Ordem e formação de padrões em sistemas com interações competitivas

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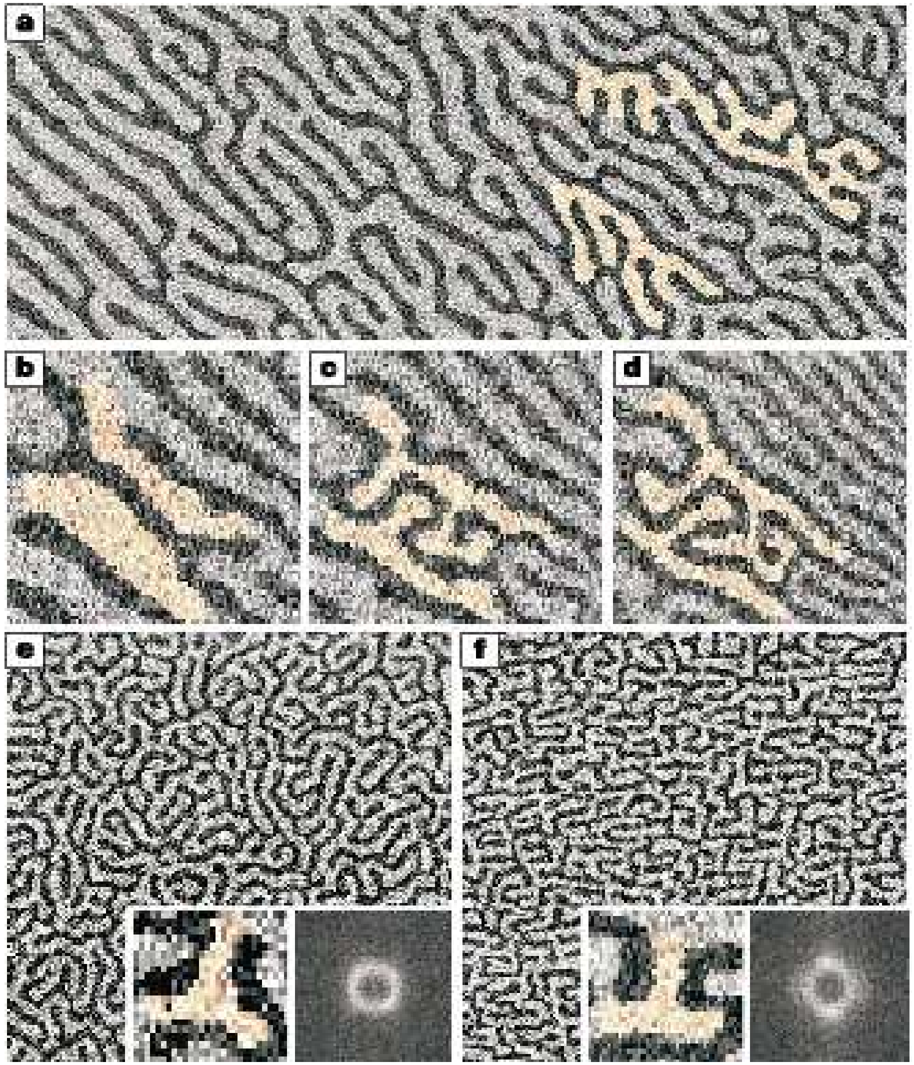
Ultrathin ferromagnetic films



Fe/Cu(001) ultrathin films, imaged with SEMPA. In a) the effective temperature increases from left to right. Image width=155 μm, thickness varies from 2.06ML to 1.98ML, T=294K. In b)-d) the temperature is increased from left to right for the same region. width=36μm, thickness$\approx$2.19ML, $T_b \approx$298K, $T_c \approx$304K and $T_d \approx$305K. e)The labyrinthine pattern. In the inset a triangular V disclination and the structure factor. Thickness$\approx$1.97ML, T$\approx$293K f) The labyrinthine pattern on a stepped substrate. The disclination has a square symmetry, as does the structure factor. Parameters as in e), thickness $\approx$ 2.15ML.

From O. Portmann et al., Nature 422, 701 (2003)

Diblock copolymers

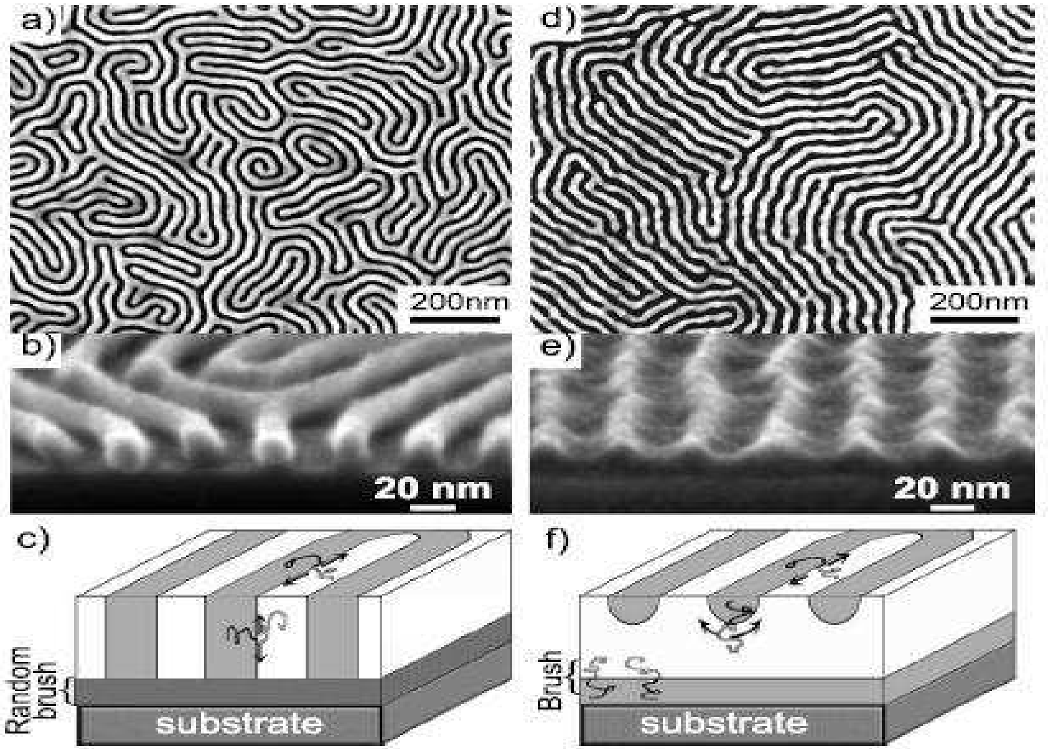


FIG. 1. Striped patterns from lamellar (left) and cylindrical (right) thin films: [(a) and (d)] top down scanning electron microscopy (SEM) of striped patterns, [(b) and (e)] cross section SEM after PMMA removal, and [(c) and (f)] cross section schematic of molecular arrangement showing possible directions for parallel diffusion (D_{par}).

R. Ruiz et el.
Phys. Rev. B 77, 054204 (2008)

Microemulsions

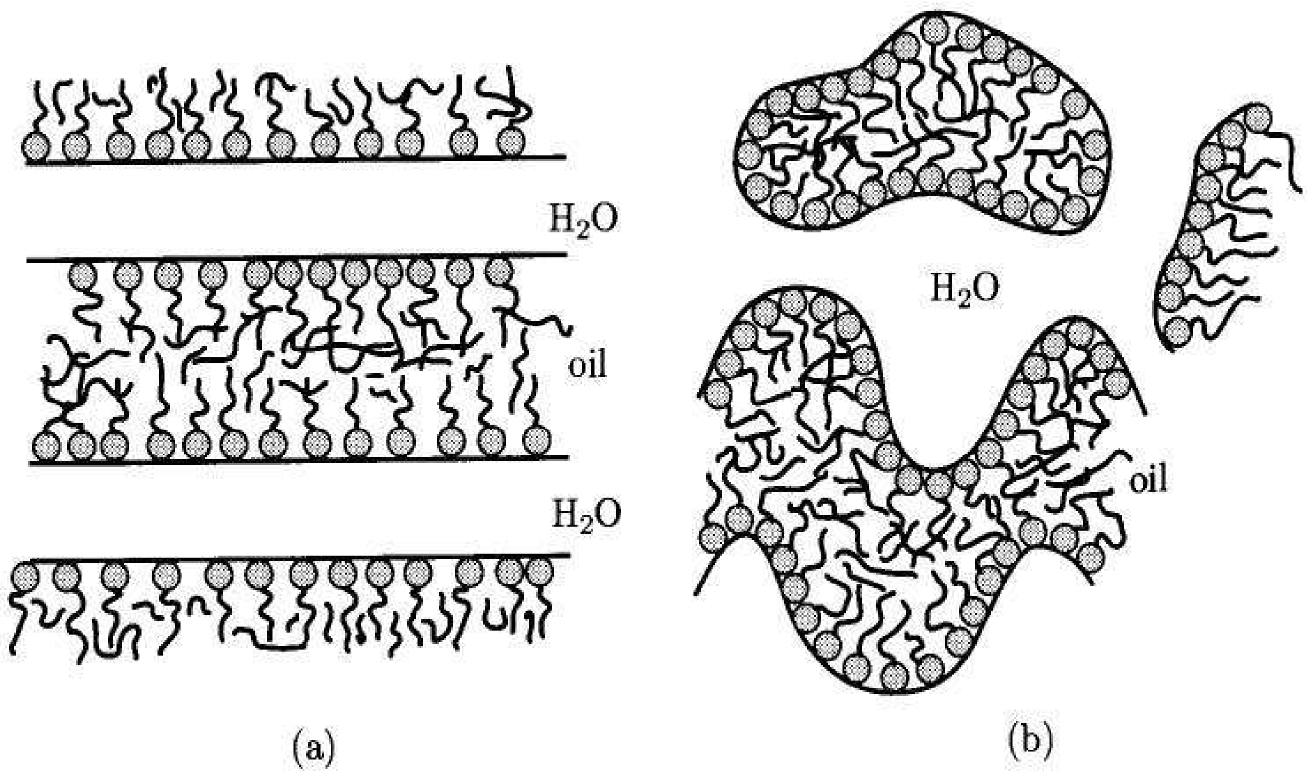


Fig. 2.7.16. (a) A lamellar microemulsion phase with water, oil, and surfactant layers. (b) Schematic representation of a random bicontinuous phase in which there is a random surfactant surface separating oil and water regions.

From P. M. Chaikin and T. C. Lubensky
Principles of Condensed Matter Physics

- Physics ruled by competition between short range and long range interactions.
- Long range interactions frustrate the tendency for long range order coming from s.r.i. and then complex structures appear as stripes, bubbles, labyrinth, tetragonal, nematic, columnar phases.
- There is also competition between positional and orientational order giving rise to phases similar to liquid crystals.
- Examples go from classical to quantum systems like ultrathin ferromagnetic films, diblock copolymers, low dimensional electronic systems, superconductors, microemulsions, Raleigh-Bénard convection, etc.

Modulated phases in condensed matter

Table 1. Illustrative list emphasizing the wide variety and diversity of 2D and 3D systems in which modulated phases have been described. Morphologies are classified as stripes (S), islands (I), and bubbles (B) in 2D and as lamellae (L), hexagonally packed cylinders (H), and cubic arrays of spheres (C) in 3D. Order parameters include composition (Φ), polarization (P), orientation of the surface reconstruction (SR), normal magnetization (M_z), the ratio $\rho = (\rho_N = \rho_{SC})$ (where ρ_N and ρ_{SC} represent, respectively, the normal and superconducting fractions of a type I superconducting film), projection of the molecular director into the plane of the layer (*c*-dir), and the chirality field $\Psi = \sin[6(\varphi - \vartheta)]$ (where φ and ϑ denote, respectively, the tilt and bond field azimuthal orientation). Sources of competition include normal polarization (P_z), polarization associated with variations in the work function (P_{wf}), magnetic field (H), and curvature (κ).

System	Typical length scale	Morphology	Order parameter	Source of competition	Ref.
Langmuir films	1–10 µm	S, B	Φ or density	P_z	(16, 17)
Adsorbates on metals	500 Å	S, I	coverage,	P_{wf}	(18)
Ferroelectric films	10 µm	S	P	Stress	(19)
Semiconductor surface	500 Å	S, I	SR	Surface stress	(20)
Ferrofluids	1 cm	S, B	Φ	H	(6)
Magnet garnets	10 µm	S, B	M_z	H	(21, 22)
Type I superconductor films	10 µm	S, B	ρ	H	(1, 23)
Membranes, vesicles	100 Å	S, B	Φ	κ	(24)
"Ripple" ($P_{\beta'}$) phase	100 Å	S	*c*-dir	κ	(25)
Freely suspended liquid crystal films	5 µm	S	Ψ	*c*-dir bend distortion	(26)
Convective patterns	1 cm	S, B			(27, 28)
Turing patterns	1 cm	S, B			(3)
Diblock copolymers	500 Å	L, H, C	Φ	(Covalent) linkage	(29–31)
Co- and homopolymer mixture					(32)
Charged diblock copolymer				Counterion entropy	(33)
Polyelectrolyte solution					(34)

M. Seul and D. Andelman
Science 267, 476 (1995)

The Brazovskii model in two dimensions: stripe solutions

$$\mathcal{H} = \frac{1}{2}\int d^2\vec{x}\,\left[\kappa(\nabla\phi(\vec{x}))^2 - r_0\phi^2(\vec{x}) + \frac{u_0}{2}\phi^4(\vec{x})\right] + \frac{g}{2}\int d^2\vec{x}\int d^2\vec{x'}\,\phi(\vec{x})G(|\vec{x}-\vec{x'}|)\phi(\vec{x'})$$

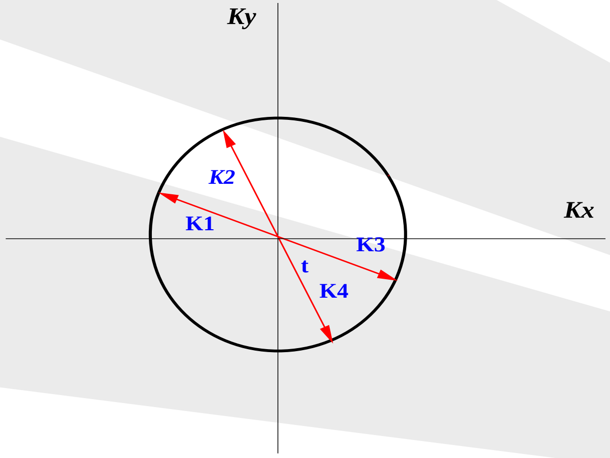
In reciprocal space $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_i$, with:

$$\mathcal{H}_0 = \int \frac{d^2k}{(2\pi)^2}\,\phi(\vec{k})\,\left(r_0 + A_2\,(k-k_0)^2 + \ldots\right)\,\phi(-\vec{k})$$

$$\mathcal{H}_i = u_0\int \frac{d^2k_1}{(2\pi)^2}\frac{d^2k_2}{(2\pi)^2}\frac{d^2k_3}{(2\pi)^2}\frac{d^2k_4}{(2\pi)^2}\,\phi(\vec{k}_1)\phi(\vec{k}_2)\phi(\vec{k}_3)\phi(\vec{k}_4)\,\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3+\vec{k}_4)$$

- First order isotropic-stripe transition induced by flucutations
- $\phi(\vec{x}) = A\,\cos(\vec{k}_0\cdot\vec{x})$
- Anglular fluctuations of $\vec{k}_0$ do not cost energy.

Brazovskii, JETP 41, 85 (1975)

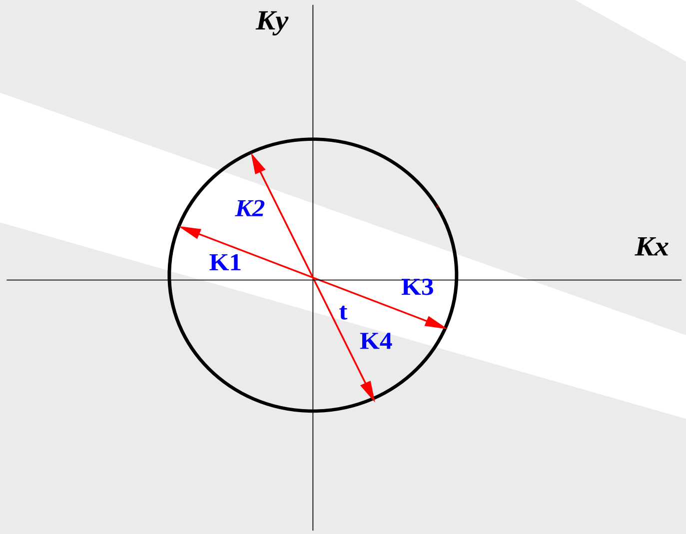


Beyond Brazovskii: nematic-like order

Consider a generic quartic interaction term of the form:

$$\mathcal{H}_i = \int \frac{d^2k_1}{(2\pi)^2}\frac{d^2k_2}{(2\pi)^2}\frac{d^2k_3}{(2\pi)^2}\frac{d^2k_4}{(2\pi)^2}\, u(\vec{k}_1,\vec{k}_2,\vec{k}_3,\vec{k}_4,)\times$$

$$\phi(\vec{k}_1)\phi(\vec{k}_2)\phi(\vec{k}_3)\phi(\vec{k}_4)\,\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3+\vec{k}_4)$$



Nematic symmetry

- Isotropy force the wave vectors to lay on the circle and hence $u(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4) = u(\theta_1, \theta_2, \theta_3, \theta_4)$.
- Conservation of total momentum allows to eliminate one of the four $k's$.
- Fixing two of them, the other two are automatically fixed by the requirement that the four vectors lay on the circle.
- Because of rotational invariance, the interaction can only depend on the difference between the two independent angles:

$$u(\theta_1, \theta_2, \theta_1 + \pi, \theta_2 + \pi) = u(\theta_1 - \theta_2) = u(\theta)$$

- Finally, index permutation invariance in $u(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4)$ implies $u(\theta) = u(\theta + \pi)$, which is a signature of nematic symmetry. Then u can be expanded in Fourier series:

$$u(\theta) = u_0 + u_2 \cos(2\theta) + u_4 \cos(4\theta) + \dots$$

Nematic energy

Up to the first two terms in the expansion, the interaction energy with nematic symmetry can be written as:

$$\mathcal{H}_4 = \int d^2x \left\{ u_0\, \phi^4(\vec{x}) + u_2\, tr\, \hat{Q}^2 \right\}$$

with

$$\hat{Q}_{ij}(\vec{x}) = \phi(\vec{x}) \left(\nabla_i \nabla_j - \frac{1}{2} \nabla^2 \right) \phi(\vec{x})$$

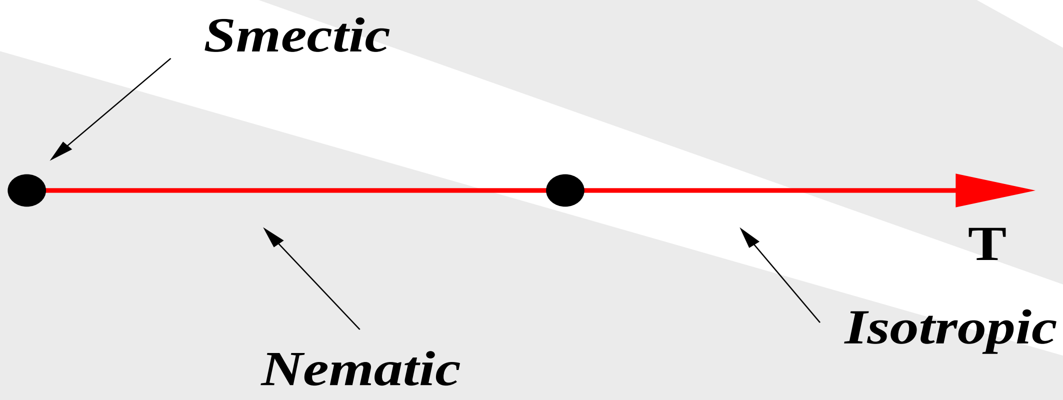
a traceless symmetric tensor which can also be represented in two dimensions as a complex nematic order parameter

$$Q = \alpha\, e^{i2\theta}$$

The order parameter is a *quadrupolar moment*.

Mean field results

- If $u_2 = 0 \Longrightarrow$ Brazovskii "smectic" solution. Nevertheless, this solution is unstable to fluctuations in $d = 2$ and $T_{Br} \to 0$.
- $u_2 > 0$ corresponds to repulsive quadrupole interactions. In this case the only solution is $\alpha(T) = 0$.
- $u_2 < 0$ corresponds to attractive quadrupole interactions.
  In this case mean field gives a second order isotropic-nematic transition, with $\alpha(T) \simeq |T_c - T|^{1/2}$ and $T_c = 2/(mk_0^2)\,\sqrt{u_0/|u_2|}$.



D.A.Barci and D.A.S., PRL 98, 200604 (2007)

Quasi-long-range order and experimental scales

The continuous symmetry implies angular fluctuations in the director cost little energy and can destroy long range nematic order. Evaluating local fluctuations:

$$\delta F \equiv F(\hat{Q}(x)) - F(<\hat{Q}>) = \rho_s(T) \int d^2x\, |\vec{\nabla}\varphi(x)|^2 \qquad KT\ energy$$

Now, the mean field nematic order parameter is given by

$$\begin{aligned} <\hat{Q}> &= \alpha\ \exp(-W) \\ W &= \frac{T}{4\pi\rho_s} \ln\left(\frac{L}{a}\right) \qquad Debye-Waller\ factor \end{aligned}$$

with $T_{KT} = (\pi/8)\rho_s(T_{KT})$.
Thermodynamic limit: $L/a \rightarrow \infty \Rightarrow <\hat{Q}> \rightarrow 0$, no nematic long range order.
For the Fe/Cu(001) films in O. Portmann et al., Nature **422**, 701 (2003)

$$L \approx 10^{-3} m \qquad a \approx 10^{-10} m \qquad W \approx 7/32$$

$$<\hat{Q}> \approx 0.8\ \alpha \qquad Long\ range\ nematic\ order should\ be\ observable!$$