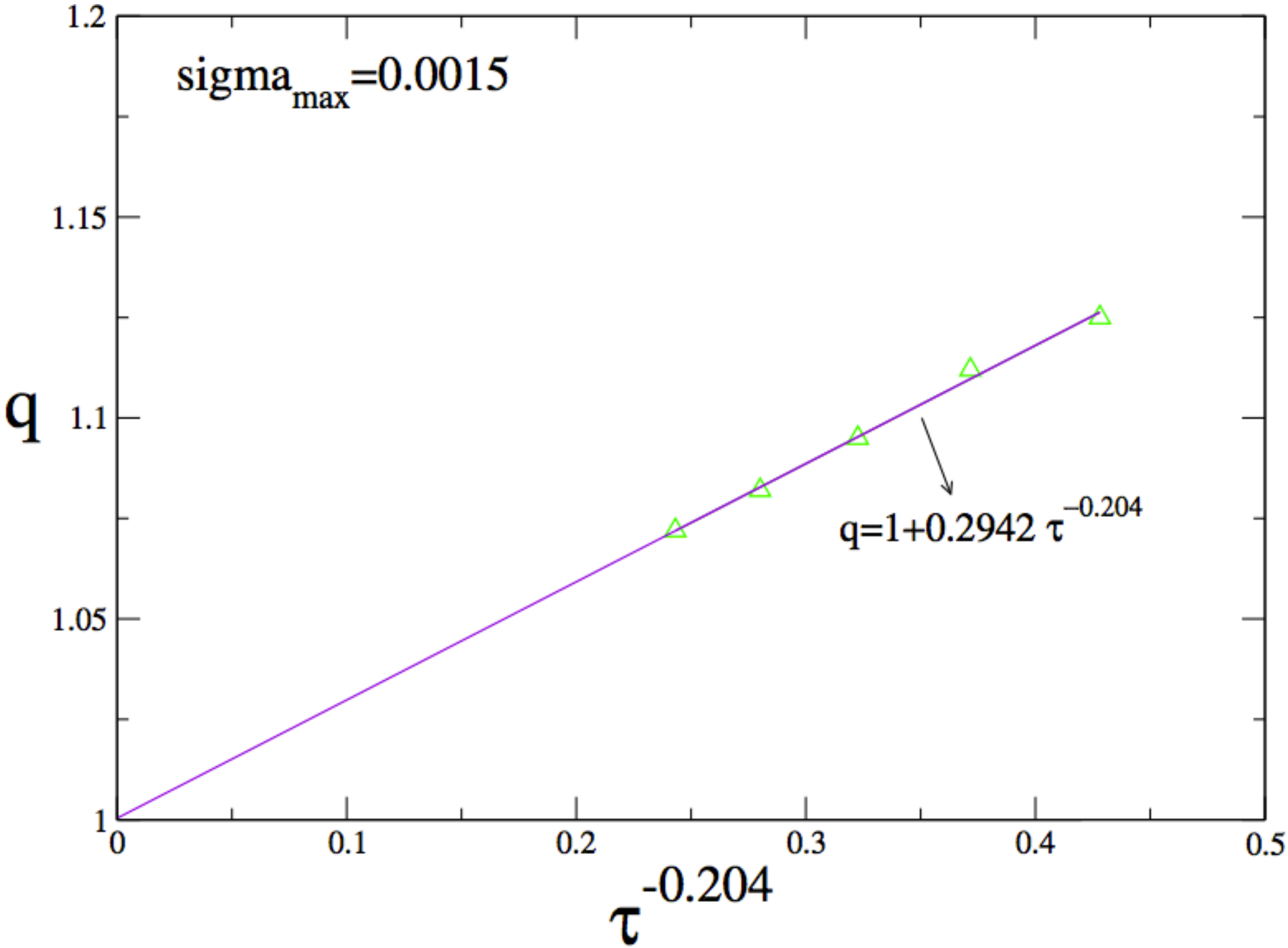
Edge of chaos: $\mu = 1.4011551$



A. Pluchino, A. Rapisarda and C. T., Phys Rev E **87**, 022910 (2013)



U. Tirnakli and C. T. (2013)



U. Tirnakli and C. T. (2013)

Physics Reports 515 (2012) 115–226

Contents lists available at SciVerse ScienceDirect

Physics Reports

journal homepage: www.elsevier.com/locate/physrep

Physical approach to complex systems

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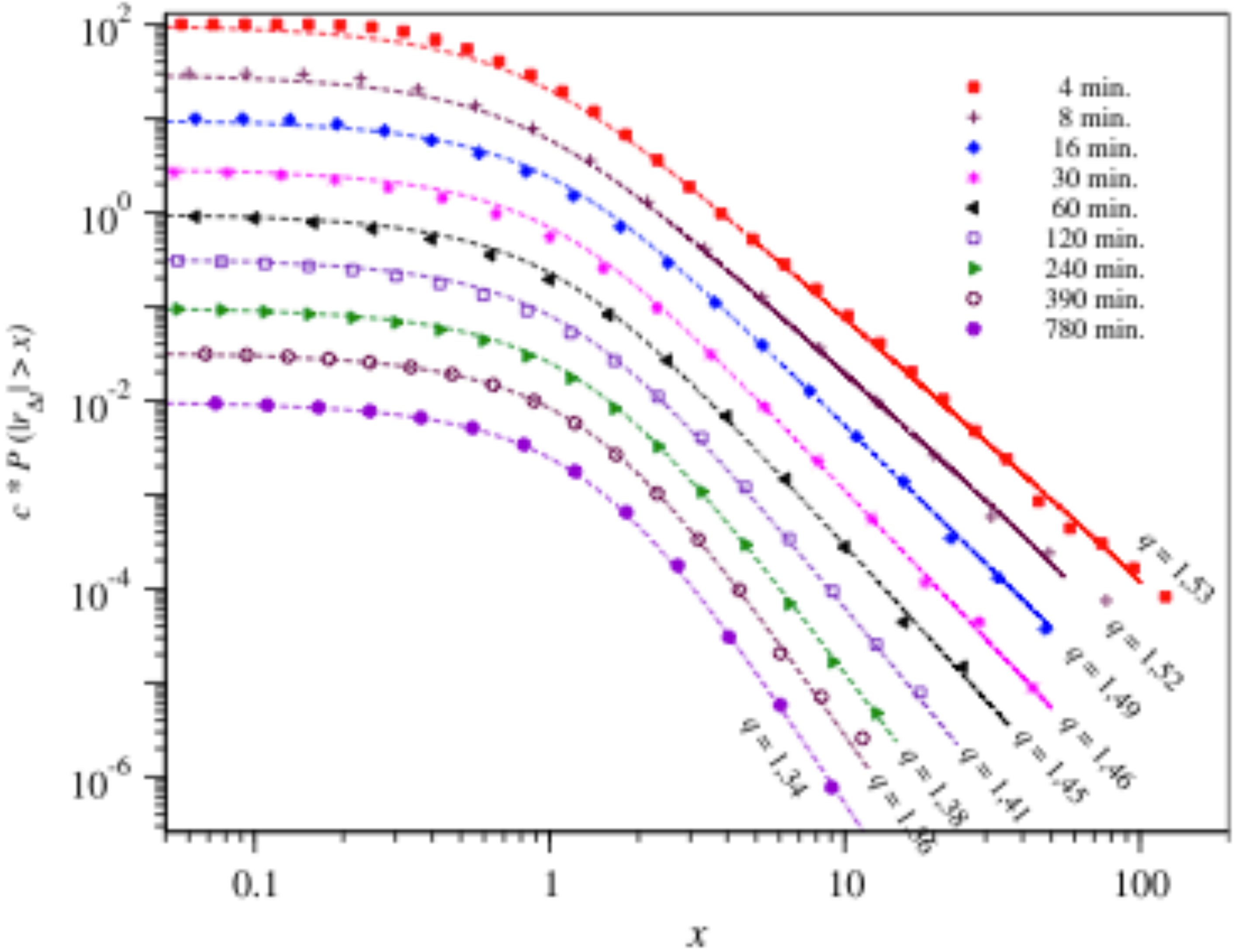
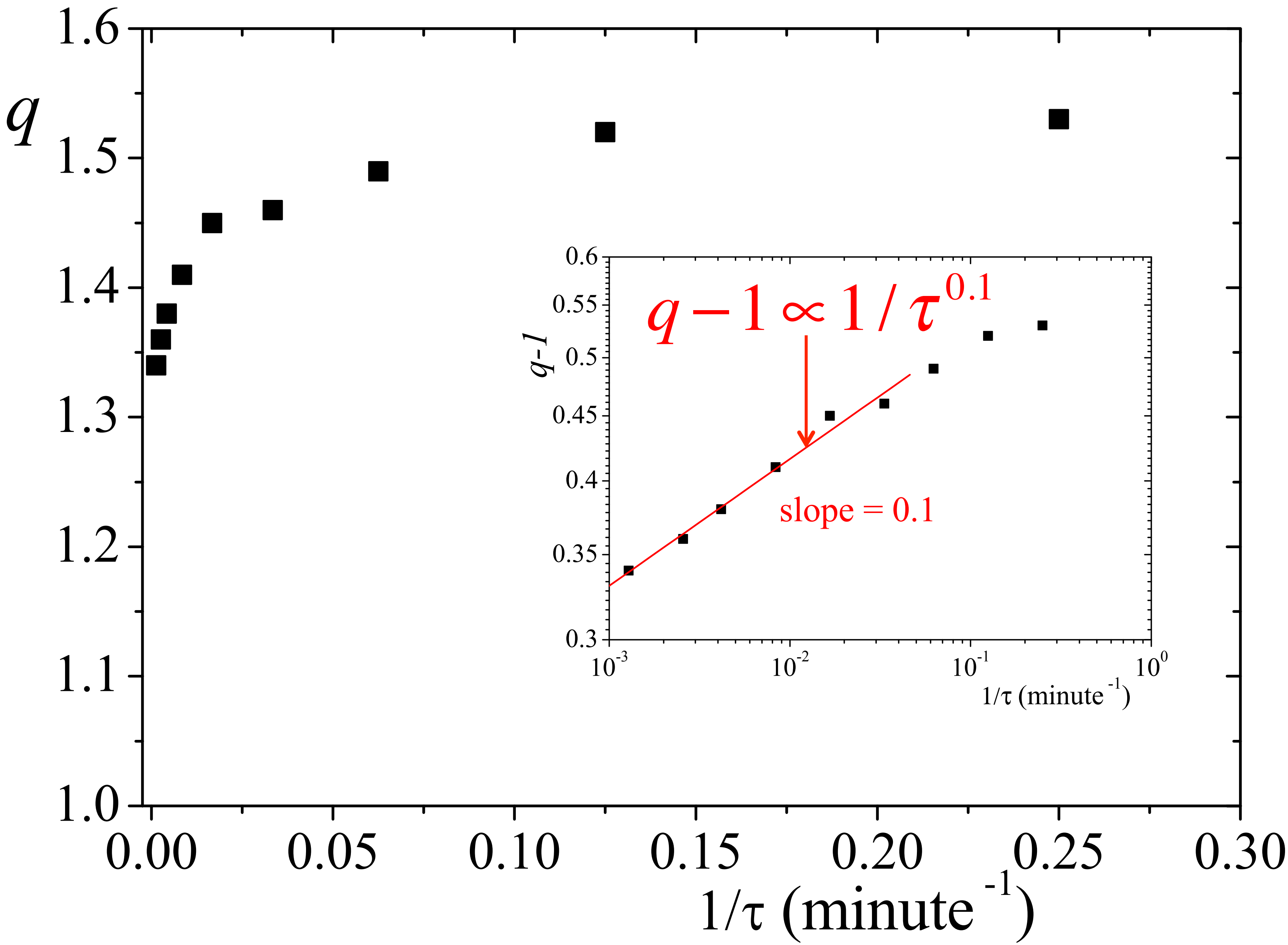
Typically, complex systems are natural or social systems which consist of a large number of nonlinearly interacting elements. These systems are open, they interchange information or mass with environment and constantly modify their internal structure and patterns of activity in the process of self-organization. As a result, they are flexible and easily adapt to variable external conditions. However, the most striking property of such systems is the existence of emergent phenomena which cannot be simply derived or predicted solely from the knowledge of the systems' structure and the interactions among their individual elements. This property points to the holistic approaches which require giving parallel descriptions of the same system on different levels of its organization. There is strong evidence – consolidated also in the present review – that different, even apparently disparate complex systems can have astonishingly similar characteristics both in their structure and in their behaviour. One can thus expect the existence of some common, universal laws that govern their properties.

Physics methodology proves helpful in addressing many of the related issues. In this review, we advocate some of the computational methods which in our opinion are especially fruitful in extracting information on selected – but at the same time most representative – complex systems like human brain, financial markets and natural language, from the time series representing the observables associated with these systems. The properties we focus on comprise the collective effects and their coexistence with noise, long-range interactions, the interplay between determinism and flexibility in evolution, scale invariance, criticality, multifractality and hierarchical structure. The methods described either originate from "hard" physics – like the random matrix theory – and then were transmitted to other fields of science via the field of complex systems research, or they originated elsewhere but turned out to be very useful also in physics — like, for example, fractal geometry. Further methods discussed borrow from the formalism of complex networks, from the theory of critical phenomena and from nonextensive statistical mechanics. Each of these methods is helpful in analyses of specific aspects of complexity and all of them are mutually complementary.



Kwapien and Drozdz, Phys Rep **515**, 115 (2012)

Fig. 61. Cumulative distributions of absolute normalized returns corresponding to different time scales Δt for the 100 American companies with the highest market capitalization, together with the fitted cumulative q-Gaussian distributions. Each q-Gaussian was labelled by the associated value of the Tsallis parameter q. In order to better visualize the results, each q-Gaussian was multiplied by a positive factor $c \neq 1$.



q
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.00
0.05
0.10
0.15
0.20
0.25
0.30
1/τ (minute -1)
q-1
0.6
0.55
0.5
0.45
0.4
0.35
0.3
$q-1 \propto 1/\tau^{0.1}$
slope = 0.1
10^{-3}
10^{-2}
10^{-1}
10^{0}
1/τ (minute -1)

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Astronomy & Astrophysics

Nonextensive distributions of asteroid rotation periods and diameters

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ABSTRACT

Context. We investigate the distribution of asteroid rotation periods from different regions of the solar system and diameter distributions of near-Earth asteroids (NEAs).
Aims. We aim to verify if nonextensive statistics satisfactorily describes the data.
Methods. Light curve data were taken from the Planetary Database System (PDS) with Rel ≥ 2. We also considered the taxonomic class and region of the solar system. Data of NEA were taken from the Minor Planet Center.
Results. The rotation periods of asteroids follow a q-Gaussian with $q = 2.6$ regardless of taxonomy, diameter, or region of the solar system of the object. The distribution of rotation periods is influenced by observational bias. The diameters of NEAs are described by a q-exponential with $q = 1.3$. According to this distribution, there are expected to be 994 ± 30 NEAs with diameters greater than 1 km.

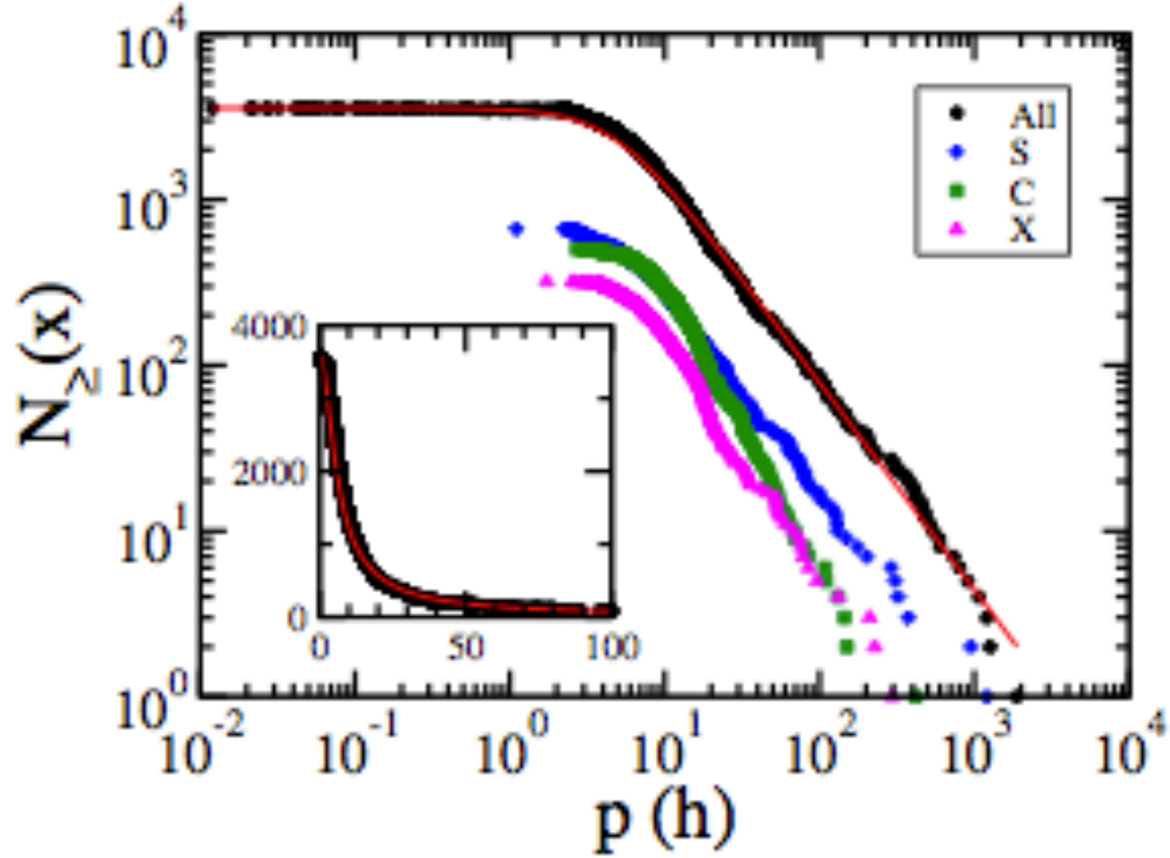

Fig. 3. Log-log plot of the decreasing cumulative distribution of periods of 3567 asteroids (dots) with Rel ≥ 2 taken from the PDS (NASA) and a q-Gaussian distribution ($N_{\geq}(p) = M \exp_q(-\beta_q p^2)$) (solid line), with $q = 2.6$, $\beta_q = 0.025$ h^{-2}, $M = 3567$. The other curves are 663 S-complex asteroids (diamonds, blue online), 503 C-complex asteroids (squares, green online), 321 X-complex asteroids (triangles, magenta online). Inset shows the 3567 asteroids and the q-Gaussian in a linear-linear plot.

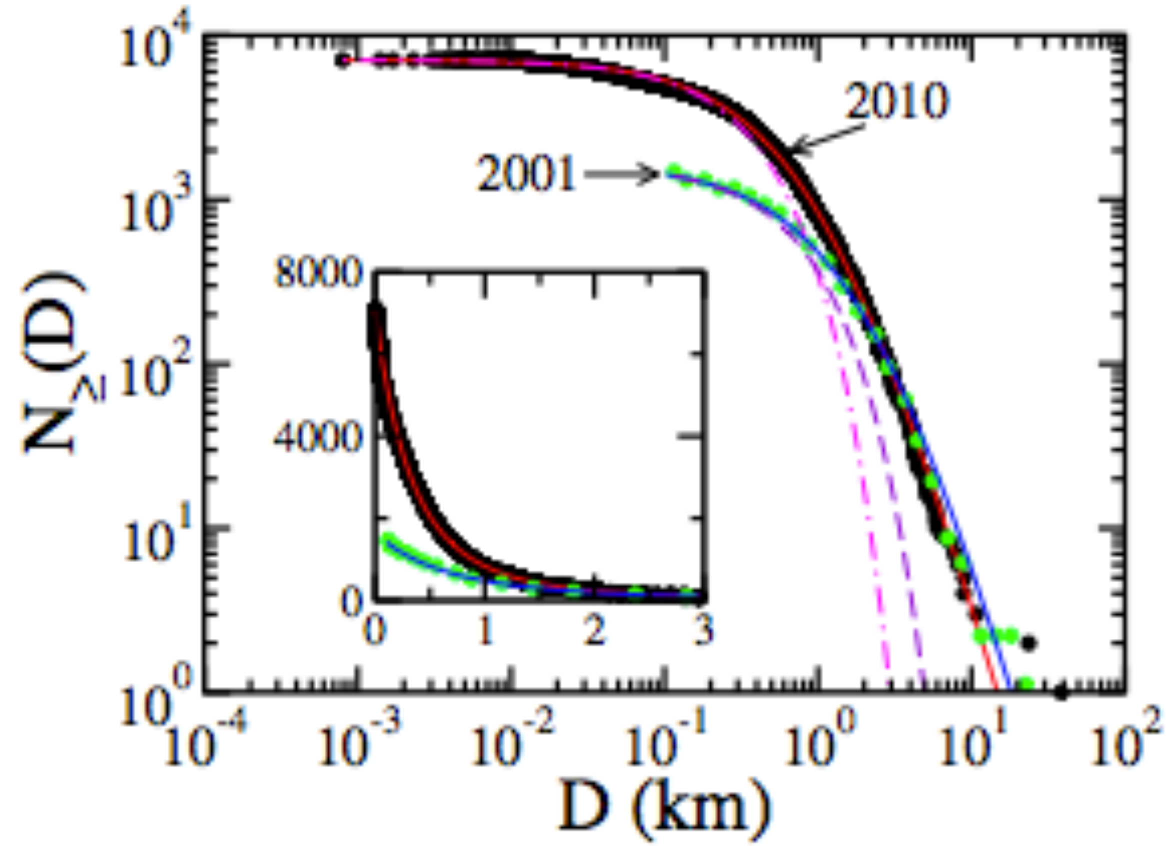

Fig. 4. Decreasing cumulative distribution of diameters of known NEAs in 2001 (1649 objects, green dots) and in 2010 (7078 objects, black dots). Solid lines are best fits of q-exponentials ($N_{\geq}(D) = M \exp_q(-\beta_q D)$). Blue line (2001): $q = 1.3$, $\beta_q = 1.5$ km^{-1}, $M = 1649$, red line (2010): $q = 1.3$, $\beta_q = 3$ km^{-1}, $M = 7078$. Normal exponentials ($q = 1$) are displayed in the main panel for comparison (dashed violet, with $\beta_1 = 1.5$ km^{-1}, $M = 1649$, and dot-dashed magenta, with $\beta_1 = 3$ km^{-1}, $M = 7078$).

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Physics Letters A

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Unified long-memory mesoscopic mechanism consistent with nonextensive statistical mechanics

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ABSTRACT

We unify two long-memory Fokker–Planck mechanisms. Stationary solutions of the equation $\frac{\partial p(x,t)}{\partial t} = -\frac{\partial}{\partial x}[F(x)p(x,t)] + \frac{1}{2}D\frac{\partial^2}{\partial x^2}[\phi(x,p)p(x,t)]$ ($D \in \mathcal{R}$; $F(x) = -\partial V(x)/\partial x$) exist for a wide class of systems, namely for $\phi(x,p) = [A + BV(x)]^\theta [p(x,t)]^\eta$, (A, B, θ, η) being constants. We obtain that, for $\theta \neq 1$ and arbitrary confining potential $V(x)$, $p(x,\infty) \propto \{1-\beta(1-q)V(x)\}^{1/(1-q)} \equiv e_q^{-\beta V(x)}$, where $q = 1 + \eta/(\theta - 1)$.

Fokker–Planck	Linear ($\eta \to 0$)	Nonlinear ($\eta \neq 0$)
Homogeneous ($\theta = 0$)	$q = 1$	$q = 1 - \eta$
Inhomogeneous ($\theta \neq 0$)	If $\eta \sim \alpha(\theta - 1)^\delta$ and $\theta \to 1$, $q = 1$ for $\delta > 1$ $q = 1 + \alpha$ for $\delta = 1$ If η is strictly 0 and $\theta \to 1$, $q = \frac{2(1+BD)}{2+BD}$	$q = 1 + \frac{\eta}{\theta - 1}$

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The large deviation approach to statistical mechanics

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ABSTRACT

The theory of large deviations is concerned with the exponential decay of probabilities of large fluctuations in random systems. These probabilities are important in many fields of study, including statistics, finance, and engineering, as they often yield valuable information about the large fluctuations of a random system around its most probable state or trajectory. In the context of equilibrium statistical mechanics, the theory of large deviations provides exponential-order estimates of probabilities that refine and generalize Einstein's theory of fluctuations. This review explores this and other connections between large deviation theory and statistical mechanics, in an effort to show that the mathematical language of statistical mechanics is the language of large deviation theory. The first part of the review presents the basics of large deviation theory, and works out many of its classical applications related to sums of random variables and Markov processes. The second part goes through many problems and results of statistical mechanics, and shows how these can be formulated and derived within the context of large deviation theory. The problems and results treated cover a wide range of physical systems, including equilibrium many-particle systems, noise-perturbed dynamics, nonequilibrium systems, as well as multifractals, disordered systems, and chaotic systems. This review also covers

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Physics Letters A

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Towards a large deviation theory for strongly correlated systems

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ABSTRACT

A large-deviation connection of statistical mechanics is provided by N independent binary variables, the $(N \to \infty)$ limit yielding Gaussian distributions. The probability of $n \neq N/2$ out of N throws is governed by e^{-Nr}, r related to the entropy. Large deviations for a strong correlated model characterized by indices (Q, γ) are studied, the $(N \to \infty)$ limit yielding Q-Gaussians ($Q \to 1$ recovers a Gaussian). Its large deviations are governed by $e_q^{-Nr_q}$ ($\propto 1/N^{1/(q-1)}$, $q > 1$), $q = (Q-1)/(\gamma[3-Q]) + 1$. This illustration opens the door towards a large-deviation foundation of nonextensive statistical mechanics.

Physics Letters A 377 (2013) 491–495

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Physics Letters A

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Comment

Reply to Comment on "Towards a large deviation theory for strongly correlated systems"

Guiomar Ruiz [a,b,*], Constantino Tsallis [a,c]

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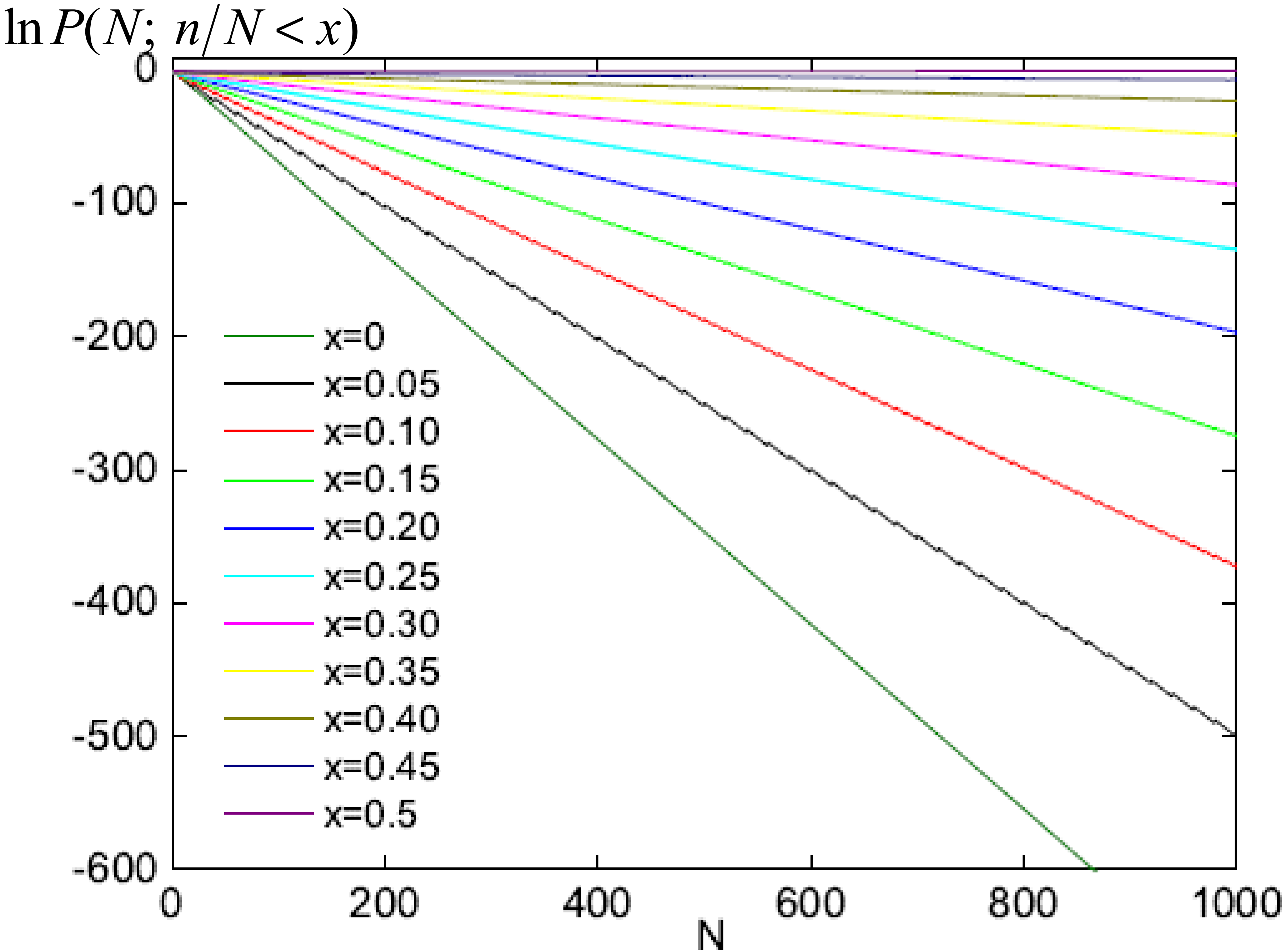
Probability theory
Statistical mechanics
Entropy

A B S T R A C T

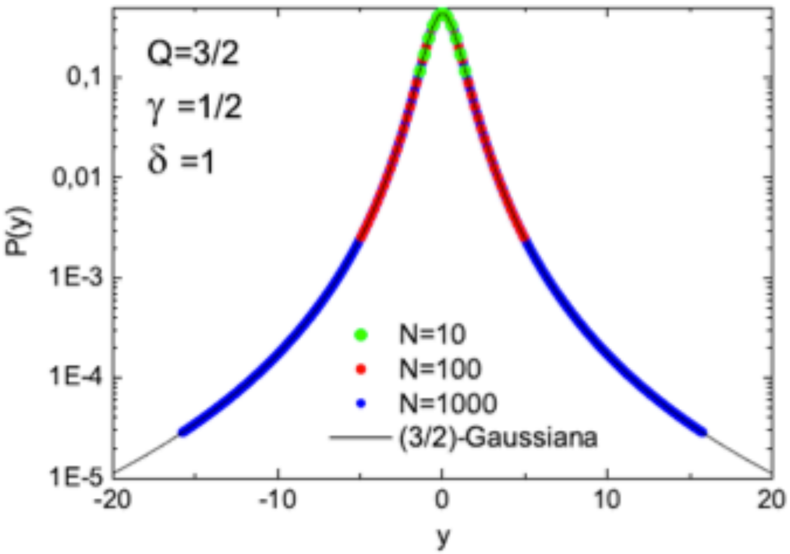
The computational study commented by Touchette opens the door to a desirable generalization of standard large deviation theory for special, though ubiquitous, correlations. We focus on three inter-related aspects: (i) numerical results strongly suggest that the standard exponential probability law is asymptotically replaced by a power-law dominant term; (ii) a subdominant term appears to reinforce the thermodynamically extensive entropic nature of q-generalized rate function; (iii) the correlations we discussed, correspond to Q-Gaussian distributions, differing from Lévy's, except in the case of Cauchy–Lorentz distributions. Touchette has agreeably discussed point (i), but, unfortunately, points (ii) and (iii) escaped to his analysis. Claiming the absence of connection with q-exponentials is unjustified.

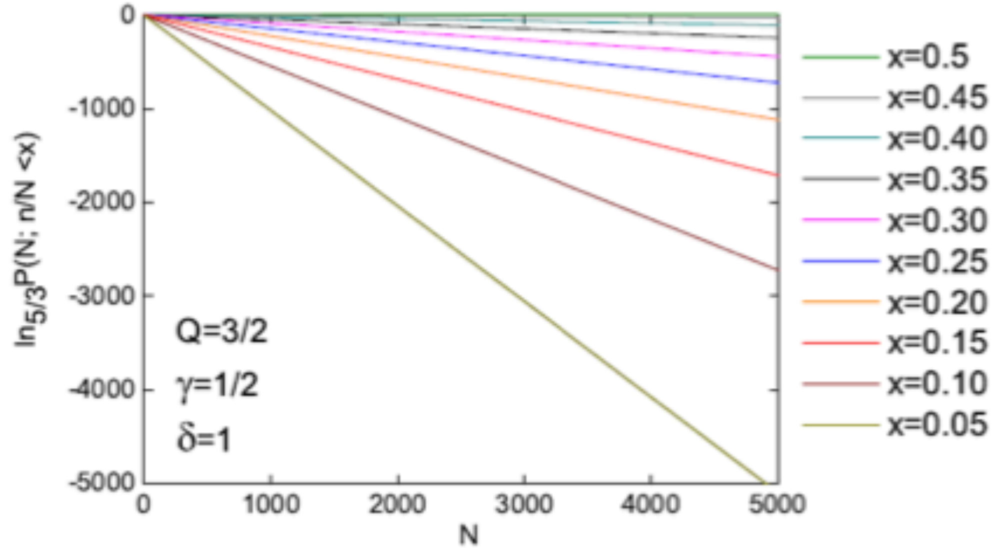
	PHYSICS (Statistical mechanics)	MATHEMATICS (Large deviation theory)
$q=1$ (quasi-independent)	$p_N \propto e^{-\beta H_N} = e^{-\left[\beta \frac{H_N}{N}\right] N}$	$P_N(x) \sim e^{-N\, r(x)}$ $(N \to \infty)$
$q \neq 1$ (strongly correlated)	$p_N \propto e_q^{-\beta_q H_N} = e_q^{-\left[(\beta_q N^*) \frac{H_N}{N N^*}\right] N}$	$P_N(x) \sim e_q^{-N\, r_q(x)}$ $(N \to \infty)$

G. Ruiz and C. T., Phys Lett A **377**, 491 (2013)

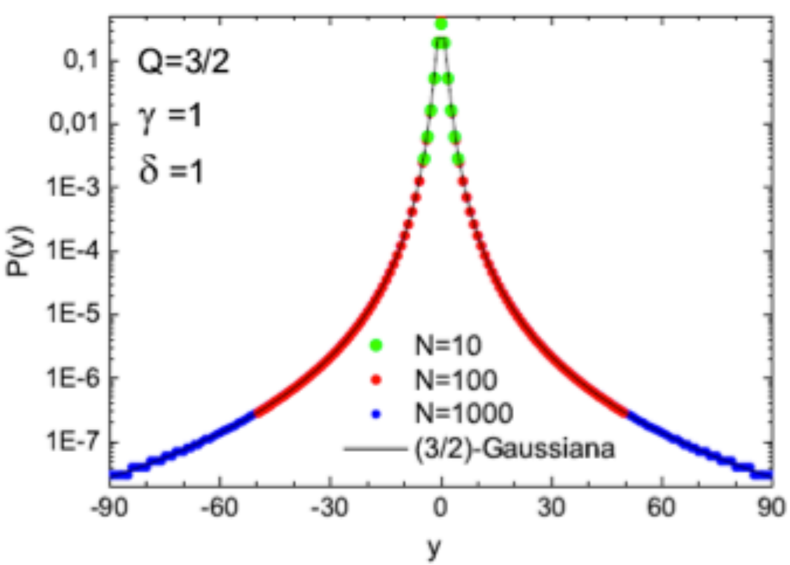


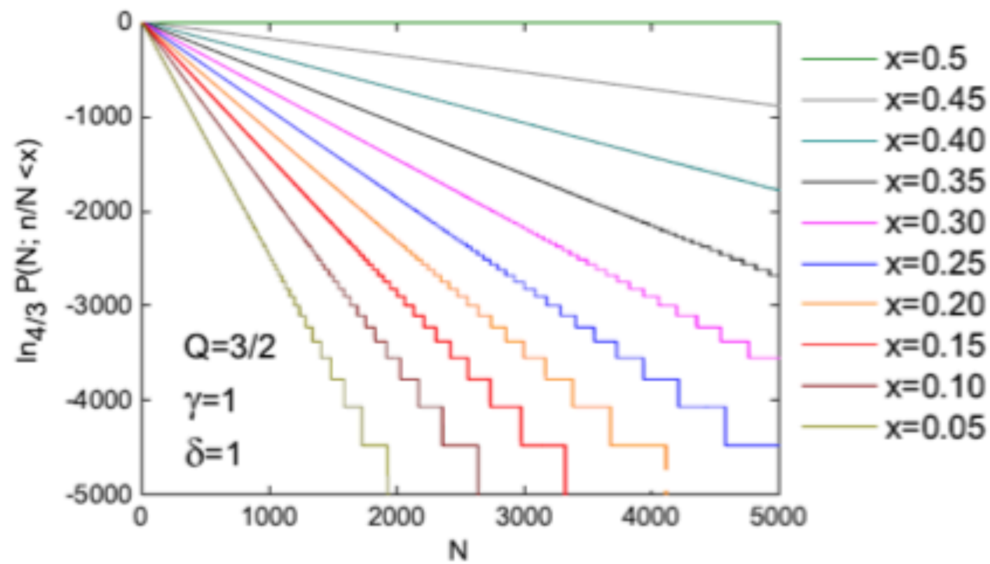
G. Ruiz and C. T., Phys Lett A **376**, 2451 (2012)



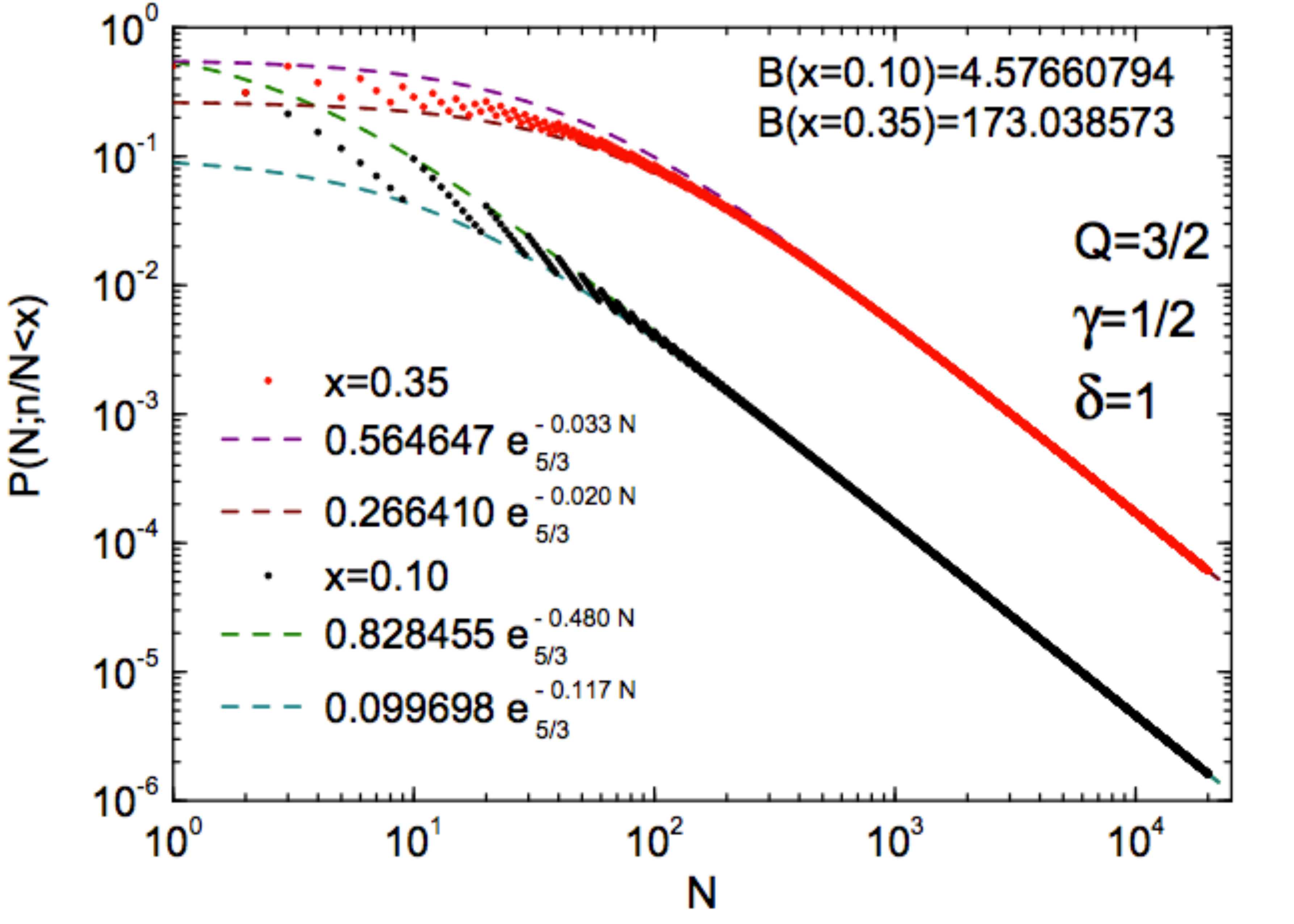


$$q = 1 + \frac{Q-1}{\gamma(3-Q)}$$

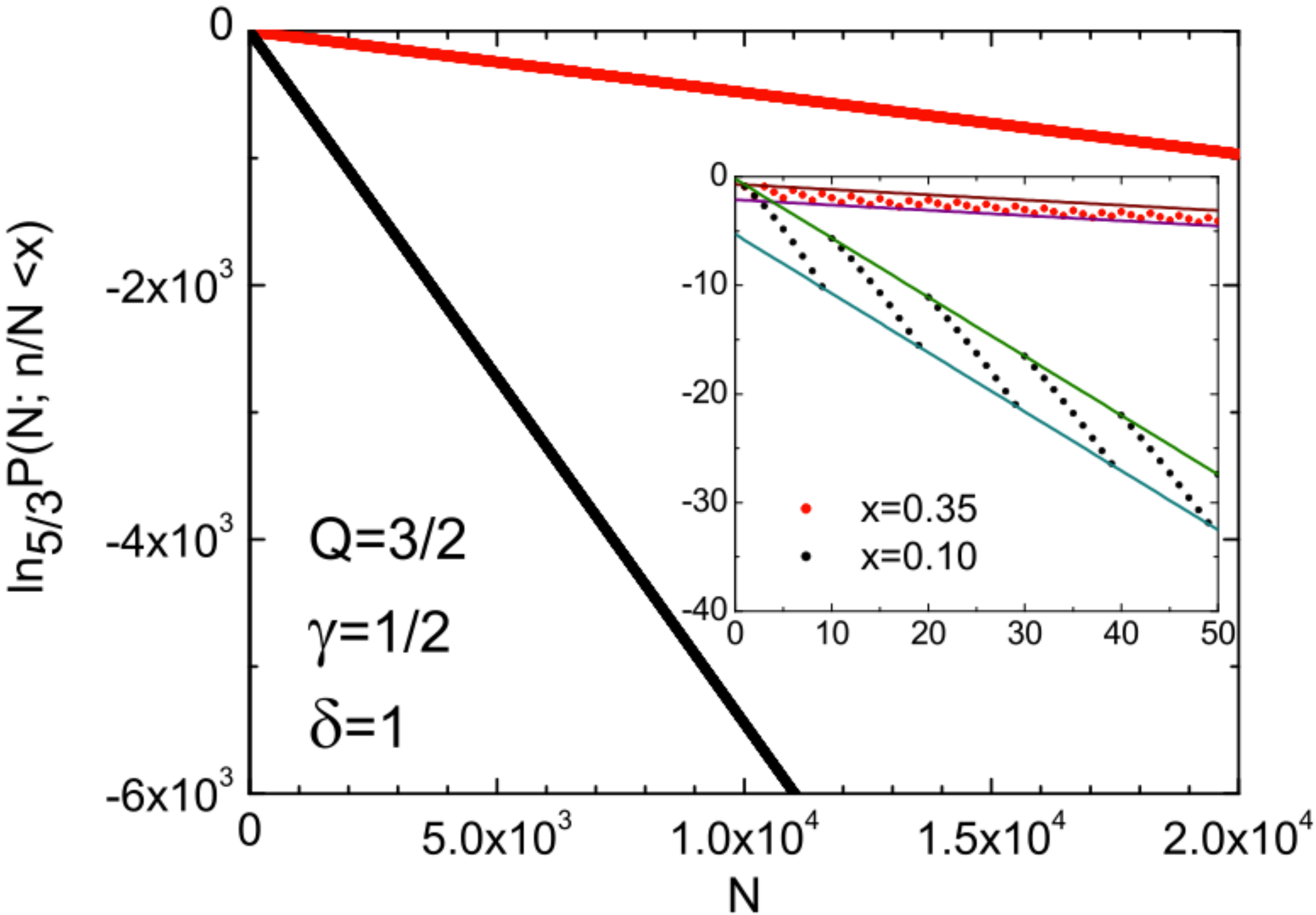




G. Ruiz and C. T., Phys Lett A **376**, 2451 (2012)



G. Ruiz and C. T., Phys Lett A **377**, 491 (2013)



G. Ruiz and C. T., Phys Lett A **377**, 491 (2013)

CONJECTURE:

For all strongly correlated systems whose CLT attractors are Q-Gaussians ($Q>1$), a set $\left[q>1,\ B(x)>0,\ r_q^{(lower\ bound)}(x)>0,\ r_q^{(upper\ bound)}(x)>0\right]$ exists such that generically

$$B(x)\left[(q-1)\, r_q^{(lower\ bound)}(x)\right]^{\frac{1}{q-1}} e_q^{-r_q^{(lower\ bound)}(x)\, N}$$

$$=\frac{B(x)}{N^{\frac{1}{q-1}}}\left[1-\frac{1}{(q-1)^2 r_q^{(lower\ bound)}(x)}\frac{1}{N}+o(1/N^2)\right]$$

$$\leq P(N;\ n/N<x)$$

$$\leq B(x)\left[(q-1)\, r_q^{(upper\ bound)}(x)\right]^{\frac{1}{q-1}} e_q^{-r_q^{(upper\ bound)}(x)\, N}$$

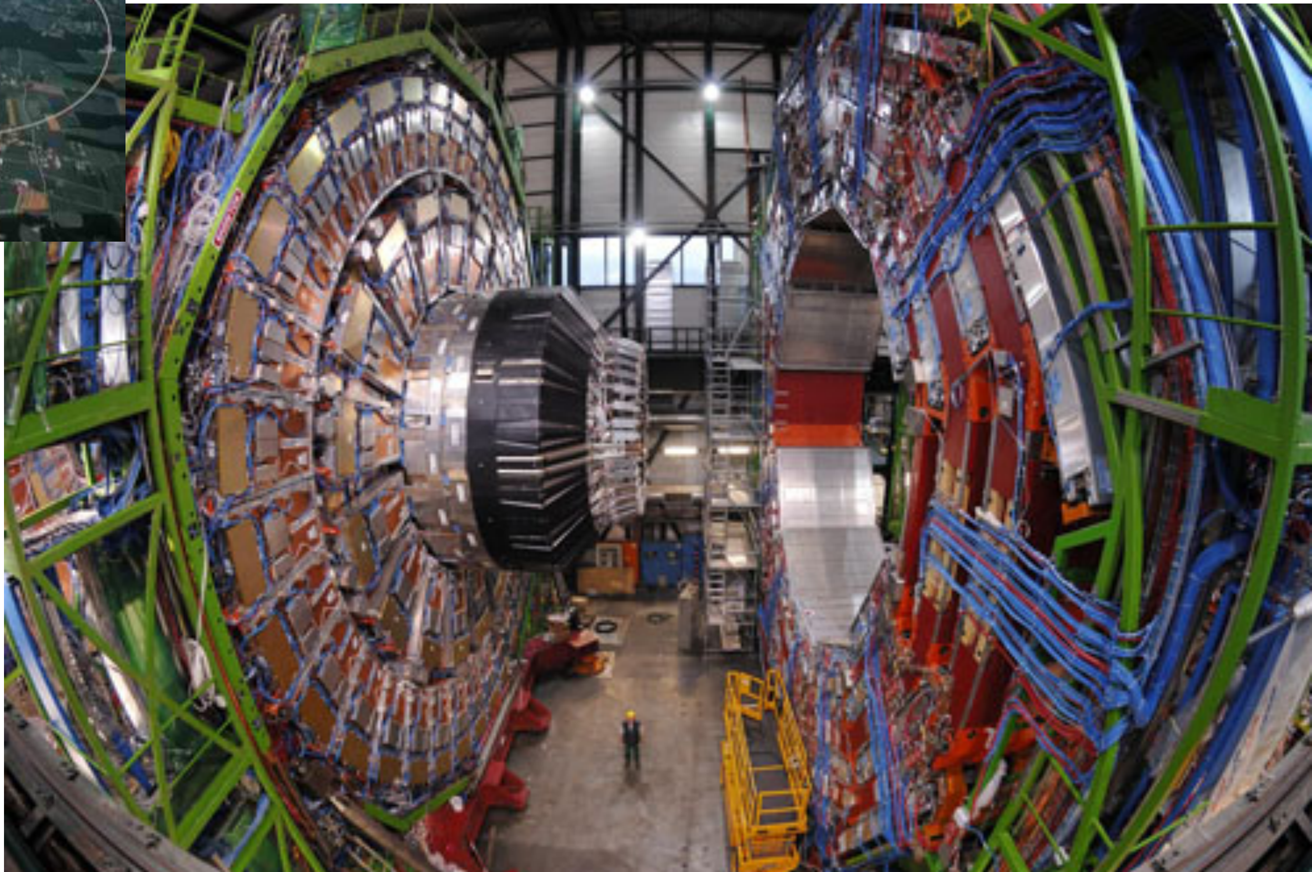
$$=\frac{B(x)}{N^{\frac{1}{q-1}}}\left[1-\frac{1}{(q-1)^2 r_q^{(upper\ bound)}(x)}\frac{1}{N}+o(1/N^2)\right]$$

G. Ruiz and C. T., Phys Lett A **377**, 491 (2013)

LHC (Large Hadron Collider)

CMS (Compact Muon Solenoid) detector

~ 2500 scientists/engineers from 183 institutions of 38 countries



PRL 105, 022002 (2010) PHYSICAL REVIEW LETTERS week ending 9 JULY 2010

Transverse-Momentum and Pseudorapidity Distributions of Charged Hadrons in pp Collisions at $\sqrt{s} = 7$ TeV

V. Khachatryan *et al.**
(CMS Collaboration)

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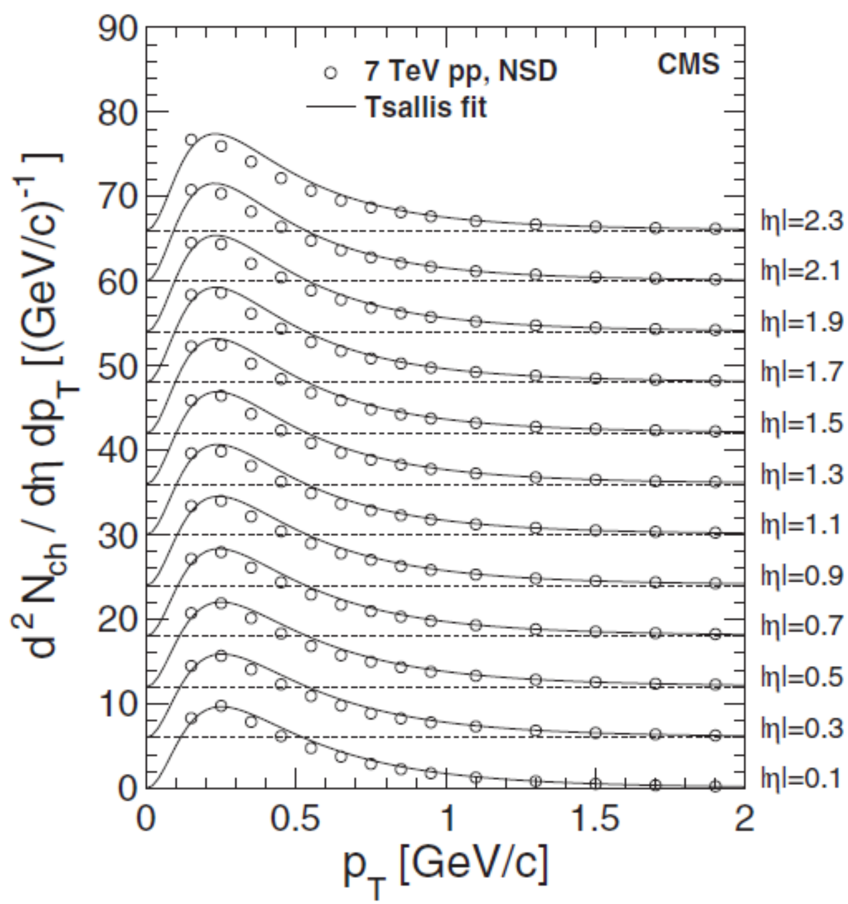
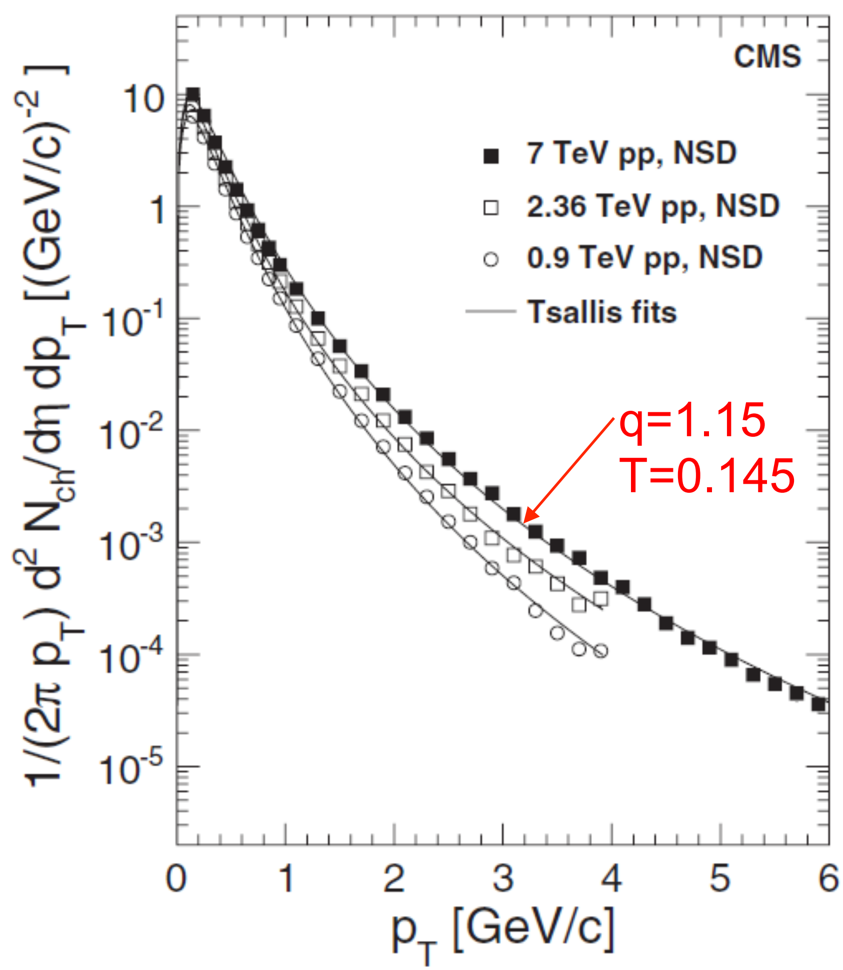
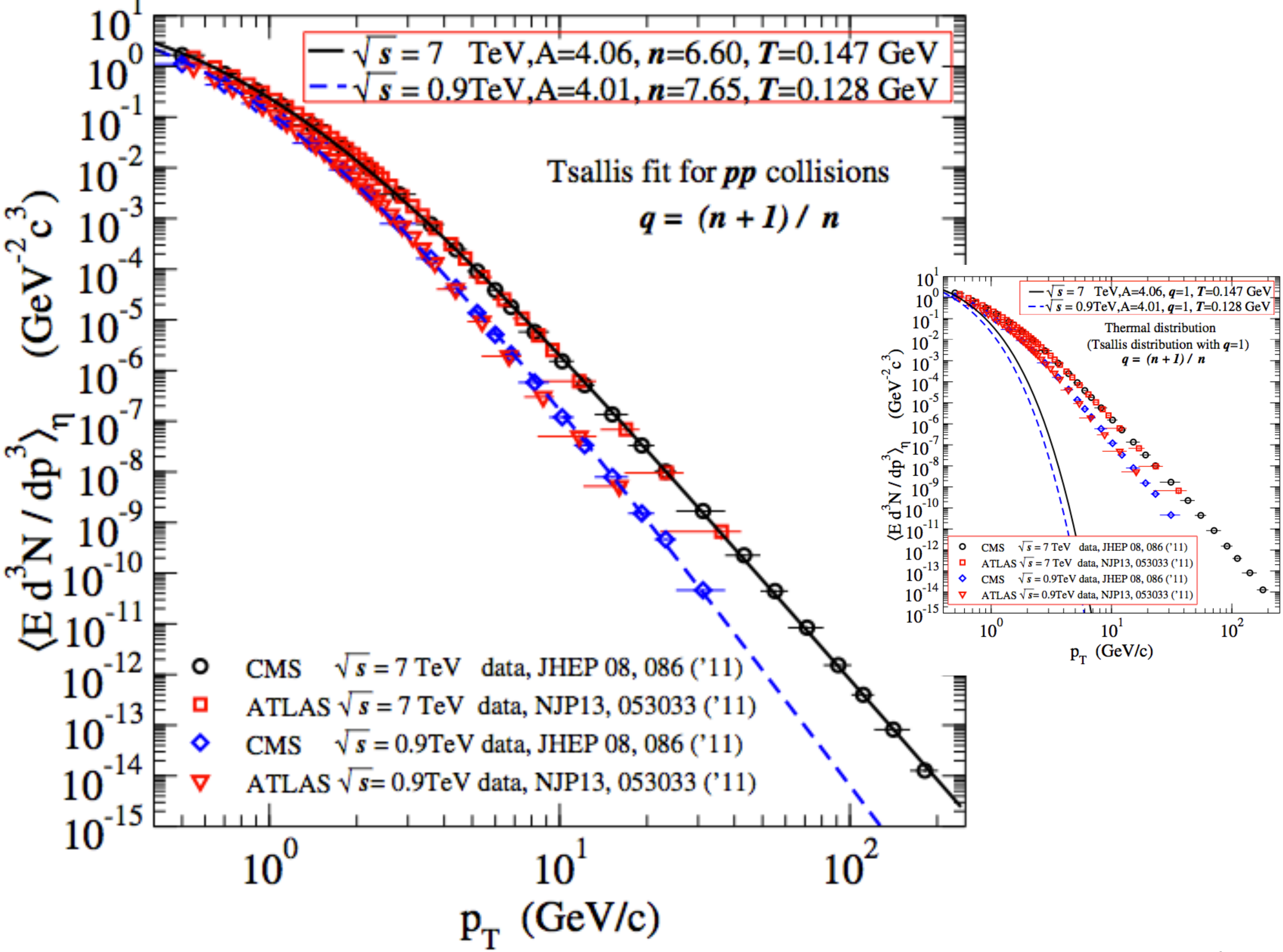
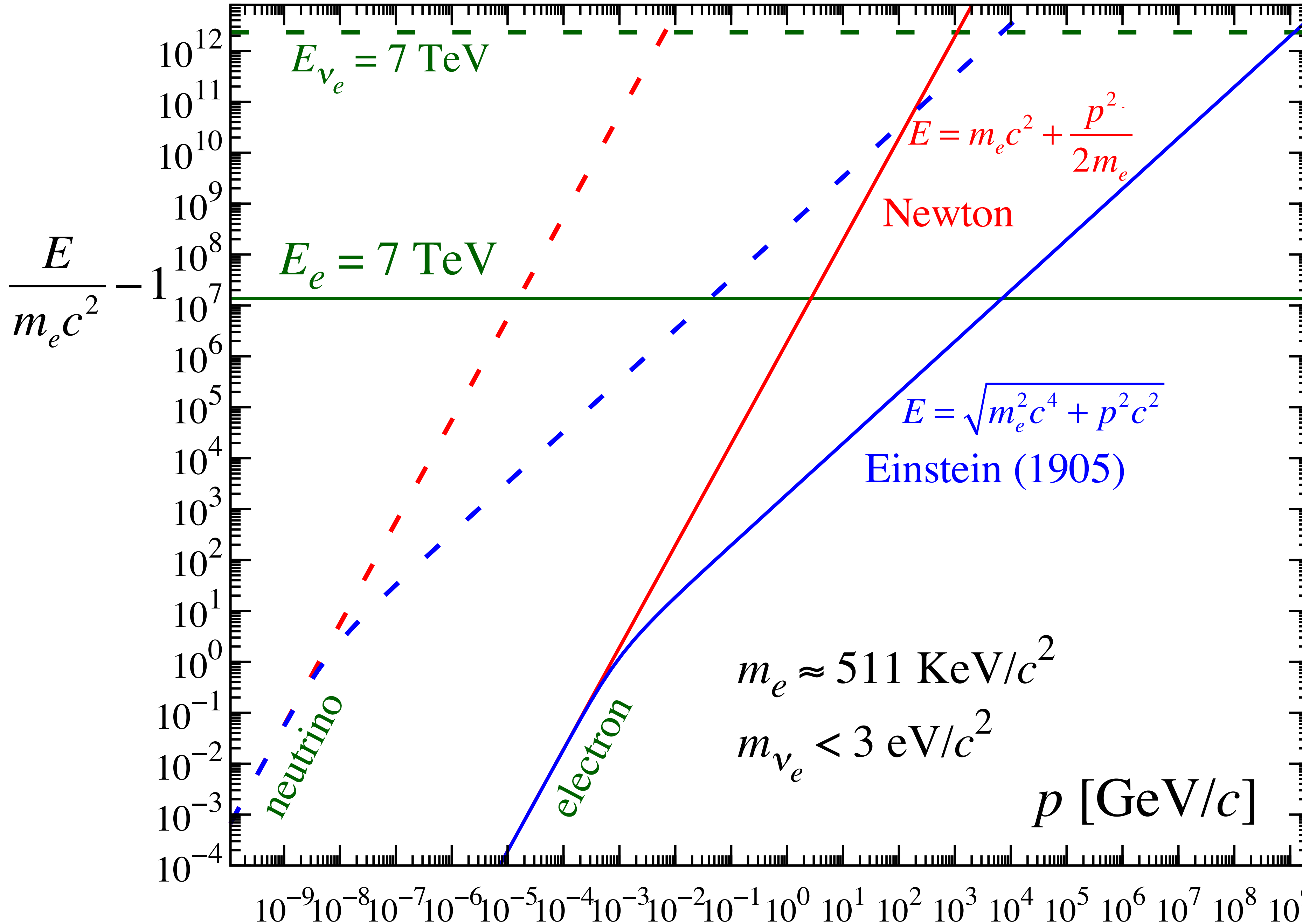






Wong and Wilk, Acta Phys Polon B **43**, 2047 (2012)



L.J.L. Cirto and C. T. (2012)

Nonlinear Relativistic and Quantum Equations with a Common Type of Solution

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Generalizations of the three main equations of quantum physics, namely, the Schrödinger, Klein-Gordon, and Dirac equations, are proposed. Nonlinear terms, characterized by exponents depending on an index q, are considered in such a way that the standard linear equations are recovered in the limit $q \to 1$. Interestingly, these equations present a common, solitonlike, traveling solution, which is written in terms of the q-exponential function that naturally emerges within nonextensive statistical mechanics. In all cases, the well-known Einstein energy-momentum relation is preserved for arbitrary values of q.

See also:

R.N. Costa Filho, M.P. Almeida, G.A. Farias and J.S. Andrade, Phys Rev A **84**, 050102(R) (2011)

F.D. Nobre, M.A. Rego-Monteiro and C. T., EPL **97**, 41001 (2012)

S.H. Mazharimousavi, Phys Rev A **85**, 034102 (2012)

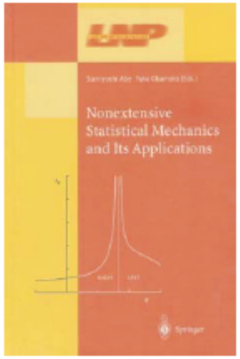
A.R. Plastino and C. T., J Math Phys. **54**, 041505 (2013)

R.N. Costa Filho, G. Alencar, B.S. Skagerstam and J.S. Andrade, EPL **101**, 10009 (2013)

B.G. Costa and E.P. Borges, preprint (2013)

BOOKS AND SPECIAL ISSUES ON NONEXTENSIVE STATISTICAL MECHANICS

1999



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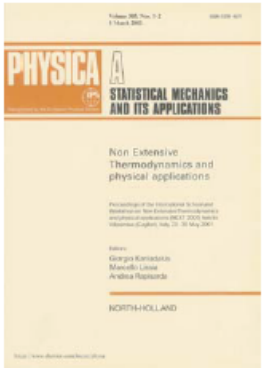

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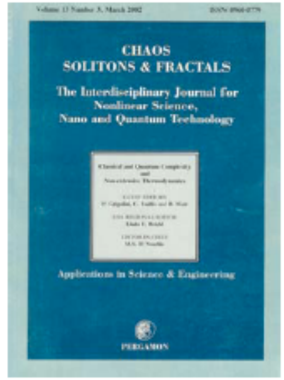

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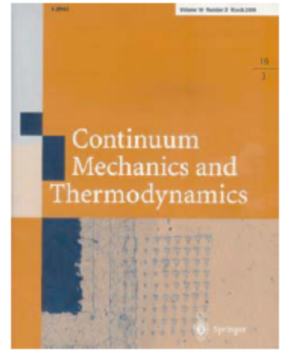

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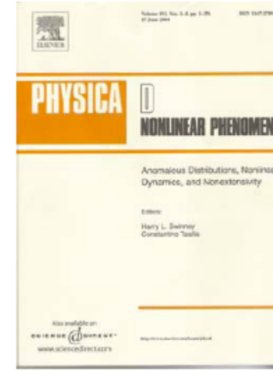
2004



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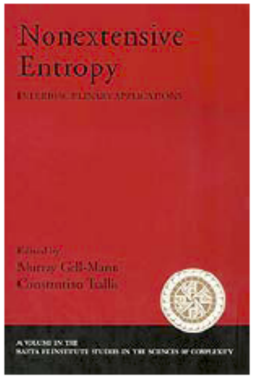

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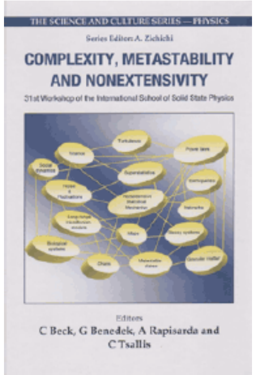
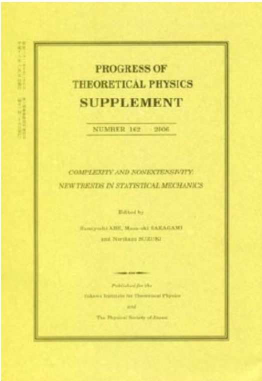


2005

2005

2006



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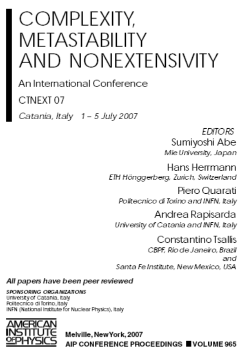
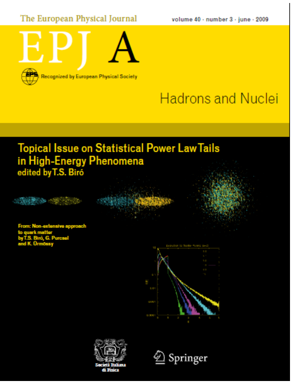


2007



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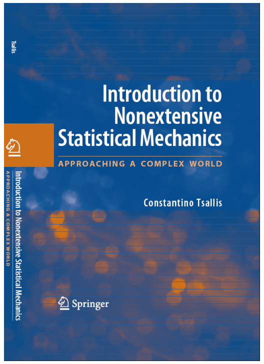

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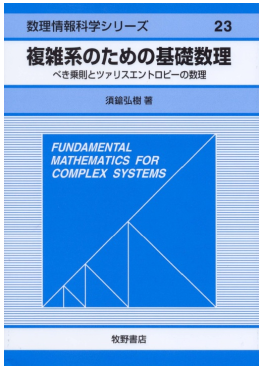

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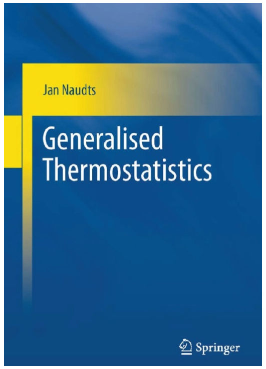

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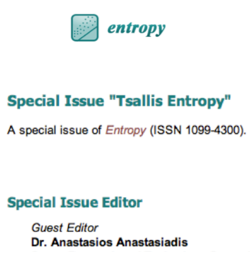

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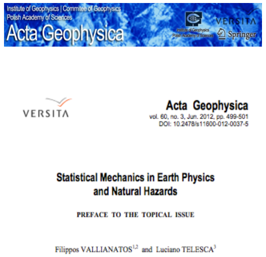

2012

Full bibliography (regularly updated):

http://tsallis.cat.cbpf.br/biblio.htm

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