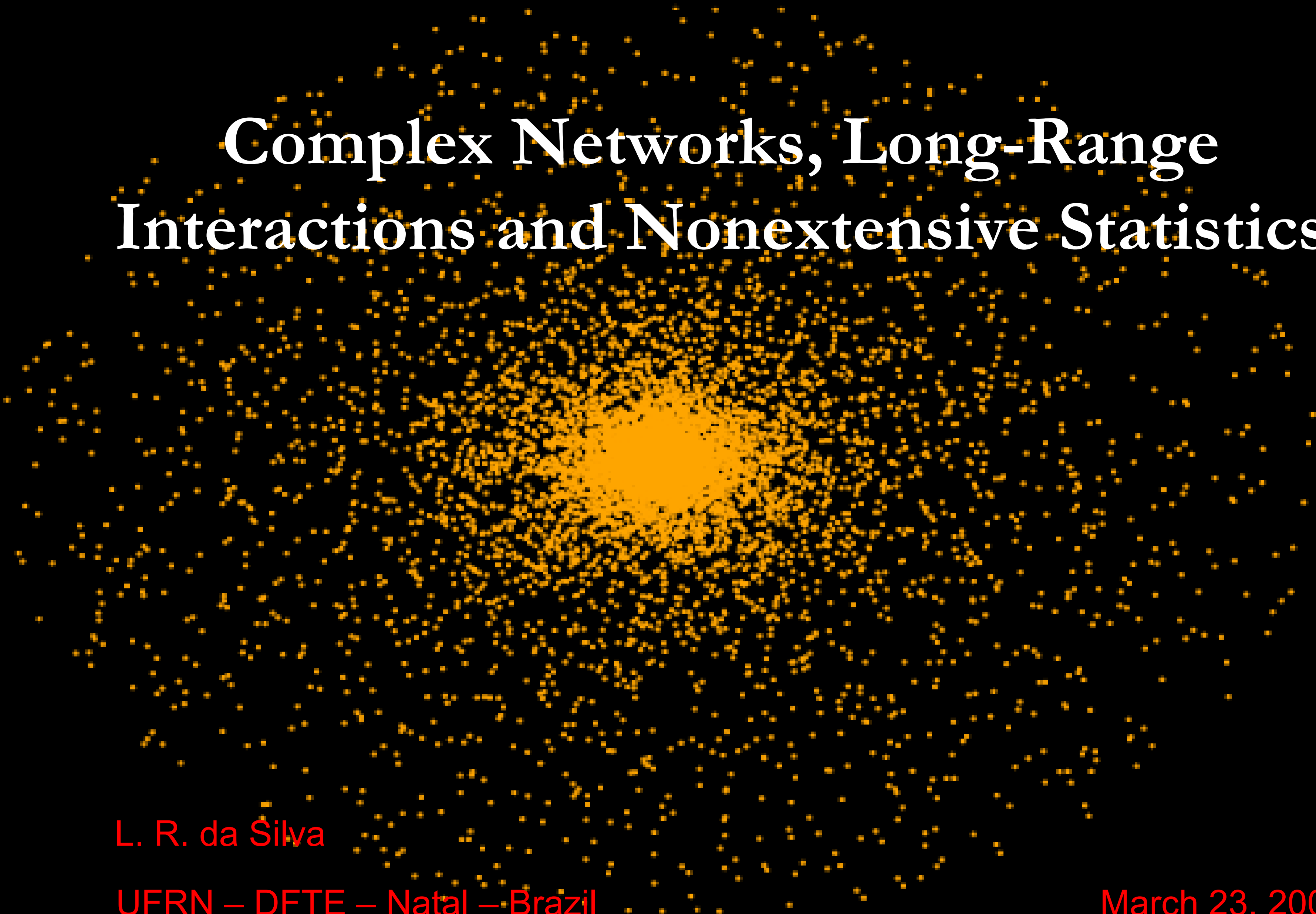
INCT-SISTEMAS COMPLEXOS -RIO-2009

Complex Networks, Long-Range Interactions and Nonextensive Statistics



L. R. da Silva

UFRN – DFTE – Natal – Brazil

March 23, 2009

OUR GOALS

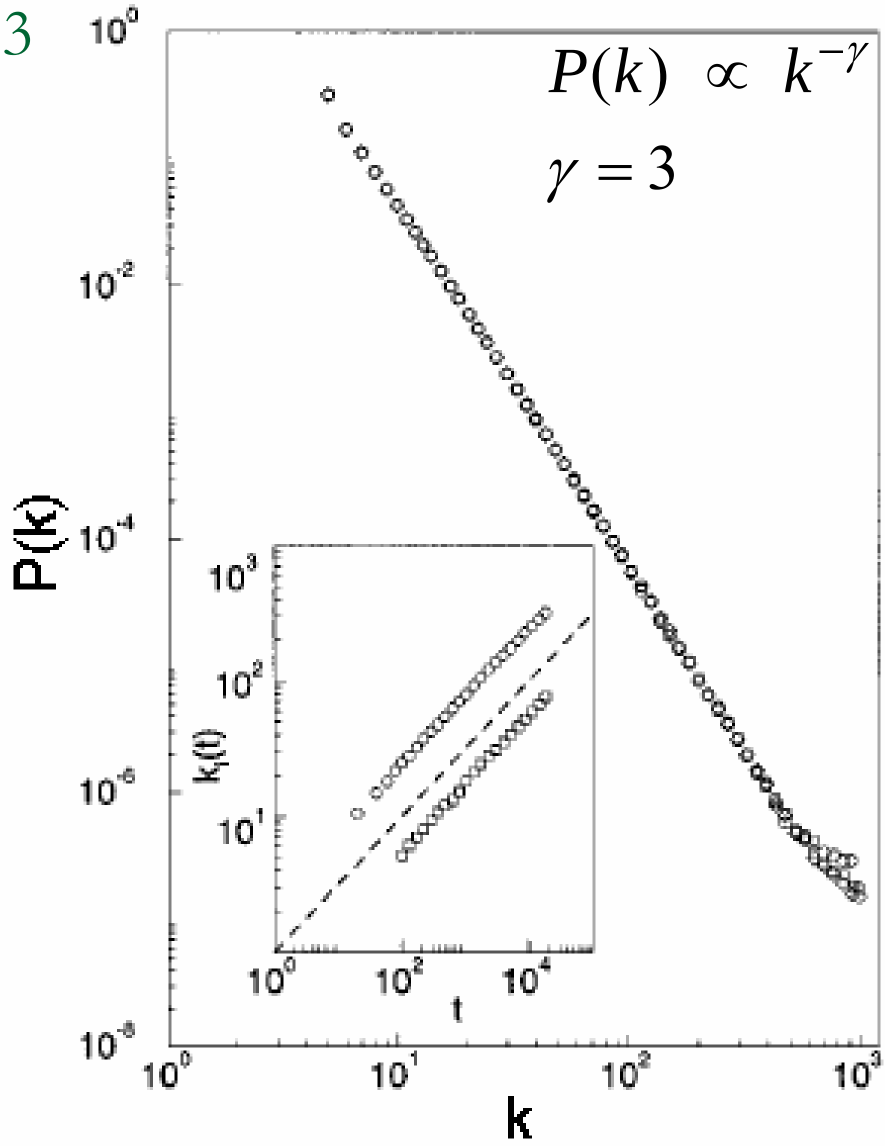
- Growth of an asymptotically scale-free network including metrics.
- Growth of a geographically localized network (around its baricenter).
- To exhibit effects of competition between metrical neighborhood, connectivity and fitness.
- To analyze the influences of considering a fitness power-law distributed.
- To analyze the influences of considering a acquaintance over a net using fitness uniform distributed.
- Last but not least, to exhibit the connection between scale-free networks and nonextensive statistics.

Scale-free Networks[1,2,3]

- Barabási and Albert[1];

$$\Pi(k_i) = \frac{k_i}{\sum_{j=1}^{N'} k_j} \qquad (01)$$

$$< k_i > = \left(\frac{t}{i}\right)^{\beta} \qquad (02)$$



[1]Science **286**, 509 (1999) ; Rev. Mod Phys. **74**, 47 (2002)

[2]M. Boguñá and R. Pastor-Satorras, Physical Review E **68**, 036112 (2003)

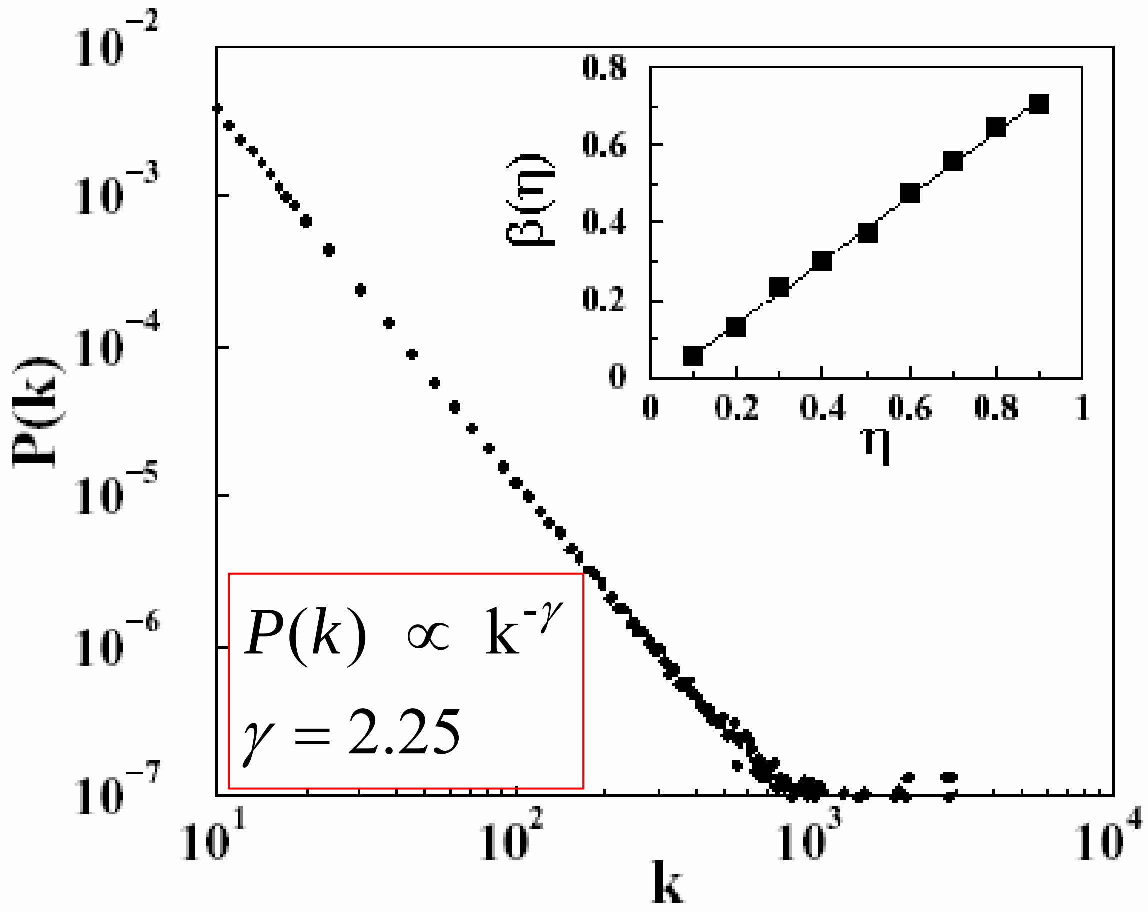
[3]S. Thurner and C. Tsallis, Europhycs Letters **72**, 197 (2005)

Fitness Model

- Bianconi and Barabási[4];
- Albert and Barabási[5];

$$\Pi(k_i) = \frac{k_i \eta_i}{\sum_{j=1}^{N'} k_j \eta_j} \quad (04)$$

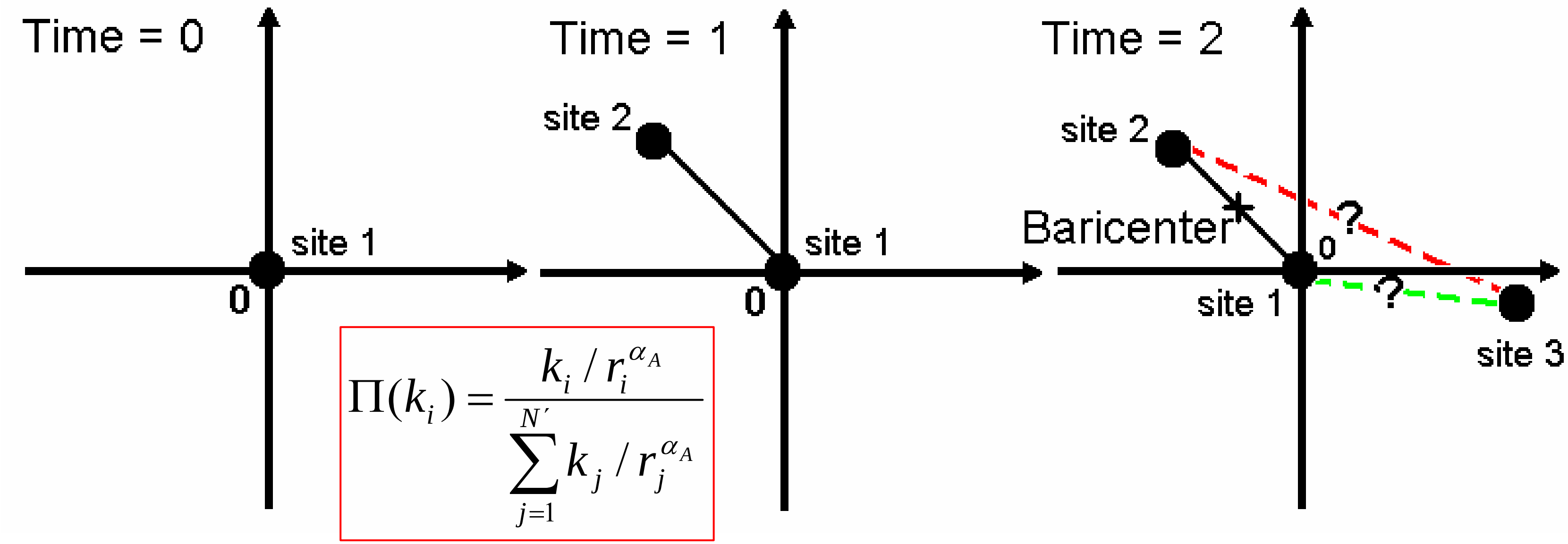
$$< k_i > = \left(\frac{t}{i} \right)^{\beta(\eta_i)} \quad (05)$$



[4]Europhys. Lett. **54**,436 (2001) ; [5]Rev. Mod Phys. **74**, 47 (2002)

Barabási-Albert Model with Euclidean Distance Power-law Distributed

Network Construction:



$$\Pi(k_i) = \frac{k_i / r_i^{\alpha_A}}{\sum_{j=1}^{N'} k_j / r_j^{\alpha_A}}$$

$$P(r) \propto r^{-\gamma_G}$$

$$\gamma_G > 1$$

where

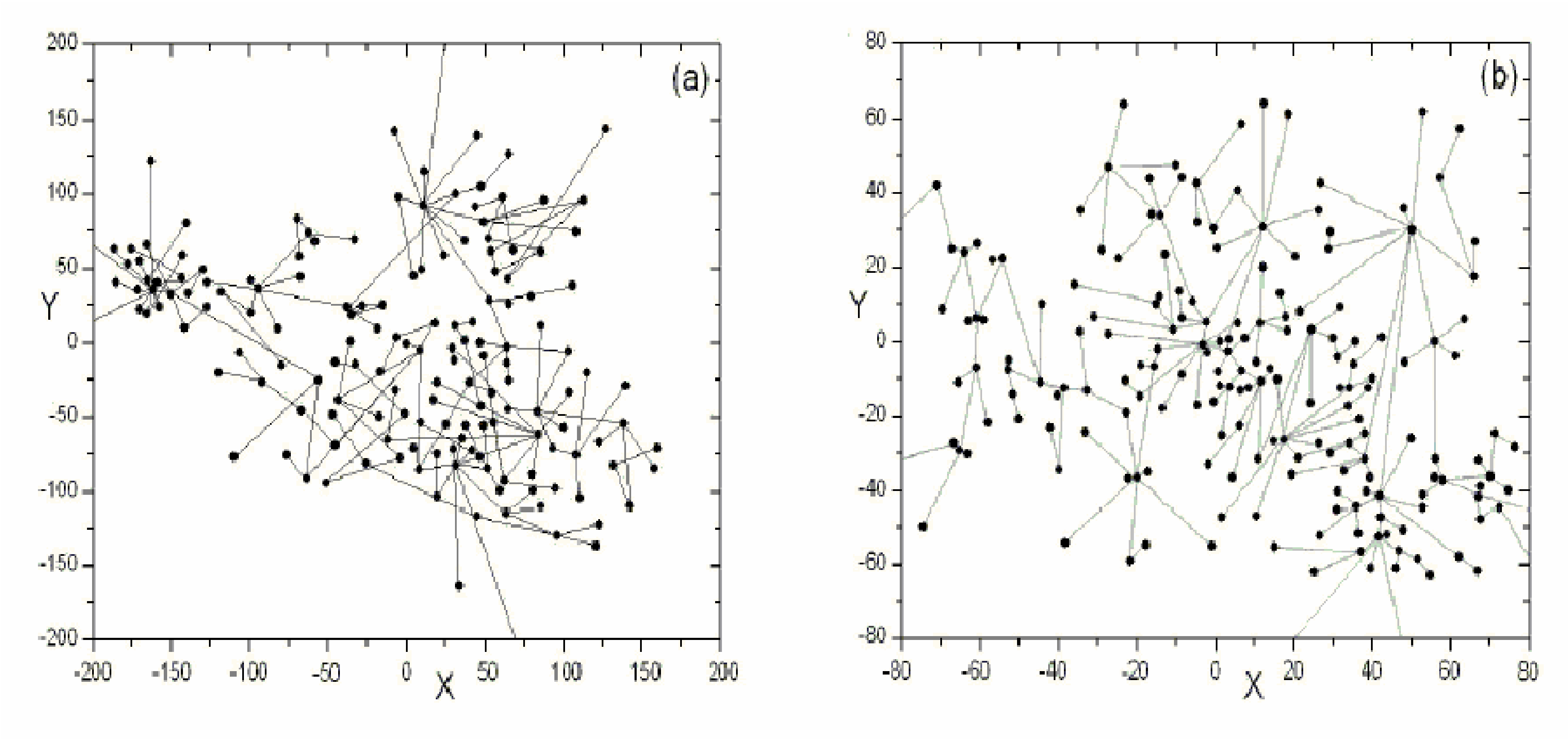
$$\gamma_G = \frac{3+\alpha_G}{2+\alpha_G}$$

$$\alpha_G \geq 0$$

$$r = (1-\xi')^{-(2+\alpha_G)}$$

$$\theta = 2\pi\xi'$$

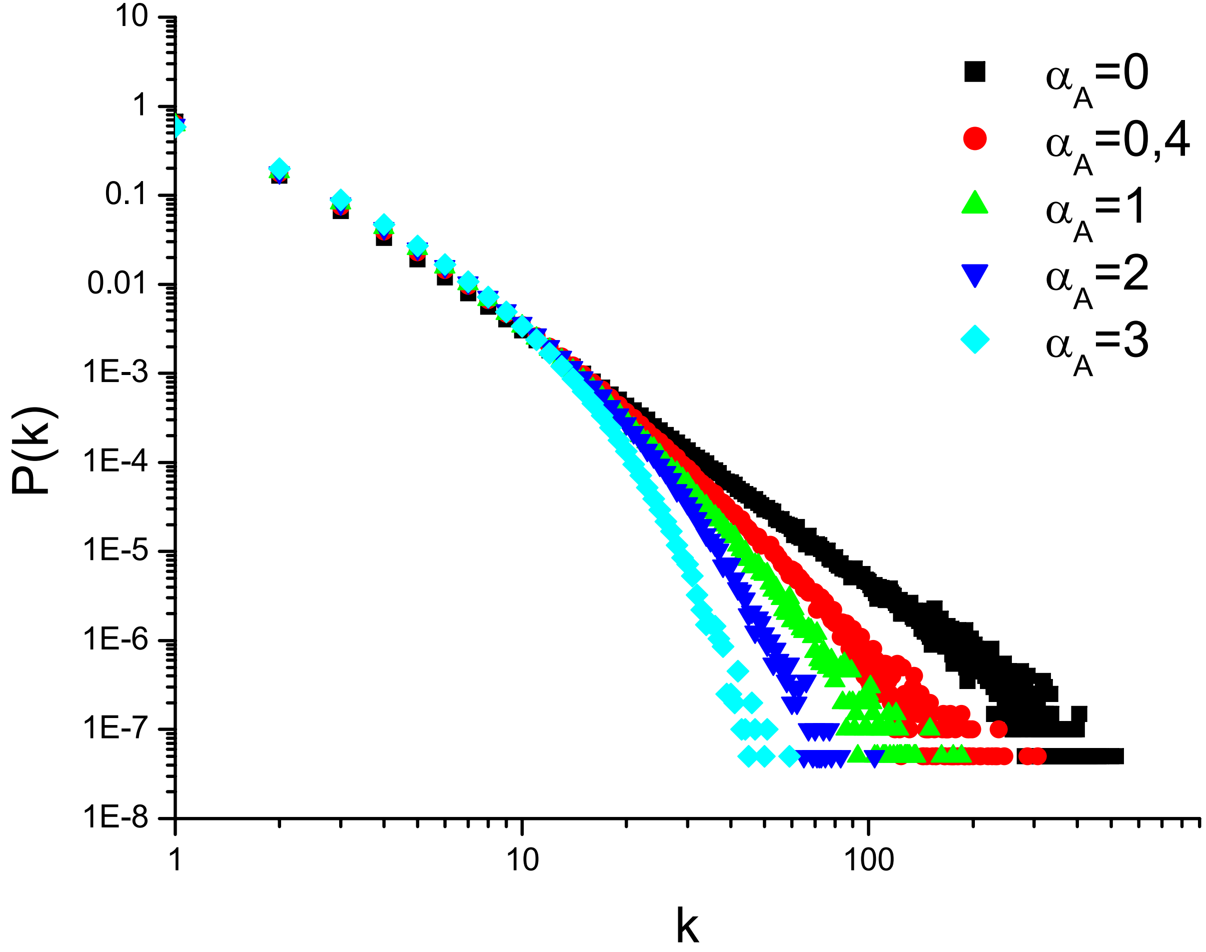
Examples



N = 250 nodes (a) $(\alpha_G, \alpha_A) = (1, 0)$ and (b) $(\alpha_G, \alpha_A) = (1, 4)$.
The starting site is at $(X, Y) = (0, 0)$. Notice the spontaneous emergence of hubs.

Barabási-Albert Model with Euclidean Distance Power-law Distributed

$$\Pi(k_i) = \frac{k_i / r_i^{\alpha_A}}{\sum_{j=1}^{N'} k_j / r_j^{\alpha_A}} \quad (03)$$



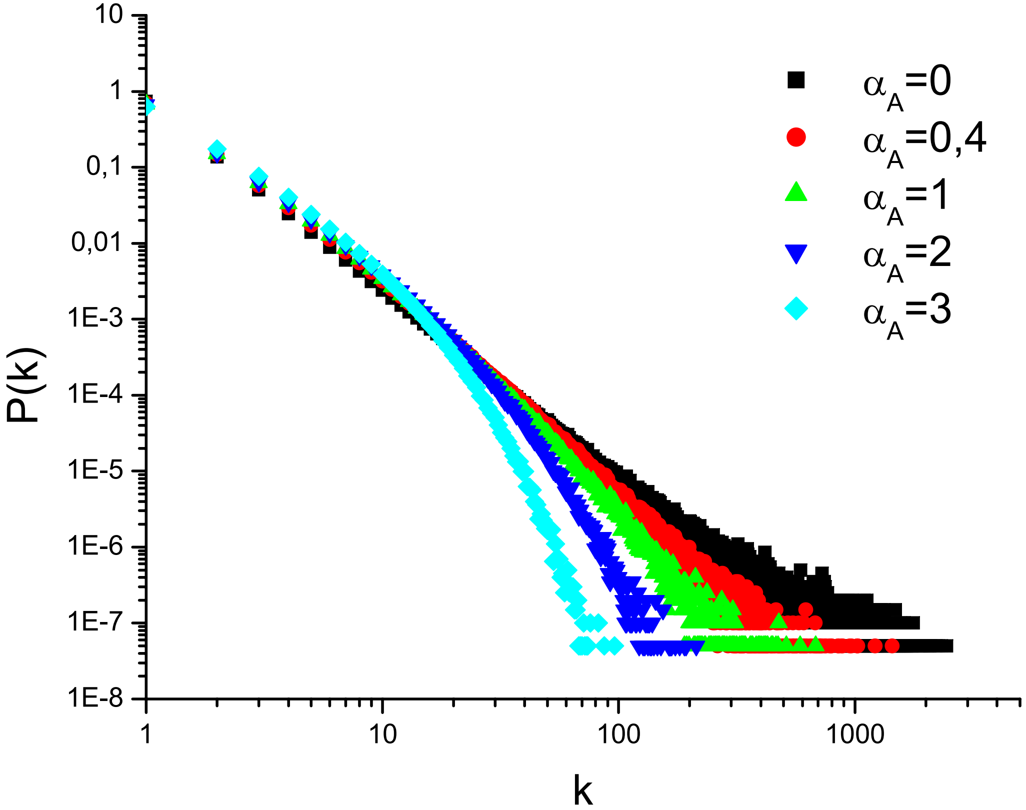
Soares, Tsallis, Mariz and da Silva, Europhys. Lett. **70,** 70 (2005)

Fitness Model with Euclidean Distance Power-law Distributed

- Inspired in the works of Soares, Tsallis, Mariz and da Silva[3],

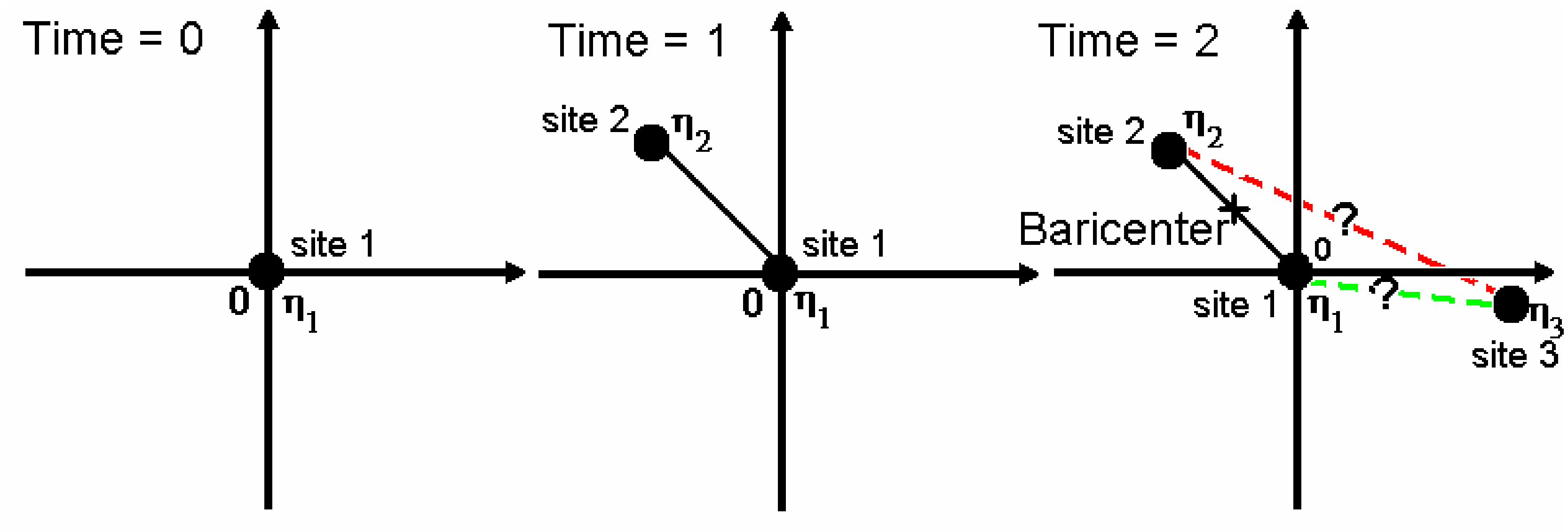
  and Bianconi and Barabasi[4].

$$\Pi(k_i) = \frac{k_i \eta_i / r_i^{\alpha_A}}{\sum_{j=1}^{N'} k_j \eta_j / r_j^{\alpha_A}} \quad (06)$$



Meneses, Cunha, Soares, and da Silva,
Progress of Theoretical Physics Supplement **162,** 131 (2006)

Network Construction



$$P(r)\ \alpha\ r^{-\gamma_G}$$

$$\gamma_G > 1$$

where

$$\gamma_G = \frac{3+\alpha_G}{2+\alpha_G}$$

$$\alpha_G \geq 0$$

$$r = (1-\xi')^{-(2+\alpha_G)}$$

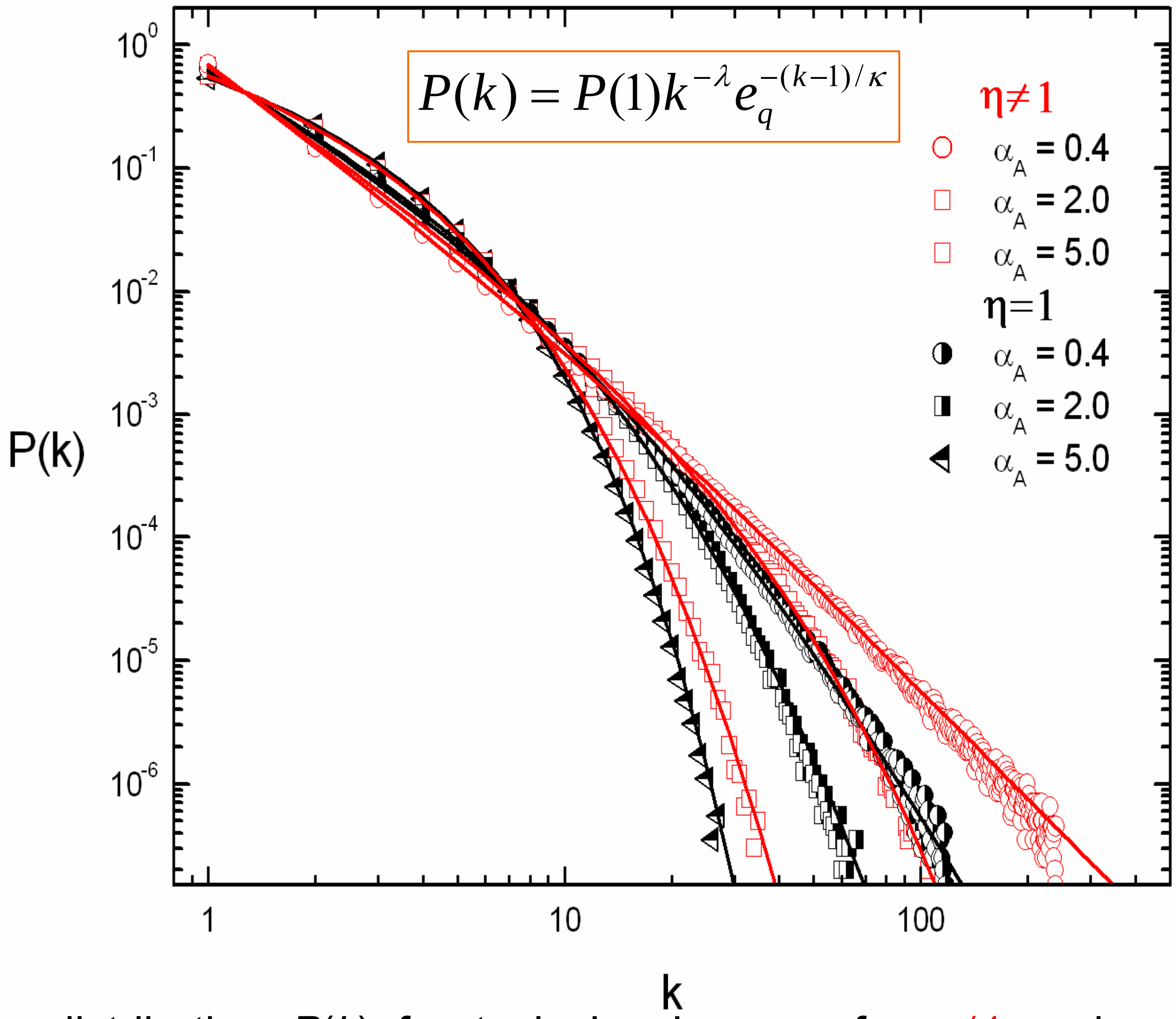
$$\theta = 2\pi\xi'$$

Tsallis Nonextensive Statistical Mechanics

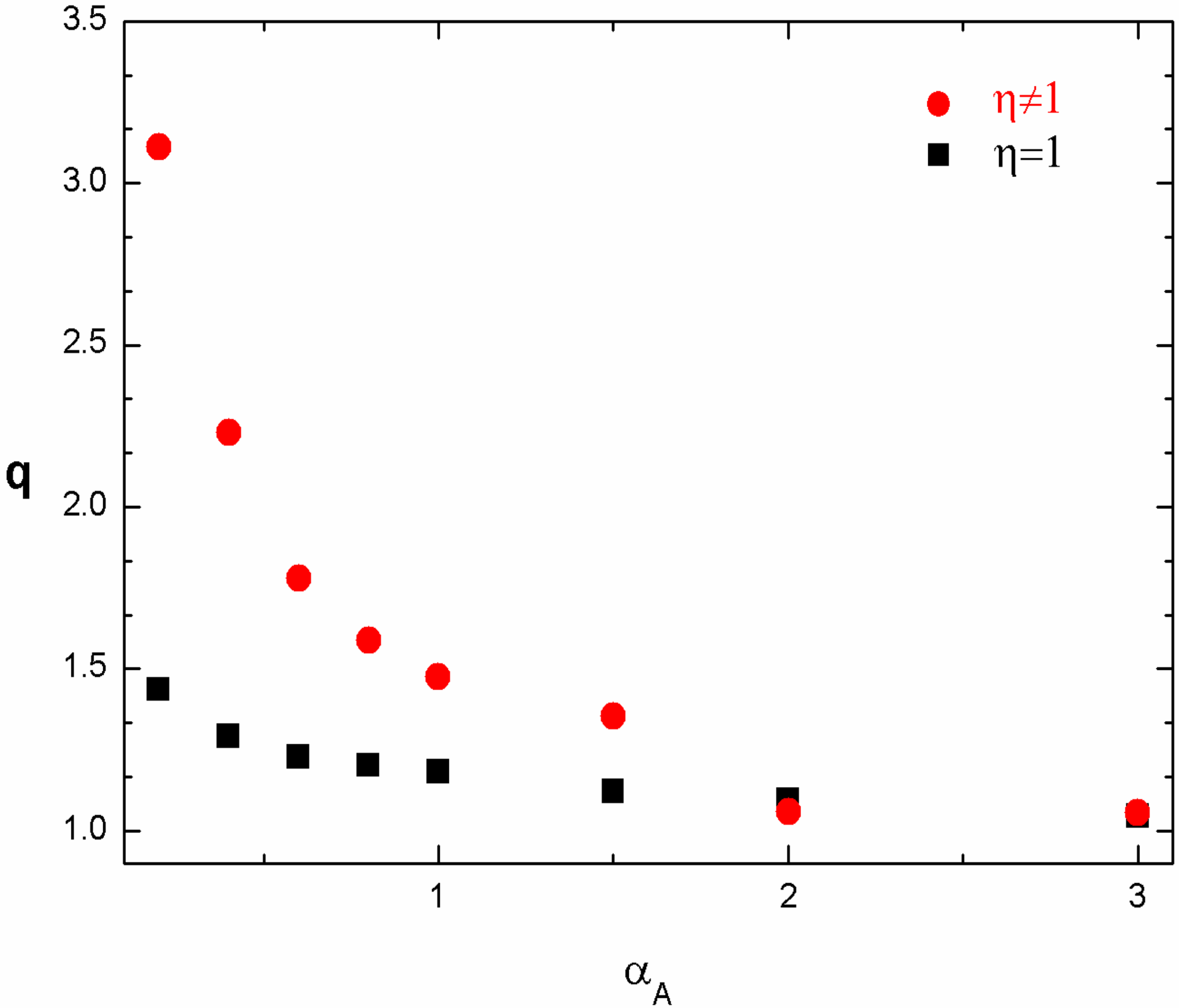
$$s_q = \frac{1 - \int dk [P(k)]^q}{q - 1} \qquad (q \in \Re; S_1 = S_{BG})$$

$$P(k) = P(1) k^{-\lambda} e_q^{-(k-1)/\kappa} \qquad (07)$$

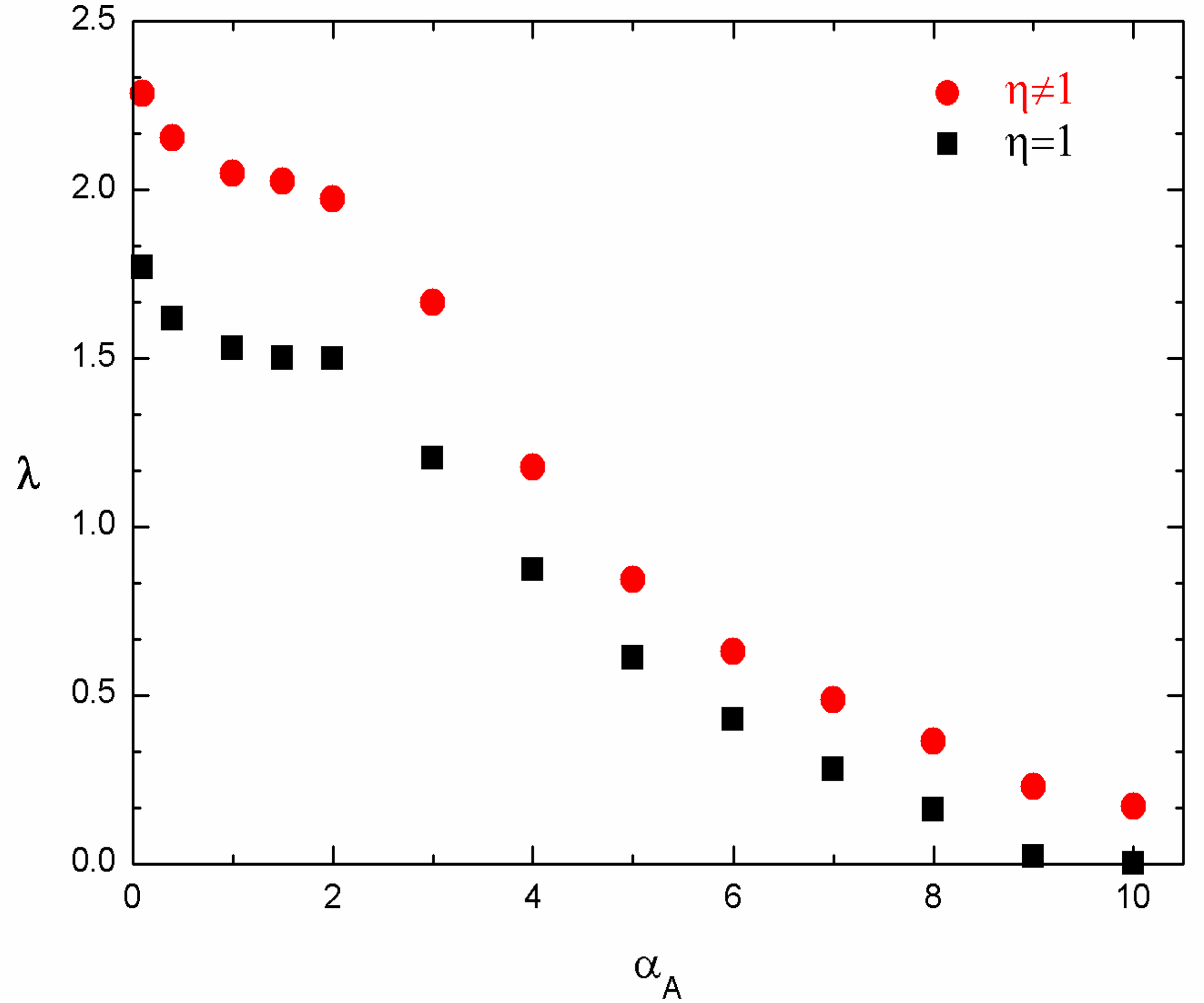
$$e_q^x \equiv [1 + (1-q)x]^{1/(1-q)}$$



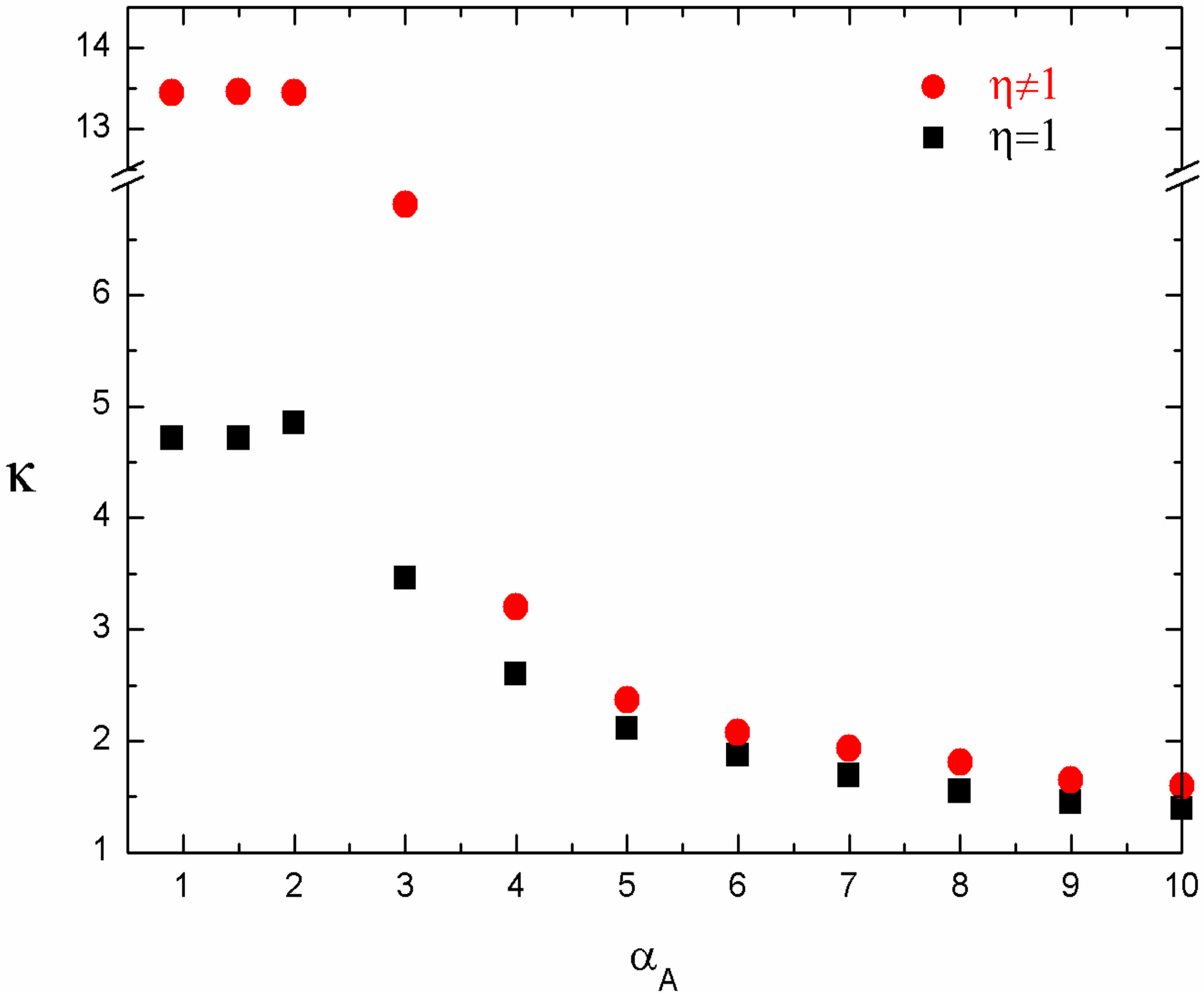
Connectivity distribution *P*(*k*) for typical values α_A for η≠1 and η=1 models. The symbols are numerical results and continuous lines are the best fits according to P(k).



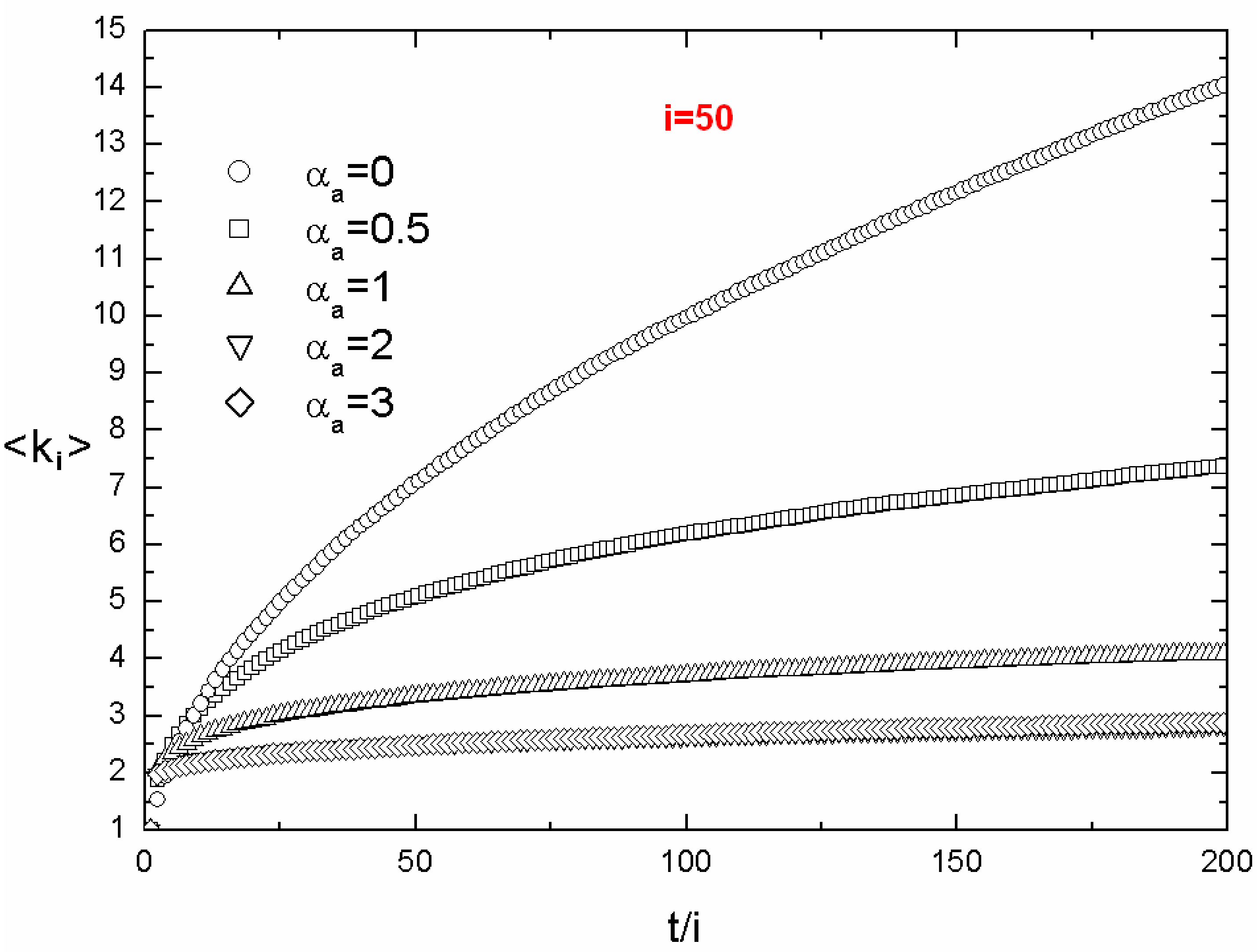
α_A-dependence of q for both η≠1 and η=1 models. In this graph we observe some kinds of changements of regimes at α_A = 2 (which coincides with the space dimensionality).



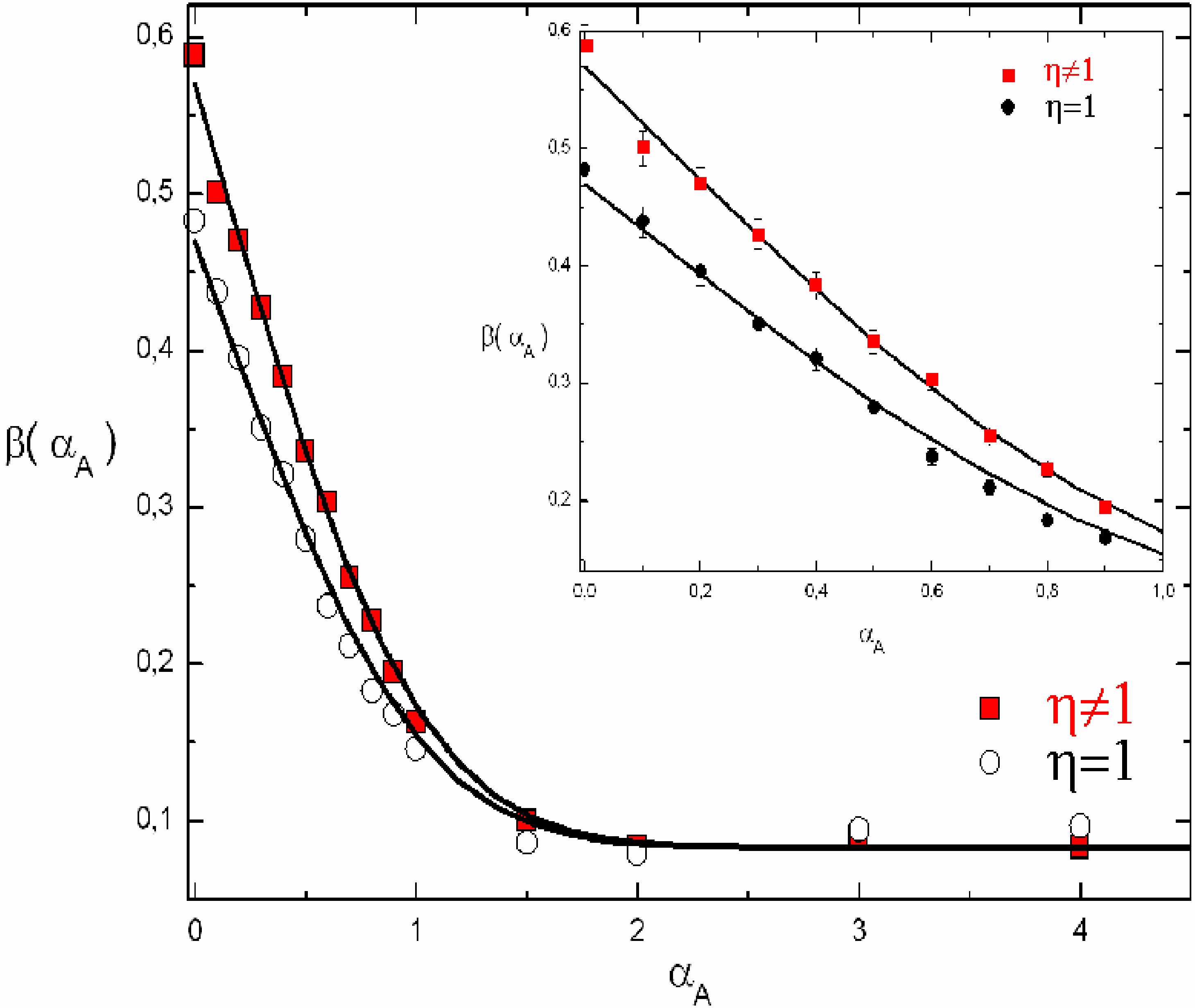
α_A-dependence of λ for both η≠1 and η=1 models. In this graph we observe some kinds of changements of regimes at $\alpha_A = 2$ (which coincides with the space dimensionality).



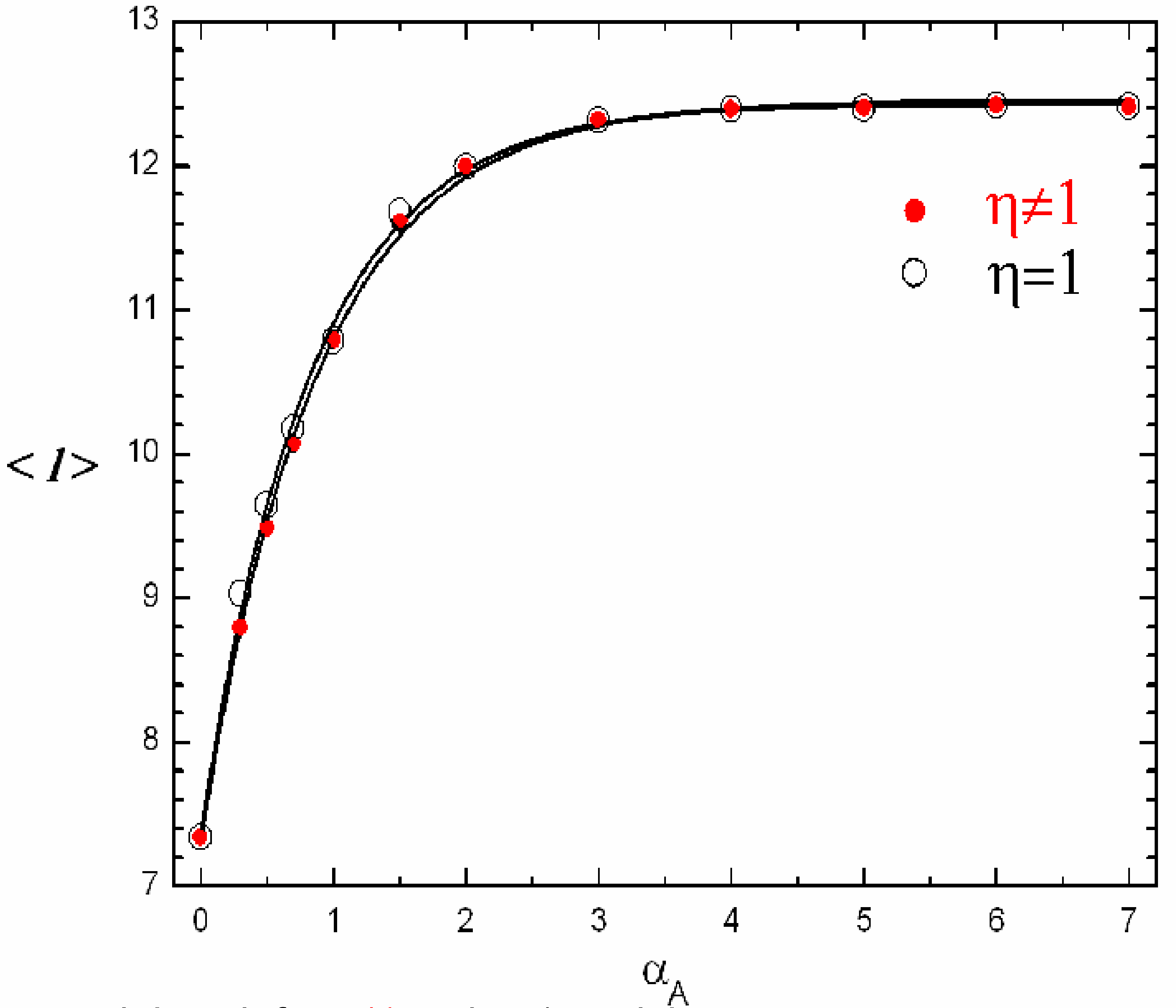
α_A-dependence of q for both η≠1 and η=1 models. In this graph we observe some kinds of changements of regimes at α_A = 2 (which coincides with the space dimensionality).



Temporal dependence of the average connectivity for $\eta \neq 1$, in 2000 samples.

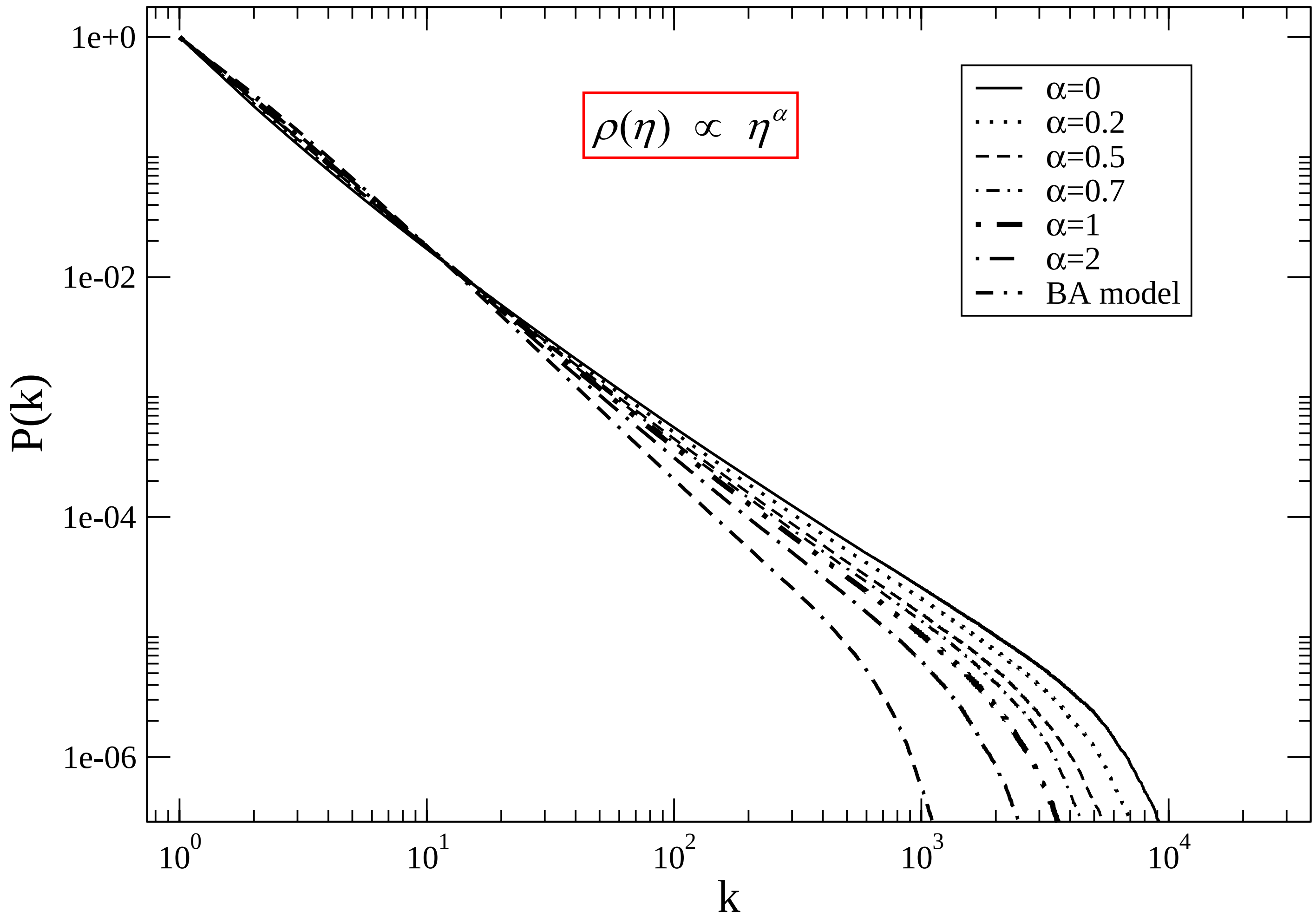


Average connectivity exponent for α_A values relative to measures on node *i* = 50.



Average path length for η≠1 and η=1 models.

Model with Fitness Power-law Distributed



$2.25 < \gamma < 3$

Generalized Model: Fitness and Euclidean Distance Power-law Distributed

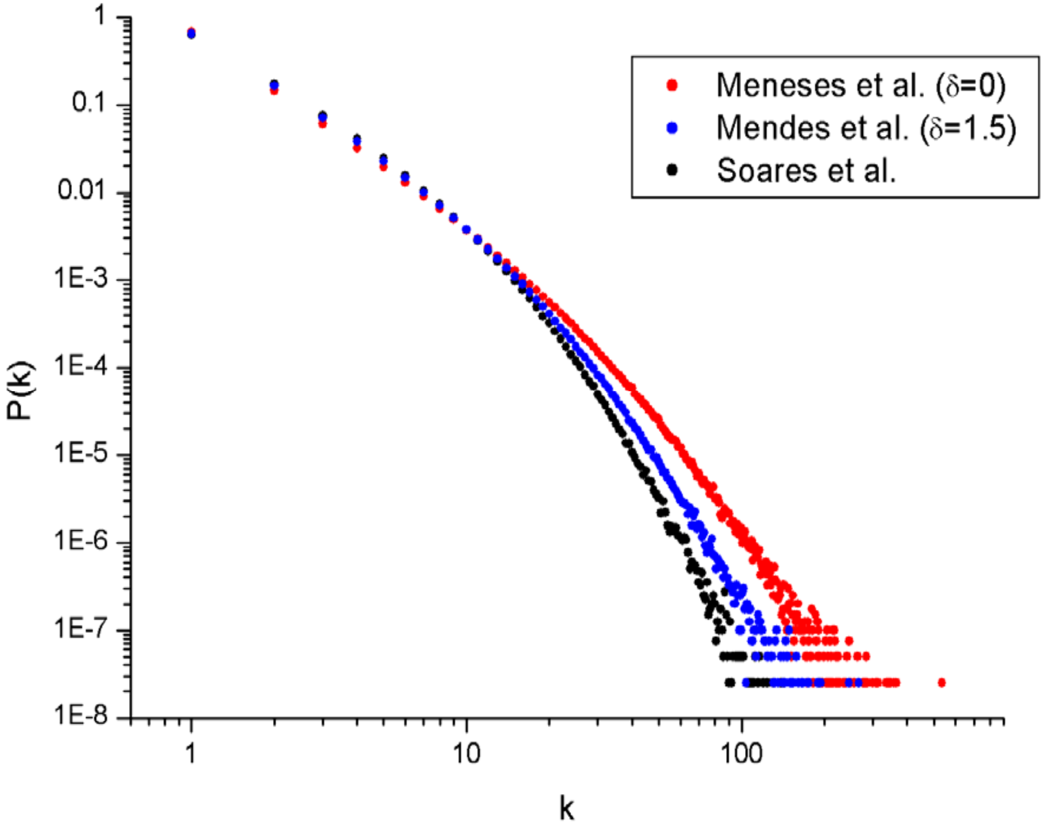
- Inspired in the works of Meneses et al;
- Mendes et al;

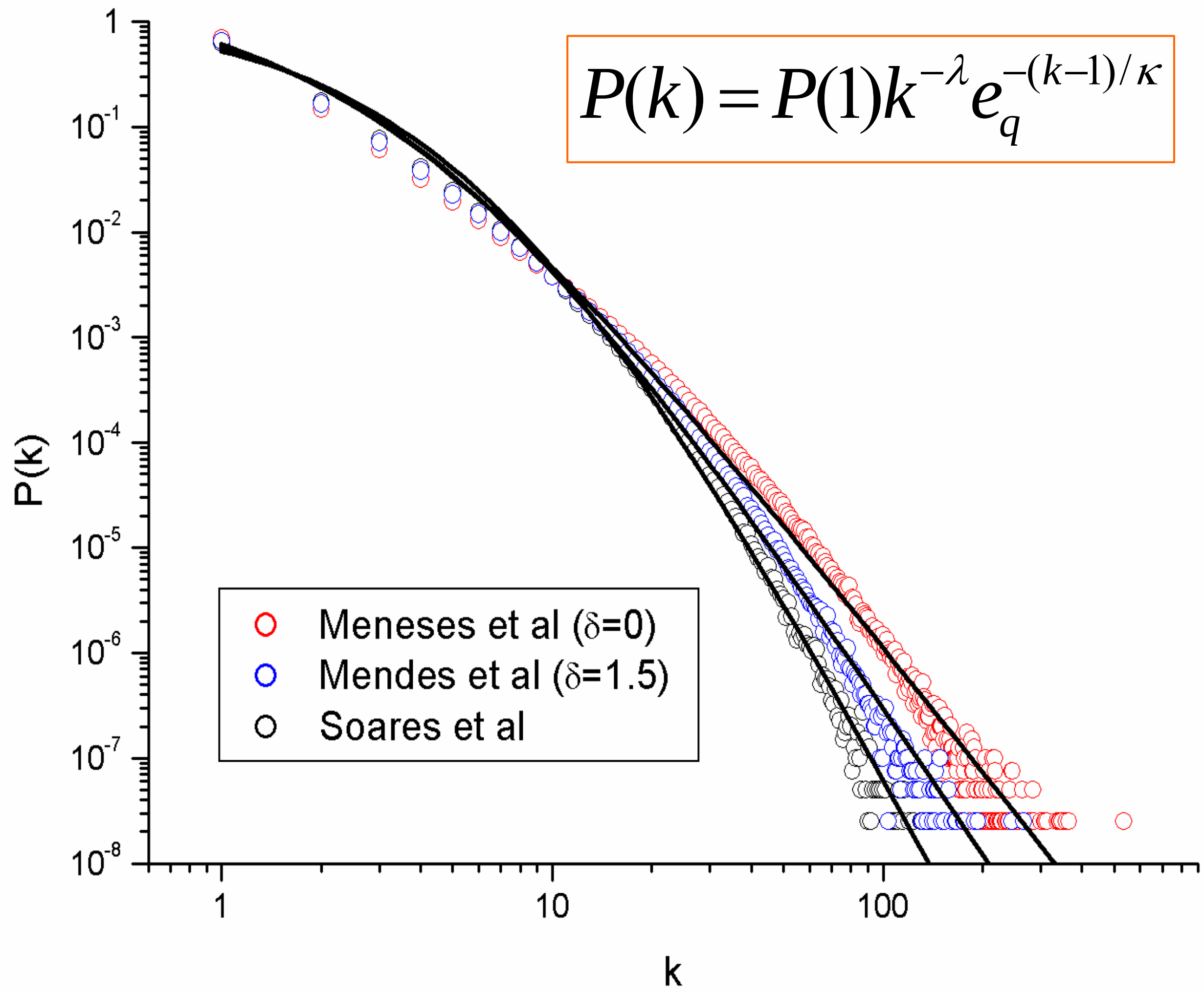
$$\Pi(k_i) = \frac{k_i \eta_i / r_i^{\alpha_A}}{\sum_{j=1}^{N'} k_j \eta_j / r_j^{\alpha_A}} \qquad (08)$$

with

$$\rho(\eta) \propto \eta^{\delta}$$

$$\delta \geq 0$$

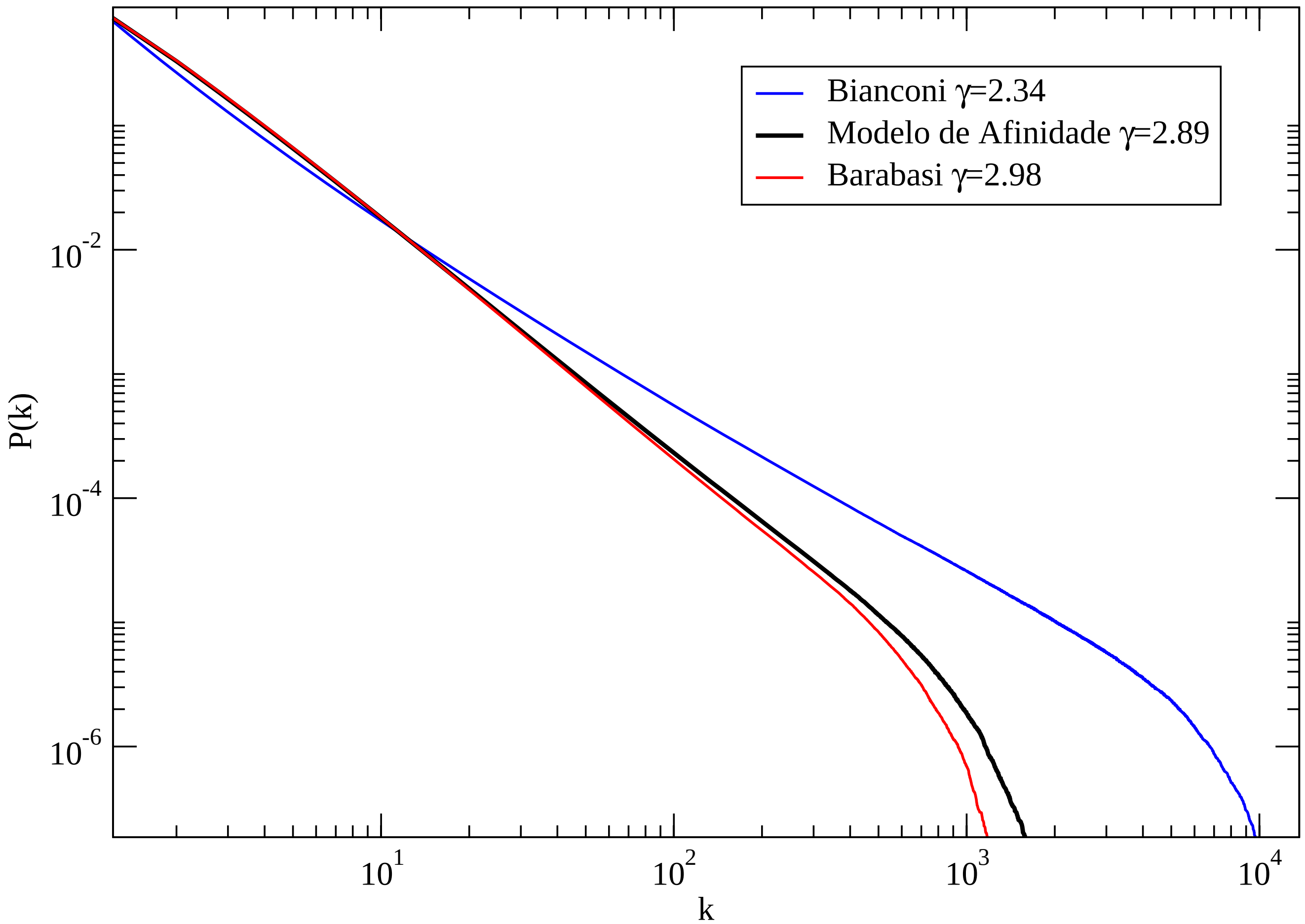




Conectivity distribution $P(k)$ for α_A=2 for Meneses et al, Mendes et al and Soares et al models. The symbols are numerical results and continuous lines are the best fits in according to P(k).

Acquaintance Model

$$\Pi_{i \to j} = \frac{\left\{1 - \left|\eta_i - \eta_j\right|\right\} k_j}{\sum_{j=1}^{N} \left\{1 - \left|\eta_i - \eta_j\right|\right\} k_j}$$



Summary

(a)

- The present model contains the five previous models:

Model	CONECTIVITY	FITNESS	METRIC
Barabási-Albert	YES	NO	NO
Bianconi et al	YES	UNIFORM	NO
Soares et al	YES	NO	POWER-LAW
Meneses et al	YES	UNIFORM	POWER-LAW
Mendes et al	YES	POWER-LAW	NO
Present	YES	POWER-LAW	POWER-LAW

Summary

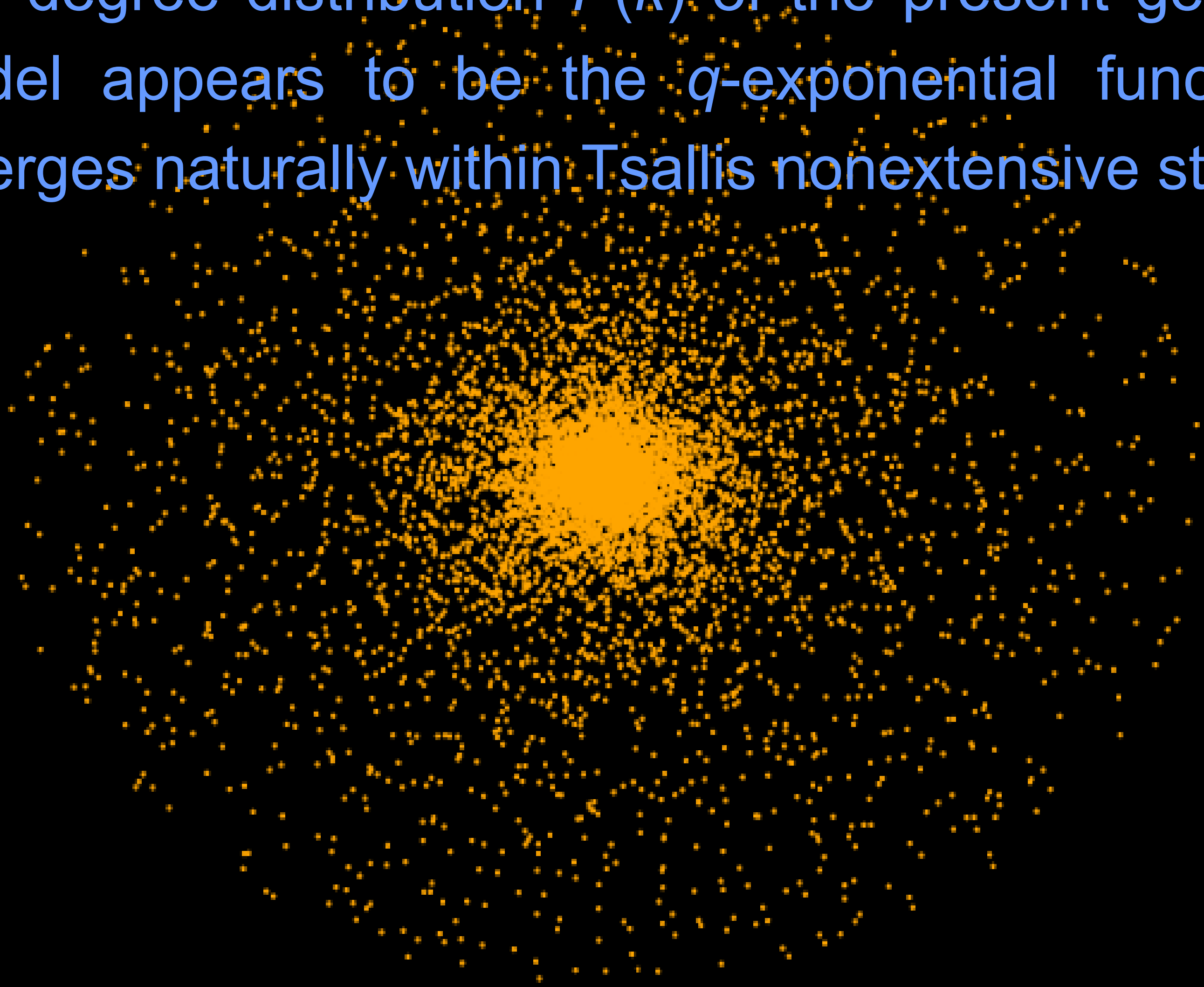
(b)

- We study the effect of competition between the relevant variables: connectivity k, fitness η and metrics r.
- The fitness may give the possibility to the younger nodes to compete equally with the older ones, when the younger node gets a high fitness.
- By including metrics favors the linking between first neighbors.
- The average connectivity $\langle k_i \rangle$ is appreciably influenced by metrics and by fitness, while the average path length $\langle l \rangle$ keeps approximatively the same.

Summary

(c)

- The degree distribution $P(k)$ of the present generalized model appears to be the q-exponential function that emerges naturally within Tsallis nonextensive statistics.



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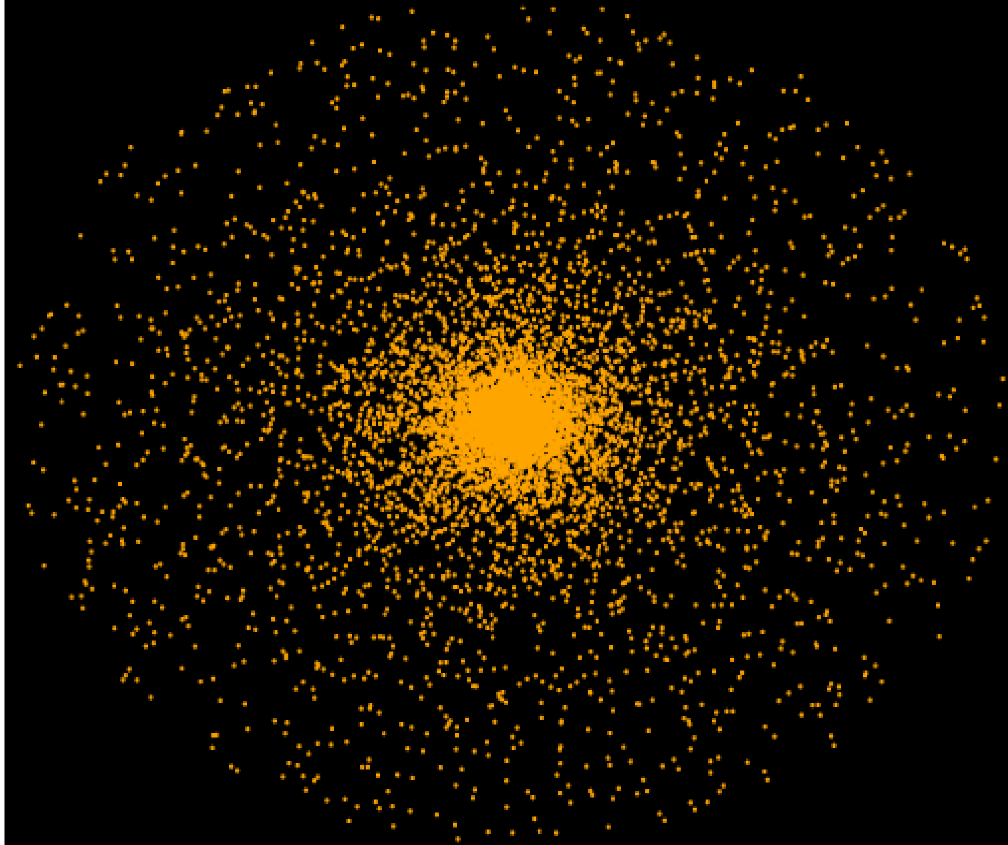

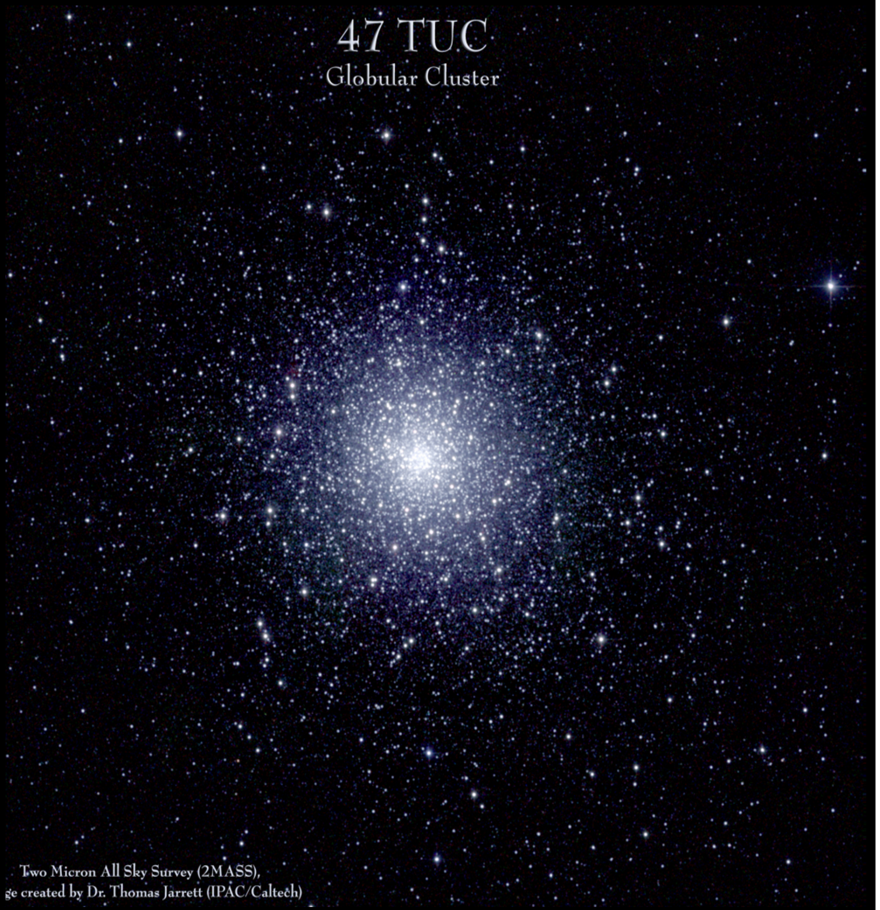

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1. D. J. B. Soares, C. Tsallis, A. M..Mariz, and L. R. da Silva.
"Preferential Attachment Growth Model and Noextensive Statistical Mechanics"
Europhysics Letters **70**, 70 (2005)

2. J. S. Andrade Jr., H. J. Herrmann, R.F Andrade and L. R. da Silva.
Apollonian Networks: Simultaneously Scale-free, Small World, Euclidean Space Filling and with Matching Graphs.
Physical Review Letters **94**, 018702 (2005).

3. M. D. de Meneses, Sharon D. da Cunha, D.J.B. Soares and L. R. da Silva.
"Preferential Attachment Scale-free Growth Model with Random Fitness and Connection with Tsallis Statistics"
Progress of Theoretical Physics Supplement **162** 131 (2006)

4. D. J. B. Soares, J. S. Andrade Jr., H. J. Herrmann and L. R. da Silva
"Three Dimension Apollonian Networks"
International Journal of Modern Physics **17** 1219 (2006)

5. P. G. Lind, L. R. da Silva, J. S. Andrade Jr. and H. J. Herrmann.
"The Spread of Gossip in American Schools"
Europhysics Letters **78**, 68005 (2007)

References

6. P. G. Lind, L. R. da Silva, J. S. Andrade Jr. and H. J. Herrmann.
"Spreading Gossip in Social Networks"
Physical Review E **76**, 036117 (2007)

7. G. A. Mendes and L. R. da Silva (2009)
Generating more realistic complex networks from power-law distribution of fitness

(in progress)

8. S. B. Jácome, A. A. Moreira, J. S. Andrade L. R. da Silva and H. J. Herrmann
"Decomposition in Scale-free Networks"

9. M. L. de Almeida, G. A. Mendes and L. R. da Silva
A fitness model for acquaintance network (master thesis)

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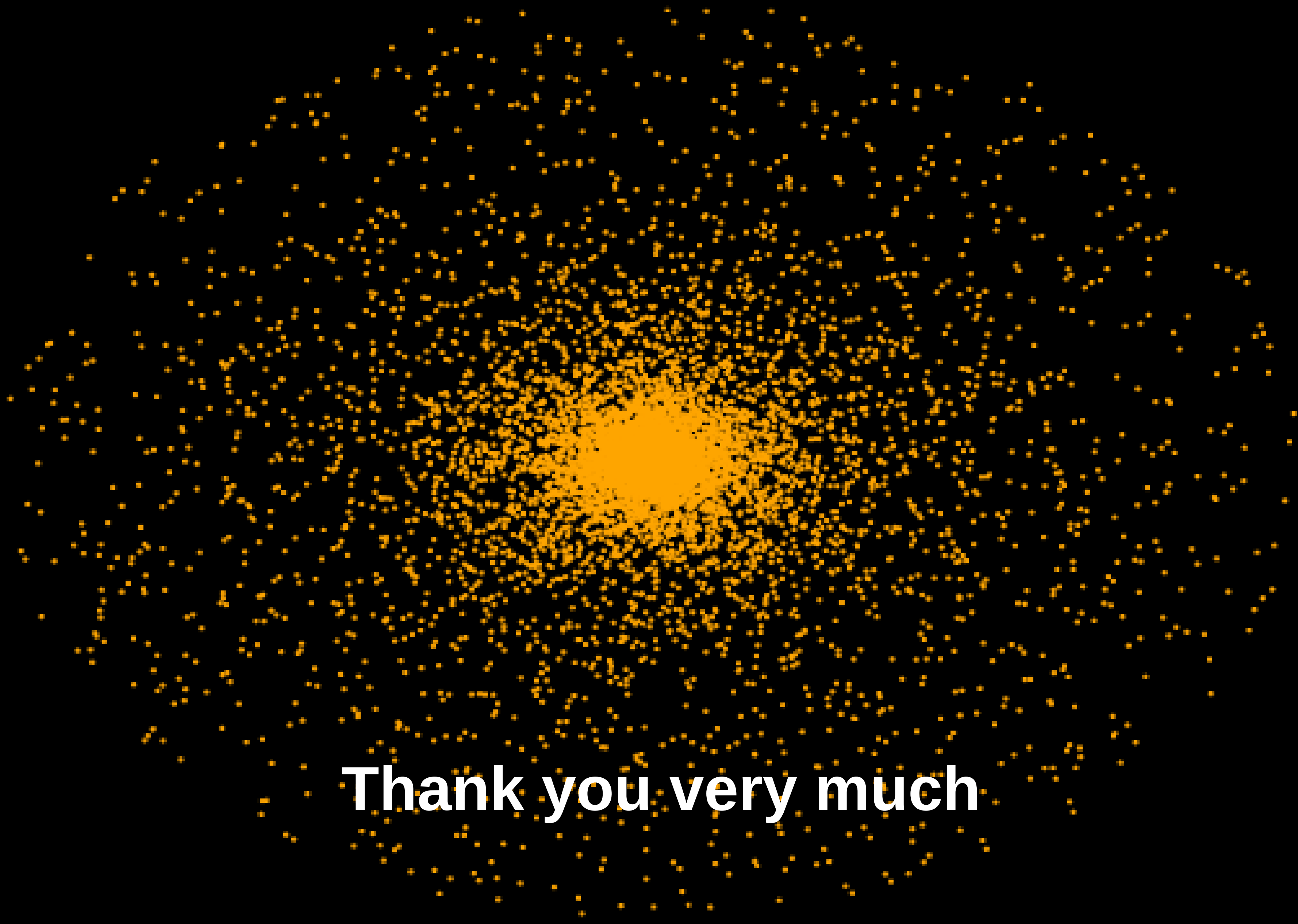
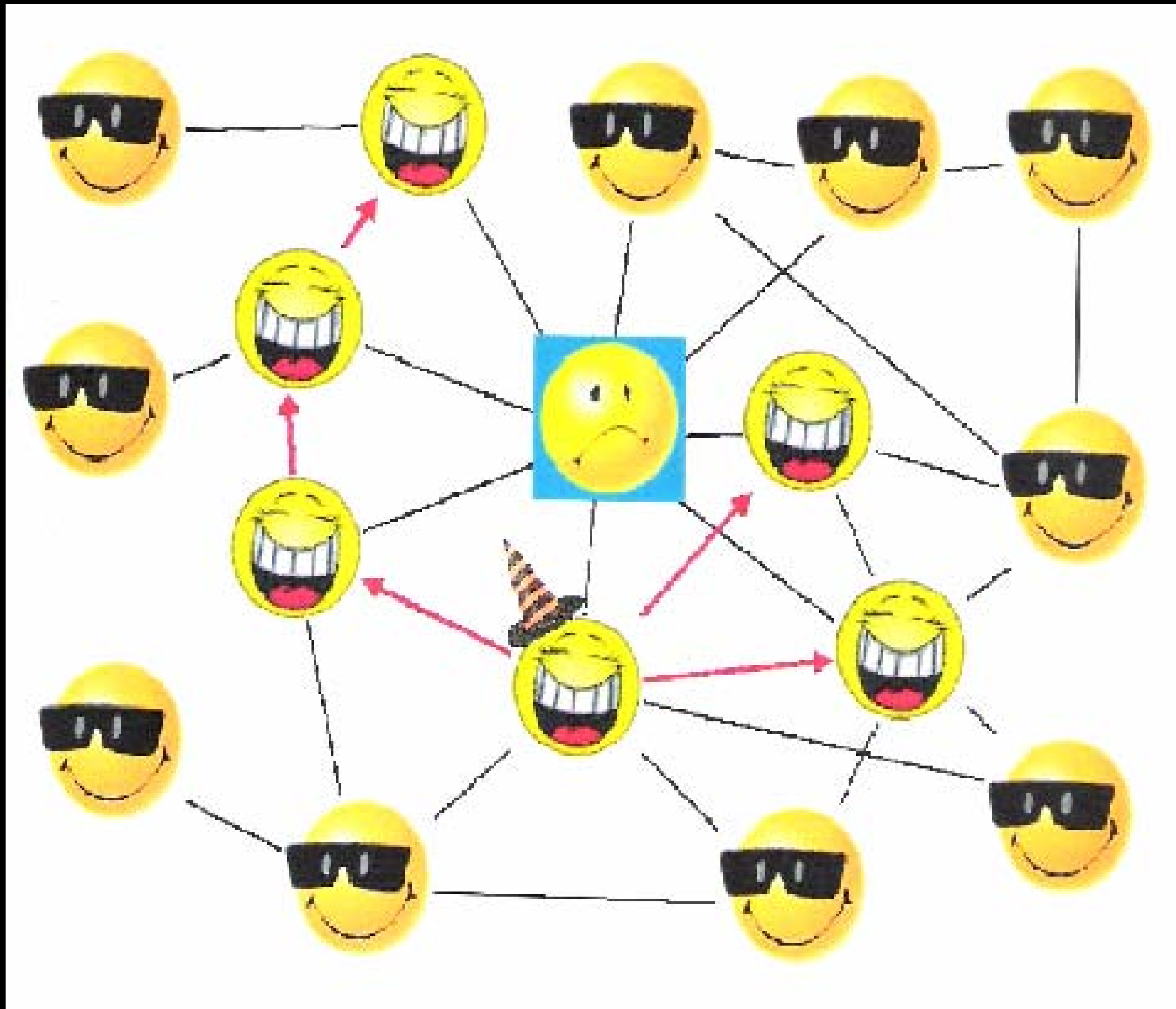
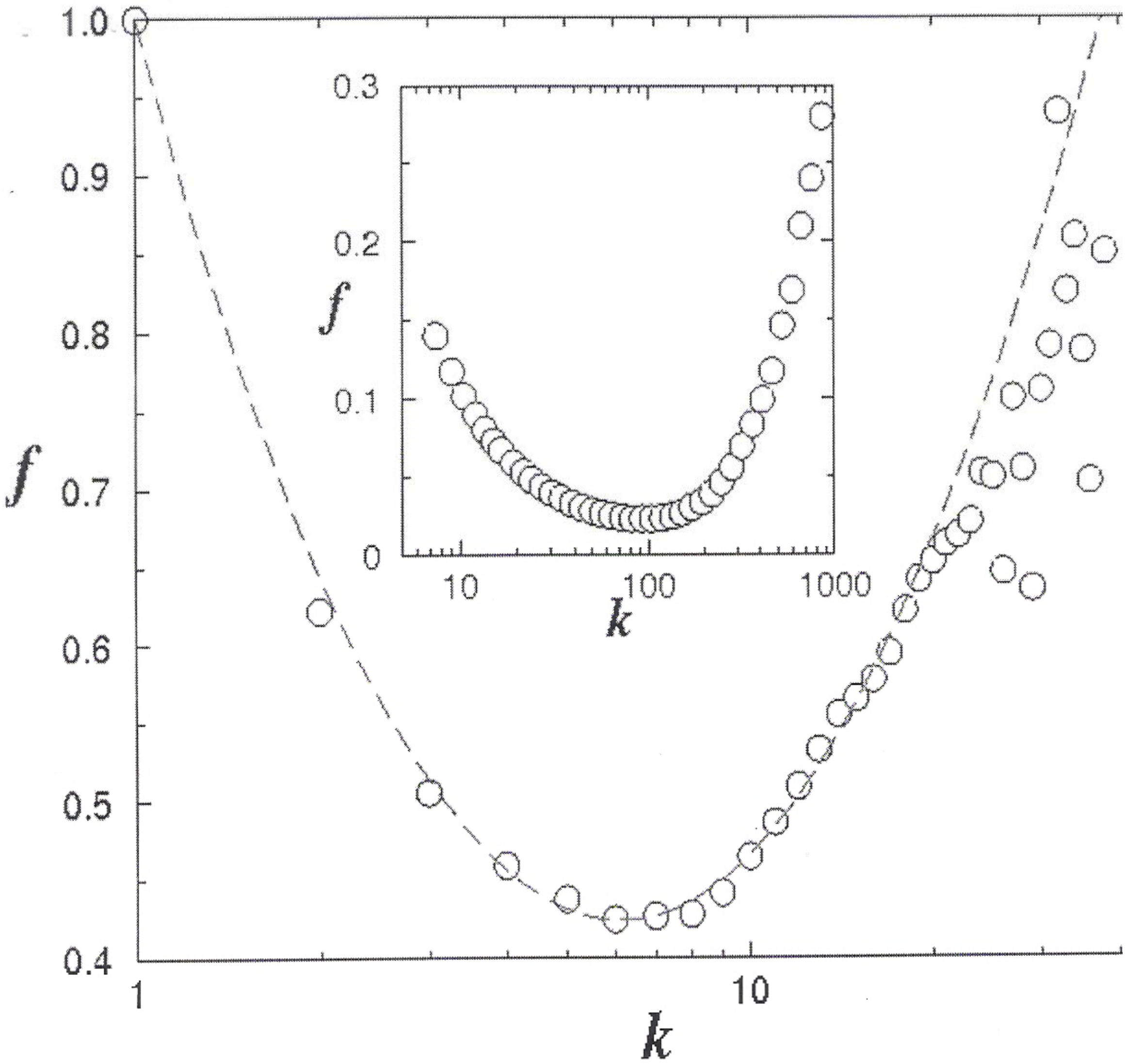


6. How gossip propagates



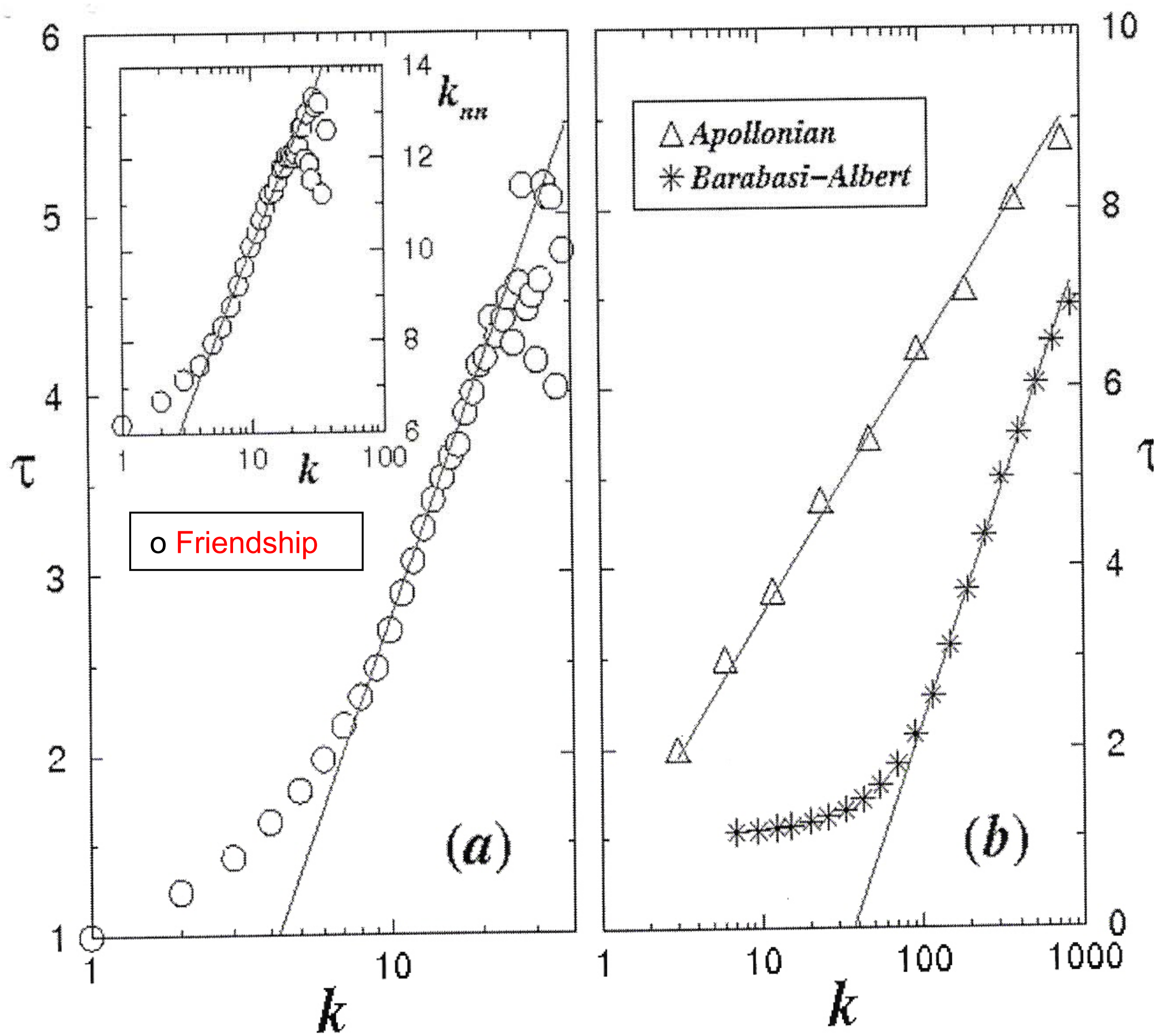
Europhysics Letters **78**, 68005 (2007)

Spread Factor ($f=n_f/k$)

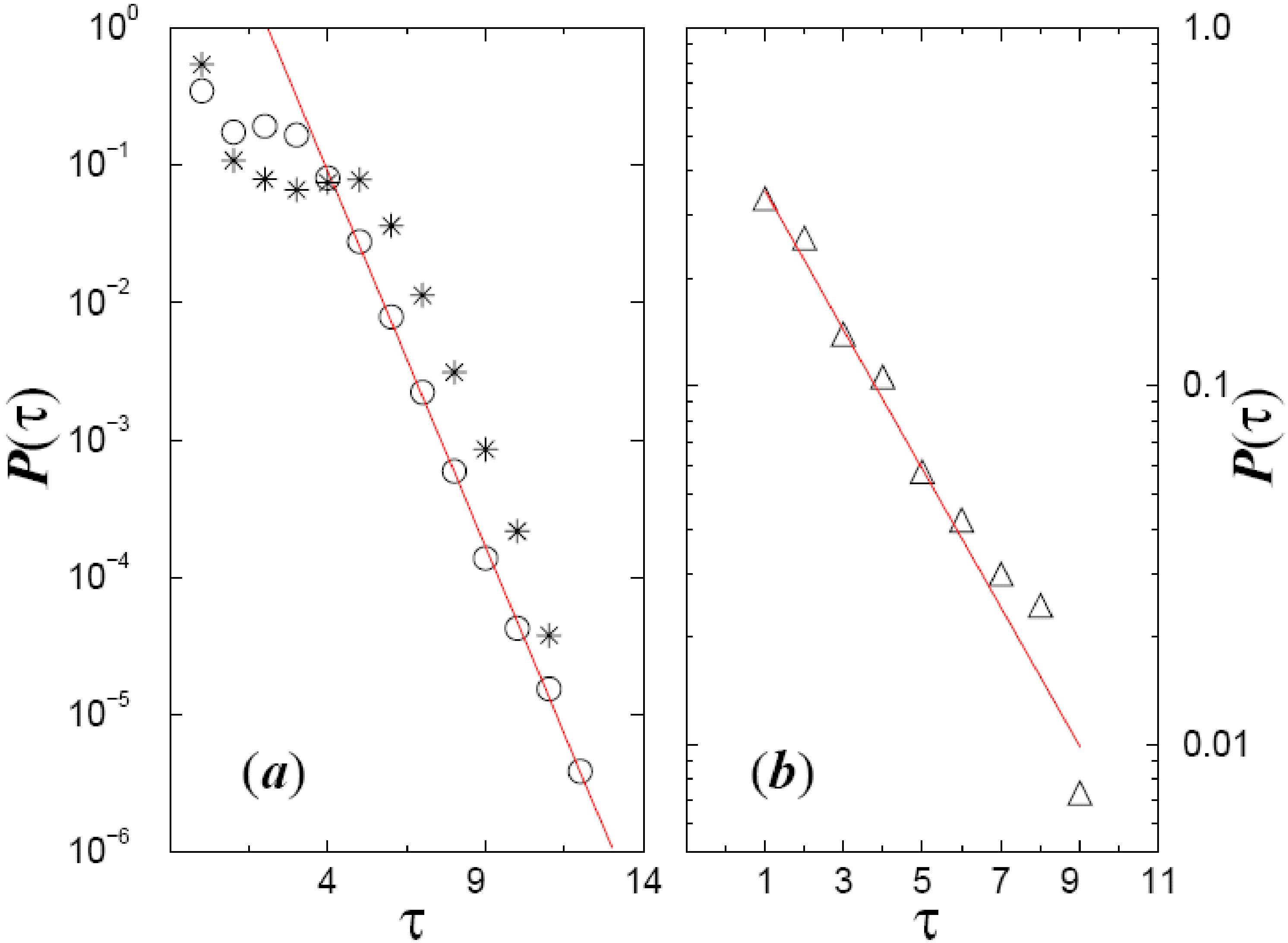


U.S Friend Schools

Inset graph: Barabási-Albert



$$\tau = A + B\log(k)$$



o Friendship

* BA; m=7

Δ Apollonian

GAS-LIKE (NODE COLLAPSING) NETWORK:

S. Thurner and C. T., Europhys Lett **72**, 197 (2005)

Number *N* of nodes fixed (*chemostat*); i=1, 2, …, *N*

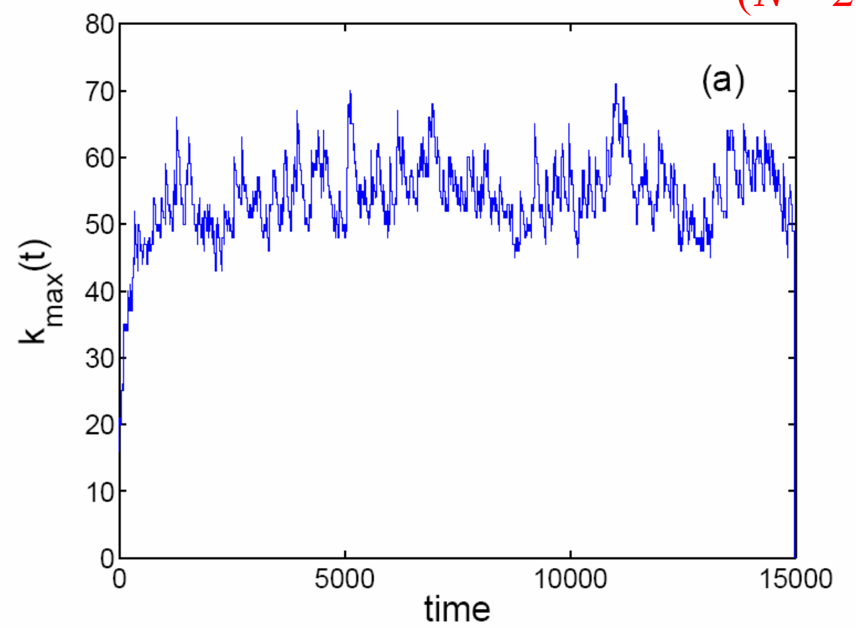
$$Merging\ probability\ \ p_{ij} \propto \frac{1}{d_{ij}{}^{\alpha}} \quad (\alpha \geq 0)$$

$$d_{ij} \equiv shortest\ path\ (chemical\ distance)\ connecting\ nodes\ i\ and\ j\ on\ the\ network$$

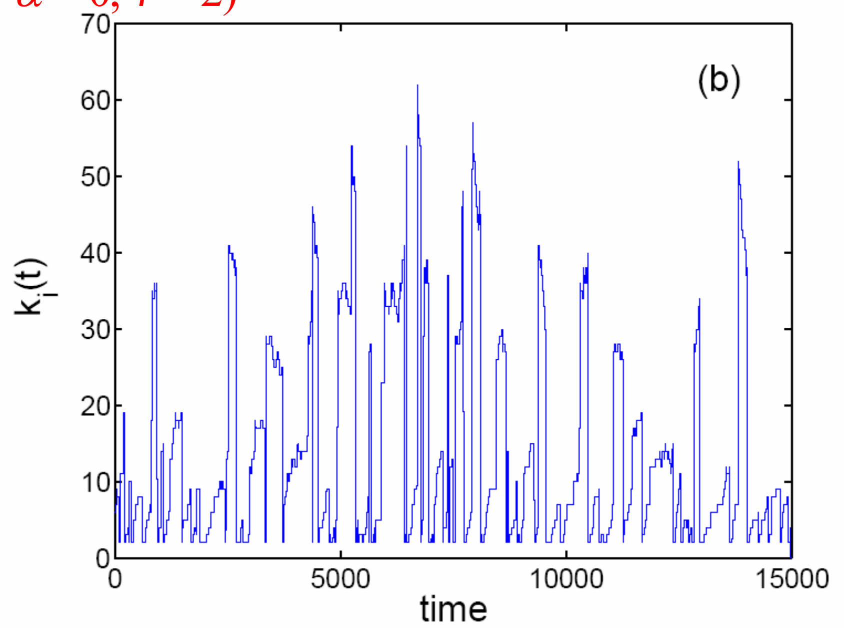
$\alpha = 0$ *and* $\alpha \to \infty$ *recover the random and the neighbor schemes respectively*

(Kim, Trusina, Minnhagen and Sneppen, *Eur. Phys. J. B* 43 (2005) 369)

$(N = 2^7;\ \alpha = 0;\ r = 2)$

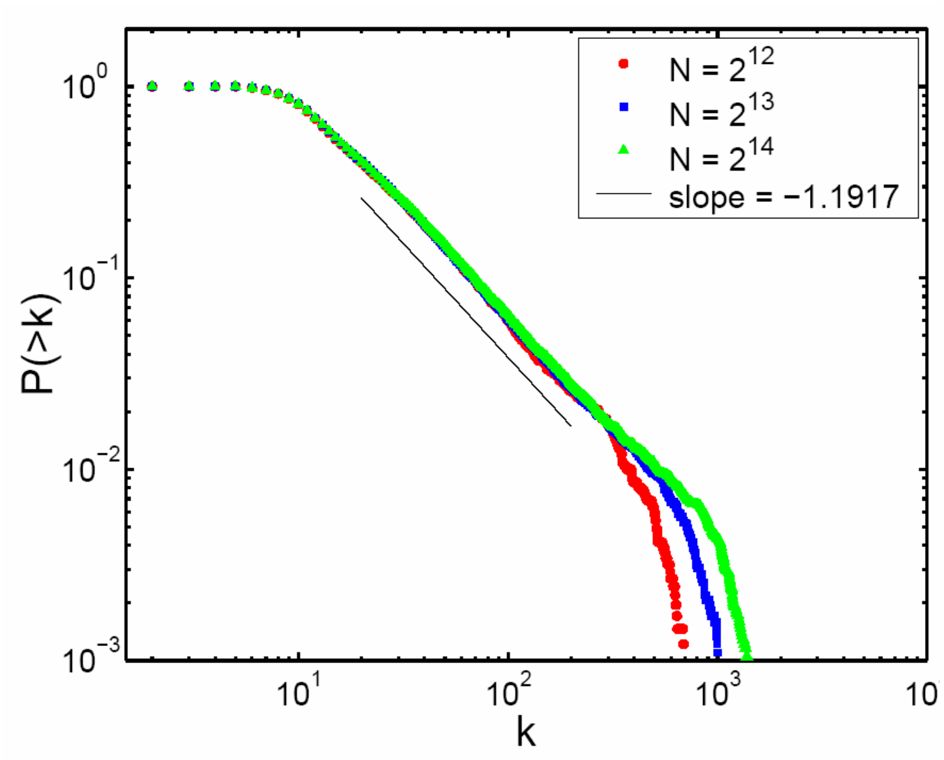


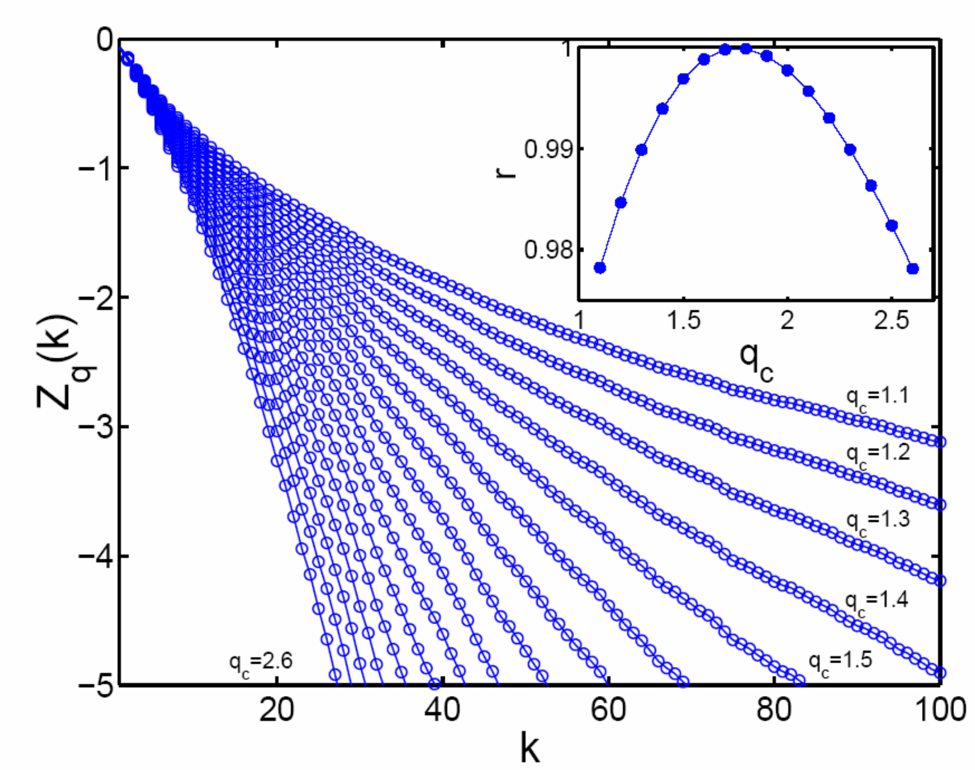
Degree of the most connected node



Degree of a randomly chosen node

$$(\alpha \to \infty\ ;\ \ <r>=8)$$



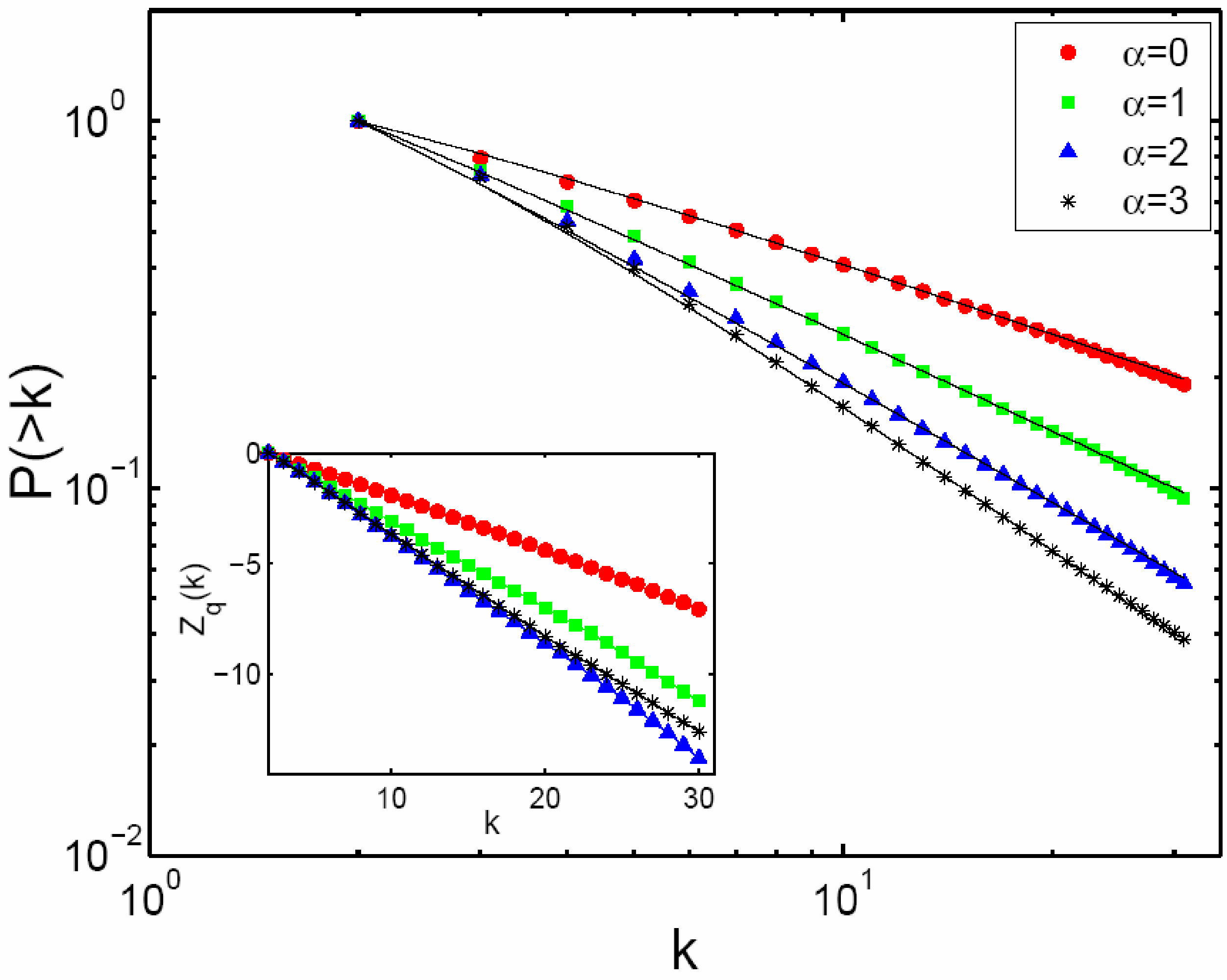


$$Z_q(k) \equiv \ln_q\left[P(>k)\right] \equiv \frac{\left[P(>k)\right]^{1-q}-1}{1-q}$$

$$(optimal\ q_c = 1.84)$$

S. Thurner and C. T., Europhys Lett **72**, 197 (2005)

$(N = 2^9;\ r = 2)$

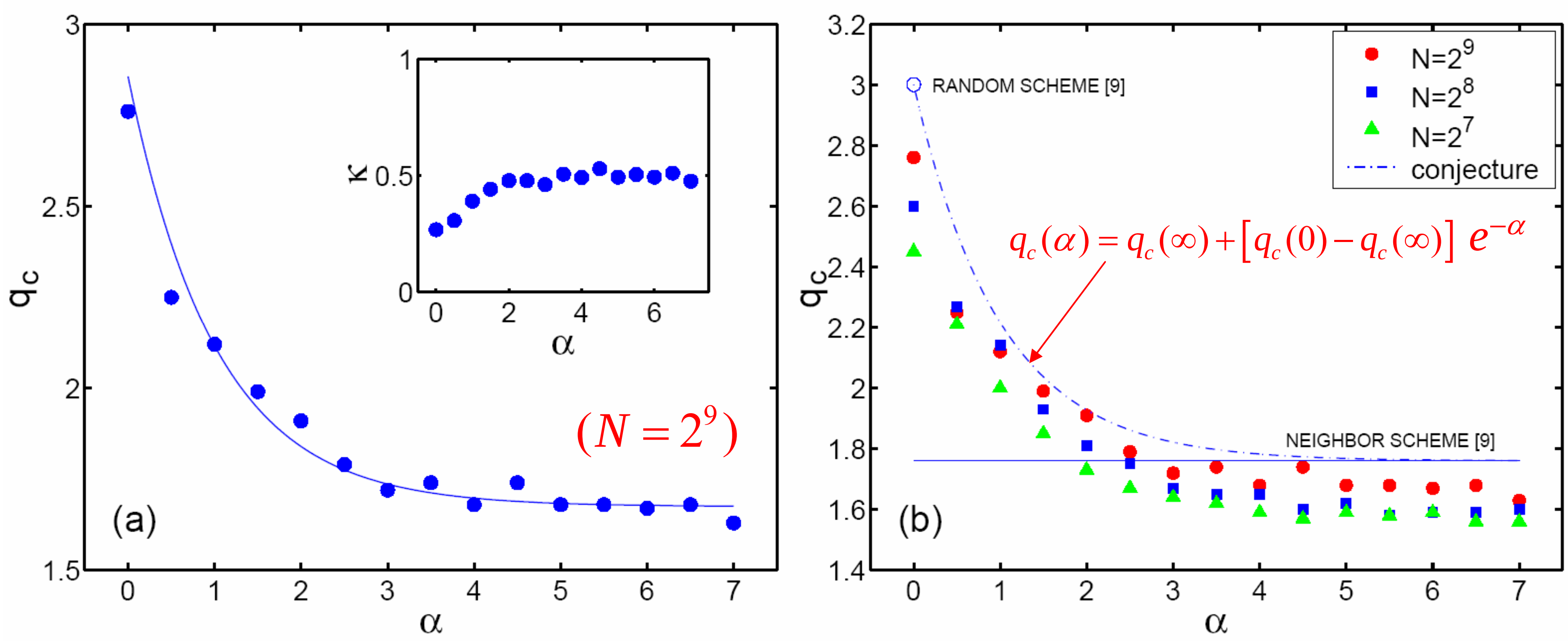


$$P(\geq k) = e_{q_c}^{-(k-2)/\kappa} \quad (k = 2,\ 3,\ 4, \ldots)$$

$$linear\ correlation \in [0.999901, 0.999976]$$

S. Thurner and C. T., Europhys Lett **72**, 197 (2005)

$$(r = 2)$$



S. Thurner and C. T., Europhys Lett **72**, 197 (2005)