Domain growth: from ants to liquid crystals

Jeferson J. Arenzon
Instituto de Física and INCT-SC – UFRGS
Porto Alegre
Brazil

Fluidos Complexos
Complex Fluids

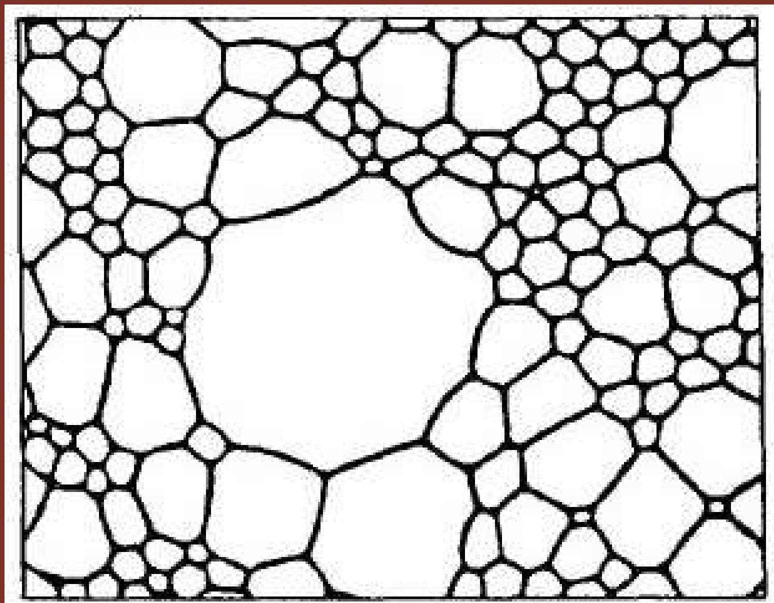
Instituto de Física

UFRGS
UNIVERSIDADE FEDERAL DO RIO GRANDE DO SUL

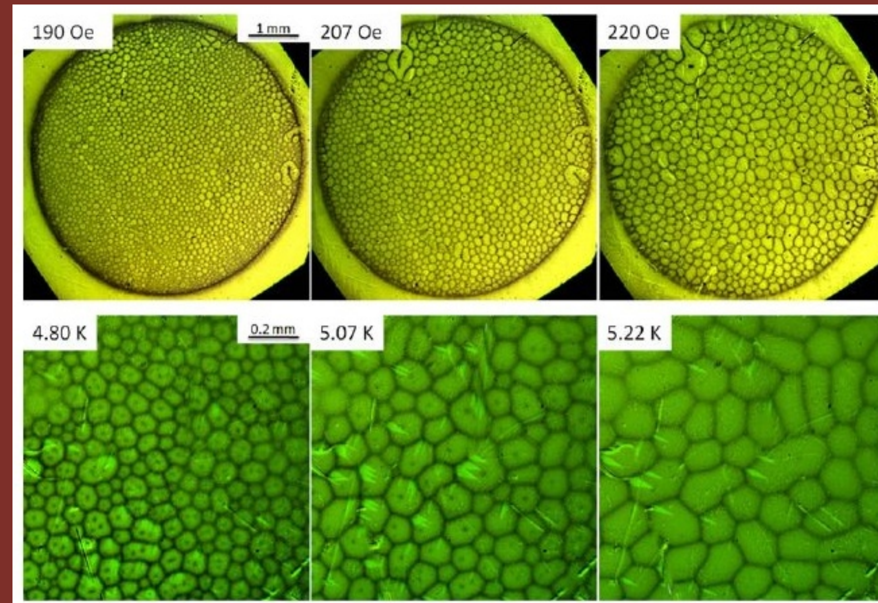
Collaboration with

- Prof. Alan J. Bray (Manchester, UK)
- Prof. Leticia F. Cugliandolo (LPTHE, Paris)
- Prof. Ingo Dierking (Manchester, UK)
- Alberto Sicilia (Paris)
- Yoann Sarrazin (Paris)
- Marcos Paulo Loureiro (Porto Alegre)

Domain Growth

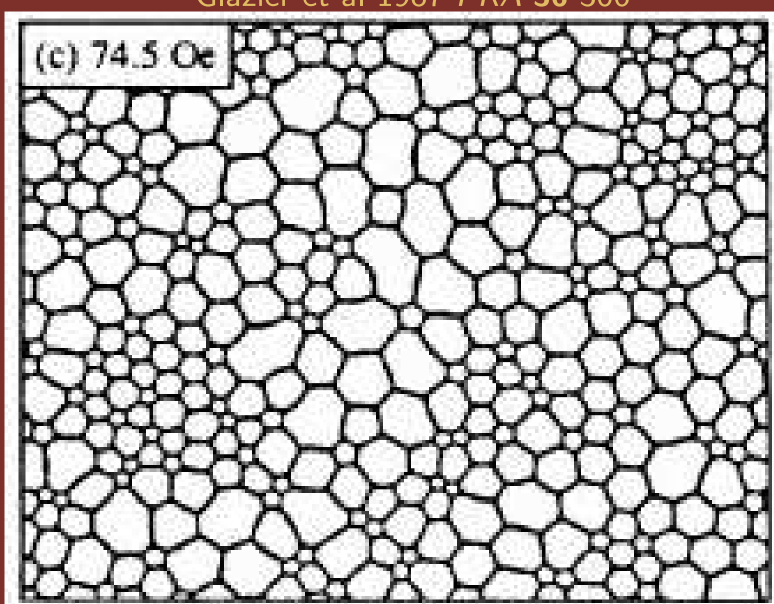


Glazier et al 1987 *PRA* **36** 306

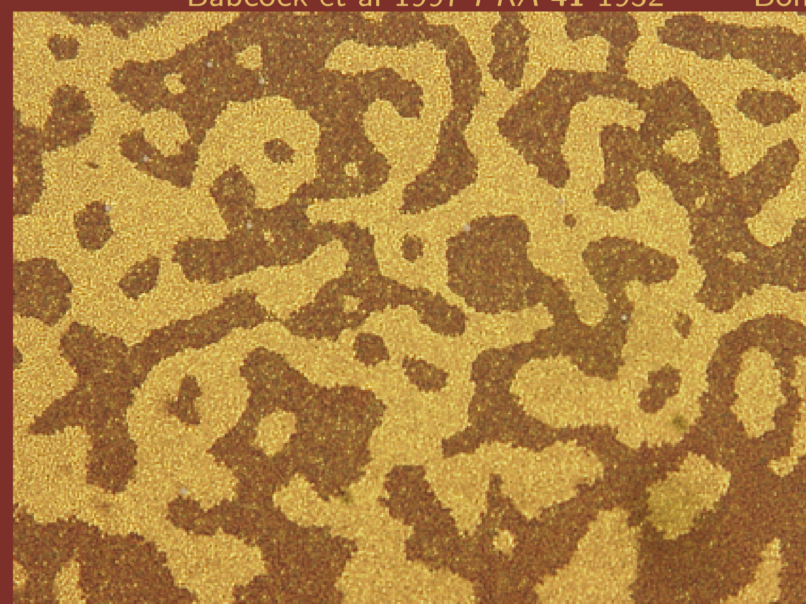


Babcock et al 1997 *PRA* **41** 1952

Bonabeau et al 1998 *PRE* **57** 4568



Prozorov et al 2008 *Nature Physics* **4** 327



Sicilia et al 2008 *PRL* **101** 197801

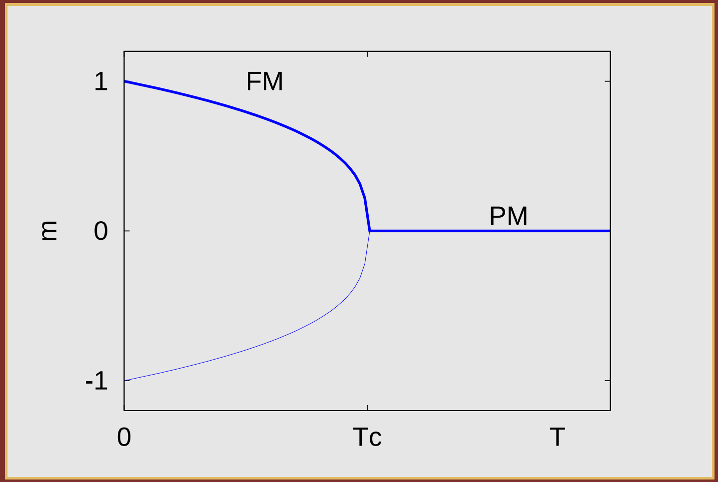
The kinetic Ising Model

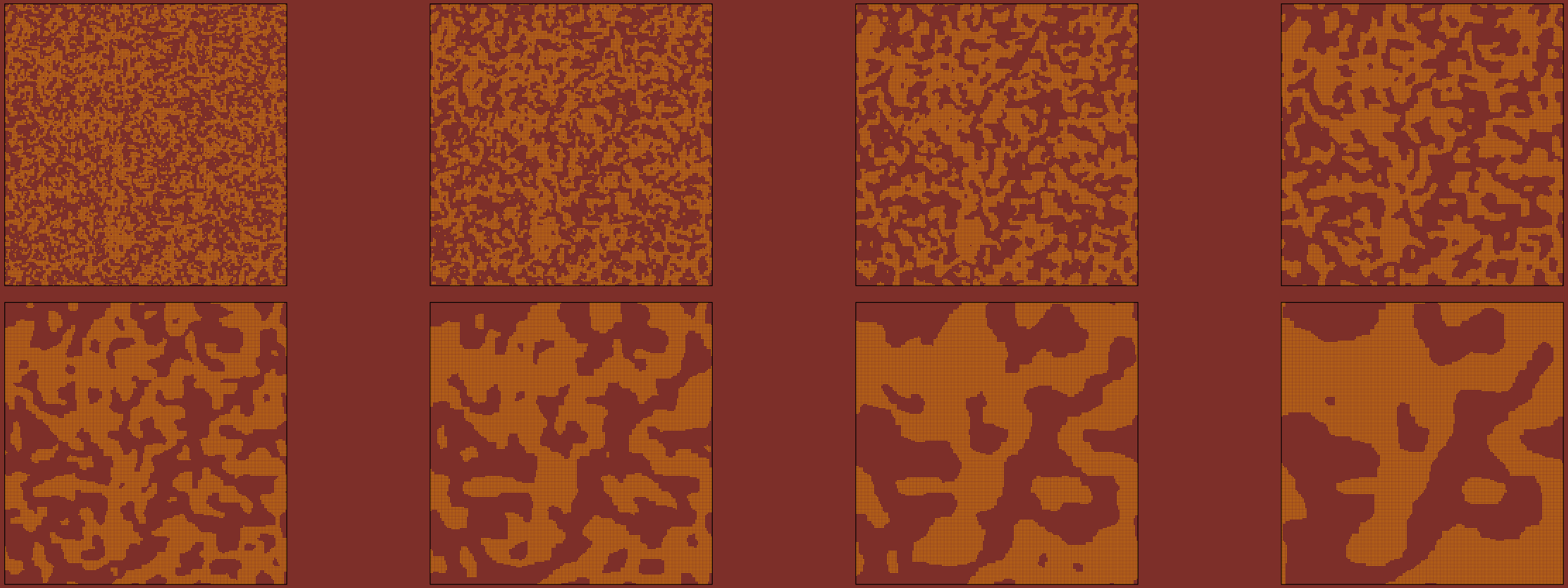
$$\mathcal{H} = -J \sum_{<ij>} S_i S_j$$

$$S_i = \pm 1$$

plus

stochastic dynamics

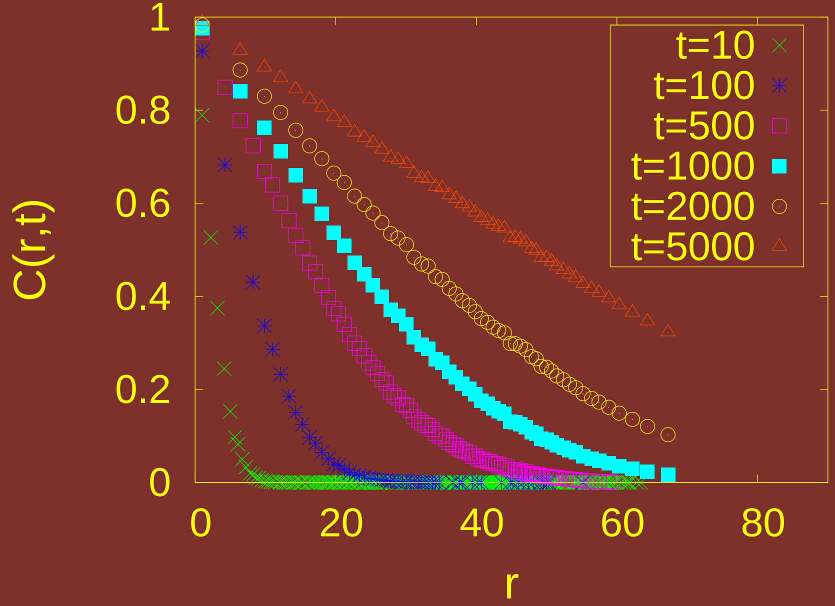




Dynamic Scaling Hypothesis

- Correlation function:

$$C(r,t) = \frac{1}{N}\sum_{i=1}^{N}\langle\, S_i(t)S_j(t)\,\rangle\big|_{|\vec{r}_i-\vec{r}_j|=r}$$



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Bray 1994 *Adv. Phys.* 43 357

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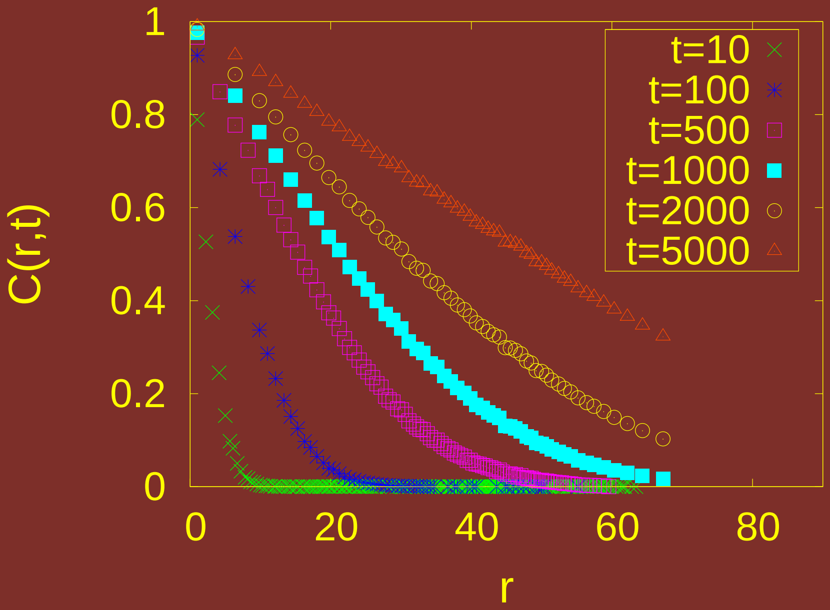
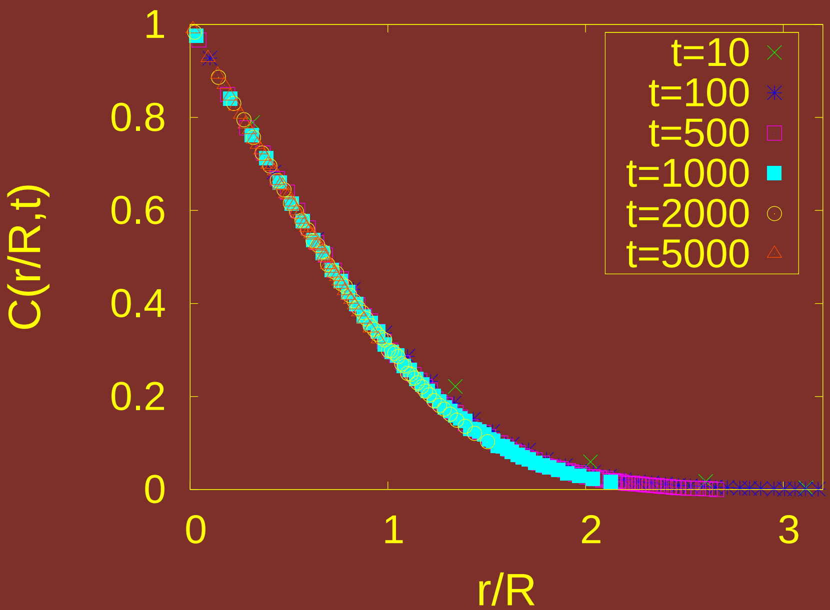


- Dynamic scaling:

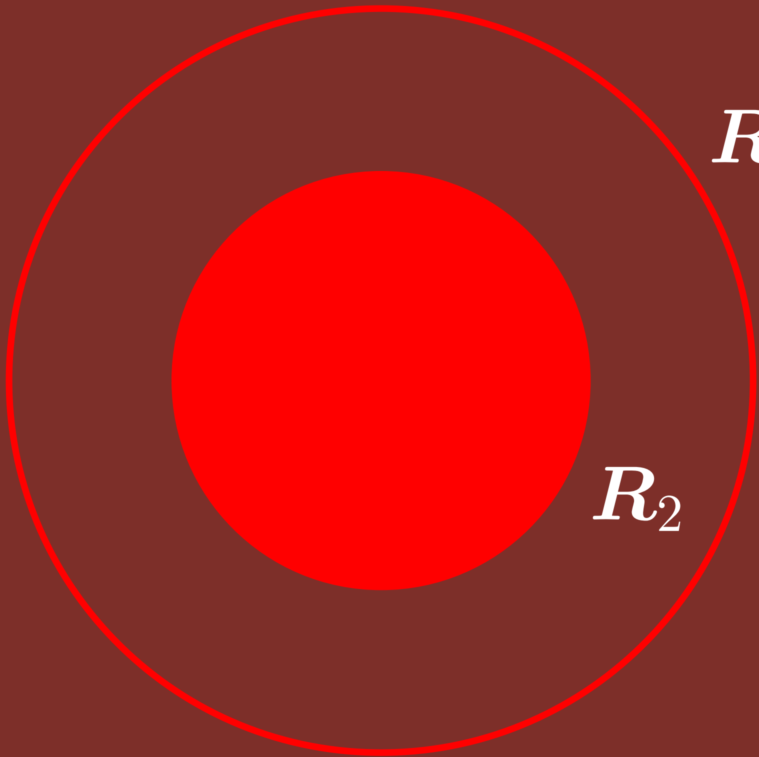
$$C(r,t) \sim f\left(\frac{r}{R(t)}\right)$$



- Characteristic length:

$$R(t) \sim t^{1/z}$$

Hulls and Domains



Two domains:

$$
\begin{aligned}
A_d^{(1)} &= \pi(R_1^2 - R_2^2) \\
A_d^{(2)} &= \pi R_2^2
\end{aligned}
$$

Two hulls:

$$
\begin{aligned}
A_h^{(1)} &= \pi R_1^2 \\
A_h^{(2)} &= \pi R_2^2
\end{aligned}
$$

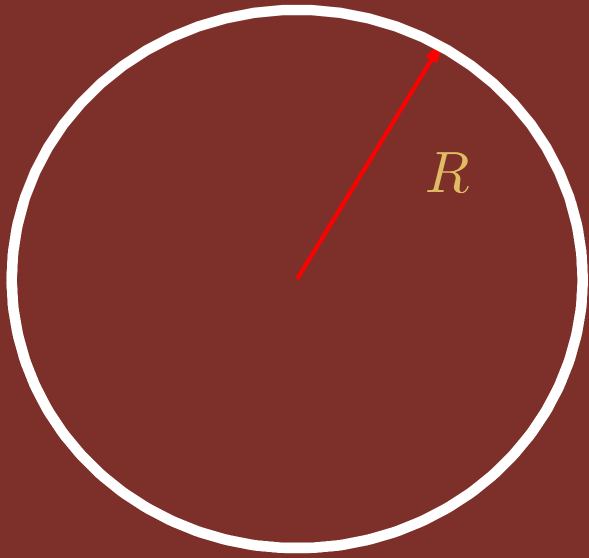
Spherical Hull ($d = 2$)

The time-variation of the **hull area** is

$$A = \pi R^2 \Longrightarrow \frac{dA}{dt} = 2\pi R \frac{dR}{dt}$$

For non conserved fields, the motion of domain walls is determined by the local curvature κ:

$$v = -\frac{\lambda}{2\pi}\kappa \quad (\mathrm{Allen-Cahn\ \ equation})$$



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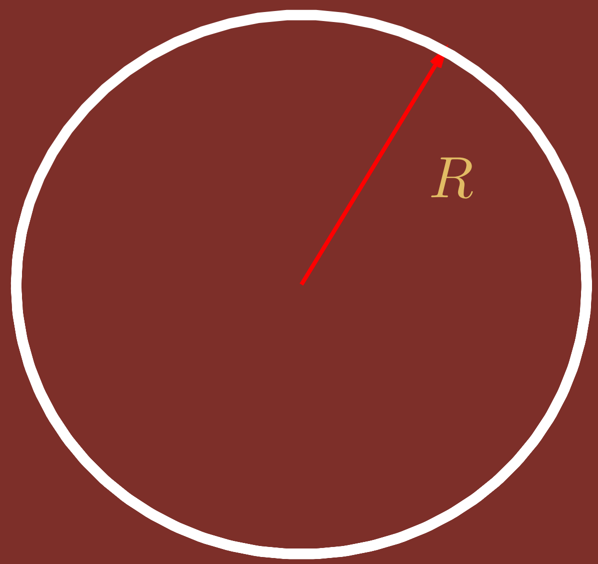


For non conserved fields, the motion of domain walls is determined by the local curvature κ:

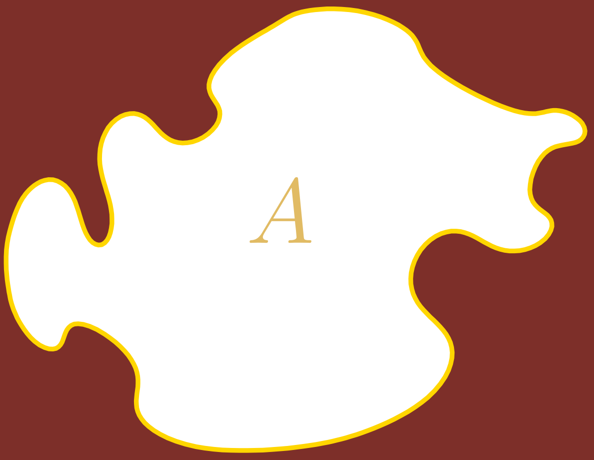
$$\begin{aligned} v &= -\frac{\lambda}{2\pi}\kappa \quad (\text{Allen} - \text{Cahn} \;\; \text{equation}) \\ &= -\frac{\lambda}{2\pi}\frac{1}{R} \end{aligned}$$

Thus we get

$$\boxed{\frac{dA}{dt} = -\lambda}$$

General Hull ($d = 2$)

$$\frac{dA}{dt} = \oint v d\ell = -\frac{\lambda}{2\pi} \underbrace{\oint \kappa d\ell}_{2\pi \text{ (Gauss–Bonnet)}} = -\lambda$$



$$\boxed{\frac{dA}{dt} = -\lambda}$$

- all hulls disappear at the same rate $-\lambda$
- hulls with initial area smaller than λt will have disappeared at t
- hulls with initial area larger than λt will have decreased by λt

The full hull area distribution is advected uniformly to the left at rate λ.

The number of hulls, per unit area of the system, with area greater than A satisfies

$$\boxed{N_h(A, t) = N_h(A + \lambda t, 0)}$$

General Hull ($d = 2$)

The number of hulls, per unit area of the system, with area greater than A satisfies

$$\boxed{N_h(A,t) = N_h(A+\lambda t, 0)}$$

If the system is quenched from equilibrium states either at $T = \infty$ or $T = T_c$, the initial conditions (Cardy and Ziff, 2003) are:

$$N_h(A,0) = \begin{cases} \dfrac{2c_h}{A}, & \text{critical percolation} \\[2ex] \dfrac{c_h}{A}, & \text{critical Ising} \end{cases}$$

where

$$c_h = \frac{1}{8\pi\sqrt{3}} \simeq 0.022972$$

Thus:

$$\boxed{N_h(A,t) = \frac{(2)c_h}{A+\lambda t}}$$

General Hull $(d = 2)$

JJA, Bray, Cugliandolo, and Sicilia 2007 *Phys. Rev. Lett.* 98 145701

The number of hulls, per unit area, having area in the interval $(A, A + dA)$ is

$$\boxed{n_h(A,t) = \frac{(2)c_h}{(A+\lambda t)^2}}$$

It has the expected scaling form:

$$n_h(A,t) = t^{-2} f\left(\frac{A}{t}\right)$$

That is:

$$A(t) \sim t \Longrightarrow R(t) \sim t^{1/2}$$

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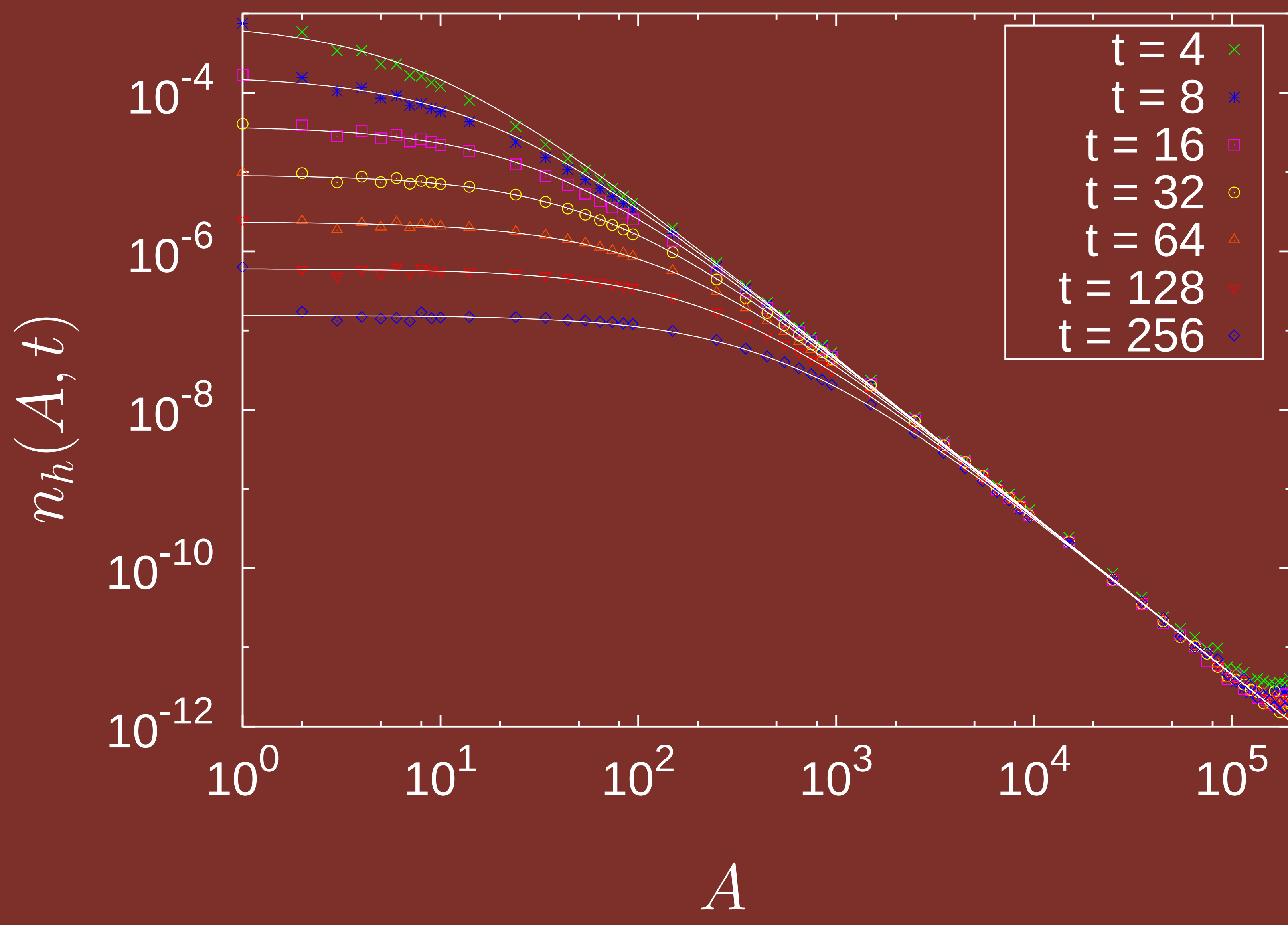
dynamical scaling proved in $2d$!

Hull's distribution

JJA, Bray, Cugliandolo, and Sicilia 2007 *Phys. Rev. Lett.* **98** 145701



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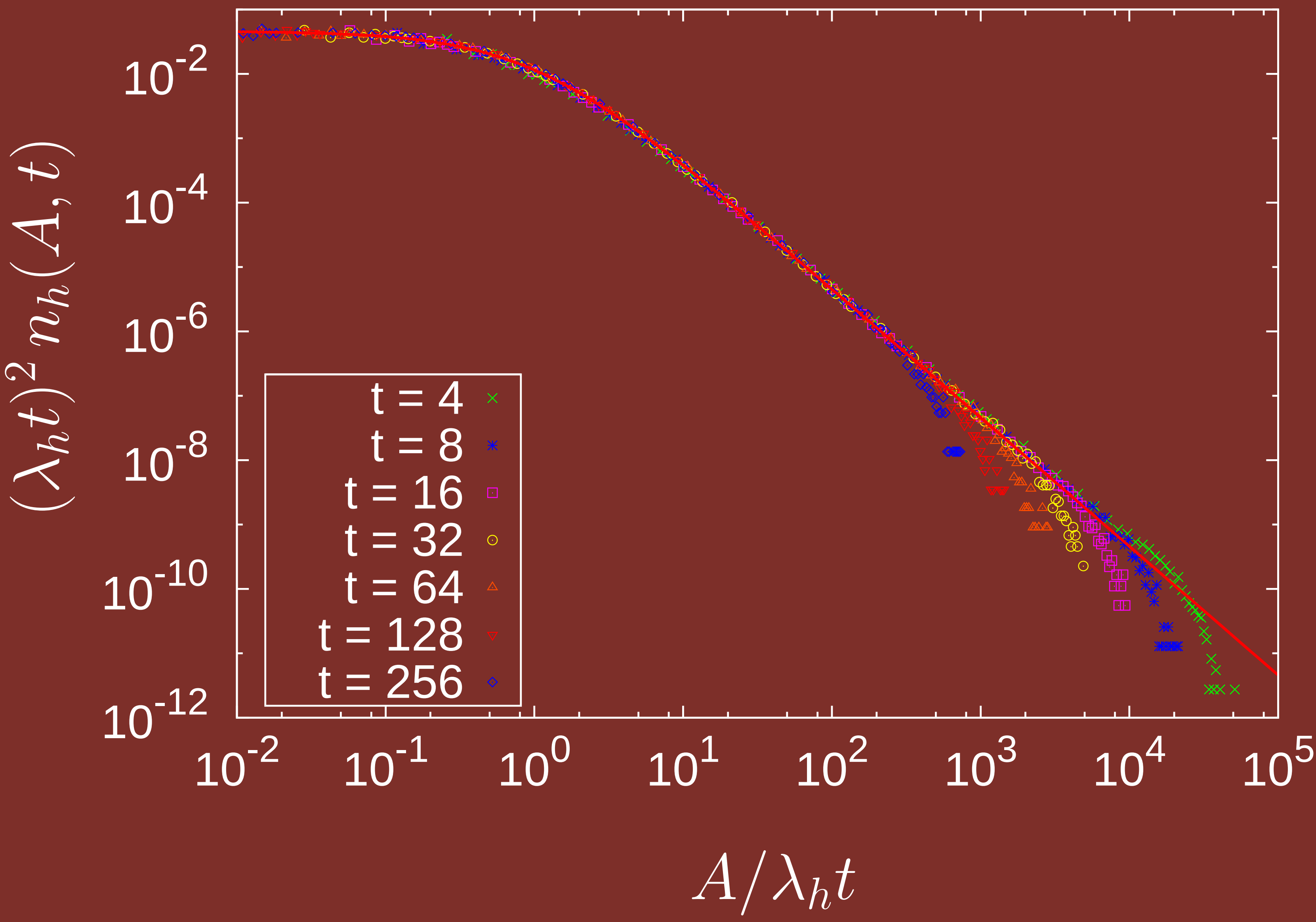
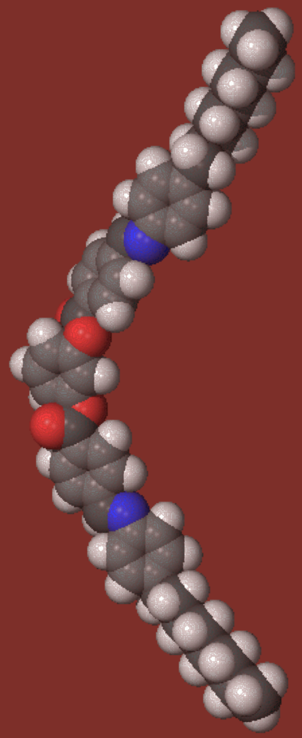
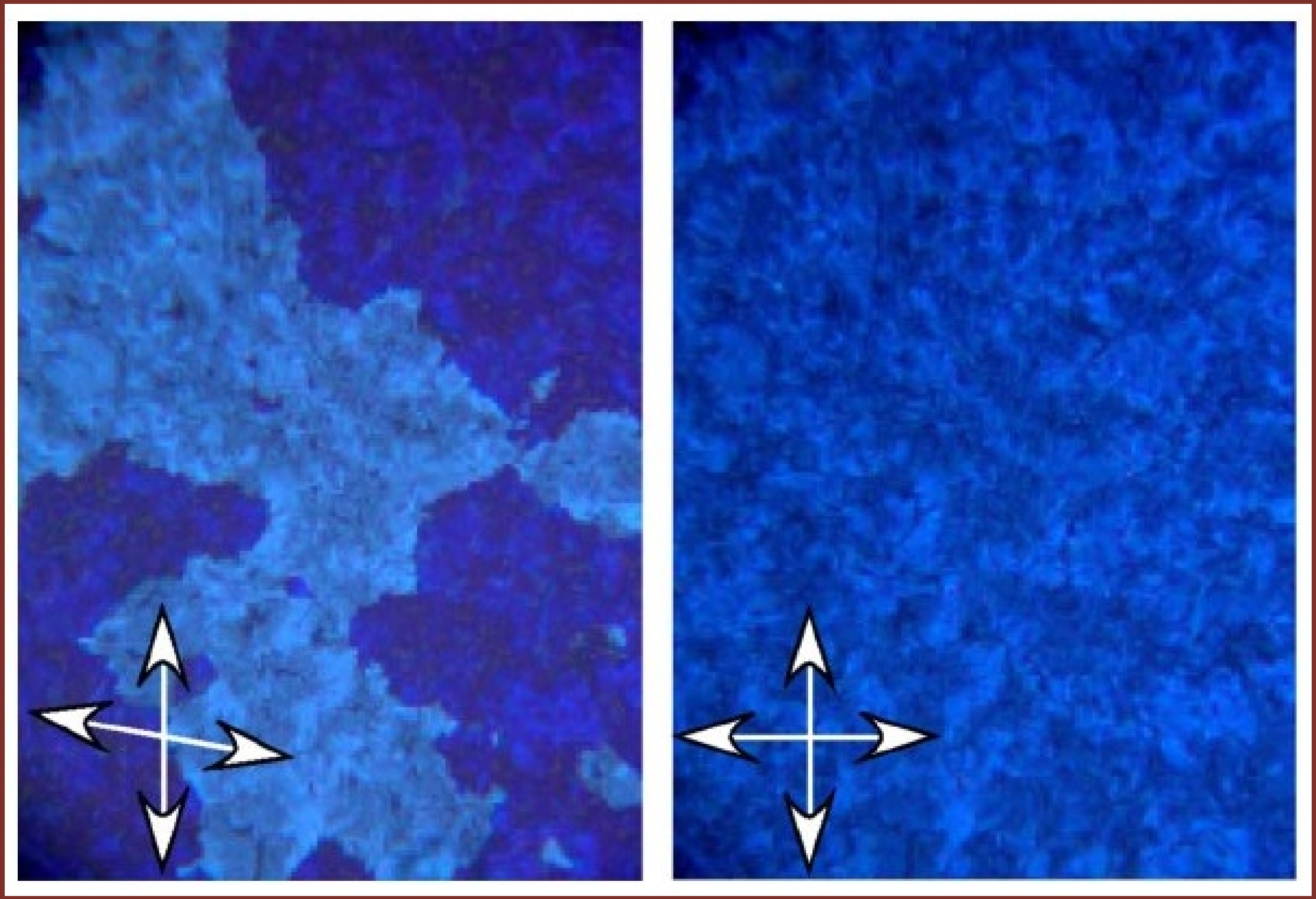
JJA, Bray, Cugliandolo, and Sicilia 2007 *Phys. Rev. Lett.* 98 145701



Banana-shaped (bent-core) Liquid Crystals



(achiral)



Earl et al, 2005 *Phys. Rev. E* **71** 021706

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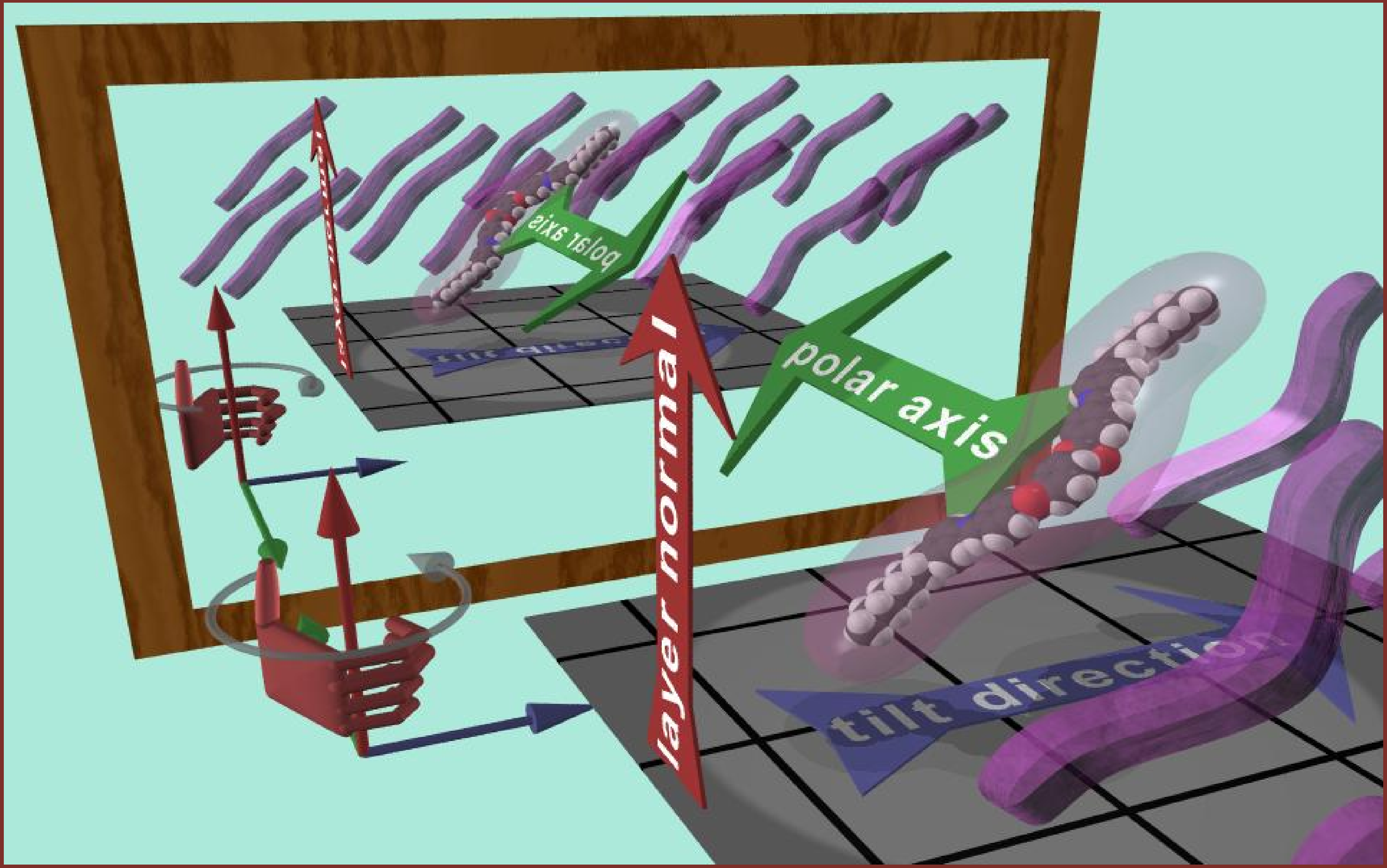
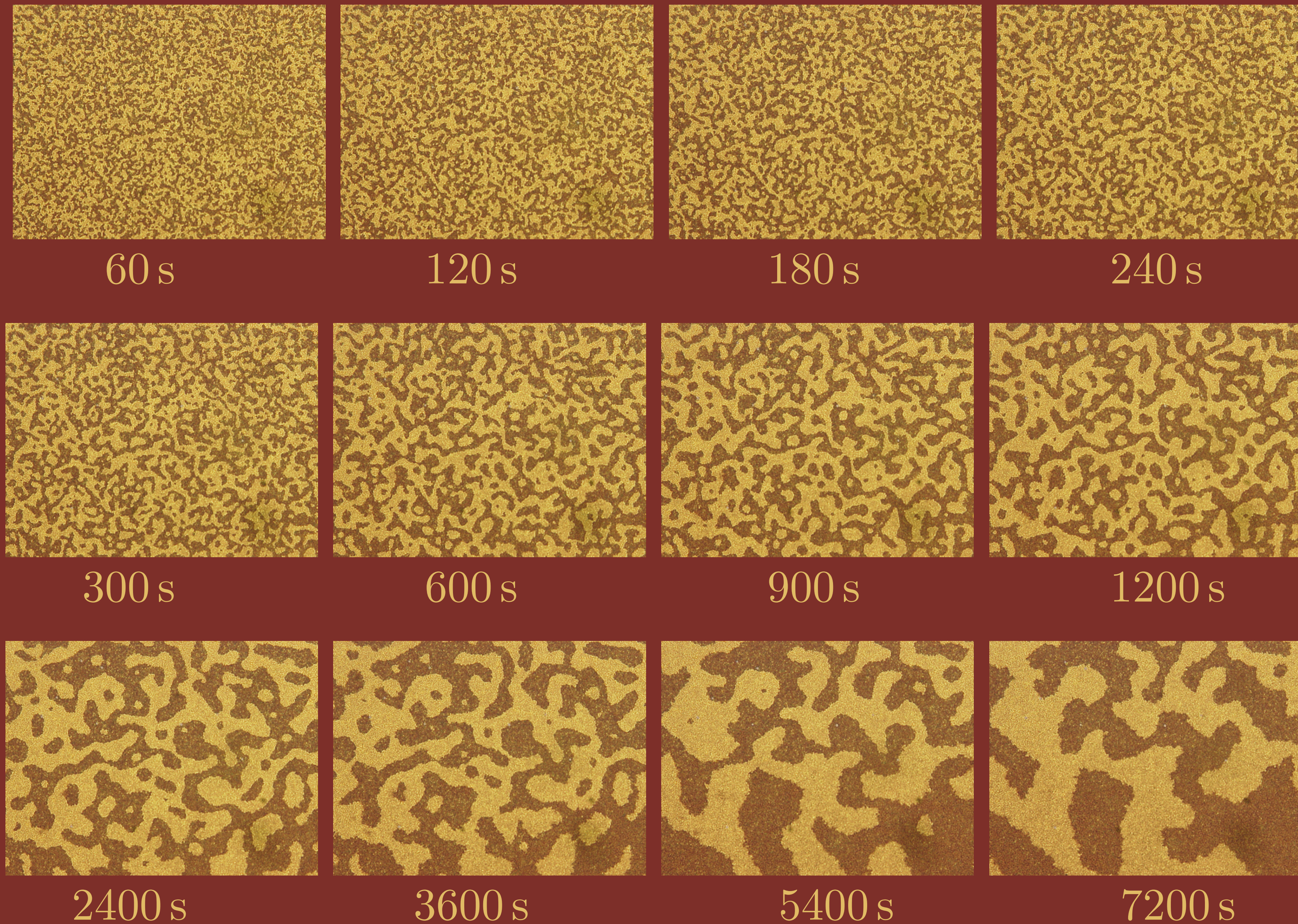


http://www2.tu-berlin.de/∼insi/ag_heppke/banana/contents.html

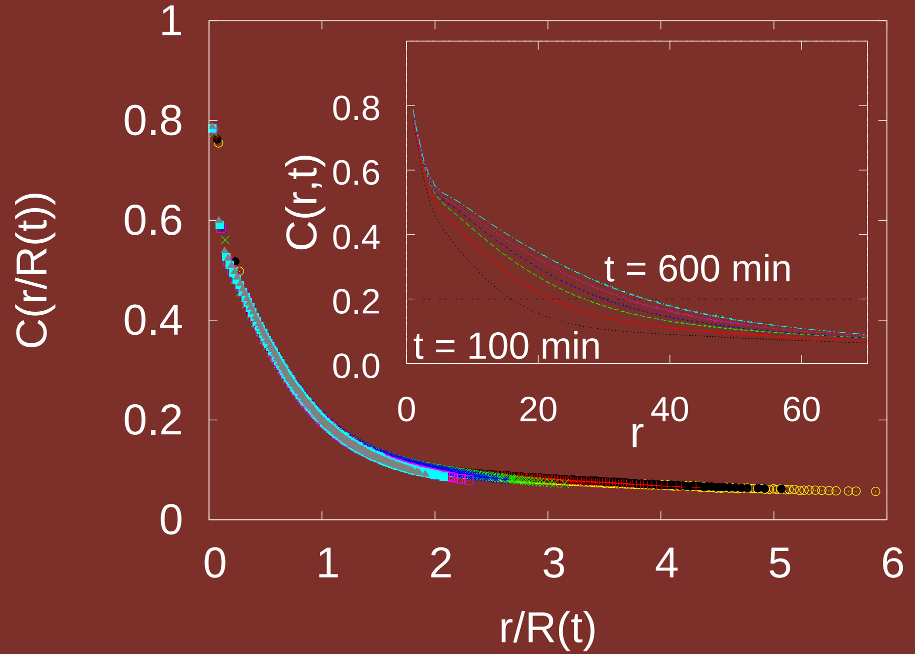
Liquid Crystals

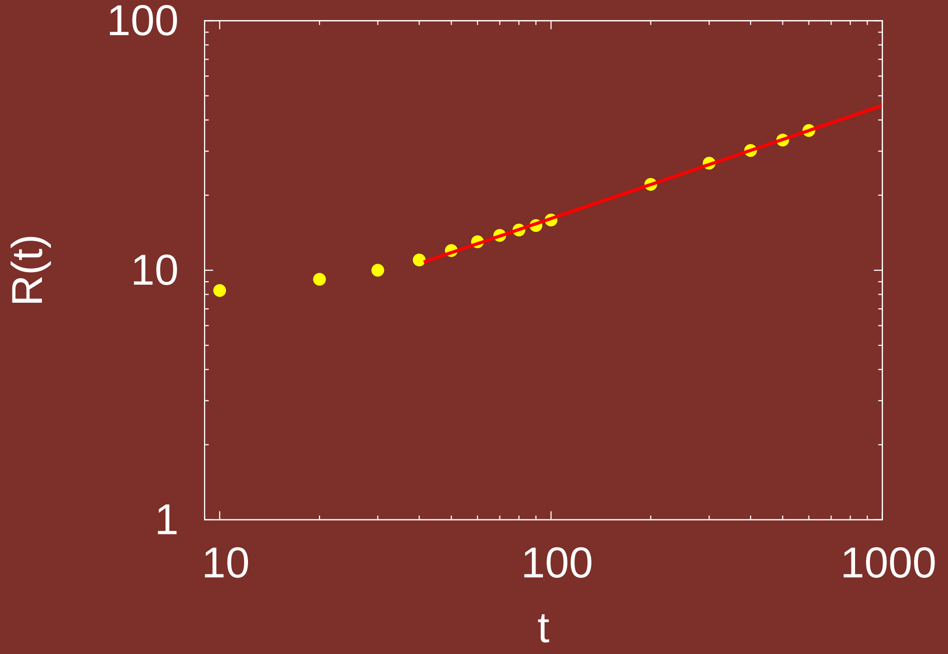
Sicilia et al, 2008 *Phys. Rev. Lett.* **101** 197801



Liquid Crystals

Sicilia et al, 2008 *Phys. Rev. Lett.* **101 197801**





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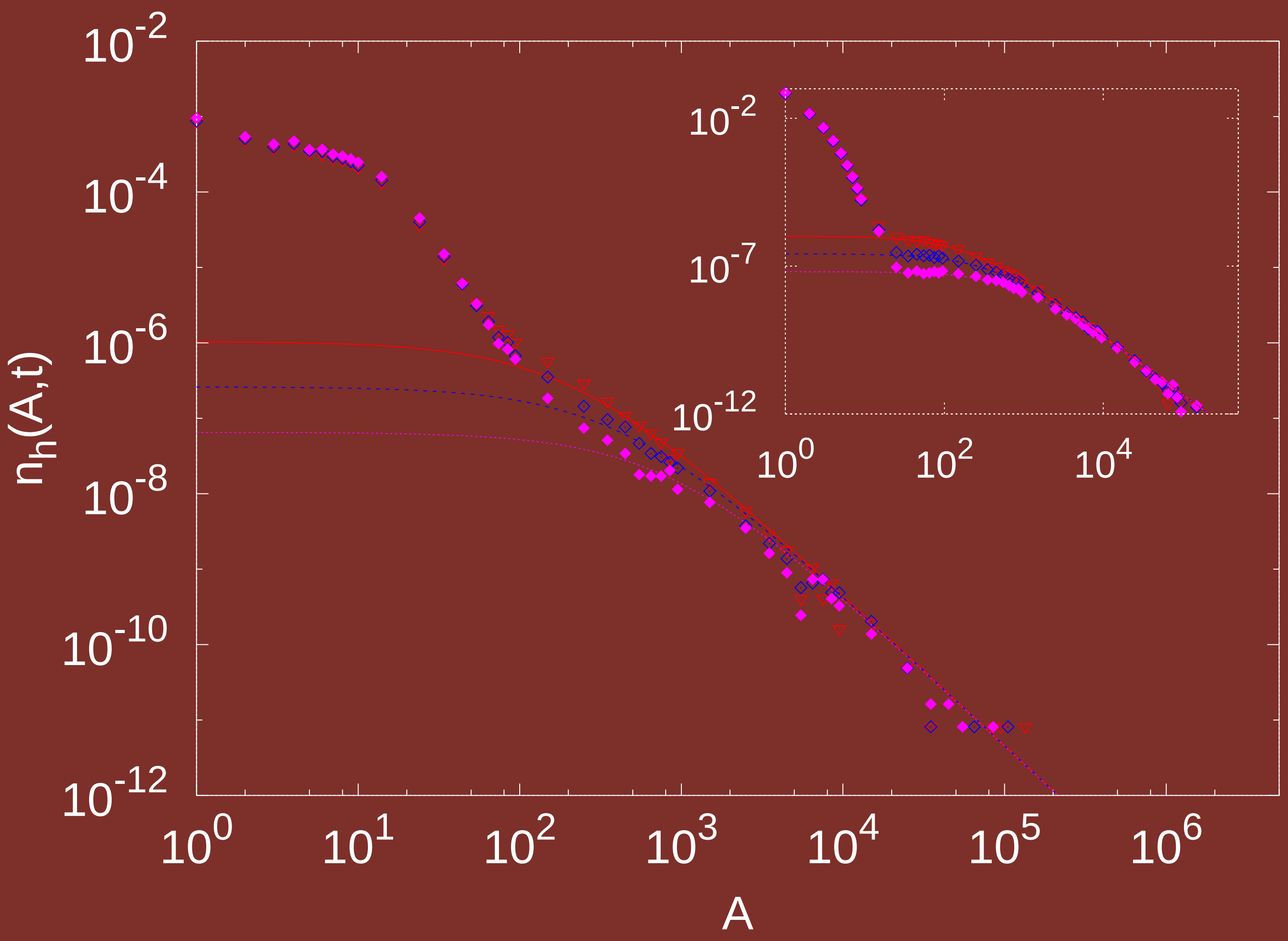

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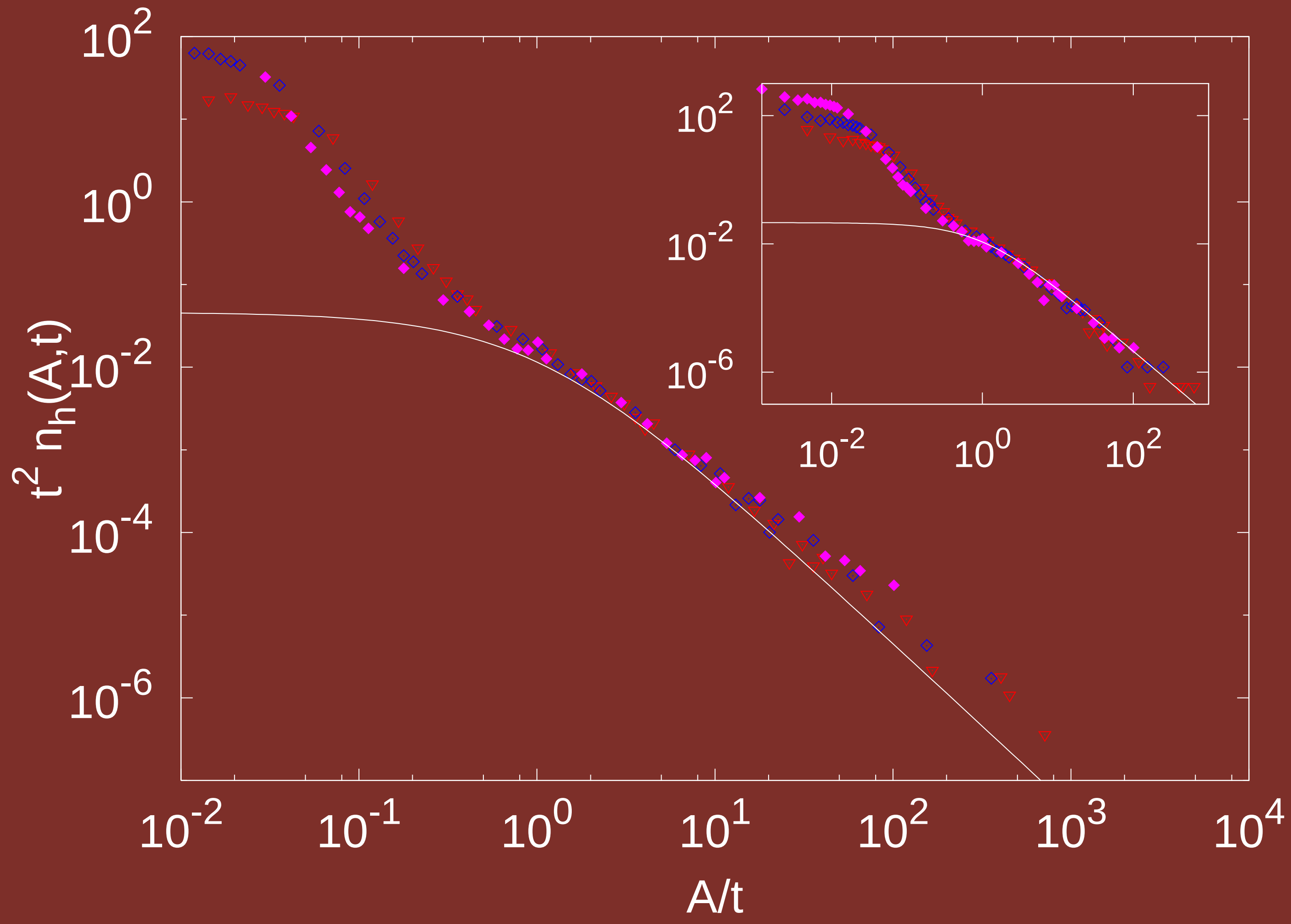

Conclusions

- The first proof of dynamical scaling in $d = 2$ (for hull enclosed areas distributions).
- The theory (continuous) is correct to a very high precision for the Ising $2d$ on a lattice, with and without (weak) disorder.
- We showed that the deracemization process (at least for the considered liquid crystal) is curvature driven and belongs to the Allen-Cahn universality class.
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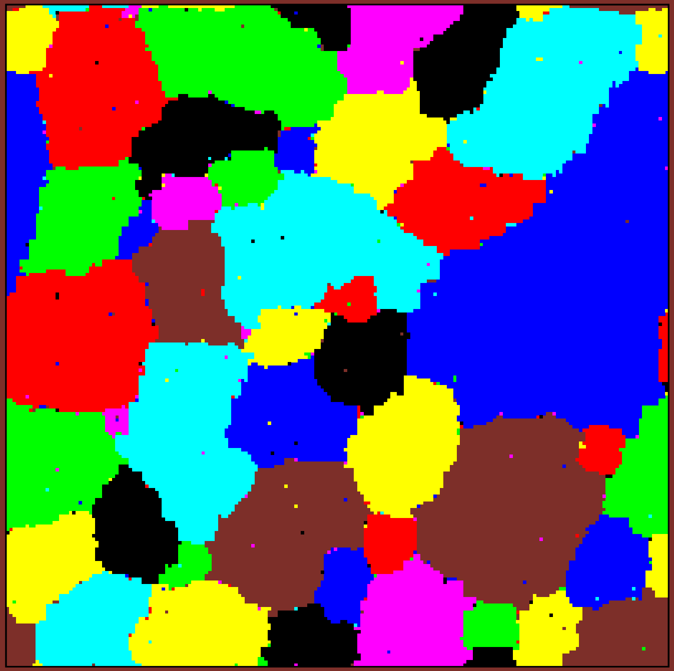
- Potts model:



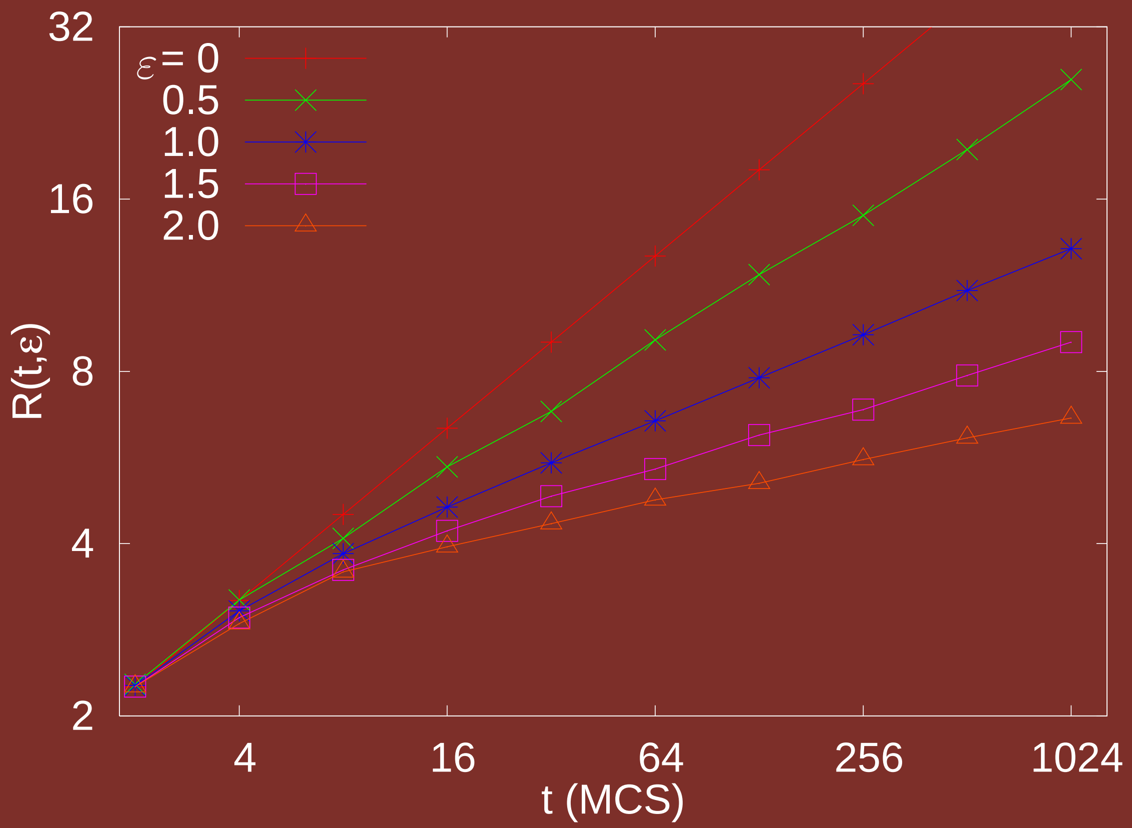
Random Bond Ising Model

Sicilia, JJA, Bray, and Cugliandolo, 2008 *EPL* 82 10001

Weak quenched disorder:

$$\mathcal{H} = -\sum_{<ij>} J_{ij} S_i S_j$$

where $J_{ij} \in [1-\varepsilon/2, 1+\varepsilon/2]$, with $0 \leq \varepsilon \leq 2$.



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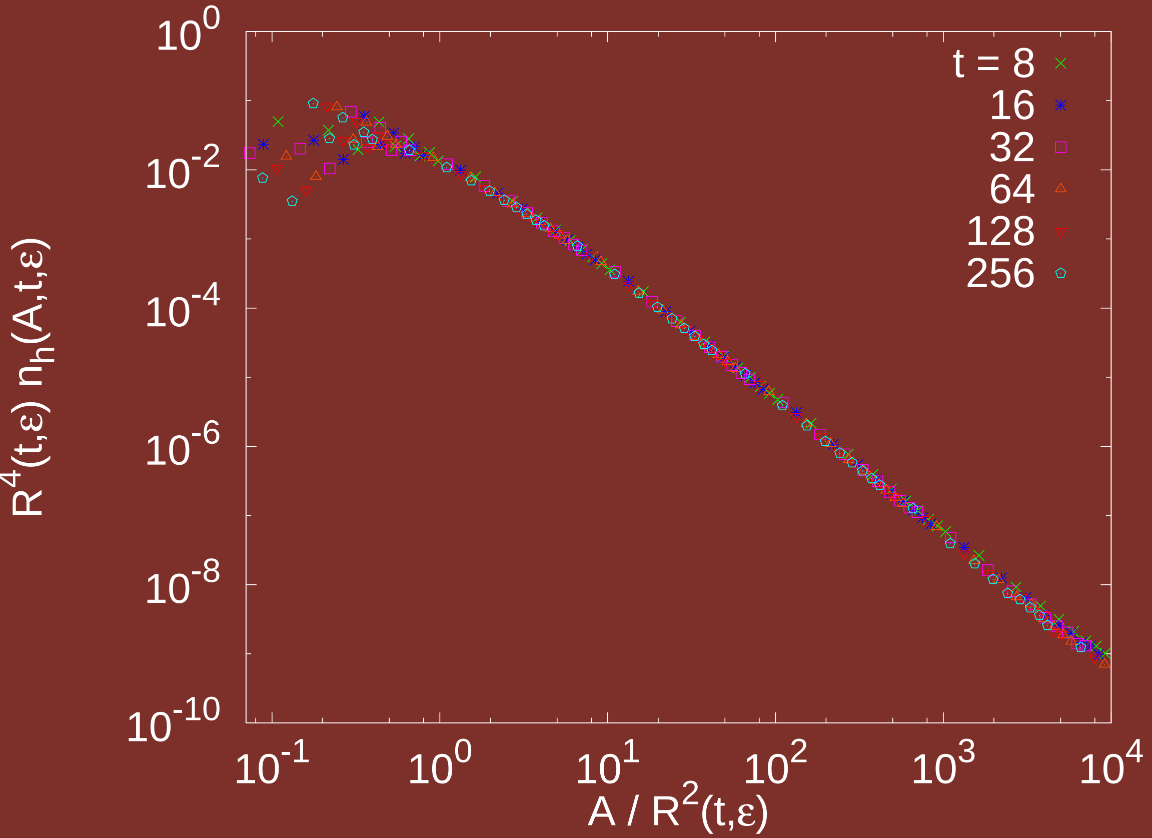

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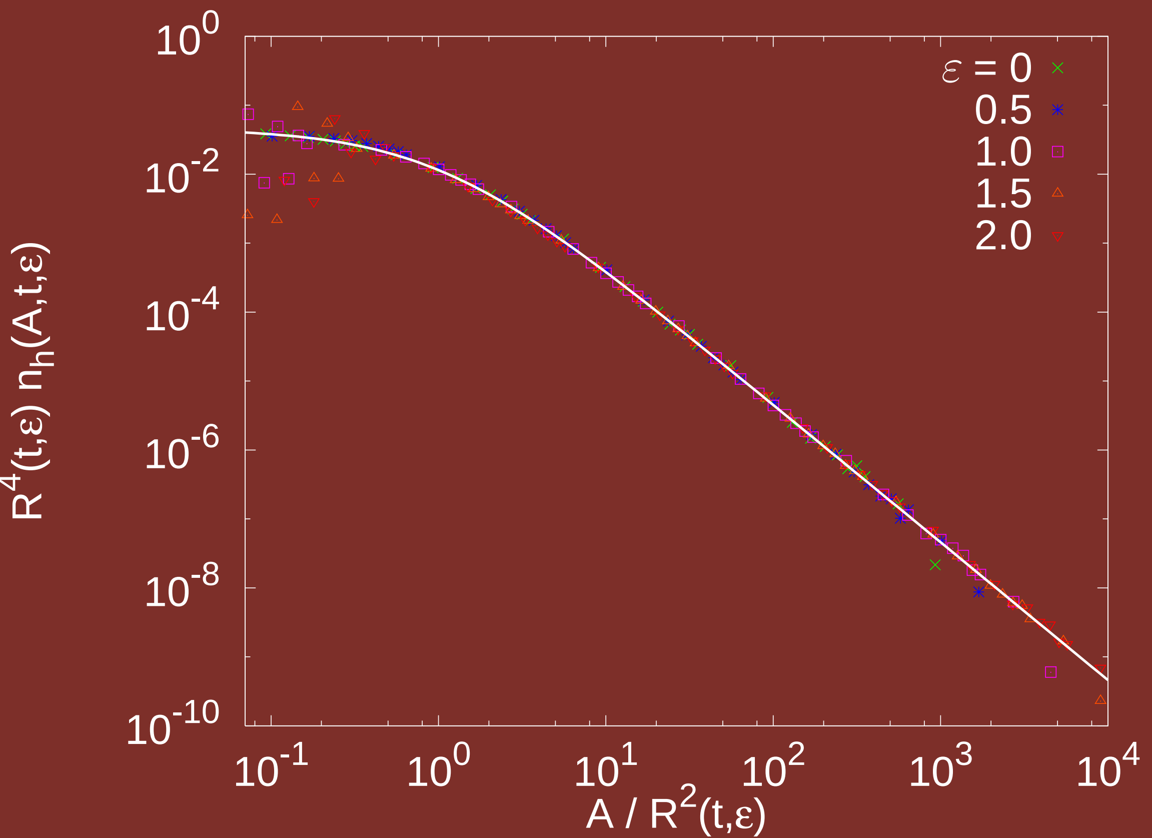

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Super-universality: scaling functions are independent of the (weak) disorder strength (Fisher and Huse)

Sicilia, JJA, Bray, and Cugliandolo, 2007 *Phys. Rev. E* 76 061116

$$\mathcal{H} = -J \sum_i s_i s_{i+1}$$

- $t = 0$ $(T = \infty)$:

$$n(\ell, 0) = 2^{-\ell}$$

- Equilibrium distribution:

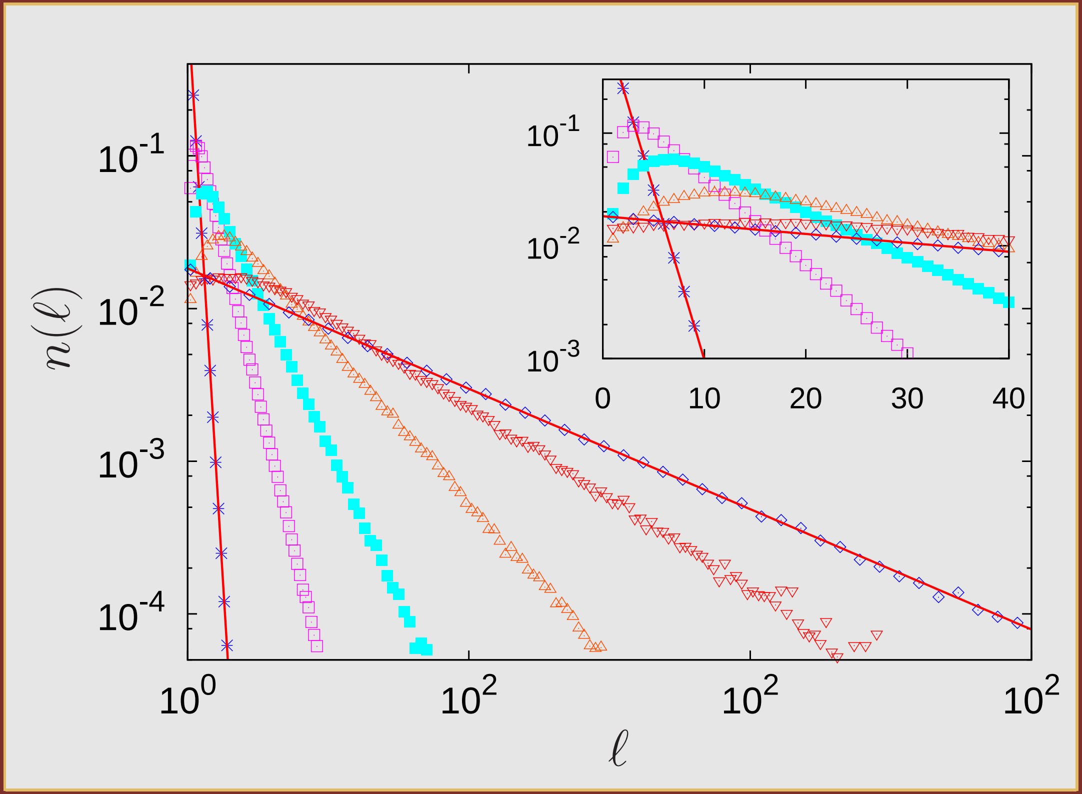
$$n_{\rm eq}(\ell) = r(1-r)^{\ell-1}$$

with $r = [1 + \exp(2\beta)]^{-1}$

- Coarsening: $T = \infty \to 0$

$$n(\ell, t) = \langle \ell(t) \rangle^{-1} f\left(\frac{\ell}{\langle \ell(t) \rangle}\right)$$

where $\langle \ell(t) \rangle = 2\sqrt{\pi t}$



$$f(x) = \begin{cases} \pi x & x \ll 1\,, \\ \exp(-Ax + B) & x \gtrsim 1\,, \end{cases}$$

(A and B are exactly known)

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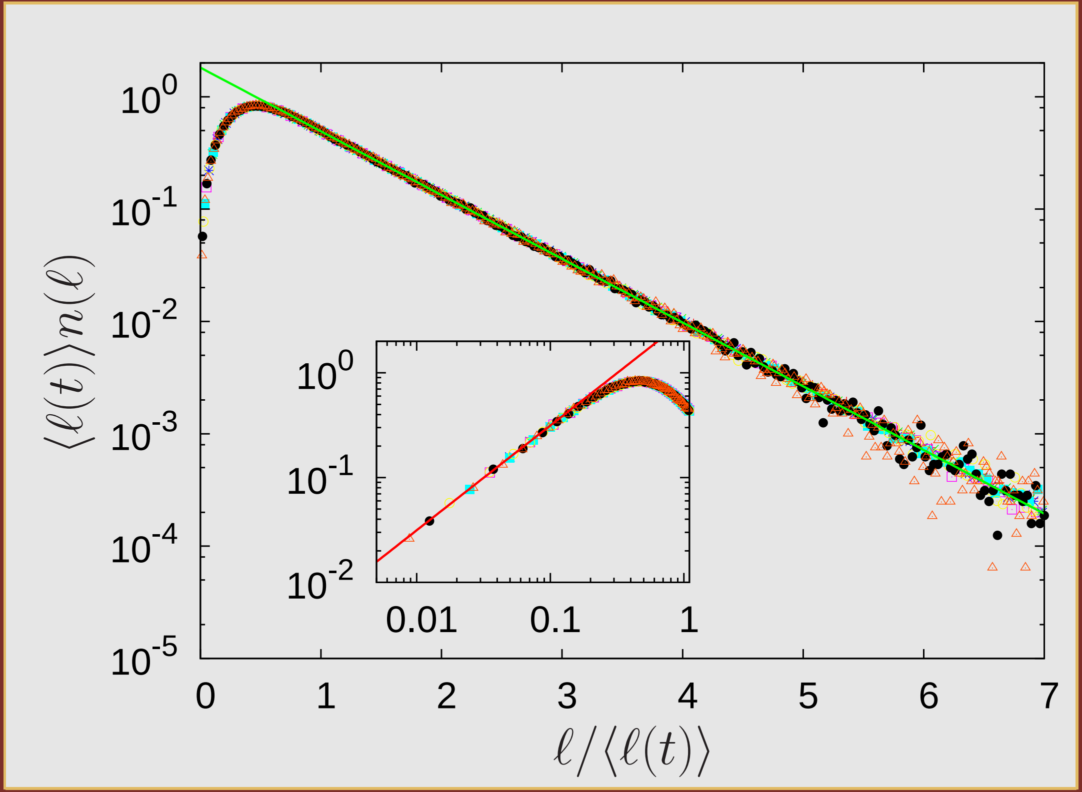

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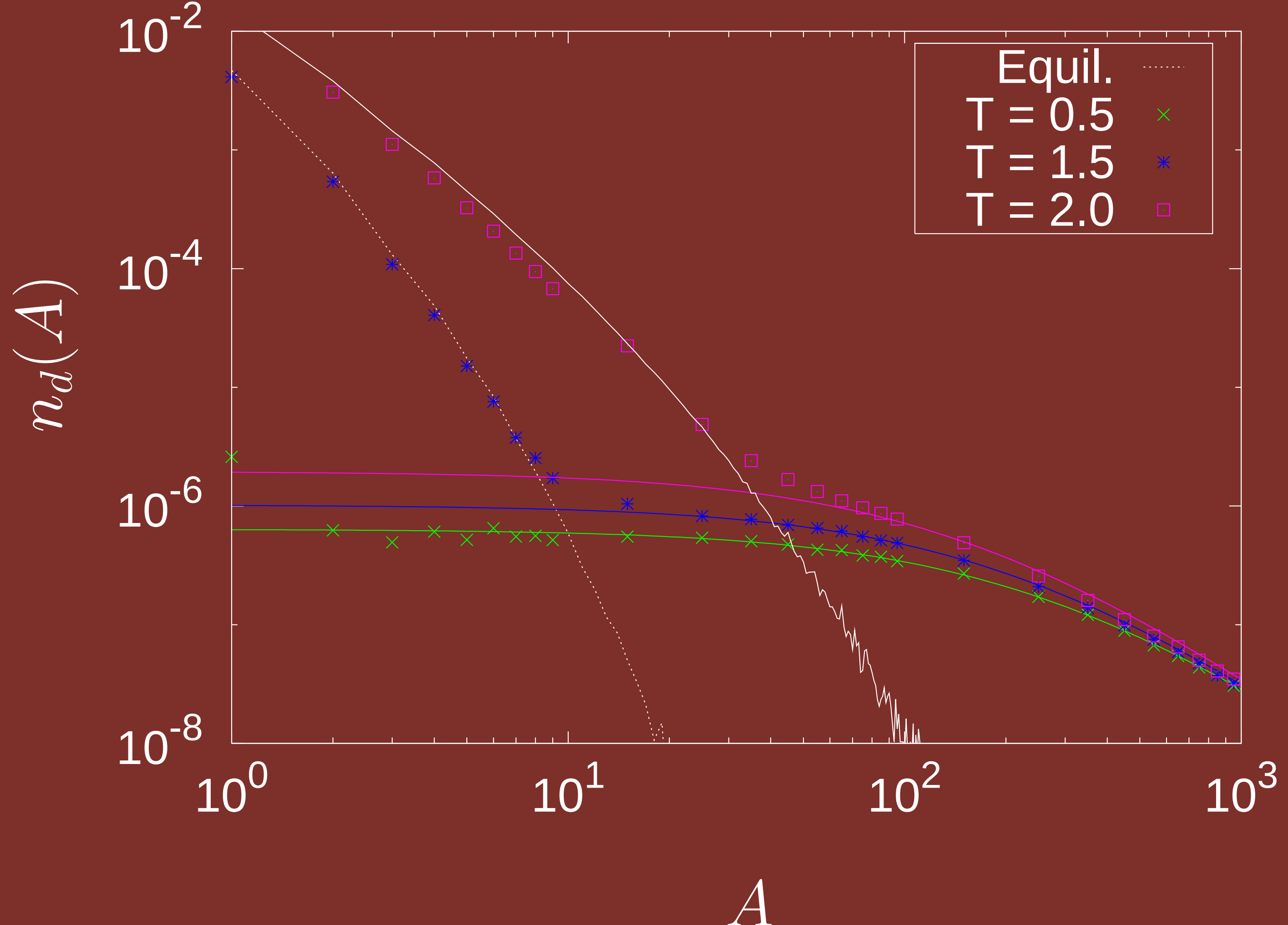
Temperature

Temperature induces thermal fluctuations and wall roughening.



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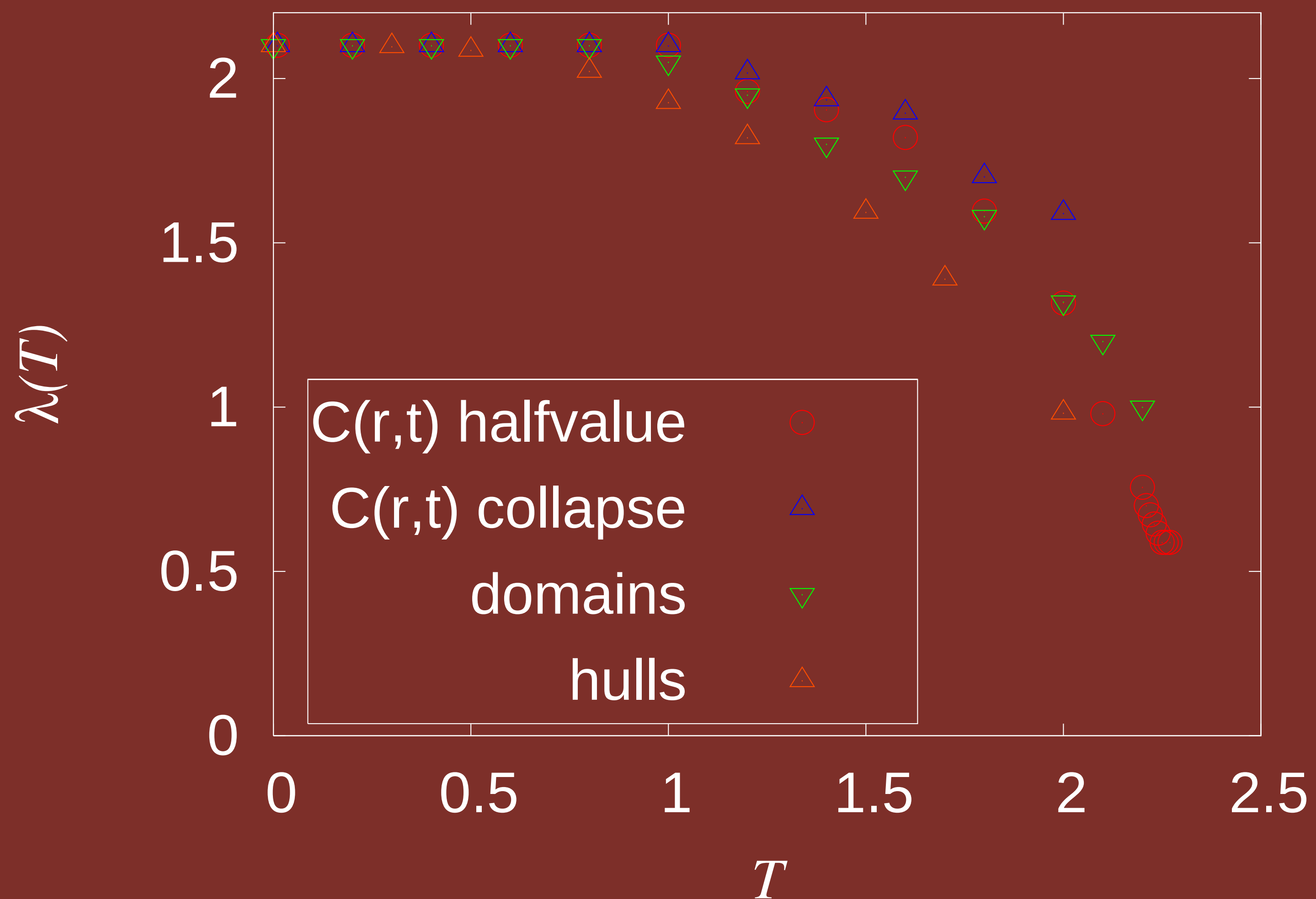


3d

For a spherical bubble:

$$\frac{dV}{dt} = 4\pi R^2 \frac{dR}{dt}$$

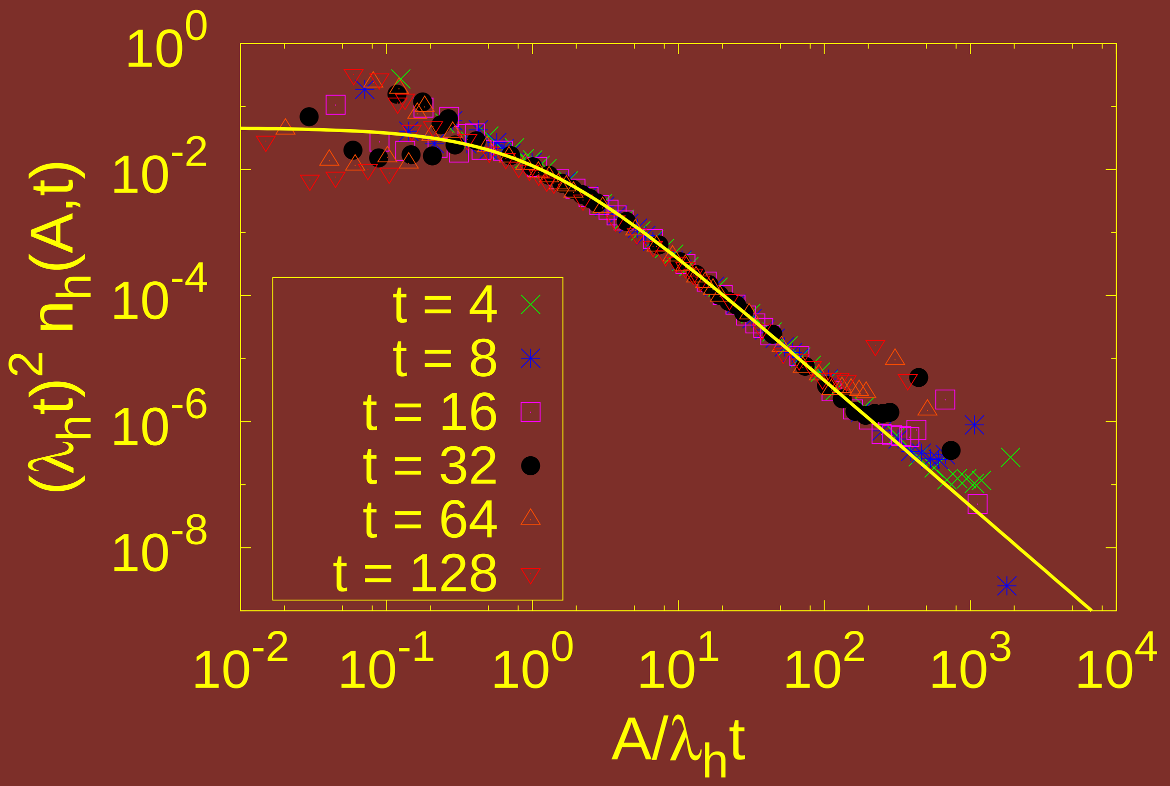
The domain wall velocity is

$$\frac{dR}{dt} = v = -\frac{\lambda_h}{2\pi}\kappa$$

where $\kappa = 2/R$ for a sphere. Thus

$$\frac{dV}{dt} = -4\lambda_h R$$

For 2d slices:



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  2008 *submitted*
- **Curvature-driven coarsening in the two dimensional Potts model**
  M.P. Loureiro, JJA, L.F. Cugliandolo, A. Sicilia and A.J. Bray
  2009 in preparation

Conserved Order Parameter

Surface diffusion (short times):

$$R(t) \sim t^{1/4}$$

Bulk diffusion:

$$R(t) \sim t^{1/3}$$

Lifshitz-Slyozov-Wagner $(3d)$:

- small concentration of the minority phase
- growth is limited by matter diffusion through the supersaturated bulk
- characteristic length scale $R_c(t) \sim t^{1/3}$ such that
  - i-th bubble grows (disappears) if $R_i > R_c$ $(R_i < R_c)$
- size distribution (minority phase):
  - upper cutoff $R_{\text{MAX}} \sim t^{1/3}$
  - $R_{\text{MAX}} = \frac{3}{2} R_c$
  - exponential decay close to R_{MAX}
  - dynamical scaling: $n(R,t) \sim R_c^{-4} f\,(R/R_c)$

Conserved Order Parameter

Sarrazin et al 2009 *submitted*

- equal concentrations of the two phases
- dynamic scaling: $n_{h,d}(A,t) = t^{-4/3} f_{h,d}(A/t^{2/3})$
- no cutoff
- $f(x)$ is a power-law
- the small-argument behavior of the scaling function recovers the LSW result

Geometric Domains

Sicilia, JJA, Bray, and Cugliandolo, 2007 *Phys. Rev. E* **76 061116**

Surface Distributions

Sicilia, JJA, Bray, and Cugliandolo, 2007 *Phys. Rev. E* **76 061116**