Generalized Allee effect model

Lindomar S. dos Santos[1], Brenno C. T. Cabella[2] Alexandre S. Martinez[3]

[1]lsoaressantos@gmail.com [2]brennousp@usp.br [3]asmartinez@ffclrp.usp.br

[4]Departamento de Física (DF), Faculdade de Filosofia, Ciências e Letras de Ribeirão Preto (FFCLRP), Universidade de São Paulo (USP), Ribeirão Preto - São Paulo, Brazil

Abstract

The Allee effect consists of a positive correlation between very small population size and fitness. Offering a new view point on the weak and strong demographic Allee effect, we propose to combine them with the Richards growth model. Model validation with ecological data is presented for some special situations.

Growth and Allee effect

The *induced saturation function* $G(N)$ [3] represents the *per capita* growth rate $d\ln N/dt = G(N)$. For $G(N) = \kappa_0 > 0$, where κ_0 is the constant intrinsic growth rate, the population grows unbounded (Malthus model). The intraspecific competition can be modeled including the Richards term on $G(p)$: $d\ln(p)/d\tau = -\ln_{\tilde{q}}(p)$, where $p = N/K$, $\tau = \kappa_0 t$ and $\ln_{\tilde{q}}(p)$ is $\tilde{q}$-logarithm function.

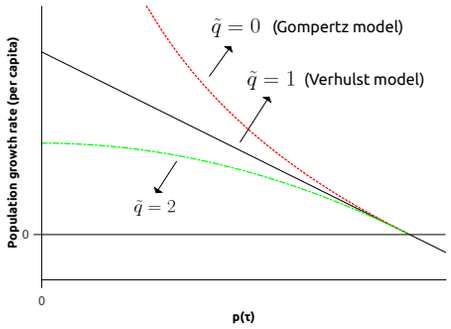

Figure: *Per capita* population growth rate of populations that follow the Richards model. The negative slope of the curves characterizes the intraspecific competition.

Strong and Weak Allee effect

The *strong Allee effect* has a critical population size, the Allee threshold. Populations that exhibit the weak Allee effect have their growth slowed down as the population sizes decreases, but it does not become negative [2] (see Fig. 2).

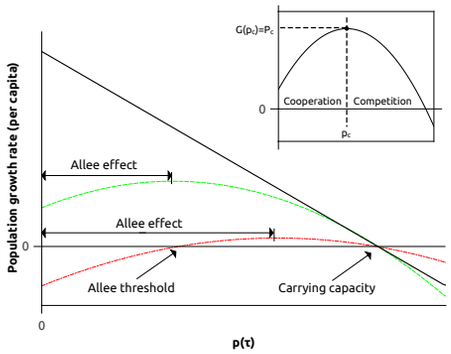

Figure: Curves for the *per capita* population growth rates. The continuous line represents the logistic growth (no Allee effect), the dashed and the dot-dashed lines represent the weak and the strong Allee effects, respectively. Graphically, the Allee effect occurs in the range where the derivative of *per capita* population growth rate with respect to population size is positive. In the weak Allee effect, the *per capita* growth rate curve has a positive y-intercept. Otherwise, the strong Allee effect has a negative y-intercept and the Allee threshold, that delimits the transition between population survival and extinction. Inset: on the left hand side of the dashed vertical line, the benefits of the cooperation outweigh the harms from the intraspecific competition (Allee effect). On the right hand side of the dashed line, the competition outweigh the cooperation. The growth rate is maximum when the effect of the cooperation is balanced by the effect of competition.

Generalized Allee effect

To model the weak and strong Allee effect, consider a population size dependent intrinsic growth rate $\kappa(p) = \kappa_0[1 - A(Kp/K_A - 1)]$, where $K_A > 0$ and A are constants, therefore:

$$G(p) = \frac{d\ln p}{d\tau} = -\left[1 - A\left(\frac{Kp}{K_A} - 1\right)\right]\ln_{\tilde{q}}(p) \ . \quad (1)$$

The Allee effect is suppressed for $A = 0$ and Eq. (1) retrieves the Richards model. Therefore, the parameter A is related to the intensity of intraspecific cooperative effect on the Richards model.

Counts of

Muskox (*Ovibos moschatus*)

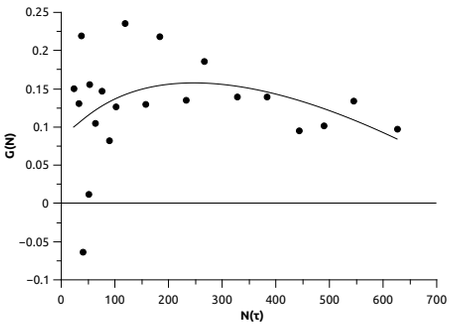

Figure: The graph shows a special case between the weak and strong demographic Allee effect, characterized by the maximum cooperation regime for population that reaches the brink of extinction when its size tends to zero, and the data related to the points were taken directly from the graph of the Ref. [4], which used a database with 1198 population time series from major taxonomic groups and biomes. Trend line represents the fit made in QtiPlot version 0.9.8.8-3 using Eq. (1) without the scaling, i.e., replacing $p(\tau)$ by $N(\tau)$ to make explicit the values of K and K_A. The fitting parameters are: $A \simeq -1$, $\tilde{q} \simeq -0.2$, $K_A \simeq 2,362$ and $K \simeq 846$. It was obtained $R^2 \simeq 0.11$.

Free-living Vancouver Island marmots (*Marmota vancouverensis*)

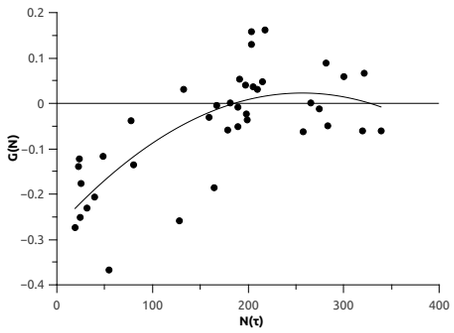

Figure: Data related to the points cover the period between 1970 and 2007 and were taken directly from the graph of the Ref. [5]. The graph shows a range with positive correlation of the *per capita* growth rate with population size - demographic Allee effect - and a negative y-intercept - strong Allee effect. Trend line represents the fit made using Eq. (1) in QtiPlot version 0.9.8.8-3. Fixing $\tilde{q} = 1$, the fitting parameters are: $A \simeq -1.27$, $K_A \simeq 1524$ and $K \simeq 186$. It was obtained $R^2 \simeq 0.55$.

Conclusions

We have presented a simple analytical model that unifies the weak and strong demographic Allee effects and the Richards growth model, which has the Verhulst and Gompertz models as particular cases. Our generalized model expands the range of descriptive possibilities of species population dynamics that exhibit Allee effect, since it permits a new view point based on a broad and well established growth models framework.

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