

MECÂNICA ESTATÍSTICA NÃO EXTENSIVA: DESAFIOS ATUAIS

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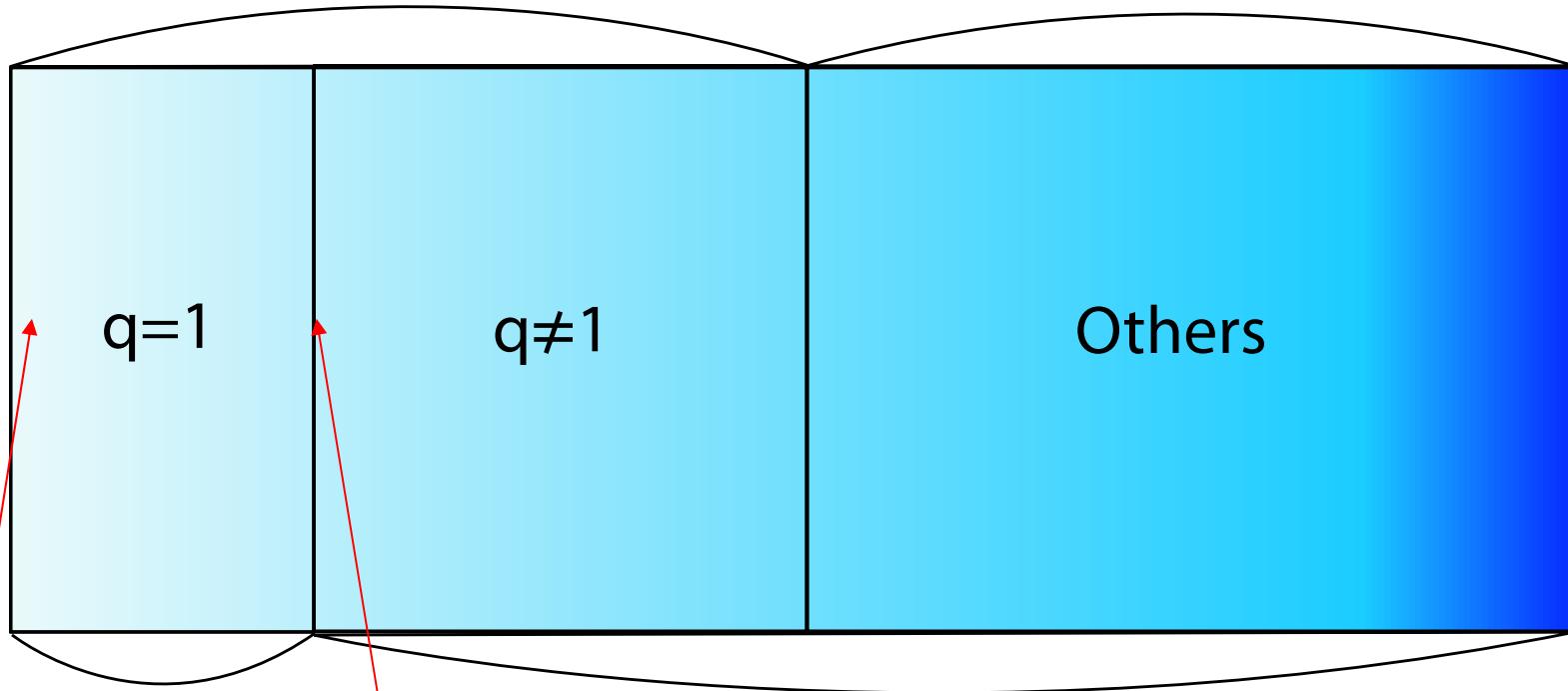
Alguns conceitos

Algumas das verificações
experimentais e computacionais

Alguns dos desafios

q-describable

non q-describable



IDEAL GAS

CRITICAL PHENOMENA

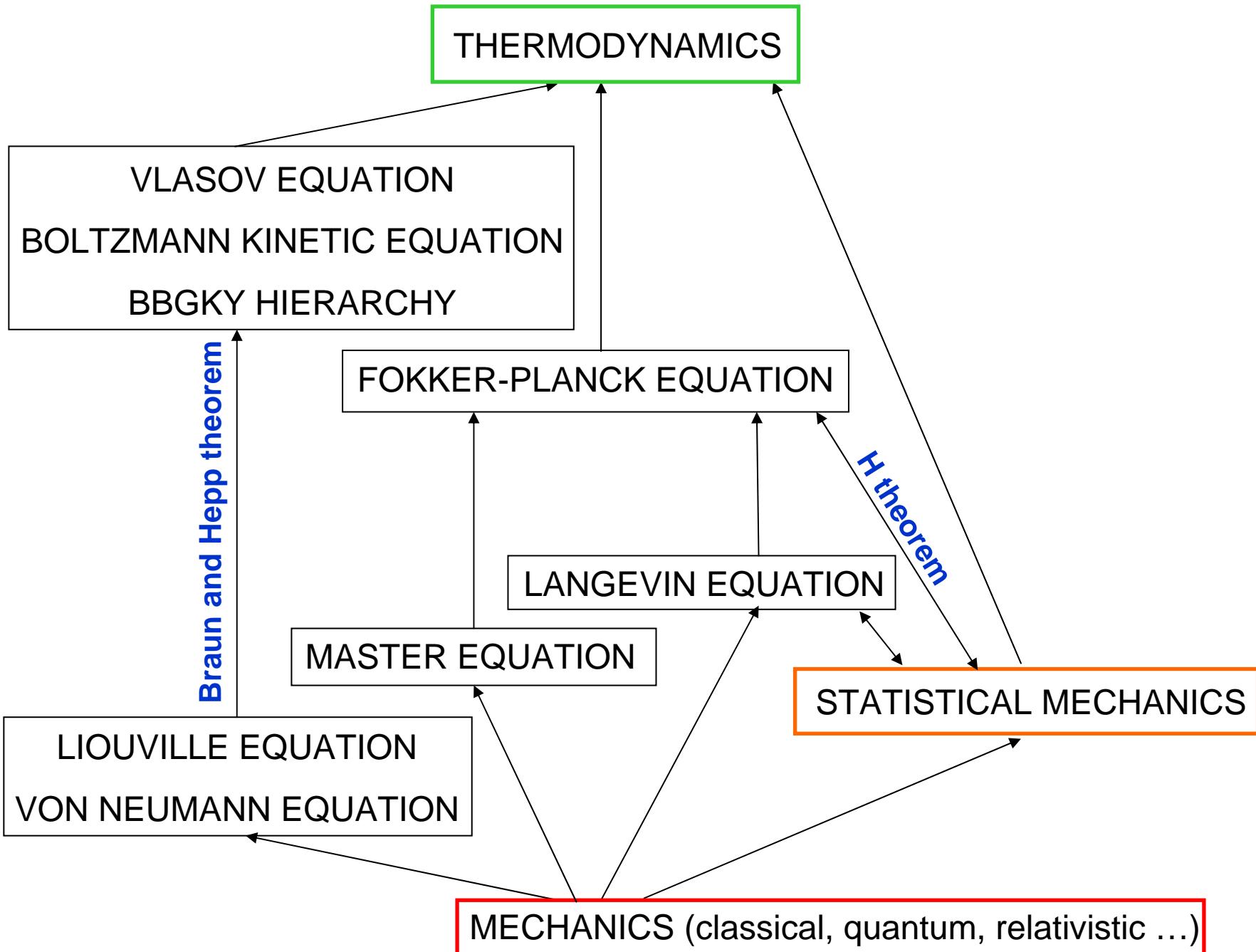
$$q = \frac{1+\delta}{2}$$

[A. Robledo, Mol Phys 103 (2005) 3025]

$$q = \frac{\sqrt{9+c^2} - 3}{c}$$

[F. Caruso and C. T., Phys Rev E 78 (2008) 021101]

C.T., M. Gell-Mann and Y. Sato
Europhysics News 36 (6), 186
(European Physical Society,
2005)



ADDITIVITY: O. Penrose, *Foundations of Statistical Mechanics: A Deductive Treatment* (Pergamon, Oxford, 1970), page 167

An entropy is **additive** if, for two **probabilistically independent** systems A and B ,

$$S(A + B) = S(A) + S(B)$$

Hence, S_{BG} and $S_q^{Renyi} (\forall q)$ are additive, and $S_q (\forall q \neq 1)$ is nonadditive.

EXTENSIVITY:

Consider a system $\Sigma \equiv A_1 + A_2 + \dots + A_N$ made of N (not necessarily independent) identical elements or subsystems A_1 and A_2, \dots, A_N . An entropy is **extensive** if

$$0 < \lim_{N \rightarrow \infty} \frac{S(N)}{N} < \infty, \text{ i.e., } S(N) \propto N \quad (N \rightarrow \infty)$$

CONSEQUENTLY:

The **additive entropies** S_{BG} and S_q^{Renyi} are **extensive if and only if** the N subsystems are (strictly or asymptotically) independent; otherwise, S_{BG} and S_q^{Renyi} are **nonextensive**.

The **nonadditive entropy** $S_q (q \neq 1)$ is **extensive for special values of q** if the subsystems are specially (globally) correlated.

Nonadditive entropy reconciles the area law in quantum systems with classical thermodynamics

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The Boltzmann–Gibbs–von Neumann entropy of a large part (of linear size L) of some (much larger) d -dimensional quantum systems follows the so-called area law (as for black holes), i.e., it is proportional to L^{d-1} . Here we show, for $d=1, 2$, that the (nonadditive) entropy S_q satisfies, for a special value of $q \neq 1$, the classical thermodynamical prescription for the entropy to be extensive, i.e., $S_q \propto L^d$. Therefore, we reconcile with classical thermodynamics the area law widespread in quantum systems. Recently, a similar behavior was exhibited in mathematical models with scale-invariant correlations [C. Tsallis, M. Gell-Mann, and Y. Sato, Proc. Natl. Acad. Sci. U.S.A. **102** 15377 (2005)]. Finally, we find that the system critical features are marked by a maximum of the special entropic index q .

SPIN $\frac{1}{2}$ XY FERROMAGNET WITH TRANSVERSE MAGNETIC FIELD:

$$\hat{\mathcal{H}} = - \sum_{j=1}^{N-1} [(1 + \gamma) \hat{\sigma}_j^x \hat{\sigma}_{j+1}^x + (1 - \gamma) \hat{\sigma}_j^y \hat{\sigma}_{j+1}^y + 2\lambda \hat{\sigma}_j^z]$$

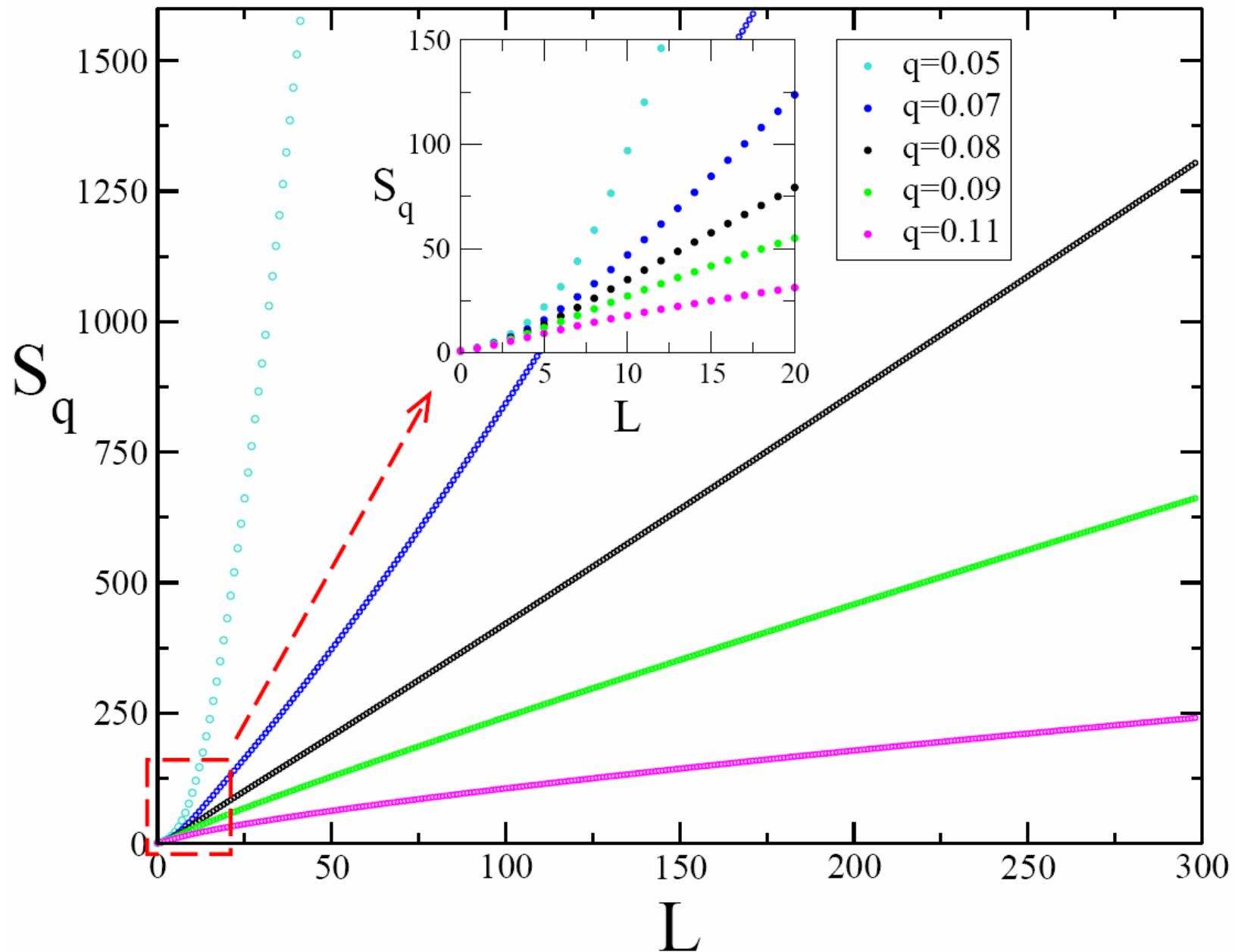
$|\gamma| = 1$ \rightarrow *Ising ferromagnet*

$0 < |\gamma| < 1$ \rightarrow *anisotropic XY ferromagnet*

$\gamma = 0$ \rightarrow *isotropic XY ferromagnet*

λ \equiv *transverse magnetic field*

L \equiv *length of a block within a $N \rightarrow \infty$ chain*



*Using a Quantum Field Theory result
in P. Calabrese and J. Cardy, JSTAT P06002 (2004)
we obtain, at the critical transverse magnetic field,*

$$q_{ent} = \frac{\sqrt{9+c^2} - 3}{c}$$

with $c \equiv$ central charge in conformal field theory

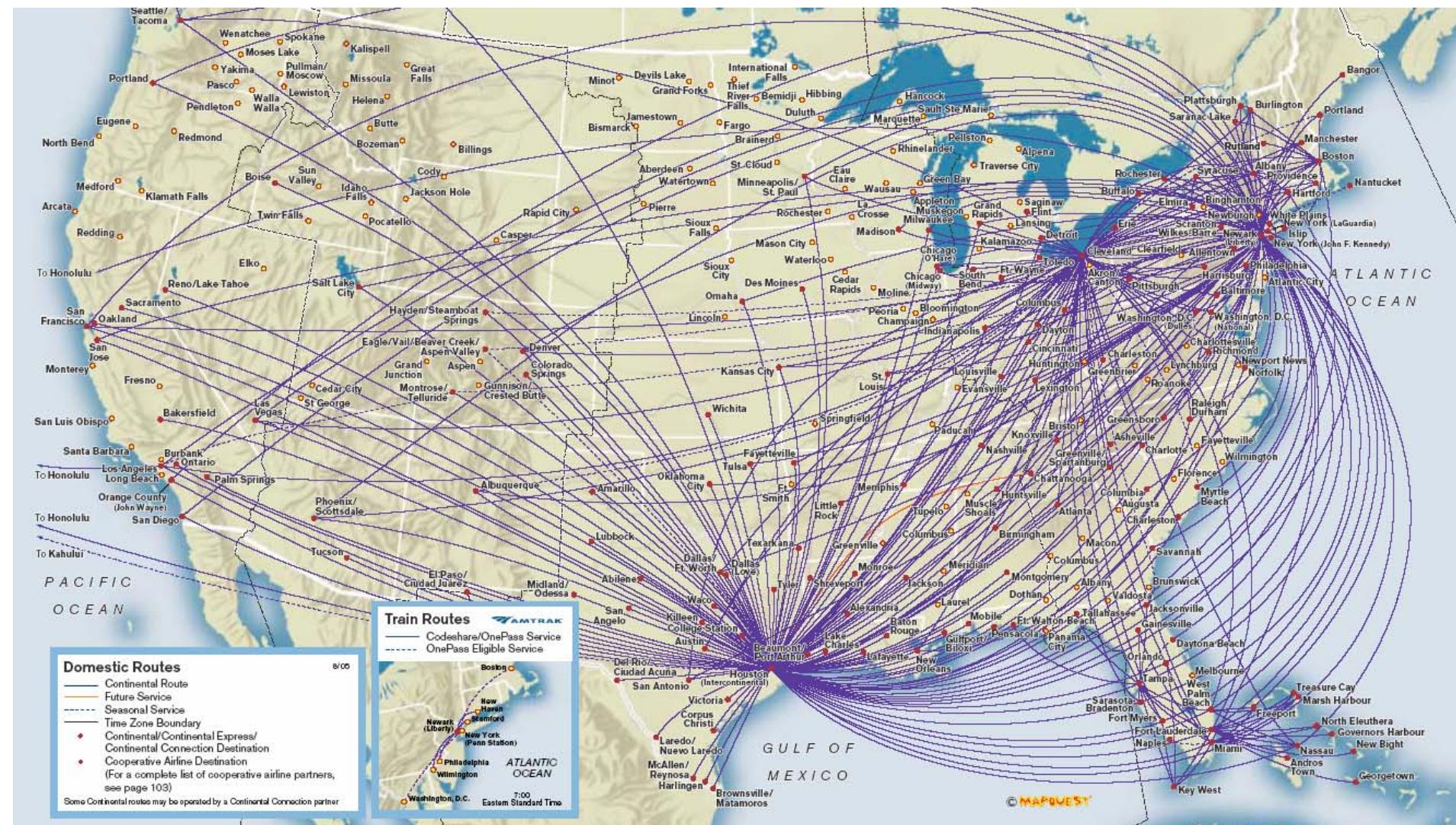
Hence

Ising and anisotropic XY ferromagnets $\Rightarrow c = \frac{1}{2} \Rightarrow q_{ent} = \sqrt{37} - 6 \approx 0.0828$

and

Isotropic XY ferromagnet $\Rightarrow c = 1 \Rightarrow q_{ent} = \sqrt{10} - 3 \approx 0.1623$

SYSTEMS	ENTROPY S_{BG} (additive)	ENTROPY S_q ($q < 1$) (nonadditive)
Short-range interactions, weakly entangled blocks, etc	EXTENSIVE	NONEXTENSIVE
Long-range interactions (QSS), strongly entangled blocks, etc	NONEXTENSIVE	EXTENSIVE



q - GENERALIZED CENTRAL LIMIT THEOREM:

S. Umarov, C.T. and S. Steinberg, Milan J Math **76**, 307 (2008)

q -Fourier transform:

$$F_q[f](\xi) \equiv \int_{-\infty}^{\infty} e_q^{ix\xi} \otimes_q f(x) dx = \int_{-\infty}^{\infty} e_q^{ix\xi[f(x)]^{q-1}} f(x) dx$$
$$(q \geq 1)$$

(nonlinear!)

CENTRAL LIMIT THEOREM

$N^{1/[\alpha(2-q)]}$ -scaled attractor $\mathbb{F}(x)$ when summing $N \rightarrow \infty$ q -independent identical random variables

with symmetric distribution $f(x)$ with $\sigma_Q \equiv \int dx x^2 [f(x)]^Q / \int dx [f(x)]^Q$ $\left(Q \equiv 2q-1, q_1 = \frac{1+q}{3-q}\right)$

	$q = 1$ [independent]	$q \neq 1$ (i.e., $Q \equiv 2q-1 \neq 1$) [globally correlated]
$\sigma_Q < \infty$ $(\alpha = 2)$	$\mathbb{F}(x) = \text{Gaussian } G(x),$ with same σ_1 of $f(x)$ Classic CLT	$\mathbb{F}(x) = G_q(x) \equiv G_{(3q_1-1)/(1+q_1)}(x)$, with same σ_Q of $f(x)$ $G_q(x) \sim \begin{cases} G(x) & \text{if } x \ll x_c(q, 2) \\ f(x) \sim C_q / x ^{2/(q-1)} & \text{if } x \gg x_c(q, 2) \end{cases}$ $\text{with } \lim_{q \rightarrow 1} x_c(q, 2) = \infty$ S. Umarov, C. T. and S. Steinberg, Milan J Math 76, 307 (2008)
$\sigma_Q \rightarrow \infty$ $(0 < \alpha < 2)$	$\mathbb{F}(x) = \text{Levy distribution } L_\alpha(x),$ with same $ x \rightarrow \infty$ behavior $L_\alpha(x) \sim \begin{cases} G(x) & \text{if } x \ll x_c(1, \alpha) \\ f(x) \sim C_\alpha / x ^{1+\alpha} & \text{if } x \gg x_c(1, \alpha) \end{cases}$ $\text{with } \lim_{\alpha \rightarrow 2} x_c(1, \alpha) = \infty$ Levy-Gnedenko CLT	$\mathbb{F}(x) = L_{q,\alpha}$, with same $ x \rightarrow \infty$ asymptotic behavior $L_{q,\alpha} \sim \begin{cases} G_{\frac{2(1-q)-\alpha(1+q)}{2(1-q)-\alpha(3-q)}, \alpha}(x) \sim C_{q,\alpha}^* / x ^{\frac{2(1-q)-\alpha(3-q)}{2(1-q)}} & \text{(intermediate regime)} \\ G_{\frac{2\alpha q - \alpha + 3}{\alpha + 1}, 2}(x) \sim C_{q,\alpha}^L / x ^{(1+\alpha)/(1+\alpha q - \alpha)} & \text{(distant regime)} \end{cases}$ S. Umarov, C. T., M. Gell-Mann and S. Steinberg cond-mat/0606038v2 and cond-mat/0606040v2 (2008)

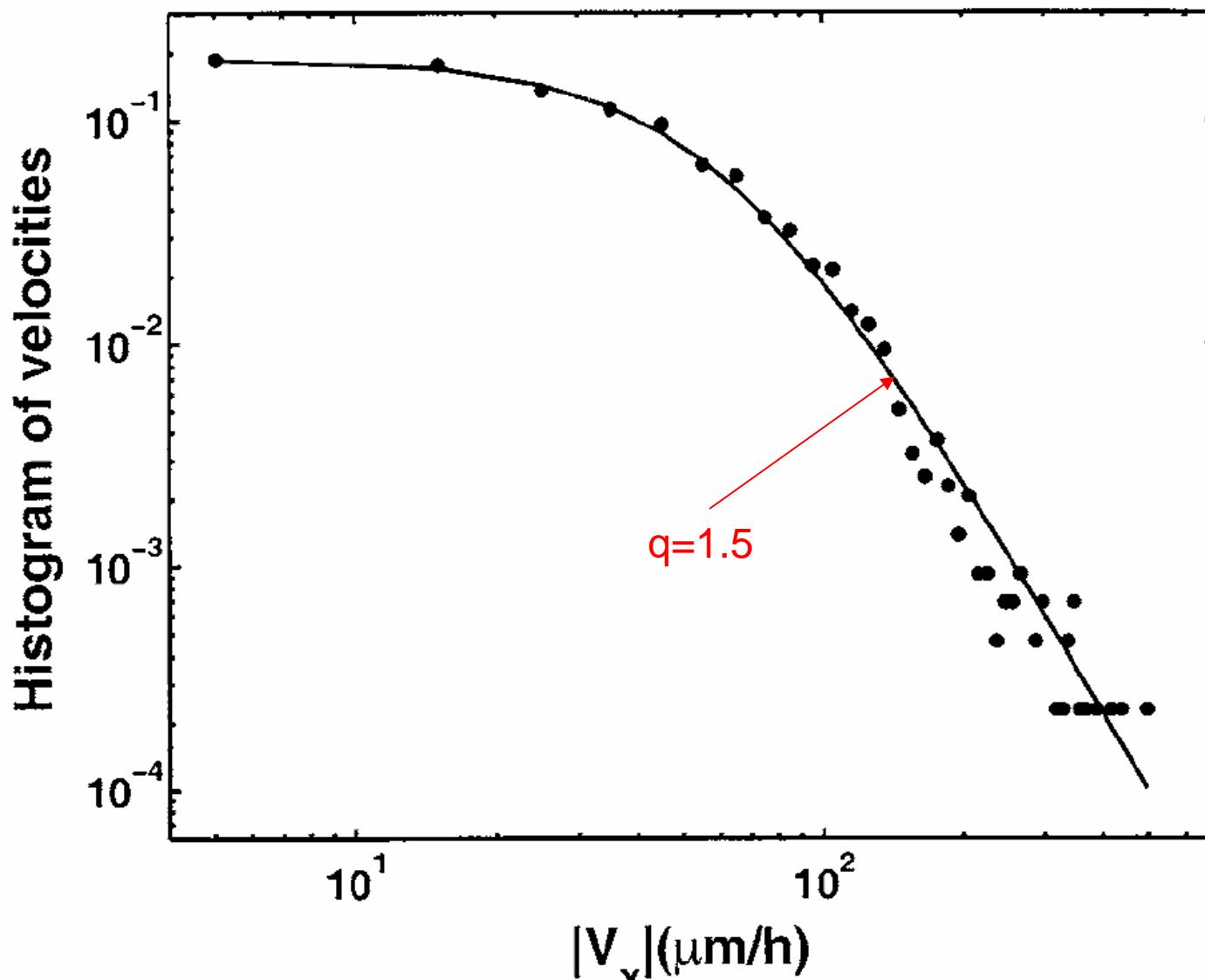
Alguns conceitos

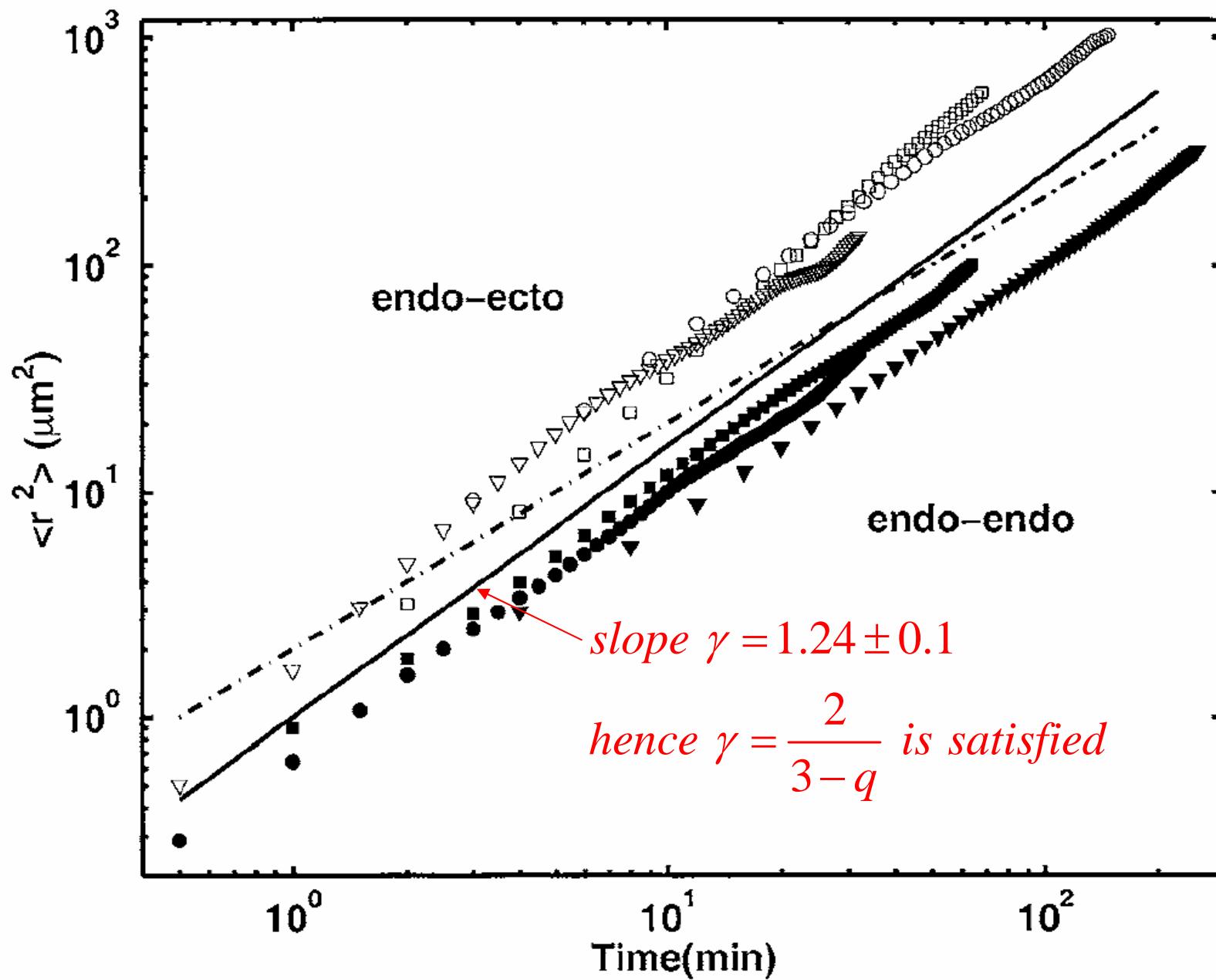
**Algumas das verificações
experimentais e computacionais**

Alguns dos desafios

Hydra viridissima:

A Upadhyaya, J-P Rieu, JA Glazier and Y Sawada, Physica A **293**, 549 (2001)





PHYSICAL REVIEW A 67, 051402(R) (2003)

Anomalous diffusion and Tsallis statistics in an optical lattice

Eric Lutz

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(Received 26 February 2003; published 27 May 2003)

We point out a connection between anomalous transport in an optical lattice and Tsallis' generalized statistics. Specifically, we show that the momentum equation for the semiclassical Wigner function which describes atomic motion in the optical potential, belongs to a class of transport equations recently studied by Borland [Phys. Lett. A 245, 67 (1998)]. The important property of these ordinary linear Fokker-Planck equations is that their stationary solutions are exactly given by Tsallis distributions. An analytical expression of the Tsallis index q in terms of the microscopic parameters of the quantum-optical problem is given and the spatial coherence of the atomic wave packets is discussed.

(i) The distribution of atomic velocities is a q -Gaussian;

$$(ii) \quad q = 1 + \frac{44E_R}{U_0} \quad \text{where} \quad E_R \equiv \text{recoil energy}$$

$U_0 \equiv$ potential depth

Experimental and computational verifications in optical lattices:

PRL 96, 110601 (2006)

PHYSICAL REVIEW LETTERS

week ending
24 MARCH 2006

Tunable Tsallis Distributions in Dissipative Optical Lattices

P. Douglas, S. Bergamini, and F. Renzoni

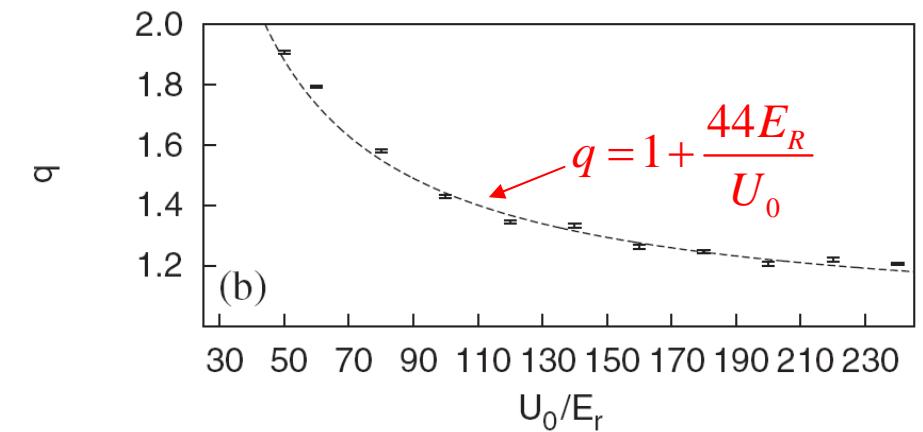
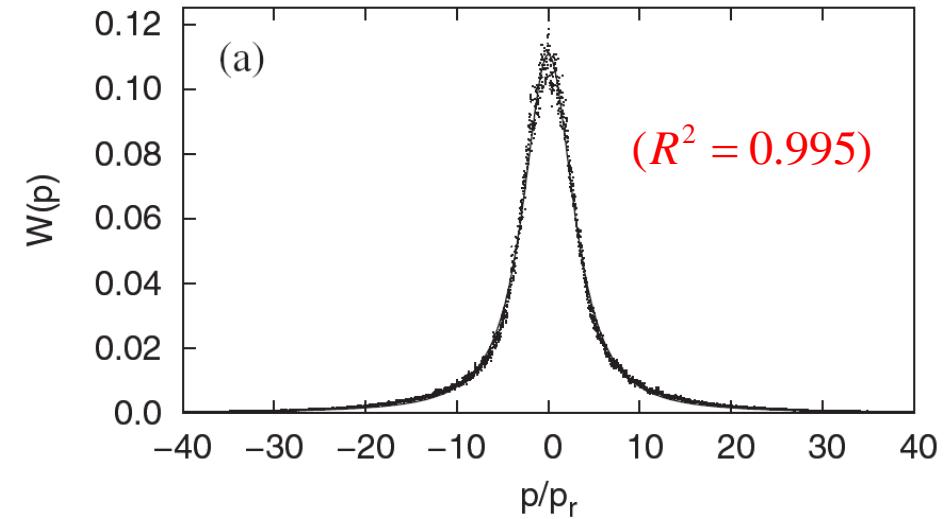
Department of Physics and Astronomy, University College London, Gower Street, London WC1E 6BT, United Kingdom

(Received 10 January 2006; published 24 March 2006)

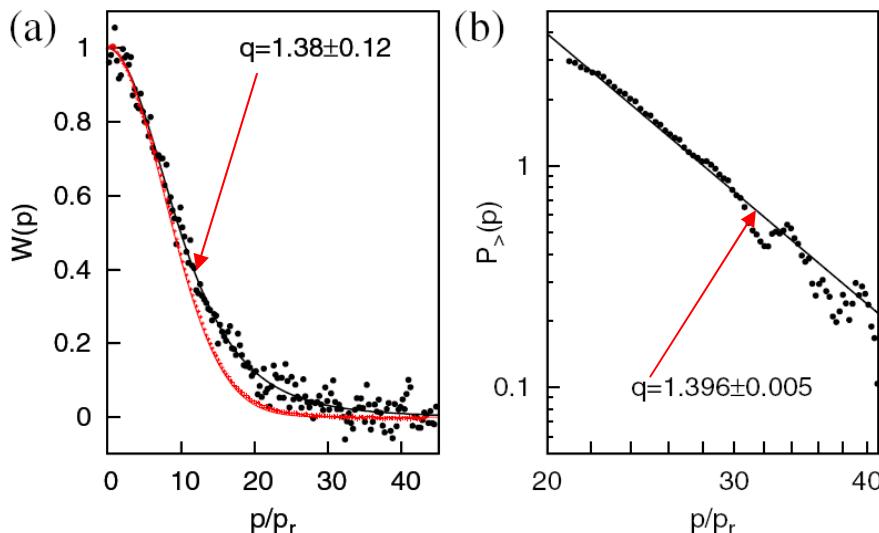
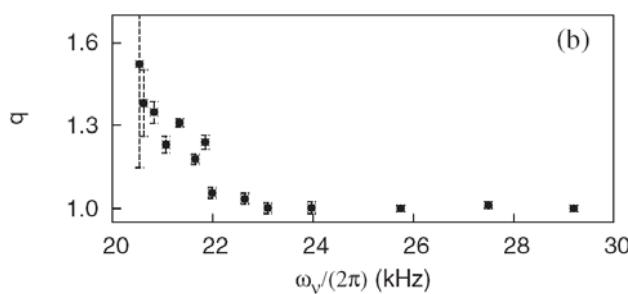
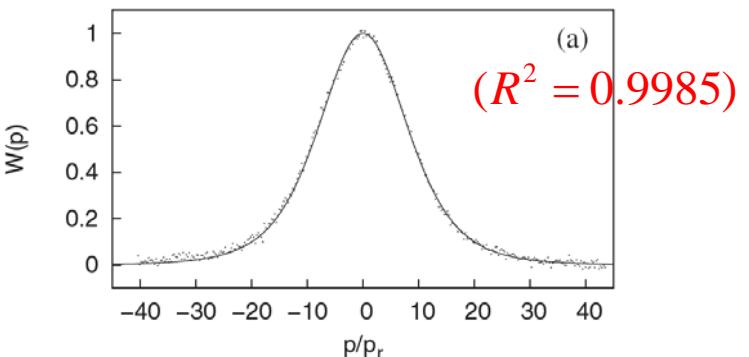
We demonstrated experimentally that the momentum distribution of cold atoms in dissipative optical lattices is a Tsallis distribution. The parameters of the distribution can be continuously varied by changing the parameters of the optical potential. In particular, by changing the depth of the optical lattice, it is possible to change the momentum distribution from Gaussian, at deep potentials, to a power-law tail distribution at shallow optical potentials.

Experimental and computational verifications

by P. Douglas, S. Bergamini and F. Renzoni, Phys Rev Lett **96**, 110601 (2006)



**(Computational verification:
quantum Monte Carlo simulations)**



(Experimental verification: Cs atoms)

Superdiffusion and Non-Gaussian Statistics in a Driven-Dissipative 2D Dusty Plasma

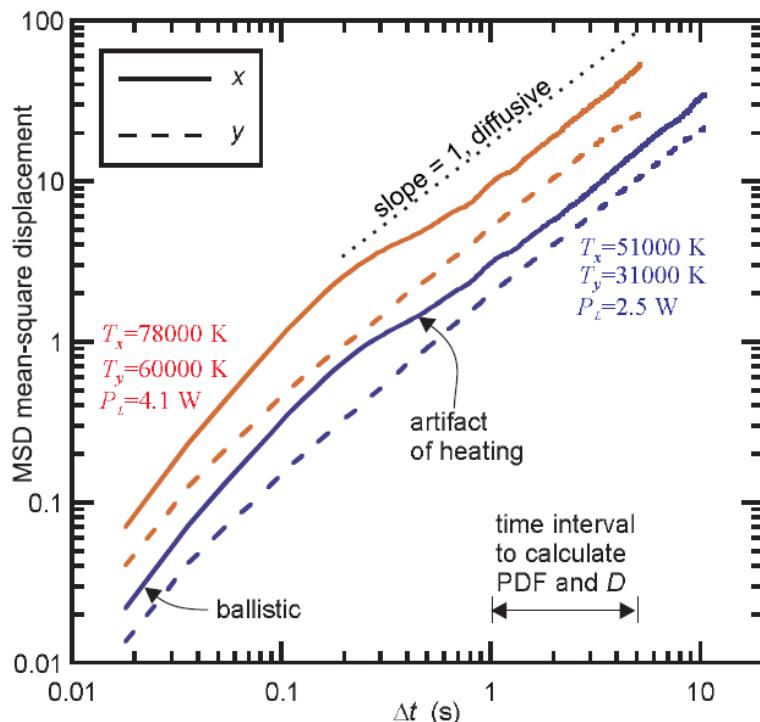
Bin Liu and J. Goree

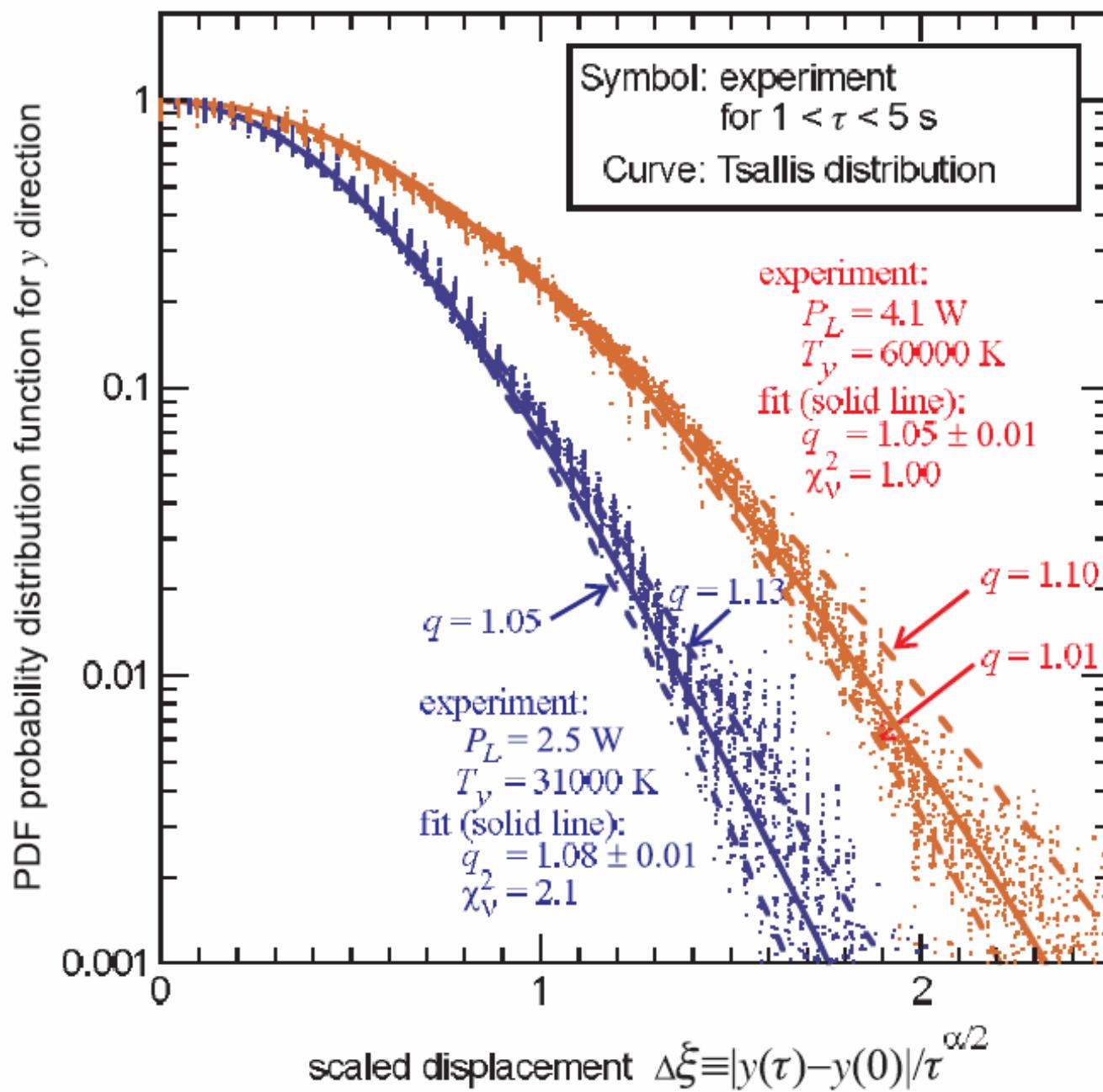
Department of Physics and Astronomy, The University of Iowa, Iowa City, Iowa 52242, USA

(Received 1 June 2007; published 6 February 2008)

Anomalous diffusion and non-Gaussian statistics are detected experimentally in a two-dimensional driven-dissipative system. A single-layer dusty plasma suspension with a Yukawa interaction and frictional dissipation is heated with laser radiation pressure to yield a structure with liquid ordering. Analyzing the time series for mean-square displacement, superdiffusion is detected at a low but statistically significant level over a wide range of temperatures. The probability distribution function fits a Tsallis distribution, yielding q , a measure of nonextensivity for non-Gaussian statistics.

$$\langle r^2 \rangle \propto t^\alpha$$





Power-Law Distributions for a Trapped Ion Interacting with a Classical Buffer Gas

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(Received 3 November 2008; published 10 February 2009)

Classical collisions with an ideal gas generate non-Maxwellian distribution functions for a single ion in a radio frequency ion trap. The distributions have power-law tails whose exponent depends on the ratio of buffer gas to ion mass. This provides a statistical explanation for the previously observed transition from cooling to heating. Monte Carlo results approximate a Tsallis distribution over a wide range of parameters and have *ab initio* agreement with experiment.

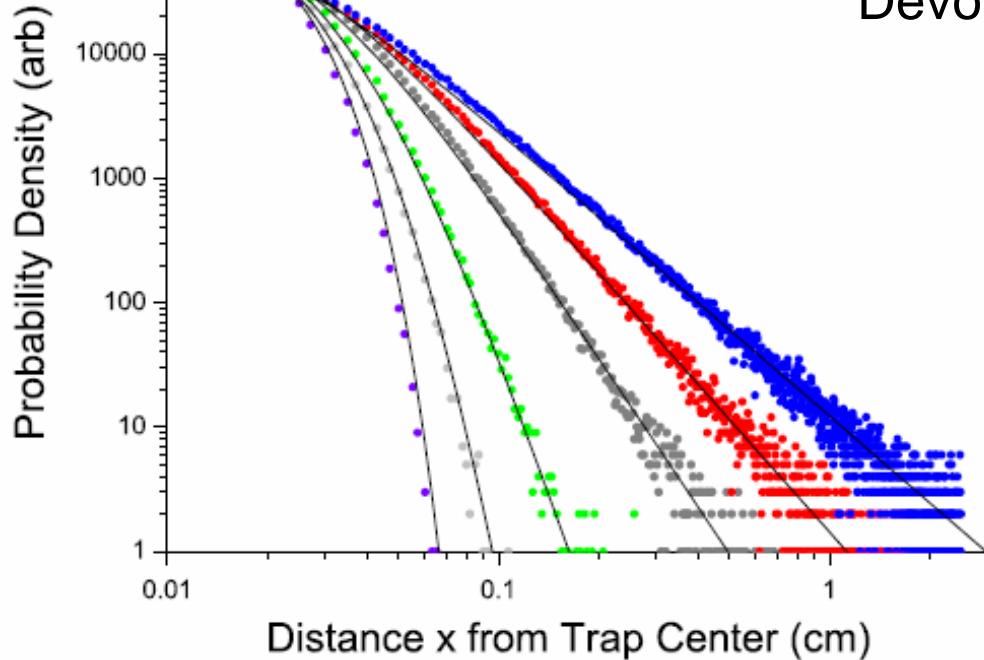


FIG. 1 (color online). Monte Carlo distributions for a single $^{136}\text{Ba}^+$ ion cooled by six different buffer gases at 300 K ranging from $m_B = 4$ (left) to $m_B = 200$ (right). Note the evolution from Gaussian to power law (straight line) as the mass increases. The solid lines are Tsallis functions [Eq. (7)] with fixed $\sigma = 0.0185$ cm and the exponents of Table I.

$$T(x) = \frac{T(0)}{\left[1 + (q-1)\left(\frac{x}{\sigma}\right)^2\right]^{\frac{1}{q-1}}}$$

TABLE I. Tsallis parameters n and q_T fit from Fig. 1.

Buffer gas	m_I/m_B	n	q_T
He	34.5	>60	1.03
Ar	3.40	8.2	1.12
Kr	1.70	3.8	1.26
Xe	1.0	1.98	1.51
170	0.80	1.50	1.80
200	0.68	1.15	1.87

Generalized Spin-Glass Relaxation

R. M. Pickup,¹ R. Cywinski,^{2,*} C. Pappas,³ B. Farago,⁴ and P. Fouquet⁴

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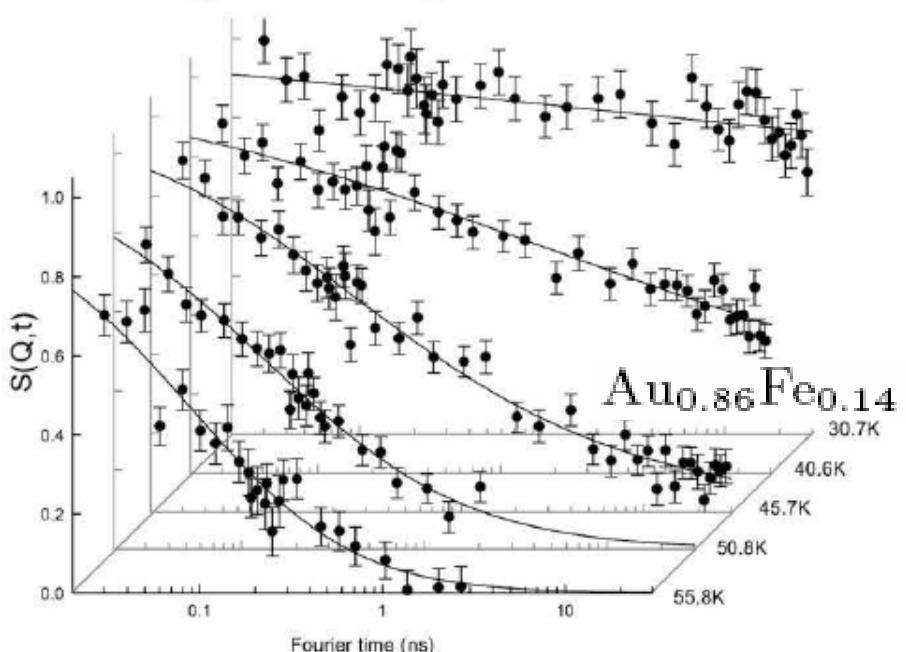
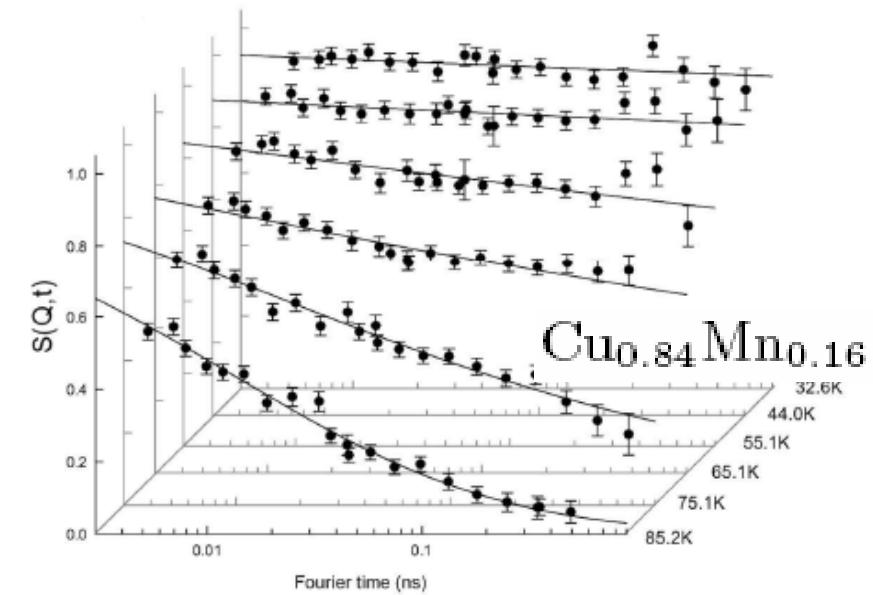
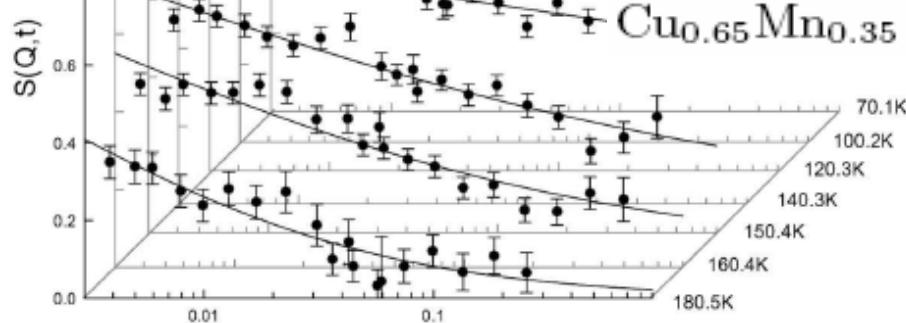
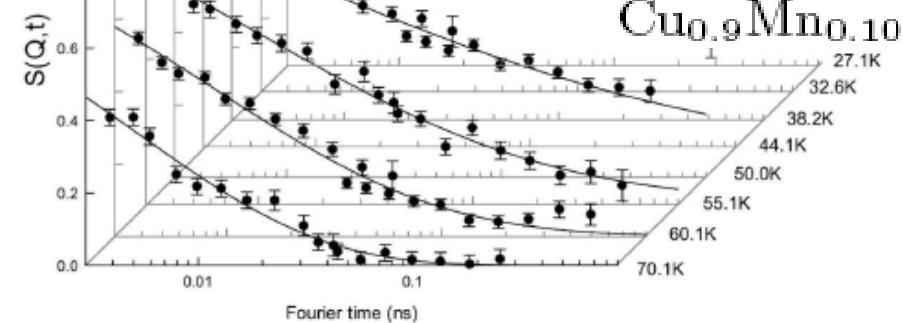
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(Received 18 July 2008; published 4 March 2009)

Spin relaxation close to the glass temperature of CuMn and AuFe spin glasses is shown, by neutron spin echo, to follow a generalized exponential function which explicitly introduces hierarchically constrained dynamics and macroscopic interactions. The interaction parameter is directly related to the normalized Tsallis nonextensive entropy parameter q and exhibits universal scaling with reduced temperature. At the glass temperature $q = 5/3$ corresponding, within Tsallis' q statistics, to a mathematically defined critical value for the onset of strong disorder and nonlinear dynamics.

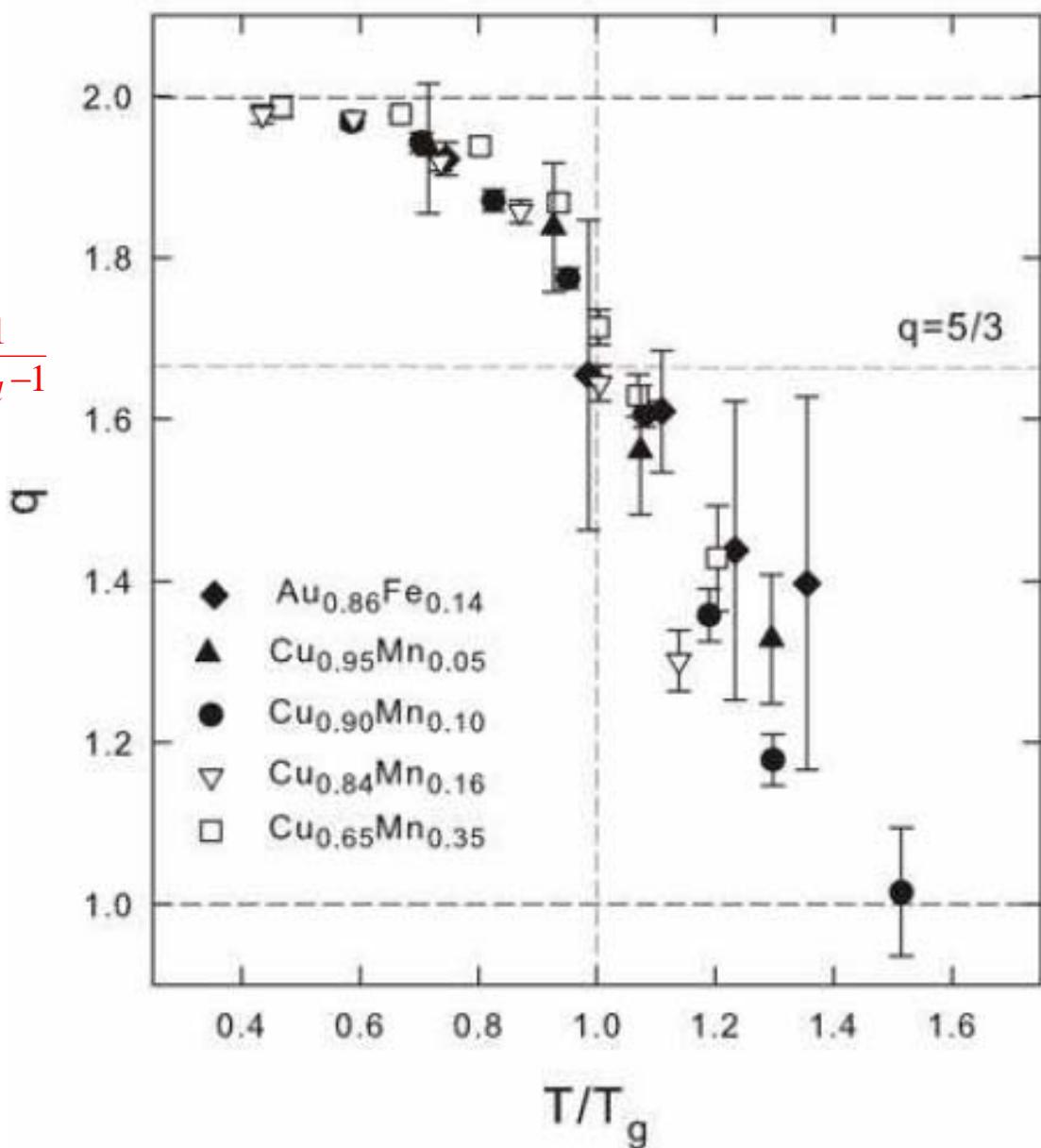


SPIN RELAXATION IN SPIN GLASSES (NEUTRON SPIN ECHO):

$$\phi(t) = \left[1 + \frac{q-1}{2-q} \left(\frac{t}{\tau} \right)^\beta \right]^{-\frac{2-q}{q-1}}$$

$$\equiv \left[1 + (q_{rel} - 1) \left(\frac{t}{\tau} \right)^\beta \right]^{-\frac{1}{q_{rel}-1}}$$

$$q_{rel} \equiv \frac{1}{2-q}$$



Alguns conceitos

Algumas das verificações experimentais e computacionais

Alguns dos desafios



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Physica A 356 (2005) 375–384

PHYSICA A

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Triangle for the entropic index q of non-extensive statistical mechanics observed by Voyager 1 in the distant heliosphere

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Greenbelt, MD 20771, USA*

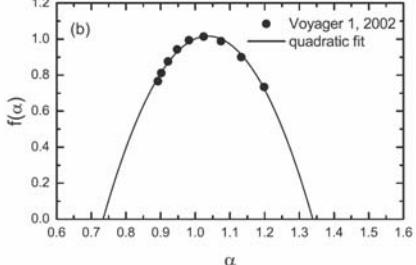
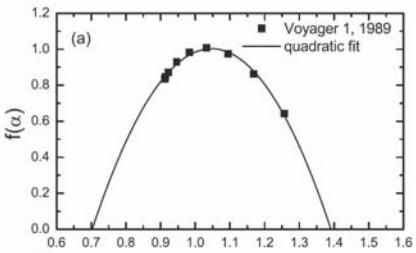
Received 10 June 2005

Available online 11 July 2005

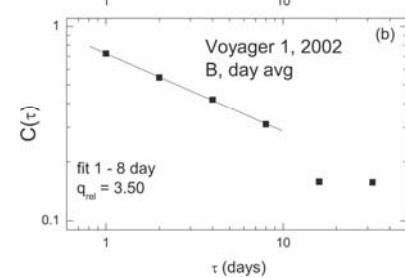
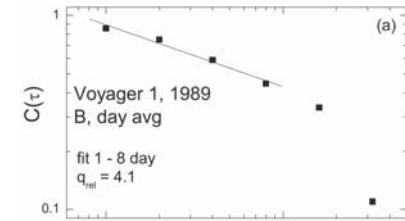
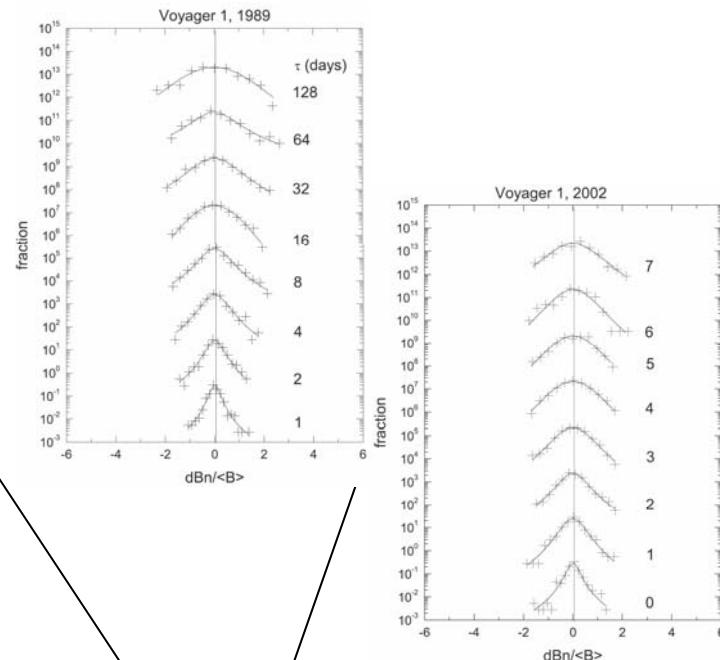
SOLAR WIND: Magnetic Field Strength

L.F. Burlaga and A. F.-Vinas (2005) / NASA Goddard Space Flight Center; Physica A **356**, 375 (2005)

[Data: Voyager 1 spacecraft (1989 and 2002); 40 and 85 AU; **daily averages**]



$$q_{sen} = -0.6 \pm 0.2$$



$$q_{rel} = 3.8 \pm 0.3$$

$$q_{stat} = 1.75 \pm 0.06$$

Playing with additive duality ($q \rightarrow 2 - q$)

and with multiplicative duality ($q \rightarrow 1/q$)

(and using numerical results related to the q -generalized central limit theorem)

we conjecture

$$q_{rel} + \frac{1}{q_{sen}} = 2 \quad \text{and} \quad q_{stat} + \frac{1}{q_{rel}} = 2$$

$$\text{hence} \quad 1 - q_{sen} = \frac{1 - q_{stat}}{3 - 2 q_{stat}}$$

hence only one independent!

Burlaga and Vinas (NASA) most precise value of the q -triplet is

$$q_{stat} = 1.75 = 7/4$$

$q_{sen} = -0.5 = -1/2$ (consistent with $q_{sen} = -0.6 \pm 0.2$!)

and $q_{rel} = 4$ (consistent with $q_{rel} = -3.8 \pm 0.3$!)

$$\varepsilon_{sen} \equiv 1 - q_{sen} = 1 - (-1/2) = 3/2$$

$$\varepsilon_{rel} \equiv 1 - q_{rel} = 1 - 4 = -3$$

$$\varepsilon_{stat} \equiv 1 - q_{stat} = 1 - 7/4 = -3/4$$

We verify

$$\varepsilon_{stat} = \frac{\varepsilon_{sen} + \varepsilon_{rel}}{2} \quad (\text{arithmetic mean!})$$

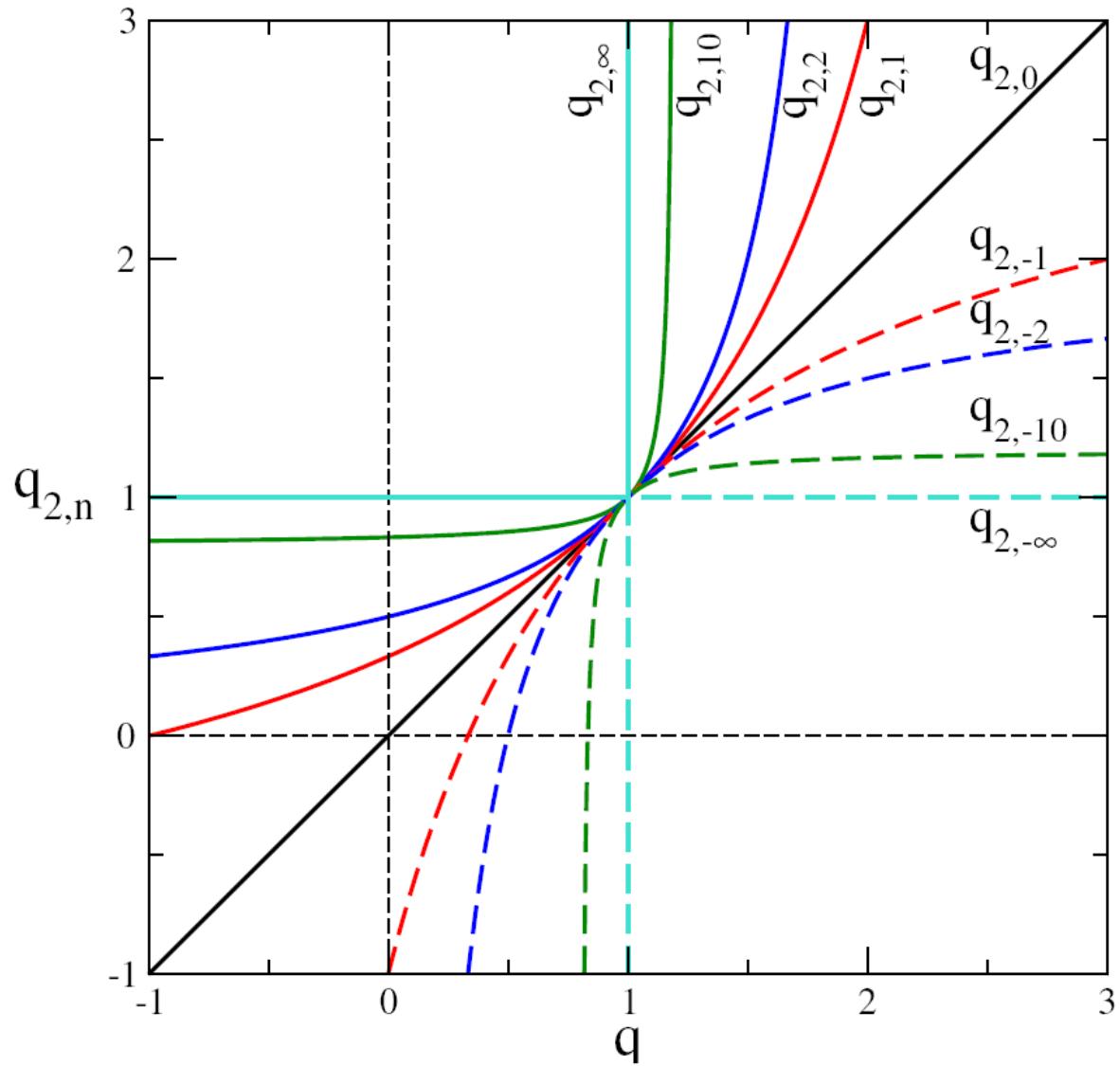
$$\varepsilon_{sen} = \sqrt{\varepsilon_{stat} \varepsilon_{rel}} \quad (\text{geometric mean!})$$

$$\varepsilon_{rel}^{-1} = \frac{\varepsilon_{sen}^{-1} + \varepsilon_{stat}^{-1}}{2} \quad (\text{harmonic mean!})$$

ALGEBRA ASSOCIATED WITH q -GENERALIZED CENTRAL LIMIT THEOREMS:

$$\frac{\alpha}{1-q_{\alpha,n}} = \frac{\alpha}{1-q} + n$$

$(n = 0, \pm 1, \pm 2, \dots)$



EDGE OF CHAOS OF THE LOGISTIC MAP:

$$q\text{-triplet} \left\{ \begin{array}{lcl} q_{sensitivity} & = & q_{ent\ production} = 0.244487701341282066198... \\ \\ q_{relaxation} & = & 2.249784109... \\ \\ q_{stationary\ state} & = & 1.65 \pm 0.05 \end{array} \right.$$

WHAT IS THE PHYSICAL MEANING OF q -INDEPENDENCE? IS IT CONSISTENT WITH (STRICT OR ASYMPTOTIC) SCALE INVARIANCE? IF YES, IS IT SUFFICIENT? NECESSARY?

CANDIDATE MODELS FOR q -INDEPENDENCE:

- 1) N compact-support continuous variables with correlation introduced through a N -variate covariance matrix **(strictly scale-invariant)**

W. Thistleton, J.A. Marsh, K. Nelson and C. T., unpublished (2007)
(see H.J. Hilhorst and G. Schehr, J Stat Mech (2007) P06003)

- 2) N binary variables with correlation introduced through the q -product **(strictly scale-invariant)**

L.G. Moyano, C. T. and M. Gell-Mann, Europhys Lett **73** (2006) 813
(see H.J. Hilhorst and G. Schehr, J Stat Mech (2007) P06003)

- 3) N binary variables with correlation introduced through a family of triangles generalizing the Leibnitz one **(strictly scale-invariant)**

A. Rodriguez, V. Schwammle and C. T., J Stat Mech (2008) P09006
R. Hanel, S. Thurner and C. T. (2008)

- 4) N -binary-discretized q -Gaussians **(asymptotically scale-invariant)**

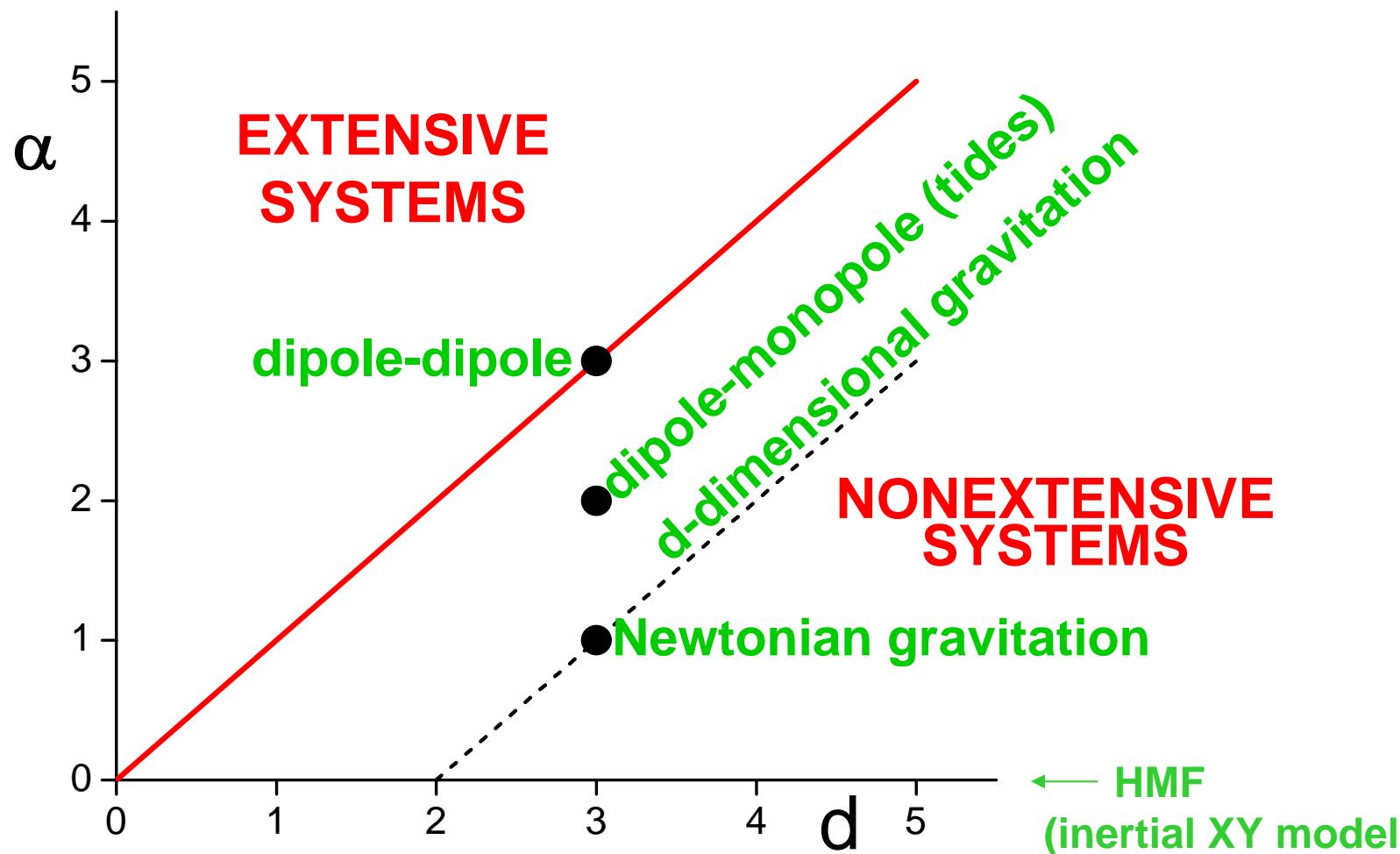
A. Rodriguez, V. Schwammle and C. T., J Stat Mech (2008) P09006

CLASSICAL LONG-RANGE-INTERACTING MANY-BODY HAMILTONIAN SYSTEMS

$$V(\vec{r}) \sim -\frac{A}{r^\alpha} \quad (r \rightarrow \infty) \quad (A > 0, \quad \alpha \geq 0)$$

integrable if $\alpha / d > 1$ (short-ranged)

non-integrable if $0 \leq \alpha / d \leq 1$ (long-ranged)



d -DIMENSIONAL CLASSICAL INERTIAL XY FERROMAGNET:

(We illustrate with the XY (i.e., $n=2$) model; the argument holds however true for any $n>1$ and any d -dimensional Bravais lattice)

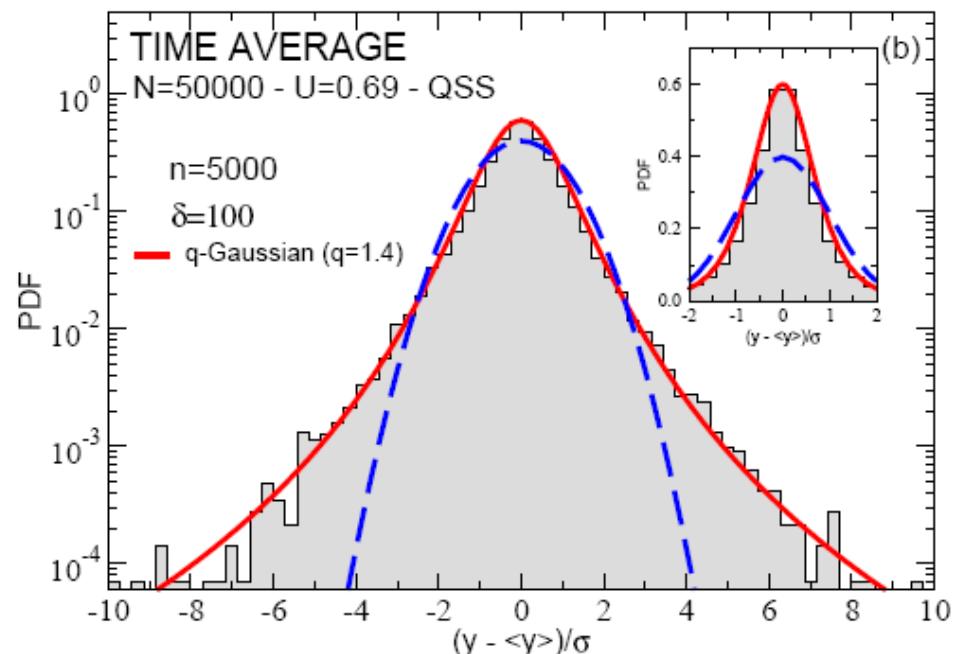
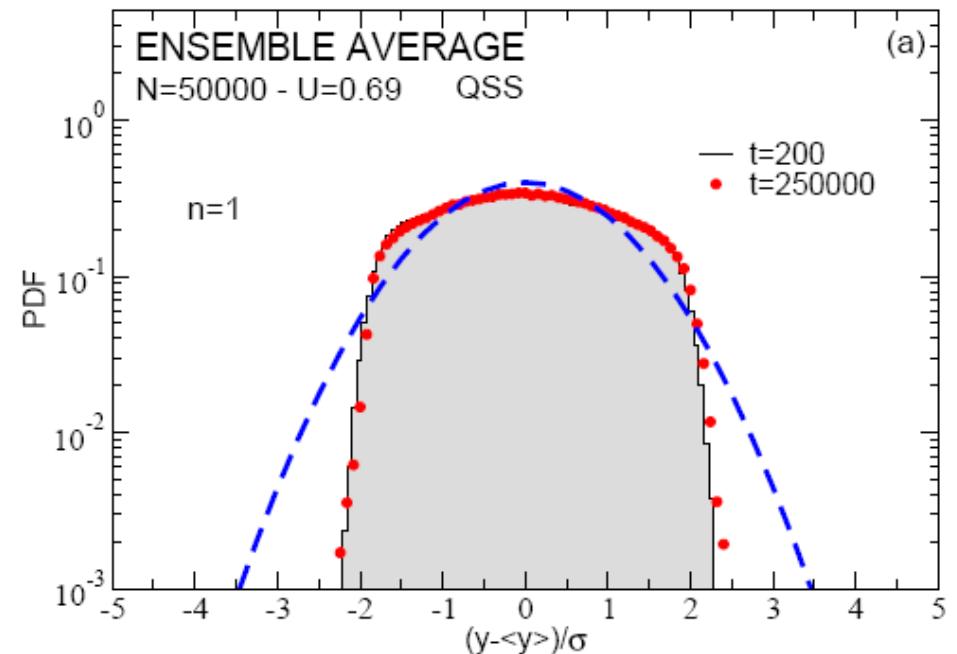
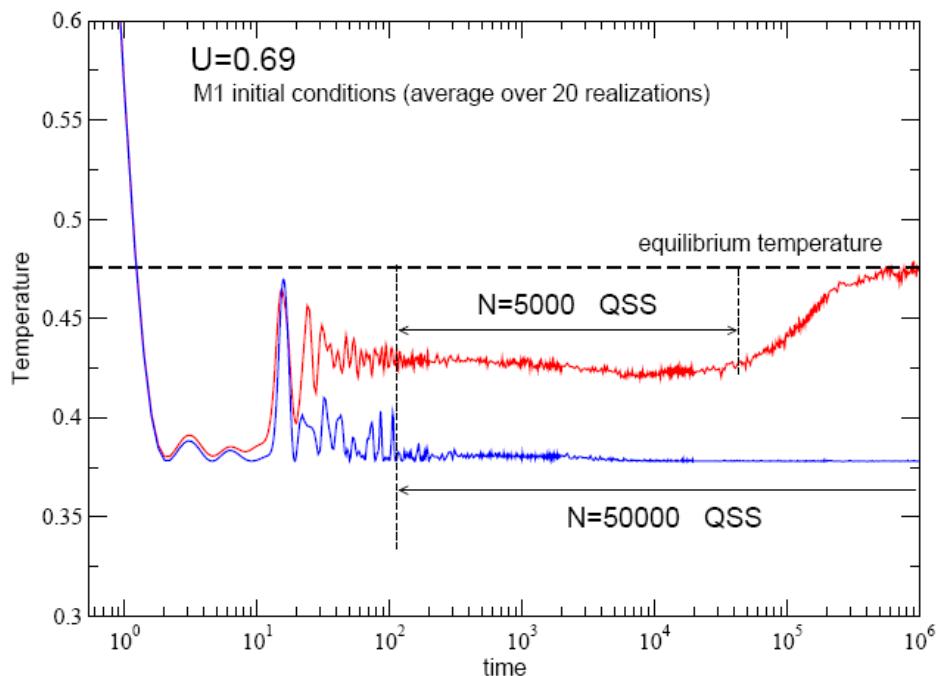
$$H = K + V = \frac{1}{2I} \sum_{i=1}^N L_i^2 + \frac{J}{\mathfrak{A}} \sum_{i,j} \frac{1 - \cos(\vartheta_i - \vartheta_j)}{r_{ij}^\alpha} \quad (I > 0, J > 0)$$

$$\text{with } \mathfrak{A} \equiv \sum_{j=1}^N r_{ij}^{-\alpha} \propto \begin{cases} N^{1-\alpha/d} & \text{if } 0 \leq \alpha/d < 1 \\ \ln N & \text{if } \alpha/d = 1 \\ \text{constant} & \text{if } \alpha/d > 1 \end{cases}$$

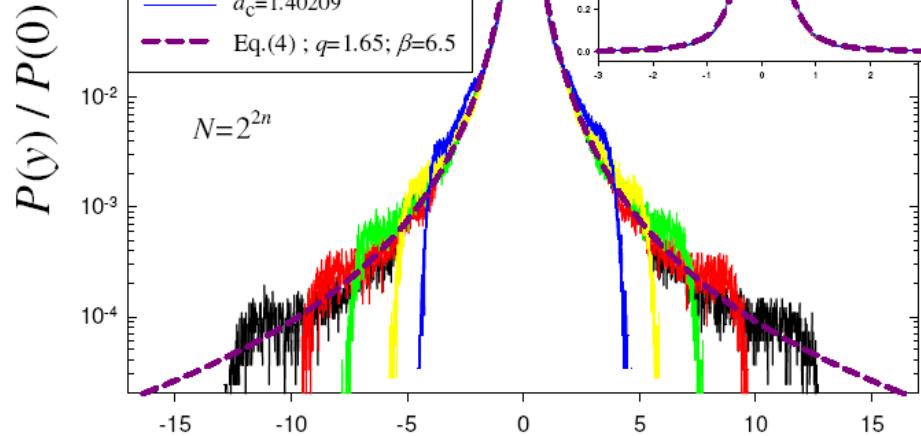
and periodic boundary conditions.

[*The HMF model corresponds to $\alpha/d = 0$*]

HMF MODEL



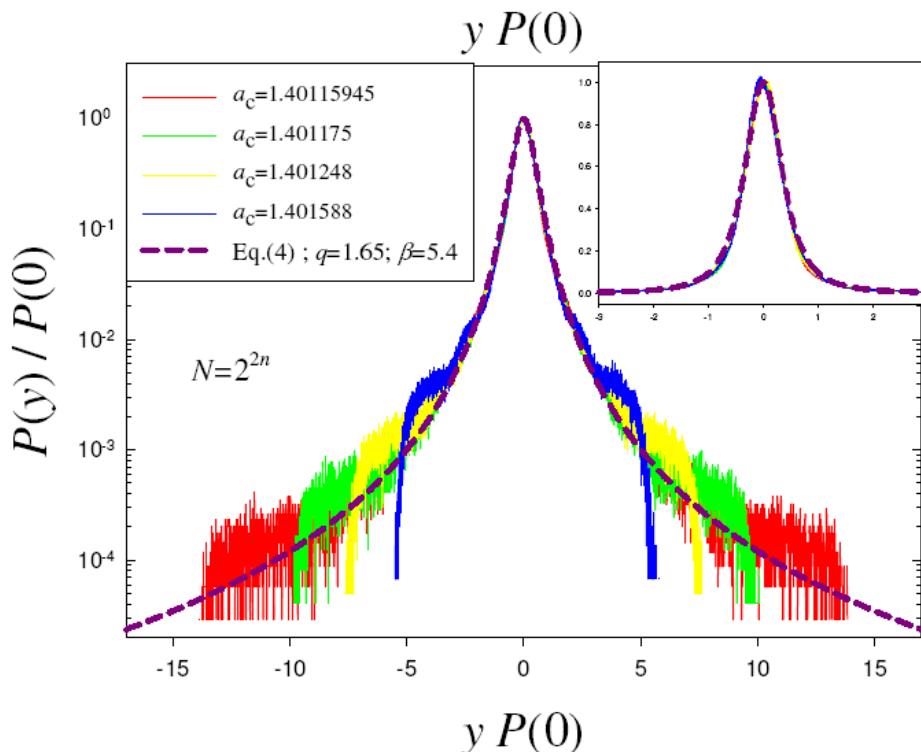
LOGISTIC MAP: EDGE OF CHAOS



odd $2n$

$q=1.65$

$\beta=6.5$



even $2n$

$q=1.65$

$\beta=5.4$

U. Tirnakli, C. Beck and C. T.
Phys. Rev. E 75, 040106(R) (2007)
U. Tirnakli, C. T. and C. Beck (2008)

Introduction to Nonextensive Statistical Mechanics

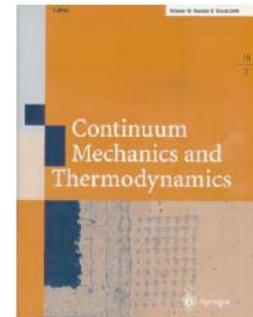
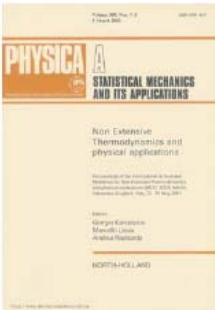
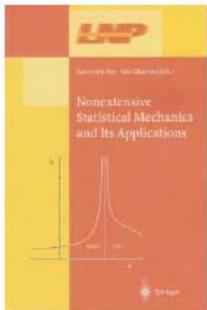
APPROACHING A COMPLEX WORLD

Constantino Tsallis

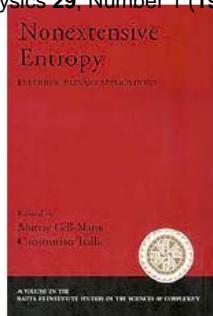


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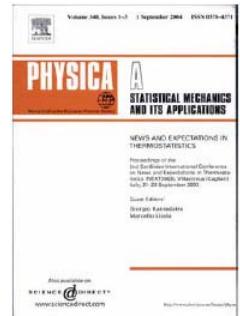
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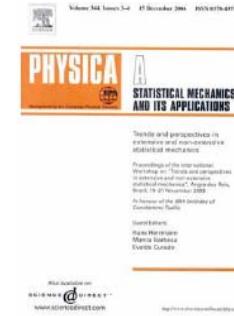
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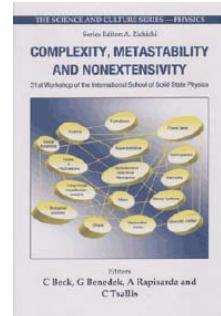
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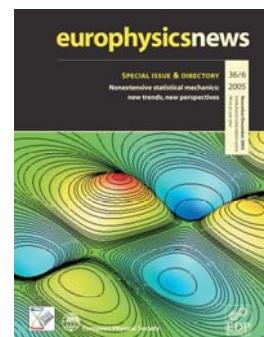
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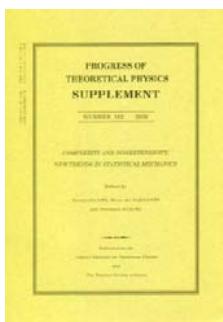
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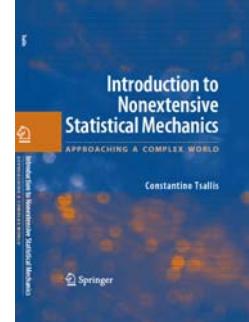
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