Coexistence and Competition in Population Models

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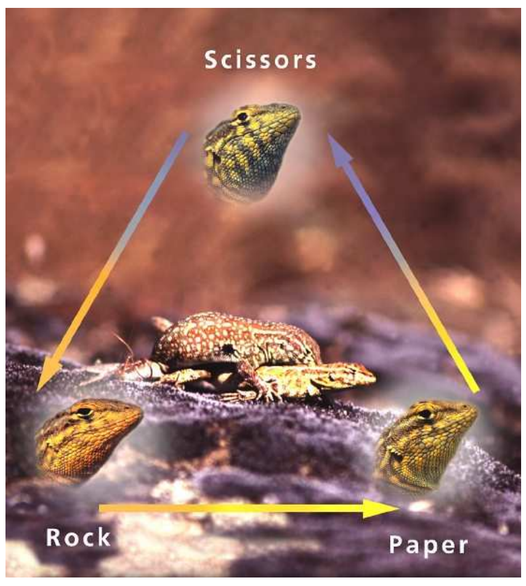
Motivation



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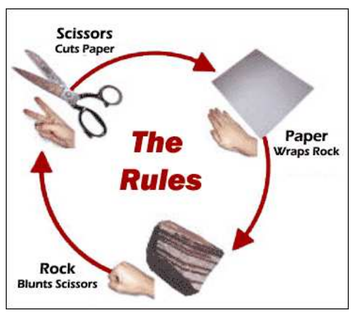
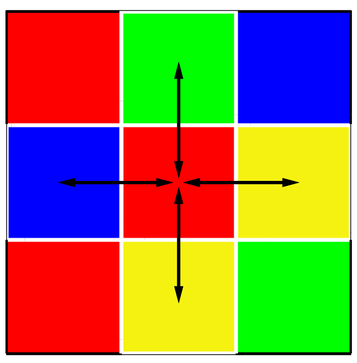
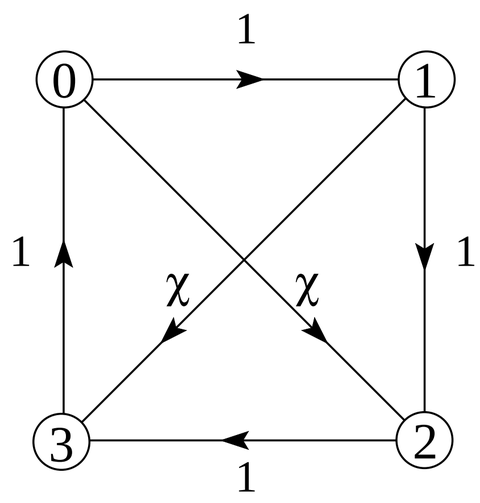
- Cyclical competions play an important role in the evolution of biological systems.
- Rock-paper-scissors-like (RSP) games have interactions like those.
- Some examples are the lizards *Uta stansburiana*, coral reef environments and competing grasses.



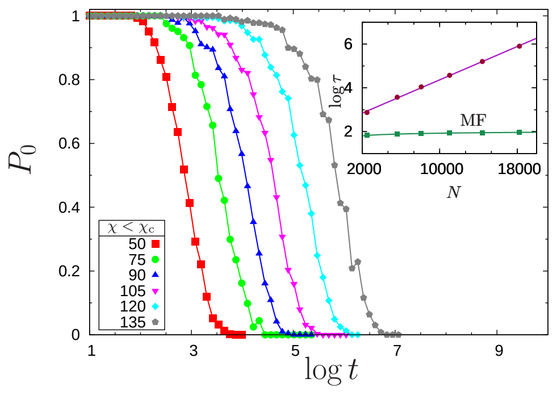
Model

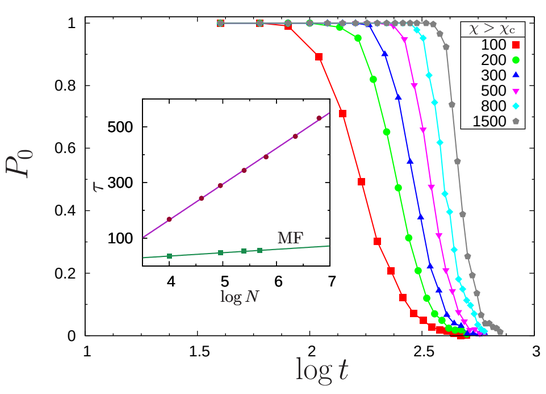
- Each site interacts with one of the nearest neighbors, acording to the interacting graph
- 4 strategies in a square lattice

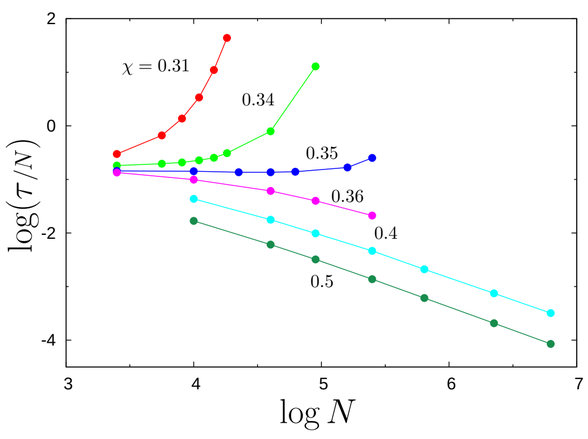




Solutions







Mean Field Aproximation

- Neglecting spatial correlations, the time variations in species densities are described by the mean field equations:

$$\dot{\rho}_i = \sum_{j=0}^{3} I_{ij}\rho_i\rho_j$$

- Each element of the interaction matrix, I_{ij}, is the invasion probability of species i over species j

$$\boldsymbol{I} = \begin{bmatrix} 0 & 1 & \chi & -1 \\ -1 & 0 & 1 & \chi \\ -\chi & -1 & 0 & 1 \\ 1 & -\chi & -1 & 0 \end{bmatrix}$$

Solutions

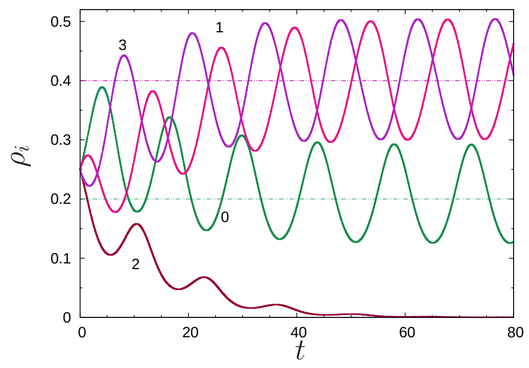
Stable fixed points:

- for $\chi = 0$

$$\left(c_o, \frac{1}{2} - c_o, c_o, \frac{1}{2} - c_o\right)$$

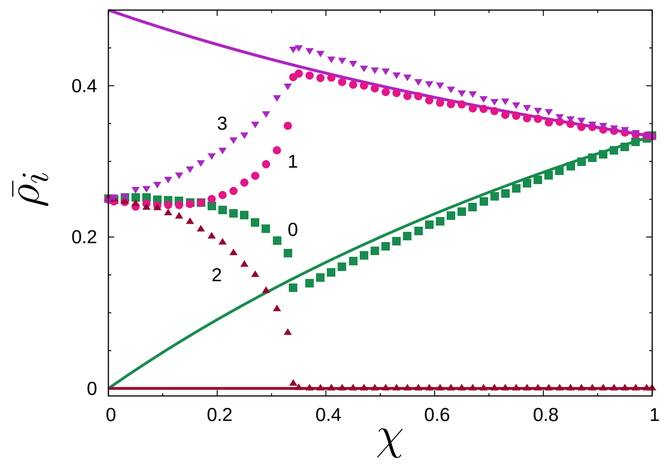
- for $\chi > 0$

$$\left(\frac{\chi}{2+\chi}, \frac{1}{2+\chi}, 0, \frac{1}{2+\chi}\right)$$



Solutions

Comparison between the mean field aproximation and lattice simulations.



Conclusions

- Mean field describes well the asymptotic behavior of the system, for $\chi > \chi_c \simeq 0.35$, where one of the four species is extinct.
- The transition in $\chi \simeq 0.35$ is not verified by the mean field aproximation.
- For $\chi < \chi_c$ the spatial correlation allows the coexistence of all species.

References

- **Evolutionary Games on Graphs**, G. Szabó and G. Fáth, *Phys. Rep.* **446** (2007) 97
- **Evolutionary Dynamics: Exploring the Equation of Life**, M. A. Nowak (2006)
- **Intransitivity and coexistence in four species cyclic games**, A. F. Lütz, S. Risau-Gussman and J. J. Arenzon, *J. Theor. Biol.* **317** (2013) 286